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Higher order derivative coupling to gravity and its cosmological implications

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We show that the $R^{(3)}\delta K$ operator in effective field theory is significant for avoiding the instability of nonsingular bounce, where $R^{(3)}$ and $K_{\mu\nu}$ are the three-dimensional Ricci scalar and the extrinsic curvature on the spacelike hypersurface, respectively. We point out that the covariant Lagrangian of $R^{(3)}\delta K$, i.e., $L_{R^{(3)}\delta K}$, has the second order derivative couplings of scalar field to gravity which do not appear in Horndeski theory or its extensions but does not bring the Ostrogradski ghost. We also discuss the possible effect of $L_{R^{(3)}\delta K}$ on the primordial scalar perturbation in the inflation scenario.

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I. INTRODUCTION

Recently, the studies of the origin of the Universe and the current accelerated expansion have greatly promoted the development of gravity theories beyond general relativity (GR); see Refs. [1–3] for recent reviews. How to design a theory without an extra degree of freedom (DOF) has acquired persistent attention.

The Horndeski theory was proposed in the 1970s [4]; see also Refs. [5,6], in which the equations of motion have at most second order time derivatives, which avoids the extra DOF (so the Ostrogradski ghost). However, it seems that equations of motion with higher order time derivatives do not necessarily suggest the presence of extra DOF. The discoveries of the beyond Horndeski theory [7–9] and degenerate higher order scalar-tensor (DHOST) theory [10–13] have confirmed this possibility and greatly enriched our understanding of gravity. In Horndeski and DHOST theories, the Lagrangian involves only the nonminimal couplings $f(\phi, X)R$ and $\phi^{\mu\nu}R_{\mu\nu}$, where $X = \phi_\mu\phi^\mu$, $\phi_\mu = \nabla_\mu\phi$, and $\phi^{\mu\nu} = \nabla^\mu\nabla^\nu\phi$.

Along a different line, the effective field theory (EFT) of cosmological perturbations has been developed for investigating inflation [14,15] and current cosmological acceleration [16–18]; see Ref. [19] for a review. Recently, the EFT has also been applied to the nonsingular cosmologies [20–22]. It was found first in Refs. [20,21] that the operators with three-dimensional Ricci scalar $R^{(3)}$, especially $R^{(3)}\delta g^{00}$, could play a significant role in curing the gradient instability induced by a negative sound speed squared (i.e., $c_s^2 < 0$) of scalar perturbation [23,24]. Actually, as will be shown, the operator $R^{(3)}\delta K$ (K is the extrinsic curvature) could play a role similar to that of $R^{(3)}\delta g^{00}$.

We built a fully stable cosmological bounce scenario in Ref. [25] by applying a least set of operators ($(\delta g^{00})^2$ and $R^{(3)}\delta g^{00}$), namely, a "least modification." The graviton throughout the bounce behaves itself like that in GR, which could naturally avoid the strong coupling regime appearing in Ref. [26]; see also Ref. [27]. The covariant Lagrangian proposed in Ref. [25] belongs to beyond Horndeski theory (see also Ref. [28] for a different implementation of a fully stable bounce), which is a subclass of the DHOST theory, but the equations of motion still could be second order in time derivatives. This enlightens us that there might still be some space of scalar-tensor theory to be explored.

As will be pointed out, the covariant description of $R^{(3)}\delta K$ contains the second order derivative couplings of the field ϕ to gravity, such as $\phi^\mu\phi_{\mu\nu}\phi^\nu R$, $\phi^\mu\phi^\nu(\Box\phi)R_{\mu\nu}$, and $\phi^\mu\phi^\nu\phi_\rho\phi^{\rho\sigma}\phi_\sigma R_{\mu\nu}$, which do not appear in Horndeski (or even DHOST) theory. The mimetic gravity with the coupling $(\Box\phi)R$ has been proposed in Ref. [29]. In scalar-tensor theory, it is interesting to explore the possibility of such higher order derivative couplings.

In this paper, we point out that the covariant Lagrangian of $R^{(3)}\delta K$, i.e., $L_{R^{(3)}\delta K}$, has the second order derivative couplings of scalar field to gravity which do not appear in Horndeski theory or its extensions but does not bring the Ostrogradski ghost. We discuss its implication on scalar-tensor theory. We also show the interesting applications of $L_{R^{(3)}\delta K}$ in the nonsingular cosmologies and the inflation scenario.

II. HIGHER ORDER DERIVATIVE COUPLING TO GRAVITY

As was first found in Ref. [20] (see also Ref. [21]), the $R^{(3)}\delta g^{00}$ operator plays a crucial role in solving the gradient instability problem induced by $c_s^2 < 0$ (see also Ref. [30] for the unitarity problem), which suffered by the nonsingular cosmologies based on the Horndeski theory [23,24,31,32].

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In the Appendix, we point out that the $R^{(3)}\delta K$ operator actually could play a role similar to that of $R^{(3)}\delta g^{00}$. As will be shown, the covariant Lagrangian of $R^{(3)}\delta K$ contains the second order derivative of ϕ coupled to gravity, such as $\sim\phi^\mu\phi_{\mu\nu}\phi^\nu R$, $\phi^\mu\phi^\nu(\Box\phi)R_{\mu\nu}$, and $\phi^\mu\phi^\nu\phi_\rho\phi^{\rho\sigma}\phi_\sigma R_{\mu\nu}$. However, in Horndeski theory, such derivative couplings do not appear, since they will bring the Ostrogradski ghost. Thus, it is interesting to have a survey.

In this section, we will derive the covariant Lagrangian of $R^{(3)}\delta K$ in unitary gauge. The induced metric on the three-dimensional spacelike hypersurface ($\phi = \text{const}$) is $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$, where $n^\mu = -\frac{1}{\sqrt{-X}}\phi^\mu$ is the unit vector orthogonal to the hypersurface and $n_\mu n^\mu = -1$, with $X = \phi_\mu\phi^\mu$ and $\phi^\mu = \nabla^\mu\phi$. The extrinsic curvature $K_{\mu\nu}$ is defined as

$$K_{\mu\nu} = h^\sigma_\mu \nabla_\sigma n_\nu. \tag{1}$$

Since $\delta K = K - 3H$, it is straightforward to get

$$\delta K = -\frac{1}{\sqrt{-X}}\left(\Box\phi - \frac{\phi^\mu\phi_{\mu\nu}\phi^\nu}{X}\right) - 3H, \tag{2}$$

with $\phi_{\mu\nu} = \nabla_\nu\nabla_\mu\phi$. In unitary gauge $\phi = \phi(t)$, we have $H = H(t(\phi))$. Using the Gauss-Codazzi relation, we have

$$\begin{aligned} R^{(3)} = R &- \frac{\phi_{\mu\nu}\phi^{\mu\nu} - (\Box\phi)^2}{X} + \frac{2\phi^\mu\phi_{\mu\nu}\phi^{\nu\sigma}\phi_\sigma}{X^2} \\ &- \frac{2\phi^\mu\phi_{\mu\nu}\phi^\nu\Box\phi}{X^2} - \frac{2R_{\mu\nu}\phi^\mu\phi^\nu}{X}. \end{aligned} \tag{3}$$

Note that in the second line of Eq. (3), we also have $R_{\mu\nu}\phi^\mu\phi^\nu = \phi_\nu{}^\mu{}_\mu\phi^\nu - \phi^\nu{}_{\nu\mu}\phi^\mu$ with $\phi^\nu{}_{\nu\mu} = \nabla_\mu\nabla_\nu\nabla^\nu\phi$, as given in Ref. [25].

We define $S_{R^{(3)}\delta K} = \int d^4x\sqrt{-g}L_{R^{(3)}\delta K}$, with

$$\begin{aligned} L_{R^{(3)}\delta K} &= \bar{f}_5\cdot(R^{(3)}\delta K) \\ &= -\frac{\bar{f}_5}{\sqrt{-X}}\left[(\Box\phi) - \frac{\phi^\mu\phi_{\mu\nu}\phi^\nu}{X}\right]R \\ &\quad + \frac{2\bar{f}_5}{\sqrt{(-X)^3}}\left[-\phi^\mu\phi^\nu(\Box\phi) + \frac{\phi^\mu\phi^\nu\phi_\rho\phi^{\rho\sigma}\phi_\sigma}{X}\right]R_{\mu\nu} \\ &\quad + \frac{\bar{f}_5}{\sqrt{(-X)^3}}\left[(\Box\phi)^3 - (\Box\phi)\phi_{\mu\nu}\phi^{\mu\nu} - \frac{(\Box\phi)^2\phi_\mu\phi^{\mu\nu}\phi_\nu - \phi_{\mu\nu}\phi^{\mu\nu}\phi_\rho\phi^{\rho\sigma}\phi_\sigma}{X}\right] \\ &\quad + \frac{2\bar{f}_5}{\sqrt{(-X)^5}}\left[(\Box\phi)^2\phi_\mu\phi^{\mu\nu}\phi_\nu - (\Box\phi)\phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho \right. \\ &\quad \left. - \frac{(\Box\phi)(\phi_\mu\phi^{\mu\nu}\phi_\nu)^2 - \phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho\phi_\sigma\phi^{\sigma\lambda}\phi_\lambda}{X}\right] - \bar{f}_4 R^{(3)}, \end{aligned} \tag{4}$$

where the leading contribution of $R^{(3)}\delta K$ is the perturbation at quadratic order, so that $\bar{f}_5$ could be a function of ϕ, X (and even $\Box\phi$ and $\phi_\mu\phi^{\mu\nu}\phi_\nu$), and $\bar{f}_4 = 3\bar{f}_5 H(t(\phi))$. When $\bar{f}_4 = 0$ is set, $L_{R^{(3)}\delta K}$ reduces to $\sim R^{(3)}K$.

Recalling that in Horndeski theory, L_5^H contains the coupling of the second order derivative of ϕ to gravity, i.e., $f(\phi, X)G_{\mu\nu}\phi^{\mu\nu}$ (or $R_{\mu\nu}\phi^{\mu\nu}$). Here, we require that f is also X dependent; otherwise, $G_{\mu\nu}\phi^{\mu\nu}$ will be equivalent to $G_{\mu\nu}\phi^\mu\phi^\nu$, the cosmological applications of which have been studied; see, e.g., Refs. [33–36]. While in $L_{R^{(3)}\delta K}$, the couplings

$$(\Box\phi)R, \qquad \phi^\mu\phi_{\mu\nu}\phi^\nu R, \qquad \phi^\mu\phi^\nu(\Box\phi)R_{\mu\nu}, \tag{5}$$

$$\phi^\mu\phi^\nu\phi_\rho\phi^{\rho\sigma}\phi_\sigma R_{\mu\nu} \tag{6}$$

appear and are independent with $R_{\mu\nu}\phi^{\mu\nu}$. In addition, such couplings to gravity also include $\phi^{\mu\rho}\phi_\rho\phi^\nu R_{\mu\nu}$, $\phi^\mu\phi^{\nu\rho}\phi^\sigma R_{\mu\nu\rho\sigma}$, which are not independent and could be obtained by the combinations of $R_{\mu\nu}\phi^{\mu\nu}$ and (5), as pointed out in Ref. [12].

In Horndeski theory, the cubic order of $\Box\phi$ in L_5^H will induce the higher derivatives in the metric and field equations, which are actually set off by $G_{\mu\nu}\phi^{\mu\nu}$ [37]. This makes it be free from the Ostrogradski ghost. In DHOST theory [10,11], all possible terms of cubic order of the second order derivative of ϕ appear and result in higher order equations of motion, but there is still no Ostrogradski ghost due to the degeneracy.

Though the DHOST theory extends the Horndeski theory, the coupling of the second order derivative of ϕ to gravity is still only $G_{\mu\nu}\phi^{\mu\nu}$, since the derivative couplings (5) and (6) will bring the Ostrogradski ghost (higher derivatives in the equations of motion). However, in $L_{R^{(3)}\delta K}$, the Ostrogradski ghost could be dispelled by the combination of $(\Box\phi)^3$, $(\Box\phi)\phi_{\mu\nu}\phi^{\mu\nu}$, etc., and $R^{(3)}$; see (4).

In principle, we could merge the Horndeski (even DHOST) theory and $L_{R^{(3)}\delta K}$ into a (second order) derivative coupling theory with all independent couplings [$R_{\mu\nu}\phi^{\mu\nu}$, Eqs. (5) and (6)] of the second order derivative of ϕ to gravity. In such a theory, the background equations of motion could be set only by the Horndeski (DHOST) theory, since $L_{R^{(3)}\delta K}$ only contributes $(\partial\zeta)^2, (\partial^2\zeta)^2$ at leading order.

The quadratic coupling of the second order derivative of ϕ to R, such as $(\Box\phi)^2 R$, might be obtained in $L \sim K R^{(3)}\delta K$ or equivalently $\bar{f}_5(\Box\phi, \phi_\mu\phi^{\mu\nu}\phi_\nu)R^{(3)}\delta K$, where all coefficients must be fixed as (4).

In mimetic gravity [38,39] (see, e.g., Ref. [40] for a review), since $\delta g^{00} = 0$, instead of $R^{(3)}\delta g^{00}$, the operator $R^{(3)}\delta K$ might be significant for curing the instabilities pointed out in Refs. [41–43]. Here, since the mimetic constraint $g^{\mu\nu}\phi_\mu\phi_\nu + 1 = 0$ suggests $X = -1$, we have

$$R^{(3)} = 2\phi^\mu\phi^\nu R_{\mu\nu} + R - \phi_{\mu\nu}\phi^{\mu\nu} - (\Box\phi)^2; \tag{7}$$

the covariant $L_{R^{(3)}\delta K}$ will be simpler.

It should be mentioned that at quadratic order $L_{R^{(3)}\delta K}$ also contributes $(\partial^2\zeta)^2 \sim k^4\zeta^2$, which is harmful or harmless, depending on the coefficient. However, $(\partial^2\zeta)^2$ could be removed by using $(R^{(3)})^2$ (if required), since $(R^{(3)})^2 \sim (\partial^2\zeta)^2$ at leading order. We define $S_{(R^{(3)})^2} = \int d^4x\sqrt{-g}L_{(R^{(3)})^2}$ and $L_{(R^{(3)})^2} = f_6 \cdot (R^{(3)})^2$, with

$$\begin{aligned}(R^{(3)})^2 = {} & R^2 - \frac{4\phi^\mu\phi^\nu R_{\mu\nu}R}{X} + \frac{4(\phi^\mu\phi^\nu R_{\mu\nu})^2}{X^2} \\ & + 2R\left[\frac{(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}}{X} + \frac{2\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu}{X^2} - \frac{2(\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu}{X^2}\right] \\ & + 4R_{\mu\nu}\phi^\mu\phi^\nu\left[\frac{\phi_{\rho\sigma}\phi^{\rho\sigma}}{X^2} - \frac{(\Box\phi)^2}{X^2} - \frac{2\phi_\alpha\phi^{\alpha\beta}\phi_{\beta\sigma}\phi^\sigma}{X^3} + \frac{2\Box\phi\phi_\alpha\phi^{\alpha\beta}\phi_\beta}{X^3}\right] \\ & + \frac{(\phi_{\mu\nu}\phi^{\mu\nu})^2}{X^2} - \frac{4\phi_{\mu\nu}\phi^{\mu\nu}\phi_\alpha\phi^{\alpha\beta}\phi_{\beta\sigma}\phi^\sigma}{X^3} + \frac{4(\phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho)^2}{X^4} \\ & + \frac{4(\Box\phi)\phi_{\mu\nu}\phi^{\mu\nu}\phi_\alpha\phi^{\alpha\beta}\phi_\beta}{X^3} - \frac{8(\Box\phi)\phi_\alpha\phi^{\alpha\beta}\phi_\beta\phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho}{X^4} \\ & + \frac{4(\Box\phi)^2(\phi_\mu\phi^{\mu\nu}\phi_\nu)^2}{X^4} - \frac{2(\Box\phi)^2\phi_{\mu\nu}\phi^{\mu\nu}}{X^2} + \frac{4(\Box\phi)^2\phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho}{X^3} \\ & - \frac{4(\Box\phi)^3\phi_\mu\phi^{\mu\nu}\phi_\nu}{X^3} + \frac{(\Box\phi)^4}{X^2},\end{aligned} \tag{8}$$

where both R^2 order and the coupling of $(\Box\phi)^2$ to R actually appear and f_6 is a function of ϕ and X (and even $\Box\phi$ and $\phi_\mu\phi^{\mu\nu}\phi_\nu$). In addition, $L_{(R^{(3)})^2}$ itself also has an interesting application in nonsingular cosmologies [20,44].

III. COSMOLOGICAL APPLICATIONS

A. Stable model for ekpyrotic scenario

We consider the ekpyrotic scenario [45,46]. How to build a fully stable bounce model is a significant issue. We proposed such a model with $L_{R^{(3)}\delta g^{00}}$ in Ref. [25]. In Ref. [28], Kolevatov *et al.* also proposed a different model by applying the "inverse method" adopted in Refs. [23,47]. However, with the covariant $L_{R^{(3)}\delta g^{00}}$, the design is actually simpler [25]. Here, with $L_{R^{(3)}\delta K}$, the method is similar (though slightly complicated).

We begin with the ekpyrotic Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{ekpy}} \sim {} & \underbrace{\frac{M_p^2}{2}R - X/2 + \frac{V_0}{2}e^{\phi/\mathcal{M}_1}\left[1 - \tanh\left(\frac{\phi}{\mathcal{M}_2}\right)\right]}_{\text{Contraction and expansion}} \\ & + \underbrace{\tilde{P}(\phi, X)}_{\text{Bounce (null energy condition violation)}} \quad (\text{around}\,\phi = 0) + \underbrace{L_{R^{(3)}\delta K} \quad \text{or} \quad L_{R^{(3)}\delta g^{00}}}_{\text{Removing } c_s^2<0 \text{ Removing ghost}},\end{aligned} \tag{9}$$

with constant $\mathcal{M}_1$, $\mathcal{M}_2$, V_0. $\tilde{P}_X > 1/2$ must be satisfied around $\phi \simeq 0$, so that $\dot{H} > 0$. In Ref. [25], see also Refs. [48,49], we adopted

$$\tilde{P}(\phi, X) = \frac{k_0}{(1+\kappa_1\phi^2)^2} X/2 + \frac{q_0}{(1+\kappa_2\phi^2)^2} X^2 \qquad (10)$$

with the constants k_0, κ_1 [switching the sign before $X/2$ in (9) around $\phi \simeq 0$] and q_0, κ_2 (making X^2 appear around $\phi \simeq 0$). A full ekpyrotic Lagrangian (9) also should involve a mechanism (a coupling $e^{-\frac{\lambda}{M_p}\phi}\partial^\mu\chi\partial_\mu\chi$ [50–52]) responsible for the scale invariant primordial perturbation.

The quadratic action of scalar perturbation for (9) is

$$S_\zeta^{(2)} = \int a^3 Q_s \left(\dot{\zeta}^2 - c_s^2 \frac{(\partial\zeta)^2}{a^2}\right) d^4x, \qquad (11)$$

where

$$Q_s = \frac{2\dot{\phi}^4 \tilde{P}_{XX} - M_p^2 \dot{H}}{H^2}, \qquad c_s^2 Q_s = \frac{\dot{c}_3}{a} - M_p^2, \qquad (12)$$

and $c_3 = \frac{aM_p^2}{H}(1 - \frac{2\bar{f}_5 Q_s H}{M_p^4})$; see the Appendix (or Ref. [20]) for details. In the Appendix, we have $M_2^4(t) = \dot{\phi}^4 \tilde{P}_{XX}$, $\bar{m}_5/2 = \bar{f}_5$ and $\bar{\lambda}/2 = f_6$. The quadratic action of tensor perturbation is unaffected by $L_{R^{(3)}\delta K}$ and is still that in GR.

Here, we require $2X^2\tilde{P}_{XX} > M_p^2\dot{H}$, so that $Q_s > 0$ can be obtained. If $\bar{f}_5 = 0$, around the bounce point $H \simeq 0$, we will have $c_s^2 \sim -\dot{H} < 0$. However, since $\bar{f}_5 \neq 0$ and satisfies

$$\frac{2\bar{f}_5 Q_s H}{M_p^4} = 1 - \frac{H}{aM_p^2}\int a(Q_s c_s^2 + M_p^2)dt, \qquad (13)$$

we always could set $c_s^2 \sim \mathcal{O}(1)$ with suitable $\bar{f}_5$. It should be mentioned that when $H \sim 0$, $\bar{f}_5 \sim \frac{1}{HQ_s} \sim H$ crosses 0.

In (11), $(\partial^2\zeta)^2$ has been canceled by adding $L_{(R^{(3)})^2}$ to $\mathcal{L}_{\rm ekpy}$ for simplicity, which requires

$$4f_6 = \frac{\bar{f}_5}{H} - \left(3 + \frac{Q_s}{M_p^2}\right)\frac{\bar{f}_5^2}{M_p^2}. \qquad (14)$$

Thus, a fully stable nonsingular bounce ($Q_s > 0$ and $c_s^2 = 1$) can be designed by using (9) with $\bar{f}_5$ given by (13), and $f_4 = 3\bar{f}_5 H$, and f_6 given by (14). With (10), the calculation is similar to that in Ref. [25].

B. Slow-roll inflation with modified c_s^2

We consider the inflation scenario. Here, the covariant $L_{R^{(3)}\delta K}$ and also $L_{R^{(3)}\delta g^{00}}$ only affect the sound speed c_s of scalar perturbation, but the background and the tensor perturbation are unaffected. The effect of modified c_s^2 may be encoded in the power spectrum of primordial scalar perturbation, which might be observable.

The Lagrangian is

$$\mathcal{L} \sim \frac{M_p^2}{2}R + L_{\rm inf} + L_{R^{(3)}\delta K} + L_{(R^{(3)})^2}, \qquad (15)$$

where $L_{\rm inf} = -\phi_\mu\phi^\mu/2 - V(\phi)$ is responsible for the inflation. We set the slow-roll parameter $\epsilon = -\dot{H}/H^2 =$ const > 0 for simplicity. The quadratic action of scalar perturbation is given in (A4) of the Appendix with $M_2 = \tilde{m}_4 = 0$. We have $Q_s = \epsilon M_p^2$ and

$$c_s^2 = 1 - \frac{\bar{m}_5 H}{M_p^2} - \frac{\dot{\bar{m}}_5}{M_p^2}, \qquad (16)$$

$$c_4 \simeq \frac{3\bar{m}_5^2}{M_p^2} - \frac{2\bar{m}_5}{H} + 8\bar{\lambda}. \qquad (17)$$

Here, $L_{R^{(3)}\delta K}$ modifies c_s^2. We require $c_4 = 0$, which suggests that $\bar{\lambda}$ in (17) is determined by $\bar{m}_5$ and H.

The equation of motion for ζ is

$$u'' + \left(c_s^2 k^2 - \frac{z_s''}{z_s}\right)u = 0 \qquad (18)$$

with the definition $u = z_s\zeta$ and $z_s = \sqrt{2a^2\epsilon M_p^2}$, and the superscript $'$ is the derivative with respect to $\tau = \int dt/a$. The initial state of the perturbation mode is $u = \frac{1}{\sqrt{2c_s k}} e^{-ic_s k\tau}$. The power spectrum of ζ is

$$P_{\mathcal{R}} = \frac{k^3}{2\pi^2}\left|\frac{u}{z_s}\right|^2. \qquad (19)$$

We have $P_{\mathcal{R}}^{\rm inf} = \frac{H_{\rm inf}^2}{8\pi^2 M_p^2\epsilon}\left(\frac{k}{aH}\right)^{-2\epsilon}$ for slow-roll inflation ($c_s^2 = 1$). Here, if $c_s^2 = $ const < 1 is required, $\dot{\bar{m}}_5 = 0$ in (16) should be satisfied. This will result in $c_s^2 = 1 - \frac{\bar{m}_5 H_{\rm inf}}{M_p^2} \simeq 1$, since $H_{inf} \ll M_p$ while $\bar{m}_5 \lesssim M_p$. Thus, the case with $c_s^2 \neq$ const might be interesting.

For an example, we consider a model in which c_s^2 acquires a dip [Fig. 1(a)]. We numerically show the corresponding evolutions of $\bar{m}_5$ and $\bar{\lambda}$ in Figs. 1(b) and 1(c), according to (16) and (17), which could be rewritten as $\bar{m}_5(\phi)$ and $\bar{\lambda}(\phi)$ since $\phi = \phi(t)$. We plot P_ζ in Fig. 1(d) by solving Eq. (18); see Refs. [53–55] for a similar method. We see that the effect of $L_{R^{(3)}\delta K}$ on c_s^2 could be encoded in the power spectrum of scalar perturbation.

The phenomenological effect of $L_{R^{(3)}\delta K}$ is very similar to that of $L_{R^{(3)}\delta g^{00}}$ at quadratic order, if the contribution of

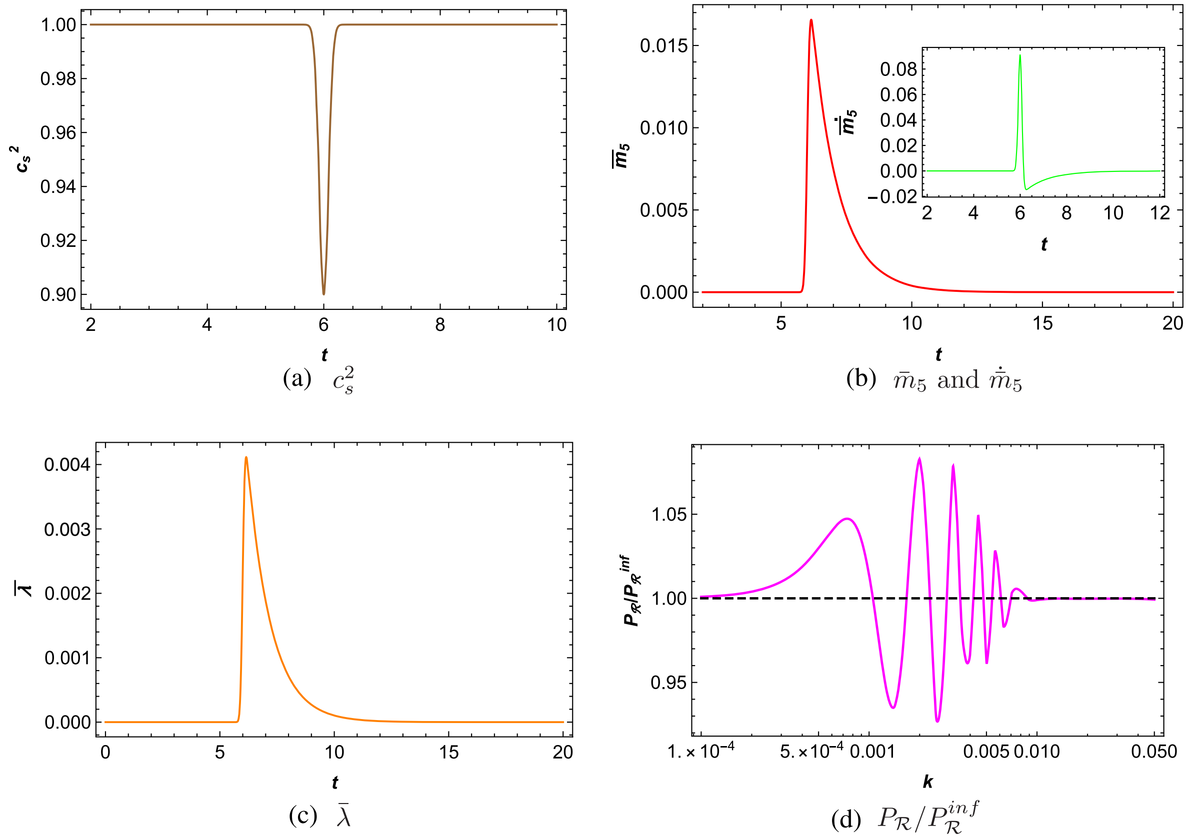

FIG. 1. The background is the slow-roll inflation with $\epsilon = 0.003$. We set $c_s^2 = 1 - \mathcal{A}_* e^{-\mathcal{B}_*(t-t_*)^2}$ with $\mathcal{A}_* = 0.1$, $\mathcal{B}_* = 80$ and $t_* = 6$.

$L_{R^{(3)}\delta K}$ to term $\sim k^4\zeta^2$ in the quadratic action is totally canceled by $L_{(R^{(3)})^2}$, i.e., $c_4 = 0$, which requires $\bar{\lambda} = \bar{\lambda}_0$ with $\bar{\lambda}_0 \simeq \frac{3\bar{m}_5^2}{8M_p^2} - \frac{\bar{m}_5}{4H}$. However, when the condition $c_4 = 0$ is violated, Eq. (18) should be modified to $u'' + (c_{s,\mathrm{eff}}^2 k^2 - z_s''/z_s)u = 0$ where $c_{s,\mathrm{eff}}^2 = c_s^2 - 2c_4 k^2/z_s^2$ (for simplicity, we will focus on the cases in which $c_4 = 0$ initially so that the initial state of the perturbation mode is still $u = \frac{1}{\sqrt{2c_s k}} e^{-ic_s k\tau}$).

Phenomenologically, we could distinguish the operator $L_{R^{(3)}\delta K}$ from $L_{R^{(3)}\delta g^{00}}$. First, when $c_4 \neq 0$, the frequency of the oscillations in the power spectrum will increase with k, while the frequency of the oscillations is nearly constant for $c_4 = 0$; see Fig. 2(b). Second, when $c_4 \neq 0$ (even when $\bar{\lambda}$ slightly deviates from $\bar{\lambda}_0$), $c_{s,\mathrm{eff}}^2$ may induce a larger amplitude of oscillations than that of c_s^2 in the power spectrum, as numerically shown in Fig. 2, unless c_s^2 has more drastic (or fine-tuned) variation.

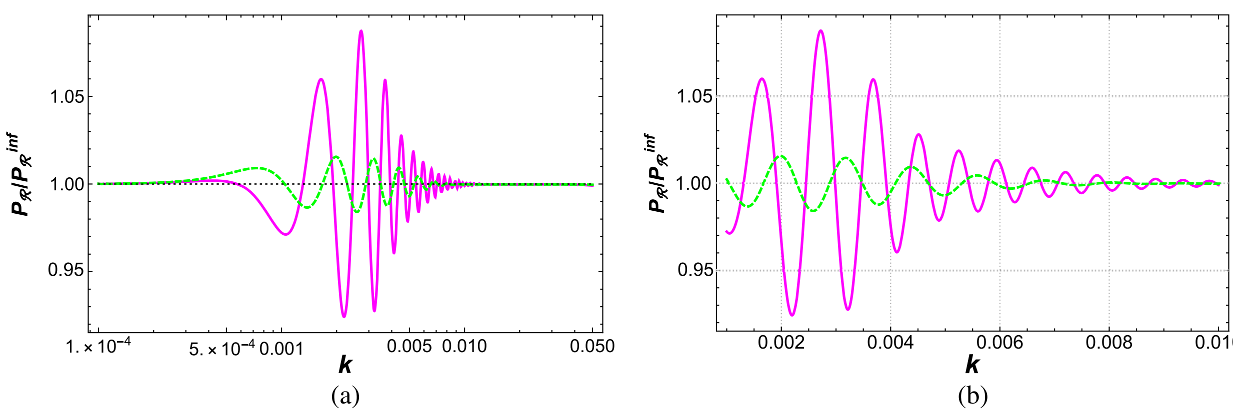

FIG. 2. The background is the slow-roll inflation with $\epsilon = 0.003$. We set $c_s^2 = 1 - \mathcal{A}_* e^{-\mathcal{B}_*(t-t_*)^2}$ with $\mathcal{A}_* = 0.02$, $\mathcal{B}_* = 80$ and $t_* = 6$ for both (a) and (b), while we set $\bar{\lambda} = \bar{\lambda}_0$ (i.e., $c_4 = 0$) for the green dashed curves and $\bar{\lambda} = 0.997\bar{\lambda}_0$ (i.e., $c_4 \neq 0$) for the magenta solid curves.

The effect of varying c_s^2 on scalar perturbations has been also studied in Refs. [56–61], but based on $P(\phi, X)$ (or equivalent EFT).

IV. DISCUSSION

Recently, it has been found in Refs. [20,21] that the operators with a three-dimensional Ricci scalar $R^{(3)}$ in EFT, especially $R^{(3)}\delta g^{00}$, are significant for solving the problem of $c_s^2 < 0$, which is suffered by the nonsingular cosmologies. Here, we find that the $R^{(3)}\delta K$ operator actually could play a role similar to that of $R^{(3)}\delta g^{00}$.

We derived the covariant Lagrangian of $R^{(3)}\delta K$. The covariant $L_{R^{(3)}\delta K}$ has the second order derivative coupling of the field ϕ to gravity, such as (5) and (6) (which do not appear in Horndeski and DHOST theory), but does not bring the Ostrogradski ghost. This suggests that the Horndeski (or even DHOST) theory and $L_{R^{(3)}\delta K}$ might be merged into a second order derivative coupling theory with all possible independent couplings, i.e., $G_{\mu\nu}\phi^{\mu\nu}$ (or $R_{\mu\nu}\phi^{\mu\nu}$), Eqs. (5) and (6), of the second order derivative of ϕ to gravity. Here, how (5) and (6) consistently appear in such a theory is just what is told by the covariant description of the $R^{(3)}\delta K$ operator.

With $L_{R^{(3)}\delta K}$, we built a fully stable cosmological model for the ekpyrotic scenario, by applying similar method used in Ref. [25]. Our work indicates that with the covariant $L_{R^{(3)}\delta g^{00}}$ (proposed in Ref. [25]) or $L_{R^{(3)}\delta K}$, the stable nonsingular bounce scenario could be concisely designed. Here, our study is motivated straightly by the EFT operators, e.g., Ref. [20]. However, other studies based on modified gravity will also be interesting [62–68], especially their stabilities.

We also studied the possible effect of $L_{R^{(3)}\delta K}$ on the primordial scalar perturbation in the inflation scenario, which might be encoded in the TT spectrum of cosmic microwave background. We will come back to the relevant issues elsewhere.

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APPENDIX: THE EFT

As pointed out in Refs. [47,22], the cubic Galileon only moves the period of $c_s^2 < 0$ to the outside of the null energy condition violating phase but cannot dispel it completely; see also the earlier discussion [69] on this point.

In this Appendix, we briefly review the EFT for nonsingular cosmologies and show how the $R^{(3)}\delta g^{00}$ and $R^{(3)}\delta K$ operators play crucial roles in solving the problem of $c_s^2 < 0$.

With the ADM line element, we have

$$g_{\mu\nu} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix}, \qquad g^{\mu\nu} = \begin{pmatrix} -N^{-2} & \frac{N^j}{N^2} \\ \frac{N^i}{N^2} & h^{ij} - \frac{N^i N^j}{N^2} \end{pmatrix}, \tag{A1}$$

and $\sqrt{-g} = N\sqrt{h}$, where $N_i = h_{ij}N^j$. The induced metric on the three-dimensional hypersurface is $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$, where $n_\mu = n_0(dt/dx^\mu) = (-N, 0, 0, 0)$, $n^\nu = g^{\mu\nu}n_\mu = (1/N, -N^i/N)$ is orthogonal to the spacelike hypersurface, and $n_\mu n^\mu = -1$. Thus,

$$h_{\mu\nu} = \begin{pmatrix} N_k N^k & N_j \\ N_i & h_{ij} \end{pmatrix}, \qquad h^{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & h^{ij} \end{pmatrix}. \tag{A2}$$

The EFT action is

$$\begin{aligned} S = \int d^4x\sqrt{-g}\Big[& \frac{M_p^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \\ & + \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta K \delta g^{00} \\ & - m_4^2(t)(\delta K^2 - \delta K_{\mu\nu}\delta K^{\mu\nu}) + \frac{\tilde{m}_4^2(t)}{2} R^{(3)}\delta g^{00} \\ & - \bar{m}_4^2(t)\delta K^2 + \frac{\bar{m}_5(t)}{2} R^{(3)}\delta K + \frac{\bar{\lambda}(t)}{2}(R^{(3)})^2 + \cdots \\ & - \frac{\tilde{\lambda}(t)}{M_p^2}\nabla_i R^{(3)} \nabla^i R^{(3)} + \cdots \Big], \end{aligned} \tag{A3}$$

where $\delta g^{00} = g^{00} + 1$, $R^{(3)}$ is the three-dimensional Ricci scalar, $K_{\mu\nu} = h_\mu^\sigma \nabla_\sigma n_\nu$ is the extrinsic curvature and $\delta K_{\mu\nu} = K_{\mu\nu} - h_{\mu\nu}H$. The first row describes the background, while the rest are for the perturbations. We always could set $f = 1$, which implies $c(t) = -M_p^2\dot{H}$ and $c(t) + \Lambda(t) = 3M_p^2 H^2$. See, e.g., Ref. [20] for the details.

Here, we only consider the coefficients set $(M_2, \tilde{m}_4, \bar{m}_5, \bar{\lambda})$ and set other coefficients in (A3) equal to 0. Only with $(M_2, \tilde{m}_4, \bar{m}_5, \bar{\lambda}) \neq 0$, the quadratic action of scalar perturbation ζ is (see, e.g., our Ref. [20])

$$S_\zeta^{(2)} = \int d^4x a^3 Q_s \left[\dot{\zeta}^2 - c_s^2\frac{(\partial\zeta)^2}{a^2} + \frac{c_4}{a^4 Q_s}(\partial^2\zeta)^2\right], \tag{A4}$$

where

$$Q_s = \frac{2M_2^4}{H^2} - \frac{\dot{H}M_p^2}{H^2}, \tag{A5}$$

$$c_s^2 Q_s = \frac{\dot{c}_3}{a} - c_2 \tag{A6}$$

$$c_2 = M_p^2, \tag{A7}$$

$$c_3 = -\frac{2aM_2^4\bar{m}_5}{H^2M_p^2} + \frac{a\dot{H}\bar{m}_5}{H^2} + \frac{aM_p^2}{H} + \frac{2a\tilde{m}_4^2}{H}, \tag{A8}$$

$$c_4 = \frac{2M_2^4\bar{m}_5^2}{H^2M_p^4} - \frac{\dot{H}\bar{m}_5^2}{H^2M_p^2} - \frac{2\bar{m}_5}{H} + \frac{3\bar{m}_5^2}{M_p^2} - \frac{4\bar{m}_5\tilde{m}_4^2}{HM_p^2} + 8\bar{\lambda}. \tag{A9}$$

Only if $Q_s > 0$ and $c_s^2 > 0$ is the nonsingular cosmological model healthy. In models with the operator $(\delta g^{00})^2$, $Q_s > 0$ can be obtained, since $(\delta g^{00})^2$ contributes $\dot{\zeta}^2$, while $c_s^2 < 0$ can be avoided since $R^{(3)}\delta g^{00}$ or $R^{(3)}\delta K$ contributes $(\partial\zeta)^2$.

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[1] V. A. Rubakov, Usp. Fiz. Nauk **184**, 137 (2014) [Phys. Usp. **57**, 128 (2014)].
[2] A. Joyce, B. Jain, J. Khoury, and M. Trodden, Phys. Rep. **568**, 1 (2015).
[3] S. Nojiri, S. D. Odintsov, and V. K. Oikonomou, Phys. Rep. **692**, 1 (2017).
[4] G. W. Horndeski, Int. J. Theor. Phys. **10**, 363 (1974).
[5] C. Deffayet, X. Gao, D. A. Steer, and G. Zahariade, Phys. Rev. D **84**, 064039 (2011).
[6] T. Kobayashi, M. Yamaguchi, and J. Yokoyama, Prog. Theor. Phys. **126**, 511 (2011).
[7] M. Zumalacrregui and J. Garca-Bellido, Phys. Rev. D **89**, 064046 (2014).
[8] J. Gleyzes, D. Langlois, F. Piazza, and F. Vernizzi, Phys. Rev. Lett. **114**, 211101 (2015).
[9] J. Gleyzes, D. Langlois, F. Piazza, and F. Vernizzi, J. Cosmol. Astropart. Phys. 02 (2015) 018.
[10] D. Langlois and K. Noui, J. Cosmol. Astropart. Phys. 02 (2016) 034.
[11] D. Langlois and K. Noui, J. Cosmol. Astropart. Phys. 07 (2016) 016.
[12] J. Ben Achour, M. Crisostomi, K. Koyama, D. Langlois, K. Noui, and G. Tasinato, J. High Energy Phys. 12 (2016) 100.
[13] D. Langlois, M. Mancarella, K. Noui, and F. Vernizzi, J. Cosmol. Astropart. Phys. 05 (2017) 033.
[14] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, and L. Senatore, J. Cosmol. Astropart. Phys. 03 (2008) 014.
[15] S. Weinberg, Phys. Rev. D **77**, 123541 (2008).
[16] G. Gubitosi, F. Piazza, and F. Vernizzi, J. Cosmol. Astropart. Phys. 02 (2013) 032.
[17] J. Gleyzes, D. Langlois, F. Piazza, and F. Vernizzi, J. Cosmol. Astropart. Phys. 08 (2013) 025.
[18] F. Piazza and F. Vernizzi, Classical Quantum Gravity **30**, 214007 (2013).
[19] A. Naskar, S. Choudhury, A. Banerjee, and S. Pal, arXiv:1706.08051.
[20] Y. Cai, Y. Wan, H. G. Li, T. Qiu, and Y. S. Piao, J. High Energy Phys. 01 (2017) 090.
[21] P. Creminelli, D. Pirtskhalava, L. Santoni, and E. Trincherini, J. Cosmol. Astropart. Phys. 11 (2016) 047.
[22] Y. Cai, H. G. Li, T. Qiu, and Y. S. Piao, Eur. Phys. J. C **77**, 369 (2017).
[23] M. Libanov, S. Mironov, and V. Rubakov, J. Cosmol. Astropart. Phys. 08 (2016) 037.
[24] T. Kobayashi, Phys. Rev. D **94**, 043511 (2016).
[25] Y. Cai and Y. S. Piao, J. High Energy Phys. 09 (2017) 027.
[26] A. Ijjas and P. J. Steinhardt, Phys. Lett. B **764**, 289 (2017).
[27] D. Yoshida, J. Quintin, M. Yamaguchi, and R. H. Brandenberger, Phys. Rev. D **96**, 043502 (2017).
[28] R. Kolevatov, S. Mironov, N. Sukhov, and V. Volkova, J. Cosmol. Astropart. Phys. 08 (2017) 038.
[29] Y. Zheng, L. Shen, Y. Mou, and M. Li, J. Cosmol. Astropart. Phys. 08 (2017) 040.
[30] C. de Rham and S. Melville, Phys. Rev. D **95**, 123523 (2017).
[31] R. Kolevatov and S. Mironov, Phys. Rev. D **94**, 123516 (2016).
[32] S. Akama and T. Kobayashi, Phys. Rev. D **95**, 064011 (2017).
[33] K. Feng, T. Qiu, and Y. S. Piao, Phys. Lett. B **729**, 99 (2014).
[34] H. Mohseni Sadjadi and P. Goodarzi, Phys. Lett. B **732**, 278 (2014).
[35] N. Yang, Q. Fei, Q. Gao, and Y. Gong, Classical Quantum Gravity **33**, 205001 (2016); Y. Zhu and Y. Gong, Int. J. Mod. Phys. D **26**, 1750005 (2016).
[36] T. Harko, F. S. N. Lobo, E. N. Saridakis, and M. Tsoukalas, Phys. Rev. D **95**, 044019 (2017); J. B. Dent, S. Dutta, E. N. Saridakis, and J. Q. Xia, J. Cosmol. Astropart. Phys. 11 (2013) 058.
[37] C. Deffayet, S. Deser, and G. Esposito-Farese, Phys. Rev. D **80**, 064015 (2009).
[38] A. H. Chamseddine, V. Mukhanov, and A. Vikman, J. Cosmol. Astropart. Phys. 06 (2014) 017.
[39] A. H. Chamseddine and V. Mukhanov, J. Cosmol. Astropart. Phys. 03 (2017) 009.
[40] L. Sebastiani, S. Vagnozzi, and R. Myrzakulov, Adv. High Energy Phys. **2017**, 3156915 (2017).
[41] A. Ijjas, J. Ripley, and P. J. Steinhardt, Phys. Lett. B **760**, 132 (2016).
[42] H. Firouzjahi, M. A. Gorji, and A. Hosseini Mansoori, J. Cosmol. Astropart. Phys. 07 (2017) 031.
[43] S. Hirano, S. Nishi, and T. Kobayashi, J. Cosmol. Astropart. Phys. 07 (2017) 009.

[44] Y. Misonoh, M. Fukushima, and S. Miyashita, Phys. Rev. D **95**, 044044 (2017).
[45] J. Khoury, B. A. Ovrut, P. J. Steinhardt, and N. Turok, Phys. Rev. D **64**, 123522 (2001).
[46] J. L. Lehners, Phys. Rep. **465**, 223 (2008).
[47] A. Ijjas and P. J. Steinhardt, Phys. Rev. Lett. **117**, 121304 (2016).
[48] M. Koehn, J. L. Lehners, and B. Ovrut, Phys. Rev. D **93**, 103501 (2016).
[49] M. Koehn, J. L. Lehners, and B. A. Ovrut, Phys. Rev. D **90**, 025005 (2014).
[50] M. Li, Phys. Lett. B **724**, 192 (2013).
[51] A. Fertig, J. L. Lehners, and E. Mallwitz, Phys. Rev. D **89**, 103537 (2014).
[52] A. Ijjas, J. L. Lehners, and P. J. Steinhardt, Phys. Rev. D **89**, 123520 (2014).
[53] Y. Cai, Y. T. Wang, and Y. S. Piao, Phys. Rev. D **93**, 063005 (2016).
[54] Y. Cai, Y. T. Wang, and Y. S. Piao, Phys. Rev. D **91**, 103001 (2015).
[55] Y. Cai, Y. T. Wang, and Y. S. Piao, J. High Energy Phys. 02 (2016) 059.
[56] M. Nakashima, R. Saito, Y. i. Takamizu, and J. Yokoyama, Prog. Theor. Phys. **125**, 1035 (2011).
[57] M. Park and L. Sorbo, Phys. Rev. D **85**, 083520 (2012).
[58] N. Bartolo, D. Cannone, and S. Matarrese, J. Cosmol. Astropart. Phys. 10 (2013) 038.
[59] A. Achucarro, V. Atal, B. Hu, P. Ortiz, and J. Torrado, Phys. Rev. D **90**, 023511 (2014).
[60] R. Saito and Y. i. Takamizu, J. Cosmol. Astropart. Phys. 06 (2013) 031.
[61] S. Mizuno, R. Saito, and D. Langlois, J. Cosmol. Astropart. Phys. 11 (2014) 032.
[62] S. Banerjee and E. N. Saridakis, Phys. Rev. D **95**, 063523 (2017).
[63] T. Biswas, E. Gerwick, T. Koivisto, and A. Mazumdar, Phys. Rev. Lett. **108**, 031101 (2012); T. Biswas, A. S. Koshelev, A. Mazumdar, and S. Y. Vernov, J. Cosmol. Astropart. Phys. 08 (2012) 024.
[64] M. Vasilic, Phys. Rev. D **95**, 123506 (2017).
[65] Y. B. Li, J. Quintin, D. G. Wang, and Y. F. Cai, J. Cosmol. Astropart. Phys. 03 (2017) 031.
[66] S. D. Odintsov and V. K. Oikonomou, Phys. Rev. D **92**, 024016 (2015); **91**, 064036 (2015); **90**, 124083 (2014).
[67] S. H. Hendi, M. Momennia, B. Eslam Panah, and M. Faizal, Astrophys. J. **827**, 153 (2016); S. H. Hendi, M. Momennia, B. Eslam Panah, and S. Panahiyan, Phys. Dark Universe **16**, 26 (2017).
[68] M. Giovannini, Phys. Rev. D **95**, 083506 (2017).
[69] D. A. Easson, I. Sawicki, and A. Vikman, J. Cosmol. Astropart. Phys. 11 (2011) 021.