

Axionic black branes in the k -essence sector of the Horndeski modelAdolfo Cisterna,^{1,2,*} Mokhtar Hassaine,^{3,†} Julio Oliva,^{4,‡} and Massimiliano Rinaldi^{2,5,§}¹*Vicerrectoría Académica, Universidad Central de Chile, Toesca 1783, Santiago, Chile*²*Dipartimento di Fisica, Università di Trento, Via Sommarive 14, 38123 Povo (TN), Italy*³*Instituto de Matemática y Física, Universidad de Talca, Casilla 747, Talca, Chile*⁴*Departamento de Física, Universidad de Concepción, Casilla 160-C, Concepción, Chile*⁵*TIFPA—INFN, Via Sommarive 14, 38123 Trento, Italy*

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We construct new black brane solutions in the context of Horndeski gravity, in particular, in its K -essence sector. These models are supported by axion scalar fields that depend only on the horizon coordinates. The dynamics of these fields is determined by a K -essence term that includes the standard kinetic term X and a correction of the form X^k . We find both neutral and charged exact and analytic solutions in D -dimensions, which are asymptotically anti-de Sitter. Then, we describe in detail the thermodynamical properties of the four-dimensional solutions and we compute the dual holographic DC conductivity.

DOI: [10.1103/PhysRevD.96.124033](https://doi.org/10.1103/PhysRevD.96.124033)**I. INTRODUCTION**

The observed current Universe is not only expanding but also accelerating because of the presence of a source to the Einstein equations that differs from the usual mixture of dark matter, baryonic matter, and radiation. In fact, the simplest phenomenological explanation for the acceleration is the presence of a cosmological constant Λ in the Einstein-Hilbert action. At the quantum level, such a constant can be interpreted as a renormalized vacuum energy. The standard model of cosmology assumes that the current Universe is dominated by the vacuum energy together with a large amount of cold dark matter and a tiny fraction of baryonic matter and it is called Λ CMD model.

From a fundamental point of view, however, the cosmological constant has a series of fundamental and conceptual issues, which makes alternatives rather appealing [1]. In general terms, one can replace the cosmological constant with a dynamical degree of freedom that is often modeled as a fluid with special properties that goes under the name of “dark energy.” In this way, the matter sector of the theory is implemented by a fluid with unusual but still reasonable properties, whose dynamics dominates at late times (for a comprehensive review, see e.g. [2]). Finally, the acceleration of the Universe could also be driven by the dynamics of the classical counterparts of the standard model fields [3].

There is a somewhat more radical approach to the problem of dark energy that relies upon a modification of general relativity (GR) in the infrared. In other words, this means that the Einstein equations are different at

cosmological scales. Several models of modified gravity have been explored during the last decade [4,5]. One of the most popular is the so-called scalar-tensor theories of gravity (STT), first proposed in the late sixties by Brans-Dicke [6] (for a modern review see e.g. [7]). STT represent the simplest way to describe a diffeomorphism invariant theory in four dimensions that avoids Ostrogradski instabilities, which typically arise in higher order theories [8,9]. The price to pay is to introduce new degrees of freedom in the form of one or more dynamical scalar fields. Through a suitable Weyl rescaling of the fields, it is always possible to write STT in terms of modified gravity actions where the Ricci scalar R is replaced by some arbitrary function of it, $f(R)$. At least at the classical level, STT and $f(R)$ gravity are perfectly equivalent [10].

In STT, gravity is described by the graviton spin-two field and one or more spin-zero particles, represented by scalar fields. In order to avoid possible violations of the Einstein equivalence principle the usual prescription is that the scalar fields are only coupled to the metric and not to matter particles. This means that, in the matter action, there is no coupling between the new degrees of freedom and ordinary matter. However, due to the nonlinearity of Einstein equations, scalar fields produce a backreaction on the metric that, in turn, affects the motion of test particles. Therefore the dynamics of matter fluids is influenced by scalar fields even in the case of minimal coupling [7,11].

The most general STT constructed in four dimensions and yielding at most second order equations of motion is known as the Horndeski model [12], which is better known in its modern version as the Galileon theory [13]. The latter is a STT coming from the generalization of the decoupling limit of the brane-inspired Dvali-Gabadadze-Porrati model [14]. Galileon theory, which exhibits Galilean symmetry in

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Minkowski spacetime, was further covariantized in [15] and it was finally shown to be equivalent to the original Horndeski action; see [16].

The Galileon action exhibits shift symmetry and, in its covariant form, is given by

$$\begin{aligned} \mathcal{L} = & \mathcal{K}(X) - G_3(X)\square\phi + G_4(X)R \\ & + G_{4,X}(X)[(\square\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] + G_5(X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ & - \frac{G_{5,X}}{6}[(\square\phi)^3 + 2(\nabla_\mu\nabla_\nu\phi)^3 - 3\square\phi(\nabla_\mu\nabla_\nu\phi)^2]. \quad (1) \end{aligned}$$

Here, X represents the canonical kinetic term for the scalar field ϕ , while \mathcal{K} and G_i are arbitrary functions of X . Each function can be generalized to the case in which it also explicitly depends on the scalar field itself. Nevertheless, in such a case, the shift invariance of theory is lost and this considerably complicates the integration of the field equations.

Many sectors of the theory (1) have been investigated in cosmology. For instance, it was shown that (1) contains a subset which possesses a self-tuning mechanism that allows to circumvent Weinberg's theorem on the cosmological constant [17]. Moreover, the sector defined by the non-minimal kinetic coupling controlled by the Einstein tensor exhibits interesting inflationary properties without the need of *ad hoc* potential terms [18–22]. Likewise, the non-minimal coupling between the Einstein tensor and the scalar field kinetic term can, on large scales, mimic cold dark matter and also flatten the rotational curves of galaxies [23]. Finally, several works have been devoted to the study of cosmological perturbations with the aim of finding observable deviations from GR in large-scale structures and the conditions on the parameter space that avoid too large gravitational instabilities [24].

Technically speaking, the so-called K-essence models of dark energy belong to the class of gravitational theories represented by (1). In K-essence, the acceleration of the Universe (both at early and late times) can be driven by the kinetic energy instead of the potential energy of the scalar field [25]. The model was first introduced in [26] and then specifically used as dark energy models in [27–32]. These models are characterized by a nonlinear kinetic term for the scalar field and are expressed in (1) by the arbitrary function $\mathcal{K}(X)$ (together with $G_4 = 1$ and $G_3 = G_5 = 0$). Quantum and classical stability of K-essence have been investigated [33]. In particular, the classical stability and their perturbations are crucial to discriminate the model from standard GR in view of the forthcoming Euclid mission [34].

One fundamental step that may put the theory on a solid theoretical foot is the construction of black hole solutions. In principle, there is a no-hair theorem that prevents the existence of nontrivial black hole solutions in Galileon gravity [35]. However, there are ways to get around this theorem and several black hole solutions have been found

for particular sectors of (1), in particular, the one containing the nonminimal coupling between the Einstein tensor and the kinetic term. Spherically symmetric solutions were found in [36–39] where their thermodynamical properties were also studied.¹ Moreover, anti-de Sitter (AdS), asymptotically flat stealth and Lifshitz solutions with a self-tuned effective cosmological constant were found making use of a time-dependent scalar field in [44,45]. Charged solutions were found in [46,47]. Recently also, for the sectors of (1) controlled by G_3 and G_4 , analytical and numerical solutions have been found [48,49].²

There is still one sector of (1), where black holes solutions are little known, that is the K-essence sector governed by $\mathcal{K}(X)$. This work aims to fill this gap, at least partially, by exploring black hole configurations in the sector of (1) that contains terms like X^k , in addition to the usual kinetic term.³ To construct our solutions, instead of considering a spherically symmetric scalar field, we use axion fields which depend linearly on the Cartesian coordinates along the flat horizon. We see later that these kinds of configurations are used in the context of dual condensed matter systems due to the fact that they break the translational invariance of the dual field theory [52]. This is an easy way to circumvent the no-hair theorem [35], which is mostly based on the fact that the equation of motion for the scalar field derived from (1) is given by a current conservation law of the form $\nabla_\mu J^\mu = 0$. For the case of spherically symmetric scalar fields, this current is given by the component J^r whose modulus diverges at the horizon. In the case studied here, our axion fields yield a finite current on the black hole horizon while simultaneously satisfying the Klein-Gordon equation. Moreover, the contribution to the equations of motion coming from the K-essence term is still spherically symmetric and the energy of the solutions remains finite.⁴

In Ref. [53], a static black brane with axionic charge generated by the presence of two 3-form fields was presented. The symmetry of the solution is endowed with a planar horizon with a lapse function mimicking that of a hyperbolic black hole in AdS. This apparent discrepancy between the horizon topology and the metric behavior was shown to be due to the presence of the axionic charges, which play the role of an effective curvature term. The thermodynamical properties and the possibility of phase transitions were also reported in [53]. These ideas were also applied to construct black branes with a source given by a

¹This model has been also thoroughly investigated in the context of astrophysical configurations such as neutron and boson stars [40–43].

²In [50] solutions have been obtained considering a kinetic coupling controlled by the Gauss-Bonnet invariant.

³These kind of Lagrangians are used to obtain cosmological models with equation of state parameter satisfying $\omega < 1$ [51].

⁴This is similar to what happens in the case of the linearly time-dependent scalar fields considered in [44].

scalar field nonminimally coupled to gravity [54,55].⁵ Planar/toroidal black holes with a scalar field are of special interest in the context of the AdS/CFT correspondence [57] due, in particular, to their applications in nonconventional superconductor systems [58,59]. Within this approach, the nonzero condensate behavior of the unconventional superconductors can be reproduced by means of a hairy black hole at low temperature with a hair that should disappear as the temperature increases. Usually, planar/toroidal solutions suffer from singular behaviors due to the lack of a curvature scale on the horizon. Nevertheless, this situation is successfully circumvented using axion fields which are homogeneously distributed along the horizon coordinates, providing in this way an effective curvature scale which makes the spacetime nonsingular. Several solutions with these ingredients have been reported in order to study different aspects of their holographic dual systems [52,60–66]. A very interesting application is the construction of homogeneous black string and black p-branes with negative cosmological constant, with no more ingredients than minimally coupled scalar fields [67]. Moreover, recently these ideas have been applied to the case of Horndeski theory, specifically to the nonminimal kinetic coupled sector [68–70].

The paper is organized as follows: Section II is devoted to the description of our model, its principal properties and the equations of motion. In Sec. III we construct asymptotically AdS black brane solutions, including the case where electric and magnetic monopole charges are considered. The particular case of vanishing cosmological constant is also studied. Section IV is devoted to the thermodynamical analysis of the AdS solutions while in Sec. V we present some holographic applications. Finally in Sec. VI we give some final remarks and outline some possible extensions. In the appendix, we report the higher-dimensional extension of the solution.

II. THE MODEL

We consider the following K-essence Lagrangian in four dimensions,

$$\mathcal{L} = \mathcal{K}(X_1, X_2), \quad (2)$$

where the two scalar fields, with their kinetic terms X_1 and X_2 , correspond to the two axion fields. As considered below, the axion fields are homogeneously distributed along the coordinates of the planar horizon. This explains the reasons for considering two axion fields in four dimensions. As mentioned before, we study the case in which the dynamics of each scalar field is governed by a standard

⁵A new planar solution with a conformally coupled scalar field is presented in [56]. This solution represents a novel generalization of the Bekenstein black hole plus cosmological constant, without self interaction and free of self-tuned parameters.

kinetic term plus a nonlinear contribution given by an arbitrary power of X . More precisely, we consider a \mathcal{K} -term of the form $\mathcal{K}(X_i) = -\sum_{i=1}^2 (X_i + \gamma X_i^k)$, and hence our four-dimensional action reads

$$\begin{aligned} \mathcal{I}[g_{\mu\nu}, \phi_i] &= \int \left[\kappa(R - 2\Lambda) - \sum_{i=1}^2 \left(\frac{1}{2} \nabla^\mu \phi_i \nabla_\mu \phi_i + \gamma \left(\frac{1}{2} \nabla^\mu \phi_i \nabla_\mu \phi_i \right)^k \right) \right] \\ &\times d^4x \sqrt{-g}, \end{aligned} \quad (3)$$

where we have defined $X_i = \frac{1}{2} \nabla^\mu \phi_i \nabla_\mu \phi_i$ with $i = 1, 2$. The coupling γ (with mass dimension $4 - 4k$) is supposed to be positive in order to avoid phantom contributions.⁶ For $\gamma = 0$ we recover the case of two minimally coupled scalar fields studied in [52,53]. Even if the solutions can be constructed in arbitrary dimensions we focus our attention on the four-dimensional case, leaving the D-dimensional extension to Appendix A. The variation of the action with respect to the metric yields the following Einstein equations,

$$\begin{aligned} \kappa(G_{\mu\nu} + \Lambda g_{\mu\nu}) &= \frac{1}{2} \sum_i [\partial_\mu \phi_i \partial_\nu \phi_i - g_{\mu\nu} X_i \\ &+ \gamma(k X_i^{k-1} \partial_\mu \phi_i \partial_\nu \phi_i - g_{\mu\nu} X_i^k)], \end{aligned} \quad (4)$$

while the Klein-Gordon equation takes the form

$$[(1 + \gamma k X_i^{k-1}) g^{\mu\nu} + \gamma k(k-1) X_i^{k-2} \nabla^\mu \phi_i \nabla^\nu \phi_i] \nabla_\mu \nabla_\nu \phi_i = 0. \quad (5)$$

We now impose the planar metric ansatz

$$ds^2 = -F(r) dt^2 + \frac{dr^2}{G(r)} + r^2(dx_1^2 + dx_2^2), \quad (6)$$

and we assume that the axion fields depend on the coordinates (x_1, x_2) only.⁷ In the case $F = G$, the Klein-Gordon equations are easily solved by

$$\phi_1 = \lambda x_1, \quad \phi_2 = \lambda x_2. \quad (7)$$

Note that these scalar fields can be dualized to construct solutions with backreacting 2-forms, $B_{(2)}$, by setting

⁶Recently, solutions for the minimally coupled case with phantom axion fields were studied in [71].

⁷In [53] the authors considered Einstein gravity with a source given by two 3-form fields (whose Hodge duals can be identified with the exterior derivatives of two scalar fields). A Birkoff's-like theorem was established where it is shown that each of the two 3-form fields must depend on one for the transverse spatial coordinate.

$$H_{(3)}^{(i)} = dB_{(2)}^{(i)} = \star d\phi_i, \quad i = 1, 2, \quad (8)$$

where \star denotes the Hodge dual.

In order to ensure that the solutions of the previous equations do not generate ghosts, it is important to check whether or not they satisfy the null energy condition given by

$$T_{\mu\nu}n^\mu n^\nu \geq 0, \quad i = 1, 2. \quad (9)$$

Classical stability is instead guaranteed by a positive sound speed, namely

$$c_s^2 = \frac{\mathcal{K}_{,X_i}}{\mathcal{K}_{,X_i} + 2X_i\mathcal{K}_{,X_iX_i}} > 0. \quad (10)$$

In our model a sufficient condition to satisfy simultaneously both requirements is $k > 1/2$. As we discuss below, the further restriction $k > 3/2$ also guarantees that the solutions asymptotically match the GR ones and have finite ADM mass.

III. K-ESSENCE BLACK HOLES WITH AXIONS

Under the conditions described above Eqs. (4) and (5) have the following exact black brane solution:

$$F(r) = G(r) = \frac{r^2}{l^2} - \frac{2M}{r} - \frac{\lambda^2}{2\kappa} + \frac{\gamma\lambda^{2k}}{2^k(2k-3)\kappa} r^{2(1-k)} \quad (11)$$

$$\phi_1 = \lambda x_1, \quad \phi_2 = \lambda x_2.$$

The case $k = 3/2$ needs to be integrated separately and it yields a logarithmic branch as reported in Appendix A. Looking at this four-dimensional solution we observe that for $1/2 < k < 3/2$, the asymptotic behavior of our AdS solutions differs from the standard ones defined in [72], and, as a consequence, configurations with infinite mass could be obtained. Thus, from now on we consider only the case $k > 3/2$, which allows the use of standard methods to compute the mass of our solutions.

It is evident that the effect of the axion fields is to include an effective hyperbolic curvature scale on the metric proportional to the axion parameter λ . By setting $\gamma = 0$, we find the solution first described in [53].

These solutions can be easily generalized to include electric and magnetic monopole charges. In order to do this it is sufficient to include in the action (3) the standard Maxwell term

$$S = -\frac{1}{4} \int F_{\mu\nu} F^{\mu\nu} d^4x \sqrt{-g}. \quad (12)$$

Then, the Maxwell equation

$$\nabla_\mu F^{\mu\nu} = 0 \quad (13)$$

is easily solved by

$$A = -\frac{Q_e}{r} dt + \frac{Q_m}{2} (x_1 dx_2 - x_2 dx_1), \quad (14)$$

where Q_e and Q_m are the electric and magnetic monopole charges. Finally, the general charged solution of the Einstein equations reads

$$F(r) = G(r) = \frac{r^2}{l^2} - \frac{2M}{r} - \frac{\lambda^2}{2\kappa} + \gamma \frac{\lambda^{2k}}{2^k(2k-3)\kappa} r^{2(1-k)} + \frac{1}{4\kappa r^2} (Q_e^2 + Q_m^2). \quad (15)$$

Solutions (11) and (15) are the neutral and charged K-essence generalization of the solutions found in [52,53], which are known to possess interesting holographic properties. We discuss a particular application in Sec. V.

A very interesting case is the one corresponding to $\Lambda = 0$. In particular, the uncharged solution takes the form

$$F(r) = G(r) = -\frac{2M}{r} - \frac{\lambda^2}{2\kappa} + \gamma \frac{\lambda^{2k}}{2^k(2k-3)\kappa} r^{2(1-k)}. \quad (16)$$

This solution can have two horizons. To show this in a simple way let us consider the case $k = 2$. Then, the horizon location can be found algebraically by solving the equation

$$F(r) = G(r) = -\frac{2M}{r} - \frac{\lambda^2}{2\kappa} + \gamma \frac{\lambda^4}{4\kappa r^2}. \quad (17)$$

The two distinct solutions are

$$r_1 = -\frac{2M\kappa + \sqrt{4M^2\kappa^2 + 2\lambda^6\gamma}}{2\lambda^2}, \quad (18)$$

$$r_2 = \frac{-2M\kappa + \sqrt{4M^2\kappa^2 + 2\lambda^6\gamma}}{2\lambda^2}. \quad (19)$$

For $\gamma > 0$ there is just one positive root that corresponds to a cosmological horizon ($r = r_c = r_2$) which surrounds a curvature singularity located at the horizon. However, if both γ and M are negative, it is possible to find two horizons. This is evident upon the substitutions $M \rightarrow -|M|$ and $\gamma \rightarrow -|\gamma|$, which gives the location of an event and a cosmological horizon located respectively at $r = r_h$ and $r = r_c$, with

$$r_c = \frac{2|M|\kappa + \sqrt{4|M|^2\kappa^2 - 2\lambda^6|\gamma|}}{2\lambda^2}, \quad (20)$$

$$r_h = \frac{2|M|\kappa - \sqrt{4|M|^2\kappa^2 - 2\lambda^6|\gamma|}}{2\lambda^2}. \quad (21)$$

As we mentioned above, negative values of γ could induce violations of the null energy condition or nonhyperbolicity of the Klein-Gordon equation. However, this violation may be hidden behind the event horizon provided the condition

$$|M| > \frac{7\sqrt{3}\lambda^3}{12\kappa} \sqrt{|\gamma|} \quad (22)$$

is satisfied.

IV. THERMODYNAMICAL PROPERTIES OF ADS K-ESSENCE BLACK BRANES

In order to explore some holographic applications, we first provide a complete and detailed analysis of the thermodynamic features of the electrically charged AdS solutions. Note that such studies have been done for Horndeski black holes with sources given by scalar fields; see e.g. [73].

In our case, the thermodynamics analysis is carried out through the Euclidean approach. In this case, the partition function for a thermodynamical ensemble is identified with the Euclidean path integral in the saddle point approximation around the classical Euclidean solution [74]. Since we are interested in a static metric with a planar base manifold, it is enough to consider the following class of metric,

$$ds^2 = N(r)^2 F(r) d\tau^2 + \frac{dr^2}{F(r)} + r^2(dx_1^2 + dx_2^2),$$

where τ is the periodic Euclidean time related to the Lorentzian time by $\tau = it$, and the radial coordinate $r_h \leq r < \infty$. Now, in order to have a well-defined reduced action principle with a Euclidean action depending only on the radial coordinate, some precautions must be taken. Indeed, in the present case, we are interested in configurations where the scalar fields ϕ_i do not depend on the radial coordinate but rather on the planar coordinates. Nevertheless, since the scalar fields only appear in the action through their derivatives that are constants, we can “artificially” introduce radial scalar fields and their associated “conjugate momentum” as

$$\begin{aligned} \Psi_i(r) &:= \int_0^r \partial_i \phi_i dr, & \Pi_{(i)} &:= -\frac{1}{2} \partial_r \Psi_i(r), \\ \hat{\Psi}_i(r) &:= \int_0^r N \partial_r \Psi_i dr, & \Psi_{i,k}(r) &:= \int_0^r (\partial_i \phi_i)^k dr, \\ \Pi_{(i,r)} &:= -\frac{1}{2} \partial_r \Psi_{i,k}, & \hat{\Psi}_{i,k}(r) &:= \int_0^r \frac{N}{2^{k-1}} r^{2(1-k)} \partial_r \Psi_{i,k} dr. \end{aligned} \quad (23)$$

Under this prescription, the Euclidean action I_E is given by

$$\begin{aligned} I_E = \sigma \beta \int_{r_h}^R \sum_{i=1}^2 \left\{ N \left[2\Pi_{(i)}^2 + \frac{2\gamma}{2^{k-1}} r^{2(1-k)} \Pi_{(i,k)}^2 + \frac{1}{2r^2} \Pi_A^2 \right. \right. \\ \left. \left. + 2\kappa r F' + 2\kappa F - \frac{6\kappa r^2}{l^2} \right] \right. \\ \left. - 2\hat{\Psi}_i \Pi'_{(i)} - 2\gamma \hat{\Psi}_{i,k} \Pi'_{(i,k)} - A \Pi'_A \right\} dr + B_E, \quad (24) \end{aligned}$$

where β is the inverse of the temperature, σ stands for the volume of the two-dimensional compact flat space and Π_A denotes the conjugate momentum to the vector potential A ,

$$\Pi_A = -\frac{r^2 A'}{N}.$$

The Euclidean action is obtained in the limit $R \rightarrow \infty$ and the boundary term B_E is fixed by requiring that the action has a well-defined extremum, i.e. $\delta I_E = 0$. It is easy to check that the field equations obtained by varying the reduced action yield the electrically charged AdS solution (15) with $Q_m = 0$. In fact the variations with respect to F and A give respectively $2\kappa r N' = 0$ and $\Pi'_A = 0$. The first equation implies that N is constant and without loss of generality, can be taken to be $N = 1$. The second equation imposes the electric potential to have the Coulomb form $A_i = \frac{Q_i}{r}$. On the other hand, the variations with respect to the conjugate momenta $\Pi_{(i)}$, $\Pi_{(i,k)}$ and Π_A yield equations that are trivially satisfied while those obtained by variation with respect to $\hat{\Psi}_i$ and $\hat{\Psi}_{i,k}$ can be easily solved by choosing

$$\Pi_{(i)} = -\frac{1}{2} \lambda, \quad \Pi_{(i,k)} = -\frac{1}{2} \lambda^k \Rightarrow \phi_i = \lambda x_i.$$

Finally, the equation obtained by varying N ,

$$2\Pi_{(i)}^2 + \frac{2\gamma}{2^{k-1}} r^{2(1-k)} \Pi_{(i,k)}^2 + \frac{1}{2r^2} \Pi_A^2 + 2\kappa r F' + 2\kappa F - \frac{6\kappa r^2}{l^2} = 0,$$

gives rise to a differential equation for the metric function F whose integration yields (15).

Now, in order to compute the boundary term, we consider the formalism of the grand canonical ensemble where the temperature β^{-1} as well as the “potentials” at the horizon $A(r_h)$, $\hat{\Psi}_i(r_h)$ and $\hat{\Psi}_{i,k}(r_h)$ are fixed. The extremal condition $\delta I_E = 0$ implies that the contribution of the boundary term must be given by

$$\begin{aligned} \delta B_E = \left[\sum_{i=1}^2 (-2\kappa \sigma \beta N r \delta F + 2\sigma \beta \hat{\Psi}_i \delta \Pi_{(i)} + 2\sigma \beta \gamma \hat{\Psi}_{i,k} \delta \Pi_{(i,k)} \right. \\ \left. + \sigma \beta A \delta \Pi_A \right]_{r=r_h}^{r=R}. \end{aligned}$$

Without loss of generality, we can set again $N = 1$, and the contribution at infinity reduces to

$$\delta B_E(R) = 4\kappa\sigma\beta\delta M \Rightarrow B_E(R) = 4\kappa\sigma\beta M.$$

At the horizon, in order to avoid the conical singularities, the variation of the metric function at the horizon is given by $\delta F|_{r_h} = -\frac{4\pi}{\beta}\delta r_h$, and hence one gets

$$B_E(r_h) = 4\pi\kappa\sigma r_h^2 + \sigma\beta A(r_h)Q_e - \sigma\beta \sum_i (\lambda\hat{\Psi}_i(r_h) + \gamma\lambda^k\hat{\Psi}_{i,k}(r_h)).$$

Finally, the boundary term becomes

$$B_E = 4\kappa\sigma\beta M - 4\pi\kappa\sigma r_h^2 - \sigma\beta A(r_h)Q_e + \sigma\beta \sum_i (\lambda\hat{\Psi}_i(r_h) + \gamma\lambda^k\hat{\Psi}_{i,k}(r_h)). \quad (25)$$

The Euclidean action is related to the Gibbs free energy \mathcal{G} by

$$I_E = \beta\mathcal{G} = \beta\mathcal{M} - \mathcal{S} - \beta A(r_h)Q_e - \beta \sum_i (\hat{\Psi}_i Q_i + \hat{\Psi}_{i,k} Q_{i,k}).$$

The mass \mathcal{M} is given by

$$\begin{aligned} \mathcal{M} &= \left(\frac{\partial I_E}{\partial \beta}\right)_{\hat{\Psi}_i, \hat{\Psi}_{i,k}} - \frac{\hat{\Psi}_i}{\beta} \left(\frac{\partial I_E}{\partial \hat{\Psi}_i}\right)_\beta - \frac{\hat{\Psi}_{i,k}}{\beta} \left(\frac{\partial I_E}{\partial \hat{\Psi}_{i,k}}\right)_\beta = 4\kappa\sigma M \\ &= 4\kappa\sigma \left(\frac{r_h^3}{2l^2} - \frac{\lambda^2 r_h}{4\kappa} + \frac{\gamma\lambda^{2k} r_h^{3-2k}}{2^{k+1}(2k-3)\kappa} + \frac{Q_e^2}{8\kappa r_h} \right), \end{aligned} \quad (26)$$

while the entropy \mathcal{S} , the electric charge Q_e and the axion charges Q_i , $Q_{i,k}$ are defined by

$$\begin{aligned} \mathcal{S} &= \beta \left(\frac{\partial I_E}{\partial \beta}\right)_{\hat{\Psi}_i, \hat{\Psi}_{i,k}} - I_E = 4\pi\kappa\sigma r_h^2, \\ Q_e &= -\frac{1}{\beta} \left(\frac{\partial I_E}{\partial A(r_h)}\right)_\beta = \sigma Q_e, \\ Q_i &= -\frac{1}{\beta} \left(\frac{\partial I_E}{\partial \hat{\Psi}_i}\right)_\beta = -\sigma\lambda, \\ Q_{i,k} &= -\frac{1}{\beta} \left(\frac{\partial I_E}{\partial \hat{\Psi}_{i,k}}\right)_\beta = -\sigma\gamma\lambda^k. \end{aligned} \quad (27)$$

With these results it is trivial to see that the first law holds, namely

$$d\mathcal{M} = Td\mathcal{S} + A(r_h)dQ_e + \sum_i (\hat{\Psi}_i(r_h)dQ_i + \hat{\Psi}_{i,k}(r_h)dQ_{i,k}).$$

We conclude this section by comparing our results with those obtained recently for a similar model with phantom axion fields [71]. In this reference, the thermodynamics analysis of the phantom black hole solution is carried out without considering the axion parameter constant λ as an

axion charge. Because of that, the thermal properties of the phantom solution are quite analogous to those of the Schwarzschild-AdS black hole. The importance of considering axion parameter constant λ as an axion charge is particularly important when holographic applications and phase transitions are studied.

V. HOLOGRAPHIC DC CONDUCTIVITY

Charged black brane solutions provide a perfect setup to compute holographic conductivities⁸ [52,68,76–79]. This can be done by constructing a conserved current with radial dependence from which it is possible to obtain the holographic properties on the boundary in terms of the black hole horizon data. Here, we are interested in the effects of the nonlinear kinetic term, controlled by the coupling constant γ , on the conductivity of the dual field theory. Along the lines of [78], we introduce a perturbation of the fields of the form

$$\begin{aligned} ds^2 &= -F(r)dt^2 + \frac{dr^2}{G(r)} + r^2(dx_1^2 + dx_2^2) \\ &\quad + 2\epsilon r^2 h_{tx_1}(r) dt dx_1 + 2\epsilon r^2 h_{rx_1}(r) dr dx_1 \end{aligned} \quad (28)$$

for the metric tensor,

$$A = \mu \left(1 - \frac{r_0}{r}\right) dt - \epsilon E dt + \epsilon a_{x_1}(r) dx \quad (29)$$

for the gauge field, and

$$\phi_1 = \mathring{\phi}_1 + \epsilon \frac{\Phi(r)}{\lambda} \quad (30)$$

for one of the axion fields, with the background axion field fixed by $\mathring{\phi}_1 = \lambda x_1$. Here $\mu = Q_e/r_0$ is the chemical potential. Plugging this in the field equations and keeping the linear terms in ϵ , Maxwell equations allow one to construct the current density in terms of the horizon radius r_h as

$$J = \frac{\lambda^2 r_h^2 + \mu^2 r_h^2 + 2^{1-k} \lambda^{2k} r_h^{4-2k} \gamma k}{\lambda^2 r_h^2 + 2^{1-k} \lambda^{2k} r_h^{4-2k} \gamma k} E, \quad (31)$$

which trivially leads to a DC conductivity of the form

$$\sigma = \frac{\partial J}{\partial E} = \frac{\lambda^2 r_h^2 + \mu^2 r_h^2 + 2^{1-k} \lambda^{2k} r_h^{4-2k} \gamma k}{\lambda^2 r_h^2 + 2^{1-k} \lambda^{2k} r_h^{4-2k} \gamma k}. \quad (32)$$

Note that when $\gamma = 0$, this expression coincides with the result obtained in Eq. (4.4) of Ref. [78] for minimally coupled axions with standard kinetic terms in $D = 4$ (see

⁸See also [75] for the computation of thermoelectric transport coefficients of systems which are dual to five-dimensional, charged black holes with horizons modeled by Thurston geometries.

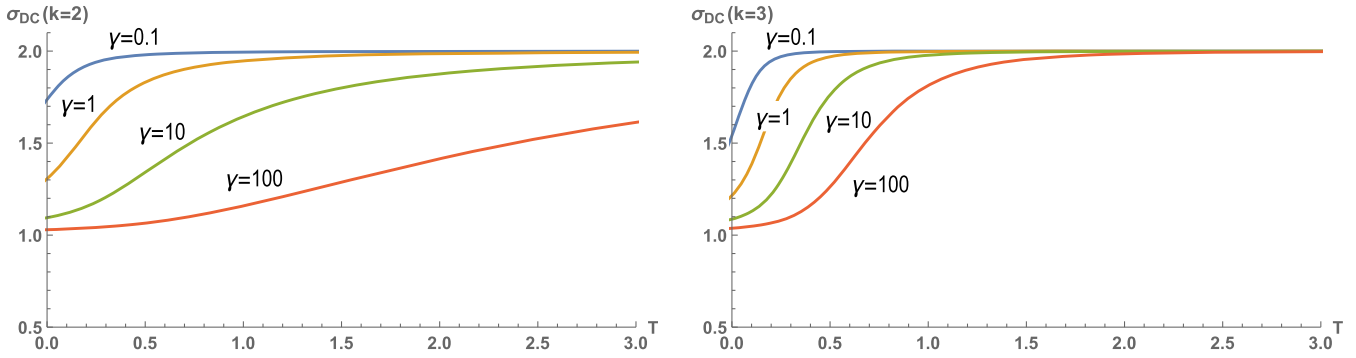


FIG. 1. DC conductivities as a function of temperature for the cases $k = 2$ (left panel) and $k = 3$ (right panel). Different curves on each panel correspond to different values of the derivative self-interaction coupling γ . We have set the chemical potential $\mu = 1$, as well as the AdS radius $l = 1$, the axions constant $\lambda = 1$ and $\kappa = 1$.

also [52,68]). Note that in such a case, in terms of the chemical potential, the DC conductivity remains constant as the radius of the horizon changes. Figure 1 shows the behavior of the DC conductivity for a quadratic derivative self-interaction ($k = 2$) and for a cubic one ($k = 3$). The plots show that in the limit $T \rightarrow 0$ the DC conductivity goes to a constant, which shows that at low temperatures the dual system presents a metallic phase. For large temperatures, the dual system approaches the behavior of the dual system with a minimally coupled axion; i.e. the conductivities saturate to a constant which coincides with the one obtained with $\gamma = 0$. As the strength of the nonlinearities of the axions (controlled by γ) increases, the conductivity at low temperature decreases, and one can see that in such a case larger temperatures are required to recover the result with minimally coupled, free axions.

VI. CONCLUDING REMARKS

As we know, the Horndeski model [12] is the most general STT we can construct with second order equations of motion in four dimensions. In its shift invariant form, it is given by the covariant version of Galileon gravity [15] whose Lagrangian is given by (1). Black hole solutions with spherical symmetry and with flat asymptotic behavior are forbidden by the no-hair results of Hui and Nicolis [35]. Their argument relies on the shift invariance of (1) which forces the scalar field equation of motion to be written as a current conservation law. Then, by demanding that the norm of this current is finite on the horizon of the hypothetical black hole solution, it is possible to show that, for spherically symmetric solutions with flat asymptotic geometry, the scalar field must be trivial. In spite of this, for the particular model of the nonminimal coupling between the Einstein tensor and the kinetic term of the scalar field, there are two ways to circumvent the no-hair conjecture. The first one is to relax the asymptotic flatness of the solutions allowing (A)dS behaviors [36–38]. The second one is to consider scalar fields that do not share the

same symmetries of the metric, but are nevertheless non-trivial. In the latter case, the simplest way is to consider scalar fields linearly dependent on time [44].

In this work we have applied the second strategy in order to construct black brane solutions in the K-essence sector of Horndeski/Galileon gravity, specifically in the model in which, along with the standard kinetic term, a nonlinear contribution of the form X^k is included. This sector represents scalar fields with nonlinear kinetic terms without need of any coupling between the scalar field and the curvature.⁹

Specifically speaking, to construct our solutions we have considered scalar fields that depend linearly on the coordinates of a flat horizon of $(D - 2)$ dimensions. The scalar fields are homogeneously distributed along these flat directions, implying the inclusion of $i = (D - 2)$ scalar fields on the theory. It follows that each scalar current norm $|J_i|$ does not diverge on the horizon and, at the same time, satisfies the continuity equation, $\nabla_\mu J_i^\mu = 0$, with a non-trivial profile for the scalar field. In order to satisfy the null energy condition and to ensure the hyperbolicity of the Klein-Gordon equation we have constrained the possible values of k to be greater than $(D - 1)/2$. This is a sufficient condition to satisfy both requirements. It is interesting to note that this choice of k endows our solutions with the same asymptotic behavior of GR without affecting the behavior of the mass term at infinity. Our solutions possess flat horizon; however the inclusion of the axion fields provides a new curvature scale including a noncanonical hyperbolic term on the metric. The electrically and magnetically charged extension is shown to exist as well as the higher-dimensional extension. We observe that in the case in which $\Lambda = 0$ solutions possessing a cosmological horizon are also possible provided both mass and coupling γ are negative. We have analyzed the thermodynamical

⁹This could be interesting due to recent results which indicate that the inclusion of nonminimal couplings with the curvature might induce problems when defining a well-posed initial value problem in Horndeski theory [80].

properties of the asymptotically AdS solutions in order to study the possible holographic applications and we have provided an explicit computation for the DC conductivities in the holographic dual theory of the electrically charged configurations. It would be also interesting to see whether or not these solutions violate the reverse isoperimetric inequality as is the case of the Horndeski black brane solutions with axions recently constructed in [68] and studied in [69]. Another possible extension of this work would be to see how the nonlinear contribution for the scalar field affects the realization of the momentum dissipation phenomena extensively studied for minimally coupled scalar fields [52] and also recently in the Einstein coupled sector of Horndeski gravity [68].

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APPENDIX: D-DIMENSIONAL SOLUTIONS

The higher-dimensional neutral extension of the solution is given by

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 d\Sigma_{D-2}^2 \quad (\text{A1})$$

where $d\Sigma_{D-2}$ stands for a (D-2) base manifold with null curvature, and where

$$F(r) = \frac{r^2}{l^2} - \frac{2M}{r^{D-3}} - \frac{\lambda^2}{2(D-3)\kappa} - \gamma \frac{\lambda^{2k}}{2^k(2k+1-D)} r^{2(1-k)}. \quad (\text{A2})$$

Here, the axion fields are $\phi_i = \lambda x_i$ and $l^{-2} := -\frac{2\Lambda}{(D-2)(D-1)}$. As it was also clear in four dimensions with $k = 3/2$, there exists a logarithmic branch for $k = (D-1)/2$.¹⁰ In four dimensions, the logarithmic branch reads

$$F(r) = \frac{r^2}{l^2} - \frac{2M}{r} - \frac{\lambda^2}{2\kappa} - \gamma \frac{\sqrt{2}\lambda^3 \ln(r/r_0)}{4\kappa r}. \quad (\text{A3})$$

It is easy to see that these solutions can be easily extended to the charged cases.

¹⁰As we state on the four-dimensional case please note that these solutions have a standard AdS asymptotic for $k > (d-1)/2$.

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