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Discovering the interior of black holes

Ram Brustein,[1,*] A. J. M. Medved,[2,3,†] and K. Yagi[4,‡]

[1]*Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel*
[2]*Department of Physics & Electronics, Rhodes University, Grahamstown 6140, South Africa*
[3]*National Institute for Theoretical Physics (NITheP), Western Cape 7602, South Africa*
[4]*Department of Physics, Princeton University, Princeton, New Jersey 08544, USA*

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The detection of gravitational waves (GWs) from black hole (BH) mergers provides an inroad toward probing the interior of astrophysical BHs. The general-relativistic description of the BH interior is that of empty spacetime with a (possibly) singular core. Recently, however, the hypothesis that the BH interior does not exist has been gaining traction, as it provides a means for resolving the BH information-loss problem. Here, we propose a simple method for answering the following question: Does the BH interior exist and, if so, does it contain some distribution of matter or is it mostly empty? Our proposal is premised on the idea that, similar to the case of relativistic, ultracompact stars, any BH-like object whose interior has some matter distribution should support fluid modes in addition to the conventional spacetime modes. In particular, the Coriolis-induced Rossby (r-) modes, whose spectrum is mostly insensitive to the composition of the interior matter, should be a universal feature of such BH-like objects. In fact, the frequency and damping time of these modes are determined by only the object's mass and speed of rotation. The r-modes oscillate at a lower frequency, decay at a slower rate, and produce weaker GWs than do the spacetime modes. Hence, they imprint a model-insensitive signature of a nonempty interior in the GW spectrum resulting from a BH merger. We find that future GW detectors, such as Advanced LIGO with its design sensitivity, have the potential of detecting such r-modes if the amount of GWs leaking out quantum mechanically from the interior of a BH-like object is sufficiently large.

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I. INTRODUCTION

The view in general relativity (GR) of a black hole (BH) as a region of empty space except for a highly dense and classically singular core of matter has recently been presented with a formidable challenge—it appears to be in contradiction with the laws of quantum mechanics! The modern point of view for diffusing this crisis is that the interior does not exist on account of spacetime ending at the BH horizon. There is, however, some divergence of views on how spacetime terminates. Some argue that it ends with a "firewall" of high-energy particles surrounding the horizon [1] (also [2–4]). Others argue that part of the geometry simply does not exist as in the fuzzball model of BHs [5–8] (also see [9] and, more recently, [10]).

But what if the BH interior does exist and is filled with some distribution of matter? The first obvious obstacle is how to prevent the inevitable fate of gravitational collapse that awaits any matter distribution whose size is approaching its gravitational radius [11]. What is then required is some exotic spacetime containing equally exotic matter which can be stored in an ultracompact object that is able to withstand gravitational collapse. This object must, at the same time, exhibit all of the standard properties of BHs when viewed from the outside. We will refer to such spacetimes collectively as "BH-like objects". One example for such an object is described by our recent proposal that a BH should be modeled as a bound and metastable state of highly energetic, interacting, long, closed strings; figuratively, a collapsed polymer [12,13]. (Here, we are using "collapsed" as it is meant in the polymer literature, e.g., [14], and not gravitationally so.)

One can understand on a from a physics perspective how such a stringy object might evade gravitational collapse. A hot bath of closed strings will entropically favor a state with just a few long loops. These long strings can be effectively described as performing a random walk whose linear size, for a fixed total length of the strings, scales in four dimensions with the square root of the total length of the string. In the case of the polymer model, this means that the linear size of the region occupied by the strings scales with the Schwarzschild radius. We are then assuming that this effective and repulsive random-walk "force" is enough to overcome the would-be gravitational collapse. We are also assuming that, like any other polymer, a fluidlike description should be applicable, if only in a macroscopic, coarse-grained sense.

Here the collapsed-polymer model is meant only as an illustrative example of a possibly more general situation; namely, a BH proxy that is composed of fluidlike matter. It

*ramyb@bgu.ac.il
†j.medved@ru.ac.za
‡kyagi@princeton.edu

will eventually become clear that the analysis applies for this broader range of models.

Putting such claims to the test need no longer be limited to the purview of thought experiments and computer simulations. Thanks to the recent advancement in gravitational wave (GW) astronomy, brought to the forefront by the celebrated observation of GW150914 [15] and its companions [16,17], there are reasons to be optimistic about the prospects for future detections. Indeed, the current observations have already proven their utility for constraining deviations from the GR model of BHs [18,19].

Let us briefly review as to why GWs can be expected to carry information about their BH sources. (A longer discussion appears in [20].) The post-merger stage of a BH collision is that of a single BH settling down into a state of equilibrium. As is typical for partially open systems, the return to equilibrium is associated with a set of ringdown modes whose characteristic frequencies are determined by the system's size, shape and composition. These modes are necessarily damped and often called quasinormal modes (QNMs). BHs are partially open in the sense that matter can enter but not exit whereas, normally, the opposite is true. This makes BHs quite different from other partially open systems because the modes are not escaping from the BH itself, which is of course an impossibility. Rather, spacetime modes propagating in from infinity are reflected back from the surrounding gravitational potential barrier or, otherwise, transmitted through it. Whereas the transmitted modes continue on past the horizon and are gone forever, some of the reflected modes constitute the observed GWs.

The frequencies and damping times for the reflected modes are determined by the properties of the gravitational potential barrier and, therefore, by only a handful of BH parameters. Provided that the BH carries no net charges (nor any exotic "hair"), the only relevant parameters are its mass M and angular velocity Ω. For further reading, one can start with the excellent review articles [21–23] and then, for example, [24–31].

The arguments in the current paper are premised on the idea that a BH-like object—which is assumed to contain a nontrivial matter distribution rather than just a singular core—has some resemblance to a relativistic star. As such, a BH-like object will have a collection of fluid modes in addition to the previously described set of spacetime modes, just like a relativistic star has both. In the relevant literature, the spacetime modes are called w-modes. As for the fluid modes, there are many different classes, with each class representing a different restoring force acting on the star to return it to equilibrium. An incomplete list includes pressure (p-) modes, buoyancy or gravitational-restoring (g-) modes, shear (s-) modes and torsional (t-) modes. For most of these cases, the frequencies and damping times of the modes are sensitive to the precise composition of the stellar object.

Our current interest is the spectra of the so-called r modes (e.g., [32–34]). These are nonradial modes whose amplitudes grow from zero at the center of the star to a maximal value at the surface. Their leading-order frequencies are insensitive to the interior composition and, just like the spacetime modes of a BH, depend only on the mass and rotational speed of the stellar body. So that, if one wants an answer to a simple binary question—"does a BH-like object contain a nontrivial matter distribution or does it not?"—these modes are just what is needed.

The r modes are Rossby (planetarylike) waves that arise due to the effects of the Coriolis force; these being the dominant effects of rotation provided that the object's radial velocity is smaller than the speed of light c. This is because the Coriolis force is proportional to Ω, whereas the effects of the centrifugal force are proportional to Ω^2. As a consequence, a stellar body that is rotating slower than the speed of light can be treated, approximately, as a spherically symmetric rotator. In a case where the axis of rotation points north, the Coriolis force induces counterclockwise motion for fluid initially flowing to the north pole from the equator and clockwise motion in the opposite case. One complete cycle defines the characteristic frequency of the mode, which scales linearly with Ω.

One might wonder about the other types of fluid modes. These would also be interesting for the purposes of discriminating between different models. But, as other types of internal modes do depend on the composition of the object, they would not have the same type of universality that is being exploited here. One might also wonder about the spacetime modes. But these, by definition, do not know about the details of the internal composition, as they depend strictly on the exterior geometry and boundary conditions at the outer surface. The former is the same for any BH-like object, whereas the latter is a model-dependent consideration; for instance, some models are supposed to produce "echoes" (see below). Nonetheless, a sufficiently compact object can be expected to produce modes that are similar to the predominant modes of a Kerr BH.

In the remainder of the paper, we review some basic facts about r modes, both in general and in the current context (Sec. II), determine the characteristic properties of the resulting GWs (Sec. III), present a gravitational waveform along with a plot of the associated spectrum (Sec. IV), discuss the prospects for detecting r modes in the near future (Sec. V), and then conclude (Sec. VI).

Before proceeding any further, it is important to emphasize that our compact objects of interest are those whose outer surfaces act (at least effectively) as BH horizons in that they inhibit the leakage of matter from inside to outside when only the effects of general relativity are considered. Our collapsed polymer model has just such a "quantum horizon"; its outer surface does not permit matter to escape by classical means but is otherwise only partially opaque for finite $\hbar$ [13]. This is because matter can only escape as a result of string interactions, which is controlled by the string coupling, a strictly quantum parameter. More generally, a quantum horizon refers to the outer surface of a BH-like object for which the escape of matter is a quantum process—quantum in the sense that it can be parametrized by a small, dimensionless parameter which would not be present for the BHs of general relativity. We commonly refer to this small parameter as a "dimensionless $\hbar$", which is simply the square of the string coupling for the collapsed-polymer model. (It is also assumed that the fundamental spacetime modes of the object are close enough to those of a Kerr BH so as to not yet be ruled out by the observational data.) And it is this quantum transparency that will allow for the internal modes to couple to external GWs; albeit with an appropriate suppression. This point is discussed further in Sec. III, although a full explanation will be deferred until a later article [35], where the same picture is considered from the perspective of both an internal and external observer.

Let us also take note of a different approach [36–38] (also [39,40]) which argues that, for "exotic compact objects" without horizons, there is a new class of modes that are absent in the classical-GR BH case and analogous to echoes (i.e., modes trapped between the object's outer surface and potential barrier for a finite time). The basic idea is to model the interior of the object as a wormhole, as the inner light ring of a wormhole captures the essence of an echo chamber. Given this setup, one finds that the damping times of the trapped modes depend on a certain power of the log of the throat location relative to the Schwarzschild radius [41]. Due to this large power, such a deviation in the damping times from the BH case effectively enters as a power-law deviation [38]. As shown in a companion article [20], the collapsed-polymer model also predicts power-law deviation to the damping times, albeit with a much different expansion parameter.

II. THE r MODES OF A ROTATING BLACK-HOLE-LIKE OBJECT

A rotating BH-like object can be treated, approximately and to leading order in Ω, as a spherically symmetric rotator with a constant angular velocity. Such a rotator naturally supports r modes. Corrections to the leading order in Ω are expected to be of order Ω^2. We will argue later that for the cases of interest, such corrections are small and therefore justify this approximation. Since our goal is to demonstrate how one could discriminate a fluid-filled interior from others in simple terms, we will confine ourselves to the nonrelativistic approximation that allows us to obtain closed form expressions for the frequency and life-time of the r modes. A more precise analysis may be required for the purpose of making definitive quantitative predictions.

Closely following [33], let us now review how these r modes come about.

The starting point is the hydrodynamic momentum equation in the co-rotating frame of reference (e.g., [42]). To leading order in the angular velocity $\vec{\Omega}$, this can be written as[1]

$$\frac{\partial \vec{u}}{\partial t} = -\vec{\nabla}\delta\Phi - \frac{1}{\rho}\vec{\nabla}\delta p + \frac{\delta\rho}{\rho^2}\vec{\nabla}p - 2\vec{\Omega}\times\vec{u}, \qquad (1)$$

where $\delta\Phi$ represents a perturbation of the gravitational potential, p and δp are the pressure and its perturbation, ρ and $\delta\rho$ are the energy density and its perturbation, and $\vec{u}$ is the velocity of the fluid. It will be assumed that $\vec{\Omega} = \Omega\hat{z}$ with $\Omega > 0$.

Let us now consider the radial component of the curl of Eq. (1). With the approximations that nonradial motion dominates over radial motion, $u_r \ll u_\perp$, and that $\vec{\nabla}\cdot\vec{u}$ is at least linear order in Ω (in fact, it scales as Ω^3 for the r modes [33]), the resulting equation is $\frac{\partial Z}{\partial t} = -2(\vec{u}_\perp\cdot\vec{\nabla}_\perp)\vec{\Omega}_r$, where $Z = (\vec{\nabla}\times\vec{u})_r$ is the radial component of the vorticity and a subscript $\perp$ stands for the nonradial components of the vector. We also used the fact that p for the background only acts radially. Since $\vec{\Omega}$ does not depend on time explicitly,

$$\frac{d}{dt}(Z + 2\Omega_r) = 0, \qquad (2)$$

to linear perturbative order. The quantity in the brackets is the radial component of the vorticity in an inertial frame, and so Eq. (2) makes it clear that this component is conserved.

One can deduce from Eq. (2) the nature of the induced oscillations. Working in the co-rotating frame, let us suppose that a fluid element starts out at the equator ($\theta = \pi/2$) where it is moving north. Then, initially, Z is a constant because $\Omega_r = \Omega\cos\theta = 0$ and we choose $Z = 0$ for simplicity. Now, as the fluid element proceeds upwards, Ω_r increases because of the factor of $\cos\theta$. From Eq. (2), it follows that a negative vorticity is generated, corresponding to a clockwise rotation of the fluid element. The element then rotates in such a way that it eventually returns to the equator and continues its motion downwards, only to come back up to the equator and so on. This type of motion is described in several nice movies [45].

[1]We are taking some liberty in using a nonrelativistic (Newtonian) equation to calculate the r-mode spectrum of BH-like objects. Our justification being that the production of r modes is, at leading order, a surface effect and thus insensitive to what lies inside. For reference, relativistic corrections only affect the r-mode frequency of a neutron star by 8%–20% [43,44], although these corrections would be enhanced for the case of a BH. Ultimately, one would have to resort to the numerical analysis of the relativistic equations to make definitive predictions. Such a study is outside the scope of the current paper, which is meant to convey the basic idea of using GWs to discriminate between fluid-filled interiors and other models.

A more formal approach allows one to deduce the actual relationship between the r-mode frequency and Ω. As an r mode is toroidal at leading order, its velocity vector in the co-rotating frame can be approximately decomposed as [46]

$$\vec{u} \simeq i\omega r K_{\ell m}\left(0, \frac{1}{\sin\theta}\frac{\partial Y_\ell^m}{\partial\phi}, -\frac{\partial Y_\ell^m}{\partial\theta}\right)e^{i\omega t}, \tag{3}$$

where ℓ, m are the angular-momentum quantum numbers, the Y's are spherical harmonics and $K_{\ell m}$ is some smooth function of r which is not relevant to our purposes. When substituting Eq. (3) into Eq. (2), one finds that the leading-order frequency of the r modes in the co rotating frame is $\omega = \frac{2m\Omega}{\ell(\ell+1)}$. In an inertial frame, the frequency translates into $\omega = \Omega(-m + \frac{2m}{\ell(\ell+1)})$.

Our main interest is the case of $\ell = 2$, $m = 2$, for which

$$\omega_{r-\mathrm{mode}} = -\frac{4}{3}\Omega. \tag{4}$$

The fact that the frequency is negative is significant and may, under some circumstances, result in an instability which amplifies the r modes [47]. This possibility will not be discussed any further and the negative sign will be left off.

To determine the value of Ω for rotating BH-like objects, we may borrow some of the standard results for Kerr BHs (e.g., [48]). This is because, as far as their external properties are concerned, BH-like objects and the BHs of GR should—by our previous assumptions and definition for the compact objects of interest—be similar and, in some cases like the collapsed-polymer model, indistinguishable. In what follows, v is the rotational speed of the object and u indicates the speed of a mode.

For spinning BHs, the frequency of rotation is parametrized by the measure of spin $a = 2v/c$ (a is the dimensionless Kerr parameter),

$$M\Omega = \frac{a}{2(1+\sqrt{1-a^2})}. \tag{5}$$

Then, for the r modes (with $\ell = m = 2$),

$$M\omega_{r-\mathrm{mode}} = \frac{2}{3}\frac{a}{(1+\sqrt{1-a^2})}. \tag{6}$$

In merger events for which the masses of the two colliding BHs are approximately equal and their initial (total) spin is small compared to their angular momentum, the final spin parameter is $a \approx 0.7$ and depends weakly on the ratio of the masses (see [49] for details). Also, as reported in [16], this value of a is approximately what was measured in the three recently detected events assuming classical-GR BHs. Then, with this choice,

$$M\Omega = 0.20 \tag{7}$$

for the BH-like object and

$$M\omega_{r-\mathrm{mode}} = 0.27 \tag{8}$$

for the frequency of the r modes with $\ell = m = 2$ [cf., Eq. (4)].

For such cases, the relativistic corrections due to the centrifugal force or to any additional relativistic corrections are governed by the small number $\frac{v^2}{c^2} = \frac{a^2}{4} = 0.12(\frac{a}{0.7})^2$. The velocity of an r mode is somewhat larger than the rotational velocity of the object but still quite nonrelativistic, $\frac{u^2_{r-\mathrm{mode}}}{c^2} = \frac{\omega^2_{r-\mathrm{mode}}}{\Omega^2}\frac{v^2}{c^2} = \frac{16}{9}\frac{v^2}{c^2} \simeq 0.22(\frac{a}{0.7})^2$. This means that the expected relativistic corrections are less than about 25% of the nonrelativistic values. At the level of accuracy of this paper, this is sufficient. To obtain more precise results one has to resort to better analytic and numerical analysis that will take into account also the relativistic corrections.

We now want to compare the frequency of the r modes in Eq. (8) to that of the slowest-oscillating spacetime modes ω_{st}. The value of the latter frequency for the case of $a = 0.7$ can be found in, e.g., Table II of [50],

$$M\omega_{st} = 0.53. \tag{9}$$

It follows that the frequency of an r mode is about half that of the lowest-frequency spacetime modes in the $a = 0.7$ case,

$$\frac{\omega_{r-\mathrm{mode}}}{\omega_{st}} \simeq 0.5, \tag{10}$$

up to a small (known) dependence on the ratio of the masses of the colliding BHs.

III. FREQUENCY, DECAY TIME, AND AMPLITUDE OF THE EMITTED GRAVITATIONAL WAVES

We would now like to determine the three quantities that characterize the additional emission of GWs due to the r modes: frequency, decay time and amplitude. We find that the frequency, which is the most robust prediction, scales roughly as $\omega_{r-\mathrm{mode}} \sim \omega_{st} v/c$. The decay time scales as $1/\tau_{r-\mathrm{mode}} \sim (1/\tau_{st})(v/c)^2$ and is less robust. The amplitude scales as $h_{r-\mathrm{mode}} \sim h_{st}(v/c)^3$ and is the least robust prediction. (Here, we have been using $u_{r-\mathrm{mode}} \propto v$.) Each of the three quantities will be discussed in turn.

Let us first recall what was found for the frequency. For GWs that are sourced by r-mode oscillations, this is given by Eq. (6) in general and, for values of the spin parameter close to $a = 0.7$, by Eqs. (8) and (10). In the latter case, we recall that $M\omega_{r-\mathrm{mode}} = 0.27$ or, equivalently, $\omega_{r-\mathrm{mode}}/\omega_{st} \simeq 0.5$. We will thus use the scaling relation

$$\omega_{r-\mathrm{mode}} \sim \frac{u_{r-\mathrm{mode}}}{c}\omega_{st} \simeq 0.5\omega_{st}\left(\frac{a}{0.7}\right). \qquad (11)$$

Although such a relation is based on only a single choice of a (namely, $a = 0.7$), it can be checked that Eq. (11) recovers the correct value of $\omega_{r-\mathrm{mode}}$ in Eq. (6) for the choice of, e.g., $a = 0.5$ to within 5% accuracy.

Let us next move on to the decay time. In general, the decay time τ of a mode can be estimated by the ratio of its dissipated energy $\frac{dE}{dt}$ to its total energy E, $1/\tau = \frac{1}{E}\frac{dE}{dt}$. The decay time of the r modes and, therefore, of their corresponding GWs is determined by the shortest dissipation time of three possibly important sources of dissipation: (i) the emission of GWs which reduces the energy of the r modes accordingly, (ii) the leakage of r modes away from the BH-like object by processes that differ from the emission of GWs (for instance, by coupling to other types of matter) and (iii) the intrinsic dissipation within the interior matter.

First, the decay time of the r modes due to emission of GWs scales as the light-crossing time R/c divided by a factor of $(M\omega_{r-\mathrm{mode}})^6$ (e.g., [47]). This is much too long a time scale to be of any relevance to our discussion.

Second, the time scale for leakage can be estimated by calculating the imaginary part of the QNM frequencies. As explained in detail in [20], when the modes are nonrelativistic, the imaginary part of the frequency ω_I is parametrically smaller than the real part ω because of the scaling $\omega_I \sim \frac{u}{c}\omega$. Then it follows from Eq. (11) that the imaginary part of the r-mode frequency is doubly suppressed relative to that of the spacetime QNMs,

$$\omega_{I\, r-\mathrm{mode}} \sim \left(\frac{u_{r-\mathrm{mode}}}{c}\right)^2 \omega_{I\, st}, \qquad (12)$$

where the value of ω_{Ist} for $a = 0.7$ is given in, e.g., Table II of [50], $M\omega_{I\, st} = 0.08$. Equivalently,

$$\tau_{r-\mathrm{mode}} \sim \left(\frac{u_{r-\mathrm{mode}}}{c}\right)^{-2} \tau_{st} \simeq 4.6\tau_{st}\left(\frac{a}{0.7}\right)^{-2}. \qquad (13)$$

The third source of energy loss is the intrinsic dissipation, whose time scale can be estimated following [47]. As will be shown, unless the ratio of the shear viscosity η to the entropy density s is the smallest that it can be—an approximate saturation of the KSS bound $\eta/s \sim 1$ [51]—then the intrinsic dissipation is too large and it is likely that the modes will decay too quickly to ever be detected. In the case of the polymer model, the interior matter does indeed saturate the KSS bound [13], and a simple argument (based on reinterpreting the KSS bound as an upper limit on the entropy [52]) suggests that this must be generally true for other models as well. Following [47], one then finds that the intrinsic-dissipation time $\tilde{\tau}$ for the r modes is given by

$$\frac{1}{\tilde{\tau}_{r-\mathrm{mode}}} \sim \frac{\eta}{\rho R^2} \sim \left(\frac{u_{r-\mathrm{mode}}}{c}\right)^2 \frac{1}{\tau_{st}}, \qquad (14)$$

where we have used the fact that $\eta/\rho \sim R$ for KSS-saturating matter with relativistic modes and η/ρ effectively scales like $(\frac{u}{c})^2$ for nonrelativistic modes [20] so that $\eta/\rho \sim (\frac{u}{c})^2 R$. If η/ρ is parametrically larger than R, as is the case for all known forms of nonexotic matter, then the decay time would be much smaller than that of the longest-lived spacetime modes, meaning that the detection of the r modes would no longer be feasible.

Conversely, if $\eta/\rho \sim R$ as expected, then both the leakage time and the intrinsic-dissipation time are parametrically longer than the decay time of the spacetime QNMs by a factor of $(u/c)^{-2}$,

$$\tau_{r-\mathrm{mode}} \sim \tilde{\tau}_{r-\mathrm{mode}} \sim \left(\frac{u_{r-\mathrm{mode}}}{c}\right)^{-2} \tau_{st} \simeq 4.6\tau_{st}\left(\frac{a}{0.7}\right)^{-2}. \qquad (15)$$

Let us now consider the amplitude of the emitted GWs. Our approach is to use Einstein's celebrated quadrupole formula, while taking into account that the matter in some models can be surrounded by a (possibly semi-transparent) horizon. The latter consideration can be incorporated by parametrizing the strength of the coupling of the fluid modes to the emitted GWs. For any specific case, this coupling is determined by the details of the model. For example, if the matter within a BH-like object is not surrounded by any horizon, this coupling can be estimated by treating the background spacetime as fixed and (essentially) flat [53]. Then, $h \propto d^2Q/dt^2$, where h is the gravitational waveform and Q is the quadrupole moment of the energy density.

Now suppose that some quadrupole moment does exist in a confined region of space. Just how much of this moment contributes to the production of outgoing GWs? If the region is surrounded by a classical horizon, the answer is none. In this case, the horizon is completely opaque and nothing can escape from inside. On the other hand, the region will be semi-transparent if surrounded by a "quantum horizon" because then some GWs can escape to the outside. The fraction of those escaping is proportional to the dimensionless $\hbar$ of the problem, $\tilde{\hbar} < 1$. For example, in the polymer model, the relevant dimensionless parameter for a certain class of fluid modes is $\tilde{\hbar} = g_s^2$ [20], where the string coupling g_s is the ratio between the Planck length and the string length scale. The numerical value of g_s is expected to be small but not extremely small. For instance, the string coupling cannot be too much smaller than unity given that the expected grand unification of forces at the Planck energy is correct. In cases like the wormhole model [36], the region is not surrounded by any horizon.

We will cover this broad spectrum of cases by introducing a "transparency" or transmission coefficient $T_{\rm hor}$ that ranges from 0 (a classical horizon) to 1 (no horizon). An r mode can now be characterized as follows: Its frequency and lifetime are fixed by the frequency of rotation (equivalently, the Kerr parameter a) of the BH-like object, whereas its amplitude additionally depends on a model-dependent parameter $T_{\rm hor}$ for which $0 \le T_{\rm hor} \le 1$.

Let us briefly comment on how $T_{\rm hor}$ can generally be estimated (see [35] for a detailed discussion). One can assign a width to a given quantum horizon of $\Delta R_S = \tilde{\hbar} R_S$ (R_S is the object's Schwarzschild radius). The width ΔR_S can, when the BH is out of equilibrium, be expected to be macroscopically large and still well within the potential barrier at about $\frac{3}{2}R_S$. This is because ΔR_S scales with the product of the horizon radius and a simple, positive power of the dimensionless $\hbar$ which need only be smaller than unity. The width ΔR_S implies that the GWs corresponding to some fluid mode will first appear in the exterior at a radius where the Tolman redshift factor is $\sqrt{\tilde{\hbar}}$. Using this redshift along with the quadrupole formula, one finds that the amplitude of the GWs, by the time they reach the potential barrier, will be suppressed by some power of $\tilde{\hbar}$—it is this suppression that should be identified with $T_{\rm hor}$, a number that is less than one but, at the same time, need not be unobservably small. On the other hand, in cases like the wormhole model for which there is no horizon, one can view $T_{\rm hor}$ as some power of the redshift factor at the location of the object's outermost surface or its throat.

The redshift factor describes how an external observer, who believes that the fluid modes originate from outside of the horizon, is able to reconcile the suppression factor $T_{\rm hor}$ with her knowledge of general relativity. From an internal perspective, the suppression can be attributed to quantum effects being the primary source of mode leakage. One should not combine these two sources of suppression, as this would amount to a double counting. The consistency between the internal and external perspectives and that these provide complementary pictures will be exposed in the aforementioned treatment [35].

Putting all of these ingredients together and recognizing that the r modes induce velocity perturbations, one can find an appropriate estimate of the GW amplitude in [53] (also [47]). Let us first express the r-mode waveform as

$$h_{r-\rm mode} = A_{r-\rm mode} e^{-t/\tau_{r-\rm mode}} \sin(\omega_{r-\rm mode} t - \phi_r), \qquad (16)$$

with ϕ_r representing the constant phase and $A_{r-\rm mode}$, the dimensionless strain amplitude. Then

$$A_{r-\rm mode} \sim \alpha_{r-\rm mode} T_{\rm hor} \frac{M}{r_s} \left(\frac{u_{r-\rm mode}}{c}\right)^3, \qquad (17)$$

where $\alpha_I < 1$ parametrizes the amount of energy that the merger injects into the $I^{\rm th}$ class of mode perturbations and r_s is the radial distance from the center of the source. The factor $(\frac{u_{r-\rm mode}}{c})^3$ is a product of a factor of $(\frac{u_{r-\rm mode}}{c})^2$ originating from the two time derivatives in the quadrupole formula ($d/dt \sim \omega_{r-\rm mode} \propto u_{r-\rm mode}$) and additional factor of $\frac{u_{r-\rm mode}}{c}$ that can be attributed to the waves being sourced by velocity perturbations.

This amplitude should be compared to that of the spacetime modes, which scales as

$$A_{st} \sim \alpha_{st} \frac{M}{r_s}. \qquad (18)$$

In the recently detected events, the fraction of radiant energy in the form of GWs was found to be a few percent of the system's total mass, which is consistent with prior estimates of about $\alpha_{st} \sim 0.1$ corresponding to a gravitational radiant energy of around 3% of M [54,55]. It is likely that $\alpha_{r-\rm mode}$ and α_{st} are of similar magnitudes, in which case the suppression of the r-mode amplitude is determined solely by $T_{\rm hor}(u_{r-\rm mode}/c)^3$,

$$A_{r-\rm mode} \sim T_{\rm hor} \left(\frac{u_{r-\rm mode}}{c}\right)^3 A_{st} \sim 0.1 T_{\rm hor} A_{st} \left(\frac{a}{0.7}\right)^3. \qquad (19)$$

IV. GRAVITATIONAL WAVEFORM AND SPECTRUM

Let us now look at the gravitational waveform for the r modes in both the time and Fourier domain, beginning with the former. The case of primary interest is when the final spin is $a = 0.7$, which corresponds to the merger of two nonspinning, equal-mass BHs. From the results of the previous section, the following picture emerges: In a BH-merger event, the r modes produce a GW signal at a lower frequency, $\omega_{r-\rm mode} \sim 0.5\omega_{st}$, with a longer decay time, $\tau_{r-\rm mode} \simeq 4.6\tau_{st}$, and with a suppressed amplitude, $h_{r-\rm mode} \sim 0.1 h_{st}$, in comparison to the standard spacetime-mode signal. One can also anticipate some additional delay in the emission of GWs due to the reduction in frequency, as there is an expected delay of about one oscillatory period. (This allows time for the mode to reach the outer surface.) Figure 1 depicts the waveform of GWs emitted from a BH-merger—if the r modes do exist—for a final spin of $a = 0.7$, $v/c = 0.35$, $\omega_{r-\rm mode} = 0.5\omega_{st}$, $\tau_{r-\rm mode} = 5\tau_{st}$ and $h_{r-\rm mode} = 0.1 h_{st}$, along with a delay of about one period.

We next consider the GW spectrum in the Fourier domain, as this is important for calculating the signal-to-noise ratio (SNR) later. The Fourier transform of a damped sinusoid is given by [20,56]

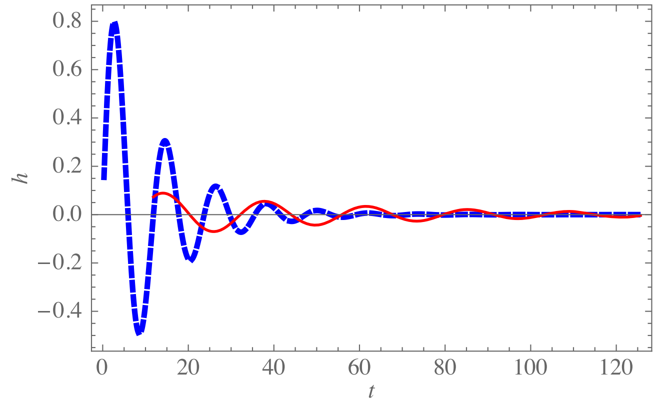

FIG. 1. Gravitational waves emitted during the ringdown phase of a BH merger with the parameters listed in the text. The blue (thick, dashed) line depicts h_{st} in arbitrary units as a function of time in units of M, while the red (thin, solid) line depicts $h_{r-\mathrm{mode}}$.

$$|\tilde{h}(f)| = A_{r-\mathrm{mode}}\tau_{r-\mathrm{mode}}\left|\frac{2f_r^2 Q_r \cos\phi_r - f_r(f_r - 2ifQ_r)\sin\phi_r}{f_r^2 - 4iff_rQ_r + 4(f_r^2 - f^2)Q_r^2}\right|, \tag{20}$$

where $f_r \equiv \omega_{r-\mathrm{mode}}/(2\pi)$ is the r-mode frequency and $Q_r \equiv \pi f_r \tau_{r-\mathrm{mode}}$. To make this transform explicit, Eqs. (11) and (13) can be used to determine how the r-mode frequency and damping time scale with respect to those of the spacetime mode. Meanwhile, the fitting function for the spacetime-mode parameters in terms of M and a can be found in [50]. Similarly, Eq. (19) can be used for the amplitude scaling, where the spacetime-mode amplitude A_{st} can be obtained from [20].

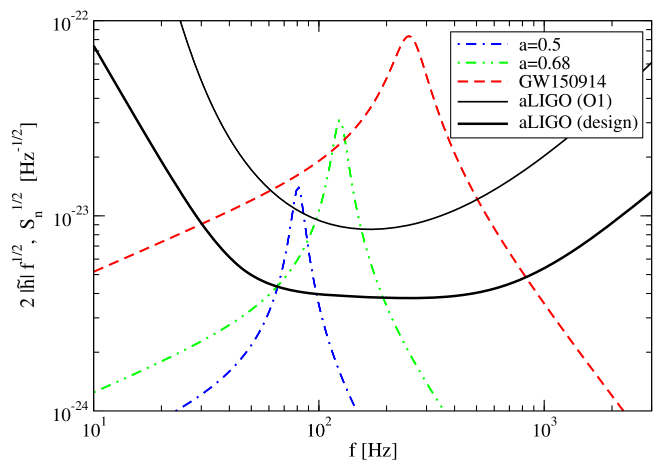

FIG. 2. Sky-averaged GW spectrum of the r mode for BH-like objects with $a = 0.5$ (blue, dotted-dashed) and $a = 0.68$ (green, double dotted-dashed). We choose $M = 62.3\ M_\odot$, $D_L = 410$ Mpc and $T_{\mathrm{hor}} = 1$. For reference, the spectrum corresponding to the observed ringdown for GW150914 (red, dashed) is included. Also shown are the noise spectral density of aLIGO in the O1 run (thin, black, solid) and for its design sensitivity (thick, black, solid). The ratio between the signal and noise roughly corresponds to the SNR and the signal is detectable if this ratio is above the threshold (~5). We stress that the results presented here are not robust and should be understood as only rough estimates.

Figure 2 presents such spectra for $a = 0.5$ and $a = 0.68$. Here, we have set $T_{\mathrm{hor}} = 1$, $\phi_r = 0$, depicted the sky-averaged amplitude at a luminosity distance of $D_L = 410$ Mpc and chosen $M = 62.3\ M_\odot$, where the last two values correspond to those of GW150914 [15,16]. The relation between a and the symmetric mass ratio η of a BH binary [57] has been adopted to rewrite the radiation efficiency in A_{st} (with the pre-merger BH spins set to 0 for simplicity) in terms of a. One can observe how the amplitude, frequency and the width of the peak all grow with increasing a. For reference, we have included the spectrum of the spacetime mode for GW150914; as well as the noise spectral density of Advanced LIGO (aLIGO), both for its O1 run and for its design sensitivity.

V. PROSPECTS FOR DETECTION

Let us now discuss the future prospects for detecting r modes. In [20], we derive an upper bound on the amplitude of the secondary ringdown mode relative to the primary one assuming that the former was not detected in the GW150914 observation. Applying that result to the current analysis and choosing $a = 0.68$ (the final spin of the remnant BH for GW150914 [15,16]), we then obtain $h_{r-\mathrm{mode}}/h_{st} < 0.26$. This inequality can, using Eq. (19), be mapped to one on T_{hor}, leading to $T_{\mathrm{hor}} \lesssim 2.6$. This should be regarded as only a rough bound, as it is based on scaling relations for the amplitude, frequency and damping time which neglect any $O(1)$ prefactors. Rough or otherwise, such a bound is not really useful because T_{hor} cannot be any larger than unity.

Our main interest is in the scaling relation for the minimum T_{hor} that is required for detecting r modes (which we denote $T_{\mathrm{hor}}^{(\mathrm{min})}$) in terms of M, a, D_L and detector sensitivity. The starting point is the calculation of the SNR, which is obtained from

$$\mathrm{SNR}^2 = 4\int_{f_{\min}}^{f_{\max}} \frac{|\tilde{h}(f)|^2}{S_n(f)}df, \tag{21}$$

where $f_{\min}$ and $f_{\max}$ are the minimum and maximum frequencies, while S_n is the noise spectral density. Then using Eqs. (19)–(21), along with $u_{r-\mathrm{mode}}/c \propto a$ and $df \sim 1/\tau_{r-\mathrm{mode}}$, one finds that

$$\mathrm{SNR} \propto A_{r-\mathrm{mode}}\sqrt{\tau_{r-\mathrm{mode}}} \propto T_{\mathrm{hor}}A_{st}a^3\sqrt{\tau_{r-\mathrm{mode}}} \tag{22}$$

for a white-noise background. It is worth noting that the SNR scales as $(v/c)^2$ since $A_{r-\mathrm{mode}} \propto (v/c)^3$ and $\tau_{r-\mathrm{mode}} \propto (v/c)^{-2}$. One can now derive the minimum

$T_{\rm hor}$ for detection by equating this calculation to the threshold SNR. However, our main interest is still in the scaling behavior of $T_{\rm hor}^{\rm (min)}$. For instance, since D_L only appears in Eq. (22) through A_{st} as $A_{st} \propto 1/D_L$, $T_{\rm hor}^{\rm (min)}$ is linearly proportional to D_L.

Let us next look at the M dependence of $T_{\rm hor}^{\rm (min)}$. Recalling that $A_{st} \propto M$ and $\tau_{r-{\rm mode}} \propto M$, one can see from Eq. (22) that SNR $\propto M^{3/2}$. Thus, setting this expression for the SNR equal to the threshold SNR of 5, one can deduce that $T_{\rm hor}^{\rm (min)}$ is proportional to $M^{-3/2}$. The top panel of Fig. 3 shows, for a fixed set of a values, the M dependence of $T_{\rm hor}^{\rm (min)}$ as calculated *directly* from Eq. (21) [i.e., without imposing the white-noise assumption or using Eq. (22)] for a sky-averaged waveform. One can compare this figure to the fit proportional to $M^{-3/2}$ (which is also plotted) and observe that the numerical values follow the anticipated $M^{-3/2}$ dependence for the smaller values of M. For larger M, the peak frequency of the GW spectrum in Fig. 2 shifts to a lower frequency and, as a result, the white-noise assumption becomes less valid. Thus, the minimum $T_{\rm hor}$ for detection deviates from its expected $M^{-3/2}$ dependence in this regime of larger mass.

Finally, we can consider the a dependence of $T_{\rm hor}^{\rm (min)}$. For one thing, A_{st} is proportional to the radiation efficiency, which is further proportional to the symmetric mass ratio η

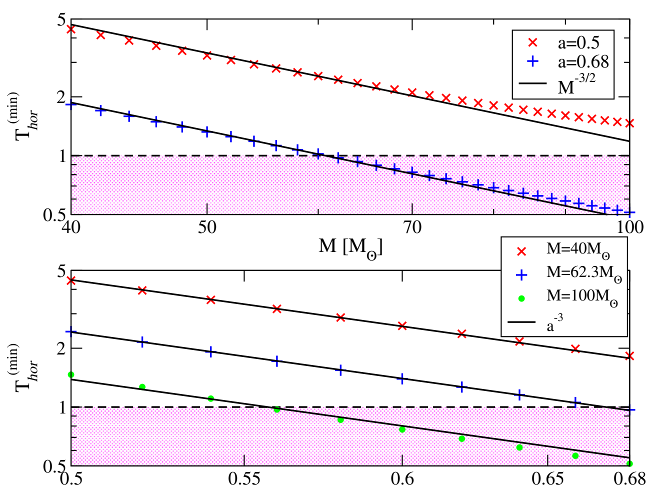

FIG. 3. The minimum $T_{\rm hor}$, which characterizes the amount of quantum leakage of the r-mode GWs through the horizon, that is needed for aLIGO at Hanford and Livingston with its design sensitivity to detect r-mode GWs from equal-mass BH binaries at $D_L = 410$ Mpc. We show the minimum $T_{\rm hor}$ as a function of M (top) and a (bottom). Solid lines are the fits proportional to $M^{-3/2}$ and a^{-3}. The shaded region ($T_{\rm hor} \leq 1$) corresponds to the theoretically allowed range of $T_{\rm hor}$. Observe that $T_{\rm hor}^{\rm (min)}$ for massive and rapidly spinning sources falls into this range. We stress that the bounds presented here are not robust and should be understood as only rough estimates.

[49], which is roughly proportional to a [57]. For another, $\tau_{r-{\rm mode}} \propto a^{-2}$, and hence Eq. (22) indicates that SNR $\propto a^3$. It then follows, in analogy to the M-dependence argument, that $T_{\rm hor}^{\rm (min)}$ is proportional to a^{-3}. The bottom panel of Fig. 3 depicts how $T_{\rm hor}^{\rm (min)}$ depends on a as calculated from Eq. (21) for a set of fixed M values. Also shown is the fit proportional to a^{-3}. Once again, the numerical values nicely follow the anticipated dependence when M is smaller but deviate from expectations when M is larger. The logic underlying this behavior is, of course, the same as that discussed in the previous paragraph.

In light of its dependence on M, a and D_L, one can roughly estimate the minimum $T_{\rm hor}$ for detection as

$$T_{\rm hor}^{\rm (min)} \approx 0.97 \left(\frac{M}{62.3\ M_\odot}\right)^{-3/2} \left(\frac{a}{0.68}\right)^{-3} \left(\frac{D_L}{410\ {\rm Mpc}}\right) \times \left(\frac{N_s}{1}\right)^{-1/2} \left(\frac{N_d}{2}\right)^{-1/2} \left(\frac{\sqrt{S_n(f_0)}}{4\times 10^{-24}\ {\rm Hz}^{-1/2}}\right), \tag{23}$$

where we also included the dependence on the number of (identical) GW sources N_s and the number of (identical) GW detectors N_d. See, for instance, [58] on how to coherently stack small-SNR signals from different GW sources. Additionally, $\sqrt{S_n(f_0)}$ is the detector sensitivity at $f_0 = 200$ Hz and is merely a representative parameter for an overall sensitivity scaling [as $T_{\rm hor}^{\rm (min)}$ depends on $\sqrt{S_n(f)}$ and not just $\sqrt{S_n(f_0)}$].

Let us study the prospect for the detection of r modes in more detail. Equation (23) and Fig. 3 imply that the detectability increases for sufficiently massive, rapidly spinning and close-enough objects. For such sources, $T_{\rm hor}^{\rm (min)}$ becomes smaller than unity and falls into the theoretically allowed range of $T_{\rm hor}$, as indicated by the magenta shaded regions in Fig. 3. For example, a mass of $M = 100\ M_\odot$ allows one to detect an r mode with $T_{\rm hor}$ as small as $\sim$0.5. If Virgo, KAGRA and LIGO-India further come online ($N_d = 5$), an r mode can be detected with $T_{\rm hor} \gtrsim 0.3$. On the other hand, third-generation GW detectors, such as the Einstein Telescope and Cosmic Explorer, will have $\sim$10 times better sensitivity than aLIGO. Hence, an r mode can be detected with $T_{\rm hor}$ as small as $\sim$0.1 for the fiducial parameters in Eq. (23) when using third-generation detectors. Alternatively, such detectors may find $\sim 10^3$ GW sources having a similar SNR to that of GW150914 ($\sim$20). Setting D_L ($\sqrt{S_n(f_0)}$) to be 10 times larger (smaller) and $N_s = 10^3$ in Eq. (23), one finds that r modes can be detected with $T_{\rm hor} \gtrsim 0.03$.

VI. CONCLUSION

We have argued that a BH-like object—an object that resembles a BH from the outside but with a different composition for its interior—can be discriminated from the

BHs of GR on the basis of its r modes. This follows from the observation that, just like a relativistic star, the r-mode frequency and damping time should be essentially independent of the object's composition, depending only on its mass and speed of rotation v to leading order in v/c.

Under suitable circumstances, the GWs originating from the r modes should stand out clearly in the data, as their frequencies scale with the rotational speed of the BH-like object and their lifetimes are enhanced by a factor of $(v/c)^{-2}$. However, because the wave amplitude drops off quickly by a factor of $(v/c)^3$, one is faced with two competing effects: The easier it is to distinguish the r-mode-sourced GWs from those sourced by the spacetime modes, the weaker is the r-mode signal. The GW spectrum also drops out of the detector band for smaller v/c, making the detection of such lower-frequency waves even more difficult. More optimistically, we have shown that, given aLIGO's design sensitivity and a sufficiently massive, rapidly rotating and close-enough source, the minimum value of $T_{\rm hor}$—this being a parameter which characterizes the quantum leakage of the r-mode GWs through the horizon—that is needed for detection is below unity, which is the theoretical upper bound on $T_{\rm hor}$. The prospect for detection increases as the detector sensitivity improves, more detectors come online and the number of GW sources increases. Alternatively, the absence of any r modes would allow one to place upper bounds on $T_{\rm hor}$. Such a bound would enable one to rule out some of the proposed models for the BH interior.

Here, we mainly focused on answering the binary question: Are the BHs in Nature those of GR or are they not? If the latter is indeed true, further discrimination will be possible by looking at other classes of fluid modes, as most of these carry information about the interior composition already at leading order in frequency. In these cases, however, the theoretical predictions will necessarily be model dependent. A detailed discussion of this topic from the perspective of the collapsed-polymer model [12] can be found in [20]. Other relevant works in this direction include [36,37,39,40,59,60].

Finally, one might be concerned as to (i) how interior fluid modes can couple to external GWs in models with a horizon, albeit a horizon with a quantum disposition, and (ii) how an external observer would perceive this class of GWs in a way that is consistent with classical GR (which certainly maintains its validity in the exterior part of spacetime). As these are important issues in their own right, we intend to address them in a separate discussion [35].

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