

# Analysis of strong decays of charmed mesons $D_2^*(2460)$ , $D_0(2560)$ , $D_2(2740)$ , $D_1(3000)$ , $D_2^*(3000)$ , and their spin partners $D_1^*(2680)$ , $D_3^*(2760)$ , and $D_0^*(3000)$

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Using the effective Lagrangian approach, we examine the recently observed charm states  $D_J^*(2460)$ ,  $D_J(2560)$ ,  $D_J(2740)$ ,  $D_J(3000)$ , and their spin partners  $D_J^*(2680)$ ,  $D_J^*(2760)$ , and  $D_J^*(3000)$  with  $J^P$  states  $1P_{\frac{3}{2}}2^+$ ,  $2S_{\frac{1}{2}}0^-$ ,  $1D_{\frac{3}{2}}2^-$ ,  $2P_{\frac{1}{2}}1^+$ , and  $2S_{\frac{3}{2}}1^-$ ,  $1D_{\frac{3}{2}}3^-$ ,  $2P_{\frac{1}{2}}0^+$  respectively. We study their two body strong decays, coupling constants and branching ratios with the emission of light pseudo-scalar mesons ( $\pi$ ,  $\eta$ ,  $K$ ). We also analyze the newly observed charm state  $D_2^*(3000)$  and suggest it to be either  $1F(2^+)$  or  $2P(2^+)$  state and justify one of them to be the most favorable assignment for  $D_2^*(3000)$ . We study the partial and the total decay width of unobserved states  $D(1^1F_3)$ ,  $D_s(1^1F_3)$  and  $D_s(1^1F_2)$  as the spin and the strange partners of the  $D_2^*(3000)$  charmed meson. The branching ratios and the coupling constants  $g_{TH}$ ,  $\tilde{g}_{HH}$ ,  $g_{YH}$ ,  $\tilde{g}_{SH}$ , and  $g_{ZH}$  calculated in this work can be confronted with the future experimental data.

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## I. INTRODUCTION

The excitation spectrum of ( $c\bar{q}$ ) heavy-light charmed mesons have received considerable theoretical and experimental attention, as it provide opportunities to study the QCD properties within the context of different models. Recently, LHCb collaboration have used the Dalitz plot analysis to study the resonant substructures  $B^- \rightarrow D^+\pi^-\pi^-$  decays in the pp collision at a center-of-mass energy 7 TeV. The masses and the widths of charm resonances with spins 1, 2 and 3 at high  $D^+\pi^-$  masses are determined [1]. The study gives indication that, these resonances are mainly coming from the contribution of the  $D_2^*(2460)$ ,  $D_1^*(2680)$ ,  $D_3^*(2760)$ , and  $D_2^*(3000)$  charmed mesons. The measured Breit-Wigner masses and widths of these charmed mesons are

$$D_2^*(2460): M = 2463.7 \pm 0.4 \pm 0.4 \pm 0.6 \text{ MeV},$$

$$\Gamma = 47.0 \pm 0.8 \pm 0.9 \pm 0.3 \text{ MeV}, \quad (1)$$

$$D_1^*(2680): M = 2681.1 \pm 5.6 \pm 4.9 \pm 13.1 \text{ MeV},$$

$$\Gamma = 186.7 \pm 8.5 \pm 8.6 \pm 8.2 \text{ MeV}, \quad (2)$$

$$D_3^*(2760): M = 2775.5 \pm 4.5 \pm 4.5 \pm 4.7 \text{ MeV},$$

$$\Gamma = 95.3 \pm 9.6 \pm 7.9 \pm 33.1 \text{ MeV}, \quad (3)$$

$$D_2^*(3000): M = 3214 \pm 29 \mp 33 \mp 36 \text{ MeV},$$

$$\Gamma = 186 \pm 38 \pm 34 \pm 63 \text{ MeV} \quad (4)$$

In 2010 and 2013, a great achievement have been made by *BABAR* and LHCb collaboration. LHCb collaboration observed two natural parity resonances  $D_J^*(2650)^0$ ,  $D_J^*(2760)^0$  and two unnatural parity resonances  $D_J(2580)^0$  and  $D_J(2740)^0$  by studying the  $D^+\pi^-$ ,  $D^0\pi^+$ , and  $D^{*+}\pi^-$  invariant mass spectra [2]. Along with these states, LHCb has also observed  $D_J(3000)^0$  in the  $D^{*+}\pi^-$  final state and  $D_J^*(3000)^+$  and  $D_J^*(3000)^0$  in the  $D^0\pi^+$  and  $D^+\pi^-$  mass spectra respectively. *BABAR* collaboration in 2010, observed  $D_J(2560)^0$ ,  $D_J(2600)^0$ ,  $D_J(2600)^+$ ,  $D_J(2750)^0$ ,  $D_J^*(2760)^+$ , and  $D_J^*(2760)^0$  in the inclusive  $e^+e^- \rightarrow c\bar{c}$  interaction [3]. Masses and the widths of charm states predicted by *BABAR* and LHCb are so close, that they are considered to be in the same  $J^P$  state. Masses and widths of these charm states observed by various collaborations are presented in Table I.

It is very crucial to assign a proper  $J^P$  to the heavy-light system in a given spectra, as large amount of experimental information like decay width, branching ratios, and hyperfine splitting are based on their  $J^P$ . Various theoretical models have suggested different  $J^P$  states to the observed charm mesons. In this paper, we analyze the available theoretical and experimental data on the excited charm states and specify their proper  $J^P$ . In our analysis, we mentioned  $D_2^*(2460)$  to be the well-established state having  $J^P = 2^+$  in the charm spectra [4]. The information provided by *BABAR* (2010) and LHCb (2013) for the states  $D_J^*(2680)$  and  $D_J^*(2760)$  were

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TABLE I. The experimental results from LHCb(2016) [1], LHCb(2013) [2], and BABAR(2010) [3] of nonstrange charm mesons. Values corresponding to M: and  $\Gamma$ : represents mass and decay width of the states. All the values are in MeV unit.

Charm State	LHCb(2013) [2]	BABAR(2010) [3]	LHCb(2016) [1]	Decay Channel
$D_2^*(2460)$			M: $2463.7 \pm 0.4 \pm 0.4$ $\Gamma$ : $47.0 \pm 0.8 \pm 0.9$	$D^{*+}\pi^-$
$D_J^*(2650)^0$	M: $2649.2 \pm 3.5 \pm 3.5$ $\Gamma$ : $140.2 \pm 17.1 \pm 18.6$	M: $2608.7 \pm 2.4 \pm 2.5$ $\Gamma$ : $93 \pm 6 \pm 13$	M: $2681.1 \pm 5.6 \pm 4.9$ $\Gamma$ : $186.7 \pm 8.5 \pm 8.6$	$D^{*+}\pi^-$
$D_J^*(2760)^0$	M: $2761.1 \pm 5.1 \pm 6.5$ $\Gamma$ : $74.4 \pm 3.4 \pm 37.0$	M: $2763.3 \pm 2.3 \pm 2.3$ $\Gamma$ : $60.9 \pm 5.1 \pm 3.6$	M: $2775.5 \pm 4.5 \pm 4.5$ $\Gamma$ : $5.3 \pm 9.6 \pm 7.9$	$D^{*+}\pi^-$
$D_J(2560)^0$	M: $2579.5 \pm 3.4 \pm 5.5$ $\Gamma$ : $177.4 \pm 17.8 \pm 46.0$	M: $2539.4 \pm 4.5 \pm 6.8$ $\Gamma$ : $130 \pm 12 \pm 13$		$D^{*+}\pi^-$
$D_J(2740)^0$	M: $2737.0 \pm 3.5 \pm 11.24$ $\Gamma$ : $73.2 \pm 13.4 \pm 25.0$	M: $2752.4 \pm 1.7 \pm 2.7$ $\Gamma$ : $71 \pm 6 \pm 11$		$D^{*+}\pi^-$
$D_J(3000)^0$	M: $2971.8 \pm 8.7$ $\Gamma$ : $188.1 \pm 44.8$			$D^{*+}\pi^-$
$D_J^*(2760)^0$	M: $2760.1 \pm 1.1 \pm 3.7$ $\Gamma$ : $74.4 \pm 3.4 \pm 19.1$			$D^+\pi^-$
$D_J^*(3000)^0$	M: $3008.1 \pm 4.0$ $\Gamma$ : $110.5 \pm 11.5$			$D^+\pi^-$
$D_2^*(3000)$			M: $3214 \pm 29 \pm 33 \pm 36$ $\Gamma$ : $186 \pm 38 \pm 34 \pm 63$	$D^+\pi^-$
$D_J^*(2760)^+$	M: $2771.7 \pm 1.7 \pm 3.8$ $\Gamma$ : $66.7 \pm 6.6 \pm 10.5$			$D^0\pi^+$
$D_J^*(3000)^+$	M: $3008.1$ $\Gamma$ : $110.5$			$D^0\pi^+$

confirmed in 2016 by LHCb, which had provided their  $J$  values as 1 and 3 respectively. Theoretical study of these two states concluded their  $J^P$  to be  $1^-$  for  $n = 2$  S-wave and  $3^-$  for  $n = 1$  D wave respectively [5–9]. States  $D_J(2560)^0$  and  $D_J(2740)^0$  being the spin partners of  $D_J^*(2680)^0$  and  $D_J^*(2760)^0$ , are assigned  $J^P = 0^-$  for S-wave ( $n = 2$ ) and  $2^-$  for D-wave ( $n = 1$ ) respectively. Higher charm states  $D_J^*(3000)$  and  $D_J(3000)$  were studied by various models like  $^3P_0$  model, heavy quark effective theory, but their  $J^{P'}$ s are not yet confirmed. Authors in [10] assigned  $D_J^*(3000)$  as the  $1F_{\frac{3}{2}}2^+$  or  $1F_{\frac{7}{2}}4^+$  state and  $D_J(3000)$  as the  $1F_{\frac{7}{2}}3^+$  or  $2P_{\frac{1}{2}}1^+$  state, but Ref. [11] have suggested various other possibilities for the  $J^{P'}$ s of ( $D_J^*(3000)$ ), ( $D_J(3000)$ ) and concluded  $2P(0^+, 1^+)$  to be the most favorable  $nLJ^{P'}$ s in the charm spectra by studying their branching ratio.

Now, the main interest of theorists is on the newly predicted  $D_2^*(3000)$  state, whose mass and decay width is comparable with the former  $D_J^*(3000)$  state. It is suggested by Zhi-Gang Wang in Ref. [12], that the energy gap between  $D_2^*(3000)^0$  and  $D_J^*(3000)^0$  is 206 MeV ( $M_{D_2^*(3000)^0} - M_{D_J^*(3000)^0} = 206$  MeV), which indicates them to be different particles. On the basis of the charm masses predicted by relativistic quark model [13], Wang suggested  $D_2^*(3000)$  to be  $1F_{\frac{3}{2}}2^+$  state [5,13]. Using the  $^3P_0$  model, they also suggested the most plausible assignment of  $D_2^*(3000)$  to be the  $3P_{\frac{3}{2}}2^+$  state, but then the other possibility like  $2F_{\frac{3}{2}}2^+$  may not be completely excluded [14]. Thus, the clear picture of the  $J^P$  of  $D_2^*(3000)$  is not

yet available. This unclear picture is the motivation for our present work.

On the basis of masses predicted by various theoretical models [8,13,15–18], we assume the two most favorable  $J^P$  states for  $D_2^*(3000)$  to be either  $1F(2^+)$  or  $2P(2^+)$ .  $D_2^*(3000)$  is observed in the decay channel  $D^+\pi^-$  but not in  $D^{*+}\pi^-$ , and hence  $D^{*+}\pi^-$  decay mode must be suppressed. By analyzing the branching ratio  $\text{BR} = \frac{\Gamma(D_2^*(3000) \rightarrow D^+\pi^-)}{\Gamma(D_2^*(3000) \rightarrow D\pi)}$  with their masses and strong decay widths, we further choose one of them as the best possible  $J^P$  state for the  $D_2^*(3000)$  and have determine its strong coupling constant. We use the HQET model for studying the decay widths at the leading order approximations, because the mass and the spin degeneracy of heavy hadrons appears as approximate internal symmetry of the Lagrangian. Beside the fact that HQET in conjugation with the chiral perturbation theory, has been successfully applied to the strong decays of the heavy hadrons [19,20]. Heavy quark symmetry helps in reducing the parameters by imposing constraints on these constants, like the range of the strong coupling constants is constrained to be within 0 and 1 by studying the decay widths and branching ratios of ground state charm mesons [21]. The strong couplings can also be retrieved by comparing the strong decay widths with the experimental available decay widths and masses. The paper is arranged as follows: Section II gives the brief review of the HQET model (For the detailed review refer Refs. [22–25]). In Sec. III, we study the strong decays and the branching ratios of the  $D_J^*(2460)$ ,  $D_J(2560)$ ,  $D_J(2740)$ ,  $D_J(3000)$ , and their

spin partners  $D_j^*(2680)$ ,  $D_j^*(2760)$ , and  $D_j^*(3000)$  with  $J^P$  states  $1P_{\frac{3}{2}}2^+$ ,  $2S_{\frac{3}{2}}0^-$ ,  $1D_{\frac{3}{2}}2^-$ ,  $2P_{\frac{1}{2}}1^+$ , and  $2S_{\frac{1}{2}}1^-$ ,  $1D_{\frac{3}{2}}3^-$ ,  $2P_{\frac{3}{2}}0^+$ , respectively and discusses their strong coupling constants involved. We also analyze the newly observed charm state  $D_2^*(3000)$  and suggest it to be either  $1F(2^+)$  or  $2P(2^+)$  state. And by studying the decay behavior and the branching ratio for both these  $nLJ^P$ 's, we justify one of them to be the most favorable assignment for  $D_2^*(3000)$ . In addition to this, we also study the strong decays for the unobserved spin and the strange partners of  $D_2^*(3000)$  i.e.,  $D(1^1F_3)$ ,  $D_s(1^1F_3)$  and  $D_s(1^1F_2)$  in the framework of the HQET, which are experimentally unobserved but theoretically predicted. Section IV presents the conclusion of our work.

## II. FRAMEWORK

In the heavy quark limit  $m_Q \gg \Lambda_{\text{QCD}} \gg m_q$ , Compton wave-length of the heavy quark  $\lambda_Q \approx 1/m_Q$  is much smaller than the hadronic distance 1 fm. The strong interactions of such a heavy quark with light quarks and gluons can be described by an effective theory, which is invariant with flavor and the spin of the heavy quark. This effective theory involves the corrections at the order of  $1/m_Q$  order. The theoretical framework for such analysis is provided by the so-called heavy quark effective theory. Also, the mass and spin degeneracy of the heavy hadrons appears as approximate internal symmetries of the Lagrangian. It is an effective QCD theory for  $N_f$  heavy quarks  $Q$  with their four velocity fixed. In this theory, spin and parity of the heavy quark decouples from the light degrees of freedom as they interact through the exchange of soft gluons. Heavy mesons are classified in doublets, in relation to the total conserved angular momentum, i.e.,  $s_l = s_{\bar{q}} + l$ , where  $s_{\bar{q}}$  and  $l$  are the spin and orbital angular momentum of the light degree of freedom respectively. For  $l = 0$  (S-wave), the doublet is represented by  $(P, P^*)$  with  $J_{s_l}^P = (0^-, 1^-)_{\frac{1}{2}}$ , which for  $l = 1$  (P-wave), there are two doublets represented by  $(P_0^*, P_1')$  and  $(P_1, P_2^*)$  with  $J_{s_l}^P = (0^+, 1^+)_{\frac{1}{2}}$  and  $(1^+, 2^+)_{\frac{3}{2}}$  respectively. Two doublets of  $l = 2$  (D-wave) are represented by  $(P_1^*, P_2)$  and  $(P_2', P_3^*)$  belonging to  $J_{s_l}^P = (1^-, 2^-)_{\frac{3}{2}}$  and  $(2^-, 3^-)_{\frac{5}{2}}$  respectively. And the doublets of  $l = 3$  (F-wave) are represented by  $(P_2^*, P_3)$  and  $(P_3', P_4^*)$  for  $J_{s_l}^P = (2^+, 3^+)_{\frac{5}{2}}$  and  $(3^+, 4^+)_{\frac{7}{2}}$  respectively. These doublets are described by the effective superfield  $H_a, S_a, T_a, X_a, Y_a$ , and  $Z_a$  [26,27].

$$H_a = \frac{1 + \not{v}}{2} \{ P_{a\mu}^* \gamma^\mu - P_a \gamma_5 \} \quad (5)$$

$$S_a = \frac{1 + \not{v}}{2} \{ P_{1a}^\mu \gamma_\mu \gamma_5 - P_{0a}^* \} \quad (6)$$

$$T_a^\mu = \frac{1 + \not{v}}{2} \left\{ P_{2a}^{*\mu\nu} \gamma_\nu - P_{1a\nu} \sqrt{\frac{3}{2}} \gamma_5 \left[ g^{\mu\nu} - \frac{\gamma^\nu (\gamma^\mu - v^\mu)}{3} \right] \right\} \quad (7)$$

$$Y_a^{\mu\nu} = \frac{1 + \not{v}}{2} \left\{ P_{3a}^{*\mu\nu\sigma} \gamma_\sigma - P_{2a}^{\alpha\beta} \sqrt{\frac{5}{3}} \gamma_5 \right. \\ \left. \times \left[ g_\alpha^\mu g_\beta^\nu - \frac{g_\beta^\nu \gamma_\alpha (\gamma^\mu - v^\mu)}{5} - \frac{g_\alpha^\mu \gamma_\beta (\gamma^\nu - v^\nu)}{5} \right] \right\} \quad (8)$$

$$Z_a^{\mu\nu} = \frac{1 + \not{v}}{2} \left\{ P_{3a}^{\mu\nu\sigma} \gamma_5 \gamma_\sigma - P_{2a}^{*\alpha\beta} \sqrt{\frac{5}{3}} \right. \\ \left. \times \left[ g_\alpha^\mu g_\beta^\nu - \frac{g_\beta^\nu \gamma_\alpha (\gamma^\mu + v^\mu)}{5} - \frac{g_\alpha^\mu \gamma_\beta (\gamma^\nu + v^\nu)}{5} \right] \right\} \quad (9)$$

Here the field  $H_a$  describe the  $(P, P^*)$  doublet, i.e., S-wave,  $S_a$  and  $T_a$  fields represents the P-wave doublets  $(0^+, 1^+)_{\frac{1}{2}}$  and  $(1^+, 2^+)_{\frac{3}{2}}$  respectively. The mentioned indices a or b in the subsequent fields and Lagrangian are  $SU(3)$  flavor index (u, d or s). P and  $P^*$  in field  $H_a$  represents  $D^0, D^+, D_s^+$  and  $D^{*0}, D^{*+}, D_s^{*+}$ , respectively. The heavy meson field  $P^{(*)}$  contain a factor  $\sqrt{m_Q}$  with mass dimension of  $\frac{1}{2}$ . For the radially excited states with radial quantum number  $n = 2$ , these states are replaced by  $\tilde{P}, \tilde{P}^*$  and so on. The properties of the hadrons are invariant under  $SU(2N_f)$  transformations, hence heavy quark spin and flavor symmetries provide a clear picture for the study of the heavy-light mesons in heavy quark physics. The light pseudoscalar mesons are described by the fields  $\xi = \exp \frac{i\mathcal{M}}{f_\pi}$ , where  $\mathcal{M}$  is defined as

$$\mathcal{M} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} \quad (10)$$

The pion octet is introduced by the vector and axial vector combinations  $V^\mu = \frac{1}{2}(\xi \partial^\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi)$  and  $A^\mu = \frac{1}{2}(\xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi)$ . We choose  $f_\pi = 130$  MeV. Here, all traces are taken over Dirac spinor indices, light quark  $SU(3)_V$  flavor indices  $a = u, d, s$  and heavy quark flavor indices  $Q = c, b$ . The Dirac structure of the chiral Lagrangian is given by the velocity vector  $v/c$ . At the leading order approximation, the heavy meson chiral Lagrangians  $L_{HH}, L_{SH}, L_{TH}, L_{YH}, L_{ZH}$  for the two-body strong interactions through light pseudoscalar mesons are written as:

$$L_{HH} = g_{HH} \text{Tr} \{ \bar{H}_a H_b \gamma_\mu \gamma_5 A_{ba}^\mu \} \quad (11)$$

$$L_{SH} = g_{SH} \text{Tr} \{ \bar{H}_a S_b \gamma_\mu \gamma_5 A_{ba}^\mu \} + \text{H.c.} \quad (12)$$

$$L_{TH} = \frac{g_{TH}}{\Lambda} \text{Tr} \{ \bar{H}_a T_b^\mu (iD_\mu \mathbf{A} + i\not{D} A_\mu)_{ba} \gamma_5 \} + \text{H.c.} \quad (13)$$

$$L_{YH} = \frac{1}{\Lambda^2} \text{Tr} \{ \bar{H}_a Y_b^{\mu\nu} [k_1^Y \{ D_\mu, D_\nu \} A_\lambda \\ + k_2^Y (D_\mu D_\lambda A_\nu + D_\nu D_\lambda A_\mu) ]_{ba} \gamma^\lambda \gamma_5 \} + \text{H.c.} \quad (14)$$

TABLE II. Numerical value of the meson masses used in this work [4].

States	$D^0$	$D^\pm$	$D^{*+}$	$D^{*0}$	$D_S^+$	$D_S^{*+}$
Masses(MeV)	1864.86	1869.62	2010.28	2006.98	1968.49	2112.30
States	$\pi^\pm$	$\pi^0$	$\eta$	$K^+$	$K^0$	
Masses(MeV)	139.57	134.97	547.85	493.67	497.61	

$$L_{ZH} = \frac{1}{\Lambda^2} \text{Tr} \{ \bar{H}_a Z_b^{\mu\nu} [k_1^Z \{ D_\mu, D_\nu \} A_\lambda + k_2^Z (D_\mu D_\lambda A_\nu + D_\nu D_\lambda A_\mu) ]_{ba} \gamma^\lambda \gamma_5 \} + \text{H.c.} \quad (15)$$

In these equations  $D_\mu = \partial_\mu + V_\mu$ ,  $\{D_\mu, D_\nu\} = D_\mu D_\nu + D_\nu D_\mu$  and  $\{D_\mu, D_\nu D_\rho\} = D_\mu D_\nu D_\rho + D_\mu D_\rho D_\nu + D_\nu D_\mu D_\rho + D_\nu D_\rho D_\mu + D_\rho D_\mu D_\nu + D_\rho D_\nu D_\mu$ .  $\Lambda$  is the chiral symmetry breaking scale taken as 1 GeV.  $g_{HH}$ ,  $g_{SH}$ ,  $g_{TH}$ ,  $g_{YH} = k_1^Y + k_2^Y$  and  $g_{ZH} = k_1^Z + k_2^Z$  are the strong coupling constants involved. The above equations describe the interactions of higher excited charm states to the ground state positive and negative parity charm mesons along with the emission of light pseudo-scalar mesons ( $\pi$ ,  $\eta$ ,  $K$ ). Using the Lagrangians  $L_{HH}$ ,  $L_{SH}$ ,  $L_{TH}$ ,  $L_{YH}$ ,  $L_{ZH}$ , the two body strong decays of  $Q\bar{q}$  heavy-light charm mesons are given as  $(0^-, 1^-) \rightarrow (0^-, 1^-) + M$

$$\Gamma(1^- \rightarrow 1^-) = C_M \frac{g_{HH}^2 M_f P_M^3}{3\pi f_\pi^2 M_i} \quad (16)$$

$$\Gamma(1^- \rightarrow 0^-) = C_M \frac{g_{HH}^2 M_f P_M^3}{6\pi f_\pi^2 M_i} \quad (17)$$

$$\Gamma(0^- \rightarrow 1^-) = C_M \frac{g_{HH}^2 M_f P_M^3}{2\pi f_\pi^2 M_i} \quad (18)$$

$$(0^+, 1^+) \rightarrow (0^-, 1^-) + M$$

$$\Gamma(1^+ \rightarrow 1^-) = C_M \frac{g_{SH}^2 M_f (P_M^2 + m_M^2) P_M}{2\pi f_\pi^2 M_i} \quad (19)$$

$$\Gamma(0^+ \rightarrow 0^-) = C_M \frac{g_{SH}^2 M_f (P_M^2 + m_M^2) P_M}{2\pi f_\pi^2 M_i} \quad (20)$$

$$(1^+, 2^+) \rightarrow (0^-, 1^-) + M$$

$$\Gamma(2^+ \rightarrow 1^-) = C_M \frac{2g_{TH}^2 M_f P_M^5}{5\pi f_\pi^2 \Lambda^2 M_i} \quad (21)$$

$$\Gamma(2^+ \rightarrow 0^-) = C_M \frac{4g_{TH}^2 M_f P_M^5}{15\pi f_\pi^2 \Lambda^2 M_i} \quad (22)$$

$$\Gamma(1^+ \rightarrow 1^-) = C_M \frac{2g_{TH}^2 M_f P_M^5}{3\pi f_\pi^2 \Lambda^2 M_i} \quad (23)$$

$$(2^-, 3^-) \rightarrow (0^-, 1^-) + M$$

$$\Gamma(2^- \rightarrow 1^-) = C_M \frac{4g_{YH}^2 M_f}{15\pi f_\pi^2 \Lambda^4 M_i} [P_M^7] \quad (24)$$

$$\Gamma(3^- \rightarrow 0^-) = C_M \frac{4g_{YH}^2 M_f}{35\pi f_\pi^2 \Lambda^4 M_i} [P_M^7] \quad (25)$$

$$\Gamma(3^- \rightarrow 1^-) = C_M \frac{16g_{YH}^2 M_f}{105\pi f_\pi^2 \Lambda^4 M_i} [P_M^7] \quad (26)$$

$$(2^+, 3^+) \rightarrow (0^-, 1^-) + M$$

$$\Gamma(2^+ \rightarrow 1^-) = C_M \frac{8g_{ZH}^2 M_f}{75\pi f_\pi^2 \Lambda^4 M_i} [P_M^5 (m_M^2 + P_M^2)] \quad (27)$$

$$\Gamma(2^+ \rightarrow 0^-) = C_M \frac{4g_{ZH}^2 M_f}{25\pi f_\pi^2 \Lambda^4 M_i} [P_M^5 (m_M^2 + P_M^2)] \quad (28)$$

$$\Gamma(3^+ \rightarrow 1^-) = C_M \frac{4g_{ZH}^2 M_f}{25\pi f_\pi^2 \Lambda^4 M_i} [P_M^5 (m_M^2 + P_M^2)] \quad (29)$$

In the above decay widths,  $M_i$  and  $M_f$  stands for initial and final meson mass,  $p_M$  and  $m_M$  are the final momentum and mass of the light pseudoscalar meson respectively. The coefficient  $C_{\pi^\pm}$ ,  $C_{K^\pm}$ ,  $C_{K^0}$ ,  $C_{\bar{K}^0} = 1$ ,  $C_{\pi^0} = \frac{1}{2}$ , and  $C_\eta = \frac{2}{3}$  or  $\frac{1}{6}$ . Different values of  $C_\eta$  corresponds to the initial state being  $c\bar{u}$ ,  $c\bar{d}$ , or  $c\bar{s}$  respectively. All hadronic coupling constants depends on the radial quantum number. For the decay within  $n = 1$  they are notated as  $g_{HH}$ ,  $g_{SH}$  etc, and the decay from  $n = 2$  to  $n = 1$  they are represented by  $\tilde{g}_{HH}^2$ ,  $\tilde{g}_{SH}^2$ . Higher order corrections for spin and flavor violation of order  $\frac{1}{m_Q}$  are excluded to avoid new unknown coupling constants. Equations (16)–(29) shows that the decay width of any state depends on the initial and final meson masses, their strong coupling constants, pion decay constant, energy scale  $\Lambda$ , mass and momentum of light pseudo-scalar mesons. Unknown coupling constants in these widths, can either be theoretically predicted or can be determined indirectly from the known experimental values of the decay widths. Theoretically, lattice QCD [28], QCD sum rules [29] have successfully predicted some of these coupling constants. The numerical masses of various mesons used in the calculation are listed in Table II.

### III. NUMERICAL ANALYSIS

Assigning a proper  $J^P$  to the experimentally available states are essential, as it helps in retrieving many properties

TABLE III. Strong decay width of newly observed charm mesons  $D_2^*(2460)$ ,  $D_0(2560)$ ,  $D_2(2740)$ ,  $D_1^*(2680)$ ,  $D_3^*(2760)$ ,  $D_1(3000)$ , and  $D_0^*(3000)$ . Ratio in 5th column represents the  $\hat{\Gamma} = \frac{\Gamma}{\Gamma(D_j^* \rightarrow D^{*+}\pi^-)}$  for the mesons. Fraction gives the percentage of the partial decay width with respect to the total decay width.

State	$nL_S J^P$	Decay channel	Decay Width(MeV)	Ratio	Fraction	Experimental value(MeV)	
$D_2^*(2460)$	$1P_{3/2}2^+$	$D^{*+}\pi^-$	$56.55g_{TH}^2$	1	20.05		
		$D^{*+}\pi^0$	$29.76g_{TH}^2$	0.52	10.55		
		$D^{*+}\eta$	-	-	0		
		$D^+\pi^-$	$128.40g_{TH}^2$	2.27	45.52		
		$D^+\pi^0$	$67.06g_{TH}^2$	1.18	23.77		
		$D^+\eta$	$0.26g_{TH}^2$	0	0		
		Total	$282.04g_{TH}^2$				$47.00 \pm 0.80$ [1]
$D_0(2560)$	$2S_{1/2}0^-$	$D^{*+}\pi^-$	$867.32g_{HH}^2$	1	65.99		
		$D^{*+}\pi^0$	$443.03g_{HH}^2$	0.51	33.71		
		$D^{*+}\eta$	$3.858g_{HH}^2$	0	0.29		
		Total	$1314.22g_{HH}^2$				$177.40 \pm 17.80$ [2]
$D_1^*(2680)$	$2S_{1/2}1^-$	$D^{*+}\pi^-$	$889.34g_{HH}^2$	1	32.41		
		$D^{*+}\pi^0$	$4451.87g_{HH}^2$	0.50	16.56		
		$D^{*+}\eta$	$31.07g_{HH}^2$	0.03	1.13		
		$D_s^{*+}K^-$	$78.40g_{HH}^2$	0.08	2.87		
		$D^+\pi^-$	$682.53g_{HH}^2$	0.76	25.01		
		$D^+\pi^0$	$346.56g_{HH}^2$	0.38	12.70		
		$D^+\eta$	$48.05g_{HH}^2$	0.05	1.76		
		$D_s^+K^-$	$200.49g_{HH}^2$	0.22	7.34		
		Total	$2728.35g_{HH}^2$				$186.70 \pm 8.50$ [1]
		$D_2(2740)$	$1D_{5/2}2^-$	$D^{*+}\pi^-$	$127.35g_{YH}^2$		1
$D^{*+}\pi^0$	$65.96g_{YH}^2$			0.51	33.55		
$D^{*+}\eta$	$1.30g_{YH}^2$			0.01	0.97		
$D_s^{*+}K^-$	$1.92g_{YH}^2$			0.01	0.97		
Total	$196.55g_{YH}^2$					$73.20 \pm 13.40$ [2]	
$D_3^*(2760)$	$1D_{5/2}3^-$	$D^{*+}\pi^-$	$100.15g_{YH}^2$	1	21.10		
		$D^{*+}\pi^0$	$51.73g_{YH}^2$	0.51	10.90		
		$D^{*+}\eta$	$1.53g_{YH}^2$	0.01	0.32		
		$D_s^{*+}K^-$	$2.88g_{YH}^2$	0.02	0.60		
		$D^+\pi^-$	$191.14g_{YH}^2$	1.90	40.28		
		$D^+\pi^0$	$98.82g_{YH}^2$	0.98	20.82		
		$D^+\eta$	$7.05g_{YH}^2$	0.07	1.48		
		$D_s^+K^-$	$21.14g_{YH}^2$	0.21	4.45		
		Total	$474.47g_{YH}^2$				$95.30 \pm 9.60$ [1]
		$D_1(3000)$	$2P_{1/2}1^+$	$D^{*+}\pi^-$	$3325.52g_{SH}^2$		1
$D^{*+}\pi^0$	$1674.26g_{SH}^2$			0.50	21.12		
$D^{*+}\eta$	$516.82g_{SH}^2$			0.15	6.52		
$D_s^{*+}K^-$	$2408.76g_{SH}^2$			0.72	30.39		
Total	$7925.36g_{SH}^2$					$188.10 \pm 44.60$ [2]	
$D_0^*(3000)$	$2P_{1/2}0^+$	$D^+\pi^-$	$2315.81g_{SH}^2$	0.50	20.26		
		$D^+\pi^0$	$4598.65g_{SH}^2$	1	40.24		
		$D^+\eta$	$748.382g_{SH}^2$	0.16	6.54		
		$D_s^+K^-$	$3763.23g_{SH}^2$	0.81	32.93		
		Total	$11426.10g_{SH}^2$				$110.50 \pm 11.50$ [2]

TABLE IV. Value of various coupling constants obtained in the literature.

Coupling constant	Our calculation	Work in [26]	Work in [9]
$g_{TH}$	$0.40 \pm 0.01$	$0.43 \pm 0.05$	$0.43 \pm 0.01$
$\tilde{g}_{HH}$	$0.31 \pm 0.05$	$0.14 \pm 0.03$	$0.28 \pm 0.01$
$g_{YH}$	$0.61 \pm 0.05$	$0.53 \pm 0.13$	$0.42 \pm 0.02$
$\tilde{g}_{SH}$	$0.12 \pm 0.03$	...	...

like decay width, strong coupling constant, branching ratios, etc. of these states. In this paper, we reanalyze the previously available theoretical and experimental data on the charm states  $D_J^*(2460)$ ,  $D_J(2560)$ ,  $D_J(2740)$ ,  $D_J^*(2680)$ ,  $D_J^*(2760)$ ,  $D_J(3000)$ , and  $D_J^*(3000)$ . This analysis is based on the available information on J values taken from LHCb in 2016. Hence we identify these states as:

$$D_J^*(2460) = (2^+)_{\frac{3}{2}} \quad \text{with } n = 1, L = 1, \quad (30)$$

$$(D_J(2560), D_J^*(2680)) = (0^-, 1^-)_{\frac{1}{2}} \quad \text{with } n = 2, L = 0, \quad (31)$$

$$(D_J(2740), D_J^*(2760)) = (2^-, 3^-)_{\frac{3}{2}} \quad \text{with } n = 1, L = 2, \quad (32)$$

$$D_J^*(3000), (D_J(3000)) = (0^+, 1^+)_{\frac{1}{2}} \quad \text{with } n = 2, L = 1. \quad (33)$$

The numerical value of the partial decay widths and the ratios for the charm states  $D_2^*(2460)$ ,  $D_0(2560)$ ,  $D_2(2740)$ ,

$D_1^*(2680)$ ,  $D_3^*(2760)$ ,  $D_1(3000)$ , and  $D_0^*(3000)$  are listed in Table III. We equate the calculated decay widths with the experimental data in Table III to obtain the coupling constants which are listed in Table IV. The couplings  $\tilde{g}_{HH}$ ,  $\tilde{g}_{SH}$  are obtained by averaging the values obtained from  $(D_0(2560), D_1^*(2680))$  and  $(D_1(3000), D_0^*(3000))$  respectively. We have neglected the small value of the coupling  $g_{YH} = 0.10$ , in comparison with its other theoretically predicted values [26]. The range in the coupling constant, comes from the error-bar in the experimental mass and decay width values.

On the basis of the theoretically predicted masses [8,13,15–18],  $D_2^*(3000)$  is assumed to belong to either  $1F_{\frac{5}{2}}(2^+)$  or  $2P_{\frac{3}{2}}(2^+)$  state. The partial and the total decay widths for both these states are shown in Table V. To clear out the  $J^P$  state for  $D_2^*(3000)$  between  $1F(2^+)$  and  $2P(2^+)$ , we have observed the BR =  $\frac{\Gamma(D_2^*(3000) \rightarrow D^*\pi)}{\Gamma(D_2^*(3000) \rightarrow D\pi)}$  for both these states with their masses. The graph for the BR with the masses for the two  $J^P$  states are shown in Fig. 1. The graph 1(a) shows, the value of BR for  $2P_{\frac{3}{2}}(2^+)$  is equal to 1.06 corresponding to the mass 3214 MeV, predicting  $D^*\pi$  to be dominant mode as compared to  $D\pi$ . And the graph 1(b) depicts the value of BR for  $1F_{\frac{5}{2}}(2^+)$  state to be 0.40 for mass 3214 MeV, predicting  $D\pi$  to be the dominant mode. Since the  $D^*\pi$  decay channel for  $D_2^*(3000)$  is experimentally suppressed, therefore  $1F(2^+)$  is considered to be the most favorable  $J^P$  for  $D_2^*(3000)$ .

Along with the decay channels mentioned in Table V,  $D_2^*(3000)$  being  $1F(2^+)$  also decays to  $1P(1^+)$ ,  $1P'(1^+)$ ,  $1D(2^-)$  and  $1D'(2^-)$  states along with pseudoscalar

TABLE V. Strong decay width of  $D_2^*(3000)$  with the  $J^P$  assignment as  $1F_{\frac{5}{2}}(2^+)$  and  $2P_{\frac{3}{2}}(2^+)$ . Ratio represents  $\hat{\Gamma} = \frac{\Gamma}{\Gamma(D_2^*(3000) \rightarrow D^*\pi)}$  for  $D_2^*(3000)$ . Fraction gives the percentage of the particular decay width with respect to the total decay width.

$nLs_J P$	Decay channel	Decay Width(MeV)	Ratio	Fraction	Experimental Value(MeV)
$1F_{5/2}(2^+)$	$D^{*+}\pi^-$	$1046.53g_{ZH}^2$	1	13.60	$186 \pm 38$
	$D^{*+}\pi^0$	$531.26g_{ZH}^2$	0.50	6.90	
	$D^{*+}\eta$	$109.14g_{ZH}^2$	0.10	1.41	
	$D_s^{*+}K^-$	$422.87g_{ZH}^2$	0.40	5.49	
	$D^+\pi^-$	$2630.35g_{ZH}^2$	2.51	34.20	
	$D^+\pi^0$	$1338.14g_{ZH}^2$	1.27	17.39	
	$D^+\eta$	$307.35g_{ZH}^2$	0.29	3.99	
	$D_s^+K^-$	$1304.87g_{ZH}^2$	1.24	16.96	
	Total				
$2P_{3/2}(2^+)$	$D^{*+}\pi^-$	$4075.15\tilde{g}_{TH}^2$	1	24.69	$186 \pm 38$
	$D^{*+}\pi^0$	$2060.89\tilde{g}_{TH}^2$	0.50	12.48	
	$D^{*+}\eta$	$387.99\tilde{g}_{TH}^2$	0.09	2.35	
	$D_s^{*+}K^-$	$1754.17\tilde{g}_{TH}^2$	0.43	10.62	
	$D^+\pi^-$	$1952.32\tilde{g}_{TH}^2$	0.94	23.36	
	$D^+\pi^0$	$3856.13\tilde{g}_{TH}^2$	0.47	11.83	
	$D^+\eta$	$413.76\tilde{g}_{TH}^2$	0.10	2.50	
	$D_s^+K^-$	$2002.65\tilde{g}_{TH}^2$	0.49	12.13	
	Total				

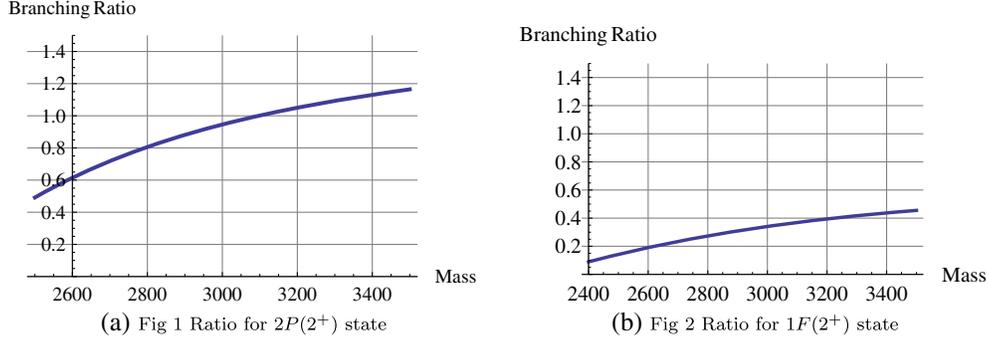


FIG. 1. Branching ratio  $\Gamma(D_2^*(3000)) \rightarrow \frac{D_s^* \pi}{D\pi}$  for two possible  $J^P$ 's for  $D_2^*(3000)$  state.

mesons ( $\pi, \eta, K$ ). Since these decays occur via relative F-wave and D-wave, the contribution of their phase space to the decay widths are negligible. And therefore, these channels are suppressed. Considering the decay channels mentioned in Table V to be the only dominant decay modes, the total decay width of  $D_2^*(3000)$  comes out to be  $7690.53g_{ZH}^2$ . Along with the partial decay widths, Table V shows the ratio  $\hat{\Gamma} = \frac{\Gamma}{\Gamma(D_2^*(3000) \rightarrow D^{*+}\pi^-)}$  and the branching fraction for the decay channels of  $D_2^*(3000)$  state. The results in Table V reveal that, for  $D_2^*(3000)$  state  $D^{*+}\pi^-$  and  $D^0\pi^0$  are the main decay modes as compared to the  $D^{*+}\pi^-$  mode. The decay width obtained in this work is finally compared with the experimental result, and the coupling constant  $g_{ZH}$  is obtained as

$$g_{ZH} = 0.15 \pm 0.02. \quad (34)$$

The information on the value of coupling  $g_{ZH}$  is very limited in the literature, so extracting its value will be useful for the theory, in finding partial and the total decay widths of unobserved charm states  $D(1^1F_3)$ ,  $D_s(1^1F_3)$ , and  $D_s(1^3F_2)$ . Until now, the experimental information on the strong decay widths of  $D(1^1F_3)$ ,  $D_s(1^1F_3)$ , and  $D_s(1^3F_2)$  states is unavailable, so the prediction of their partial and total decay widths will be a motivation for future experiments. Mass of  $D(1^1F_3)$  is predicted to be  $3099 \pm 25$  MeV Refs. [13,16–18]. OZI allowed decay channels of  $D(1^1F_3)$  are listed in the Table VI. Column 4 of the Table VI gives the ratio of the partial decay widths for  $D(1^1F_3)$  with respect to its partial decay width  $D^{*+}\pi^-$ . Apart from the decay channels listed in Table VI,  $D(1^1F_3)$  also decays to P-wave charm meson states through the light pseudoscalar meson, the decay occurs via F-wave, and due to small phase space, these modes are suppressed and not considered in the present work. From the listed decay channels,  $D^{*+}\pi^-$  comes out to be the dominant decay mode for  $D(1^1F_3)$  with branching fraction 51.84%. Hence, the decay channel  $D^{*+}\pi^-$  is suitable for the experimental search for the missing charm state  $D(1^1F_3)$  in future. Using the value of the coupling constant  $g_{ZH}$  obtained from Eq. (34),

the total decay width of the charm state  $D(1^1F_3)$  is obtained as 55.40 MeV. The partial decay widths predicted in this paper are comparable with the values predicted in Ref. [8].

We have also studied the decay behavior of strange partners of  $D_2^*(3000)$  and  $D_3(3099)$  charm states, i.e.,  $(D_{s2}^*, D_{s3}) = (2^+, 3^+)_{\frac{3}{2}}$  with  $n = 1$  and  $L = 3$ . Masses for these strange charm states are taken as  $3220.66 \pm 9$  MeV and  $3232.50 \pm 33$  MeV from the theoretical work [13,16–18]. OZI allowed two body strong decay channels of these two states are also listed in Table VI. For  $D_{s2}^*$  state, we observe,  $D^0K^-$  to be the dominant decay mode with branching fraction 25.94% and for  $D_{s3}$  state,  $D^{*0}K^-$  to be

TABLE VI. Strong decay width of  $D(1^1F_3)$ ,  $D_s(1^1F_3)$ , and  $D_s(1^3F_2)$  charm mesons being the spin and strange partners of  $1F(2^+)$ . Ratio depicts the value  $\hat{\Gamma} = \frac{\Gamma}{\Gamma(D_j^* \rightarrow D^{*+}\pi^-)}$  for  $D(1^1F_3)$  and  $\hat{\Gamma} = \frac{\Gamma}{\Gamma(D_{s,j}^* \rightarrow D^{*0}K^+)}$  for  $D_s(1^1F_3)$  and  $D_s(1^3F_2)$ . Last column gives the branching fraction for these states.

$nLs_l J^P$	Decay channel	Decay Width(MeV)	Ratio	Branching Fraction
$1F_{5/2}(3^+)$	$D^{*+}\pi^-$	29.03	1	51.84
	$D^{*+}\pi^0$	14.78	0.50	26.38
	$D^{*+}\eta$	2.57	0.09	4.75
	$D_s^{*+}K^-$	9.00	0.32	17.01
	Total	55.40	...	100
$1F_{s5/2}(3^+)$	$D^{*+}K^0$	42.41	0.97	35.15
	$D^{*0}K^+$	43.38	1	35.95
	$D^{*+}\eta$	14.81	0.34	12.27
	$D_s^{*+}\pi^0$	20.04	0.46	16.61
	Total	120.66	...	100
$1F_{s5/2}(2^+)$	$D^{*+}K^0$	16.61	0.97	9.29
	$D^{*0}K^+$	17.00	1	9.50
	$D^{*+}\eta$	5.78	0.34	3.23
	$D_s^{*+}\pi^0$	7.86	0.46	4.39
	$D^+K^0$	45.30	2.66	25.33
	$D^0K^+$	46.37	2.72	25.94
	$D^+\eta$	19.37	1.08	10.29
	$D_s^+\pi^0$	21.47	1.26	12.01
	Total	178.79	...	100

the dominant mode with branching fraction 35.95%. These strange states also decays to P-wave charm meson states, but due to small phase space, these modes are suppressed in our study. Using above  $g_{ZH}$ , the total decay width for  $D_{s2}^*$  comes out to be 178.79 MeV and for  $D_{s3}$  it is 120.66 MeV. Taking sum of the partial decay widths to be the total decay width for these strange states,  $D_{s2}^*$  state is observed to be a broader state as compared to its spin partner  $D_{s3}$ .

#### IV. CONCLUSION

In the present article, we have examined the charm states  $D_J^*(2460)$ ,  $D_J(2560)$ ,  $D_J^*(2680)$ ,  $D_J(2740)$ ,  $D_J^*(2760)$ ,  $D_J(3000)$ , and  $D_J^*(3000)$  with  $J^P$  states  $1P_{\frac{3}{2}}2^+$ ,  $2S_{\frac{3}{2}}0^-$ ,  $2S_{\frac{1}{2}}1^-$ ,  $1D_{\frac{3}{2}}2^-$ ,  $1D_{\frac{3}{2}}3^-$ ,  $2P_{\frac{1}{2}}1^+$ , and  $2P_{\frac{1}{2}}0^+$  respectively. Here we have used the HQET Lagrangian at the leading order approximation, and studied their two body strong decay behavior with the emission of light pseudoscalar mesons ( $\pi, \eta, K$ ). We have computed the branching ratios and the coupling constants  $g_{TH}$ ,  $\tilde{g}_{HH}$ ,  $g_{YH}$ ,  $\tilde{g}_{SH}$  for the above states,

that can be useful for the future experimental data to compare with.

Along with this, we have also tentatively identified the  $J^P$  for  $D_2^*(3000)$  charm meson which is recently observed by the LHCb in 2016 [1]. We studied the branching ratio for this state and concluded its  $J^P$  to be  $1F_{\frac{3}{2}}2^+$ , and correspondingly obtained the coupling constant  $g_{ZH} \approx 0.15$ . The obtained coupling constant helps in calculating the strong decay channels for the experimentally missing  $D(1^1F_3)$ ,  $D_S(1^1F_3)$ , and  $D_S(1^3F_2)$  states. Thus, the observation of  $D_2^*(3000)$  as  $1F_{\frac{3}{2}}2^+$  has opened a window to investigate the higher excitations of charm mesons at the LHCb, BABAR, BESIII.

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