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Ghost-free higher derivative unimodular gravity

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The unimodular version of the ghost-free higher derivative gravity is obtained. It is the unimodular reduction of some particular Lagrangians quadratic in curvature.

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I. INTRODUCTION

Unimodular gravity (UG) is an interesting truncation of general relativity (GR), where the spacetime metric is restricted to be unimodular,

$$g \equiv \det g_{\mu\nu} = -1. \tag{1.1}$$

It is convenient to implement the truncation through the (noninvertible) map

$$g_{\mu\nu} \to |g|^{-1/n} g_{\mu\nu}. \tag{1.2}$$

The resulting theory is not diffeomorphism (Diff) invariant anymore, but only TDiff invariant. Transverse diffeomorphisms are those whose generator is transverse, that is,

$$\partial_\mu \xi^\mu = 0. \tag{1.3}$$

The ensuing action of unimodular gravity (cf., [1] for a recent review with references to previous literature) reads

$$S_{UG} \equiv \int d^n x \mathcal{L}_{UG} \equiv -M_P^{n-2} \int |g|^{1/n} \left(R + \frac{(n-1)(n-2)}{4n^2} \frac{g^{\mu\nu} \nabla_\mu g \nabla_\nu g}{g^2} \right). \tag{1.4}$$

It can be easily shown using Bianchi identities that the classical equations of motion (EM) of unimodular gravity coincide with those of general relativity with an arbitrary cosmological constant. The main difference at this level between both theories is that a constant value for the matter potential energy has no weight at all, which solves part of the cosmological constant problem (namely, why the cosmological constant is not much bigger that observed). This property is preserved by quantum corrections.

While the nature of the cosmological constant makes unimodular gravity an appealing alternative to general relativity, it is still an effective field theory for low energies as it has the same problems with renormalizability.

On the other hand, it has long been known [2] that quadratic theories of gravity are quite interesting. They are renormalizable (even asymptotically free) and they are in many senses the closest analogues to Yang-Mills theories. The problem however is with unitarity or, equivalently, with a ghostly state in the spectrum. This problem can in turn be traced to the quartic propagators, which contradict the Källen-Lehmann spectral representation.

It has been suggested by Siegel [3], however, that string theory provides a natural way out, namely, exponential falling off of the propagators, of the type

$$L = \phi \Box e^{-\frac{\Box}{2M^2}} \phi + \phi T. \tag{1.5}$$

Building on these ideas, in [4,5] general actions of the type

$$S = \int d(\text{vol}) R_{\mu\nu\rho\sigma} \mathcal{O}^{\mu\nu\rho\sigma}_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \tag{1.6}$$

(where $\mathcal{O}$ is a differential operator) in the quadratic approximation in the weak field expansion $g_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}$ are analyzed. Using the same notation as in [5] this yields the action

$$S = -\int d^n x \left\{ \frac{1}{2} h_{\mu\nu} a(\Box) \Box h^{\mu\nu} + h^\sigma_\mu b(\Box) \partial_\sigma \partial_\nu h^{\mu\nu} + hc(\Box) \partial_\mu \partial_\nu h^{\mu\nu} + \frac{1}{2} hd(\Box) \Box h + h^{\lambda\sigma} \frac{f(\Box)}{\Box} \partial_\sigma \partial_\lambda \partial_\mu \partial_\nu h^{\mu\nu} \right\}. \tag{1.7}$$

Here all coefficients a, b, c, d, f are dimensionless functions of the d'Alembert operator.

The aim of the present work is to see if it is possible to extend this idea to the unimodular theory. As is well known [6] flat space UG is a consistent theory propagating spin 2 only, with no admixture of spin zero, in the same foot as

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(and inequivalent to) Fierz Pauli, with the curious property that it does not admit massive deformations.

We see in the sequel that all ghost-free quadratic theories admit unimodular cousins.

II. UNIMODULAR REDUCTION

In [7] we have demonstrated that there is a (noninjective) map that we call unimodular reduction (UR) that yields the flat space unimodular theory out of the ordinary Fierz-Pauli one. Similar mappings can also be introduced also in the full nonlinear case. To be specific, the UR

$$UR\colon\ h_{\mu\nu} \to h_{\mu\nu} - \frac{1}{n} h\eta_{\mu\nu} \tag{2.1}$$

of the action (1.7) reads

$$\begin{aligned} S_{UG} \equiv & UR[S] \\ = & -\int d(\mathrm{vol})\left\{\frac{1}{2}h_{\mu\nu}a(\Box)\Box h^{\mu\nu} + h^\sigma_\mu b(\Box)\partial_\sigma\partial_\nu h^{\mu\nu}\right. \\ & + h^{\lambda\sigma}\frac{f(\Box)}{\Box}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\mu\nu} - \frac{2}{n}h^{\alpha\beta}(b(\Box)+f(\Box))\partial_\alpha\partial_\beta h \\ & \left. + h\left[\frac{1}{n^2}f(\Box) - \frac{1}{2n}a(\Box) + \frac{1}{n^2}b(\Box)\right]\Box h\right\}. \end{aligned} \tag{2.2}$$

Please note that the UG action is independent of $c(\Box)$ and $d(\Box)$. Moreover, the local (two-derivative) unimodular gravity in [6] corresponds to constant values of the functions, namely,

$$a = \frac{1}{2} \qquad b = -\frac{1}{2} \qquad f = 0. \tag{2.3}$$

When computing the UR of the EM it is important to realize [6] that the unimodular reduction does not commute with the variation, that is,

$$[UR, EM] \neq 0. \tag{2.4}$$

From now on, we work in momentum space, where the different functions $a(\Box)$ etc. in (2.2) are functions of k^2. Actually, the EM stemming from the action S can be written as

$$K_{\mu\nu\rho\sigma}h^{\rho\sigma} = 0, \tag{2.5}$$

where

$$\begin{aligned} K_{\mu\nu\rho\sigma} = & \frac{a}{4}k^2(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}) \\ & + \frac{b}{4}(k_\sigma k_\nu \eta_{\mu\rho} + k_\rho k_\nu \eta_{\mu\sigma} + k_\mu k_\sigma \eta_{\nu\rho} + k_\mu k_\rho \eta_{\nu\sigma}) \\ & + \frac{c}{2}(\eta_{\rho\sigma}k_\mu k_\nu + \eta_{\mu\nu}k_\rho k_\sigma) \\ & + \frac{d}{2}k^2\eta_{\mu\nu}\eta_{\rho\sigma} + f\frac{k_\mu k_\nu k_\rho k_\sigma}{k^2}. \end{aligned} \tag{2.6}$$

It is important to note that the EM are symmetrized, id est,

$$K_{\mu\nu\rho\sigma} = K_{\rho\sigma\mu\nu}, \qquad K_{\mu\nu\rho\sigma} = K_{\nu\mu\rho\sigma}. \tag{2.7}$$

It is clear that the unimodular equations of motion cannot be the unimodular reduction of the Fierz-Pauli ones, since $c(\Box)$ does not disappear, whereas it is not even present in the UG action. To obtain the latter, there is a general procedure explained in [6]. Define

$$K_{\mu\nu} \equiv K_{\mu\nu\rho\sigma}\eta^{\rho\sigma}, \qquad K \equiv K_{\mu\nu}\eta^{\mu\nu}. \tag{2.8}$$

Then

$$K^{UG}_{\mu\nu\rho\sigma} \equiv K_{\mu\nu\rho\sigma} - \frac{1}{n}\left(K_{\mu\nu}\eta_{\rho\sigma} + K_{\rho\sigma}\eta_{\mu\nu} - \frac{1}{n}K\eta_{\mu\nu}\eta_{\rho\sigma}\right), \tag{2.9}$$

where this operator is built in such a way that it inherits the previous symmetries,

$$K^{UG}_{\mu\nu\rho\sigma} = K^{UG}_{\rho\sigma\mu\nu}, \qquad K^{UG}_{\mu\nu\rho\sigma} = K^{UG}_{\nu\mu\rho\sigma}, \tag{2.10}$$

plus an extra one,

$$K^{UG}_{\mu\nu\rho\sigma}\eta^{\rho\sigma} = 0. \tag{2.11}$$

This yields

$$\begin{aligned} K^{UG}_{\mu\nu\rho\sigma} = & \frac{1}{4}a(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho})k^2 \\ & - \frac{b+f}{n}(k_\mu k_\nu \eta_{\rho\sigma} + k_\rho k_\sigma \eta_{\mu\nu}) \\ & + \frac{1}{4}b(k_\rho k_\nu \eta_{\mu\sigma} + k_\rho k_\mu \eta_{\nu\sigma} + k_\sigma k_\nu \eta_{\mu\rho} + k_\mu k_\sigma \eta_{\nu\rho}) \\ & + \frac{2(b+f) - na}{2n^2}k^2\eta_{\mu\nu}\eta_{\rho\sigma} + f\frac{k_\mu k_\nu k_\rho k_\sigma}{k^2}. \end{aligned} \tag{2.12}$$

It is plain that

$$\begin{aligned} k^\mu K^{UG}_{\mu\nu\rho\sigma} = & \frac{a}{4}(k_\rho \eta_{\nu\sigma} + k_\sigma \eta_{\mu\rho})k^2 - \frac{b+f}{n}(k^2 k_\nu \eta_{\rho\sigma} + k_\nu k_\rho k_\sigma) \\ & + \frac{b}{4}(2k_\sigma k_\rho k_\nu + k^2 k_\rho \eta_{\nu\sigma} + k^2 k_\sigma \eta_{\nu\rho}) \\ & + \frac{2(b+f) - na}{2n^2}k^2 k_\nu \eta_{\rho\sigma} + f k_\rho k_\nu k_\sigma. \end{aligned} \tag{2.13}$$

The Bianchi identity implies that $K_{\mu\nu\rho\sigma}$ is transverse, so that the source term in [5] must also be conserved,

$$\partial_\nu T^{\mu\nu} = 0. \tag{2.14}$$

This means that the source term after UR, which is the traceless piece of the energy-momentum tensor, namely,

$$T^T_{\mu\nu} \equiv T_{\mu\nu} - \frac{1}{n} T\eta_{\mu\nu} \tag{2.15}$$

(where $T \equiv \eta^{\mu\nu} T_{\mu\nu}$), is not transverse anymore, but rather

$$\partial_\nu T^T_{\mu\nu} = \frac{1}{n}\partial_\mu T. \tag{2.16}$$

This is a nontrivial constraint, which is true only when the functions a and b are such that

$$a + b = 0$$

in which case the trace is given by

$$T = \left(\frac{(n-2)a + 2(n-1)f}{2n} k^2 \eta_{\rho\sigma} + \frac{2(n-1)f - (n-2)a}{2} k_\sigma k_\rho \right) h^{\rho\sigma}. \tag{2.17}$$

There are no constraints in the unimodular case on the function $f(k^2)$.

III. PROPAGATORS

$$K^{UG} = a_1 P_1 + a_2 P_2 + a_s P_0^s + a_w P_0^w + a_\times P_0^\times, \tag{3.1}$$

where

$$\begin{aligned} a_1 &= 0, \\ a_2 &= \frac{1}{2} a k^2, \\ a_s &= \frac{2(n-1)f - (n-2)a}{2n^2} k^2 = L k^2, \\ a_w &= \frac{2(n-1)^2 f - (n-1)(n-2)a}{2n^2} k^2 \\ &= (n-1)Lk^2 = (n-1)a_s, \\ a_\times &= \sqrt{n-1}\frac{(n-2)a - 2(n-1)f}{2n^2} k^2 \\ &= -\sqrt{n-1}Lk^2 = -\sqrt{n-1}a_s, \end{aligned} \tag{3.2}$$

where we have defined

$$L \equiv \frac{2(n-1)f - (n-2)a}{2n^2}. \tag{3.3}$$

The discriminant vanishes,

$$\Delta \equiv a_s a_w - a_\times^2 = 0. \tag{3.4}$$

This means that we have to introduce a *TDiff* gauge fixing,

$$K_1^{\text{gf}} \equiv \alpha_1 P_1, \tag{3.5}$$

and besides another one for Weyl's symmetry, namely,

$$K_2^{\text{gf}} \equiv \alpha_2 (P_0^w + (n-1)P_0^s + \sqrt{n-1} P_0^\times). \tag{3.6}$$

The full operator is then

$$K_{gf}^{UG} = \alpha_1 P_1 + a_2 P_2 + (a_s + (n-1)\alpha_2) P_0^s + (a_w + \alpha_2) P_0^w + (\alpha_2 \sqrt{n-1} + a_\times) P_0^\times. \tag{3.7}$$

Using the formulas in the appendix, the propagator is given by

$$K_{UG}^{-1} = \frac{1}{\alpha_1} P_1 + \left\{ \frac{2}{a} P_2 + \frac{1}{n^2 \alpha_2 L} ((Lk^2 + (n-1)\alpha_2) P_0^w + (L(n-1)k^2 + \alpha_2) P_0^s + \sqrt{n-1}(Lk^2 - \alpha_2) P_\times) \right\} \frac{1}{k^2}. \tag{3.8}$$

The interaction energy between external, gauge invariant sources is a physical quantity. In our case this is given by the coupling of the graviton to the traceless piece of the energy-momentum tensor,

$$\int d(\text{vol}) T^T_{\mu\nu} h^{\mu\nu}. \tag{3.9}$$

The source then is not transverse, but rather

$$\partial_\mu (T^T)^{\mu\nu} = \frac{1}{n}\partial^\nu T. \tag{3.10}$$

The only projectors that do not vanish when sandwiched between physical sources are

$$T^{T\mu\nu} \cdot (P_2)_{\mu\nu\rho\sigma} \cdot T^{T\rho\sigma} = T^2_{\mu\nu} - \frac{1}{n-1} T^2, \tag{3.11}$$

$$T^{T\mu\nu} \cdot (P_1)_{\mu\nu\rho\sigma} \cdot T^{T\rho\sigma} = 0, \tag{3.12}$$

$$T^{T\mu\nu} \cdot (P_0^s)_{\mu\nu\rho\sigma} \cdot T^{T\rho\sigma} = \frac{1}{n^2(n-1)} T^2, \tag{3.13}$$

$$T^{T\mu\nu} \cdot (P_0^x)_{\mu\nu\rho\sigma} \cdot T^{T\rho\sigma} = -\frac{2}{n^2\sqrt{n-1}} T^2, \tag{3.14}$$

$$T^{T\mu\nu} \cdot (P_0^w)_{\mu\nu\rho\sigma} \cdot T^{T\rho\sigma} = \frac{1}{n^2} T^2. \tag{3.15}$$

This yields the value for the free energy in the linear limit of our theory to be

$$W = \frac{1}{k^2} \left(\frac{2}{a} T^2_{\mu\nu} + \frac{a - 2n^2 L}{a(n-1)n^2 L} T^2 \right). \tag{3.16}$$

In order not to disagree with the classical solar system GR tests, this must be proportional to the well-known result (which is the Fierz-Pauli result, reproduced also by UG)

$$W_{FP} = \frac{1}{k^2}\left(T_{\mu\nu}^2 - \frac{1}{n-2}T^2\right). \tag{3.17}$$

This determines

$$L = -\frac{a(n-2)}{2n^2}, \tag{3.18}$$

that is, f is given in terms of a through

$$f(k) = 0. \tag{3.19}$$

It is known that the function $f(z)$ must be an entire function of the complex variable z. This condition is trivially satisfied here.

IV. CONCLUSIONS

We have demonstrated here that the complete set of higher-derivative ghost-free theories has a related unimodular theory that can be easily obtained from the parent theory through unimodular reduction.

These theories, as also happens with the usual unimodular gravity constructed out of the Einstein-Hilbert Lagrangian, do not couple the constant vacuum energy to gravity, which makes them an interesting alternative to the standard ones.

Further work is needed to investigate if one can get different physical predictions from the unimodular reduction than those of the parent theory. In spite of some work [8], no differences were found at tree level between unimodular gravity and general relativity other than the role of the vacuum energy just mentioned.

Let us finish with a word of caution. The suggestion has been made [9] that in spite of the original claims there are hidden ghosts in the quantum version of these theories. This question asks for a more detailed analysis. Let us point out, finally, that it is interesting to generalize the bootstrap mechanism to the unimodular case, in particular, for higher order in curvature theories. Some suggestions have been made in [10]. Work on this topic will be reported in due time.

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APPENDIX: SPIN PROJECTORS

We start with the longitudinal and transverse projectors

$$\theta_{\alpha\beta} \equiv \eta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}, \qquad \omega_{\alpha\beta} \equiv \frac{k_\alpha k_\beta}{k^2}. \tag{A1}$$

They obey

$$\theta_\mu^\nu + \omega_\mu^\nu = \delta_\mu^\nu, \qquad \theta_\alpha^\beta \theta_\beta^\gamma = \theta_\alpha^\gamma, \qquad \omega_\alpha^\beta \omega_\beta^\gamma = \omega_\alpha^\gamma, \tag{A2}$$

as well as

$$\mathrm{tr}\theta_\mu^\nu = n-1, \qquad \mathrm{tr}\omega_\mu^\nu = 1. \tag{A3}$$

The four-indices projectors are

$$\begin{aligned}
(P_2)_{\mu\nu\rho\sigma}) &\equiv \frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho}) - \frac{1}{n-1}\theta_{\mu\nu}\theta_{\rho\sigma}, \\
(P_1)_{\mu\nu\rho\sigma} &\equiv \frac{1}{2}(\theta_{\mu\rho}\omega_{\nu\sigma} + \theta_{\mu\sigma}\omega_{\nu\rho} + \theta_{\nu\rho}\omega_{\mu\sigma} + \theta_{\nu\sigma}\omega_{\mu\rho}), \\
(P_0^s)_{\mu\nu\rho\sigma} &\equiv \frac{1}{n-1}\theta_{\mu\nu}\theta_{\rho\sigma}, \\
(P_0^w)_{\mu\nu\rho\sigma} &\equiv \omega_{\mu\nu}\omega_{\rho\sigma}, \\
(P_0^{sw})_{\mu\nu\rho\sigma} &\equiv \frac{1}{\sqrt{n-1}}\theta_{\mu\nu}\omega_{\rho\sigma}, \\
(P_0^{ws})_{\mu\nu\rho\sigma} &\equiv \frac{1}{\sqrt{n-1}}\omega_{\mu\nu}\theta_{\rho\sigma}.
\end{aligned} \tag{A4}$$

They obey

$$\begin{aligned}
P_i^a P_j^b &= \delta_{ij}\delta^{ab} P_i^b, \qquad P_i^a P_j^{bc} = \delta_{ij}\delta^{ab} P_j^{ac}, \\
P_i^{ab} P_j^c &= \delta_{ij}\delta^{bc} P_j^{ac}, \qquad P_i^{ab} P_j^{cd} = \delta_{ij}\delta^{bc}\delta^{ad} P_j^a,
\end{aligned} \tag{A5}$$

as well as

$$\begin{aligned}
&\mathrm{tr}((P_2)_{\mu\nu\rho\sigma}) \equiv \eta^{\mu\nu}(P_2)_{\mu\nu\rho\sigma} = 0, \\
&\mathrm{tr}((P_0^s)_{\mu\nu\rho\sigma}) \equiv \eta^{\mu\nu}(P_0^s)_{\mu\nu\rho\sigma} = \theta_{\rho\sigma}, \\
&\mathrm{tr}((P_0^w)_{\mu\nu\rho\sigma}) \equiv \eta^{\mu\nu}(P_0^w)_{\mu\nu\rho\sigma} = \omega_{\rho\sigma}, \\
&\mathrm{tr}((P_1)_{\mu\nu\rho\sigma}) \equiv \eta^{\mu\nu}(P_1)_{\mu\nu\rho\sigma} = 0, \\
&\mathrm{tr}((P_0^{sw})_{\mu\nu\rho\sigma}) \equiv \eta^{\mu\nu}(P_0^{sw})_{\mu\nu\rho\sigma} = \sqrt{n-1}\,\omega_{\rho\sigma}, \\
&\mathrm{tr}((P_0^{ws})_{\mu\nu\rho\sigma}) \equiv \eta^{\mu\nu}(P_0^{ws})_{\mu\nu\rho\sigma} = \frac{1}{\sqrt{n-1}}\theta_{\rho\sigma}, \\
&(P_2)_{\mu\nu}^{\rho\sigma} + (P_1)_{\mu\nu}^{\rho\sigma} + (P_0^w)_{\mu\nu}^{\rho\sigma} + (P_0^s)_{\mu\nu}^{\rho\sigma} = \frac{1}{2}(\delta_\mu^\nu \delta_\rho^\sigma + \delta_\mu^\sigma \delta_\rho^\nu).
\end{aligned} \tag{A6}$$

Any symmetric operator can be written symbolically as

$$K = a_2 P_2 + a_1 P_1 + a_w P_0^w + a_s P_0^s + a_\times P_0^\times \tag{A7}$$

(where $P_0^\times \equiv P_0^{ws} + P_0^{sw}$). Then

$$K^{-1} = \frac{1}{a_2} P_2 + \frac{1}{a_1} P_1 + \frac{a_s}{a_s a_w - a_\times^2} P_0^w + \frac{a_w}{a_s a_w - a_\times^2} P_0^s - \frac{a_\times}{a_s a_w - a_\times^2} P_0^\times. \tag{A8}$$

Sometimes the action of those projectors on trace-free tensors is needed. Defining the trace-free projector

$$(P_{\rm tr})_{\rho\sigma}{}^{\lambda\delta} \equiv \frac{1}{2}(\delta^\lambda_\rho \delta^\delta_\sigma + \delta^\delta_\rho \delta^\lambda_\sigma) - \frac{1}{n}\eta_{\rho\sigma}\eta^{\lambda\delta}. \tag{A9}$$

It is a fact that

$$\begin{aligned}
P_2 P_{\rm tr} &= P_2,\\
P_0^s P_{\rm tr} &= P_0^s - \frac{n-1}{n} P_0^s - \frac{\sqrt{n-1}}{n} P_0^{sw},\\
P_0^w P_{\rm tr} &= P_0^w - \frac{\sqrt{n-1}}{n} P_0^{ws} - \frac{1}{n} P_0^w,\\
P_1 P_{\rm tr} &= P_1,\\
P_0^{sw} P_{\rm tr} &= P_0^{sw} - \frac{\sqrt{n-1}}{n} P_0^{ws} - \frac{1}{n} P_0^w,\\
P_0^{ws} P_{\rm tr} &= P_0^{ws} - \frac{\sqrt{n-1}}{n} P_0^{sw} - \frac{n-1}{n} P_0^s.
\end{aligned} \tag{A10}$$

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