

Charming new physics in rare B decays and mixing?

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We conduct a systematic study of the impact of new physics in quark-level $b \rightarrow c\bar{c}s$ transitions on B physics, in particular rare B decays and B -meson lifetime observables. We find viable scenarios where a sizable effect in rare semileptonic B decays can be generated, compatible with experimental indications and with a possible dependence on the dilepton invariant mass, while being consistent with constraints from radiative B decay and the measured B_s width difference. We show how, if the effect is generated at the weak scale or beyond, strong renormalization-group effects can enhance the impact on semileptonic decays while leaving radiative B decay largely unaffected. A good complementarity of the different B -physics observables implies that precise measurements of lifetime observables at LHCb may be able to confirm, refine, or rule out this scenario.

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I. INTRODUCTION

Rare B decays are excellent probes of new physics at the electroweak scale and beyond, due to their strong suppression in the Standard Model (SM). Interestingly, experimental data on rare branching ratios [1,2] and angular distributions for $B \rightarrow K^{(*)}\mu^+\mu^-$ decay [2,3] may hint at a beyond-SM (BSM) contact interaction of the form $(\bar{s}_L\gamma^\mu b_L)(\bar{\mu}\gamma_\mu\mu)$, which would destructively interfere with the corresponding SM (effective) coupling C_9 [4–6], although the significance of the effect is somewhat uncertain because of form-factor uncertainties as well as uncertain long-distance virtual charm contributions [7]. However, if the BSM interpretation is correct, it requires reducing C_9 by $\mathcal{O}(20\%)$ in magnitude. Such an effect might arise from new particles (see e.g. [8]), which might in turn be part of a more comprehensive new dynamics. Noting that in the SM, about half of C_9 comes from (short-distance) virtual-charm contributions, in this article we ask whether new physics affecting the quark-level $b \rightarrow c\bar{c}s$ transitions could cause the anomalies, affecting rare B

decays through a loop. The bulk of these effects would also be captured through an effective shift $\Delta C_9(q^2)$, with a possible dependence on the dilepton mass q^2 . At the same time, such a scenario offers the exciting prospect of confirming the rare B -decay anomalies through correlated effects in hadronic B decays into charm, with “mixing” observables such as the B_s -meson width difference standing out as precisely measured [9] and under reasonable theoretical control. This is in contrast with the Z' and leptoquark models usually considered, where correlated effects are typically restricted to other rare processes and are highly model dependent. Specific scenarios of hadronic new physics in the B widths have been considered previously [10], while the possibility of virtual charm BSM physics in rare semileptonic decay has been raised in [11] (see also [12]). As we will show, viable scenarios exist, which can mimic a shift $\Delta C_9 = -\mathcal{O}(1)$ while being consistent with all other observables. In particular, very strong renormalization-group effects can generate large shifts in the (low-energy) effective C_9 coupling from small $b \rightarrow c\bar{c}s$ couplings at a high scale without conflicting with the measured $\bar{B} \rightarrow X_s\gamma$ decay rate [13].

II. CHARMING NEW PHYSICS SCENARIO

We consider a scenario where new physics affects the $b \rightarrow c\bar{c}s$ transitions. This could be the case in models containing new scalars or new gauge bosons, or strongly coupled new physics. Such models will typically affect other observables, but in a model-dependent manner. For this paper, we restrict ourselves to studying the new effects

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induced by modified $b \rightarrow c\bar{c}s$ couplings, leaving construction and phenomenology of concrete models for future work. We refer to this as the ‘‘charming BSM’’ (CBSM) scenario. As long as the mass scale M of new physics satisfies $M \gg m_B$, the modifications to the $b \rightarrow c\bar{c}s$ transitions can be accounted for through a local effective Hamiltonian,

$$\mathcal{H}_{\text{eff}}^{c\bar{c}} = \frac{4G_F}{\sqrt{2}} V_{cs}^* V_{cb} \sum_{i=1}^{10} (C_i^c Q_i^c + C_i^{c'} Q_i^{c'}). \quad (1)$$

We choose our operator basis and renormalization scheme to agree with [14] upon the substitution $d \rightarrow b$, $\bar{s} \rightarrow \bar{c}$, $\bar{u} \rightarrow \bar{s}$:

$$\begin{aligned} Q_1^c &= (\bar{c}_L^i \gamma_\mu b_L^j) (\bar{s}_L^j \gamma^\mu c_L^i), & Q_2^c &= (\bar{c}_L^i \gamma_\mu b_L^i) (\bar{s}_L^j \gamma^\mu c_L^j), \\ Q_3^c &= (\bar{c}_R^i b_L^j) (\bar{s}_L^j c_R^i), & Q_4^c &= (\bar{c}_R^i b_L^i) (\bar{s}_L^j c_R^j), \\ Q_5^c &= (\bar{c}_R^i \gamma_\mu b_R^j) (\bar{s}_L^j \gamma^\mu c_L^i), & Q_6^c &= (\bar{c}_R^i \gamma_\mu b_R^i) (\bar{s}_L^j \gamma^\mu c_L^j), \\ Q_7^c &= (\bar{c}_L^i b_R^j) (\bar{s}_L^j c_R^i), & Q_8^c &= (\bar{c}_L^i b_R^i) (\bar{s}_L^j c_R^j), \\ Q_9^c &= (\bar{c}_L^i \sigma_{\mu\nu} b_R^j) (\bar{s}_L^j \sigma^{\mu\nu} c_R^i), & Q_{10}^c &= (\bar{c}_L^i \sigma_{\mu\nu} b_R^i) (\bar{s}_L^j \sigma^{\mu\nu} c_R^j). \end{aligned} \quad (2)$$

The $Q_i^{c'}$ are obtained by changing all the quark chiralities. We leave a discussion of such ‘‘right-handed current’’ effects for future work [15] and discard the $Q_i^{c'}$ below. We split the Wilson coefficients into SM and BSM parts,

$$C_i^c(\mu) = C_i^{c,\text{SM}}(\mu) + \Delta C_i(\mu), \quad (3)$$

where $C_i^{c,\text{SM}} = 0$ except for $i = 1, 2$ and μ is the renormalization scale.

III. RARE B DECAYS

The leading-order (LO), one-loop CBSM effects in radiative and rare semileptonic decays may be expressed through ‘‘effective’’ Wilson coefficient contributions $\Delta C_9^{\text{eff}}(q^2)$ and $\Delta C_7^{\text{eff}}(q^2)$ in an effective local Hamiltonian,

$$\mathcal{H}_{\text{eff}}^{rsl} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} (C_7^{\text{eff}}(q^2) Q_{7\gamma} + C_9^{\text{eff}}(q^2) Q_{9V}), \quad (4)$$

where q^2 is the dilepton mass and

$$\begin{aligned} Q_{7\gamma} &= \frac{em_b}{16\pi^2} (\bar{s}_L \sigma_{\mu\nu} b_R) F^{\mu\nu}, \\ Q_{9V} &= \frac{\alpha}{4\pi} (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \ell). \end{aligned}$$

For q^2 small (in particular, well below the charm resonances), $\Delta C_9^{\text{eff}}(q^2)$ and $\Delta C_7^{\text{eff}}(q^2)$ govern the theoretical predictions for both exclusive ($B \rightarrow K^{(*)} \ell^+ \ell^-$,

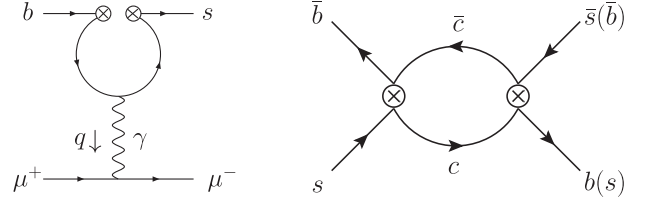


FIG. 1. Leading CBSM contributions to rare decays (left), and to width difference $\Delta\Gamma_s$ and lifetime ratio $\tau(B_s)/\tau(B_d)$ (right).

$B_s \rightarrow \phi \ell^+ \ell^-$, etc.) and inclusive $B \rightarrow X_s \ell^+ \ell^-$ decay, up to $\mathcal{O}(\alpha_s)$ QCD corrections and power corrections to the heavy-quark limit that we neglect in our leading-order analysis. Similarly, $\Delta C_7^{\text{eff}}(0)$ determines radiative B -decay rates. We will neglect the small CKM combination $V_{us}^* V_{ub}$, implying $V_{cs}^* V_{cb} = -V_{ts}^* V_{tb}$, and focus on real (CP -conserving) values for the C_i^c . From the diagram shown in Fig. 1 (left), we then obtain

$$\Delta C_9^{\text{eff}}(q^2) = \left(C_{1,2}^c - \frac{C_{3,4}^c}{2} \right) h - \frac{2}{9} C_{3,4}^c, \quad (5)$$

$$\Delta C_7^{\text{eff}}(q^2) = \frac{m_c}{m_b} \left[(4C_{9,10}^c - C_{7,8}^c) y + \frac{4C_{5,6}^c - C_{7,8}^c}{6} \right], \quad (6)$$

with $C_{x,y}^c = 3\Delta C_x + \Delta C_y$ and the loop functions

$$h(q^2, m_c, \mu) = -\frac{4}{9} \left[\ln \frac{m_c^2}{\mu^2} - \frac{2}{3} + (2+z)a(z) - z \right], \quad (7)$$

$$y(q^2, m_c, \mu) = -\frac{1}{3} \left[\ln \frac{m_c^2}{\mu^2} - \frac{3}{2} + 2a(z) \right], \quad (8)$$

where $a(z) = \sqrt{|z-1|} \arctan \frac{1}{\sqrt{z-1}}$ and $z = 4m_c^2/q^2$. Our numerical evaluation employs the charm pole mass.

We note that only the four Wilson coefficients $\Delta C_{1\dots 4}$ enter $\Delta C_9^{\text{eff}}(q^2)$. Conversely, $\Delta C_7^{\text{eff}}(q^2)$ is given in terms of the other six Wilson coefficients $\Delta C_{5\dots 10}$. The appearance of a one-loop, q^2 -dependent contribution to C_7^{eff} is a novel feature in the CBSM scenario. Numerically, the loop function $a(z)$ equals one at $q^2 = 0$ and vanishes at $q^2 = (2m_c)^2$. The constant terms and the logarithm accompanying $y(q^2, m_c)$ partially cancel the contribution from $a(z)$ and they introduce a sizable dependence on the renormalization scale μ and the charm quark mass. Since a shift of $\Delta C_7^{\text{eff}}(q^2)$ is strongly constrained by the measured $B \rightarrow X_s \gamma$ decay rate, we do not consider the coefficients $\Delta C_{5\dots 10}$ in the remainder and focus on the four coefficients $\Delta C_{1\dots 4}$, which do not contribute to $B \rightarrow X_s \gamma$ at 1-loop order. Higher-order contributions can be important if new physics generates ΔC_i at the weak scale or beyond, as is typically expected. In this case, large logarithms $\ln M/m_B$ occur, requiring resummation. To leading-logarithmic accuracy, we find

$$\Delta C_7^{\text{eff}} = 0.02\Delta C_1 - 0.19\Delta C_2 - 0.01\Delta C_3 - 0.13\Delta C_4, \quad (9)$$

$$\Delta C_9^{\text{eff}} = 8.48\Delta C_1 + 1.96\Delta C_2 - 4.24\Delta C_3 - 1.91\Delta C_4, \quad (10)$$

if ΔC_i are understood to be renormalized at $\mu = M_W$ and $\Delta C_{7,9}^{\text{eff}}$ at $\mu = 4.2$ GeV. It is clear that ΔC_1 and ΔC_3 contribute (strongly) to rare semileptonic decay but only weakly to $B \rightarrow X_s \gamma$.

IV. MIXING AND LIFETIME OBSERVABLES

A distinctive feature of the CBSM scenario is that nonzero ΔC_i affect not only radiative and rare semileptonic decays, but also tree-level hadronic $b \rightarrow c\bar{c}s$ transitions. While the theoretical control over exclusive $b \rightarrow c\bar{c}s$ modes is very limited at present, the decay width difference $\Delta\Gamma_s$ and the lifetime ratio $\tau(B_s)/\tau(B_d)$ stand out as being calculable in a heavy-quark expansion [16]; see Fig. 1 (right). For both observables, the heavy-quark expansion gives rise to an operator product expansion in terms of local $\Delta B = 2$ (for the width difference) or $\Delta B = 0$ (for the lifetime ratio) operators. The formalism is reviewed in [17] and applies to both SM and CBSM contributions. For the B_s width difference, we have [18] $\Delta\Gamma_s = 2|\Gamma_{12}^{\text{s,SM}} + \Gamma_{12}^{\text{c}\bar{c}}| \cos\phi_{12}^s$, where the phase ϕ_{12}^s is small. Neglecting the strange-quark mass, we find

$$\begin{aligned} \Gamma_{12}^{\text{c}\bar{c}} = & -G_F^2 (V_{cs}^* V_{cb})^2 m_b^2 M_{B_s} f_{B_s}^2 \frac{\sqrt{1-4x_c^2}}{576\pi} \\ & \times \{ [16(1-x_c^2)(4C_2^{c,2} + C_4^{c,2}) + 8(1-4x_c^2) \\ & \times (12C_1^{c,2} + 8C_1^c C_2^c + 2C_3^c C_4^c + 3C_3^{c,2}) - 192x_c^2 \\ & \times (3C_1^c C_3^c + C_1^c C_4^c + C_2^c C_3^c + C_2^c C_4^c)] B + 2(1+2x_c^2) \\ & \times (4C_2^{c,2} - 8C_1^c C_2^c - 12C_1^{c,2} - 3C_3^{c,2} - 2C_3^c C_4^c + C_4^{c,2}) \tilde{B}'_s \}, \end{aligned} \quad (11)$$

with $x_c = m_c/m_b$. B , \tilde{B}'_s are defined through

$$\langle B_s | (\bar{s}_L \gamma_\mu b_L) (\bar{s}_L \gamma^\mu b_L) | B_s \rangle = \frac{2}{3} M_{B_s}^2 f_{B_s}^2 B, \quad (12)$$

$$\langle B_s | (\bar{s}_L^i b_R^j) (\bar{s}_L^j b_R^i) | B_s \rangle = \frac{1}{12} M_{B_s}^2 f_{B_s}^2 \tilde{B}'_s, \quad (13)$$

with values taken from [19]. For our numerical evaluation of $\Gamma_{12}^{\text{c}\bar{c}}$, we split the Wilson coefficients according to (3), subtract from the LO expression (11) the pure SM contribution and add the NLO SM expressions from [20]. In general, a modification of $\Gamma_{12}^{\text{c}\bar{c}}$ also affects the semileptonic CP asymmetries. However, since we consider CP -conserving new physics in this paper and since the corresponding experimental uncertainties are still large, the semi-leptonic asymmetries will not lead to an additional constraint.

In a similar manner, for the lifetime ratio, we find

$$\frac{\tau_{B_s}}{\tau_{B_d}} = \left(\frac{\tau_{B_s}}{\tau_{B_d}} \right)_{\text{SM}} + \left(\frac{\tau_{B_s}}{\tau_{B_d}} \right)_{\text{NP}}, \quad (14)$$

where the SM contribution is taken from [21] and

$$\begin{aligned} & \left(\frac{\tau_{B_s}}{\tau_{B_d}} \right)_{\text{NP}} \\ & = G_F^2 |V_{cb} V_{cs}|^2 m_b^2 M_{B_s} f_{B_s}^2 \tau_{B_s} \frac{\sqrt{1-4x_c^2}}{144\pi} \\ & \times \{ (1-x_c^2) [(4C_{1,2}^{c,2} + C_{3,4}^{c,2}) B_1 + 6(4C_2^{c,2} + C_4^{c,2}) \epsilon_1] \\ & - 12x_c^2 (C_{1,2}^c C_{3,4}^c B_1 + 6C_2^c C_4^c \epsilon_1) - \frac{M_{B_s}^2 (1+2x_c^2)}{(m_b + m_s)^2} \\ & \times [(4C_{1,2}^{c,2} + C_{3,4}^{c,2}) B_2 + 6(4C_2^{c,2} + C_4^{c,2}) \epsilon_2] \}, \end{aligned} \quad (15)$$

subtracting the SM part and defining B_1 , B_2 , ϵ_1 , ϵ_2 as

$$\langle B_s | (\bar{b}_L \gamma_\mu s_L) (\bar{s}_L \gamma^\mu b_L) | B_s \rangle = \frac{1}{4} f_{B_s}^2 M_{B_s}^2 B_1, \quad (16)$$

$$\langle B_s | (\bar{b}_R s_L) (\bar{s}_L b_R) | B_s \rangle = \frac{1}{4} \left[\frac{M_{B_s}}{(m_b + m_s)} \right]^2 f_{B_s}^2 M_{B_s}^2 B_2, \quad (17)$$

$$\langle B_s | (\bar{b}_L \gamma_\mu T^A s_L) (\bar{s}_L \gamma^\mu T^A b_L) | B_s \rangle = \frac{1}{4} f_{B_s}^2 M_{B_s}^2 \epsilon_1, \quad (18)$$

$$\langle B_s | (\bar{b}_R T^A s_L) (\bar{s}_L T^A b_R) | B_s \rangle = \frac{1}{4} \left[\frac{M_{B_s}}{(m_b + m_s)} \right]^2 f_{B_s}^2 M_{B_s}^2 \epsilon_2, \quad (19)$$

with values taken from [22]. We interpret the quark masses as $\overline{\text{MS}}$ parameters at $\mu = 4.2$ GeV.

V. RARE DECAYS VERSUS LIFETIMES—LOW-SCALE SCENARIO

We are now in a position to confront the CBSM scenario with rare decay and mixing observables, as long as we consider renormalization scales $\mu \sim m_B$. Then the logarithms inside the h function entering (5) are small and our leading-order calculation should be accurate. Such a scenario is directly applicable if the mass scale M of the physics generating the ΔC_i is not too far above m_B , such that $\ln(M/m_B)$ is small. Fig. 2 (left) shows the experimental 1σ allowed regions for the width difference and lifetime ratio (from the web update of [23]) in the $(\Delta C_1, \Delta C_2)$ plane. The central values are attained on the brown (solid) and green (dashed) curves, respectively. The measured lifetime ratio and the width difference measurement can be simultaneously accommodated for different values of the Wilson coefficients: in the ΔC_1 - ΔC_2 plane, we find the SM solution, as well as a solution around $\Delta C_1 = -0.5$ and $\Delta C_2 \approx 0$. In the ΔC_3 - ΔC_4 plane, we have

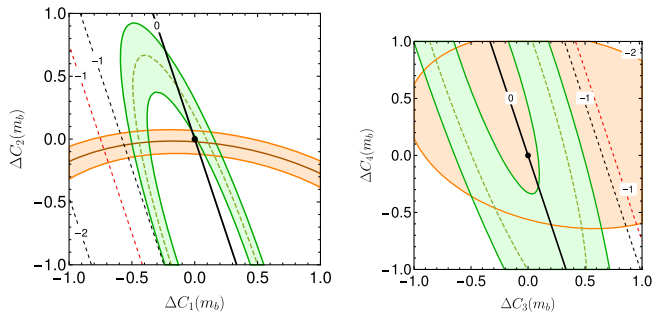


FIG. 2. Mixing observables versus rare decays in the CBSM scenario. Left: $(\Delta C_1, \Delta C_2)$ plane, Right: $(\Delta C_3, \Delta C_4)$ plane. In each case, all Wilson coefficients are renormalized at $\mu = 4.2$ GeV and those not corresponding to either axis set to zero. The black dot corresponds to the SM, i.e. $\Delta C_i = 0$. The measured central value for the width difference is shown as brown (solid) line together with the 1σ allowed region. The lifetime ratio measurement is depicted as green (dashed) line and band. Overlaid are contours of $\Delta C_9^{\text{eff}}(5 \text{ GeV}^2) = -1, -2$ (black, dashed) and $\Delta C_9^{\text{eff}}(2 \text{ GeV}^2) = -1, -2$ (red, dotted), as computed from (5), and of $\Delta C_9^{\text{eff}} = 0$ (black, solid).

a relatively broad allowed range, roughly covering the interval $[-0.9, +0.7]$ for ΔC_3 and $[-0.6, +1.1]$ for ΔC_4 . For further conclusions, a considerably higher precision in experiment and theory is required for $\Delta\Gamma_s$ and τ_{B_s}/τ_{B_d} . Also shown in the plot are contour lines for the contribution to the effective semileptonic coefficient $\Delta C_9^{\text{eff}}(q^2)$, both for $q^2 = 2 \text{ GeV}^2$ and $q^2 = 5 \text{ GeV}^2$. We see that sizable negative shifts are possible while respecting the measured width difference and the lifetime ratio. For example, a shift $\Delta C_9^{\text{eff}} \sim -1$ as data may suggest could be achieved through $\Delta C_1 \sim -0.5$ alone. Such a value for ΔC_1 may well be consistent with CP -conserving exclusive $b \rightarrow c\bar{c}s$ decay data, where no accurate theoretical predictions exist. On the other hand, ΔC_9^{eff} only exhibits a mild q^2 -dependence. Distinguishing this from possible long-distance contributions would require substantial progress on the theoretical understanding of the latter.

We can also consider other Wilson coefficients, such as the pair $(\Delta C_3, \Delta C_4)$ (right panel in Fig. 2). A shift $\Delta C_9^{\text{eff}} \sim -1$ is equally possible and consistent with the width difference, requiring only $\Delta C_3 \sim 0.5$.

VI. HIGH-SCALE SCENARIO AND RGE

A. RG enhancement of ΔC_9^{eff}

If the CBSM operators are generated at a high scale then large logarithms $\ln M/m_B$ appear. Their resummation is achieved by evolving the initial (matching) conditions $C_i(\mu_0 \sim M)$ to a scale $\mu \sim M_B$ according to the coupled renormalization-group equations (RGE),

$$\mu \frac{dC_j}{d\mu}(\mu) = \gamma_{ij}(\mu) C_i(\mu), \quad (20)$$

where γ_{ij} is the anomalous-dimension matrix. As is well known, the operators Q_i^c mix not only with Q_7 and Q_9 , but also with the 4 QCD penguin operators $P_{3\dots 6}$ and the chromodipole operator Q_{8g} (defined as in [24]), which in turn mix into Q_7 . Hence the index j runs over 11 operators with $\Delta B = -\Delta S = 1$ flavor quantum numbers in order to account for all contributions to $C_7(\mu)$ that are proportional to $\Delta C_i(\mu_0)$. Most entries of γ_{ij} are known at LO [14,24–30]; our novel results are ($i = 3, 4$)

$$\begin{aligned} \gamma_{Q_i^c \tilde{Q}_9}^{(0)} &= \left(\frac{4}{3}, \frac{4}{9} \right)_i, & \gamma_{Q_i^c P_4}^{(0)} &= \left(0, -\frac{2}{3} \right)_i, \\ \gamma_{Q_i^c Q_7}^{\text{eff}(0)} &= \left(0, \frac{224}{81} \right)_i, \end{aligned}$$

where $\tilde{Q}_9 = (4\pi/\alpha_s)Q_{9V}(\mu)$ and $\gamma_{Q_i^c Q_7}^{\text{eff}(0)}$ requires a two-loop calculation. (See appendix for further technical information.) Solving the RGE for $\mu_0 = M_W$, $\mu = 4.2$ GeV, and $\alpha_s(M_Z) = 0.1181$, results in the CBSM contributions to ΔC_7^{eff} and ΔC_9^{eff} in (9), (10) as well as

$$\begin{pmatrix} \Delta C_1(\mu) \\ \Delta C_2(\mu) \\ \Delta C_3(\mu) \\ \Delta C_4(\mu) \end{pmatrix} = \begin{pmatrix} 1.12 & -0.27 & 0 & 0 \\ -0.27 & 1.12 & 0 & 0 \\ 0 & 0 & 0.92 & 0 \\ 0 & 0 & 0.33 & 1.91 \end{pmatrix} \begin{pmatrix} \Delta C_1(\mu_0) \\ \Delta C_2(\mu_0) \\ \Delta C_3(\mu_0) \\ \Delta C_4(\mu_0) \end{pmatrix}. \quad (21)$$

A striking feature are the large coefficients in the ΔC_9^{eff} case, which are $\mathcal{O}(1/\alpha_s)$ in the logarithmic counting. The largest coefficients appear for ΔC_1 and ΔC_3 , which at the same time practically do not mix into C_7^{eff} . This means that small values $\Delta C_1 \sim -0.1$ or $\Delta C_3 \sim 0.2$ can generate $\Delta C_9^{\text{eff}}(\mu) \sim -1$ while having essentially no impact on the $B \rightarrow X_s \gamma$ decay rate. Conversely, values for ΔC_2 or ΔC_4 that lead to $\Delta C_9^{\text{eff}} \sim -1$ lead to large effects in C_7^{eff} and $B \rightarrow X_s \gamma$.

B. Phenomenology for high NP scale

The situation in various two-parameter planes is depicted in Fig. 3, where the 1σ constraint from $B \rightarrow X_s \gamma$ is shown as blue, straight bands. (We implement it by splitting $\text{BR}(B \rightarrow X_s \gamma)$ into SM and BSM parts and employ the numerical result and theory error from [31] for the former. The experimental result is taken from the web update of [23].) The top row corresponds to Fig. 2, but contours of given ΔC_9 lie much closer to the origin. All six panels testify to the fact that the SM is consistent with all data when leaving aside the question of rare semileptonic B decays—the largest pull stems from the fact that the experimental value for τ_{B_s}/τ_{B_d} is just under 1.5 standard deviations below the SM expectation, such that the black (SM) point is less than 0.5σ outside the green area. Our

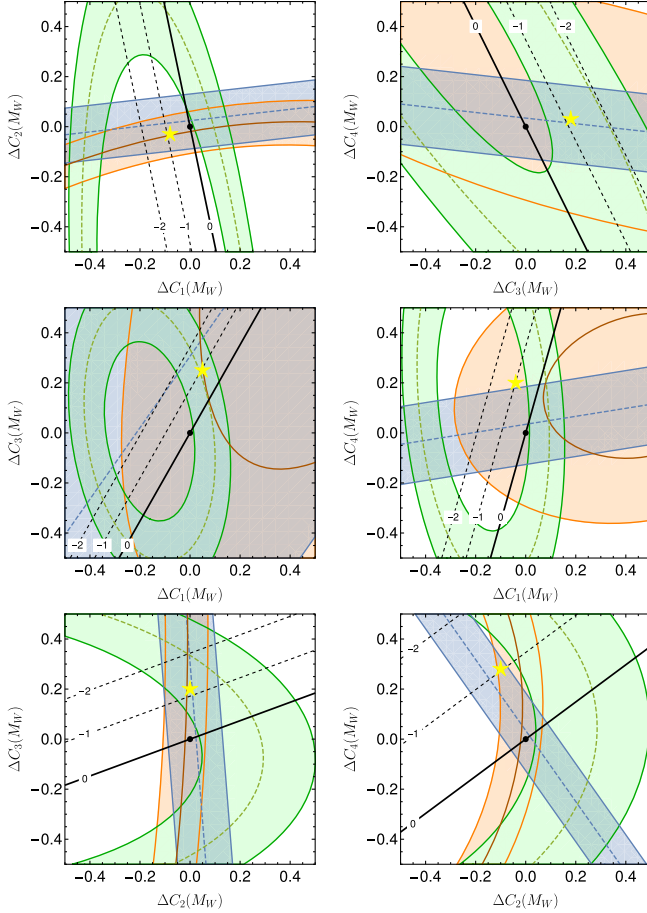


FIG. 3. Mixing observables versus rare decays, for ΔC_i renormalized at $\mu_0 = M_W$. Color coding as in Fig. 2, $B \rightarrow X_s \gamma$ constraint shown in addition (straight blue bands).

main question is now: can we have a new contribution $\Delta C_9^{\text{eff}} \sim -1$ to rare semileptonic decays, while being consistent with the bounds stemming from $b \rightarrow s \gamma$, $\Delta \Gamma_s$ and τ_{B_s}/τ_{B_d} ? This is clearly possible (indicated by the yellow star in the plots) if we have a new contribution $\Delta C_3 \approx 0.2$, see the three plots of the $\Delta C_i - \Delta C_3$ planes in Fig. 3 (right on the top row, left on the middle row and left on the lower row). In these cases, the $\Delta C_9^{\text{eff}} \sim -1$ solution is even favored compared to the SM solution. A joint effect in $\Delta C_2 \approx -0.1$ and $\Delta C_4 \approx 0.3$ can also accommodate our desired scenario, see the right plot on the lower row, while new BSM effects in the pairs $\Delta C_1, \Delta C_2$ and $\Delta C_1, \Delta C_4$ alone are less favored. One could also consider three or all four ΔC_i simultaneously.

C. Implications for UV physics

Our model-independent results are well suited to study the rare B -decay and lifetime phenomenology of ultraviolet (UV) completions of the Standard Model. Any such completion may include extra UV contributions to $C_7(M)$ and $C_9(M)$, correlations with other flavor

observables, collider phenomenology, etc.; the details are highly model-dependent and beyond the scope of our model-independent analysis. Here we restrict ourselves to some basic sanity checks.

Taking the case of $\Delta C_1(M) \sim -0.1$ corresponds to a naive ultraviolet scale

$$\Lambda \sim \left(\frac{4G_F}{\sqrt{2}} |V_{cs}^* V_{cb}| \times 0.1 \right)^{-1/2} \sim 3 \text{ TeV}.$$

This effective scale could arise in a weakly-coupled scenario from tree-level exchange of new scalar or vector mediators, or at loop level in addition from fermions; or the effective operator could arise from strongly-coupled new physics. For a tree-level exchange, $\Lambda \sim M/g_*$ where $g_* = \sqrt{g_1 g_2}$ is the geometric mean of the relevant couplings. For weak coupling $g_* \sim 1$, this then gives $M \sim 3 \text{ TeV}$. Particles of such mass are certainly allowed by collider searches if they do not couple (or only sufficiently weakly) to leptons and first-generation quarks. Multi-TeV weakly coupled particles also generically are not in violation of electroweak precision tests of the SM. Loop-level mediation would require mediators close to the weak scale which may be problematic and would require a specific investigation; this is of course unsurprising given that $b \rightarrow c \bar{c} s$ transitions are mediated at tree level in the SM. The same would be true in a BSM scenario that mimics the flavor suppressions in the SM (such as MFV models). Conversely, in a strongly-coupled scenario we would have $M \sim g_* \Lambda \sim 4\pi \Lambda \sim 30 \text{ TeV}$. This is again safe from generic collider and precision constraints, and a model-specific analysis would be required to say more.

Finally, as all CBSM effects are lepton-flavor-universal, they cannot on their own account for departures of the lepton flavor universality parameters $R_{K^{(*)}}$ [32] from the SM values as suggested by current experimental measurements [33]. However, even if those departures are real, they may still be caused by direct UV contributions to ΔC_9 . For example, as shown in [5], a scenario with a muon-specific contribution $\Delta C_9^\mu = -\Delta C_{10}^\mu \sim -0.6$ and in addition a lepton-universal contribution $\Delta C_9 \sim -0.6$, which may have a CBSM origin, is perfectly consistent with all rare- B -decay data, and in fact marginally preferred.

VII. PROSPECTS AND SUMMARY

The preceding discussion suggests that a precise knowledge of width difference and lifetime ratio, as well as $\text{BR}(B \rightarrow X_s \gamma)$, can have the potential to identify and discriminate between different CBSM scenarios, or rule them out altogether. This is illustrated in Fig. 4, showing contour values for future precision both in mixing and lifetime observables. In each panel, the solid (brown and green) contours correspond to the SM central values of the width difference and lifetime ratio (respectively). The spacing of the accompanying contours is such that the area

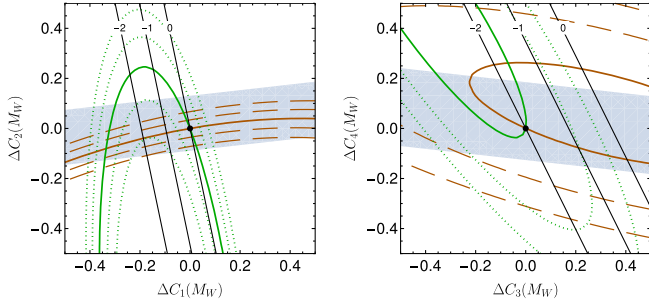


FIG. 4. Future prospects for mixing observables. Dashed: contours of constant width difference, dotted: contours of constant lifetime ratio. See text for discussion.

between any two neighboring contours corresponds to a prospective 1σ -region, assuming a combined (theoretical and experimental) error on the lifetime ratio of 0.001 and a combined error on $\Delta\Gamma_s$ of 5%. The assumed future errors are ambitious but seem feasible with expected experimental and theoretical progress. Overlaid is the (current) $B \rightarrow X_s\gamma$ constraint (blue). The figure indicates that a discrimination between the SM and the scenario where $\Delta C_9 \approx -1$, while $\text{BR}(B \rightarrow X_s\gamma)$ is SM-like is clearly possible. A crucial role is played by the lifetime ratio τ_{B_s}/τ_{B_d} : in e.g. the $\Delta C_3 - \Delta C_4$ case a 1σ deviation of the lifetime ratio almost coincides with the $\Delta C_9 = -1$ contour line; a further precise determination of $\Delta\Gamma_s$ could then identify the point on this line chosen by nature. Further progress on $B \rightarrow X_s\gamma$ in the Belle II era would provide complementary information.

In summary, we have given a comprehensive, model-independent analysis of BSM effects in partonic $b \rightarrow c\bar{c}s$ transitions (CBSM scenario) in the CP conserving case, focusing on those observables that can be computed in a heavy-quark expansion. An effect in rare semileptonic B decays compatible with hints from current LHCb and B -factory data can be generated, while satisfying the $B \rightarrow X_s\gamma$ constraint. It can originate from different combinations of $b \rightarrow c\bar{c}s$ operators. The required Wilson coefficients are so small that constraints from B decays into charm are not effective, particularly if new physics enters at a high scale; then large renormalization-group enhancements are present. Likewise, there are no obvious model-independent conflicts with collider searches or electroweak precision observables. A more precise measurement of mixing observables and lifetime ratios, at a level achievable at LHCb, may be able to confirm (or rule out) the CBSM scenario, and to discriminate between different BSM couplings. Finally, all CBSM effects are lepton-flavor-universal; the current R_K and R_{K^*} anomalies would either have to be mismeasurements or require additional lepton-flavor-specific UV contribution to C_9 ; such a combined scenario has been shown elsewhere to be consistent with all rare B -decay data and also presents the most generic way for UV physics to affect rare decays. With the stated caveats, our conclusions are rather model independent. It

would be interesting to construct concrete UV realizations of the CBSM scenario, which almost certainly will affect other observables in a correlated, but model-dependent manner.

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APPENDIX: TECHNICAL ASPECTS OF THE ANOMALOUS-DIMENSION CALCULATION

Here we provide additional technical information regarding our results on anomalous dimensions entering in the RGE (20).

A set of Wilson coefficients that contains C_7 , C_9 , and $C_{1\dots 4}^c$ and is closed under renormalization necessarily also contains four QCD-penguin coefficients C_{P_i} multiplying the operators $P_{3\dots 6}$ (we define them as in [24]) and the chromodipole coefficient C_{8g} , resulting in an 11×11 anomalous-dimension matrix γ . If the rescaled semileptonic operator $\tilde{Q}_9(\mu) = (4\pi/\alpha_s(\mu))Q_{9V}(\mu)$ is used then to leading order $\gamma_{ij}(\mu) = \alpha_s(\mu)/(4\pi)\gamma_{ij}^{(0)}$, with constant $\gamma_{ij}^{(0)}$. As is well known, this matrix is scheme-dependent already at LO [28]. A scheme-independent matrix $\gamma^{\text{eff}(0)}$ can be achieved by replacing C_7 and C_8 by the scheme-independent combinations

$$C_7^{\text{eff}} = C_7 + \sum_i y_i C_i, \tag{A1}$$

$$C_8^{\text{eff}} = C_8 + \sum_i z_i C_i, \tag{A2}$$

where

$$\langle s\gamma|Q_i|b \rangle = y_i \langle s\gamma|Q_{7\gamma}|b \rangle, \tag{A3}$$

$$\langle sg|Q_i|b \rangle = z_i \langle s\gamma|Q_{8g}|b \rangle, \tag{A4}$$

to lowest order and the sums run over all four-quark operators. We find that y_i and z_i vanish for $Q_{1\dots 4}^c$, leaving only the known coefficients $y_{P_i} = (-1/3, -4/9, -20/3, -80/9)_i$ and $z_{P_i} = (1, -1/6, 20, -10/3)_i$ ($i = 3\dots 6$) [24]. The BSM correction ΔC_9^{eff} in (5), (10) coincides with the (BSM correction to the) coefficient C_9 of Q_{9V} to LL accuracy.

Many of the elements of $\gamma^{\text{eff}(0)}$ are known [14,25–28], except for $\gamma_{Q_i^c Q_{7\gamma}}^{\text{eff}(0)}$, $\gamma_{Q_i^c Q_{8g}}^{\text{eff}(0)}$, $\gamma_{Q_i^c P_j}^{\text{eff}(0)}$, and $\gamma_{Q_i^c \tilde{Q}_9}^{\text{eff}(0)}$, for $i = 3, 4$. The latter can be read off from the logarithmic terms in (5), and

the mixing into P_i follows from substituting gauge coupling and color factors in diagram Fig. 1 (left). This gives

$$\gamma_{Q_i^c \bar{Q}_9}^{(0)} = \left(-\frac{8}{3}, -\frac{8}{9}, \frac{4}{3}, \frac{4}{9} \right)_i, \quad \gamma_{Q_i^c P_4}^{(0)} = \left(0, \frac{4}{3}, 0, -\frac{2}{3} \right)_i,$$

for $i = 1, 2, 3, 4$, with the mixing into $C_{P_{3,5,6}}$ vanishing.

The leading mixing into C_7^{eff} arises at two loops [29] and is the technically most challenging aspect of this work. Our calculation employs the 1PI (off-shell) formalism and the method of [30] for computing UV divergences, which involves an infrared-regulator mass and the appearance of a set of gauge-non-invariant counterterms. The result is

$$\gamma_{Q_i^c Q_7}^{\text{eff}(0)} = \left(0, \frac{416}{81}, 0, \frac{224}{81} \right)_i \quad (i = 1, 2, 3, 4).$$

Our stated results for $i = 1, 2$ agree with the results in [24,26], which constitutes a cross-check of our calculation.

We have not obtained the 2-loop mixing of $C_{3,4}^c$ into C_{8g} and set these anomalous dimension elements to zero. For the case of $C_{1,2}^c$ where this mixing is known, the impact of neglecting $\gamma_{i8}^{\text{eff}(0)}$ on $\Delta C_7^{\text{eff}}(\mu)$ is small [the only change being $-0.19\Delta C_2 \rightarrow -0.18\Delta C_2$ in (9)]. We expect a similarly small error in the case of $\Delta C_{3,4}$.

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