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Two-form supergravity, superstring couplings, and Goldstino superfields in three dimensions

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We develop off-shell formulations for $\mathcal{N}=1$ and $\mathcal{N}=2$ anti-de Sitter supergravity theories in three spacetime dimensions that contain gauge two-forms in the auxiliary field sector. These formulations are shown to allow consistent couplings of supergravity to the Green-Schwarz superstring with $\mathcal{N}=1$ or $\mathcal{N}=2$ spacetime supersymmetry. In addition to being κ-symmetric, the Green-Schwarz superstring actions constructed are also invariant under super-Weyl transformations of the target space. We also present a detailed study of models for spontaneously broken local supersymmetry in three dimensions obtained by coupling the known off-shell $\mathcal{N}=1$ and $\mathcal{N}=2$ supergravity theories to nilpotent Goldstino superfields.

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I. INTRODUCTION

The Green-Schwarz superstring action with $\mathcal{N}=1$ or $\mathcal{N}=2$ supersymmetry [1] exists for spacetime dimensions $D=3$, 4, 6 and 10. However, its light-cone quantization breaks Lorentz invariance unless either $D=10$ (see, e.g., [2]), which corresponds to critical superstring theory, or $D=3$ [3,4]. Due to the exceptional status of the $D=3$ case, it is of interest to study in more detail three-dimensional (3D) superstring actions in supergravity backgrounds. In order for such a coupling to supergravity to be consistent, the superstring action must possess a local fermionic invariance (known as the κ-symmetry) which was first discovered in the cases of massive [5,6] and massless [7] superparticles.[1] The κ-symmetry, in its turn, requires the superstring action to include a Wess-Zumino term associated with a closed super three-form in curved superspace such that (i) it is the field strength of a gauge super two-form, and (ii) it reduces to a nonvanishing invariant super three-form in the flat superspace limit. The latter requirement means that only certain supergravity formulations are suitable to describe string propagation in curved superspace. The constraints on the geometry of curved $D=3$, 4, 6, 10 superspace, which are required for the coupling of supergravity to the Green-Schwarz superstring, were studied about 30 years ago [10–13]. Nevertheless, there still remain some open questions and unexplored cases, as can be seen from the recent work by Tseytlin and Wulff [14] that determined the precise constraints imposed on the 10D target superspace geometry by the requirement of classical κ-symmetry of the Green-Schwarz superstring. In regard to the 3D case, it should be kept in mind that at the time when Refs. [12,13] were written, those off-shell formulations for $\mathcal{N}=1$ and $\mathcal{N}=2$ supergravity theories, which are suitable to describe consistent superstring propagation, had not been described in the literature. One such theory, the so-called $\mathcal{N}=2$ two-form supergravity, was formulated six years ago [15]. A new $\mathcal{N}=2$ supergravity theory will be given in the present paper.

The present work aims at developing (i) $\mathcal{N}=1$ and $\mathcal{N}=2$ anti-de Sitter (AdS) supergravity theories that contain gauge two-forms in the auxiliary field sector; (ii) consistent couplings of these supergravity theories to the Green-Schwarz superstring with $\mathcal{N}=1$ or $\mathcal{N}=2$ supersymmetry; and (iii) models for spontaneously broken 3D supergravity obtained by coupling the off-shell $\mathcal{N}=1$ or $\mathcal{N}=2$ supergravity theories to Goldstino superfields. The first two goals are related to the above discussion. As to point (iii), it requires additional comments.

In the last 3 years, there has been considerable interest in models for spontaneously broken $\mathcal{N}=1$ local supersymmetry in four dimensions [16–27], including the models for off-shell supergravity coupled to nilpotent Goldstino superfields. One of the reasons for this interest is that a positive contribution to the cosmological constant is generated once the local supersymmetry becomes spontaneously broken. For instance, if the supergravity multiplet is coupled to an irreducible Goldstino superfield [20,25,28–30] (with the Volkov-Akulov Goldstino [31,32] being the only independent component field of the superfield), a *universal* positive contribution to the cosmological constant is generated,[2]

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[1]For reviews of various aspects of the κ-symmetry, see, e.g., [8,9].

[2]The gravitino becomes massive in accordance with the super-Higgs effect [33–35].

which is proportional to f^2, with the parameter f setting the scale of supersymmetry breaking. The same positive contribution is generated by the reducible Goldstino superfields used in the models studied in [18,19,26].[3] There is one special reducible Goldstino superfield, the nilpotent three-form multiplet introduced in [24,27], which yields a dynamical positive contribution to the cosmological constant.

Since our Universe is characterized by a positive cosmological constant, and a theoretical explanation for this positivity is required, 4D supergravity theories with nilpotent Goldstino superfields deserve further studies. In this respect, it is also of some interest to construct models for spontaneously broken local 3D $\mathcal{N}=1$ and $\mathcal{N}=2$ supersymmetry that are obtained by coupling off-shell 3D supergravity to nilpotent superfields. This is one of the objectives of the present work.

This paper is organized as follows. Sections II and III provide thorough discussions of the $\mathcal{N}=1$ and $\mathcal{N}=2$ off-shell supergravity theories, respectively. Section IV describes consistent couplings of the two-form supergravity theories to the Green-Schwarz superstring with $\mathcal{N}=1$ or $\mathcal{N}=2$ spacetime supersymmetry. The nilpotent Goldstino superfields and their couplings to various off-shell supergravity theories are presented in Sec. V. Here we introduce only those reducible Goldstino superfields that are defined in the presence of conformal supergravity without making use of any conformal compensator. Section VI contains concluding comments and a brief discussion of the results obtained. The main body of the paper is accompanied by three technical appendices which are devoted to the analysis of the component structure of several Goldstino superfield models in the flat superspace limit.

II. TWO-FORM MULTIPLET IN $\mathcal{N}=1$ SUPERGRAVITY

In this section we describe two off-shell formulations for $\mathcal{N}=1$ AdS supergravity, with $4+4$ off-shell degrees of freedom, which differ from each other by their auxiliary fields. One of them is known since the late 1970s (see [36] for a review), and its auxiliary field is a scalar. The other formulation is obtained by replacing the auxiliary scalar field with the field strength of a gauge two-form, which requires the use of a different compensating supermultiplet. As was pointed out in [12,13], the latter formulation is required for consistent coupling to the Green-Schwarz superstring. However, the technical details of this formulation have not been described in the literature, to the best of our knowledge.

[3] The notion of irreducible and reducible Goldstino superfields was introduced in [25].

We follow the notation and make use of the results of [37]. Every supergravity theory will be realized as a super-Weyl invariant coupling of conformal supergravity to a compensating supermultiplet.

A. Conformal supergravity

Consider a curved $\mathcal{N}=1$ superspace, $\mathcal{M}^{3|2}$, parametrized by local real coordinates $z^M=(x^m,\theta^\mu)$, with $m=0$, 1, 2 and $\mu=1$, 2, of which x^m are bosonic and θ^μ fermionic. We introduce a preferred basis of one-forms $E^A=(E^a,E^\alpha)$ and its dual basis $E_A=(E_a,E_\alpha)$,

$$E^A=\mathrm{d}z^M E_M{}^A, \qquad E_A=E_A{}^M\partial_M, \tag{2.1}$$

which will be referred to as the supervielbein and its inverse, respectively.

The superspace structure group is $\mathrm{SL}(2,\mathbb{R})$, the double cover of the connected Lorentz group $\mathrm{SO}_0(2,1)$. The covariant derivatives have the form

$$\mathcal{D}_A=(\mathcal{D}_a,\mathcal{D}_\alpha)=E_A+\Omega_A, \tag{2.2}$$

where

$$\Omega_A=\frac{1}{2}\Omega_A{}^{bc}M_{bc}=-\Omega_A{}^bM_b=\frac{1}{2}\Omega_A{}^{\beta\gamma}M_{\beta\gamma} \tag{2.3}$$

is the Lorentz connection. The Lorentz generators with two vector indices ($M_{ab}=-M_{ba}$), one vector index (M_a), and two spinor indices ($M_{\alpha\beta}=M_{\beta\alpha}$) are related to each other by the rules $M_a=\frac{1}{2}\varepsilon_{abc}M^{bc}$ and $M_{\alpha\beta}=(\gamma^a)_{\alpha\beta}M_a$. These generators act on a vector V_c and a spinor Ψ_γ as follows:

$$M_{ab}V_c=2\eta_{c[a}V_{b]}, \qquad M_{\alpha\beta}\Psi_\gamma=\varepsilon_{\gamma(\alpha}\Psi_{\beta)}. \tag{2.4}$$

The covariant derivatives are characterized by graded commutation relations

$$[\mathcal{D}_A,\mathcal{D}_B\}=T_{AB}{}^C\mathcal{D}_C+\frac{1}{2}R_{AB}{}^{cd}M_{cd}, \tag{2.5}$$

where $T_{AB}{}^C$ and $R_{AB}{}^{cd}$ are the torsion and curvature tensors, respectively. To describe supergravity, the covariant derivatives have to obey certain torsion constraints [36] such that the algebra (2.5) takes the form

$$\{\mathcal{D}_\alpha,\mathcal{D}_\beta\}=2\mathrm{i}\mathcal{D}_{\alpha\beta}-4\mathrm{i}\mathcal{S}M_{\alpha\beta}, \tag{2.6a}$$

$$[\mathcal{D}_a,\mathcal{D}_\beta]=(\gamma_a)_\beta{}^\gamma[\mathcal{S}\mathcal{D}_\gamma-C_{\gamma\delta\rho}M^{\delta\rho}] -\frac{2}{3}[\mathcal{D}_\beta\mathcal{S}\delta_a^c-2\varepsilon_{ab}{}^c(\gamma^b)_{\beta\gamma}\mathcal{D}^\gamma\mathcal{S}]M_c, \tag{2.6b}$$

$$[\mathcal{D}_a, \mathcal{D}_b] = \varepsilon_{abc}\bigg\{ \mathrm{i}\left[\frac{1}{2}(\gamma^c)_{\alpha\beta}C^{\alpha\beta\gamma} - \frac{2}{3}(\gamma^c)^{\beta\gamma}\mathcal{D}_\beta\mathcal{S}\right]\mathcal{D}_\gamma + \bigg[\frac{\mathrm{i}}{2}(\gamma^c)^{\alpha\beta}(\gamma^d)^{\gamma\delta}\mathcal{D}_{(\alpha}C_{\beta\gamma\delta)} + \left(\frac{2\mathrm{i}}{3}\mathcal{D}^2\mathcal{S} + 4\mathcal{S}^2\right)\eta^{cd}\bigg]M_d\bigg\}. \tag{2.6c}$$

Here the scalar $\mathcal{S}$ is real, while the symmetric spinor $C_{\alpha\beta\gamma} = C_{(\alpha\beta\gamma)}$ is *imaginary*. The dimension-2 Bianchi identities imply that

$$\mathcal{D}_\alpha C_{\beta\gamma\delta} = \mathcal{D}_{(\alpha}C_{\beta\gamma\delta)} - \mathrm{i}\varepsilon_{\alpha(\beta}\mathcal{D}_{\gamma\delta)}\mathcal{S} \Rightarrow \mathcal{D}^\gamma C_{\alpha\beta\gamma} = -\frac{4\mathrm{i}}{3}\mathcal{D}_{\alpha\beta}\mathcal{S}. \tag{2.7}$$

Throughout this section we make use of the definition $\mathcal{D}^2 \coloneqq \mathcal{D}^\alpha\mathcal{D}_\alpha$.

The definition of the torsion and curvature tensors, Eq. (2.5), can be recast in the language of superforms, which will be used in Sec. IV. Starting from the Lorentz connection Ω_A given by (2.3), we introduce the connection one-form

$$\Omega = E^C\Omega_C, \qquad \Omega V_A = \Omega_A{}^B V_B = E^C\Omega_{CA}{}^B V_B, \qquad V_A = (V_a, \Psi_\alpha). \tag{2.8}$$

Then the torsion and curvature two-forms are

$$T^C \coloneqq \frac{1}{2}E^B \wedge E^A T_{AB}{}^C = -\mathrm{d}E^C + E^B \wedge \Omega_B{}^C, \tag{2.9a}$$

$$R_C{}^D \coloneqq \frac{1}{2}E^B \wedge E^A R_{ABC}{}^D = \mathrm{d}\Omega_C{}^D - \Omega_C{}^E \wedge \Omega_E{}^D. \tag{2.9b}$$

The gauge group of conformal supergravity includes local transformations of the form

$$\delta_\mathcal{K}\mathcal{D}_A = [\mathcal{K}, \mathcal{D}_A], \qquad \mathcal{K} = \xi^C E_C + \frac{1}{2}K^{cd}M_{cd}, \tag{2.10}$$

with the gauge parameters $\xi^C(z)$ and $K^{bc}(z)$ obeying natural reality conditions but otherwise arbitrary. Here the supervector field $\xi = \xi^C E_C$ describes a general coordinate transformation, and K^{cd} a local Lorentz transformation. The transformation (2.10) acts on a tensor superfield T as follows:

$$\delta_\mathcal{K} T = \mathcal{K}T. \tag{2.11}$$

The algebra of covariant derivatives is invariant under super-Weyl transformations

$$\delta_\sigma\mathcal{D}_\alpha = \frac{1}{2}\sigma\mathcal{D}_\alpha + \mathcal{D}^\beta\sigma M_{\alpha\beta}, \tag{2.12a}$$

$$\delta_\sigma\mathcal{D}_a = \sigma\mathcal{D}_a + \frac{\mathrm{i}}{2}(\gamma_a)^{\gamma\delta}\mathcal{D}_\gamma\sigma\mathcal{D}_\delta + \varepsilon_{abc}\mathcal{D}^b\sigma M^c, \tag{2.12b}$$

with the parameter σ being a real unconstrained superfield, provided the torsion superfields transform as

$$\delta_\sigma\mathcal{S} = \sigma\mathcal{S} - \frac{\mathrm{i}}{4}\mathcal{D}^2\sigma, \qquad \delta_\sigma C_{\alpha\beta\gamma} = \frac{3}{2}\sigma C_{\alpha\beta\gamma} - \frac{1}{2}\mathcal{D}_{(\alpha\beta}\mathcal{D}_{\gamma)}\sigma. \tag{2.13}$$

The super-Weyl transformation of the vielbein is

$$\delta_\sigma E^a = -\sigma E^a, \tag{2.14a}$$

$$\delta_\sigma E^\alpha = -\frac{1}{2}\sigma E^\alpha - \frac{\mathrm{i}}{2}E^b(\gamma_b)^{\alpha\beta}\mathcal{D}_\beta\sigma. \tag{2.14b}$$

The gauge group of conformal supergravity is generated by the local transformations (2.10) and (2.12). Due to the super-Weyl invariance, the above geometry describes the Weyl multiplet of $\mathcal{N} = 1$ conformal supergravity [38], which consists of the vielbein $e_m{}^a(x)$ and the gravitino $\psi_m{}^\alpha(x)$ (no auxiliary fields).[4]

A tensor superfield T is said to be (super-Weyl) primary of weight w if its super-Weyl transformation law is

$$\delta_\sigma T = w\sigma T. \tag{2.15}$$

Such superfields will be of primary importance in what follows.

The action for conformal supergravity was constructed for the first time by van Nieuwenhuizen [38] using the $\mathcal{N} = 1$ superconformal tensor calculus. More recently, it was reformulated in superspace [39], as well as within the superform approach [39,40]. The interested reader is referred to these publications for the technical details.

B. Supersymmetric action

To construct a locally supersymmetric and super-Weyl invariant action [37], one needs a real scalar Lagrangian $\mathcal{L}$ that is super-Weyl primary of weight $+2$,

$$\delta_\sigma\mathcal{L} = 2\sigma\mathcal{L}. \tag{2.16}$$

The action is

$$S = \mathrm{i}\int \mathrm{d}^3x\mathrm{d}^2\theta E\mathcal{L}, \qquad E = \mathrm{Ber}(E_M{}^A). \tag{2.17}$$

The action is super-Weyl invariant, since the super-Weyl transformation of E proves to be $\delta_\sigma E = -2\sigma E$.

Instead of defining the action using the superspace integration, an alternative approach is to construct a

[4]The super-Weyl transformation of $\mathcal{S}$ implies that its lowest component $\mathcal{S}|_{\theta=0}$ is a pure gauge.

dimensionless super three-form $\Xi_3[\mathcal{L}]$ which is given in terms of $\mathcal{L}$ and possesses the following properties: (i) $\Xi_3[\mathcal{L}]$ is closed, $\mathrm{d}\Xi_3[\mathcal{L}]=0$; and (ii) $\Xi_3[\mathcal{L}]$ is super-Weyl invariant, $\delta_\sigma\Xi_3[\mathcal{L}]=0$.[5] Modulo an overall numerical factor, these conditions prove to completely determine $\Xi_3[\mathcal{L}]$ to be

$$\Xi_3[\mathcal{L}]=\frac{\mathrm{i}}{2}E^\gamma\wedge E^\beta\wedge E^a(\gamma_a)_{\beta\gamma}\mathcal{L} +\frac{1}{4}E^\gamma\wedge E^b\wedge E^a\varepsilon_{abc}(\gamma^c)_\gamma{}^\delta\mathcal{D}_\delta\mathcal{L} -\frac{1}{24}E^c\wedge E^b\wedge E^a\varepsilon_{abc}(\mathrm{i}\mathcal{D}^2+8\mathcal{S})\mathcal{L}. \tag{2.18}$$

This super three-form was originally constructed in [43,45]; however its super-Weyl invariance was first described in [39]. The action (2.17) is recast via $\Xi_3[\mathcal{L}]$ as follows:

$$S=\int_{\mathcal{M}^3}\Xi_3[\mathcal{L}], \tag{2.19}$$

where the integration is carried out over a spacetime $\mathcal{M}^3$ being homotopic to the bosonic body of the curved superspace $\mathcal{M}^{3|2}$ obtained by switching off the Grassmann variables.

C. AdS supergravity

Both AdS and Poincaré supergravity theories can be realized as super-Weyl invariant systems describing the coupling of conformal supergravity to a compensating multiplet. The standard choice for compensator is a nowhere vanishing scalar superfield φ, such that φ^{-1} exists, with the super-Weyl transformation

$$\delta_\sigma\varphi=\frac{1}{2}\sigma\varphi. \tag{2.20}$$

The action for $\mathcal{N}=1$ AdS supergravity is given by

$$S_{\rm SG}=-\frac{4}{\kappa}\mathrm{i}\int\mathrm{d}^3x\mathrm{d}^2\theta E\{\mathrm{i}\mathcal{D}^\alpha\varphi\mathcal{D}_\alpha\varphi-2\mathcal{S}\varphi^2+\lambda\varphi^4\}, \tag{2.21}$$

where κ is the gravitational coupling constant, and the parameter λ determines the cosmological constant. Setting $\lambda=0$ in (2.21) gives the action for $\mathcal{N}=1$ Poincaré supergravity.

The equation of motion for the compensator is

$$\mathbb{S}=\lambda,\qquad \mathbb{S}:=\varphi^{-3}\left(\frac{\mathrm{i}}{2}\mathcal{D}^2+\mathcal{S}\right)\varphi. \tag{2.22a}$$

For completeness we also give the equation of motion for the gravitational superfield (which is the $\mathcal{N}=1$ supersymmetric analog of the gravitational field)

$$\mathbb{C}_{\alpha\beta\gamma}=0,\qquad \mathbb{C}_{\alpha\beta\gamma}:=-\frac{1}{2}\varphi^{-1}(\mathcal{D}_{(\alpha\beta}\mathcal{D}_{\gamma)}-2C_{\alpha\beta\gamma})\varphi^{-2}. \tag{2.22b}$$

See [46] for the technical details. The specific feature of $\mathbb{S}$ and $\mathbb{C}_{\alpha\beta\gamma}$ is that they are super-Weyl invariant. Note that it is possible to choose a super-Weyl gauge in which $\varphi=1$ and, therefore, $\mathbb{S}$ and $\mathbb{C}_{\alpha\beta\gamma}$ coincide with $\mathcal{S}$ and $C_{\alpha\beta\gamma}$, respectively. In this gauge, Eqs. (2.22) describe, locally, the $\mathcal{N}=1$ AdS superspace [47].

The action (2.21) can readily be reduced to components. In the super-Weyl gauge $\varphi=1$ we obtain

$$S_{\rm SG}=\frac{1}{\kappa}\int\mathrm{d}^3xe\left\{\frac{1}{2}\mathcal{R}-4S^2+8S\lambda\right\}+\text{fermions},\qquad e=\det(e_m{}^a), \tag{2.23}$$

where $e_m{}^a(x):=E_m{}^a|_{\theta=0}$ and $S(x):=\mathcal{S}|_{\theta=0}$. Integrating out the auxiliary field S turns the action into

$$S_{\rm SG}=\frac{1}{\kappa}\int\mathrm{d}^3xe\left\{\frac{1}{2}\mathcal{R}-\Lambda_{\rm AdS}\right\}+\text{fermions},\qquad \Lambda_{\rm AdS}=-4\lambda^2. \tag{2.24}$$

D. Two-form supergravity

In this section we introduce a variant formulation for $\mathcal{N}=1$ AdS supergravity which is obtained by replacing the conformal compensator φ^4 with a two-form multiplet.[6]

Let us first consider a massless two-form multiplet coupled to conformal supergravity. It is described by a real scalar superfield defined by

$$L=\mathcal{D}^\alpha\Lambda_\alpha, \tag{2.25}$$

where the prepotential Λ_α is a primary real spinor superfield of dimension 3/2,

$$\delta_\sigma\Lambda_\alpha=\frac{3}{2}\sigma\Lambda_\alpha. \tag{2.26}$$

This super-Weyl transformation implies that L is primary of dimension 2,

$$\delta_\sigma L=2\sigma L. \tag{2.27}$$

[5] See [41–44] for the construction of locally supersymmetric invariants in D spacetime dimensions by using closed super D-forms.

[6] In the case of Minkowski superspace, the two-form multiplet was described in [36].

The superfield L defined by (2.25) is a gauge-invariant field strength with respect to gauge transformations of the form

$$\delta_\zeta \Lambda_\alpha = \frac{\mathrm{i}}{2}\mathcal{D}^\beta \mathcal{D}_\alpha \zeta_\beta + 2\mathcal{S}\zeta_\alpha, \qquad \mathcal{D}^\alpha \delta_\zeta \Lambda_\alpha = 0, \qquad (2.28)$$

where the gauge parameter ζ_α is an arbitrary real spinor superfield. The gauge invariance of L follows from the identity

$$\mathcal{D}^\beta \mathcal{D}_\alpha \mathcal{D}_\beta = 4\mathrm{i}\mathcal{S}\mathcal{D}_\alpha - \frac{8\mathrm{i}}{3}(\mathcal{D}^\beta \mathcal{S})M_{\alpha\beta} - 2\mathrm{i}C_{\alpha\beta\gamma}M^{\beta\gamma}. \qquad (2.29)$$

The gauge parameter in (2.28) is defined modulo arbitrary shifts of the form

$$\zeta_\alpha \to \zeta'_\alpha = \zeta_\alpha + \mathrm{i}\mathcal{D}_\alpha \xi, \qquad \bar{\xi} = \xi, \qquad (2.30)$$

in the sense that $\delta_{\zeta'}\Lambda_\alpha = \delta_\zeta \Lambda_\alpha$. This property means that the two-form multiplet is a gauge theory with linearly dependent generators, in accordance with the terminology of the Batalin-Vilkovisky quantization [48].

We now assume L to be nowhere vanishing, such that L^{-1} exists. Then L can be used as a conformal compensator corresponding to a variant formulation of AdS supergravity. Upon replacement $\varphi \to L^{1/4}$, the supergravity action (2.21) turns into

$$S_{\mathrm{SG}} = -\frac{4}{\kappa}\mathrm{i}\int \mathrm{d}^3x \mathrm{d}^2\theta E \sqrt{L}\left\{\frac{\mathrm{i}}{16}\mathcal{D}^\alpha \ln L \mathcal{D}_\alpha \ln L - 2\mathcal{S}\right\}. \qquad (2.31)$$

The supersymmetric cosmological term in (2.21) does not contribute, since φ^4 turns into $L = \mathcal{D}^\alpha \Lambda_\alpha$, which is a total derivative. Hence, the $\mathcal{N} = 1$ two-form supergravity does not allow for a supersymmetric cosmological term. This is analogous to the new minimal formulation for $\mathcal{N} = 1$ supergravity in four dimensions [49–51]. However, the difference from the new minimal supergravity is that a cosmological term is now generated dynamically.

For the theory with action (2.31), the equation of motion for the compensator is

$$\mathcal{D}_\alpha \mathbb{S} = 0, \qquad \mathbb{S} := L^{-\frac{3}{4}}\left(\frac{\mathrm{i}}{2}\mathcal{D}^2 + \mathcal{S}\right)L^{\frac{1}{4}}, \qquad (2.32)$$

and therefore

$$\mathbb{S} = \lambda = \text{const.} \qquad (2.33)$$

If a solution with $\lambda \neq 0$ is chosen, it describes an AdS background. Unlike the supergravity formulation (2.21), the action (2.31) does not contain a free parameter. The negative cosmological constant is generated dynamically. It should be pointed out that the equation of motion for the gravitational superfield, which corresponds to (2.31), is obtained from (2.22b) by replacing $\varphi \to L^{\frac{1}{4}}$.

E. Superform formulation for the two-form multiplet

In this subsection we present a superform formulation for the three-form multiplet coupled to conformal supergravity, as an extension of the flat-superspace construction given in [36]. Let us consider a gauge super two-form

$$B_2 = \frac{1}{2}\mathrm{d}z^N \wedge \mathrm{d}z^M B_{MN} = \frac{1}{2}E^B \wedge E^A B_{AB}, \qquad (2.34)$$

which is defined modulo gauge transformations of the form

$$B_2 \to B_2 + \mathrm{d}A_1, \qquad A_1 = \mathrm{d}z^N A_N = E^B A_B, \qquad (2.35)$$

where the gauge parameter A_1 is an arbitrary super one-form. Associated with the potential B_2 is the gauge-invariant field strength

$$\begin{aligned} H_3 := \mathrm{d}B_2 &= \frac{1}{2}\mathrm{d}z^P \wedge \mathrm{d}z^N \wedge \mathrm{d}z^M \partial_M B_{NP} \\ &= \frac{1}{2}E^C \wedge E^B \wedge E^A \{\mathcal{D}_A B_{BC} - T_{AB}{}^D B_{DC}\}. \end{aligned} \qquad (2.36)$$

By construction, H_3 is an exact super three-form, and hence it is closed, $\mathrm{d}H_3 = 0$.

We are interested in a closed super three-form H_3 such that (i) its components are descendants of a scalar primary superfield L, and (ii) its lowest nonzero component is constrained to be $H_{\alpha\beta\gamma} = \mathrm{i}(\gamma_a)_{\beta\gamma}L$. It turns out that the closure condition, $\mathrm{d}H_3 = 0$, completely determine the entire super three-form to be

$$\begin{aligned} H_3[L] = &\frac{\mathrm{i}}{2}E^\gamma \wedge E^\beta \wedge E^a (\gamma_a)_{\beta\gamma} L \\ &+ \frac{1}{4}E^\gamma \wedge E^b \wedge E^a \varepsilon_{abc}(\gamma^c)_\gamma{}^\delta \mathcal{D}_\delta L \\ &- \frac{1}{24}E^c \wedge E^b \wedge E^a \varepsilon_{abc}(\mathrm{i}\mathcal{D}^2 + 8\mathcal{S})L, \end{aligned} \qquad (2.37)$$

which is obtained from (2.18) by replacing $\mathcal{L} \to L$. In general, if L is an arbitrary scalar superfield, the superform H_3 given by (2.37) is closed but not exact. However, if we choose $L := \mathcal{D}^\alpha \Lambda_\alpha$ in (2.37) then H_3 turns out to be exact. In fact, the following super two-form

$$B_2[\Lambda_\alpha] = -\mathrm{i}E^\beta \wedge E^a (\gamma_a)_\beta{}^\gamma \Lambda_\gamma - \frac{1}{4}E^b \wedge E^a \varepsilon_{abc}(\gamma^c)^{\rho\tau}\mathcal{D}_\rho \Lambda_\tau, \qquad (2.38)$$

is such that

$$\mathrm{d}B_2[\Lambda_\alpha] = H_3[\mathcal{D}^\alpha\Lambda_\alpha]. \qquad (2.39)$$

This proves that, if we consider the two- and three-forms

$$B_{ab} = -\frac{1}{2}\varepsilon_{abc}(\gamma^c)^{\rho\tau}\mathcal{D}_\rho\Lambda_\tau, \qquad (2.40a)$$

$$H_{abc} = -\frac{1}{4}\varepsilon_{abc}(\mathrm{i}\mathcal{D}^2 + 8\mathcal{S})\mathcal{D}^\delta\Lambda_\delta, \qquad (2.40b)$$

the latter is the field strength of the former,

$$H_{abc} = 3\mathcal{D}_{[a}B_{bc]} + 2\varepsilon_{abc}(\mathcal{D}^\alpha\mathcal{S})\Lambda_\alpha. \qquad (2.41)$$

Using the super-Weyl transformation laws (2.14) and (2.26), one can show that the superform (2.38) is super-Weyl invariant,

$$\delta_\sigma B_2[\Lambda_\alpha] = 0 \Rightarrow \delta_\sigma H_3[\mathcal{D}^\alpha\Lambda_\alpha] = 0. \qquad (2.42)$$

This result will be important for our analysis in Sec. IV A.

Choosing B_2 in the form (2.38) corresponds to a partial fixing of the gauge freedom (2.35). The residual gauge freedom is given by

$$\delta_\zeta B_2[\Lambda_\alpha] = B_2[\delta_\zeta\Lambda_\alpha] \Rightarrow \mathrm{d}\delta_\zeta B_2[\Lambda_\alpha] = 0, \qquad (2.43)$$

where $\delta_\zeta\Lambda_\alpha$ is defined by (2.28).

III. TWO-FORM MULTIPLETS IN $\mathcal{N} = 2$ SUPERGRAVITY

It is well known that the 3D AdS group is reducible,

$$\mathrm{SO}(2,2) \cong (\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R}))/\mathbb{Z}_2,$$

and so are its supersymmetric extensions, $\mathrm{OSp}(p|2;\mathbb{R}) \times \mathrm{OSp}(q|2;\mathbb{R})$. This implies that $\mathcal{N}$-extended AdS supergravity exists in several versions [52]. These are known as the (p,q) AdS supergravity theories where the non-negative integers $p \geq q$ are such that $\mathcal{N} = p + q$.[7] In this section we choose $\mathcal{N} = 2$ and describe four off-shell formulations for (1,1) AdS supergravity and one for (2,0) AdS supergravity. Only one of these five off-shell supergravity theories is new, the so-called complex two-form supergravity; the others were presented in [15].

A. Conformal supergravity

We consider a curved $\mathcal{N} = 2$ superspace, $\mathcal{M}^{3|4}$, parametrized by local bosonic (x^m) and fermionic $(\theta^\mu, \bar{\theta}_\mu)$ coordinates $z^M = (x^m, \theta^\mu, \bar{\theta}_\mu)$, where $m = 0, 1, 2$ and $\mu = 1, 2$. The Grassmann variables θ^μ and $\bar{\theta}_\mu$ are related to each other by complex conjugation: $\overline{\theta^\mu} = \bar{\theta}^\mu$. The supervielbein $E^A = (E^a, E^\alpha, \bar{E}_\alpha)$ and its inverse $E_A = (E_a, E_\alpha, \bar{E}^\alpha)$ are defined similarly to (2.1).

Within the superspace formulation for $\mathcal{N} = 2$ conformal supergravity proposed in [53] and fully developed in [37], the structure group is $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{U}(1)$. The covariant derivatives have the form

$$\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^\alpha) = E_A + \boldsymbol{\Omega}_A, \qquad \boldsymbol{\Omega}_A := \Omega_A + \mathrm{i}\Phi_A\mathcal{J}. \qquad (3.1)$$

We recall that the Lorentz connection Ω_A can be written in several equivalent forms (2.3). The U(1) generator acts on the covariant derivatives as follows:

$$[\mathcal{J}, \mathcal{D}_\alpha] = \mathcal{D}_\alpha, \qquad [\mathcal{J}, \bar{\mathcal{D}}^\alpha] = -\bar{\mathcal{D}}^\alpha. \qquad (3.2)$$

In general, the covariant derivatives have graded commutation relations of the form

$$[\mathcal{D}_A, \mathcal{D}_B\} = T_{AB}{}^C\mathcal{D}_C + \boldsymbol{R}_{AB}, \qquad \boldsymbol{R}_{AB} := \frac{1}{2}R_{AB}{}^{cd}M_{cd} + \mathrm{i}R_{AB}\mathcal{J}. \qquad (3.3)$$

In order to describe the multiplet of conformal supergravity, certain constraints should be imposed on the torsion tensor [53]. Solving these constraints leads to the following algebra of covariant derivatives:

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = -4\bar{R}M_{\alpha\beta}, \qquad (3.4a)$$

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_\beta\} = -2\mathrm{i}(\gamma^c)_{\alpha\beta}\mathcal{D}_c - 2\mathcal{C}_{\alpha\beta}\mathcal{J} - 4\mathrm{i}\varepsilon_{\alpha\beta}\mathcal{S}\mathcal{J} + 4\mathrm{i}\mathcal{S}M_{\alpha\beta} - 2\varepsilon_{\alpha\beta}\mathcal{C}^{\gamma\delta}M_{\gamma\delta}, \qquad (3.4b)$$

$$\begin{aligned}[\mathcal{D}_a, \mathcal{D}_\beta] = {}& \mathrm{i}\varepsilon_{abc}(\gamma^b)_\beta{}^\gamma\mathcal{C}^c\mathcal{D}_\gamma + (\gamma_a)_\beta{}^\gamma\mathcal{S}\mathcal{D}_\gamma - \mathrm{i}(\gamma_a)_{\beta\gamma}\bar{R}\bar{\mathcal{D}}^\gamma - (\gamma_a)_\beta{}^\gamma\boldsymbol{C}_{\gamma\delta\rho}M^{\delta\rho} \\ & -\frac{1}{3}(2\mathcal{D}_\beta\mathcal{S} + \mathrm{i}\bar{\mathcal{D}}_\beta\bar{R})M_a - \frac{2}{3}\varepsilon_{abc}(\gamma^b)_\beta{}^\alpha(2\mathcal{D}_\alpha\mathcal{S} + \mathrm{i}\bar{\mathcal{D}}_\alpha\bar{R})M^c \\ & -\frac{1}{2}\left((\gamma_a)^{\alpha\gamma}\boldsymbol{C}_{\alpha\beta\gamma} + \frac{1}{3}(\gamma_a)_\beta{}^\gamma(8\mathcal{D}_\gamma\mathcal{S} + \mathrm{i}\bar{\mathcal{D}}_\gamma\bar{R})\right)\mathcal{J}, \end{aligned} \qquad (3.4c)$$

[7] For any values of p and q allowed, the pure (p,q) AdS supergravity was constructed in [52] as a Chern-Simons theory with the gauge group $\mathrm{OSp}(p|2;\mathbb{R}) \times \mathrm{OSp}(q|2;\mathbb{R})$.

$$
\begin{aligned}
[\mathcal{D}_a,\mathcal{D}_b] &= \frac{1}{2}\varepsilon_{abc}(\gamma^c)^{\alpha\beta}\varepsilon^{\gamma\delta}\left(-\mathrm{i}\bar{\boldsymbol{C}}_{\alpha\beta\delta}+\frac{4\mathrm{i}}{3}\varepsilon_{\delta(\alpha}\bar{\mathcal{D}}_{\beta)}\mathcal{S}+\frac{2}{3}\varepsilon_{\delta(\alpha}\mathcal{D}_{\beta)}R\right)\mathcal{D}_\gamma \\
&\quad+\frac{1}{2}\varepsilon_{abc}(\gamma^c)^{\alpha\beta}\varepsilon^{\gamma\delta}\left(-\mathrm{i}\boldsymbol{C}_{\alpha\beta\delta}+\frac{4\mathrm{i}}{3}\varepsilon_{\delta(\alpha}\mathcal{D}_{\beta)}\mathcal{S}-\frac{2}{3}\varepsilon_{\delta(\alpha}\bar{\mathcal{D}}_{\beta)}\bar{R}\right)\bar{\mathcal{D}}_\gamma \\
&\quad-\varepsilon_{abc}\left(\frac{1}{4}(\gamma^c)^{\alpha\beta}(\gamma_d)^{\tau\delta}(\mathrm{i}\mathcal{D}_{(\tau}\bar{\boldsymbol{C}}_{\delta\alpha\beta)}+\mathrm{i}\bar{\mathcal{D}}_{(\tau}\boldsymbol{C}_{\delta\alpha\beta)})+\frac{1}{6}\delta^c_d(\mathcal{D}^2R+\bar{\mathcal{D}}^2\bar{R})\right. \\
&\quad\left.+\frac{2}{3}\mathrm{i}\delta^c_d\mathcal{D}^\alpha\bar{\mathcal{D}}_\alpha\mathcal{S}-4\mathcal{C}^c\mathcal{C}_d-4\delta^c_d(\bar{R}R+\mathcal{S}^2)\right)M^d \\
&\quad+\mathrm{i}\varepsilon_{abc}\left(\frac{1}{2}(\gamma^c)^{\alpha\beta}[\mathcal{D}_\alpha,\bar{\mathcal{D}}_\beta]\mathcal{S}-\varepsilon^{cef}\mathcal{D}_e\mathcal{C}_f-4\mathcal{S}\mathcal{C}^c\right)\mathcal{J}.
\end{aligned}
\tag{3.4d}
$$

The algebra involves four dimension-one torsion superfields: a real scalar $\mathcal{S}$, a complex scalar R, and its conjugate $\bar{R}$, and a real vector $\mathcal{C}_a$. The U(1) charge of R is -2. These torsion superfields obey differential constraints implied by the Bianchi identities, which are

$$\bar{\mathcal{D}}_\alpha R = 0, \tag{3.5a}$$

$$(\bar{\mathcal{D}}^2-4R)\mathcal{S}=0, \tag{3.5b}$$

$$\mathcal{D}_\alpha\mathcal{C}_{\beta\gamma}=\mathrm{i}\boldsymbol{C}_{\alpha\beta\gamma}-\frac{1}{3}\varepsilon_{\alpha(\beta}(\bar{\mathcal{D}}_{\gamma)}\bar{R}+4\mathrm{i}\mathcal{D}_{\gamma)}\mathcal{S}). \tag{3.5c}$$

In this paper we make use of the definitions

$$\mathcal{D}^2 := \mathcal{D}^\alpha\mathcal{D}_\alpha, \qquad \bar{\mathcal{D}}^2 := \bar{\mathcal{D}}_\alpha\bar{\mathcal{D}}^\alpha. \tag{3.6}$$

As follows from (3.5c), the complex dimension-3/2 symmetric spinor $\boldsymbol{C}_{\alpha\beta\gamma}$, which appears in (3.4), is a descendant of the torsion three-vector $\mathcal{C}_a$, $\boldsymbol{C}_{\alpha\beta\gamma}=-\mathrm{i}\mathcal{D}_{(\alpha}\mathcal{C}_{\beta\gamma)}$.

The definition of the torsion and curvature tensors, Eq. (3.3), can be recast in the superform notation, which will be used in Sec. IV. Associated with the connection $\boldsymbol{\Omega}_A$, Eq. (3.1), is the connection one-form $\boldsymbol{\Omega}=E^C\boldsymbol{\Omega}_C$. Its action on a real super-vector

$$V_A=(V_a,\Psi_\alpha,\bar{\Psi}^\alpha), \qquad \mathcal{J}\Psi_\alpha=\Psi_\alpha \tag{3.7}$$

is given by

$$\boldsymbol{\Omega}V_A=\boldsymbol{\Omega}_A{}^BV_B=\Omega_A{}^BV_B+\mathrm{i}\Phi_A{}^BV_B, \tag{3.8}$$

with $\Omega_A{}^B$ and $\Phi_A{}^B$ being the Lorentz and U(1) connections, respectively. Using the definitions given, the torsion and curvature two-forms are

$$T^C := \frac{1}{2}E^B\wedge E^AT_{AB}{}^C=-\mathrm{d}E^C+E^B\wedge\boldsymbol{\Omega}_B{}^C, \tag{3.9a}$$

$$\boldsymbol{R}_C{}^D := \frac{1}{2}E^B\wedge E^AR_{ABC}{}^D=\mathrm{d}\boldsymbol{\Omega}_C{}^D-\boldsymbol{\Omega}_C{}^E\wedge\boldsymbol{\Omega}_E{}^D. \tag{3.9b}$$

The important property of the algebra (3.4) is that its form is preserved under super-Weyl transformations of the covariant derivatives [15,37]

$$\delta_\sigma\mathcal{D}_\alpha=\frac{1}{2}\sigma\mathcal{D}_\alpha+\mathcal{D}^\gamma\sigma M_{\gamma\alpha}-\mathcal{D}_\alpha\sigma\mathcal{J}, \tag{3.10a}$$

$$\delta_\sigma\bar{\mathcal{D}}_\alpha=\frac{1}{2}\sigma\bar{\mathcal{D}}_\alpha+\bar{\mathcal{D}}^\gamma\sigma M_{\gamma\alpha}+\bar{\mathcal{D}}_\alpha\sigma\mathcal{J}, \tag{3.10b}$$

$$
\begin{aligned}
\delta_\sigma\mathcal{D}_a &= \sigma\mathcal{D}_a-\frac{\mathrm{i}}{2}(\gamma_a)^{\gamma\delta}\mathcal{D}_{(\gamma}\sigma\bar{\mathcal{D}}_{\delta)}-\frac{\mathrm{i}}{2}(\gamma_a)^{\gamma\delta}\bar{\mathcal{D}}_{(\gamma}\sigma\mathcal{D}_{\delta)} \\
&\quad+\varepsilon_{abc}\mathcal{D}^b\sigma M^c-\frac{\mathrm{i}}{8}(\gamma_a)^{\gamma\delta}[\mathcal{D}_\gamma,\bar{\mathcal{D}}_\delta]\sigma\mathcal{J}
\end{aligned}
\tag{3.10c}
$$

and the torsion tensors

$$\delta_\sigma\mathcal{S}=\sigma\mathcal{S}+\frac{\mathrm{i}}{4}\mathcal{D}^\alpha\bar{\mathcal{D}}_\alpha\sigma, \tag{3.10d}$$

$$\delta_\sigma\mathcal{C}_a=\sigma\mathcal{C}_a+\frac{1}{8}(\gamma_a)^{\gamma\delta}[\mathcal{D}_\gamma,\bar{\mathcal{D}}_\delta]\sigma, \tag{3.10e}$$

$$\delta_\sigma R=\sigma R+\frac{1}{4}\bar{\mathcal{D}}^2\sigma. \tag{3.10f}$$

Here the super-Weyl parameter σ is an unconstrained real scalar superfield. It follows from (3.10) that the super-Weyl transformation law of the supervielbein is

$$\delta_\sigma E^a=-\sigma E^a, \tag{3.11a}$$

$$
\begin{aligned}
\delta_\sigma E^\alpha &= -\frac{1}{2}\sigma E^\alpha+\frac{\mathrm{i}}{2}E^b(\gamma_b)^{\alpha\gamma}\bar{\mathcal{D}}_\gamma\sigma, \\
\delta_\sigma\bar{E}_\alpha &= -\frac{1}{2}\sigma\bar{E}_\alpha+\frac{\mathrm{i}}{2}E^b(\gamma_b)_{\alpha\gamma}\mathcal{D}^\gamma\sigma.
\end{aligned}
\tag{3.11b}
$$

The group of super-Weyl transformations must be a subgroup of the supergravity gauge group in order for the superspace geometry under consideration to describe the multiplet of $\mathcal{N}=2$ conformal supergravity.

A tensor superfield T of a U(1) charge q, $\mathcal{J}T = qT$, is said to be super-Weyl primary if its super-Weyl transformation law is

$$\delta_\sigma T = w\sigma T, \tag{3.12}$$

for some constant parameter w which will be referred to as the super-Weyl weight of T.

The action for $\mathcal{N} = 2$ conformal supergravity was constructed for the first time by Roček and van Nieuwenhuizen [54] using the $\mathcal{N} = 2$ superconformal tensor calculus. More recently, it was reformulated within the superform approach [40]. The interested reader is referred to these publications for the technical details.

B. Supersymmetric actions

As in the 4D $\mathcal{N} = 1$ case, there are two (closely related) locally supersymmetric and super-Weyl invariant actions in 3D $\mathcal{N} = 2$ supergravity [37].

Given a real scalar Lagrangian $\mathcal{L} = \bar{\mathcal{L}}$ with the super-Weyl transformation law

$$\delta_\sigma \mathcal{L} = \sigma \mathcal{L}, \tag{3.13}$$

the action

$$S = \int \mathrm{d}^3x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} E \mathcal{L}, \qquad E = \mathrm{Ber}(E_M{}^A) \tag{3.14}$$

is invariant under the supergravity gauge group. It is also super-Weyl invariant due to the transformation law

$$\delta_\sigma E = -\sigma E. \tag{3.15}$$

Given a covariantly chiral scalar Lagrangian $\mathcal{L}_c$ of super-Weyl weight two,

$$\bar{\mathcal{D}}_\alpha \mathcal{L}_c = 0, \qquad \mathcal{J}\mathcal{L}_c = -2\mathcal{L}_c, \qquad \delta_\sigma \mathcal{L}_c = 2\sigma \mathcal{L}_c, \tag{3.16}$$

the following chiral action

$$S_c = \int \mathrm{d}^3x \mathrm{d}^2\theta \mathcal{E} \mathcal{L}_c \tag{3.17}$$

is locally supersymmetric and super-Weyl invariant. Action (3.17) involves integration over the chiral subspace of the full superspace, with $\mathcal{E}$ the chiral density possessing the properties

$$\mathcal{J}\mathcal{E} = 2\mathcal{E}, \qquad \delta_\sigma \mathcal{E} = -2\sigma \mathcal{E}. \tag{3.18}$$

The explicit expression for $\mathcal{E}$ in terms of the supergravity prepotentials is given in [55]. Alternatively, the chiral density can be read off using the general formalism of integrating out fermionic dimensions, which was developed in [56].

The two actions, (3.14) and (3.17), are related to each other as follows:

$$\int \mathrm{d}^3x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} E \mathcal{L} = \int \mathrm{d}^3x \mathrm{d}^2\theta \mathcal{E} \mathcal{L}_c, \qquad \mathcal{L}_c := -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\mathcal{L}. \tag{3.19}$$

This relation shows that the chiral action, or its conjugate antichiral action, is more fundamental than (3.14).

The chiral projection operator in (3.19) defined by

$$\bar{\Delta} := -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R) \tag{3.20}$$

plays a fundamental role in $\mathcal{N} = 2$ supergravity. Among its most important properties is the following: given a primary complex scalar ψ satisfying

$$\mathcal{J}\psi = (2 - w)\psi, \qquad \delta_\sigma \psi = (w - 1)\sigma\psi, \tag{3.21}$$

for some constant super-Weyl weight w, its descendant

$$\phi = \bar{\Delta}\psi \tag{3.22}$$

is a primary chiral superfield of super-Weyl weight w,

$$\bar{\mathcal{D}}_\alpha \phi = 0, \qquad \mathcal{J}\phi = -w\phi, \qquad \delta_\sigma \phi = w\sigma\phi. \tag{3.23}$$

For every primary chiral scalar superfield, its super-Weyl weight w and U(1) charge q are related to each other as $w + q = 0$, in accordance with [37]. Any superfield ϕ with the properties (3.23) will be referred to as a weight-w chiral scalar.

The chiral action, Eq. (3.17), can be represented as an integral over the full superspace,

$$S_c = \int \mathrm{d}^3x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} E \mathfrak{C} \mathcal{L}_c, \tag{3.24}$$

if we make use of an *improved complex linear* superfield $\mathfrak{C}$ defined by the two properties:

(i) $\mathfrak{C}$ obeys the constraint

$$\bar{\Delta}\mathfrak{C} = 1; \tag{3.25a}$$

(ii) the transformation properties of $\mathfrak{C}$ are

$$\delta_\sigma \mathfrak{C} = -\sigma\mathfrak{C}, \qquad \mathcal{J}\mathfrak{C} = 2\mathfrak{C}. \tag{3.25b}$$

A possible choice for $\mathfrak{C}$ is

$$\mathfrak{C} = \frac{\bar{\eta}}{\bar{\Delta}\bar{\eta}}, \qquad \bar{\mathcal{D}}_\alpha \eta = 0, \qquad \delta_\sigma \eta = \frac{1}{2}\sigma\eta, \tag{3.26}$$

for some covariantly chiral superfield η such that $\Delta\eta$ is nowhere vanishing. In case $\mathfrak{C}$ is not required to be super-Weyl primary, it can be identified with R^{-1},

$$S_c = \int d^3x d^2\theta d^2\bar{\theta} E \frac{\mathcal{L}_c}{R}, \tag{3.27}$$

provided R is nowhere vanishing. This representation is analogous to that discovered by Siegel [57] and Zumino [58] in 4D $\mathcal{N}=1$ supergravity.

The chiral action can also be described using the super three-form constructed in [59]

$$\begin{aligned}\Xi_3[\mathcal{L}_c] = &-2\bar{E}_\gamma \wedge \bar{E}_\beta \wedge E^a (\gamma_a)^{\beta\gamma} \mathcal{L}_c \\ &-\frac{i}{2}\bar{E}_\gamma \wedge E^b \wedge E^a \varepsilon_{abd}(\gamma^d)^{\gamma\delta}\mathcal{D}_\delta \mathcal{L}_c \\ &+\frac{1}{24}E^c \wedge E^b \wedge E^a \varepsilon_{abc}(\mathcal{D}^2 - 16\bar{R})\mathcal{L}_c.\end{aligned} \tag{3.28}$$

This superform is closed and super-Weyl invariant,

$$d\Xi_3[\mathcal{L}_c] = 0, \qquad \delta_\sigma \Xi_3[\mathcal{L}_c] = 0. \tag{3.29}$$

The chiral action is equivalently represented as

$$S_c = \int_{\mathcal{M}^3} \Xi_3[\mathcal{L}_c], \tag{3.30}$$

where the integration is carried out over a spacetime $\mathcal{M}^3$ being homotopic to the bosonic body of the curved superspace $\mathcal{M}^{3|4}$ obtained by switching off the Grassmann variables.

C. AdS supergravity

There are two off-shell formulations for (1,1) AdS supergravity developed in [15], minimal and nonminimal ones, which do not have gauge two-forms in the sector of auxiliary fields.

1. (1,1) AdS supergravity

In the minimal case, the conformal compensators are a weight-1/2 chiral scalar Φ, $\bar{\mathcal{D}}_\alpha \Phi = 0$, and its conjugate $\bar{\Phi}$. Of course, Φ has to be nowhere vanishing, such that Φ^{-1} exists, in order to serve as a conformal compensator. The supergravity action is

$$\begin{aligned}S^{\rm minimal}_{(1,1)\rm SG} = &-\frac{4}{\kappa}\int d^3x d^2\theta d^2\bar{\theta} E \bar{\Phi}\Phi \\ &+\left\{\frac{\mu}{\kappa}\int d^3x d^2\theta \mathcal{E}\Phi^4 + \text{c.c.}\right\},\end{aligned} \tag{3.31}$$

where μ is a complex parameter. The second terms in the action is the supersymmetric cosmological term. Using the component results of [59], for the cosmological constant one obtains

$$\Lambda_{\rm AdS} = -4|\mu|^2. \tag{3.32}$$

The above minimal formulation for (1,1) AdS supergravity (which was called type I minimal supergravity in [15]) is the 3D analog of the old minimal formulation for 4D $\mathcal{N}=1$ supergravity [60–62].

For the supergravity theory with action (3.31), the equation of motion for the chiral compensator is

$$\mathbb{R} = \mu, \qquad \mathbb{R} := \Phi^{-3}\bar{\Delta}\,\bar{\Phi}. \tag{3.33a}$$

We also reproduce the equation of motion for the $\mathcal{N}=2$ gravitational superfield[8]

$$\mathbb{C}_{\alpha\beta} = 0, \qquad \mathbb{C}_{\alpha\beta} := -\frac{1}{4}([\mathcal{D}_{(\alpha}, \bar{\mathcal{D}}_{\beta)}] - 4\mathcal{C}_{\alpha\beta})(\Phi\bar{\Phi})^{-1}, \tag{3.33b}$$

see [46] for the technical details. The specific feature of $\mathbb{R}$ and $\mathbb{C}_{\alpha\beta}$ is that they are super-Weyl invariant. The super-Weyl and local U(1) transformations can be used to choose the gauge $\Phi = 1$, which implies that $\mathcal{S} = 0$ and $\mathbb{R}$ and $\mathbb{C}_{\alpha\beta}$ coincide with the torsion superfields R and $\mathcal{C}_{\alpha\beta}$, respectively. In this gauge, every solution to the equations (3.33) is locally diffeomorphic to the (1,1) AdS superspace [47].

Within the nonminimal formulation for (1,1) AdS supergravity [15], the conformal compensators are an improved complex linear scalar Γ and its conjugate $\bar{\Gamma}$. The former has the transformation properties

$$\delta_\sigma \Gamma = -\sigma\Gamma, \qquad \mathcal{J}\Gamma = 2\Gamma \tag{3.34a}$$

and obeys the improved linear constraint

$$\bar{\Delta}\Gamma = \mu = \text{const}, \tag{3.34b}$$

compare with (3.25). The supergravity action is

$$S^{\rm nonminimal}_{(1,1)\rm SG} = -\frac{2}{\kappa}\int d^3x d^2\theta d^2\bar{\theta} E (\bar{\Gamma}\Gamma)^{-1/2}. \tag{3.35}$$

As demonstrated in [15], this theory is dual to the minimal AdS supergravity, Eq. (3.31). The theory under consideration is the 3D analog of the nonminimal $\mathcal{N}=1$ AdS supergravity in four dimensions [64]. Both formulations lead to the (1,1) AdS superspace [15,47] as the maximally supersymmetric solution.

[8]The $\mathcal{N}=2$ gravitational superfield was introduced in [55,63].

2. (2,0) AdS supergravity

The conformal compensator for (2,0) AdS supergravity is a *linear multiplet* [15,37,53] describing the field strength of an Abelian vector multiplet. It is realized in terms of a real scalar superfield $L = \bar{L}$ subject to the constraint

$$\bar{\Delta} L = 0 \Leftrightarrow \Delta L = 0, \tag{3.36}$$

which is consistent with the super-Weyl transformation law

$$\delta_\sigma L = \sigma L. \tag{3.37}$$

The constraint (3.36) is solved in terms of a real unconstrained prepotential V,

$$L = \mathrm{i}\mathcal{D}^\alpha \bar{\mathcal{D}}_\alpha V, \qquad \bar{V} = V, \tag{3.38}$$

which is defined modulo gauge transformations of the form

$$\delta_\lambda V = \lambda + \bar{\lambda}, \qquad \bar{\mathcal{D}}_\alpha \lambda = 0. \tag{3.39}$$

To reproduce the super-Weyl transformation (3.37), it suffices to choose

$$\delta_\sigma V = 0. \tag{3.40}$$

In order to be used as a conformal compensator, L has to be nowhere vanishing, such that L^{-1} exists. The action for (2,0) AdS supergravity was constructed in [15]. It is

$$S_{(2,0)\mathrm{SG}} = \frac{4}{\kappa} \int \mathrm{d}^3x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} E \{ L \ln L - 4V\mathcal{S} + 4\xi V L \}, \tag{3.41}$$

where the parameter ξ determines the cosmological constant. The equations of motion for this theory can be written in the form [46]

$$\mathbb{S} = \xi, \qquad \mathbb{S} := -\frac{\mathrm{i}}{4} L^{-1} (\mathcal{D}^\gamma \bar{\mathcal{D}}_\gamma \ln L + 4\mathrm{i}\mathcal{S}), \tag{3.42a}$$

$$\mathbb{C}_{\alpha\beta} = 0, \qquad \mathbb{C}_{\alpha\beta} := -\frac{1}{4} ([\mathcal{D}_{(\alpha}, \bar{\mathcal{D}}_{\beta)}] - 4\mathcal{C}_{\alpha\beta}) L^{-1}, \tag{3.42b}$$

with $\mathbb{S}$ and $\mathbb{C}_{\alpha\beta}$ being super-Weyl invariant.[9] The super-Weyl gauge freedom can be used to set $L = 1$, which implies $R = 0$, and then $\mathbb{S}$ and $\mathbb{C}_{\alpha\beta}$ turn into the torsion superfields $\mathcal{S}$ and $\mathcal{C}_{\alpha\beta}$, respectively. Under the gauge condition chosen, every solution to the equations (3.42) is locally diffeomorphic to the (2,0) AdS superspace [15,47].

The above supergravity theory (called type II minimal supergravity in [15]) is the 3D analog of the new minimal for $\mathcal{N} = 1$ supergravity in four dimensions [49–51]. The latter theory is known to allow no supersymmetric cosmological term. Such a supersymmetric cosmological term does exist in the 3D case, and it is given by the Chern-Simons ξ-term in (3.41). For $\xi \neq 0$ the theory possesses a maximally supersymmetric solution, which is the (2,0) AdS superspace [15,47] corresponding to the (2,0) AdS supersymmetry [52].

[9]The vector superfield $\mathbb{C}_{\alpha\beta}$ should not be confused with (3.33b).

D. Two-form supergravity

There is one more variant off-shell formulation for (1,1) AdS supergravity proposed in [15]. Its conformal compensator is the so-called two-form multiplet, which is the 3D cousin of the well-known three-form multiplet in 4D $\mathcal{N} = 1$ supersymmetry, which was proposed by Gates [65] and reviewed in [36,66].

In curved superspace, the two-form multiplet is described by a real unconstrained scalar prepotential $P = \bar{P}$ which enters any action functional, $S = S[\Pi, \bar{\Pi}]$, only via the covariantly chiral descendant

$$\Pi = \bar{\Delta} P \tag{3.43}$$

and its conjugate $\bar{\Pi}$. In order for Π to be a primary superfield, the prepotential P should possess the super-Weyl transformation law

$$\delta_\sigma P = \sigma P, \tag{3.44}$$

which implies

$$\delta_\sigma \Pi = 2\sigma \Pi, \qquad \mathcal{J}\Pi = -2\Pi. \tag{3.45}$$

The chiral scalar (3.43) is a gauge-invariant field strength for gauge transformations of the form

$$\delta_L P = L, \qquad \bar{\Delta} L = 0, \qquad \bar{L} = L. \tag{3.46}$$

Here the linear gauge parameter can be expressed via an unconstrained superfield V as in (3.38). Since V is defined modulo gauge transformations (3.39), we conclude that any system with action $S = S[\Pi, \bar{\Pi}]$, which describes the dynamics of the two-form multiplet, is a gauge theory with linearly dependent generators.

Lagrangian quantization of the two-form multiplet can be carried out similarly to that of the 4D $\mathcal{N} = 1$ three-form multiplet coupled to supergravity [67] (see [68] for a review).

Upon replacement $\Phi^4 \to \Pi$ in (3.31) the supergravity action turns into

$$S^{\text{two-form}}_{(1,1)\text{SG}} = -\frac{4}{\kappa}\int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}E\left\{(\bar{\Pi}\Pi)^{\frac{1}{4}} - \frac{1}{2}mP\right\}$$
$$= -\frac{4}{\kappa}\int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}E(\bar{\Pi}\Pi)^{\frac{1}{4}}$$
$$+\left\{\frac{m}{\kappa}\int \mathrm{d}^3x\mathrm{d}^2\theta\mathcal{E}\Pi + \text{c.c.}\right\}, \qquad (3.47)$$

where m is a real parameter. In the second form, the action is manifestly invariant under gauge transformations (3.46). The equation of motion for the compensator is

$$\mathbb{R} + \bar{\mathbb{R}} = 2m, \qquad \mathbb{R} \coloneqq \Pi^{-3/4}\bar{\Delta}\bar{\Pi}^{1/4}, \qquad (3.48)$$

and therefore

$$\mathbb{R} = \mu = \text{const}. \qquad (3.49)$$

The action for type I minimal supergravity (3.31) involves two real parameters, Reμ and Imμ, which appear in the supersymmetric cosmological term. The action for two-form supergravity (3.47) contains only one real parameter, m, which determines the corresponding supersymmetric cosmological term. As is seen from (3.49), the second parameter Imμ is generated dynamically. At the component level, the cosmological constant in the theory (3.49) is given by (3.32).

The two-form supergravity theory described above is the 3D analog of the variant formulation for 4D $\mathcal{N}=1$ supergravity known as three-form supergravity. The latter was proposed for the first time by Gates and Siegel [66] and fully developed at the component level in [69,70]. The super-Weyl invariant formulation for the three-form supergravity was given in [71]. Our formulation of the 3D two-form supergravity is similar to [71].

E. Superform formulation for the two-form multiplet

We now present a geometric formulation for the two-form multiplet used in the previous section. Let us introduce a super two-form B_2 defined by

$$B_2[P] = -\bar{E}_\alpha \wedge E^\alpha P + \frac{\mathrm{i}}{2}E^\beta \wedge E^a(\gamma_a)_{\beta\gamma}\mathcal{D}^\gamma P$$
$$+\frac{\mathrm{i}}{2}\bar{E}_\beta \wedge E^a(\gamma_a)^{\beta\gamma}\bar{\mathcal{D}}_\gamma P$$
$$-\frac{1}{16}\varepsilon_{abc}E^b \wedge E^a((\gamma^c)^{\rho\tau}[\mathcal{D}_\rho, \bar{\mathcal{D}}_\tau] - 8\mathcal{C}^c)P \qquad (3.50)$$

and consider its exterior derivative $H_3 \coloneqq \mathrm{d}B_2$. It is not difficult to check that H_3 is given by the following expression:

$$H_3[\Pi] = -\mathrm{i}\bar{E}_\gamma \wedge \bar{E}_\beta \wedge E^a(\gamma_a)^{\beta\gamma}\Pi - \mathrm{i}E^\gamma \wedge E^\beta \wedge E^a(\gamma_a)_{\beta\gamma}\bar{\Pi}$$
$$+\frac{1}{4}\bar{E}_\gamma \wedge E^b \wedge E^a\varepsilon_{abd}(\gamma^d)^{\gamma\delta}\mathcal{D}_\delta\Pi$$
$$-\frac{1}{4}E^\gamma \wedge E^b \wedge E^a\varepsilon_{abd}(\gamma^d)_{\gamma\delta}\bar{\mathcal{D}}^\delta\bar{\Pi}$$
$$+\frac{\mathrm{i}}{48}E^c \wedge E^b \wedge E^a\varepsilon_{abc}$$
$$\times((\mathcal{D}^2 - 16\bar{R})\Pi - (\bar{\mathcal{D}}^2 - 16R)\bar{\Pi}), \qquad (3.51)$$

and, hence, it is constructed solely in terms of the compensator Π and its conjugate $\bar{\Pi}$, with Π being related to P as in (3.43).

The relation $H_3[\Pi] = \mathrm{d}B_2[P]$ implies that the top components of $B_2[P]$ and $H_3[\Pi]$,

$$B_{ab} = -\frac{1}{8}\varepsilon_{abc}((\gamma^c)^{\rho\tau}[\mathcal{D}_\rho, \bar{\mathcal{D}}_\tau] - 8\mathcal{C}^c)P, \qquad (3.52\text{a})$$

$$H_{abc} = -\frac{\mathrm{i}}{8}\varepsilon_{abc}((\bar{\mathcal{D}}^2 - 16R)\Pi - (\mathcal{D}^2 - 16\bar{R})\bar{\Pi}), \qquad (3.52\text{b})$$

are connected to each other as

$$H_{abc} = 3\mathcal{D}_{[a}B_{bc]} + \varepsilon_{abc}(\mathrm{i}\mathcal{D}^\alpha R - 2\bar{\mathcal{D}}^\alpha\mathcal{S})\mathcal{D}_\alpha P$$
$$+ \varepsilon_{abc}(\mathrm{i}\bar{\mathcal{D}}^\alpha\bar{R} + 2\mathcal{D}^\alpha\mathcal{S})\bar{\mathcal{D}}_\alpha P. \qquad (3.53)$$

Equations (3.52) and (3.53) tell us that the imaginary part of the top component field of the chiral superfield Π, defined by $F = -\frac{1}{4}\mathcal{D}^2\Pi|$, is the field strength of a gauge two-form.

The gauge transformation (3.46) of the prepotential P is equivalent to the following transformation of the super two-form (3.50):

$$\delta_L B_2[P] = B_2[L] \Rightarrow \delta_L H_3[\Pi] = 0. \qquad (3.54)$$

This allows us to interpret $B_2[P]$ as a gauge two-form and $H_3[\Pi]$ as its gauge-invariant field strength. The closed super two-form $B_2[L]$ in (3.54) is actually exact, $B_2[L] = \mathrm{d}A_1$, where A_1 is the gauge potential of a vector multiplet.

Using the super-Weyl transformation laws (3.11) and (3.44), one can check that the superform (3.50) is invariant under arbitrary super-Weyl transformations,

$$\delta_\sigma B_2[P] = 0 \Rightarrow \delta_\sigma H_3[\Pi] = 0. \qquad (3.55)$$

This property will be important for our analysis in Sec. IV B.

Let us recall the closed super three-form $\Xi_3[\mathcal{L}_c]$, defined by Eq. (3.28), which generates the supersymmetric invariant (3.30). If we choose $\mathcal{L}_c = \Pi$, with Π given by (3.43), then the exact super three-form $H_3[\Pi]$ proves to be the imaginary part of $\Xi_3[\Pi]$,

$$H_3[\Pi] = \frac{\mathrm{i}}{2}\Xi_3[\Pi] - \frac{\mathrm{i}}{2}\bar{\Xi}_3[\bar{\Pi}]. \qquad (3.56)$$

The real part of $\Xi_3[\Pi]$, on the other hand, is not exact and generates a nontrivial supersymmetric invariant, which may be realized as the full superspace integral (3.14), with P playing the role of the Lagrangian $\mathcal{L}$.

The local U(1) and super-Weyl transformations may be used to choose the gauge $\Pi = 1$. This condition implies that $\mathcal{S} = 0$ and the algebra of covariant derivatives reduces to that of type I minimal supergravity [15,37] with one extra constraint: the imaginary part of R is now the divergence of a vector (related, by Poincaré duality, to a two-form potential). To see this it suffices to write the super three-form $H_3[\Pi]$ in the gauge $\Pi = 1$

$$\begin{aligned} H_3 &= -\mathrm{i}E^\gamma \wedge E^\beta \wedge E^a(\gamma_a)_{\beta\gamma} - \mathrm{i}\bar{E}_\gamma \wedge \bar{E}_\beta \wedge E^a(\gamma_a)^{\beta\gamma} \\ &\quad + \frac{1}{3}E^c \wedge E^b \wedge E^a\varepsilon_{abc}\mathrm{i}(R - \bar{R}), \end{aligned} \qquad (3.57)$$

keeping in mind that $H_3 = \mathrm{d}B_2[P]$. Note that a similar constraint appears in the case of the 4D $\mathcal{N} = 1$ three-form supergravity where $\mathrm{i}(R - \bar{R})$ is also the divergence of a vector [69,70,72].

F. Complex two-form supergravity

In the framework of 4D $\mathcal{N} = 1$ Poincaré supersymmetry, the complex three-form multiplet was introduced by Gates and Siegel [66] as a conformal compensator for the Stelle-West formulation for 4D $\mathcal{N} = 1$ supergravity [61], in which the complex auxiliary field F was realized as the field strength of a complex gauge three-form. The name "complex three-form multiplet" was coined in [36]. This multiplet was recently used in [73] (under the name of "double three-form multiplet") to construct a super-Weyl invariant formulation for the complex three-form supergravity of [61], in the spirit of the super-Weyl invariant formulation [71] for three-form supergravity [66,70]. Here we propose a 3D $\mathcal{N} = 2$ cousin of the complex three-form multiplet.

A complex two-form multiplet coupled to conformal supergravity is described in terms of a covariantly chiral scalar Υ and its conjugate $\bar{\Upsilon}$, with Υ being defined by

$$\Upsilon = \bar{\Delta}\,\bar{\Sigma}, \qquad (3.58)$$

where Σ is a *complex linear* superfield constrained by

$$\bar{\Delta}\Sigma = 0. \qquad (3.59)$$

In general, if Σ is chosen to transform homogeneously under the super-Weyl transformations, its U(1) charge is determined by the super-Weyl weight [37]

$$\delta_\sigma\Sigma = w_\Sigma\sigma\Sigma \Rightarrow \mathcal{J}\Sigma = (1 - w_\Sigma)\Sigma. \qquad (3.60)$$

We wish the chiral scalar Υ to be super-Weyl primary, which means

$$\delta_\sigma\Upsilon = w_\Upsilon\sigma\Upsilon, \qquad \mathcal{J}\Upsilon = -w_\Upsilon\Upsilon, \qquad (3.61)$$

in accordance with (3.23). The transformation properties (3.60) and (3.61) are consistent with (3.58) only if $w_\Sigma = 1$, and therefore

$$\delta_\sigma\Upsilon = 2\sigma\Upsilon, \qquad \mathcal{J}\Upsilon = -2\Upsilon. \qquad (3.62)$$

The chiral scalar Υ defined by (3.58) is a gauge-invariant field strength under gauge transformations of the form

$$\delta_L\bar{\Sigma} = L_1 + \mathrm{i}L_2, \qquad \bar{\Delta}L_i = 0, \qquad \bar{L}_i = L_i \qquad (3.63)$$

For many purposes such as Lagrangian quantisation, it is advantageous to work with unconstrained superfields. The antilinear superfield $\bar{\Sigma}$ can always be represented as

$$\bar{\Sigma} = \mathcal{D}^\alpha\Psi_\alpha, \qquad (3.64)$$

for some unconstrained complex spinor prepotential Ψ_α. The chiral scalar Υ defined by (3.58) is a gauge-invariant field strength under gauge transformations of the form

$$\delta\Psi_\alpha = \bar{\mathcal{D}}_\alpha Z + \mathcal{D}^\beta\Lambda_{\alpha\beta}, \qquad \Lambda_{\alpha\beta} = \Lambda_{\beta\alpha}, \qquad (3.65)$$

with unconstrained complex gauge parameters Z and $\Lambda_{(\alpha\beta)}$. Here the gauge transformation generated by $\Lambda_{\alpha\beta}$ leaves the superfield (3.64) invariant. The gauge transformation generated by Z is equivalent to (3.63) when acting on $\bar{\Sigma}$. Any dynamical system with action $S[\Upsilon, \bar{\Upsilon}]$, which is realized in terms of the unconstrained prepotentials Ψ_α and $\bar{\Psi}_\alpha$, is a gauge theory with linearly dependent generators of an infinite stage of reducibility, following the terminology of the Batalin-Vilkovisky quantization [48].

Upon replacement $\Phi^4 \to \Upsilon$ in (3.31) the supergravity action turns into

$$S^{\text{complex two-form}}_{(1,1)\text{SG}} = -\frac{4}{\kappa}\int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}E(\bar{\Upsilon}\Upsilon)^{\frac{1}{4}}. \qquad (3.66)$$

This complex two-form supergravity allows no supersymmetric cosmological term, and the action involves no free parameter, unlike the actions for type I supergravity (3.31) and two-form supergravity (3.47). However, the equation of motion for Ψ_α is

$$\mathcal{D}_\alpha\mathbb{R} = 0, \qquad \mathbb{R} := \Upsilon^{-3/4}\bar{\Delta}\bar{\Upsilon}^{1/4}, \qquad (3.67)$$

and it implies that $\mathbb{R} = \mu = \text{const}$. Thus the complex cosmological parameter μ is generated dynamically.

G. Superform formulation for the complex two-form multiplet

Similarly to the real two-form multiplet, the complex two-form multiplet has a geometric superform origin. Let us consider the following complex super two-form:

$$C_2[\bar\Sigma] = 2\mathrm{i}\bar E_\alpha \wedge E^\alpha \bar\Sigma + E^\beta \wedge E^a (\gamma_a)_{\beta\gamma}\mathcal{D}^\gamma \bar\Sigma \\ + \bar E_\beta \wedge E^a (\gamma_a)^{\beta\gamma}\bar{\mathcal{D}}_\gamma \bar\Sigma \\ + \frac{\mathrm{i}}{8}\varepsilon_{abc}E^b \wedge E^a((\gamma^c)^{\rho\tau}[\mathcal{D}_\rho, \bar{\mathcal{D}}_\tau] - 8\mathcal{C}^c)\bar\Sigma. \qquad (3.68)$$

All coefficients C_{AB} of $C_2[\bar\Sigma] = \frac{1}{2}E^B \wedge E^A C_{AB}$ are descendants of $\bar\Sigma$. For the exterior derivative of $C_2[\bar\Sigma]$ we get

$$\mathrm{d}C_2[\bar\Sigma] = -2\bar E_\gamma \wedge \bar E_\beta \wedge E^a (\gamma_a)^{\beta\gamma}\Upsilon \\ - \frac{\mathrm{i}}{2}\bar E_\gamma \wedge E^b \wedge E^a \varepsilon_{abd}(\gamma^d)^{\gamma\delta}\mathcal{D}_\delta \Upsilon \\ + \frac{1}{24}E^c \wedge E^b \wedge E^a \varepsilon_{abc}(\mathcal{D}^2 - 16\bar R)\Upsilon \equiv \Xi_3[\Upsilon]. \qquad (3.69)$$

Thus, all coefficients of $\mathrm{d}C_2[\bar\Sigma]$ are descendants of Υ.

The expression for $\Xi_3[\Upsilon]$ is obtained from (3.28) by replacement $\mathcal{L}_c \to \Upsilon$. Since both $\mathcal{L}_c$ and Υ are chiral primary superfields of the same weight, we conclude that $\Xi_3[\Upsilon]$ is super-Weyl invariant, $\delta_\sigma \Xi_3[\Upsilon] = 0$. A stronger result is that the superform (3.68) is also super-Weyl invariant

$$\delta_\sigma C_2[\bar\Sigma] = 0. \qquad (3.70)$$

Our result $\Xi_3[\Upsilon] = \mathrm{d}C_2[\bar\Sigma]$ implies that the top components of the superforms $C_2[\bar\Sigma]$ and $\Xi_3[\Upsilon]$,

$$C_{ab} = \frac{\mathrm{i}}{4}\varepsilon_{abc}((\gamma^c)^{\rho\tau}[\mathcal{D}_\rho, \bar{\mathcal{D}}_\tau] - 8\mathcal{C}^c)\bar\Sigma, \qquad (3.71\mathrm{a})$$

$$\Xi_{abc} = \frac{1}{4}\varepsilon_{abc}(\mathcal{D}^2 - 16\bar R)\Upsilon, \qquad (3.71\mathrm{b})$$

are related to each other as

$$\Xi_{abc} = 3\mathcal{D}_{[a}C_{bc]} + 2\varepsilon_{abc}(\mathcal{D}^\alpha R + 2\mathrm{i}\bar{\mathcal{D}}^\alpha \mathcal{S})\mathcal{D}_\alpha \Sigma \\ + 2\varepsilon_{abc}(\bar{\mathcal{D}}^\alpha \bar R - 2\mathrm{i}\mathcal{D}^\alpha \mathcal{S})\bar{\mathcal{D}}_\alpha \Sigma. \qquad (3.72)$$

This confirms that the F component of Υ is the field strength of a complex two-form.

The gauge transformation of $\bar\Sigma$, Eq. (3.63), can be viewed as the following superform transformation:

$$\delta_L C_2[\bar\Sigma] = C_2[L_1 + \mathrm{i}L_2] \Rightarrow \delta_L \Xi_3[\Upsilon] = 0. \qquad (3.73)$$

This allows us to interpret $C_2[\bar\Sigma]$ as a gauge complex two-form and $\Xi_3[\Upsilon]$ as the corresponding gauge-invariant field strength.

The local U(1) and super-Weyl transformations may be used to choose the gauge $\Upsilon = 1$. In this gauge, $\mathcal{S} = 0$ and the algebra of covariant derivatives reduces to that of type I minimal supergravity [15,37] with one extra constraint: the torsion R is the divergence of a vector (related, by Poincaré duality, to a complex two-form potential). This follows from the fact that $\Xi_3[\Upsilon]$ in the gauge $\Upsilon = 1$ is given by

$$\Xi_3 = \mathrm{d}C_2 = -2\bar E_\gamma \wedge \bar E_\beta \wedge E^a(\gamma_a)^{\beta\gamma} \\ - \frac{2}{3}E^c \wedge E^b \wedge E^a \varepsilon_{abc}\bar R. \qquad (3.74)$$

IV. GREEN-SCHWARZ SUPERSTRINGS COUPLED TO TWO-FORM SUPERGRAVITY

In this section we will show that the $\mathcal{N} = 1$ and $\mathcal{N} = 2$ two-form supergravity theories provide consistent backgrounds for the Green-Schwarz superstring.

A. 3D $\mathcal{N} = 1$ Green-Schwarz superstring in curved superspace

In the case of 3D $\mathcal{N} = 1$ Green-Schwarz superstring, we draw on the results obtained by Bergshoeff *et al.* [13]. To describe the dynamics of a superstring propagating in a two-form supergravity background, we propose the following action:

$$S = T_2 \int \mathrm{d}^2\xi \left\{ \frac{1}{2}\sqrt{-\gamma}\gamma^{ij} L E_i{}^a E_j{}^b \eta_{ab} - \epsilon^{ij} E_i{}^B E_j{}^A B_{AB} \right\}. \qquad (4.1)$$

Here $\xi^i = (\tau, \sigma)$ are the world-sheet coordinates, γ_{ij} is the world-sheet metric, $\gamma = \det \gamma_{ij} = \frac{1}{2}\epsilon^{ij}\epsilon^{kl}\gamma_{ik}\gamma_{jl}$ with $\epsilon^{12} = \epsilon_{21} = 1$. Both the kinetic and Wess-Zumino terms in (4.1) involve certain target space fields associated with two-form supergravity, which are the supervielbein $E_M{}^A$ entering the action via the pull-back supervielbein

$$E_i{}^A = \partial_i z^M E_M{}^A, \qquad (4.2)$$

the super two-form B_{AB} and the compensator $L = \mathcal{D}^\alpha L_\alpha$ (the dilaton superfield).

The classical consistency of the Green-Schwarz superstring action requires that it be invariant under gauge fermionic transformations (κ-symmetry) of the form

$$\delta E^a = 0, \qquad \delta E^\alpha = 2(\gamma_a)^{\alpha\beta} L^{\frac{1}{4}} E_i^a \kappa_\beta^i, \qquad (4.3)$$

where we have defined $\delta E^A \coloneqq \delta z^M E_M{}^A$. The gauge parameter κ_α^i is a real 3D spinor and also a 2D vector satisfying the self-duality condition $(\gamma^{ij} - (-\gamma)^{-\frac{1}{2}}\epsilon^{ij})\kappa_{\alpha j} = 0$.

It can be shown that the action (4.1) is invariant under the gauge transformation (4.3) provided the super three-form

$H_3 = \mathrm{d}B_2$ is given by Eq. (2.37) and the world-sheet metric transforms as

$$\delta(\sqrt{-\gamma}\gamma^{ij}) = -2\sqrt{-\gamma}L^{\frac{1}{4}}(4\mathrm{i}E_k{}^\alpha - L^{-1}(\gamma_c)^{\alpha\beta}E_k{}^c\mathcal{D}_\beta L) \times (2\gamma^{k(i}\gamma^{j)l} - \gamma^{ij}\gamma^{kl})\kappa_{l\alpha}. \tag{4.4}$$

Let us point out that one can absorb the factor of $L^{1/4}$ into κ^i_α. After this redefinition, the action (4.1) and the κ transformations (4.3) and (4.4) become similar to those in [13].

The action (4.1) is invariant under arbitrary super-Weyl transformations of the target space, as a consequence of the relations (2.14a), (2.26) and (2.42). The super-Weyl gauge freedom may be fixed by setting $L = 1$.

B. 3D $\mathcal{N} = 2$ Green-Schwarz superstring in curved superspace

Now we turn to constructing the covariant action for the 3D $\mathcal{N} = 2$ superstring in a two-form supergravity background, and make use of the results by Grisaru *et al.* [12] concerning the 10D $\mathcal{N} = 2$ superstring. We propose the following superstring action

$$S = T_2\int \mathrm{d}^2\xi\left\{\frac{1}{2}\sqrt{-\gamma}\gamma^{ij}(\Phi\bar{\Phi})^2E_i{}^aE_j{}^b\eta_{ab} - \frac{1}{2}\epsilon^{ij}E_i{}^BE_j{}^AB_{AB}\right\}, \tag{4.5}$$

where the pull-back supervielbein $E_i{}^A$ is defined similarly to (4.2). The dilaton $(\Phi\bar{\Phi})^2$ is constructed in terms of the conformal compensator described by a weight-1/2 chiral scalar superfield Φ and its conjugate $\bar{\Phi}$. The concrete structure of Φ depends on the supergravity formulation chosen. In the case of three-form supergravity, the conformal compensator is the three-from multiplet, and then $\Phi^4 = \Pi = \bar{\Delta}P$. On the other hand, the choice $\Phi^4 = \Upsilon = \bar{\Delta}\bar{\Sigma}$ corresponds to complex three-form supergravity.

Both the real and complex two-form supergravities possess a real super two-form B_2 which can be used as the Kalb-Ramond field B_{AB} in the action (4.5). For two-form supergravity, the choice of B_2 is unique, modulo an overall numerical factor, and is given by $B_2[P]$, Eq. (3.50). In the case of complex two-form supergravity, there is a whole family of possible super two-forms that can be put in a one-to-one correspondence with a circle U(1). However, all these choices are equivalent. For concreteness, we choose B_2 to be the real or imaginary part of the super two-form $C_2[\bar{\Sigma}]$ given by Eq. (3.68).

Let us show that the action (4.5) is κ symmetric once we consider a background of real or complex two-form supergravity. We postulate the following κ-symmetry transformation

$$\delta E^a = 0, \quad \delta E^\alpha = \Phi^{\frac{3}{2}}\bar{\Phi}^{-\frac{1}{2}}E^a_i(\gamma_a)^{\alpha\beta}(\gamma^{ij}\kappa_{j\beta} - (-\gamma)^{-\frac{1}{2}}\epsilon^{ij}\bar{\kappa}_{j\beta}), \tag{4.6}$$

where $\delta E^A \coloneqq \delta z^M E_M{}^A$, $\delta\bar{E}_\alpha$ is given by the complex conjugate of δE^α and $\overline{\kappa^\alpha_i} \equiv \bar{\kappa}^\alpha_i$. We point out the relation

$$\delta E_i{}^A = \partial_i\delta E^A - 2\delta E^C E_i{}^B\mathbf{\Omega}_{[BC)}{}^A + \delta E^C E_i{}^B T_{BC}{}^A, \tag{4.7}$$

where we have used the definitions (3.8) and (3.9a). Then it is not difficult to show that the variation of the action is given by the following expression (compare with [12])

$$\begin{aligned}\delta S = T_2\int \mathrm{d}^2\xi\bigg\{&\frac{1}{2}\delta(\sqrt{-\gamma}\gamma^{ij})(\Phi\bar{\Phi})^2E_i{}^aE_j{}^b\eta_{ab} \\ &- \sqrt{-\gamma}\gamma^{ij}(\Phi\bar{\Phi})^2E_i{}^B\delta E^A T_{AB}{}^cE_j{}^d\eta_{cd} \\ &+ \sqrt{-\gamma}\gamma^{ij}E_i{}^aE_j{}^b\eta_{ab}(\Phi\bar{\Phi}^2\delta E^\alpha\mathcal{D}_\alpha\Phi + \Phi^2\bar{\Phi}\delta\bar{E}_\alpha\bar{\mathcal{D}}^\alpha\bar{\Phi}) \\ &+ \epsilon^{ij}E_i{}^CE_j{}^B\delta E^A H_{ABC}\bigg\},\end{aligned} \tag{4.8}$$

where $H_3 \coloneqq \frac{1}{6}E^C\wedge E^B\wedge E^A H_{ABC} = \mathrm{d}B_2$.

Let us first consider the case of two-form supergravity, with $\Phi^4 = \Pi$. To show that the variation (4.8) vanishes, we have to make use of the geometrical data specific for the two-form supergravity. The only nonvanishing torsion appearing in the variation (4.8) is the dimension-zero torsion which is

$$T_\alpha{}^{\beta c} = -2\mathrm{i}(\gamma^c)_\alpha{}^\beta. \tag{4.9}$$

The nontrivial components of the super three-form H_3 given by Eq. (3.51), which enter the variation (4.8), are

$$H_{\alpha\beta c} = -2\mathrm{i}(\gamma_c)_{\alpha\beta}\bar{\Phi}^4, \qquad H_{ab\gamma} = -\frac{1}{2}\varepsilon_{abd}(\gamma^d)_{\gamma\delta}\bar{\mathcal{D}}^\delta\bar{\Phi}^4 \tag{4.10}$$

together with their complex conjugates. Substituting the expressions (4.9) and (4.10) into the variation (4.8) and using the identities

$$(\gamma_a)_\alpha{}^\gamma(\gamma_b)_\gamma{}^\beta = \eta_{ab}\delta^\beta_\alpha + \varepsilon_{abc}(\gamma^c)_\alpha{}^\beta, \quad \gamma^{i[j}\gamma^{k]l} = \frac{1}{2}\epsilon^{jk}\epsilon^{il}\gamma^{-1}, \tag{4.11}$$

one can show that the Green-Schwarz action is indeed invariant provided the κ-transformation law of the world-sheet metric is postulated to be

$$\begin{aligned}\delta(\sqrt{-\gamma}\gamma^{ij}) = {}&2\sqrt{-\gamma}(2\gamma^{k(i}\gamma^{j)l} - \gamma^{ij}\gamma^{kl})\Phi^{\frac{3}{2}}\bar{\Phi}^{-\frac{1}{2}} \\ &\times(2\mathrm{i}\bar{E}_{k\alpha} + \Phi^{-1}(\gamma_c)_{\alpha\beta}E_k{}^c\mathcal{D}^\beta\Phi)\kappa^\alpha_l \\ &- 2(\epsilon^{k(i}\gamma^{j)l} + \epsilon^{l(i}\gamma^{j)k})\Phi^{-\frac{1}{2}}\bar{\Phi}^{\frac{3}{2}} \\ &\times(2\mathrm{i}E_k{}^\alpha + \bar{\Phi}^{-1}(\gamma_c)^{\alpha\beta}E_k{}^c\bar{\mathcal{D}}_\beta\bar{\Phi})\kappa_{l\alpha} + \text{c.c.}\end{aligned} \tag{4.12}$$

The superstring action constructed is invariant under arbitrary super-Weyl transformations of the background fields, as follows from the transformation laws (3.11), (3.45) and (3.55).

It is clear that the analysis in the case of the complex two-form supergravity is identical to the one presented above with the only difference that Φ^4 is replaced with Υ instead of Π. In fact, in proving the κ-invariance of the action only the real closed super three-form H_3 enters the computations rather than its potential B_2. Therefore we have proven that both the real and complex two-form supergravities are consistent backgrounds for the 3D $\mathcal{N}=2$ Green-Schwarz superstring.

V. GOLDSTINO SUPERFIELDS COUPLED TO SUPERGRAVITY

In this section we present various models for spontaneously broken local $\mathcal{N}=1$ and $\mathcal{N}=2$ supersymmetry that are obtained by coupling the off-shell supergravity theories, which have been described in the previous sections, to nilpotent Goldstino superfields. It should be pointed out that the first model for spontaneously broken local $\mathcal{N}=1$ supersymmetry was constructed in 1977 [74] by coupling on-shell $\mathcal{N}=1$ supergravity to the Volkov-Akulov action.

We often make use of the notion of reducible and irreducible Goldstino superfields introduced in [25]. By definition, an irreducible Goldstino superfield contains Goldstone spin-$\frac{1}{2}$ fermion(s) as the only independent component field(s). Every reducible Goldstino superfield also contains some auxiliary field(s) along with the Goldstone spin-$\frac{1}{2}$ fermion(s).

A. $\mathcal{N}=1$ Goldstino superfields

A reducible Goldstino multiplet is described by a real scalar superfield X subject to the nilpotency constraint

$$X^2=0. \tag{5.1}$$

We also require $\mathcal{D}^2X$ to be nowhere vanishing so that $(\mathcal{D}^2X)^{-1}$ is well defined, and therefore (5.1) implies

$$X=-\frac{\mathcal{D}^\alpha X\mathcal{D}_\alpha X}{\mathcal{D}^2X}. \tag{5.2}$$

As a result, X has two independent component fields, a spinor $\psi_\alpha(x)$ and a real auxiliary scalar $F(x)$, that may be defined as $\mathrm{i}\psi_\alpha=\mathcal{D}_\alpha X|$ and $\mathrm{i}F=-\frac{1}{4}\mathcal{D}^2X|$, where F^{-1} is well defined. The lowest component of the Goldstino superfield, $X|$, is a composite field as a consequence of (5.2).

We postulate X to be super-Weyl primary of weight $1/2$, which means the super-Weyl transformation law of X is

$$\delta_\sigma X=\frac{1}{2}\sigma X. \tag{5.3}$$

The Goldstino superfield action is

$$S_X=\mathrm{i}\int \mathrm{d}^3x\mathrm{d}^2\theta E\left\{\frac{\mathrm{i}}{2}\mathcal{D}^\alpha X\mathcal{D}_\alpha X+2f'\varphi^3X\right\}, \tag{5.4}$$

for some nonzero parameter f' which characterizes the scale of supersymmetry breaking. The second term in the action involves the compensator, φ, of $\mathcal{N}=1$ AdS supergravity; see Sec. II C. The action is super-Weyl invariant.

The nilpotency constraint (5.1) is invariant under local arbitrary rescalings of X,

$$X\to\tilde{X}=\mathrm{e}^\rho X, \tag{5.5}$$

for any real scalar ρ. Such a rescaling (5.5) acts on the component fields of X as

$$\psi_\alpha\to\tilde{\psi}_\alpha=\mathrm{e}^{\rho|}\left(\psi_\alpha+\frac{\psi^2}{4F}(\mathcal{D}_\alpha\rho)|\right), \tag{5.6a}$$

$$F\to\tilde{F}=\mathrm{e}^{\rho|}\left(F-\frac{1}{2}\psi^\alpha(\mathcal{D}_\alpha\rho)|-\frac{\psi^2}{16F}(\mathcal{D}^2\rho)|\right). \tag{5.6b}$$

Each of these transformations is a local rescaling accompanied by a nilpotent shift of the field under consideration, and therefore $\tilde{F}^{-1}$ is well defined. Requiring the action (5.4) to be stationary under (5.5) (following the 4D works [26,27]) gives the constraint

$$f'\varphi^3X=\frac{\mathrm{i}}{2}X\mathcal{D}^2X=X\Delta X,\qquad \Delta:=\frac{\mathrm{i}}{2}\mathcal{D}^2+\mathcal{S}. \tag{5.7}$$

Here $X\Delta X$ is manifestly a super-Weyl primary. As follows from (5.6a) and (5.6b), the F component of the nonlinear constraint (5.7) is equivalent to a sum of the equation of motion for F and a linear combination of the equations of motion for ψ_α.

Consider an irreducible Goldstino superfield $\mathcal{X}$ constrained by

$$\mathcal{X}^2=0,\qquad f'\varphi^3\mathcal{X}=\mathcal{X}\Delta\mathcal{X}, \tag{5.8}$$

with $\Delta\mathcal{X}$ being nowhere vanishing. This superfield is irreducible because the Goldstino $\psi_\alpha=-\mathrm{i}\mathcal{D}_\alpha X|$ is the only independent component field of $\mathcal{X}$. Indeed, the second constraint in (5.8) proves to express the auxiliary field F in terms of the Goldstino; see Appendix A. The dynamics of $\mathcal{X}$ is described by the action

$$S_{\mathcal{X}}=\mathrm{i}f'\int \mathrm{d}^3x\mathrm{d}^2\theta E\varphi^3\mathcal{X}, \tag{5.9}$$

which is obtained from (5.4) by making use of the nonlinear constraint obeyed by $\mathcal{X}$. The Goldstino theories (5.4) and (5.9) prove to be equivalent, which may be shown by extending the 4D analyses given in [75–77].[10] This issue is discussed in more detail in Appendix A. The flat-superspace limit of our Goldstino theory defined by Eqs. (5.8) and (5.9) is analogous to the 2D $\mathcal{N}=1$ Goldstino model pioneered by Roček [79].

It is not difficult to check that the constraints (5.8) are satisfied if

$$\mathcal{X} = f'\varphi^3 \frac{X}{\Delta X}, \tag{5.10}$$

where X is only subject to the nilpotency constraint (5.1). The important property of $\mathcal{X}$ defined by (5.10) is that it is invariant under arbitrary local rescalings (5.5),

$$\delta_\rho X = \rho X \Rightarrow \delta_\rho \mathcal{X} = 0, \tag{5.11}$$

for arbitrary real superfield ρ, compare with the 4D analysis in [27]. This remarkable property actually can be explained by recalling at the component transformation law (5.6b) implied by (5.5). The point is the superfield transformation (5.5) implies an arbitrary local rescaling of the auxiliary field of X, $F \to e^{\rho|}F$. Since $\mathcal{X}$ does not contain an independent auxiliary field, it should remain invariant under (5.5).

Let us consider the model for spontaneously broken local supersymmetry which is obtained by coupling the supergravity theory (2.21) to the Goldstino superfield X. The dynamics of this system is described by the action

$$S = S_{\rm SG} + S_X. \tag{5.12}$$

The component structure of this theory will be discussed elsewhere. Here we only present the corresponding cosmological constant. It is obtained upon eliminating all the auxiliary fields, and is given by

$$\Lambda = \frac{1}{2}f'^2\kappa + \Lambda_{\rm AdS} = \frac{1}{2}f'^2\kappa - 4\lambda^2. \tag{5.13}$$

The supergravity-matter system (5.12) may be reformulated as a model for nilpotent supergravity. Varying (5.12) with respect to the compensator φ gives the equation

$$\mathbb{S} - \lambda = -\frac{3}{8}f'\kappa\frac{X}{\varphi}, \tag{5.14}$$

where $\mathbb{S}$ is defined by (2.22a). Since X is nilpotent, the equation of motion implies

$$(\mathbb{S} - \lambda)^2 = 0. \tag{5.15}$$

[10] Reference [76] is a considerably generalized and extended version of [78].

Making use of (5.14) in order to express X in terms of the supergravity fields, the action (5.12) can be recast as a higher-derivative supergravity theory

$$S = \frac{8{\rm i}}{3\kappa}\int {\rm d}^3x{\rm d}^2\theta E\varphi^4\left\{\mathbb{S} + \frac{\lambda}{2}\right\} - \frac{32}{(3f'\kappa)^2}\int {\rm d}^3x{\rm d}^2\theta E\varphi^2\mathcal{D}^\alpha\mathbb{S}\mathcal{D}_\alpha\mathbb{S}. \tag{5.16}$$

In four dimensions, various approaches to nilpotent $\mathcal{N}=1$ supergravity were developed, e.g., in [16,17,20,23,26, 80,81]. Our presentation here is similar to [20].

To conclude this subsection, we note that the nilpotent Goldstino superfield X can also be coupled to the two-form supergravity constructed in Sec. II D. For this we should simply replace the action (5.4) with

$$\tilde{S}_X = {\rm i}\int {\rm d}^3x{\rm d}^2\theta E\left\{\frac{\rm i}{2}\mathcal{D}^\alpha X\mathcal{D}_\alpha X + 2f'L^{3/4}X\right\}. \tag{5.17}$$

Then, the equation of motion (5.14) turns into

$$\mathcal{D}_\alpha\left(\mathbb{S} + \frac{3}{8}f'\kappa\frac{X}{L^{1/4}}\right) = 0, \tag{5.18}$$

where $\mathbb{S}$ is now defined as in (2.32).

B. Reducible $\mathcal{N}=2$ Goldstino superfields

The family of nilpotent $\mathcal{N}=2$ Goldstino superfields, both reducible and irreducible, is more populous than in the $\mathcal{N}=1$ case.[11] However practically all 3D $\mathcal{N}=2$ Goldstino superfields can be obtained from the known 4D $\mathcal{N}=1$ Goldstino supermultiplets by dimensional reduction, at least in the flat superspace case. This is why our discussion of nilpotent $\mathcal{N}=2$ Goldstino superfields will be reasonably concise. We will try to emphasize only conceptual constructions and those results that are truly new or have not received much discussion in the 4D case.

1. Nilpotent chiral scalar superfield

To begin with, we consider a 3D $\mathcal{N}=2$ locally supersymmetric counterpart of the reducible Goldstino superfield introduced by Casalbuoni *et al.* [84] and independently by Komargodski and Seiberg [75]. We choose it to be a covariantly chiral scalar X of super-Weyl weight $+1/2$,

$$\bar{\mathcal{D}}_\alpha X = 0, \qquad \delta_\sigma X = \frac{1}{2}\sigma X \Rightarrow \mathcal{J}X = -\frac{1}{2}X, \tag{5.19}$$

which is subject to the nilpotency constraint

[11] One can also introduce spinor Goldstino superfields, by analogy with the 4D $\mathcal{N}=1$ constructions given in [29,82,83]. However such superfields are not particularly useful in the supergravity framework.

$$X^2 = 0, \tag{5.20}$$

in conjunction with the requirement that the descendant $\mathcal{D}^2 X$ is nowhere vanishing. The nilpotency condition implies that X has two independent component fields, a complex Goldstino $\psi_\alpha(x)$ and a complex auxiliary field $F(x)$, which we define as $\psi_\alpha = \frac{1}{\sqrt{2}}\mathcal{D}_\alpha X|$ and $F = -\frac{1}{4}\mathcal{D}^2 X|$, respectively.

The constraints on X do not make use of any supergravity compensator, which means that X is defined in any conformal supergravity background. However, a compensator is required in order to define an action functional for the Goldstino superfield. Here we choose the chiral compensator Φ corresponding to the minimal (1,1) AdS supergravity described in Sec. III C 1. The dynamics of this supermultiplet is described by the action

$$S_X = \int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}E\bar{X}X - \left\{ f\int \mathrm{d}^3x\mathrm{d}^2\theta\mathcal{E}\Phi^3 X + \text{c.c.}\right\}, \tag{5.21}$$

in which the supersymmetry breaking parameter, f, may be chosen to be real.

We now consider a model for spontaneously broken $\mathcal{N}=2$ local supersymmetry which is obtained by coupling the Goldstino superfield X to the minimal (1,1) AdS supergravity reviewed in Sec. III C 1. The complete action is

$$S = S^{\text{minimal}}_{(1,1)\text{SG}} + S_X, \tag{5.22}$$

where the supergravity action $S^{\text{minimal}}_{(1,1)\text{SG}}$ is given by Eq. (3.31). This theory proves to generate the following cosmological constant

$$\Lambda = f^2\kappa + \Lambda_{\text{AdS}} = f^2\kappa - 4|\mu|^2. \tag{5.23}$$

Varying the action (5.22) with respect to the chiral compensator gives the equation of motion

$$\mathbb{R} - \mu = -\frac{3}{4}f\kappa\frac{X}{\Phi}, \tag{5.24}$$

where the super-Weyl neutral chiral scalar $\mathbb{R}$ is defined by (3.33a). Since X is nilpotent, the above equation implies

$$(\mathbb{R}-\mu)^2 = 0, \tag{5.25}$$

and thus the torsion superfield $(\mathbb{R}-\mu)$ becomes nilpotent. Equation (5.24) can be used to eliminate X and $\bar{X}$ from the action (5.22), resulting with the following geometric higher-derivative supergravity action:

$$S = -\frac{4}{3\kappa}\int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}E\bar{\Phi}\Phi - \left\{\frac{\mu}{3\kappa}\int \mathrm{d}^3x\mathrm{d}^2\theta\mathcal{E}\Phi^4 + \text{c.c.}\right\} + \left(\frac{4}{3f\kappa}\right)^2\int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}E\bar{\Phi}\Phi|\mathbb{R}-\mu|^2. \tag{5.26}$$

Here the expression in the first line differs from the supergravity action (3.31) only by new values for the parameters involved, $\kappa \to 3\kappa$ and $\mu \to -\mu$. The functional form of the action (5.26) differs from its 4D $\mathcal{N}=1$ counterpart derived in [20] (see also [26]) in the sense that the supersymmetric Einstein-Hilbert term completely canceled out in the latter case.

The nilpotency condition (5.20) is preserved if X is locally rescaled,

$$X \to \mathrm{e}^{\tau}X, \qquad \bar{\mathcal{D}}_\alpha\tau = 0, \tag{5.27}$$

where the parameter τ is neutral under U(1). Requiring the action (5.21) to be stationary under such rescalings of X (compare with [26]) gives the nonlinear equation

$$X\bar{\Delta}\bar{X} = f\Phi^3 X. \tag{5.28}$$

This nonlinear constraint proves to express the auxiliary field F in terms of the Goldstini ψ_α and $\bar{\psi}_\alpha$ and their derivatives, see Appendix B.

The constraints (5.19), (5.20) and (5.28) define an irreducible Goldstino superfield $\mathcal{X}$,

$$\bar{\mathcal{D}}_\alpha\mathcal{X} = 0, \qquad \delta_\sigma\mathcal{X} = \frac{1}{2}\sigma\mathcal{X}, \qquad \mathcal{X}^2 = 0, \qquad \mathcal{X}\bar{\Delta}\bar{\mathcal{X}} = f\Phi^3\mathcal{X}. \tag{5.29}$$

It is the 3D $\mathcal{N}=2$ analog of the 4D $\mathcal{N}=1$ Goldstino superfield used by Lindström and Roček [28] in their off-shell model for spontaneously broken $\mathcal{N}=1$ local supersymmetry.[12] The corresponding action can be given in two different but equivalent forms:

$$S_{\mathcal{X}} = -\int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}E\bar{\mathcal{X}}\mathcal{X} = -f\int \mathrm{d}^3x\mathrm{d}^2\theta\mathcal{E}\Phi^3\mathcal{X}. \tag{5.30}$$

So far we have considered the coupling of the nilpotent Goldstino superfield X to the minimal (1,1) AdS

[12] Reference [28] is the first work on off-shell de Sitter supergravity in four dimensions. Terminology "de Sitter supergravity" was introduced by Bergshoeff *et al.* [18]. The only difference between the supergravity models put forward in [18,28] is that they made use of different Goldstino superfields—the 4D $\mathcal{N}=1$ analogs of X and $\mathcal{X}$, respectively. The two supergravity models are equivalent on shell [25]. However, the power of the approach advocated in [18] is that the nilpotency condition $X^2=0$ is model independent, which implies that the Goldstino superfield can be readily coupled to matter multiplets.

supergravity. Its coupling to the two-form (or complex two-form) supergravity is obtained simply by replacing the chiral compensator Φ in (5.21) with $\Pi^{1/4}$ ($\Upsilon^{1/4}$ in the case of complex three-form supergravity). However, there is a different universal approach to couple a nilpotent chiral supermultiplet to any off-shell supergravity. It consists in replacing X, defined by (5.19) and (5.20), with a super-Weyl primary scalar $\mathfrak{X}$ with the properties

$$\bar{\mathcal{D}}_\alpha \mathfrak{X} = 0, \qquad \delta_\sigma \mathfrak{X} = 2\sigma \mathfrak{X}, \qquad \mathfrak{X}^2 = 0. \tag{5.31}$$

The action (5.21) has to be replaced with

$$S_{\mathfrak{X}} = \int d^3x d^2\theta d^2\bar{\theta} E \frac{\bar{\mathfrak{X}}\mathfrak{X}}{W} - \left\{ f \int d^3x d^2\theta \mathcal{E} \mathfrak{X} + \text{c.c.} \right\}, \tag{5.32}$$

where W is a real scalar primary superfield of weight $+3$ such that (i) it is nowhere vanishing, and (ii) it is a composite of the supergravity compensators. In particular, $W = (\bar{\Phi}\Phi)^3$ in the case of minimal (1,1) AdS supergravity, $W = L^3$ for (2,0) AdS supergravity, $W = (\bar{\Pi}\Pi)^{3/4}$ for the two-form supergravity, and so on.

2. Nilpotent real scalar superfield

We now introduce a 3D $\mathcal{N} = 2$ analog of the reducible Goldstino superfield proposed in [26]. It is a real scalar superfield subject to the nilpotency conditions:

$$V^2 = 0, \tag{5.33a}$$

$$V \mathcal{D}_A \mathcal{D}_B V = 0, \tag{5.33b}$$

$$V \mathcal{D}_A \mathcal{D}_B \mathcal{D}_C V = 0. \tag{5.33c}$$

The super-Weyl transformation of V is postulated to be

$$\delta_\sigma V = \sigma V. \tag{5.34}$$

We also require that the descendant $\frac{1}{2}\{\Delta, \bar{\Delta}\} V$ is nowhere vanishing. The nilpotency conditions (5.33) imply that V has three independent component fields (see Appendix B for more details) that may be chosen as follows: the complex Goldstino $\chi_\alpha(x) = \frac{1}{\sqrt{2}} \mathcal{D}_\alpha \bar{\Delta} V|$, its conjugate $\bar{\chi}_\alpha(x)$ and a real auxiliary field $D(x) = \frac{1}{2}\{\Delta, \bar{\Delta}\} V|$, with D^{-1} being well defined.

The constraints (5.33) imposed on V do not make use of any supergravity compensator, which means that V is defined in any conformal supergravity background. However, a compensator is required in order to formulate an action functional for the Goldstino superfield. As in the previous section, here we again choose the chiral compensator Φ corresponding to the minimal (1,1) AdS supergravity (minimal type I supergravity) described in Sec. III C 1. The dynamics of the nilpotent superfield V is described by the action

$$S_V = \int d^3x d^2\theta d^2\bar{\theta} E \left\{ \frac{|\Delta V|^2}{(\bar{\Phi}\Phi)^3} - 2fV \right\}, \tag{5.35}$$

with f the supersymmetry breaking parameter.[13]

The constraints (5.33) are preserved if V is locally rescaled,

$$V \to e^\rho V, \tag{5.36}$$

for any real scalar ρ. Requiring the action (5.35) to be stationary under such rescalings of V gives the nonlinear equation

$$\frac{1}{2} V \{\Phi^{-3}\bar{\Delta}, \bar{\Phi}^{-3}\Delta\} V = fV. \tag{5.37a}$$

Due to the constraints (5.33), this may equivalently be rewritten as

$$V\Phi^{-3}\bar{\Delta}(\bar{\Phi}^{-3}\Delta V) = V\bar{\Phi}^{-3}\Delta(\Phi^{-3}\bar{\Delta}V) = fV. \tag{5.37b}$$

This nonlinear constraint proves to express the auxiliary field D in terms of the Goldstini.

The constraints (5.33) and (5.37) define an irreducible Goldstino superfield $\mathcal{V}$. It is a 3D $\mathcal{N} = 2$ counterpart of the Goldstino superfield introduced in [25]. The corresponding action can be written in two equivalent forms

$$S_{\mathcal{V}} = -\int d^3x d^2\theta d^2\bar{\theta} E \frac{|\Delta \mathcal{V}|^2}{(\bar{\Phi}\Phi)^3} = -f \int d^3x d^2\theta d^2\bar{\theta} E \mathcal{V}. \tag{5.38}$$

The Goldstino models (5.35) and (5.38) are equivalent on the mass shell.

The irreducible Goldstino superfields $\mathcal{X}$ and $\mathcal{V}$ are related to each other as follows

$$f\mathcal{V} = \bar{\mathcal{X}}\mathcal{X}, \tag{5.39a}$$

$$\mathcal{X} = \Phi^{-3}\bar{\Delta}\mathcal{V}. \tag{5.39b}$$

These relations are analogous to those given in [28] in the 4D case.

3. Relating X and V

Starting from the nilpotent chiral superfield X described in Sec. V B 1, we define

$$fV := \bar{X}X, \tag{5.40}$$

[13]Had we chosen V to be an *unconstrained* real scalar superfield, the action (5.35) would have described the dynamics of a two-form multiplet (with a linear superpotential) coupled to the minimal type I supergravity.

as a generalization of (5.39). The superfield V introduced satisfies all the requirements imposed on the nilpotent Goldstino superfield V in Sec. V B 2. One of the two auxiliary fields of X does not contribute to the right-hand side of (5.40).

Implementing the field redefinition (5.40) in the Goldstino superfield action (5.35) leads to the following higher-derivative action:

$$S_{\rm HD}[X,\bar{X}] = \int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}E\left\{\frac{1}{f^2}\frac{|X\Delta X|^2}{(\bar{\Phi}\Phi)^3} - 2\bar{X}X\right\}. \tag{5.41}$$

Its important property is

$$S_{\rm HD}[\mathcal{X},\bar{\mathcal{X}}] = S_{\mathcal{X}}, \tag{5.42}$$

with $S_{\mathcal{X}}$ given by (5.30). Unlike the Goldstino action (5.21), (5.41) is invariant under the discrete transformation $X \to -X$. The model (5.41) will be studied in more detail in Appendix C.

4. Nilpotent two-form Goldstino superfield

As a generalization of the 4D $\mathcal{N}=1$ models proposed in [24,27], we introduce a nilpotent two-form Goldstino multiplet. It is described by a chiral scalar superfield

$$Y = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)U, \qquad \bar{U} = U, \tag{5.43a}$$

which is constrained to be nilpotent,

$$Y^2 = 0. \tag{5.43b}$$

The prepotential U in (5.43) is defined modulo gauge transformations of the form

$$\delta_L U = L, \qquad \bar{\Delta}L = 0, \qquad \bar{L} = L, \tag{5.44}$$

and Y and $\bar{Y}$ are gauge-invariant field strengths.

The super-Weyl transformation of the prepotential U is

$$\delta_\sigma U = \sigma U, \tag{5.45}$$

which implies

$$\delta_\sigma Y = 2\sigma Y. \tag{5.46}$$

To describe dynamics of the nilpotent two-form multiplet, we propose the action

$$S_Y = \int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}E\frac{\bar{Y}Y}{(\bar{\Phi}\Phi)^3} - \left\{f\int \mathrm{d}^3x\mathrm{d}^2\theta\mathcal{E}Y + \text{c.c.}\right\}. \tag{5.47}$$

The component structure of this model will be discussed in Appendix B.4. Here we would like just to point out that the Goldstino superfield Y contains two independent auxiliary fields, $F = H + \mathrm{i}G$, of which H is a scalar and G is the divergence of a vector. In supergravity, both H and G produce positive contributions to the cosmological constant. While the contribution from H is *universal* and uniquely determined by the parameter of the supersymmetry breaking f in (5.47), the contribution from G is *dynamical*. We believe that the latter may be used to neutralize the negative contribution from the supersymmetric cosmological term.

C. Irreducible $\mathcal{N}=2$ Goldstino superfields

Using the nilpotent chiral superfield X described in Sec. V B 1, we introduce a composite superfield

$$\Sigma = f\frac{\bar{X}}{\bar{\Delta}\bar{X}}. \tag{5.48}$$

It has the following transformation properties:

$$\delta_\sigma\Sigma = -\sigma\Sigma, \qquad \mathcal{J}\Sigma = 2\Sigma, \tag{5.49}$$

as well as it identically satisfies the improved linear constraint

$$\bar{\Delta}\Sigma = f, \tag{5.50a}$$

compare with (3.25). By construction, it is nilpotent,

$$\Sigma^2 = 0. \tag{5.50b}$$

It also obeys the nonlinear constraint

$$f\mathcal{D}_\alpha\Sigma = -\frac{1}{4}\Sigma(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha\Sigma, \tag{5.50c}$$

which is equivalent to

$$f\mathcal{D}_\alpha\Sigma = -\mathrm{i}\Sigma\mathcal{D}_{\alpha\beta}\bar{\mathcal{D}}^\beta\Sigma. \tag{5.51}$$

Thus Σ is a 3D $\mathcal{N}=2$ counterpart of the irreducible Goldstino superfield introduced in [30]. Unlike other irreducible Goldstino superfields, such as $\mathcal{X}$ and $\mathcal{V}$, the constraints obeyed by Σ, Eq. (5.50), do not make use of any supergravity compensator. In other words, Σ couples to conformal supergravity.

The remarkable feature of Σ and its conjugate is that these superfields are invariant under local rescalings of X and its conjugate, Eq. (5.27),

$$\delta_\tau X = \tau X \Rightarrow \delta_\tau\Sigma = 0, \qquad \bar{\mathcal{D}}_\alpha\tau = 0; \tag{5.52}$$

compare with [27]. In complete analogy with the 4D $\mathcal{N}=1$ case [30], every irreducible Goldstino superfield is a descendant of Σ and $\bar{\Sigma}$, for instance

$$f\mathcal{V} = (\bar{\Phi}\Phi)^3\bar{\Sigma}\Sigma. \tag{5.53}$$

Therefore we conclude that all irreducible Goldstino superfields are invariant under local rescalings (5.27).

As pointed out above, the Goldstino superfields Σ and $\bar{\Sigma}$ couple to conformal supergravity. Relation (5.53) clearly shows that the conformal compensators have to be used in order to define $\mathcal{V}$ as a composite superfield constructed from Σ and $\bar{\Sigma}$.[14]

VI. CONCLUDING COMMENTS

The results obtained in this work may lead to several interesting developments including the following:

(i) The work by Ovrut and Waldram [70] provided membrane solutions in the 4D $\mathcal{N} = 1$ three-form supergravity. In a similar way, the two-form supergravity theories described in the present paper should possess string solutions. It is of interest to derive such solutions explicitly.

(ii) In three dimensions, consistent models for massive supergravity can be constructed by adding certain higher-derivative terms to the standard supergravity action. These include $\mathcal{N} = 1$ and $\mathcal{N} = 2$ topologically massive [59,85,86] and new massive [46,87–89] supergravity theories. Coupling these theories to the Goldstino superfields described in Sec. V should give consistent models for spontaneously broken massive supergravity.

(iii) It is of interest to construct $\mathcal{N} = 3$ and $\mathcal{N} = 4$ Goldstino superfields, as an extension of the 4D results given in [26,90]. The $\mathcal{N} = 3$ case is especially interesting since it has no 4D analog.

(iv) Since we formulated the 3D Green-Schwarz superstring action, with $\mathcal{N} = 1$ and $\mathcal{N} = 2$ spacetime supersymmetry, in off-shell supergravity backgrounds, the quantum superstring analysis given in [3,4] may be extended from the Minkowski superspace to other maximally supersymmetric backgrounds including the AdS one.

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[14]However, if $\mathcal{V}$ is replaced with $\mathfrak{V} := \mathcal{V}(\bar{\Phi}\Phi)^{-3}$, then the constraints obeyed by $\mathfrak{V}$ are independent of any compensator, and therefore $\mathfrak{V}$ couples to conformal supergravity.

APPENDIX A: COMPONENT STRUCTURE OF $\mathcal{N} = 1$ GOLDSTINO MODELS

In this appendix we will discuss the component actions for the $\mathcal{N} = 1$ Goldstino models introduced in Sec. V A. For simplicity we will perform our analysis in flat superspace.

Here we specialize the superspace $\mathcal{M}^{3|2}$ of Sec. II A to be the standard $\mathcal{N} = 1$ Minkowski superspace $\mathbb{M}^{3|2}$ parametrized by Cartesian real coordinates $z^A = (x^a, \theta^\alpha)$. The covariant derivatives $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha)$ on $\mathcal{M}^{3|2}$, defined by Eq. (2.2), become the flat-superspace ones

$$D_A = (\partial_a, D_\alpha), \quad D_\alpha = \partial_\alpha + \mathrm{i}(\gamma^a)_{\alpha\beta}\theta^\beta\partial_a = \partial_\alpha + \mathrm{i}\theta^\beta\partial_{\alpha\beta}. \tag{A1}$$

Making use of the anticommutation relation

$$\{D_\alpha, D_\beta\} = 2\mathrm{i}\partial_{\alpha\beta} \tag{A2}$$

allows us to obtain a number of useful properties including the following:

$$D_\alpha D_\beta = \mathrm{i}\partial_{\alpha\beta} + \frac{1}{2}\varepsilon_{\alpha\beta}D^2, \qquad D^\alpha D_\beta D_\alpha = 0,$$
$$D^2D^2 = -4\Box. \tag{A3}$$

We recall that $D^2 = D^\alpha D_\alpha$. Given a supersymmetric action

$$S = \mathrm{i}\int \mathrm{d}^3x\mathrm{d}^2\theta\mathcal{L}, \qquad \bar{\mathcal{L}} = \mathcal{L}, \tag{A4}$$

with some superfield Lagrangian $\mathcal{L}$, the component action is computed by the rule

$$S = -\frac{\mathrm{i}}{4}\int \mathrm{d}^3xD^2\mathcal{L}|. \tag{A5}$$

As usual, the bar projection is defined by $U| := U|_{\theta=0}$, for any superfield $U(x,\theta)$.

Let us now consider a real scalar superfield X. We define its *real* component fields $\phi(x)$, $\psi_\alpha(x)$ and $F(x)$ as

$$\phi = X|, \qquad \mathrm{i}\psi_\alpha = D_\alpha X|, \qquad \mathrm{i}F = -\frac{1}{4}D^2X|. \tag{A6}$$

Introducing a free supersymmetric model with action

$$S_X = \mathrm{i}\int \mathrm{d}^3x\mathrm{d}^2\theta\left\{\frac{\mathrm{i}}{2}D^\alpha XD_\alpha X + 2f'X\right\}, \tag{A7}$$

at the component level we obtain

$$S_X = -\int d^3x\left\{\frac{1}{2}\partial^a\phi\partial_a\phi + \frac{i}{2}\psi^\alpha\partial_{\alpha\beta}\psi^\beta - 2F^2 + 2f'F\right\}. \tag{A8}$$

Let us turn to the component analysis of the $\mathcal{N}=1$ Goldstino model (5.4) in flat superspace. To describe a reducible Goldstino multiplet, we subject X to the nilpotency condition

$$X^2 = 0 \tag{A9}$$

and assume that F^{-1} is well defined. The nilpotency constraint allows us to solve for ϕ in terms of the Goldstino ψ and the auxiliary field F:

$$\phi = \frac{i\psi^2}{4F}. \tag{A10}$$

With the constraint (A9) imposed, the supersymmetric action (A7) defines a nonlinear interacting theory. Making use of (A8) and (A10) leads to the following action:

$$S_X = -\int d^3x\left\{\frac{i}{2}\psi^\alpha\partial_{\alpha\beta}\psi^\beta + \frac{1}{32}\frac{\psi^2}{F}\Box\frac{\psi^2}{F} - 2F^2 + 2f'F\right\}. \tag{A11}$$

The equation of motion for F is

$$\frac{\delta S_X}{\delta F} = 4F + \frac{1}{16}\frac{\psi^2}{F^2}\Box\frac{\psi^2}{F} - 2f' = 0. \tag{A12}$$

This equation can be solved by repeated substitution which gives

$$F = \frac{f'}{2} - \frac{1}{8f'^3}\psi^2\Box\psi^2. \tag{A13}$$

Substituting it back into (A11) gives the following action for the Goldstino:

$$\tilde{S}_X = -\int d^3x\left\{\frac{f'^2}{2} + \frac{i}{2}\psi^\alpha\partial_{\alpha\beta}\psi^\beta + \frac{1}{8f'^2}\psi^2\Box\psi^2\right\}. \tag{A14}$$

Since the auxiliary field possesses a nonvanishing expectation value, $\langle F\rangle = \frac{1}{2}f'$, the supersymmetry is spontaneously broken. The constant term in the integrand (A14) generates a positive contribution to the cosmological constant in supergravity.

Our next goal is to study the component structure of the irreducible Goldstino model $S_{\mathcal{X}}$, Eq. (5.9), in Minkowski superspace. We recall that it is obtained from the reducible Goldstino theory defined by Eqs. (A7) and (A9) by requiring the action (A7) to be stationary under local rescalings $X \to e^\rho X$. This gives the constraint

$$\frac{i}{2}XD^2X = f'X, \tag{A15}$$

which allows one to solve for the auxiliary field F in terms of ψ. Evaluating the top component of (A15) gives

$$F - \frac{f'}{2} - \frac{i}{4F}\psi^\alpha\partial_{\alpha\beta}\psi^\beta - \frac{1}{64}\frac{\psi^2}{F^2}\Box\frac{\psi^2}{F} = 0. \tag{A16}$$

This equation can be solved by repeated substitution to result with

$$F = \frac{f'}{2} + \frac{i}{2f'}\psi^\alpha\partial_{\alpha\beta}\psi^\beta - \frac{1}{4f'^3}\psi^2\partial_{\alpha\beta}\psi^\beta\partial^{\alpha\gamma}\psi_\gamma + \frac{1}{8f'^3}\psi^2\Box\psi^2. \tag{A17}$$

Plugging this into (A11) leads to the component action

$$S_{\mathcal{X}} = -\int d^3x\left\{\frac{f'^2}{2} + \frac{i}{2}\psi^\alpha\partial_{\alpha\beta}\psi^\beta - \frac{1}{4f'^2}\psi^2\partial_{\alpha\beta}\psi^\beta\partial^{\alpha\gamma}\psi_\gamma + \frac{1}{8f'^2}\psi^2\Box\psi^2\right\}. \tag{A18}$$

Comparing the two expressions for F, which are given by Eqs. (A13) and (A17) and which correspond to the models S_X and $S_{\mathcal{X}}$, respectively, we see that they are different. The final Goldstino actions (A14) and (A18) also have different quartic terms. Nevertheless, the two models are equivalent. Indeed, it was pointed out in Sec. V A that the top component of (A15) is equivalent to a sum of the equation of motion for F and a linear combination of the equations of motion for ψ_α, both equations of motion corresponding to the action (A11). One can readily check that the left-hand side of (A16) can be represented as

$$F - \frac{f'}{2} - \frac{i}{4F}\psi^\alpha\partial_{\alpha\beta}\psi^\beta - \frac{1}{64}\frac{\psi^2}{F^2}\Box\frac{\psi^2}{F} = \frac{1}{4}\left(\frac{\delta S_X}{\delta F} + \frac{\psi^\alpha}{F}\frac{\delta S_X}{\delta\psi^\alpha}\right), \tag{A19}$$

and therefore the two expressions for F coincide on the mass shell. Moreover, it may be shown that every solution to the equation of motion for the Goldstino action (A14) is a solution to the equation of motion for (A18) and vice versa. This follows from the identity

$$S_{\mathcal{X}} = \tilde{S}_X + \frac{1}{4f'^2}\int d^3x\psi^2\varepsilon^{\alpha\beta}\frac{\delta\tilde{S}_X}{\delta\psi^\alpha}\frac{\delta\tilde{S}_X}{\delta\psi^\beta}. \tag{A20}$$

APPENDIX B: COMPONENT STRUCTURE OF $\mathcal{N}=2$ GOLDSTINO MODELS

In this appendix we will discuss the component actions for $\mathcal{N}=2$ Goldstino models in flat superspace. We specialize the superspace $\mathcal{M}^{3|4}$ of Sec. III A to be the standard $\mathcal{N}=2$ Minkowski superspace $\mathbb{M}^{3|4}$ parametrized by Cartesian coordinates $z^A=(x^a,\theta^\alpha,\bar\theta_\alpha)$, with $\bar\theta^\alpha$ being the complex conjugate of θ^α. The covariant derivatives $\mathcal{D}_A=(\mathcal{D}_a,\mathcal{D}_\alpha,\bar{\mathcal{D}}^\alpha)$ on $\mathcal{M}^{3|4}$, defined by Eq. (3.1), become the flat-superspace ones $D_A=(\partial_a,D_\alpha,\bar D^\alpha)$. Here the spinor covariant derivatives have the form

$$D_\alpha=\partial_\alpha+\mathrm{i}\bar\theta^\beta(\gamma^a)_{\alpha\beta}\partial_a=\partial_\alpha+\mathrm{i}\bar\theta^\beta\partial_{\alpha\beta},\\ \bar D_\alpha=-\bar\partial_\alpha-\mathrm{i}\theta^\beta\partial_{\alpha\beta} \tag{B1}$$

and obey the anticommutation relations

$$\{D_\alpha,D_\beta\}=0,\qquad \{\bar D_\alpha,\bar D_\beta\}=0,\\ \{D_\alpha,\bar D_\beta\}=-2\mathrm{i}\partial_{\alpha\beta}. \tag{B2}$$

Given a supersymmetric action

$$S=\int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\mathcal{L}+\left\{\int \mathrm{d}^3x\mathrm{d}^2\theta\mathcal{L}_\mathrm{c}+\mathrm{c.c.}\right\},\\ \bar{\mathcal{L}}=\mathcal{L},\qquad \bar D_\alpha\mathcal{L}_\mathrm{c}=0, \tag{B3}$$

with some real $\mathcal{L}$ and chiral $\mathcal{L}_\mathrm{c}$ superfield Lagrangians, the component action is computed using the formula

$$S=\frac{1}{16}\int \mathrm{d}^3xD^2\bar D^2\mathcal{L}|-\left\{\frac{1}{4}\int \mathrm{d}^3xD^2\mathcal{L}_\mathrm{c}|+\mathrm{c.c.}\right\}. \tag{B4}$$

The contractions D^2 and $\bar D^2$ are defined as in (3.6).

1. The $\mathcal{N}=2$ chiral scalar Goldstino superfield

Let us consider a model of a chiral scalar superfield X satisfying

$$\bar D_\alpha X=0,\qquad X^2=0. \tag{B5}$$

This model defines a reducible Goldstino superfield model analogous to the 4D $\mathcal{N}=1$ chiral model studied in [75,84]. Hence, our analysis will be similar to those in [75–77]. A general chiral superfield can be written as

$$X=\phi+\sqrt{2}\theta^\alpha\psi_\alpha+\theta^2F, \tag{B6}$$

so that the components can be defined as

$$\phi=X|,\qquad \psi_\alpha=\frac{1}{\sqrt2}D_\alpha X|,\qquad F=-\frac14 D^2X|. \tag{B7}$$

The nilpotency condition $X^2=0$ gives

$$\phi=\frac{\psi^2}{2F},\qquad \bar\phi=\frac{\bar\psi^2}{2\bar F}. \tag{B8}$$

The action for X follows from (5.21)

$$S_X=\int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\,\bar XX-\left\{f\int \mathrm{d}^3x\mathrm{d}^2\theta X+\mathrm{c.c.}\right\}. \tag{B9}$$

The integral over θ and $\bar\theta$ can be performed using (B7), (B8) to give the following component action

$$S=\int \mathrm{d}^3x\left[-\frac12(\langle u\rangle+\langle\bar u\rangle)+\frac{\bar\psi^2}{2\bar F}\Box\frac{\psi^2}{2F}+F\bar F-f(F+\bar F)\right], \tag{B10}$$

where we have defined

$$\langle u\rangle=\mathrm{i}\psi^\alpha\partial_{\alpha\beta}\bar\psi^\beta,\qquad \langle\bar u\rangle=-\mathrm{i}\partial_{\alpha\beta}\psi^\beta\bar\psi^\alpha. \tag{B11}$$

The superfield X defined by (B5) describes a reducible multiplet containing the Goldstino ψ_α and an auxiliary field F.

As was explained in the previous appendix there are two approaches to define an irreducible Goldstino multiplet. We can eliminate F and $\bar F$ from the action (B10) using the equations of motion

$$F=f+\frac{\bar\psi^2}{2\bar F^2}\Box\frac{\psi^2}{2F},\qquad \bar F=f+\frac{\psi^2}{2F^2}\Box\frac{\bar\psi^2}{2\bar F}. \tag{B12}$$

Solving equations (B12) by repeated substitution yields

$$F=f+\frac14 f^{-3}\bar\psi^2\Box\psi^2-\frac{3}{16}f^{-7}\psi^2\bar\psi^2(\Box\psi^2)(\Box\bar\psi^2). \tag{B13}$$

Then the Goldstino action becomes

$$S=-\int \mathrm{d}^3x\left[f^2+\frac12(\langle u\rangle+\langle\bar u\rangle)+\frac{1}{4f^2}(\partial^a\bar\psi^2)(\partial_a\psi^2)\right.\\ \left.+\frac{1}{16f^6}\psi^2\bar\psi^2(\Box\psi^2)(\Box\bar\psi^2)\right]. \tag{B14}$$

Alternatively, we can require that the action be stationary under rescalings $X\to e^\tau X$, $\bar D_\alpha\tau=0$ which gives the constraint

$$-\frac14 X\bar D^2\bar X=fX. \tag{B15}$$

Equations (B5), (B15) define a Goldstino model (5.28) as was discussed in Sec. V. From (B15) we find the following equation for the auxiliary field:

$$F=f+\mathrm{i}\bar F^{-1}\bar\psi^\alpha\partial_{\alpha\beta}\psi^\beta-\frac14\bar F^{-2}\bar\psi^2\Box(F^{-1}\psi^2). \tag{B16}$$

Solving it by repeated substitution we obtain

$$
\begin{aligned}
F = f &+ f^{-1}\langle\bar{u}\rangle - f^{-3}\left(\langle u\rangle\langle\bar{u}\rangle + \frac{1}{4}\bar{\psi}^2\Box\psi^2\right) \\
&+ f^{-5}(\langle u\rangle^2\langle\bar{u}\rangle + \langle\bar{u}\rangle^2\langle u\rangle) \\
&+ \frac{1}{4}f^{-5}(\langle\bar{u}\rangle\psi^2\Box\bar{\psi}^2 + 2\langle u\rangle\bar{\psi}^2\Box\psi^2 + \bar{\psi}^2\Box(\psi^2\langle\bar{u}\rangle)) \\
&- 3f^{-7}\left(\langle u\rangle^2\langle\bar{u}\rangle^2 + \frac{1}{4}\psi^2\bar{\psi}^2\Box(\langle u\rangle^2 - \langle u\rangle\langle\bar{u}\rangle + \langle\bar{u}\rangle^2)\right. \\
&\left.+ \frac{1}{16}\psi^2\bar{\psi}^2\Box\psi^2\Box\bar{\psi}^2\right), \qquad \text{(B17)}
\end{aligned}
$$

where $\langle u\rangle$ is given in Eq. (B11). Comparing Eqs. (B13) and (B17) we see that the solution for F is different in our two approaches but the difference is related to the equation of motion for the Goldstino as was explained at the end of the previous appendix.

2. The $\mathcal{N}=2$ real scalar Goldstino superfield

The real scalar Goldstino superfield is defined to obey the constraints

$$V^2 = 0, \qquad VD_AD_BV = 0, \qquad VD_AD_BD_CV = 0. \qquad \text{(B18)}$$

We will start with a general $\mathcal{N}=2$ real scalar superfield

$$
\begin{aligned}
V = v &+ \sqrt{2}\theta^\alpha\lambda_\alpha + \sqrt{2}\bar{\theta}_\alpha\bar{\lambda}^\alpha + \theta^2F + \bar{\theta}^2\bar{F} + \theta^\alpha\bar{\theta}^\beta A_{\alpha\beta} \\
&+ \sqrt{2}\bar{\theta}^2\theta^\alpha\varrho_\alpha + \sqrt{2}\theta^2\bar{\theta}_\alpha\bar{\varrho}^\alpha + \theta^2\bar{\theta}^2\mathbb{D}. \qquad \text{(B19)}
\end{aligned}
$$

Here $A_{\alpha\beta}$ describes both a vector $\tilde{A}^a$ and a scalar φ:

$$A_{\alpha\beta} = (\gamma_a)_{\alpha\beta}\tilde{A}^a + \mathrm{i}\epsilon_{\alpha\beta}\varphi. \qquad \text{(B20)}$$

Imposing conditions (B18) we find that $v, \lambda_\alpha, \bar{\lambda}_\alpha, A_{\alpha\beta}, F, \bar{F}$ can be solved in terms of ϱ_α, $\bar{\varrho}_\alpha, \mathbb{D}$ as follows:

$$
\begin{aligned}
v &= \frac{\varrho^2\bar{\varrho}^2}{4\mathbb{D}^3}, \qquad \lambda_\alpha = \frac{\varrho_\alpha\bar{\varrho}^2}{2\mathbb{D}^2}, \qquad \bar{\lambda}_\alpha = \frac{\bar{\varrho}_\alpha\varrho^2}{2\mathbb{D}^2}, \\
F &= \frac{\bar{\varrho}^2}{2\mathbb{D}}, \qquad \bar{F} = \frac{\varrho^2}{2\mathbb{D}}, \qquad A_{\alpha\beta} = \frac{2\varrho_\alpha\bar{\varrho}_\beta}{\mathbb{D}}. \qquad \text{(B21)}
\end{aligned}
$$

Hence, we have explicitly shown that the model (B18) describes a reducible Goldstino multiplet $(\varrho_\alpha, \mathbb{D})$ consisting of the Goldstino ϱ_α and an auxiliary field $\mathbb{D}$.

Alternatively, we can define the Goldstino as follows. Let

$$W_\alpha = -\frac{1}{4}\bar{D}^2D_\alpha V. \qquad \text{(B22)}$$

Let us define

$$\psi_\alpha = \frac{1}{\sqrt{2}}W_\alpha|, \qquad D = -\frac{1}{4}D^\alpha W_\alpha|. \qquad \text{(B23)}$$

Since W_α satisfies

$$D^\alpha W_\alpha = \bar{D}_\alpha\bar{W}^\alpha \qquad \text{(B24)}$$

we see that D is real. Using Eqs. (B19), (B21), (B22), (B23) we obtain

$$
\begin{aligned}
\psi_\alpha &= \varrho_\alpha - \frac{\mathrm{i}}{2}\partial_{\alpha\beta}\bar{\lambda}^\beta = \varrho_\alpha - \frac{\mathrm{i}}{4}\partial_{\alpha\beta}\left(\frac{\bar{\varrho}^\beta\varrho^2}{\mathbb{D}^2}\right), \\
D &= \mathbb{D} - \frac{1}{4}\Box v = \mathbb{D} - \frac{1}{16}\Box\left(\frac{\varrho^2\bar{\varrho}^2}{\mathbb{D}^3}\right). \qquad \text{(B25)}
\end{aligned}
$$

From here we can derive the following useful relations

$$\varrho^2\bar{\varrho}^\alpha = \psi^2\bar{\psi}^\alpha, \qquad \bar{\varrho}^2\varrho^\alpha = \bar{\psi}^2\psi^\alpha, \qquad \varrho^2\bar{\varrho}^2 = \psi^2\bar{\psi}^2, \qquad \text{(B26)}$$

which, in turn, allow us to invert (B25) to get

$$\varrho_\alpha = \psi_\alpha + \frac{\mathrm{i}}{4}\partial_{\alpha\beta}\left(\frac{\bar{\psi}^\beta\psi^2}{D^2}\right), \qquad \mathbb{D} = D + \frac{1}{16}\Box\left(\frac{\psi^2\bar{\psi}^2}{D^3}\right). \qquad \text{(B27)}$$

Substituting (B27) into (B21) we obtain the components of V in terms of (ψ_α, D)

$$
\begin{aligned}
v &= \frac{\psi^2\bar{\psi}^2}{4D^3}, \qquad \lambda_\alpha = \frac{\psi_\alpha\bar{\psi}^2}{2D^2}, \qquad \bar{\lambda}_\alpha = \frac{\bar{\psi}_\alpha\psi^2}{2D^2}, \\
F &= \frac{\bar{\psi}^2}{2D} + \frac{\bar{\psi}^2}{4D^3}\langle u\rangle, \qquad \bar{F} = \frac{\psi^2}{2D} + \frac{\psi^2}{4D^3}\langle\bar{u}\rangle, \\
A_{\alpha\beta} &= \frac{2\psi_\alpha\bar{\psi}_\beta}{D} - \frac{\mathrm{i}}{2D^3}\psi^2\bar{\psi}^\gamma(\partial_{\alpha\gamma}\bar{\psi}_\beta) + \frac{\mathrm{i}}{2D^3}\bar{\psi}^2(\partial_{\beta\gamma}\psi_\alpha)\psi^\gamma \\
&\quad - \frac{1}{8D^5}\psi^2\bar{\psi}^2\partial_{\alpha\gamma}\partial_{\beta\delta}(\psi^\delta\bar{\psi}^\gamma) - \frac{1}{4D^5}\psi^2\bar{\psi}^2\partial_a\psi_\alpha\partial^a\bar{\psi}_\beta. \qquad \text{(B28)}
\end{aligned}
$$

Either $(\varrho_\alpha, \mathbb{D})$ or (ψ_α, D) can be used to describe a reducible Goldstino multiplet in this model. Relations (B25) and (B27) allow one to quickly transform from one description to another. Since the components of V are simpler when written in terms of $(\varrho_\alpha, \mathbb{D})$ below we will use this pair of fields.

The action for the Goldstino superfield can be taken as the flat superspace limit of (5.35)

$$S = \frac{1}{16}\int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}D^2V\bar{D}^2V - 2f\int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}V. \qquad \text{(B29)}$$

Using the nilpotency conditions (B18) the first term of the action (B29) can also be written as

$$-\frac{1}{4}\int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}VD^\alpha W_\alpha = \frac{1}{4}\int \mathrm{d}^3x\mathrm{d}^2\theta W^\alpha W_\alpha. \quad \text{(B30)}$$

However, we find that Eq. (B29) is more convenient to use. In terms of $(\varrho_\alpha, \mathbb{D})$ the action (B29) reads

$$\begin{aligned} S = \int \mathrm{d}^3x\Big[&\mathbb{D}^2 - 2f\mathbb{D} - \frac{\mathrm{i}}{2}\varrho^\alpha(\partial_{\alpha\beta}\bar{\varrho}^\beta) \\ &+\frac{\mathrm{i}}{2}(\partial_{\alpha\beta}\varrho^\beta)\bar{\varrho}^\alpha - \frac{\varrho^\alpha\bar{\varrho}^\beta}{4\mathbb{D}}\partial_{\alpha\beta}\partial_{\gamma\delta}\left(\frac{\varrho^\gamma\bar{\varrho}^\delta}{\mathbb{D}}\right) \\ &+\frac{1}{8}(\Box\mathbb{D})\frac{\varrho^2\bar{\varrho}^2}{\mathbb{D}^3} - \frac{1}{4}\varrho^\alpha\Box\left(\frac{\varrho_\alpha\bar{\varrho}^2}{\mathbb{D}^2}\right) - \frac{1}{4}\bar{\varrho}_\alpha\Box\left(\frac{\bar{\varrho}^\alpha\varrho^2}{\mathbb{D}^2}\right) \\ &+\frac{1}{4\mathbb{D}}\varrho^2\Box\left(\frac{\bar{\varrho}^2}{\mathbb{D}}\right) + \frac{\mathrm{i}}{16}\partial_{\alpha\beta}\left(\frac{\bar{\varrho}^\beta\varrho^2}{\mathbb{D}^2}\right)\Box\left(\frac{\varrho^\alpha\bar{\varrho}^2}{\mathbb{D}^2}\right) \\ &+\frac{1}{256\mathbb{D}^6}\varrho^2\bar{\varrho}^2\Box^2(\varrho^2\bar{\varrho}^2)\Big]. \end{aligned} \quad \text{(B31)}$$

Again, there are two approaches to define an irreducible Goldstino multiplet. We can eliminate $\mathbb{D}$ using its equation of motion:

$$\begin{aligned} \mathbb{D} = f &- \frac{1}{4}\frac{\varrho^\alpha\bar{\varrho}^\beta}{\mathbb{D}^2}\partial_{\alpha\beta}\partial_{\gamma\delta}\left(\frac{\varrho^\gamma\bar{\varrho}^\delta}{\mathbb{D}}\right) - \frac{1}{16}\Box\left(\frac{\varrho^2\bar{\varrho}^2}{\mathbb{D}^3}\right) \\ &+\frac{3}{16}(\Box\mathbb{D})\frac{\varrho^2\bar{\varrho}^2}{\mathbb{D}^4} - \frac{1}{4}\frac{\varrho^\alpha\bar{\varrho}^2}{\mathbb{D}^3}(\Box\varrho_\alpha) - \frac{1}{4}\frac{\bar{\varrho}_\alpha\varrho^2}{\mathbb{D}^3}(\Box\bar{\varrho}^\alpha) \\ &+\frac{1}{8\mathbb{D}^2}\varrho^2\Box\left(\frac{\bar{\varrho}^2}{\mathbb{D}}\right) + \frac{1}{8\mathbb{D}^2}\bar{\varrho}^2\Box\left(\frac{\varrho^2}{\mathbb{D}}\right) \\ &-\frac{\mathrm{i}}{16}\frac{\varrho^\alpha\bar{\varrho}^2}{\mathbb{D}^3}\partial_{\alpha\beta}\Box\left(\frac{\bar{\varrho}^\beta\varrho^2}{\mathbb{D}^2}\right) - \frac{\mathrm{i}}{16}\frac{\bar{\varrho}^\alpha\varrho^2}{\mathbb{D}^3}\partial_{\alpha\beta}\Box\left(\frac{\varrho^\beta\bar{\varrho}^2}{\mathbb{D}^2}\right) \\ &+\frac{3}{256\mathbb{D}^7}\varrho^2\bar{\varrho}^2\Box^2(\varrho^2\bar{\varrho}^2), \end{aligned} \quad \text{(B32)}$$

which can be solved by repeated substitutions. The second approach is to require that the action (B29) be stationary under local rescalings $V \to e^\rho V$ which yields the constraint

$$\frac{1}{32}V\{D^2, \bar{D}^2\}V = fV, \quad \text{(B33)}$$

as was discussed in Sec. V. Here for simplicity we will follow the first approach and eliminate $\mathbb{D}$ using the equation of motion (B32). From Eq. (B32) we see that, the solution for $\mathbb{D}$ has to be of the following form:

$$\begin{aligned} \mathbb{D} = f &+ \varrho^\alpha\bar{\varrho}^\beta\mathcal{A}_{\alpha\beta} + \varrho^2\mathcal{B} + \bar{\varrho}^2\bar{\mathcal{B}} + \bar{\varrho}^2\varrho^\alpha\mathcal{C}_\alpha \\ &+ \varrho^2\bar{\varrho}_\alpha\bar{\mathcal{C}}^\alpha + \varrho^2\bar{\varrho}^2\mathcal{F}, \end{aligned} \quad \text{(B34)}$$

where $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{F}$ depend on ϱ only through derivatives. Note that in Eq. (B34) there are no terms linear in Goldstino. Examining Eqs. (B31), (B34) one can show that the last three terms in (B34) do not contribute to the action and, hence, can be ignored. Keeping this in mind, we obtain

$$\begin{aligned} \mathbb{D} = f &- \frac{1}{4f^3}\varrho^\alpha\bar{\varrho}^\beta\partial_{\alpha\beta}\partial_{\gamma\delta}(\varrho^\gamma\bar{\varrho}^\delta) - \frac{1}{16f^3}\Box(\varrho^2\bar{\varrho}^2) \\ &+\frac{1}{8f^3}\varrho^2\Box(\bar{\varrho}^2) + \frac{1}{8f^3}\bar{\varrho}^2\Box(\varrho^2) + \cdots, \end{aligned} \quad \text{(B35)}$$

where the ellipsis stands for the terms which do not contribute to the action. Substituting Eq. (B35) into (B31) we find the following action for the Goldstino:

$$\begin{aligned} S = -\int \mathrm{d}^3x\Big[&f^2 + \frac{1}{2}(\langle w\rangle + \langle\bar{w}\rangle) + \frac{1}{4f^2}\varrho^\alpha\Box(\varrho_\alpha\bar{\varrho}^2) \\ &+\frac{1}{4f^2}\bar{\varrho}_\alpha\Box(\bar{\varrho}^\alpha\varrho^2) - \frac{1}{4f^2}\varrho^2\Box(\bar{\varrho}^2) \\ &+\frac{1}{4f^2}(\langle w\rangle - \langle\bar{w}\rangle)^2 + \frac{\mathrm{i}}{16f^4}\varrho^\alpha\bar{\varrho}^2\partial_{\alpha\beta}\Box(\bar{\varrho}^\beta\varrho^2) \\ &+\frac{1}{64f^6}\varrho^2\bar{\varrho}^2\Box(\langle w\rangle - \langle\bar{w}\rangle)^2 \\ &+\frac{1}{32f^6}\varrho^\alpha\bar{\varrho}^\beta\partial_{\alpha\beta}\partial_{\gamma\delta}(\varrho^\gamma\bar{\varrho}^\delta)\Box(\varrho^2\bar{\varrho}^2)\Big], \end{aligned} \quad \text{(B36)}$$

where $\langle w\rangle = \mathrm{i}\varrho^\alpha\partial_{\alpha\beta}\bar{\varrho}^\beta$.

3. From V to equivalent two-form multiplet

There is another possibility to study the model from the previous subsection. For this we will introduce

$$\Psi = -\frac{1}{4}\bar{D}^2V, \qquad \bar{\Psi} = -\frac{1}{4}D^2V. \quad \text{(B37)}$$

The action in Eq. (B29) can be equivalently written as

$$S = \int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\,\bar{\Psi}\Psi - f\int \mathrm{d}^3x\mathrm{d}^2\theta\Psi - f\int \mathrm{d}^3x\mathrm{d}^2\bar{\theta}\,\bar{\Psi}. \quad \text{(B38)}$$

Since Ψ is chiral we can define its components as

$$\phi = \Psi|, \qquad \chi_\alpha = \frac{1}{\sqrt{2}}D_\alpha\Psi|, \qquad F_1 + \mathrm{i}F_2 = -\frac{1}{4}D^2\Psi|. \quad \text{(B39)}$$

From Eq. (B18) it follows that $\Psi^2 = 0$, hence,

$$\phi = \frac{\chi^2}{2(F_1 + \mathrm{i}F_2)}. \quad \text{(B40)}$$

Note that in addition to the Goldstino χ_α, Ψ contains two auxiliary fields F_1 and F_2. However, as we will see below F_2 is a function of the Goldstino and F_1. Therefore, it is the pair (χ_α, F_1) which describes a reducible Goldstino multiplet which, of course, is equivalent to the ones studied in the previous subsection up to a nonlinear transformation which we will derive below. To express F_2 in terms of χ_α and F_1 we note that

$$D^2\Psi - \bar{D}^2\bar{\Psi} = \mathrm{i}\partial^{\alpha\beta}[D_\alpha, \bar{D}_\beta]V. \tag{B41}$$

Using the fact that

$$A_{\alpha\beta} = \frac{1}{2}[D_\alpha, \bar{D}_\beta]V| \tag{B42}$$

which follows from (B19) we find that

$$F_2 = -\frac{1}{4}\partial^{\alpha\beta}A_{\alpha\beta} = -\frac{1}{2}\partial^{\alpha\beta}\left(\frac{\varrho_\alpha\bar{\varrho}_\beta}{\mathbb{D}}\right). \tag{B43}$$

Hence, we see that F_2 is expressed in terms of the Goldstino and the remaining auxiliary field. The relation between $(\varrho_\alpha, \mathbb{D})$ and (χ_α, F_1) can be obtained using the defining equation (B37) as well as the definition of the component (B19), (B39). We get

$$\chi_\alpha = \varrho_\alpha + \frac{\mathrm{i}}{4}\partial_{\alpha\beta}\left(\frac{\bar{\varrho}^\beta\varrho^2}{\mathbb{D}^2}\right), \qquad F_1 = \mathbb{D} + \frac{1}{16}\Box\left(\frac{\varrho^2\bar{\varrho}^2}{\mathbb{D}^3}\right). \tag{B44}$$

Using the identities

$$\varrho^2\bar{\varrho}^\alpha = \chi^2\bar{\chi}^\alpha, \quad \bar{\varrho}^2\varrho^\alpha = \bar{\chi}^2\chi^\alpha, \quad \varrho^2\bar{\varrho}^2 = \chi^2\bar{\chi}^2, \tag{B45}$$

we can invert (B44):

$$\varrho_\alpha = \chi_\alpha - \frac{\mathrm{i}}{4}\partial_{\alpha\beta}\left(\frac{\bar{\chi}^\beta\chi^2}{F_1^2}\right), \qquad \mathbb{D} = F_1 - \frac{1}{16}\Box\left(\frac{\chi^2\bar{\chi}^2}{F_1^3}\right). \tag{B46}$$

Using the relations (B43) and (B46), we can express F_2 in terms of the fields χ_α and F_1. The result is

$$F_2 = -\frac{1}{8}\partial^{\alpha\beta}\left[\frac{1}{F_1}\left\{4\chi_\alpha\bar{\chi}_\beta + \mathrm{i}\chi_\alpha\partial_{\beta\gamma}\left(\frac{\chi^\gamma\bar{\chi}^2}{F_1^2}\right) + \mathrm{i}\bar{\chi}_\alpha\partial_{\beta\gamma}\left(\frac{\bar{\chi}^\gamma\chi^2}{F_1^2}\right)\right\} + \frac{1}{F_1^5}\chi^2\bar{\chi}^2\left(\partial_{\alpha\gamma}\partial_{\beta\delta}(\bar{\chi}^\gamma\chi^\delta) - \frac{1}{2}\partial^a\chi_\alpha\partial_a\bar{\chi}_\beta\right)\right]. \tag{B47}$$

Since the action (B38) is the same as the action for a chiral superfield X in (B9) it has the identical component structure:

$$S = \int \mathrm{d}^3x\left[-\frac{1}{2}(\langle v\rangle + \langle\bar{v}\rangle) + \frac{\bar{\chi}^2}{2(F_1 - \mathrm{i}F_2)}\Box\frac{\chi^2}{2(F_1 + \mathrm{i}F_2)} + F_1^2 + F_2^2 - 2fF_1\right], \tag{B48}$$

where $\langle v\rangle = \mathrm{i}\chi^\alpha\partial_{\alpha\beta}\bar{\chi}^\beta$ and F_2 is given by (B47). We will not present the final action in terms of χ_α since it is substantially more complicated than the one in Eq. (B36).

Out of the three possible Goldstino fields ϱ, ψ and χ it is ϱ which has the simplest action.

4. Nilpotent two-form Goldstino superfield

Here we will discuss the component structure of the model introduced in Sec. V B 4. As before, we will take the flat space limit. The two-form Goldstino multiplet is described by a chiral scalar superfield Y satisfying the following conditions:

$$Y = -\frac{1}{4}\bar{D}^2U, \qquad Y^2 = 0, \tag{B49}$$

where U is an *unconstrained* real superfield. Since Y is chiral we can define its components in the usual way

$$\phi = Y|, \qquad \xi_\alpha = \frac{1}{\sqrt{2}}D_\alpha Y|, \qquad F = -\frac{1}{4}D^2Y|. \tag{B50}$$

From (B49) it follows that

$$D^2Y - \bar{D}^2\bar{Y} = \mathrm{i}\partial^{\alpha\beta}[D_\alpha, \bar{D}_\beta]U. \tag{B51}$$

This means that the imaginary part of the auxiliary field F is the divergence of a vector. Let us denote $F = H + \mathrm{i}G$. Then we have $G = \partial_aC^a$, where C^a is an auxiliary vector field. The action for the superfield Y is given by the flat space limit of Eq. (5.47):

$$S_Y = \int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\,\bar{Y}\,Y - \left\{f\int \mathrm{d}^3x\mathrm{d}^2\theta Y + \mathrm{c.c.}\right\}. \tag{B52}$$

Just like in the theory of three-form multiplet in four dimensions this action has to be supplemented with the boundary term [91–93]

$$B_Y = \frac{1}{4}\int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}D^\alpha(YD_\alpha U - UD_\alpha Y) + \mathrm{c.c} = -2\int \mathrm{d}^3x\partial_a(C^aG) + \cdots, \tag{B53}$$

where the ellipsis stands for the boundary terms which do not play a role and can be set to zero.

Since the action (B52) is the same as the action for a chiral superfield it is given by

$$S_Y = \int \mathrm{d}^3x \left[\frac{\xi^2}{2F} \Box \frac{\bar{\xi}^2}{2\bar{F}} + \frac{\mathrm{i}}{2} (\partial_{\alpha\beta}\xi^\alpha)\bar{\xi}^\beta - \frac{\mathrm{i}}{2}\xi^\alpha(\partial_{\alpha\beta}\bar{\xi}^\beta) + H^2 + G^2 - 2fH \right], \tag{B54}$$

where we have used the fact that $Y^2 = 0$ and, hence, $\phi = \xi^2/(2F)$. Now we will eliminate the auxiliary fields using their equations of motion. Varying the action (B54) with respect to H and C_a gives the following equations:

$$\begin{aligned} &H - f - \frac{\bar{\xi}^2}{4\bar{F}^2}\Box\frac{\xi^2}{2F} - \frac{\xi^2}{4F^2}\Box\frac{\bar{\xi}^2}{2\bar{F}} = 0, \\ &\partial_a\left[G + \mathrm{i}\frac{\bar{\xi}^2}{4\bar{F}^2}\Box\frac{\xi^2}{2F} - \mathrm{i}\frac{\xi^2}{4F^2}\Box\frac{\bar{\xi}^2}{2\bar{F}} \right] = 0. \end{aligned} \tag{B55}$$

The second equation implies that

$$G + \mathrm{i}\left[\frac{\bar{\xi}^2}{4\bar{F}^2}\Box\frac{\xi^2}{2F} - \frac{\xi^2}{4F^2}\Box\frac{\bar{\xi}^2}{2\bar{F}} \right] = g, \tag{B56}$$

where g is an arbitrary constant. Hence, we find that

$$F = h + \frac{\bar{\xi}^2}{2\bar{F}^2}\Box\frac{\xi^2}{2F}, \qquad h = f + \mathrm{i}g. \tag{B57}$$

Solving this equation by repeated substitution yields

$$\begin{aligned} &F = h\left(1 + \frac{1}{4}|h|^{-4}\bar{\xi}^2\Box\xi^2 - \frac{3}{16}|h|^{-8}\xi^2\bar{\xi}^2\Box\xi^2\Box\bar{\xi}^2 \right), \\ &|h|^2 = f^2 + g^2. \end{aligned} \tag{B58}$$

The boundary term on the solution $G = g + \cdots$ gives $-2\int \mathrm{d}^3x(g^2 + \text{total derivative})$. Substituting Eq. (B58) into the bulk action (B54) and combining the result with the contribution from the boundary term yields the following Goldstino action:

$$\begin{aligned} S_Y + B_Y = -\int \mathrm{d}^3x \bigg[&|h|^2 - \frac{\mathrm{i}}{2}(\partial_{\alpha\beta}\xi^\alpha)\bar{\xi}^\beta + \frac{\mathrm{i}}{2}\xi^\alpha(\partial_{\alpha\beta}\bar{\xi}^\beta) \\ &+ \frac{1}{4}\frac{f^2 + 3g^2}{|h|^4}\partial^a\xi^2\partial_a\bar{\xi}^2 \\ &+ \frac{1}{16}\frac{f^2 + 7g^2}{|h|^8}\xi^2\bar{\xi}^2\Box\xi^2\Box\bar{\xi}^2 \bigg]. \end{aligned} \tag{B59}$$

APPENDIX C: GOLDSTINO MULTIPLET FROM A HIGHER-DERIVATIVE THEORY

In this appendix we will analyze the higher-derivative model (5.41) in Minkowski superspace. We first consider the case when the dynamical variable X is an unconstrained chiral superfield, $\bar{D}_\alpha X = 0$, which obeys no nilpotency condition. Then the model with action

$$S = \int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\left\{ \frac{1}{16f^2}XD^2X\bar{X}\bar{D}^2\bar{X} - 2\bar{X}X \right\} \tag{C1}$$

has two phases, one with unbroken supersymmetry, and the other with spontaneously broken one. In the unbroken phase, the equations of motion have free massless solutions

$$D^2X = 0. \tag{C2}$$

However, the kinetic term in (C1) has a wrong sign and thus the theory is ill defined at the quantum level. We therefore turn to the phase with spontaneously broken supersymmetry in which F develops a nonzero expectation value, $\langle F\rangle = f$.

Defining the components of X as in Eq. (B7) we obtain the component action:

$$\begin{aligned} S = 2\int &\mathrm{d}^3x[\partial^a\phi\partial_a\bar{\phi} + \mathrm{i}\psi^\alpha\partial_{\alpha\beta}\bar{\psi}^\beta - F\bar{F}] \\ &+ \frac{1}{f^2}\int \mathrm{d}^3x\bigg\{ F\bar{F}(\bar{\phi}\Box\phi + \phi\Box\bar{\phi}) + \phi\bar{\phi}(\Box\phi)(\Box\bar{\phi}) + (F\bar{F})^2 - \partial_a(\phi\bar{F})\partial^a(\bar{\phi}F) \\ &- \frac{3\mathrm{i}}{2}F\bar{F}\psi^\alpha\partial_{\alpha\beta}\bar{\psi}^\beta - \frac{3\mathrm{i}}{2}F\bar{F}\bar{\psi}^\alpha\partial_{\alpha\beta}\psi^\beta - \frac{\mathrm{i}}{2}\bar{F}\partial_{\alpha\beta}F\psi^\alpha\bar{\psi}^\beta + \frac{\mathrm{i}}{2}F\partial_{\alpha\beta}\bar{F}\psi^\alpha\bar{\psi}^\beta \\ &- \phi F\bar{\psi}_\alpha\Box\bar{\psi}^\alpha - \bar{\phi}\bar{F}\psi^\alpha\Box\psi_\alpha + F\partial^{\alpha\gamma}\phi\bar{\psi}_\gamma\partial_{\alpha\beta}\bar{\psi}^\beta - \bar{F}\partial^{\alpha\gamma}\bar{\phi}\psi_\gamma\partial_{\alpha\beta}\psi^\beta \\ &+ \frac{\mathrm{i}}{2}(\bar{\phi}\partial^{\alpha\gamma}\phi - \phi\partial^{\alpha\gamma}\bar{\phi})\partial_{\alpha\beta}\bar{\psi}^\beta\partial_{\gamma\delta}\psi^\delta + \frac{\mathrm{i}}{2}\phi\bar{\phi}(\partial_{\alpha\beta}\bar{\psi}^\beta\Box\psi^\alpha + \partial_{\alpha\beta}\psi^\beta\Box\bar{\psi}^\alpha) \\ &- \mathrm{i}(\phi\Box\bar{\phi})\bar{\psi}^\alpha\partial_{\alpha\beta}\psi^\beta - \mathrm{i}(\bar{\phi}\Box\phi)\psi^\alpha\partial_{\alpha\beta}\bar{\psi}^\beta - (\psi^\alpha\partial_{\alpha\beta}\bar{\psi}^\beta)(\bar{\psi}^\gamma\partial_{\gamma\delta}\psi^\delta)\bigg\}. \end{aligned} \tag{C3}$$

The equation of motion for $\bar{F}$ is

$$\begin{aligned} &-2Ff^2 + 2F^2\bar{F} + F\bar{\phi}\Box\phi + F\phi\Box\bar{\phi} - 2\mathrm{i}F\psi^\alpha\partial_{\alpha\beta}\bar{\psi}^\beta - \mathrm{i}F\bar{\psi}^\alpha\partial_{\alpha\beta}\psi^\beta + \phi\Box(\bar{\phi}F) - \mathrm{i}\partial_{\alpha\beta}F\psi^\alpha\bar{\psi}^\beta - \bar{\phi}\psi^\alpha\Box\psi_\alpha \\ &- \partial^{\alpha\gamma}\bar{\phi}\psi_\gamma\partial_{\alpha\beta}\psi^\beta = 0. \end{aligned} \tag{C4}$$

It shows that F and $\bar{F}$ are no longer auxiliary fields since they cannot be expressed in terms of the off-shell physical fields ϕ and ψ_α and their conjugates. One could try to look for F as a series in powers of the fields ϕ, ψ_α and their derivatives,

$$F = f + a_1\phi\Box\bar{\phi} + a_2\bar{\phi}\Box\phi + a_3\psi^\alpha\partial_{\alpha\beta}\bar{\psi}^\beta + a_4\bar{\psi}^\alpha\partial_{\alpha\beta}\psi^\beta + \cdots, \tag{C5}$$

where $a_1, a_2, \ldots$ are some constants which have to be found by substituting (C5) into (C4) and working order by order in perturbation theory. Such a solution would correspond to a supersymmetry breaking phase (note that $F = f$ for $\phi = 0, \psi_\alpha = 0$). However, it is not difficult to show that no solution for F exists: substitution of (C5) into (C4) yields inconsistent equations

$$2a_2 + 2\bar{a}_1 + f^{-1} = 0, \qquad 2a_1 + 2\bar{a}_2 + 2f^{-1} = 0, \tag{C6}$$

and similarly for a_3, a_4. This means that it is impossible to solve the equation of motion for the field F and substitute the solution into Eq. (C3) to find the action for the off-shell physical fields. The procedure of eliminating the auxiliary field F can be fulfilled only when the physical fields are also on shell. In other words, the equation (C4) and its conjugate have to be solved in conjunction with the equations of motion for the physical fields, and then the above inconsistencies do not occur. In doing so, we will obtain correctly normalized kinetic terms for the physical fields. Indeed, since in the supersymmetry breaking phase $F = f + \cdots$, for the relevant terms in (C3) we get

$$\begin{aligned} 2\int \mathrm{d}^3x[\partial^a\phi\partial_a\bar{\phi} + \mathrm{i}\psi^\alpha\partial_{\alpha\beta}\bar{\psi}^\beta] &+ \frac{1}{f^2}\int \mathrm{d}^3x\Big\{F\bar{F}\Big(\bar{\phi}\Box\phi + \phi\Box\bar{\phi} \\ &- \frac{3\mathrm{i}}{2}(\psi^\alpha\partial_{\alpha\beta}\bar{\psi}^\beta + \bar{\psi}^\alpha\partial_{\alpha\beta}\psi^\beta)\Big) + \phi\bar{F}\Box(\bar{\phi}F)\Big\} \\ &= -\int \mathrm{d}^3x[\partial^a\phi\partial_a\bar{\phi} + \mathrm{i}\psi^\alpha\partial_{\alpha\beta}\bar{\psi}^\beta] + \cdots \end{aligned} \tag{C7}$$

where the ellipsis stands for cubic and higher order terms in the fields ϕ, ψ_α and their conjugates.

We now restrict our study to the case of model (C1) with X chosen to be nilpotent,

$$X^2 = 0. \tag{C8}$$

Then ϕ can be expressed as in (B8) and we have a reducible Goldstino model. The component action of this model is given by (C3) with ϕ replaced according to Eq. (B8). The equation for the auxiliary field now reads

$$\begin{aligned} &-2f^2F + \frac{f^2}{2}\frac{\bar{\psi}^2}{\bar{F}^2}\Box\frac{\psi^2}{F} + \frac{1}{4}\psi^2\Box\frac{\bar{\psi}^2}{\bar{F}} - \frac{1}{4}\frac{\bar{\psi}^2}{\bar{F}^2}\Box(\bar{F}\psi^2) \\ &- \frac{1}{8}\frac{\psi^2\bar{\psi}^2}{F^2\bar{F}^3}\Box\psi^2\Box\bar{\psi}^2 + 2F^2\bar{F} + \frac{1}{4}\frac{\psi^2}{F}\Box\frac{F\bar{\psi}^2}{\bar{F}} - \frac{1}{4}\frac{F\bar{\psi}^2}{\bar{F}^2}\Box\frac{\bar{F}\psi^2}{F} \\ &- \frac{3\mathrm{i}}{2}F\psi^\alpha\partial_{\alpha\beta}\bar{\psi}^\beta - \frac{3\mathrm{i}}{2}F\bar{\psi}^\alpha\partial_{\alpha\beta}\psi^\beta - \frac{\mathrm{i}}{2}\psi^\alpha\bar{\psi}^\beta(\partial_{\alpha\beta}F) - \frac{\mathrm{i}}{2}\partial_{\alpha\beta}(F\psi^\alpha\bar{\psi}^\beta) \\ &- \frac{1}{2}\partial^{\alpha\gamma}\left(\frac{\bar{\psi}^2}{\bar{F}}\right)\psi_\gamma\partial_{\alpha\beta}\psi^\beta - \frac{1}{2}\frac{\bar{\psi}^2}{\bar{F}^2}\partial^{\alpha\gamma}(\bar{F}\psi_\gamma\partial_{\alpha\beta}\psi^\beta) - \frac{\mathrm{i}}{8}\frac{\bar{\psi}^2}{\bar{F}^2}\partial^{\alpha\gamma}\left(\frac{\psi^2}{F}\right)\partial_{\alpha\beta}\bar{\psi}^\beta\partial_{\gamma\delta}\psi^\delta \\ &- \frac{\mathrm{i}}{8}\frac{\bar{\psi}^2}{\bar{F}^2}\partial^{\alpha\gamma}\left(\frac{\psi^2}{F}\partial_{\alpha\beta}\bar{\psi}^\beta\partial_{\gamma\delta}\psi^\delta\right) - \frac{\mathrm{i}}{8F\bar{F}^2}\psi^2\bar{\psi}^2[\partial_{\alpha\beta}\bar{\psi}^\beta\Box\psi^\alpha + \partial_{\alpha\beta}\psi^\beta\Box\bar{\psi}^\alpha] \\ &+ \frac{\mathrm{i}}{4}\bar{\psi}^2\bar{F}^{-2}\Box\left[\frac{\psi^2}{F}\bar{\psi}^\alpha\partial_{\alpha\beta}\psi^\beta\right] + \frac{\mathrm{i}}{4}\bar{\psi}^2\bar{F}^{-2}\psi^\alpha\partial_{\alpha\beta}\bar{\psi}^\beta\Box\frac{\psi^2}{F} = 0. \end{aligned} \tag{C9}$$

One can show that just like in the case of Eq. (C4) it is not possible to solve this equation for F in terms of the physical fields ψ and $\bar{\psi}$. The procedure of eliminating the field F can be performed only if the Goldstino is on shell. Therefore, we will follow the other approach: instead of considering the equations of motion for F, we will require that the action (C1) be stationary under rescaling $X \to \mathrm{e}^\tau X$, which yields

$$\bar{D}^2(X\bar{X}D^2X\bar{D}^2\bar{X} - 16f^2X\bar{X}) = 0. \tag{C10}$$

Since $\bar{D}^2\bar{X}$ is nowhere vanishing, this condition is equivalent to

$$X\bar{D}^2(\bar{X}D^2X) = 16f^2X. \tag{C11}$$

The problem of solving Eqs. (C10), (C11) can be reformulated as follows. Let us define the superfield Y by the rule

$$-\frac{1}{4}X\bar{D}^2\bar{X} = fY. \tag{C12}$$

It then follows from Eq. (C10) that Y has the properties

$$
\begin{aligned}
\bar{D}_\alpha Y &= 0, \\
Y^2 &= 0, \\
-\frac{1}{4} Y \bar{D}^2 \bar{Y} &= f Y.
\end{aligned}
\tag{C13}
$$

That is Y defines an irreducible Goldstino multiplet whose auxiliary field is uniquely solved in terms of the Goldstini with the solution given in Eq. (B17). Therefore, the problem can be stated as to find X using Eq. (C12) given Y. Comparing Eqs. (C12) and (C13) we see that there is an obvious solution $X = Y$.[15] However, this solution is not unique. To show it we will examine Eq. (C11) in components. Let us consider the equation for F followed from (C11). We obtain

$$
\begin{aligned}
&-2f^2 F + 2F^2 \bar{F} - 2\mathrm{i} F \psi^\alpha \partial_{\alpha\beta} \bar{\psi}^\beta - 2\mathrm{i} F \bar{\psi}^\alpha \partial_{\alpha\beta} \psi^\beta \\
&\quad - 2\mathrm{i} (\partial_{\alpha\beta} F) \psi^\alpha \bar{\psi}^\beta + \frac{1}{2} \frac{F \bar{\psi}^2}{\bar{F}} \Box \frac{\psi^2}{F} + \frac{1}{2} \frac{\psi^2}{F} \Box \frac{F \bar{\psi}^2}{\bar{F}} \\
&\quad + \psi^\alpha \partial_{\alpha\beta} \left(\frac{\bar{\psi}^2}{\bar{F}} \partial^{\beta\gamma} \psi_\gamma \right) = 0.
\end{aligned}
\tag{C14}
$$

Note that we cannot solve this equation by repeated substitution. However, we can solve it by expanding F in powers in the Goldstino and its derivatives

$$
F = f + a_1 (\mathrm{i} \psi^\alpha \partial_{\alpha\beta} \bar{\psi}^\beta) + a_2 (\mathrm{i} \bar{\psi}^\alpha \partial_{\alpha\beta} \psi^\beta) + \cdots. \tag{C15}
$$

Since ψ is nilpotent this expansion is finite. Substituting it into (C14) we can fix the coefficients. From the analysis presented above we know that there is a solution for F given by (B17). Therefore, we will look for a solution in the form of (B17):

$$
\begin{aligned}
F = f &+ a_1 \langle u \rangle + a_2 \langle \bar{u} \rangle + a_3 \langle u \rangle \langle \bar{u} \rangle + a_4 \psi^2 \Box \bar{\psi}^2 \\
&+ a_5 \bar{\psi}^2 \Box \psi^2 + a_6 (\langle u \rangle^2 \langle \bar{u} \rangle + \langle \bar{u} \rangle^2 \langle u \rangle) + a_7 \langle \bar{u} \rangle \psi^2 \Box \bar{\psi}^2 \\
&+ a_8 \langle u \rangle \bar{\psi}^2 \Box \psi^2 + a_9 \bar{\psi}^2 \Box (\psi^2 \langle \bar{u} \rangle) + a_{10} \langle u \rangle^2 \langle \bar{u} \rangle^2 \\
&+ a_{11} \psi^2 \bar{\psi}^2 \Box (\langle u \rangle^2 - \langle u \rangle \langle \bar{u} \rangle + \langle \bar{u} \rangle^2) \\
&+ a_{12} \psi^2 \bar{\psi}^2 \Box \psi^2 \Box \bar{\psi}^2.
\end{aligned}
\tag{C16}
$$

Substituting this ansatz into (C14) we find that the coefficients a_1, a_2, a_3, a_6, a_8, a_9, a_{10}, a_{11} are fixed as in (B17), whereas the remaining coefficients satisfy

$$
\begin{aligned}
a_4 &= -\frac{1}{4} f^{-3} - a_5, \\
a_7 &= -\frac{1}{4} f^{-5} - 2f^{-2} a_5, \\
a_{12} &= \frac{1}{2} f^{-4} a_5 - a_5^2
\end{aligned}
\tag{C17}
$$

and cannot be fixed uniquely. The solution (B17) corresponds to $a_4 = 0$, $a_5 = -\frac{1}{4} f^{-3}$, $a_7 = \frac{1}{4} f^{-5}$, $a_{12} = -\frac{3}{16} f^{-7}$. The ambiguity that we can have more than one solution to (C14) is expected to be related to the fact that we can add to F and to the action terms proportional to the equations of motion as in (A19) and (A20), but we will not discuss this issue in detail in this paper.

Let us now clarify why Eq. (C11), or equivalently Eq. (C14), has a solution for F despite the fact that Eq. (C9) does not. For this we will consider the equation of motion for the superfield X. Since X is nilpotent to find it we have to add the term

$$
\int \mathrm{d}^3 x \mathrm{d}^2 \theta \lambda X^2 + \text{c.c.} \tag{C18}
$$

to the action (C1), where λ is a Lagrange multiplier. Thus, we obtain the following equation of motion for X:

$$
\begin{aligned}
&\bar{D}^2 [\bar{X} D^2 X \bar{D}^2 \bar{X}] + \bar{D}^2 D^2 [\bar{X} X \bar{D}^2 \bar{X}] - 32 f^2 \bar{D}^2 \bar{X} \\
&\quad - 128 f^2 \lambda X = 0.
\end{aligned}
\tag{C19}
$$

Multiplying it by X we get the constraint (C10). However, the equation of motion for X contains not just the equation of motion for F (C9) but also the equation for the Goldstino. Hence, in obtaining Eq. (C14) equations of motion for both F and ψ are taken into account and that is why it has a solution.

With the nilpotency condition (C8) imposed, the action (C1) can be rewritten as

$$
S = \int \mathrm{d}^3 x \mathrm{d}^2 \theta \mathrm{d}^2 \bar{\theta} \left\{ \frac{1}{16 f^2} D^\alpha X D_\alpha X \bar{D}_\beta \bar{X} \bar{D}^\beta \bar{X} - 2 \bar{X} X \right\}. \tag{C20}
$$

Similar supersymmetric higher derivative models have been considered in the literature in the case when X is an unconstrained chiral superfield. In particular, an action of the type (C20) was studied in [94]. In their case they could solve for the auxiliary field in terms of the off-shell physical scalar field provided the fermions were ignored. However, if we take into account the fermions as well we can show that it is also impossible to solve for the auxiliary field unless the fermions are on-shell. Unlike in our case, Eq. (C7), in the model studied in [94] the kinetic term for scalars completely canceled in the supersymmetry breaking phase. Ref. [95] studied a model with canonically normalized kinetic term. It is obtained from (C20) by replacement $-2\bar{X}X \to \bar{X}X$. It was shown in [95] that the resulting model cannot break supersymmetry.

[15]Note that if X is a solution to (C12) then so is $-X$. Hence, we have two supersymmetry breaking phases. For concreteness we select the phase in which $\langle F \rangle = f$.

[1] M. B. Green and J. H. Schwarz, Covariant description of superstrings, Phys. Lett. **136B**, 367 (1984).

[2] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 1987), Vols. 1 and 2.

[3] L. Mezincescu and P. K. Townsend, Anyons from Strings, Phys. Rev. Lett. **105**, 191601 (2010).

[4] L. Mezincescu and P. K. Townsend, Quantum 3D superstrings, Phys. Rev. D **84**, 106006 (2011).

[5] J. A. de Azcarraga and J. Lukierski, Supersymmetric particles with internal symmetries and central charges, Phys. Lett. **113B**, 170 (1982).

[6] J. A. de Azcarraga and J. Lukierski, Supersymmetric particles in $N = 2$ superspace: Phase space variables and Hamiltonian dynamics, Phys. Rev. D **28**, 1337 (1983).

[7] W. Siegel, Hidden local supersymmetry in the supersymmetric particle action, Phys. Lett. **128B** (1983) 397.

[8] E. Sezgin, in *Salamfestschrift*, edited by A. Ali, J. Ellis, and S. Randjbar-Daemi (World Scientific, Singapore, 1994), pp. 478–498.

[9] D. P. Sorokin, Superbranes and superembeddings, Phys. Rep. **329**, 1 (2000).

[10] M. Henneaux and L. Mezincescu, A σ-model interpretation of Green-Schwarz covariant superstring action, Phys. Lett. **152B**, 340 (1985).

[11] E. Witten, Twistor-like transform in ten dimensions, Nucl. Phys. **B266**, 245 (1986).

[12] M. T. Grisaru, P. S. Howe, L. Mezincescu, B. Nilsson, and P. K. Townsend, $N = 2$ superstrings in a supergravity background, Phys. Lett. **162B**, 116 (1985).

[13] E. Bergshoeff, E. Sezgin, and P. K. Townsend, Superstring actions in $D = 3$, 4, 6, 10 curved superspace, Phys. Lett. **169B**, 191 (1986).

[14] A. A. Tseytlin and L. Wulff, Kappa-symmetry of superstring sigma model and generalized 10d supergravity equations, J. High Energy Phys. 06 (2016) 174.

[15] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, Three-dimensional $N = 2$ (AdS) supergravity and associated supercurrents, J. High Energy Phys. 12 (2011) 052.

[16] I. Antoniadis, E. Dudas, S. Ferrara, and A. Sagnotti, The Volkov-Akulov-Starobinsky supergravity, Phys. Lett. B **733**, 32 (2014).

[17] E. Dudas, S. Ferrara, A. Kehagias, and A. Sagnotti, Properties of nilpotent supergravity, J. High Energy Phys. 09 (2015) 217.

[18] E. A. Bergshoeff, D. Z. Freedman, R. Kallosh, and A. Van Proeyen, Pure de Sitter supergravity, Phys. Rev. D **92**, 085040 (2015); Erratum, Phys. Rev. D**93**, 069901(E) (2016).

[19] F. Hasegawa and Y. Yamada, Component action of nilpotent multiplet coupled to matter in 4 dimensional $\mathcal{N} = 1$ supergravity, J. High Energy Phys. 10 (2015) 106.

[20] S. M. Kuzenko, Complex linear Goldstino superfield and supergravity, J. High Energy Phys. 10 (2015) 006.

[21] R. Kallosh and T. Wrase, de Sitter supergravity model building, Phys. Rev. D **92**, 105010 (2015).

[22] M. Schillo, E. van der Woerd, and T. Wrase, The general de Sitter supergravity component action, Fortschr. Phys. **64**, 292 (2016).

[23] I. Bandos, L. Martucci, D. Sorokin, and M. Tonin, Brane induced supersymmetry breaking and de Sitter supergravity, J. High Energy Phys. 02 (2016) 080.

[24] F. Farakos, A. Kehagias, D. Racco, and A. Riotto, Scanning of the supersymmetry breaking scale and the gravitino mass in supergravity, J. High Energy Phys. 06 (2016) 120.

[25] I. Bandos, M. Heller, S. M. Kuzenko, L. Martucci, and D. Sorokin, The Goldstino brane, the constrained superfields and matter in $\mathcal{N} = 1$ supergravity, J. High Energy Phys. 11 (2016) 109.

[26] S. M. Kuzenko, I. N. McArthur, and G. Tartaglino-Mazzucchelli, Goldstino superfields in $N = 2$ supergravity, J. High Energy Phys. 05 (2017) 061.

[27] E. I. Buchbinder and S. M. Kuzenko, Three-form multiplet and supersymmetry breaking, J. High Energy Phys. 09 (2017) 089.

[28] U. Lindström and M. Roček, Constrained local superfields, Phys. Rev. D **19**, 2300 (1979).

[29] S. Samuel and J. Wess, A superfield formulation of the nonlinear realization of supersymmetry and its coupling to supergravity, Nucl. Phys. **B221**, 153 (1983).

[30] S. M. Kuzenko and S. J. Tyler, Complex linear superfield as a model for Goldstino, J. High Energy Phys. 04 (2011) 057.

[31] D. V. Volkov and V. P. Akulov, Possible universal neutrino interaction, Pis'ma Zh. Eksp. Teor. Fiz. **16**, 621 (1972) [JETP Lett. **16**, 438 (1972]; Is the neutrino a Goldstone particle?, Phys. Lett. **46B**, 109 (1973).

[32] V. P. Akulov and D. V. Volkov, Goldstone fields with spin 1/2, Teor. Mat. Fiz. **18**, 39 (1974) [Theor. Math. Phys. **18**, 28 (1974].

[33] D. V. Volkov and V. A. Soroka, Higgs effect for Goldstone particles with spin 1/2, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 529 (1973) [JETP Lett. **18**, 312 (1973].

[34] D. V. Volkov and V. A. Soroka, Gauge fields for symmetry group with spinor parameters, Teor. Mat. Fiz. **20**, 291 (1974) [Theor. Math. Phys. **20**, 829 (1974].

[35] S. Deser and B. Zumino, Broken Supersymmetry and Supergravity, Phys. Rev. Lett. **38**, 1433 (1977).

[36] S. J. Gates Jr., M. T. Grisaru, M. Roček, and W. Siegel, *Superspace, or One Thousand and One Lessons in Supersymmetry* (Benjamin/Cummings, Reading, MA, 1983).

[37] S. M. Kuzenko, U. Lindström, and G. Tartaglino-Mazzucchelli, Off-shell supergravity-matter couplings in three dimensions, J. High Energy Phys. 03 (2011) 120.

[38] P. van Nieuwenhuizen, $D = 3$ conformal supergravity and Chern-Simons terms, Phys. Rev. D **32**, 872 (1985).

[39] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, Conformal supergravities as Chern-Simons theories revisited, J. High Energy Phys. 03 (2013) 113.

[40] D. Butter, S. M. Kuzenko, J. Novak, and G. Tartaglino-Mazzucchelli, Conformal supergravity in three dimensions: Off-shell actions, J. High Energy Phys. 10 (2013) 073.

[41] L. Castellani, R. D'Auria, and P. Fre, *Supergravity and superstrings: A Geometric perspective. Vol. 2: Supergravity* (World Scientific, Singapore, 1991), pp. 680–684.

[42] M. F. Hasler, The three-form multiplet in $N = 2$ superspace, Eur. Phys. J. C **1**, 729 (1998).

[43] S. J. Gates Jr., in *Supersymmetries and Quantum Symmetries*, edited by J. Wess and E. A. Ivanov (Springer, Berlin, 1999),

pp. 46–57; J. Wess and E. A. IvanovEctoplasm has no topology, *Nucl. Phys.* ***B541****, 615 (1999)*.

[44] S. J. Gates Jr., M. T. Grisaru, M. E. Knutt-Wehlau, and W. Siegel, Component actions from curved superspace: Normal coordinates and ectoplasm, Phys. Lett. B **421**, 203 (1998).

[45] M. Becker, D. Constantin, S. J. Gates Jr., W. D. Linch III, W. Merrell, and J. Phillips, M-theory on Spin(7) manifolds, fluxes and 3D, $\mathrm{N}=1$ supergravity, Nucl. Phys. **B683**, 67 (2004).

[46] S. M. Kuzenko, J. Novak, and G. Tartaglino-Mazzucchelli, Higher derivative couplings and massive supergravity in three dimensions, J. High Energy Phys. 09 (2015) 081.

[47] S. M. Kuzenko, U. Lindström, and G. Tartaglino-Mazzucchelli, Three-dimensional (p,q) AdS superspaces and matter couplings, J. High Energy Phys. 08 (2012) 024.

[48] I. A. Batalin and G. A. Vilkovisky, Quantization of gauge theories with linearly dependent generators, Phys. Rev. D **28**, 2567 (1983).

[49] M. F. Sohnius and P. C. West, An alternative minimal off-shell version of $N=1$ supergravity, Phys. Lett. **105B**, 353 (1981).

[50] M. F. Sohnius and P. C. West, in *Quantum Structure of Space and Time*, edited by M. J. Duff and C. J. Isham (Cambridge University Press, Cambridge, England, 1982), pp. 187–222.

[51] M. Sohnius and P. C. West, The tensor calculus and matter coupling of the alternative minimal auxiliary field formulation of $N=1$ supergravity, Nucl. Phys. **B198**, 493 (1982).

[52] A. Achúcarro and P. K. Townsend, A Chern-Simons action for three-dimensional AdS supergravity theories, Phys. Lett. B **180**, 89 (1986).

[53] P. S. Howe, J. M. Izquierdo, G. Papadopoulos, and P. K. Townsend, New supergravities with central charges and Killing spinors in $2+1$ dimensions, Nucl. Phys. **B467**, 183 (1996).

[54] M. Roček and P. van Nieuwenhuizen, $N\geq 2$ supersymmetric Chern-Simons terms as $d=3$ extended conformal supergravity, Classical Quantum Gravity **3**, 43 (1986).

[55] S. M. Kuzenko, Prepotentials for $N=2$ conformal supergravity in three dimensions, J. High Energy Phys. 12 (2012) 021.

[56] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, Different representations for the action principle in 4D $N=2$ supergravity, J. High Energy Phys. 04 (2009) 007.

[57] W. Siegel, Solution to constraints in Wess-Zumino supergravity formalism, Nucl. Phys. **B142**, 301 (1978).

[58] B. Zumino, in *Recent Developments in Gravitation, Cargèse 1978*, edited by M. Lévy and S. Deser (Plenum Press, New York, 1979), pp. 405–459.

[59] S. M. Kuzenko, U. Lindström, M. Roček, I. Sachs, and G. Tartaglino-Mazzucchelli, Three-dimensional $\mathcal{N}=2$ supergravity theories: From superspace to components, Phys. Rev. D **89**, 085028 (2014).

[60] J. Wess and B. Zumino, Superfield Lagrangian for supergravity, Phys. Lett. **74B**, 51 (1978).

[61] K. S. Stelle and P. C. West, Minimal auxiliary fields for supergravity, Phys. Lett. **74B**, 330 (1978).

[62] S. Ferrara and P. van Nieuwenhuizen, The auxiliary fields of supergravity, Phys. Lett. **74B**, 333 (1978).

[63] B. M. Zupnik and D. G. Pak, Superfield formulation of the simplest three-dimensional gauge theories and conformal supergravities, Teor. Mat. Fiz. **77**, 97 (1988) [Theor. Math. Phys. **77**, 1070 (1988)].

[64] D. Butter and S. M. Kuzenko, A dual formulation of supergravity-matter theories, Nucl. Phys. **B854**, 1 (2012).

[65] S. J. Gates, Jr., Super p-form gauge superfields, Nucl. Phys. **B184**, 381 (1981).

[66] S. J. Gates, Jr. and W. Siegel, Variant superfield representations, Nucl. Phys. **B187**, 389 (1981).

[67] I. L. Buchbinder and S. M. Kuzenko, Quantization of the classically equivalent theories in the superspace of simple supergravity and quantum equivalence, Nucl. Phys. **B308**, 162 (1988).

[68] I. L. Buchbinder and S. M. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity, Or a Walk Through Superspace* (IOP, Bristol, 1995) (revised ed. 1998).

[69] P. Binetruy, F. Pillon, G. Girardi, and R. Grimm, The 3-form multiplet in supergravity, Nucl. Phys. **B477**, 175 (1996).

[70] B. A. Ovrut and D. Waldram, Membranes and three-form supergravity, Nucl. Phys. **B506**, 236 (1997).

[71] S. M. Kuzenko and S. A. McCarthy, On the component structure of $N=1$ supersymmetric nonlinear electrodynamics, J. High Energy Phys. 05 (2005) 012.

[72] P. Binetruy, G. Girardi, and R. Grimm, Supergravity couplings: A Geometric formulation, Phys. Rep. **343**, 255 (2001).

[73] F. Farakos, S. Lanza, L. Martucci, and D. Sorokin, Three-forms in supergravity and flux compactifications, Eur. Phys. J. C **77**, 602 (2017).

[74] T. Dereli and S. Deser, Fermionic Goldstone-Higgs effect in $(2+1)$-dimensional supergravity, J. Phys. A **11**, L27 (1978).

[75] Z. Komargodski and N. Seiberg, From linear SUSY to constrained superfields, J. High Energy Phys. 09 (2009) 066.

[76] S. M. Kuzenko and S. J. Tyler, On the Goldstino actions and their symmetries, J. High Energy Phys. 05 (2011) 055.

[77] E. Bergshoeff, D. Freedman, R. Kallosh, and A. Van Proeyen, Construction of the de Sitter supergravity, arXiv:1602.01678.

[78] S. M. Kuzenko and S. J. Tyler, Relating the Komargodski-Seiberg and Akulov-Volkov actions: Exact nonlinear field redefinition, Phys. Lett. B **698**, 319 (2011).

[79] M. Roček, Linearizing the Volkov-Akulov model, Phys. Rev. Lett. **41**, 451 (1978).

[80] I. Antoniadis and C. Markou, The coupling of non-linear supersymmetry to supergravity, Eur. Phys. J. C **75**, 582 (2015).

[81] N. Cribiori, G. Dall'Agata, F. Farakos, and M. Porrati, Minimal constrained supergravity, Phys. Lett. B **764**, 228 (2017).

[82] E. Ivanov and A. Kapustnikov, General relationship between linear and nonlinear realizations of supersymmetry, J. Phys. A **11**, 2375 (1978).

[83] E. Ivanov and A. Kapustnikov, The nonlinear realization structure of models with spontaneously broken supersymmetry, J. Phys. G **8**, 167 (1982).

[84] R. Casalbuoni, S. De Curtis, D. Dominici, F. Feruglio, and R. Gatto, Non-linear realization of supersymmetry algebra from supersymmetric constraint, Phys. Lett. B **220**, 569 (1989).

[85] S. Deser and J. H. Kay, Topologically massive supergravity, Phys. Lett. **120B**, 97 (1983).

[86] S. Deser, in *Quantum Theory of Gravity*, edited by S. M. Christensen (Adam Hilger, Bristol, 1984), pp. 374–381.

[87] R. Andringa, E. A. Bergshoeff, M. de Roo, O. Hohm, E. Sezgin, and P. K. Townsend, Massive 3D supergravity, Classical Quantum Gravity **27**, 025010 (2010).

[88] E. A. Bergshoeff, O. Hohm, J. Rosseel, E. Sezgin, and P. K. Townsend, More on massive 3D supergravity, Classical Quantum Gravity **28**, 015002 (2011).

[89] G. Alkac, L. Basanisi, E. A. Bergshoeff, M. Ozkan, and E. Sezgin, Massive $\mathcal{N}=2$ supergravity in three dimensions, J. High Energy Phys. 02 (**2015**) 125.

[90] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, New nilpotent $\mathcal{N}=2$ superfields, arXiv:1707.07390.

[91] M. J. Duff, The cosmological constant is possibly zero, but the proof is probably wrong, Phys. Lett. B **226**, 36 (1989); *Conf. Proc. C* 8903131, 403 (1989).

[92] M. J. Duncan and L. G. Jensen, Four-forms and the vanishing of the cosmological constant, Nucl. Phys. **B336**, 100 (1990).

[93] K. Groh, J. Louis, and J. Sommerfeld, Duality and couplings of 3-form-multiplets in $N=1$ supersymmetry, J. High Energy Phys. 05 (2013) 001.

[94] F. Farakos and A. Kehagias, Emerging potentials in higher-derivative gauged chiral models coupled to $N=1$ supergravity, J. High Energy Phys. 11 (2012) 077.

[95] F. Farakos, S. Ferrara, A. Kehagias, and M. Porrati, Supersymmetry breaking by higher dimension operators, Nucl. Phys. **B879**, 348 (2014).