Generation of circular polarization in CMB radiation via nonlinear photon-photon interaction

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Standard cosmological models do predict a measurable amount of anisotropies in the intensity and linear polarization of the cosmic microwave background radiation (CMB) via Thomson scattering, even though these theoretical models do not predict circular polarization for CMB radiation. In other hand, the circular polarization of CMB has not been excluded in observational evidences. Here we estimate the circular polarization power spectrum $C_l^{V(S)}$ in CMB radiation due to Compton scattering and nonlinear photon-photon forward scattering via Euler-Heisenberg effective Lagrangian. We have estimated the average value of circular power spectrum is $l(l+1)C_l^{V(S)}/(2\pi) \sim 10^{-4}\ (\mu\mathrm{K})^2$ for $l \sim 300$ at present time which is smaller than recently reported data for upper limit of circular polarization (SPIDER collaboration). As a result to test our results, the ability to detect nano-Kelvin level signals of CMB circular polarization requires. We also show that the generation of B-mode polarization for CMB photons in the presence of the primordial scalar perturbation via Euler-Heisenberg interaction is possible however this contribution for B-mode polarization is not remarkable.

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I. INTRODUCTION

Photon-matter interactions can convert or generate the polarization states of photons in different situations such as Faraday rotation, Faraday conversion and so on. In some special cases, the measurement of circular polarization contribution provides very important tools to better understand the universe. In standard scenario of cosmology, CMB anisotropies are partially linearly polarized [1–6] while the generation of circular polarization is ignored, because there is not a notable mechanism to generate circular polarization in the recombination epoch. Note Compton (Thomson) scattering, as most important interaction of CMB radiation, cannot generate the circular polarization [6].

On the other hand, the circular polarization of CMB has not been excluded in observational evidences. For example, recently the SPIDER collaboration has made maps of approximately 10% of the sky with degree-scale angular resolution in 95 and 150 GHz observing bands. Data of the SPIDER group have been analyzed in [7] and a new upper limit on CMB circular polarization is obtained, so that constraints of the circular power spectrum $l(l+1)C_l^V/(2\pi)$ are reported in rang of a few hundred $(\mu\mathrm{K})^2$ at 150 GHz for a thermal CMB spectrum. Also it is worthwhile to take a look at other reports about the constraint on the circular polarizations $\frac{\Delta V}{T_{\mathrm{CMB}}}$ [8–10] and B-mode polarization [11–13].

In the case of theoretical models, there are several mechanisms, almost considering new physics interactions, which discuss the possibility of the generation of circular polarization in the CMB. For instances, the conversion of the existing linear polarization into circular one in the presence of external magnetic fields of galaxy clusters [14], the relativistic plasma remnants [15] and magnetic fields in the primordial universe [16–18] is discussed. Forward scattering of CMB radiation from the cosmic neutrino background [19], and photon-photon interactions in neutral hydrogen [20] have also been shown as potential mechanisms for the generation of CMB circular polarization. There are some mechanisms which are postulated extensions to QED such as Lorentz-invariance violating operators [16,21,22], axionlike pseudoscalar particles [23], and nonlinear photons interactions (through effective Euler-Heisenberg Lagrangian) [24]. In [25], the production of primordial circular polarization in axion inflation coupled to fermions and gauge fields, with special attention paid to reheating, has been studied. Also see a brief review of some of the mentioned mechanisms in [26].

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In this work, we focus on the generation of circular polarization due to nonlinear photon-photon interaction (via Euler-Heisenberg Lagrangian). Of course we should mention that Faraday conversion phase shift $\Delta\phi_{FC}$ due to Euler-Heisenberg Lagrangian for CMB radiation has been estimated in [24]. It is worthwhile to mention that one can calculate $\Delta\phi_{FC}$ from the below equation [see more detail in [14,27]]

$$\dot{V} = 2U\frac{d}{dt}(\Delta\phi_{FC}), \tag{1}$$

where U and V are Stokes parameters which describe linear and circular polarizations respectively. Note $\Delta\phi_{FC}$ reported in [24] is not a suitable quantity to compare with experimental data which are usually reported by circular polarization power spectrum C_l^V. So the main purpose of our work is to calculate C_l^V via Euler-Heisenberg effective interactions and make a comparison with recently data reported by the SPIDER collaboration group.

We start by a brief discussion on Stokes parameters and their definitions in terms of density matrix elements. Then we calculate time evolution of those parameters by Euler-Heisenberg consideration. In the next two sections we solve them by some estimations to calculate dominant contribution terms. These contributions come from total intensity of CMB photon contribution in comparison with linear and circular polarizations. Finally in the last section, we compute the power spectrum and B-mode spectrum of CMB photons which are generated by the Euler-Heisenberg effective Lagrangian.

II. POLARIZATION AND STOKES PARAMETERS

An ensemble of photons in a completely general mixed states is given by a normalized density matrix $\rho_{ij} \equiv (|\varepsilon_i\rangle\langle\varepsilon_j|/\mathrm{tr}\rho)$, where in the quantum mechanics description, an arbitrary polarized state of a photon with energy ($|k^0|^2 = |\mathbf{k}|^2$) propagating in the $\hat{z}$-direction is written as

$$|\varepsilon\rangle = a_1 \exp(i\theta_1)|\varepsilon_1\rangle + a_2 \exp(i\theta_2)|\varepsilon_2\rangle, \tag{2}$$

where $|\varepsilon_1\rangle$ and $|\varepsilon_2\rangle$ represent the polarization states in the $\hat{x}$- and $\hat{y}$-directions. Then the 2×2 density matrix ρ of photon polarization states are given as

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} I+Q & U-iV \\ U+iV & I-Q \end{pmatrix}, \tag{3}$$

where I, Q, U, and V are Stokes parameters, so that I-parameter is the total intensity of radiation, Q- and U-parameters indicate the intensity of linear polarization of radiation, and V-parameter determines the intensity of circular polarization of radiation. Note I and V are independently physical observable quantities of the coordinate system, while Q- and U-parameters depend on the orientation of the selected coordinate system. Linear polarization can also be characterized through a vector parameter $\mathbf{P}$ which describe by $|\mathbf{P}| \equiv \sqrt{Q^2+U^2}$ and $\alpha = \frac{1}{2}\tan^{-1}\frac{U}{Q}$ [28].

The time evolution of each Stokes parameter can be yielded through the quantum Boltzmann equation. To do this issue, ones can play with each polarization state of the CMB radiation as the phase space distribution function χ which can generally obey from the classical Boltzmann equation

$$\frac{d}{dt}\chi = \mathcal{C}(\chi). \tag{4}$$

The left-hand side of the above equation is known as the Liouville term (containing all gravitational effects), while the right-hand side one contains all possible collision terms. By considering the CMB interactions on the right-hand side of the Boltzmann equation, we can calculate the time evolution of the each polarization state of the photons. In the next section, we consider nonlinear photon-photon forward scattering via the Euler-Hesinberg Hamiltonian to compute the time evolution of each polarization sates.

III. THE EULER-HEISENBERG LAGRANGIAN AND THE PHOTONS POLARIZATIONS

The time evolution of $\rho_{ij}(k)$s as well as Stokes parameters are given by (see [6] for more detail),

$$(2\pi)^3\delta^3(0)(2k^0)\frac{d}{dt}\rho_{ij}(k) = i\langle[H_I^0(t); D_{ij}^0(k)]\rangle - \frac{1}{2}\int dt\langle[H_I^0(t);[H_I^0(0); D_{ij}^0(k)]]\rangle, \tag{5}$$

where $H_I^0(t)$ is the leading order of the photon-photon interacting via the Euler-Hiesenberg Hamiltonian. The first term on the right-handed side of above equation is called forward scattering term, and the second one is a higher order collision term which is in order of the ordinary cross section of photon-photon scattering. The Euler-Heisenberg Lagrangian is a low energy effective lagrangian describing multiple photon interactions. The first order of photon-photon interacting Hamiltonian via Euler-Heisenberg Lagrangian can be written as [29,30]

$$H_I^0(t) = -\frac{\alpha^2}{90m^4}\int d^3x\left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\right], \tag{6}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the strength of electromagnetic field and $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$, in which $\epsilon^{\mu\nu\alpha\beta}$ is an antisymmetric tensor of rank four (for example see [31,32]). Note

$$\hat{A}_\mu(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k^0}[a_r(k)\epsilon_{r\mu}(k)e^{-ik.x} + a_r^\dagger(k)\epsilon^*_{r\mu}(k)e^{ik.x}]. \tag{7}$$

where creation $a_r^\dagger$ and annihilation a_r operators satisfy the canonical commutation relation as

$$[a_r(k), a_{r'}^\dagger(k')] = (2\pi)^3 2k^0 \delta_{rr'}\delta^{(3)}(\mathbf{k}-\mathbf{k}'). \tag{8}$$

We only compute the first order of the quantum Boltzmann equation, i.e., the first term in the RHS of the Eq. (5), and neglect the second term which is in order of α^4. In principle when the first term does not have any result, in any special theory, one can try to compute the second term. It is worthwhile to mention that the contribution of $(F_{\mu\nu}\tilde{F}^{\mu\nu})^2$ for CMB polarization is given in [24], however they have just calculated Faraday conversion phase shift. Here we will consider both term of Euler-Heisenberg Lagrangian. After a tedious but straightforward calculation, using Eq. (A1), the time-evolutions of Stokes parameters Eq. (3) are obtained (find details in the Appendix). First we start with I-parameter

$$\dot{I}(\mathbf{k}) = 0, \tag{9}$$

$\dot{I}(\mathbf{k}) = 0$ implies, for each ensemble of photons like CMB, the total intensity I in any direction $\hat{\mathbf{k}}$ is constant and does not change from Euler-Heisenberg forward scattering. The above result for intensity I is expected, because the forward scattering cannot change momenta of photons which is necessary condition to change intensity in any direction. Note for the rest of paper, we do not consider the terms with linearly dependence of ρ_{ij} on the right side of the above equations, because we are interested in photon-photon forward scattering. The time evolution of linear and circular polarization parameters are given as follows

$$\dot{Q}(\mathbf{k}) = \frac{16\alpha^2}{45m^4k^0}V(\mathbf{k})\int \frac{d^3p}{(2\pi)^3 2p^0}(p^0k^0)^2[f_1(\hat{p},\hat{k})U(\mathbf{p})], \tag{10}$$

$$\dot{U}(\mathbf{k}) = \frac{8\alpha^2}{45m^4k^0}V(\mathbf{k})\int \frac{d^3p}{(2\pi)^3 2p^0}(p^0k^0)^2[f_1(\hat{p},\hat{k})I(\mathbf{p})]. \tag{11}$$

$$\dot{V}(\mathbf{k}) = \frac{8\alpha^2}{45m^4k^0}U(\mathbf{k})\int \frac{d^3p}{(2\pi)^3 2p^0}(p^0k^0)^2[f_2(\hat{p},\hat{k})I(\mathbf{p})] \tag{12}$$

where f_is are given in the Appendix. Note in the case of CMB radiation, I can be total intensity of CMB or CMB thermal anisotropy (depending of angular dependence of f_is) while the contributions of Q, U, and V are about or less than 10% of total CMB thermal anisotropy. As a result, to consider dominated contribution in our calculations Eqs. (10)–(12), we neglect terms in second order of Q, U, and V. As Eqs. (10)–(12) show, the initial circular polarization of an ensemble of photon $V(\mathbf{k})$ can be converted to linear one $U(\mathbf{k}), Q(\mathbf{k})$ and inverse due to Euler-Hisenberg interactions. To go further and calculate angular integrals most conveniently, we introduce the momentum and polarization vectors of incoming photons as follow [6]

$$\begin{aligned}
\hat{\mathbf{k}} &= (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta), \\
\vec{\epsilon}_1(\mathbf{k}) &= (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta), \\
\vec{\epsilon}_2(\mathbf{k}) &= (-\sin\phi, \cos\phi, 0).
\end{aligned} \tag{13}$$

The exactly same definition are correct for momentum and polarization vectors of target photons (denoted by $\mathbf{p}$ and $\vec{\epsilon}_s(\mathbf{p})$) just with $\theta \to \theta'$ and $\phi \to \phi'$. The angular integrals in Eqs. (10)–(12) must be done over θ' and ϕ'. As momentum and polarization vectors of photons are defined in spherical coordinate, one can expand all variables and Stokes parameters in terms of spherical harmonics Y_l^m to make angular integrals easily, so we have

$$\begin{aligned}
I(\mathbf{p}) &= \sum_{l'm'} I_{l'm'}(\mathbf{p})Y_{l'}^{m'}(\theta', \phi'), \\
(Q \pm iU)(\mathbf{p}) &= \sum_{l'm'} (Q \pm iU)_{l'm'}(\mathbf{p})Y_{l'}^{m'}(\theta', \phi'), \\
V(\mathbf{p}) &= \sum_{l'm'} V_{l'm'}(\mathbf{p})Y_{l'}^{m'}(\theta', \phi').
\end{aligned} \tag{14}$$

Also we can use above equations to expand $I(\mathbf{k})$, $Q(\mathbf{k})$, $U(\mathbf{k})$, and $V(\mathbf{k})$ in terms of spherical harmonics by replacing $\theta \to \theta'$, $\phi \to \phi'$, $l' \to l$, and $m' \to m$. So by considering the time evolution of Stokes parameters given in Eqs. (10)–(12), using expansions in Eq. (14) and adding the Compton scattering contributions to Euler-Heisenberg contributions, we have

$$\begin{aligned}
\frac{dI}{dt} &= C^I_{e\gamma}, \\
\frac{d}{dt}(Q \pm iU) &= C^\pm_{e\gamma} \mp i\dot{\kappa}_\pm V, \\
\frac{dV}{dt} &= C^V_{e\gamma} + \dot{\kappa}_U U,
\end{aligned} \tag{15}$$

where $C^I_{e\gamma}$, $C^\pm_{e\gamma}$, and $C^V_{e\gamma}$ denote contributions of Compton scattering which their expressions could be found in [6,33,34]. The Euler-Heisenberg contribution coefficients are given as follows

$$\dot{\kappa}_\pm = \frac{8\alpha^2}{45m^4k^0}\int \frac{p^2dpd\Omega'}{(2\pi)^3 2p^0}(p^0k^0)^2 \times [(-2iU(\mathbf{p}) \pm I(\mathbf{p}))f_1(\hat{p},\hat{k})] \quad (16)$$

$$\dot{\kappa}_U = \frac{8\alpha^2}{45m^4k^0}\int \frac{p^2dpd\Omega'}{(2\pi)^3 2p^0}(p^0k^0)^2[f_2(\hat{p},\hat{k})I(\mathbf{p})]. \quad (17)$$

As shown in Eq. (16), $\dot{\kappa}_\pm$ is divided into two terms which are proportional to $U(p)$ and $I(p)$. According to the earlier mentioned argument, to consider dominant contributions of Euler-Heisenberg effective Lagrangian in CMB power spectrum, we can neglect the term including $U(p)$. Then

$$\dot{\kappa}_\pm = \pm\frac{1}{15\pi}\sigma_T\frac{k}{m_e}\frac{I_0}{m_e}\left(\int\frac{d^3p}{(2\pi)^3}pf_1(\hat{p},\hat{k})\sum_{lm}Y_{l,m}\frac{I_{lm}(\mathbf{p})}{I_0}\right), = \pm\dot{\bar{\kappa}}\left(f_1^0 + \int\frac{d^3p}{(2\pi)^3}p\tilde{f}_1(\hat{p},\hat{k})\sum_{lm}Y_{l,m}\frac{I_{lm}(\mathbf{p})}{I_0}\right) \quad (18)$$

$$\dot{\kappa}_U = \frac{1}{15\pi}\sigma_T\frac{k}{m_e}\frac{I_0}{m_e}\left(\int\frac{d^3p}{(2\pi)^3}pf_2(\hat{p},\hat{k})\sum_{lm}Y_{l,m}\frac{I_{lm}(\mathbf{p})}{I_0}\right), = \dot{\bar{\kappa}}\left(f_2^0 + \int\frac{d^3p}{(2\pi)^3}p\tilde{f}_2(\hat{p},\hat{k})\sum_{lm}Y_{l,m}\frac{I_{lm}(\mathbf{p})}{I_0}\right), \quad (19)$$

where $\dot{\bar{\kappa}}=\frac{1}{15\pi}\sigma_T\frac{k}{m_e}\frac{I_0}{m_e}$ and here we separate $f_i(\hat{p},\hat{k}) = f_i^0+\tilde{f}_i(\hat{p},\hat{k})$, note f_i^0 is constant part of $f_i(\hat{p},\hat{k})$ and also

$$\int\frac{pd^3p}{(2\pi)^3}I(\mathbf{p}) = I_0(\bar{p}) \simeq \bar{p}n_\gamma. \quad (20)$$

and $\bar{p} = |\mathbf{p}|$ is the average value of the momentum of target (CMB-photons). Be ware in above equations, the term including $\tilde{f}_i(\hat{p},\hat{k})$ is in the order of CMB temperature anisotropy $\sim\frac{\delta T}{T}$ which several order of magnitude smaller than the term including f_i^0. So it is reasonable to ignore the term including $\tilde{f}_i(\hat{p},\hat{k})$ for the rest of our calculation. As a result, by considering nonlinear photon-photon interaction, a linear polarization converts to circular one while crossing through an isotopic unpolarized medium beam I_0.

To understand the above results, we can assume that linearly polarized CMB photons encounter by an isotopic background magnetic and electric fields when they cross through the unpolarized beam. By purposing the mentioned point, we can rewrite Euler-Heisenberg Hamiltonian by replacing $F_{\mu\nu} \to B_{\mu\nu} + F_{\mu\nu}$ where $B_{\mu\nu}$ indicates background fields (for example see [35])

$$H_I^0(t) = -\frac{\alpha^2}{90m_e^4}\int d^3x\bigg([(F_{\mu\nu}+B_{\mu\nu})(F^{\mu\nu}+B^{\mu\nu})]^2 + \frac{7}{4}[(F_{\mu\nu}+B_{\mu\nu})(\tilde{F}^{\mu\nu}+\tilde{B}^{\mu\nu})]^2\bigg). \quad (21)$$

Note in above equation, we just need terms with two $F_{\mu\nu}$ while terms including $(B_{\mu\nu}B^{\mu\nu})(F_{\mu\nu}F^{\mu\nu})$ and $(B_{\mu\nu}\tilde{B}^{\mu\nu})(F_{\mu\nu}\tilde{F}^{\mu\nu})$ do not affect our results. So by using Eq. (7),

$$H_I^0(t) = \frac{4\alpha^2}{90m_e^4}\int\frac{d^3p}{(2\pi)^3(2p^0)^2}\sum_{ss'}\hat{a}_s^\dagger(p)\hat{a}_{s'}(p) \times\left[p^\mu B_{\mu\nu}\epsilon_{s'}^\nu p^\lambda B_{\lambda\rho}\epsilon_s^{*\rho} - \frac{7}{4}p^\mu\tilde{B}_{\mu\nu}\epsilon_{s'}^\nu p^\lambda\tilde{B}_{\lambda\rho}\epsilon_s^{*\rho}\right]. \quad (22)$$

and by substituting the below equations

$$p^\mu B_{\mu\nu}\epsilon_s^\nu = \vec{B}.(\vec{p}\times\vec{\epsilon}_s) + p^0\vec{E}.\vec{\epsilon}_s, \quad p^\mu\tilde{B}_{\mu\nu}\epsilon_{s'}^\nu = 2\vec{E}.(\vec{p}\times\vec{\epsilon}_s) + 2p^0\vec{B}.\vec{\epsilon}_s, \quad (23)$$

in Eq. (22), we obtain

$$H_I^0(t) = \frac{4\alpha^2}{90m_e^4}\int\frac{d^3p}{(2\pi)^3(2p^0)^2}\sum_{ss'}\hat{a}_s^\dagger(p)\hat{a}_{s'}(p)([(\vec{B}.(\vec{p}\times\vec{\epsilon}_s) + p^0\vec{E}.\vec{\epsilon}_s)(\vec{B}.(\vec{p}\times\vec{\epsilon}_{s'}) + p^0\vec{E}.\vec{\epsilon}_{s'})] - 7[(\vec{E}.(\vec{p}\times\vec{\epsilon}_s) + p^0\vec{B}.\vec{\epsilon}_s)(\vec{E}.(\vec{p}\times\vec{\epsilon}_{s'}) + p^0\vec{B}.\vec{\epsilon}_{s'})]). \quad (24)$$

At the end, we have used Eqs. (5) and (24) to obtain the time evolution of Stokes parameters, here we just discuss the V-parameter

$$\dot{V}(\vec{k}) = \frac{4\alpha^2k^0}{90m_e^4}[\tilde{g}Q(\vec{k}) + \tilde{f}U(\vec{k})] \quad (25)$$

where

$$\tilde{g} = 2(\vec{B}\cdot(\hat{k}\times\hat{\epsilon}_2)\vec{B}.(\hat{k}\times\hat{\epsilon}_1) + \vec{E}\cdot\hat{\epsilon}_2\vec{B}\cdot(\hat{k}\times\hat{\epsilon}_1) + \vec{E}\cdot\hat{\epsilon}_1\vec{B}\cdot(\hat{k}\times\hat{\epsilon}_2) + \vec{E}\cdot\hat{\epsilon}_1\vec{E}\cdot\hat{\epsilon}_2) + 14(\vec{E}\cdot(\hat{k}\times\hat{\epsilon}_2)\vec{E}.(\hat{k}\times\hat{\epsilon}_1) + \vec{B}\cdot\hat{\epsilon}_2\vec{E}\cdot(\hat{k}\times\hat{\epsilon}_1) + \vec{B}\cdot\hat{\epsilon}_1\vec{E}\cdot(\hat{k}\times\hat{\epsilon}_2) + \vec{B}\cdot\hat{\epsilon}_1\vec{B}\cdot\hat{\epsilon}_2) \quad (26)$$

and

$$\tilde{f} = k^0[6((\vec{B}\cdot\epsilon_1)^2 - (\vec{B}\cdot\epsilon_2)^2) + 6((\vec{E}\cdot\epsilon_2)^2 - (\vec{E}\cdot\epsilon_1)^2) + 16((\vec{E}\cdot\epsilon_1)(\vec{B}\cdot\epsilon_2) - (\vec{B}\cdot\epsilon_1)(\vec{E}\cdot\epsilon_2))]. \quad (27)$$

Now we are ready to check the results discussed in Eq. (19). Using Eqs. (13) and considering a random direction for electric fields $\vec{E} = E(\sin\theta_E \cos\phi_E, \sin\theta_E \sin\phi_E, \cos\theta_E)$, we will rewrite the average value of $\langle\tilde{f}\rangle$ and $\langle\tilde{g}\rangle$ as follows

$$\begin{aligned}\langle\tilde{f}\rangle &= 3/4(1-\cos 2\theta)\langle E^2\rangle + \langle\tilde{f}_1(\theta_E,\phi_E)\rangle \\ &\propto 3/4(1-\cos 2\theta)I_0 + \langle\tilde{f}_1(\theta_E,\phi_E)\rangle \end{aligned} \tag{28}$$

note in the above equation $3/4(1-\cos 2\theta)\langle E^2\rangle$ is independent from the direction of electric fields as well as the polarizations of radiation ($\langle E^2\rangle \propto I_0$). But $\langle\tilde{g}\rangle$ does not include a term which can be independent from the direction of electric fields. In simple words, $\langle\tilde{f}\rangle$ has a contribution from isotropic unpolarized CMB radiation which comes from the nature of nonlinear interaction between CMB photons themselves via Euler-Heisenberg Hamiltonian.

IV. THE TIME EVOLUTION OF CMB POLARIZATIONS DUE TO EULER-HEISENBERG LAGRANGIAN AND COMPTON SCATTERING

In the present section, we consider our rest calculation in the presence of the primordial scalar perturbations indicating by (S) which we expand these perturbations in the Fourier modes characterized by a wave number $\mathbf{K}$. For each given wave number $\mathbf{K}$, it is useful to select a coordinate system with $\mathbf{K}\|\hat{\mathbf{z}}$ and $(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2) = (\hat{\mathbf{e}}_\theta, \hat{\mathbf{e}}_\phi)$. Temperature anisotropy $\Delta_I^{(S)}$, linear polarizations ($\Delta_Q^{(S)}$ and $\Delta_U^{(S)}$), and circular polarization $\Delta_V^{(S)}$ of the CMB radiation can be expanded in an appropriate spin-weighted basis as following [33]

$$\Delta_I^{(S)}(\mathbf{K},\mathbf{k},\tau) = \sum_{\ell m} a_{\ell m}(\tau,K) Y_{lm}(\mathbf{n}), \tag{29}$$

$$\Delta_P^{\pm(S)}(\mathbf{K},\mathbf{k},\tau) = \sum_{\ell m} a_{\pm 2,\ell m}(\tau,K)\, {}_{\pm 2}Y_{lm}(\mathbf{n}), \tag{30}$$

$$\Delta_V^{(S)}(\mathbf{K},\mathbf{k},\tau) = \sum_{\ell m} a_{V,\ell m}(\tau,K) Y_{lm}(\mathbf{n}), \tag{31}$$

where we define

$$\begin{aligned}\Delta_I^{(S)}(\mathbf{K},\mathbf{k},\tau) &= \left(4k\frac{\partial I_0}{\partial k}\right)^{-1}\Delta_I^{(S)}(\mathbf{K},\mathbf{k},t), \\ \Delta_P^{\pm(S)} &= \left(4k\frac{\partial I_0}{\partial k}\right)^{-1}(Q^{(S)} \pm iU^{(S)}). \end{aligned} \tag{32}$$

As usual, one can transfer the CMB temperature and polarizations $\Delta_{I,P,V}(\eta,\mathbf{K},\mu)$ in the conformal time η and describe them by multipole moments as following

$$\Delta_{I,P,V}(\eta,\mathbf{K},\mu) = \sum_{l=0}^{\infty}(2l+1)(-i)^l \Delta_{I,P,V}^l(\eta,\mathbf{K})P_l(\mu) \tag{33}$$

where $\mu = \hat{n}\cdot\hat{\mathbf{K}} = \cos\theta$, the θ is angle between the CMB photon direction $\hat{n} = \mathbf{k}/|\mathbf{k}|$ and the wave vectors $\mathbf{K}$, and $P_l(\mu)$ is the Legendre polynomial of rank l. Here we should define left-hand sides of Eq. (15) $\frac{d}{dt}$ to take into account space-time structure and gravitational effects such as redshift and so on. For each plane wave, each scattering and interaction can be described as the transport through a plane parallel medium [36,37], and finally Boltzmann equations in the presence of the primordial scalar perturbations are given as

$$\begin{aligned}&\frac{d}{d\eta}\Delta_I^{(S)} + iK\mu\Delta_I^{(S)} + 4[\dot{\psi} - iK\mu\varphi] \\ &\quad = \dot{\tau}_{e\gamma}\left[-\Delta_I^{(S)} + \Delta_I^{0(S)} + i\mu v_b + \frac{1}{2}P_2(\mu)\Pi\right] \end{aligned} \tag{34}$$

$$\begin{aligned}&\frac{d}{d\eta}\Delta_P^{\pm(S)} + iK\mu\Delta_P^{\pm(S)} \\ &\quad = \dot{\tau}_{e\gamma}\left[-\Delta_P^{\pm(S)} - \frac{1}{2}[1-P_2(\mu)]\Pi\right] \mp ia(\eta)\dot{\bar{\kappa}} f_1^0 \Delta_V^{(S)} \end{aligned} \tag{35}$$

$$\begin{aligned}&\frac{d}{d\eta}\Delta_V^{(S)} + iK\mu\Delta_V^{(S)} \\ &\quad = -\dot{\tau}_{e\gamma}\left[\Delta_V^{(S)} - \frac{3}{2}\mu\Delta_{V1}^{(S)}\right] + \frac{i}{2}\dot{\bar{\kappa}} f_2^0(\Delta_P^{-(S)} - \Delta_P^{+(S)}) \end{aligned} \tag{36}$$

where $\dot{\tau}_{e\gamma} \equiv \frac{d\tau_{e\gamma}}{d\eta}$ which $\tau_{e\gamma}$ is Compton scattering optical depth, $a(\eta)$ is normalized scale factor and $\Pi \equiv \Delta_I^{2(S)} + \Delta_P^{2(S)} + \Delta_P^{0(S)}$.

The values of $\Delta_P^{\pm(S)}(\hat{n})$ and $\Delta_V^{(S)}(\hat{n})$ at the present time η_0 and the direction $\hat{n}$ can be obtained in following general form by integrating of the Boltzmann equation [Eqs. (34)–(36)] along the line of sight [33] and summing over all the Fourier modes $\mathbf{K}$ as follows

$$\Delta_P^{\pm(S)}(\hat{\mathbf{n}}) = \int d^3\mathbf{K}\xi(\mathbf{K})\mathbf{e}^{\pm 2\mathbf{i}\phi_{\mathbf{K},\mathbf{n}}}\mathbf{\Delta}_\mathbf{P}^{\pm(\mathbf{S})}(\mathbf{K},\mathbf{k},\eta_0), \tag{37}$$

$$\Delta_V^{(S)}(\hat{\mathbf{n}}) = \int d^3\mathbf{K}\xi(\mathbf{K})\mathbf{\Delta}_\mathbf{V}^{(\mathbf{S})}(\mathbf{K},\mathbf{k},\eta_0), \tag{38}$$

where $\phi_{K,n}$ is the angle needed to rotate the $\mathbf{K}$ and $\hat{\mathbf{n}}$ dependent basis to a fixed frame in the sky, $\xi(\mathbf{K})$ is a random variable using to characterize the initial amplitude of each primordial scalar perturbations mode, and also the values of $\Delta_P^{\pm(S)}(\mathbf{K},\mathbf{k},\eta_0)$ and $\Delta_V^{(S)}(\mathbf{K},\mathbf{k},\eta_0)$ are given as

$$\begin{aligned}\Delta_P^{\pm(S)}(\mathbf{K},\mu,\eta_0) = \int_0^{\eta_0} d\eta\dot{\tau}_{e\gamma}e^{ix\mu-\tau_{e\gamma}}\Bigg[&\frac{3}{4}(1-\mu^2)\Pi(K,\eta) \\ &\mp if_1^0\frac{\dot{\bar{\kappa}}}{\dot{\tau}_{e\gamma}}\Delta_V^{(S)}\Bigg], \end{aligned} \tag{39}$$

and

$$\Delta_V^{(S)}(\mathbf{K},\mu,\eta_0) \approx \int_0^{\eta_0} d\eta \dot{\tau}_{e\gamma} e^{ix\mu-\tau_{e\gamma}} \left[\frac{3}{2}\mu\Delta_{V1}^{(S)} - if_2^0 \frac{\dot{\bar{\kappa}}}{\dot{\tau}_{e\gamma}} \Delta_P^{(S)}\right], \quad (40)$$

in which $x = K(\eta_0 - \eta)$, $f_{1,2}(\hat{p},\hat{k})$ are defined in (A2), (A3) and

$$\Delta_P^{(S)}(\mathbf{K},\mu,\eta) = \int_0^{\eta} \mathbf{d}\eta \dot{\tau}_{\mathbf{e}\gamma} \mathbf{e}^{\mathbf{i}\mathbf{x}\mu-\tau_{e\gamma}} \left[\frac{3}{4}(\mathbf{1}-\mu^{\mathbf{2}})\mathbf{\Pi}(\mathbf{K},\eta)\right]. \quad (41)$$

The differential optical depth $\dot{\tau}_{e\gamma}(\eta)$ and total optical depth $\tau_{e\gamma}(\eta)$ due to the Thomson scattering at time η are defined as

$$\dot{\tau}_{e\gamma} = a n_e \sigma_T, \qquad \tau_{e\gamma}(\eta) = \int_{\eta}^{\eta_0} \dot{\tau}_{e\gamma}(\eta) d\eta. \quad (42)$$

V. THE CONTRIBUTION OF EULER-HEISENBERG INTERACTION FOR THE CIRCULAR POWER SPECTRUM OF CMB

In the preceding section, we have prepared all instruments to calculate different power spectra $C_l^{X(S)}$s of CMB radiation due to Compton scattering and photon-photon forward scattering via Euler-Heisenberg interaction. So the power spectrum $C_l^{X(S)}$ in the presence of primordial scalar perturbation [indicated by (S)] is given as

$$C_l^{X(S)} = \frac{1}{2l+1}\sum_m \langle a_{X,lm}^* a_{X,lm}\rangle, \quad X = \{I,E,B,V\}, \quad (43)$$

where

$$a_{E,lm} = -(a_{2,lm} + a_{-2,lm})/2, \quad (44)$$

$$a_{B,lm} = i(a_{2,lm} - a_{-2,lm})/2, \quad (45)$$

$$a_{V,lm} = \int d\Omega Y_{lm}^* \Delta_V. \quad (46)$$

By using (39)–(41), the circular power spectrum $C_l^{V(S)}$ of CMB radiation can be written as follows

$$\begin{aligned} C_l^{V(S)} &= \frac{1}{2l+1}\sum_m \langle a_{V,lm}^* a_{V,lm}\rangle, \\ &\approx \frac{1}{2l+1}\int d^3\mathbf{K} P_\phi^{(S)}(\mathbf{K},\eta) \\ &\quad \times \sum_m \left| \int d\Omega Y_{lm}^* \int_0^{\eta_0} d\eta \dot{\tau}_{e\gamma} e^{ix\mu-\tau_{e\gamma}} \eta_{\text{EH}}(\eta) \Delta_P^{(S)} \right|^2, \end{aligned} \quad (47)$$

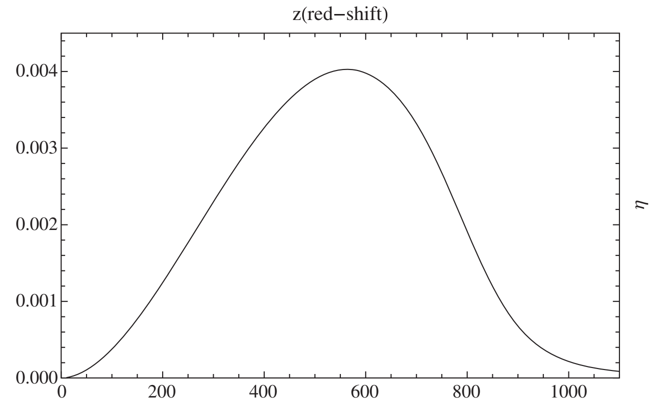

FIG. 1. $\eta_{\text{EH}}(z)$ is plotted in terms of redshift.

where $\eta_{\text{EH}}(\tau) = f_2^0 \frac{\dot{\bar{\kappa}}}{\dot{\tau}_{e\gamma}}$

$$P_\phi^{(S)}(\mathbf{K},\tau)\delta(\mathbf{K}' - \mathbf{K}) = \langle \xi(\mathbf{K})\xi(\mathbf{K}')\rangle, \quad (48)$$

and $P_\phi^{(S)}(\mathbf{K},\tau)$ is the scalar power spectrum of primordial matter perturbations.

Furthermore, as shown Eq. (47), the circular polarization cannot be generated in the scalar perturbation without considering the effects of Euler-Hiesenberg interactions. This result is in agreement with results of standard cosmology models [6]. With this knowledge that $\dot{\bar{\kappa}}$ and $\dot{\tau}_{e\gamma}$ depend on redshift, we have

$$\eta_{\text{EH}}(z) \simeq \frac{f_2^0}{15\pi} \frac{n_\gamma^0}{n_e^0} \frac{(1+z)^2}{\chi_e(z)} \left(\frac{T_{\text{CMB}}^0}{m_e}\right)^2, \quad (49)$$

where $\chi_e(z)$ is fraction of free cosmic electron, n_γ^0 and n_e^0 are number densities of CMB photons and cosmic electrons at present time, and $T_{\text{CMB}}^0 \simeq 2.7$ K. $\eta_{\text{EH}}(z)$ is plotted in terms of redshift in Fig. 1.

Now we can estimate $C_l^{V(S)}$ in terms of the linearly polarized power spectrum $C_l^{P(S)}$ and the average value of η_{EH} as

$$C_l^{V(S)} \approx (\eta_{\text{EH}}^{av})^2 C_l^{P(S)}, \quad (50)$$

where

$$\begin{aligned} C_l^{P(S)} &= \frac{1}{2l+1}\int d^3\mathbf{K} P_\phi^{(S)}(\mathbf{K},\tau) \\ &\quad \times \sum_m \left| \int d\Omega Y_{lm}^* \int_0^{\eta_0} d\eta \dot{\tau}_{e\gamma} e^{ix\mu-\tau_{e\gamma}} \Delta_P^{(S)} \right|^2, \end{aligned} \quad (51)$$

and

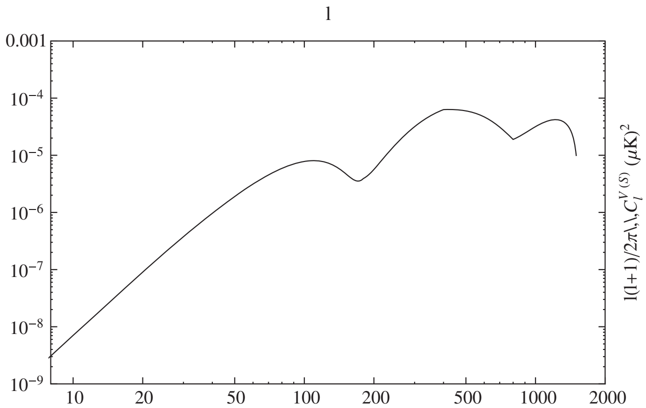

FIG. 2. The power spectrum of circular polarization $l(l+1)/2\pi$ $C_l^{V(S)}$ is plotted in terms of l and in unit $(\mu\mathrm{K})^2$ due to Compton scattering and photon-photon forward scattering via Euler-Heisenberg effective Lagrangian. This file contains the LCDM power spectra that are derived from Planck (2015) parameters and also we have modified CMBQUICK *Mathematica* code to make above plot.

$$\eta_{\mathrm{EH}}^{av} = \frac{1}{z^{lss}} \int_0^{z^{lss}} \eta_{\mathrm{EH}}(z) dz \simeq 0.0002, \qquad (52)$$

where z^{lss} indicates redshift at the last scattering surface. Using the experimental value for $C_l^{P(S)}$ which is in the order of $\sim(\mu\mathrm{K})^2$ and Eqs. (50)–(52), one can obtain an estimation on the range of $C_l^{V(S)} \sim 10\ (\mathrm{nK})^2$. Note, we just make above estimation to have a sense about the contribution of Euler-Heisenberg interactions for the power spectrum of CMB circular polarization. The more precise estimation of $l(l+1)C_l^{V(S)}/(2\pi)$ is given in Fig. 2. Let us compare our results with experimental data reported by the SPIDER group [7]. Constraints (upper bound) of the circular power spectrum $l(l+1)C_l^V/(2\pi)$ reported by the SPIDER group is in ranging from 141 to 203 $(\mu\mathrm{K})^2$ at 150 GHz for a thermal CMB spectrum and $33 < l < 307$ which is much larger than what can be found by considering nonlinear photon-photon interaction.

The Euler-Heisenberg interactions not only can generate circular polarization for CMB, but also generate the B-mode polarization in the presence of scalar metric perturbations in contrast with standard cosmology models [33,38]. Next, one can divide the CMB linear polarization in terms of the divergence-free part (B-mode $\Delta_B^{(S)}$) and the curl-free part (E-mode $\Delta_E^{(S)}$) which are defined in terms of Stokes parameters as following

$$\Delta_E^{(S)}(\hat{\mathbf{n}}) \equiv -\frac{1}{2}[\bar{\eth}^2 \Delta_P^{+(S)}(\hat{\mathbf{n}}) + \eth^2 \Delta_P^{-(S)}(\hat{\mathbf{n}})], \qquad (53)$$

$$\Delta_B^{(S)}(\hat{\mathbf{n}}) \equiv \frac{i}{2}[\bar{\eth}^2 \Delta_P^{+(S)}(\hat{\mathbf{n}}) - \eth^2 \Delta_P^{-(S)}(\hat{\mathbf{n}})], \qquad (54)$$

where $\eth$ and $\bar{\eth}$ indicate spin raising and lowering operators respectively [38]. As Eqs. (38), (43), (52), and (54) shown, the B-mode power spectrum $C_l^{B(S)}$ is given in terms of the circular polarization power spectrum $C_l^{V(S)}$ which can be estimated as

$$C_l^{B(S)} \propto \bar{\eta}^2 C_l^{V(S)} \ll (n\mathrm{K})^2. \qquad (55)$$

Note the B-mode generating by Euler-Hiesenberg interaction is very small than $(n\mathrm{K})^2$ and so that we can neglect it.

VI. CONCLUSION AND REMARKS

In this work, we have solved the first order of the quantum Boltzmann equation for the density matrix of CMB radiation by considering Compton scattering and nonlinear photon-photon forward scattering via the Euler-Heisenberg effective Lagrangian as collision terms. We have shown that propagating photons convert their linear polarizations to circular polarizations via the Euler-Heisenberg effective interaction. Also we have discussed that by considering nonlinear CMB-CMB photons interaction, CMB linear polarization converts to circular one while crossing through CMB isotopic unpolarized medium I_0. The power spectrum of circular polarization in CMB radiations $C_l^{V(S)}$ in the presence of scalar perturbations is given in terms of linearly polarized power spectrum of CMB radiation $C_l^{V(S)} \sim (\eta_{\mathrm{EH}}^{av})^2 C_l^{P(S)}$ which η_{EH} (49) is given in terms of redshift by factor $(1+z)^2/\chi_e(z)$ and also $\eta_{\mathrm{EH}}^{av} \simeq 0.0002$ (52). Also, we have estimated the average value of circular power spectrum is $l(l+1)C_l^{V(S)}/(2\pi) \sim 10^{-4}\ (\mu\mathrm{K})^2$ for $l \sim 300$ at present time which is very smaller than recently reported data for upper limit of circular polarization (SPIDER collaboration). As a result to observe our results, the ability to detect Nano-Kelvin level signals of CMB circular polarization requires. $l(l+1)C_l^{V(S)}/(2\pi)$ is plotted in Fig. 2. We also show that the generation of B-mode polarization for CMB photons in the presence of the primordial scalar perturbation via Euler-Heisenberg interaction is possible however this contribution for B-mode polarization is not remarkable. It is shown in Eq. (55) that $C_l^{B(S)} \ll (\mathrm{nK})^2$.

APPENDIX: TIME EVOLUTION OF DENSITY MATRIX

The time-evolution of the density matrix approximately obtained as

$$
\begin{aligned}
(2\pi)^3\delta^3(0)2k^0\frac{d}{dt}\rho_{ij}(k) &\approx i\langle[H_I^0(t),D_{ij}^0(k)]\rangle \\
&= -\frac{2\alpha^2 i}{45m^4}(2\pi)^3\delta^3(0)\times\int\frac{d^3p}{(2\pi)^3 2p^0}[(p.k)^2[\epsilon_s(k).\epsilon_{s'}(p)\epsilon_l(k).\epsilon_{l'}(p)] \\
&\quad\times\{-5\rho_{s'l'}(p)\rho_{is}(k)\delta^{lj}+5\rho_{s'l'}(p)\rho_{lj}(k)\delta^{si}+4\rho_{l's'}(p)\rho_{lj}(k)\delta^{si}-4\rho_{l's'}(p)\rho_{is}(k)\delta^{lj} \\
&\quad+3\rho_{l's'}(p)\rho_{sj}(k)\delta^{li}-3\rho_{l's'}(p)\rho_{il}(k)\delta^{sj}+4\rho_{s'l'}(p)\rho_{sj}(k)\delta^{li}-4\rho_{s'l'}(p)\rho_{il}(k)\delta^{sj}+9\rho_{lj}(k)\delta^{si}\delta^{s'l'} \\
&\quad-9\rho_{is}(k)\delta^{lj}\delta^{s'l'}+3\rho_{sj}(k)\delta^{s'l'}\delta^{li}-3\rho_{il}(k)\delta^{sj}\delta^{s'l'}\} \\
&\quad+[p.\epsilon_s(k)k.\epsilon_{s'}(p)p.\epsilon_l(k)k.\epsilon_{l'}(p)-2(p.k)\epsilon_s(k).\epsilon_{s'}(p)p.\epsilon_l(k)k.\epsilon_{l'}(p)] \\
&\quad\times\{8\rho_{lj}(k)\delta^{si}\delta^{s'l'}-8\rho_{is}(k)\delta^{lj}\delta^{s'l'}+4\rho_{l's'}(p)\rho_{lj}(k)\delta^{si}-4\rho_{l's'}(p)\rho_{is}(k)\delta^{lj}-4\rho_{s'l'}(p)\rho_{is}(k)\delta^{lj} \\
&\quad+4\rho_{s'l'}(p)\rho_{lj}(k)\delta^{si}+4\rho_{sj}(k)\delta^{l's'}\delta^{li}+4\rho_{sj}(k)\rho_{l's'}(p)\delta^{li}-4\rho_{il}(k)\delta^{l's'}\delta^{sj}-4\rho_{l's'}(p)\rho_{il}(k)\delta^{sj} \\
&\quad+4\rho_{s'l'}(p)\rho_{sj}(k)\delta^{li}-4\rho_{s'l'}(p)\rho_{il}(k)\delta^{sj}\}-28\epsilon^{\mu\nu\alpha\beta}\epsilon^{\sigma\nu'\gamma\beta'}k_\gamma k_\mu p_\alpha p_\sigma\epsilon_{s'\beta}(p)\epsilon_{l\nu'}(p)\epsilon_{s\nu}(k)\epsilon_{l'\beta'}(k) \\
&\quad\times[\rho_{l'j}(k)\delta^{si}-\rho_{is}(k)\delta^{l'j}+\rho_{sj}(k)\delta^{l'i}-\rho_{il'}(k)\delta^{sj}]\times[\rho_{ls'}(p)+\rho_{s'l}(p)+\delta^{s'l}]],
\end{aligned}
\tag{A1}
$$

where k and p indicate the energy-momentum states of photons and $\delta^3(0)$ will be canceled in the final expression. Here detail of abbreviated functions in Eqs. (9)–(12) are brought.

$$
\begin{aligned}
f_1(\hat{p},\hat{k}) &= 2[(\hat{p}.\hat{k})^2((\hat{\epsilon}_2(k).\hat{\epsilon}_1(p))^2-(\hat{\epsilon}_1(k).\hat{\epsilon}_1(p))^2+(\hat{\epsilon}_2(k).\hat{\epsilon}_2(p))^2-(\hat{\epsilon}_1(k).\hat{\epsilon}_2(p))^2) \\
&\quad+((\hat{p}.\hat{\epsilon}_2(k))^2-(\hat{p}.\hat{\epsilon}_1(k))^2)((\hat{k}.\hat{\epsilon}_2(p))^2+(\hat{k}.\hat{\epsilon}_1(p))^2)+2(\hat{k}.\hat{p})((\hat{\epsilon}_1(k).\hat{\epsilon}_1(p)\hat{p}.\hat{\epsilon}_1(k)-\hat{\epsilon}_2(k).\hat{\epsilon}_1(p)\hat{p}.\hat{\epsilon}_2(k))\hat{k}.\hat{\epsilon}_1(p) \\
&\quad+(\hat{\epsilon}_1(k).\hat{\epsilon}_2(p)\hat{p}.\hat{\epsilon}_1(k)-\hat{\epsilon}_2(k).\hat{\epsilon}_2(p)\hat{p}.\hat{\epsilon}_2(k))\hat{k}.\hat{\epsilon}_2(p))]
\end{aligned}
\tag{A2}
$$

$$
\begin{aligned}
f_2(\hat{p},\hat{k}) &= 2[(\hat{p}.\hat{k})^2([\hat{\epsilon}_2(k).\hat{\epsilon}_1(p)]^2-[\hat{\epsilon}_1(k).\hat{\epsilon}_1(p)]^2+[\hat{\epsilon}_2(k).\hat{\epsilon}_2(p)]^2-[\hat{\epsilon}_1(k).\hat{\epsilon}_2(p)]^2) \\
&\quad+2(\hat{p}.\hat{k})((\hat{\epsilon}_2(k).\hat{\epsilon}_2(p)\hat{p}.\hat{\epsilon}_2(k)-\hat{\epsilon}_1(k).\hat{\epsilon}_2(p)\hat{p}.\hat{\epsilon}_1(k))\hat{k}.\hat{\epsilon}_2(p)-\hat{\epsilon}_1(k).\hat{\epsilon}_1(p)\hat{p}.\hat{\epsilon}_1(k)\hat{k}.\hat{\epsilon}_1(p)) \\
&\quad+((\hat{p}.\hat{\epsilon}_1(k))^2-(\hat{p}.\hat{\epsilon}_2(k))^2)((\hat{k}.\hat{\epsilon}_1(p))^2+(\hat{k}.\hat{\epsilon}_2(p))^2)].
\end{aligned}
\tag{A3}
$$

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