

Heavy molecules and one- σ/ω -exchange modelRui Chen,^{1,2,3,*} Atsushi Hosaka,^{3,‡} and Xiang Liu^{1,2,†}¹*School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China*²*Research Center for Hadron and CSR Physics,**Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China*³*Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan*

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In the framework of the one-boson-exchange model, we explore whether the intermediate- and short-range forces from σ/ω exchange can be strong enough to bind heavy molecular states. $\Lambda_c D(\bar{D})$ and $\Lambda_c \Lambda_c(\bar{\Lambda}_c)$ systems have been studied and compared. We find that the force from σ exchange is attractive and dominant, whereas the ω -exchange force is not. As a consequence, the S-wave $\Lambda_c D$, $\Lambda_c \Lambda_c$, and $\Lambda_c \bar{\Lambda}_c$ can be possible molecular candidates. We further indicate that a one hadron-hadron system with more light quarks (u, d) can be easier to form than a bound state. As a by-product, by studying the heavy-quark mass dependence for the $\Lambda_c D(\bar{D})$ -like and $\Lambda_c \Lambda_c(\bar{\Lambda}_c)$ -like systems, we find that the charm/bottom sector can easily accommodate molecular states. Finally, the $\Lambda_c N(\bar{N})$ and $\Lambda_b N(\bar{N})$ systems are investigated. Our results indicate that they are also likely to form bound states. By including one- π -exchange forces, providing additional attraction when coupled channels are included, we expect many molecular states in heavy-quark sectors.

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I. INTRODUCTION

Ever since the observations of $X(3872)$ [1] and Θ^+ [2], much evidence has been reported for new types of structures that are beyond the minimal $\bar{q}q$ mesons and qqq baryons, although the observation of Θ^+ has been criticized by subsequent experiments [3,4]. Hence, they are called the exotic hadrons, like $X(3872)$ [1], $Z_b(10610)/Z_b(10650)$ [5], $P_c(4380)/P_c(4450)$ [6], and so on. Their unusual structure may contain more constituents, such as $\bar{q}q$ pairs or gluons. With the extra $\bar{q}q$ pairs, multi-quark configurations may form a compact structure with colored correlations, such as tetraquark and triquark, or a rather extended structure with color-singlet hadronic correlations, which is called the hadronic molecule [7–9].

Since many new findings are seen near the threshold of hadronic decays, it is natural that the hadronic molecular-like structure develops if suitable attractive interactions between the hadrons are available. The strength of the interaction between color-singlet hadrons should be weaker than that between colored objects of order Λ_{QCD} —several hundred MeV. A typical example of such hadronic molecules is an atomic nucleus whose binding energy is of order 10 or a few MeV.

For the study of hadronic molecules, the interaction between the hadrons is a crucial input. Unfortunately, not much is known for the hadron interactions, which are relevant for the recent exotic hadrons. For example, for

$X(3872)$, regarded as $D\bar{D}^*$ molecule [10–15], the realistic interaction between D and \bar{D}^* mesons is not well known, partly because there is no experimental data. Lattice QCD approaches should, in principle, be promising as the recent study for $Z_c(3900)$ [16]. However, application to various systems is rather limited. In such a situation, perhaps, the one-boson-exchange model is a reasonable theoretical approach.

According to the mass differences for the exchanged meson, the interactions from π , σ , ρ , and ω exchanges contribute in the long-range, intermediate-range, and short-range distances, respectively. Among them, the one pion exchange is the best known, as is important for the deuteron [17,18] and the $X/Y/Z$ states [10–12]. For the vector meson ρ , based on the local hidden gauge approach, it is also very essential in identifying the heavy molecular state [19]. In Ref. [20], the η exchange is proposed to form heavy hadronic molecules. Soon after, the one- η -exchange model was adopted to investigate the interaction of $\Lambda_c \bar{D}_s^*/\Sigma_c^{(*)} \bar{D}_s^*/\Xi_c^{(t,*)} \bar{D}^*$ systems in Ref. [21]. Numerical results indicate that the one- η exchange can be helpful in binding the heavy molecular pentaquarks.

For the one- σ -exchange (OSE) and one- ω -exchange (OOE) models, they have been always considered together with the other bosons (π, η, ρ) in heavy molecular states, and their effect has been submerged by the effect of the one- π -exchange model. Thus, their importance is often overlooked. Therefore, the purpose of this paper is to study systematically the role of the OSE and OOE interactions between heavy hadrons. The coupling strengths and form factors are estimated by using the quark model, where the

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sigma and omega mesons couple to light quarks in heavy hadrons. Then, we investigate if the intermediate- and short-range forces, due to the OSE and OOE models, can be strong enough to form heavy molecules, by varying model parameters within a reasonable range.

To elucidate the role of the σ and ω mesons, we consider the systems, where π, η, ρ meson exchanges are suppressed, by using the spin and isospin conservation. For instance, there is no coupling $\pi\Lambda_c\Lambda_c$ and πDD . The pion couples rather in the transitions such as $\pi\Lambda_c\Sigma_c$ and πDD^* , which leads to the coupled channel problem. In our present study, we focus exclusively on the σ and ω mesons exchange by ignoring such coupled channels. Then, the $DD, {}^1\Lambda_c D$, and $\Lambda_c\bar{\Lambda}_c$ systems are the ones that we study in this paper. We then compare the properties of those systems where the σ and ω mesons couple differently depending on the numbers of light quarks and antiquarks in the relevant hadrons.

This paper is organized as follows. After the Introduction, we derive the one-boson exchange (OBE) effective potentials in Sec. II. In Sec. III, we present the corresponding numerical results. Then, according to these conclusions, heavy-quark mass dependence is studied by varying it continuously in Sec. IV. The paper ends with a summary in Sec. V.

II. INTERACTIONS

A. Lagrangians

According to the heavy-quark symmetry, the OSE and OOE Lagrangians are constructed as

$$\mathcal{L}_{DD\sigma/\omega} = -2g_\sigma DD^\dagger\sigma + 2g_\omega DD^\dagger\mathbf{v}\cdot\boldsymbol{\omega}, \quad (1)$$

$$\mathcal{L}_{\Lambda_c\Lambda_c\sigma/\omega} = -2g'_\sigma\bar{\Lambda}_c\Lambda_c\sigma - 2g'_\omega\bar{\Lambda}_c\Lambda_c\mathbf{v}\cdot\boldsymbol{\omega}. \quad (2)$$

Here, \mathbf{v} is the four velocity, which has the form of $\mathbf{v} = (1, \mathbf{0})$.

The coupling constants in Eqs. (1) and (2) will be determined in the quark model. Since the σ and ω mesons couple dominantly to the light quarks, the relevant interaction Lagrangian for the light quarks ($q = u, d$) with σ/ω can be expressed as

$$\mathcal{L}_{qq\sigma/\omega} = -g_\sigma^q\bar{\psi}\sigma\psi - g_\omega^q\bar{\psi}\boldsymbol{\gamma}\boldsymbol{\omega}\psi. \quad (3)$$

Compared with the vertices of $D-D-\sigma/\omega$, $\Lambda_c-\Lambda_c-\sigma/\omega$, and $q-q-\sigma/\omega$, all the coupling constants in Eqs. (1)–(3) can be related, i.e.,

¹The ρ exchange can be also unsuppressed for the DD system. Here, it is considered to contrastively discuss the relation of the effective potentials from the OSE and OOE model.

²We also notice there are several former works on the $\Lambda_c\Lambda_c(\bar{\Lambda}_c)$ interactions [22–24].

TABLE I. Scattering amplitudes for all the investigated systems. Here, function $\mathcal{H}(\mathbf{q}, m)$ is defined as $\mathcal{H}(\mathbf{q}, m) = 1/(q^2 + m^2)$.

$h_1 h_2 \rightarrow h_3 h_4$	$\mathcal{M}(h_1 h_2 \rightarrow h_3 h_4)$
$DD \rightarrow DD$	$4M_D^2 [g_\sigma^2 \mathcal{H}(\mathbf{q}, m_\sigma) - g_\omega^2 \mathcal{H}(\mathbf{q}, m_\omega)]$
$D\bar{D} \rightarrow D\bar{D}$	$4M_D^2 [g_\sigma^2 \mathcal{H}(\mathbf{q}, m_\sigma) + g_\omega^2 \mathcal{H}(\mathbf{q}, m_\omega)]$
$\Lambda_c \bar{D} \rightarrow \Lambda_c \bar{D}$	$8M_D M_{\Lambda_c} \chi_3^\dagger \chi_1 [g_\sigma g'_\sigma \mathcal{H}(\mathbf{q}, m_\sigma) - g_\omega g'_\omega \mathcal{H}(\mathbf{q}, m_\omega)]$
$\Lambda_c D \rightarrow \Lambda_c D$	$8M_D M_{\Lambda_c} \chi_3^\dagger \chi_1 [g_\sigma g'_\sigma \mathcal{H}(\mathbf{q}, m_\sigma) + g_\omega g'_\omega \mathcal{H}(\mathbf{q}, m_\omega)]$
$\Lambda_c \Lambda_c \rightarrow \Lambda_c \Lambda_c$	$16M_{\Lambda_c}^2 \chi_3^\dagger \chi_4^\dagger \chi_1 \chi_2 [g_\sigma g'_\sigma \mathcal{H}(\mathbf{q}, m_\sigma) - g_\omega g'_\omega \mathcal{H}(\mathbf{q}, m_\omega)]$
$\Lambda_c \bar{\Lambda}_c \rightarrow \Lambda_c \bar{\Lambda}_c$	$16M_{\Lambda_c}^2 \chi_3^\dagger \chi_4^\dagger \chi_1 \chi_2 [g_\sigma g'_\sigma \mathcal{H}(\mathbf{q}, m_\sigma) + g_\omega g'_\omega \mathcal{H}(\mathbf{q}, m_\omega)]$

$$g_\sigma = g'_\sigma = g_\sigma^q, \quad g_\omega = g'_\omega = g_\omega^q. \quad (4)$$

In a σ model [25], the value of g_σ^q is taken as $g_\sigma^q = 3.65$. For the ω coupling g_ω^q , it is of a little uncertainty; in the Nijmegen model, $g_\omega^q = 3.45$, whereas it is equal to 5.28 in the Bonn model [26]. In Ref. [27], g_ω^q was roughly assumed to be 3.00. In the following calculation, all the possible choices will be employed.

According to the effective Lagrangians in Eqs. (1) and (2), all the relevant OBE scattering amplitudes can be collected in Table I.

Here, for the derivation of effective potentials of the $D\bar{D}$, $\Lambda_c \bar{D}$, and $\Lambda_c \bar{\Lambda}_c$ systems, the G-parity rule [28] is adopted, which relates the scattering amplitudes between the processes $a + b \rightarrow c + d$ and $a + \bar{b} \rightarrow c + \bar{d}$ by exchanging one light meson.

With the help of the Breit approximation, a relation between the effective potentials in momentum space and the scattering amplitudes is obtained, i.e.,

$$\mathcal{V}_E(\mathbf{q}) = -\frac{\mathcal{M}(h_1 h_2 \rightarrow h_3 h_4)}{\sqrt{\prod_i 2M_i \prod_f 2M_f}}. \quad (5)$$

Here, $\mathcal{M}(h_1 h_2 \rightarrow h_3 h_4)$ is defined as the scattering amplitude of the process $h_1 h_2 \rightarrow h_3 h_4$. M_i and M_f are the masses of the initial states (h_1, h_2) and final states (h_3, h_4), respectively.

B. Form factors

The effective potential in the coordinate space $\mathcal{V}(r)$ is obtained by performing the Fourier transformation as

$$\mathcal{V}_E(\mathbf{r}) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \mathcal{V}_E(\mathbf{q}) \mathcal{F}^2(q^2). \quad (6)$$

In order to manipulate the off shell effect of the exchanged mesons σ and ω and finite size effect of the interacting hadrons, we introduce a form factor $\mathcal{F}(q^2)$ at every vertex.

Generally, the form factor has the monopole, dipole, and exponential forms

$$\begin{aligned}\mathcal{F}_M(q^2) &= \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}, \\ \mathcal{F}_D(q^2) &= \frac{(\Lambda^2 - m^2)^2}{(\Lambda^2 - q^2)^2}, \\ \mathcal{F}_E(q^2) &= e^{(q^2 - m^2)/\Lambda^2}.\end{aligned}$$

Here, Λ , m , and q correspond to the cutoff, mass, and momentum of the exchanged meson, respectively. These three kinds of form factors are normalized at the on shell momentum of $q^2 = m^2$. In the low momentum limit, these form factors may be related to each other by redefining the cutoff parameter Λ such that the first terms of the Taylor expansion in powers of q^2/Λ^2 coincide. In this way, low momentum phenomena of hadronic molecules do not depend very much on different choices of form factors.

The form factor can not be uniquely determined and various forms and cutoff Λ are used phenomenologically. However, an intuitive guideline for the choice of Λ is done by relating it to the size of hadrons. In Refs. [17,18], Λ is related to the root-mean-square radius of the source hadron to which the exchanged boson (σ or ω) couples. According to the previous experience of the deuteron, the cutoff Λ in covariant-type monopole form factor is taken around 1 GeV. In the present qualitative study, we use the same form factor both for meson and baryon vertices, because both of them contain light quarks and their spatial distributions are of order 1 fm or less.

C. Effective potentials

In this subsection, we adopt the monopole form factor $\mathcal{F}_M(q^2)$, and the resulting effective potentials for the investigated systems are collected in Table II.

In Table II, we can find that the interactions from the OSE model are always attractive for these investigated systems. This is a general consequence of the scalar meson exchange with a momentum independent coupling constant, as briefly explained in the next section. The depth of the OSE effective potentials depends on the number of the light quarks and/or antiquark combinations ($q - q$, $q - \bar{q}$, $\bar{q} - \bar{q}$), where the light quark or antiquark is reserved in

TABLE II. Effective potentials for the investigated systems. The function $Y(\Lambda, m, r)$ is defined as $Y(\Lambda, m, r) = (e^{-mr} - e^{-\Lambda r})/4\pi r - (\Lambda^2 - m^2)e^{-\Lambda r}/8\pi\Lambda$.

Systems	Quarks	$\mathcal{V}(r)$
DD	$(c\bar{q})(c\bar{q})$	$-g_\sigma^2 Y(\Lambda, m_\sigma, r) + g_\omega^2 Y(\Lambda, m_\omega, r)$
$D\bar{D}$	$(c\bar{q})(\bar{c}q)$	$-g_\sigma^2 Y(\Lambda, m_\sigma, r) - g_\omega^2 Y(\Lambda, m_\omega, r)$
$\Lambda_c \bar{D}$	$(cqq)(\bar{c}q)$	$-2g_\sigma g_\sigma^q Y(\Lambda, m_\sigma, r) + 2g_\omega g_\omega^q Y(\Lambda, m_\omega, r)$
$\Lambda_c D$	$(cqq)(c\bar{q})$	$-2g_\sigma g_\sigma^q Y(\Lambda, m_\sigma, r) - 2g_\omega g_\omega^q Y(\Lambda, m_\omega, r)$
$\Lambda_c \Lambda_c$	$(cqq)(cqq)$	$-4g_\sigma^2 Y(\Lambda, m_\sigma, r) + 4g_\omega^2 Y(\Lambda, m_\omega, r)$
$\Lambda_c \bar{\Lambda}_c$	$(cqq)(\bar{c}\bar{q}\bar{q})$	$-4g_\sigma^2 Y(\Lambda, m_\sigma, r) - 4g_\omega^2 Y(\Lambda, m_\omega, r)$

different hadrons of the hadron-hadron systems, respectively. For example, according to the quark configurations as shown in the second column of the Table II, the light $\bar{q} - \bar{q}$ combination for the DD system is one, and there is only one $q - \bar{q}$ combination in the $D\bar{D}$ system.

Since $g_\sigma = g_\sigma^q$ as estimated in the quark model (4), a simple relation between the OSE effective potentials and the light-quark and/or antiquark combination numbers can be summarized as

$$\mathcal{V}_\sigma(x_{qq/\bar{q}\bar{q}}, y_{q\bar{q}}) = -(x_{qq/\bar{q}\bar{q}} + y_{q\bar{q}})g_\sigma^2 Y(\Lambda, m_\sigma, r), \quad (7)$$

where $x_{qq/\bar{q}\bar{q}}$ and $y_{q\bar{q}}$ correspond to the numbers of $qq/\bar{q}\bar{q}$ and $q\bar{q}$ ($q = u, d$) combinations, respectively. For the OOE effective potentials, a similar relation can be also written as

$$\mathcal{V}_\omega(x_{qq/\bar{q}\bar{q}}, y_{q\bar{q}}) = (x_{qq/\bar{q}\bar{q}} - y_{q\bar{q}})g_\omega^2 Y(\Lambda, m_\omega, r). \quad (8)$$

Here, we note that the sign of the OOE changes according to the charge conjugation symmetry. For example, the OOE force is repulsive for the $q - q$ and $\bar{q} - \bar{q}$ combinations, while reversed for the system with $q - \bar{q}$ combination.

For example, for the $\Lambda_c D$ system, there are two $q - \bar{q}$ combinations, thus $y_{q\bar{q}} = 2$, and its potential from σ and ω exchanges is

$$\mathcal{V}_{\Lambda_c D}(r) = -2g_\sigma^2 Y(\Lambda, m_\sigma, r) - 2g_\omega^2 Y(\Lambda, m_\omega, r). \quad (9)$$

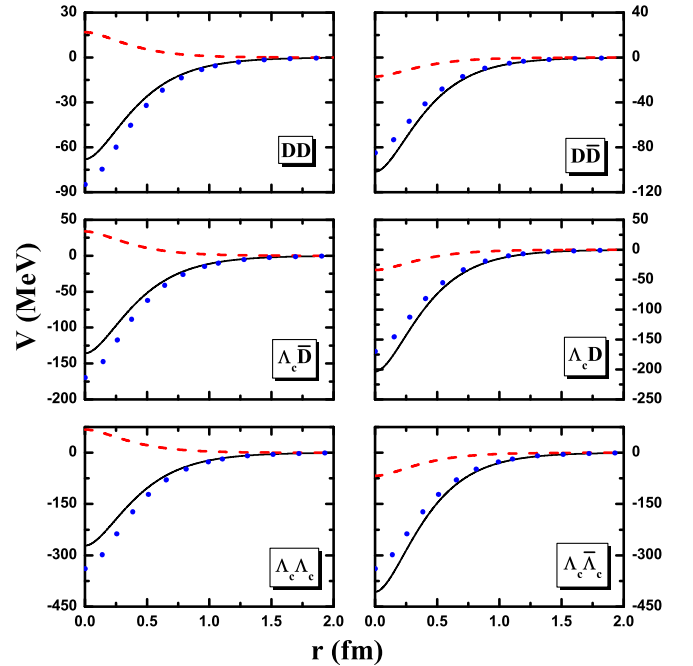


FIG. 1. Effective potentials for the $DD(\bar{D})$, $\Lambda_c D(\bar{D})$, and $\Lambda_c \Lambda_c(\bar{\Lambda}_c)$ systems with cutoff value $\Lambda = 1.00$ GeV, $g_\sigma^q = 3.65$, and $g_\omega^q = 3.00$. Here, the dotted, dashed, and solid lines correspond to the OSE, OOE, and total effective potentials, respectively.

In Fig. 1, we present the resulting potential as functions of the distance r , where the total potential is shown by the solid line, OSE by dotted lines, and OOE by dashed lines.

Here, the OSE and the OOE forces are of typical character of intermediate- and short-range force, and therefore they are suppressed when the radius r reaches 1 fm and larger. Since the force from the OSE model is the dominant, the total effective potentials for all the investigated systems are all attractive.

To summarize shortly, the OSE model can always provide an attractive force. However, the OOE force is repulsive for the system including the same light quarks or antiquarks in its components of the investigated systems. The interaction strength from the OSE and OOE models depends on the light-quark combination numbers.

III. NUMERICAL RESULTS

In this section, we discuss the role of the OSE and OOE interaction for the systems of $\Lambda_c D(\bar{D})$ and $\Lambda_c \Lambda_c(\bar{\Lambda}_c)$ by solving the Schrödinger equation for them

$$-\frac{1}{2M}\nabla^2\psi(r) + V(r)\psi(r) = E\psi(r), \quad (10)$$

where $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}$, and $M = m_1 m_2 / (m_1 + m_2)$ is the reduced mass for the investigated system composed by

TABLE III. Parameters adopted in this work [29].

Hadron	$I(J^P)$	Mass (MeV)	Hadron	$I(J^P)$	Mass (MeV)
K	$\frac{1}{2}(0^-)$	495.64	Λ	$0(\frac{1}{2}^+)$	1115.683
D	$\frac{1}{2}(0^-)$	1867.24	Λ_c	$0(\frac{1}{2}^+)$	2286.46
B	$\frac{1}{2}(0^-)$	5279.42	Λ_b	$0(\frac{1}{2}^+)$	5619.4
σ	$0(0^+)$	600	ω	$0(1^-)$	782.65

TABLE IV. Bound solutions for the S-wave $\Lambda_c D(\bar{D})$ and $\Lambda_c \Lambda_c(\bar{\Lambda}_c)$ systems. Here, the monopole form factor $\mathcal{F}_M(q^2) = (\Lambda^2 - m^2)/(\Lambda^2 - q^2)$ is adopted. The units for cutoff Λ , binding energy E , and root-mean-square radius r_{RMS} are GeV, MeV, and fm, respectively. Cases I, II, and III correspond to the numerical results by adopting the coupling constant g_ω^q taken with the value $g_\omega^q = 3.00$ in Ref. [27], 3.45 in the Nijmegen model, and 5.28 in the Bonn model [26], respectively. The notation ... stands for no bound solutions.

Cases	$\Lambda_c \bar{D}$			$\Lambda_c D$			$\Lambda_c \Lambda_c$			$\Lambda_c \bar{\Lambda}_c$		
	Λ	E	r_{RMS}	Λ	E	r_{RMS}	Λ	E	r_{RMS}	Λ	E	r_{RMS}
I	1.00	1.00	-1.61	2.79	1.00	-17.47	1.06	1.00	-50.49	0.73
	1.10	-0.04	6.27	1.10	-11.09	1.26	1.10	-31.70	0.85	1.10	-110.39	0.55
	1.20	-0.90	3.53	1.20	-27.66	0.89	1.20	-46.84	0.74	1.20	-187.32	0.46
II	1.00	1.00	-2.41	2.35	1.00	-13.52	1.16	1.00	-56.99	0.70
	1.10	1.10	-14.40	1.14	1.10	-22.74	0.96	1.10	-126.02	0.53
	1.20	1.20	-34.85	0.82	1.20	-31.46	0.85	1.20	-215.32	0.44
III	1.00	1.00	-9.32	1.36	1.00	-0.60	4.01	1.00	-97.49	0.58
	1.10	1.10	-38.72	0.80	1.10	1.10	-222.80	0.44
	1.20	1.20	-85.10	0.60	1.20	1.20	-387.71	0.36

particle 1 and particle 2. The parameters we use are summarized in Table III.

A. Solutions with covariant-type monopole form factor $\mathcal{F}_M(q^2) = (\Lambda^2 - m^2)/(\Lambda^2 - q^2)$

We summarize the properties of S-wave bound states when they exist and the binding energy and root-mean-square radii (r_{RMS}) for the S-wave $\Lambda_c D(\bar{D})$ and $\Lambda_c \Lambda_c(\bar{\Lambda}_c)$ systems in Table IV. For the coupling constant g_ω^q , we use three values (3.00 for Case I, 3.45 for II, and 5.28 for III) corresponding to the g_ω^q coupling constants of Ref. [27], of the Nijmegen model, and of the Bonn model [26], respectively. In Table IV, the cutoff parameters are chosen as 1, 1.1, and 1.2 GeV. These are the typical values for the form factor $\mathcal{F}_M(q^2)$ [17,18]. In fact, depending on detailed values of Λ and on channels, bound states may or may not appear. In this way, we discuss whether bound states appear or not and study the role of the σ and ω meson exchanges.

Before discussing details of Table IV, we make general remarks for boson exchange potentials.

- (i) The σ meson exchange provides attractive interaction. This is understood using a second-order perturbation theory for the one-boson exchange; the intermediate three particle state with σ meson has a virtual energy that is larger than the initial (or final) energy of the two particles. Moreover, due to the positive charge conjugation of the Lorentz scalar charge that the σ meson couples to, the signs of the couplings for both quark and antiquark are the same. This explains the universally attractive nature of the σ meson exchange.
- (ii) In comparison with the σ exchange, the ω meson couples to the baryonic charge, which flips its sign for quark and antiquark. This provides a repulsive interaction between quarks and attractive interaction between the quark and antiquark.

- (iii) The role of Λ is to suppress the interaction strength for larger momentum transfer and thus effectively reduce the strength of the interaction for bound states. As we will see, the results depend very much on the choice of the form factor.

For $\Lambda_c \bar{D}$, the interaction is the sum of attractive OSE and repulsive OOE, the total effective potential is attractive. As Λ is increased, the OSE becomes more prominent and a bound state appears for $\Lambda > 1.1$ GeV for case I. For cases II and III, because of slightly stronger ω exchange repulsion, we do not find any bound states. These are the results for S waves. For higher partial waves, due to the repulsive centrifugal force $l(l+1)/2Mr^2$, it is less likely to have bound states. Thus, we may conclude that, in our model with a reasonable $\Lambda \sim 1$ GeV, hidden-charm molecular pentaquarks made up by $\Lambda_c \bar{D}$ are not likely to exist. Indeed, if we increase Λ larger than 1 GeV when more attraction is expected, we do not yet find bound states or do, at most, very weakly bound states only for case I. Experimentally, our conclusion for the $\Lambda_c \bar{D}$ system is consistent with the current results of LHCb [6], where no obvious evidence of possible partners of $P_c(4380)$ and $P_c(4450)$ has been reported, in the region close to the mass of $\Lambda_c \bar{D}$.

As compared to the $\Lambda_c \bar{D}$ system, the OOE force for the $\Lambda_c D$ system is attractive, as explained above. Together with the attractive OSE force, the net attractive force for the $\Lambda_c D$ turns out to be strong enough to accommodate bound states. As shown in Table IV, for the cutoff $\Lambda \sim 1$ GeV, we find a shallow bound state with a binding energy around several MeV. Therefore, this channel may provide a good candidate of a loosely bound molecular state of the $\Lambda_c D$ system with $|^2S_{\frac{1}{2}}\rangle$. Since D and Λ_c are the lowest ground hadrons of the charmed mesons and baryons, its possible strong decay channel should be rather limited, like $\Xi_{cc}(\frac{1}{2}^+) + \pi/\eta$.

For the heavy baryon systems $\Lambda_c \Lambda_c(\bar{\Lambda}_c)$, since one more light quark (antiquark) is in the baryon $\Lambda_c(\bar{\Lambda}_c)$, the interaction strength becomes two times stronger than that in the $\Lambda_c \bar{D}$ and $\Lambda_c D$ systems. Therefore, as shown in Table IV, more bound state solutions have been found both for $\Lambda_c \Lambda_c$ and $\Lambda_c \bar{\Lambda}_c$ systems than for $\Lambda_c \bar{D}$ and $\Lambda_c D$ systems. With the same cutoff input, their binding energies reach several tens MeVs. Thus, they can be also possible molecular candidates. For their decay behaviors, the $\Xi_{cc}(\frac{1}{2}^+)N$ can be the only strong decay channel for the S-wave $\Lambda_c \Lambda_c$ bound state. The decay processes will be much more complicated for the S-wave $\Lambda_c \bar{\Lambda}_c$ molecular state, as they include open-charm and hidden-charm channels, like $\chi_{c0} + \pi\pi$, $D\bar{D}_1 + \pi$, and so on.

B. Solutions with noncovariant-type monopole form factor $\mathcal{F}_M(q^2) = \Lambda^2/(\Lambda^2 - q^2)$

So far, we have employed a covariant monopole form factor and discussed the role of OSE and OOE potentials, with some predictions for molecular candidates, the S-wave $\Lambda_c D$, $\Lambda_c \Lambda_c$, and $\Lambda_c \bar{\Lambda}_c$. In order to further see our discussions, in the following, we attempt to use a three-momentum form factor of the form of $\mathcal{F}(q^2) = \Lambda^2/(\Lambda^2 - q^2)$, which is often adopted in nuclear physics.

In the nonrelativistic kinematics, the energy transfer is neglected, and so this condition reduces to the condition of vanishing three momentum. In fact, the difference of this form factor from the monopole form factor is absorbed into the redefinition of the coupling constants as Eqs. (11) and (12)

$$f_\sigma = f'_\sigma = \left(1 - \frac{m_\sigma^2}{\Lambda^2}\right)g_\sigma = \left(1 - \frac{m_\sigma^2}{\Lambda^2}\right)g'_\sigma, \quad (11)$$

TABLE V. Bound solutions for the S-wave $\Lambda_c D(\bar{D})$ and $\Lambda_c \Lambda_c(\bar{\Lambda}_c)$ systems. Here, the form factor $\mathcal{F}(q^2) = \Lambda^2/(\Lambda^2 - q^2)$ is adopted. The units for cutoff Λ , binding energy E , and root-mean-square radius r_{RMS} are GeV, MeV, and femtometer, respectively. Cases I, II, and III correspond to the numerical results by adopted the coupling constant g_ω^q taken the value $g_\omega^q = 3.00$ in Ref. [27], 3.45 in the Nijmegen model, and 5.28 in the Bonn model [26], respectively. The notation ... stands for no bound solutions.

Cases	$\Lambda_c \bar{D}$			$\Lambda_c D$			$\Lambda_c \Lambda_c$			$\Lambda_c \bar{\Lambda}_c$		
	Λ	E	r_{RMS}	Λ	E	r_{RMS}	Λ	E	r_{RMS}	Λ	E	r_{RMS}
I	0.40	0.40	-6.94	1.81	0.40	-4.08	2.14	0.40	-45.87	0.95
	0.50	0.50	-16.55	1.28	0.50	-9.51	1.51	0.50	-88.07	0.73
	0.60	-0.15	5.79	0.60	-30.66	1.00	0.60	-17.20	1.20	0.60	-144.37	0.60
II	0.40	0.40	-9.38	1.63	0.40	-1.14	3.45	0.40	-54.98	0.90
	0.50	0.50	-21.48	1.17	0.50	-3.31	2.23	0.50	-104.66	0.69
	0.60	0.60	-38.96	0.92	0.60	-6.57	1.69	0.60	-170.72	0.56
III	0.40	0.40	-26.97	1.15	0.40	0.40	-112.02	0.72
	0.50	0.50	-55.46	0.86	0.50	0.50	-207.12	0.56
	0.60	0.60	-94.71	0.69	0.60	0.60	-331.99	0.46

$$f_\omega = f'_\omega = \left(1 - \frac{m_\omega^2}{\Lambda^2}\right)g_\omega = \left(1 - \frac{m_\omega^2}{\Lambda^2}\right)g'_\omega. \quad (12)$$

If we use the same coupling constants and cutoff Λ , the interaction strengths are larger when the three-dimensional form factor is employed. Therefore, to obtain loosely bound molecular states, we need to use smaller cutoff Λ when the coupling constants are kept unchanged. This is the reason that we show the results in Table V with smaller cutoff Λ .

In order to determine the value of cutoff in noncovariant-type monopole form factor, here, we recall the relation,

$$\langle r^2 \rangle = -6 \frac{\partial \mathcal{F}(q^2)}{\partial q^2} \Big|_{q^2 \rightarrow 0} \approx \frac{6}{\Lambda^2}. \quad (13)$$

If the form factor $\mathcal{F}(q^2)$ is introduced, in practice, the resulting cutoff parameter for \mathcal{F} turns out to be around 0.5 GeV as we discussed around Eq. (7), consistent with typical hadronic size. The results are shown in Table V for $\Lambda \sim 0.4, 0.5$, and 0.6 GeV.

Compared with the numerical results in Table IV, one can find that, if we take a value of $\Lambda = 0.5$ GeV, which is estimated by Eq. (13), the results in Table V are very similar to those in Table IV. Having these results together with those of different form factors, we find that the intermediate-range and short-range force from OSE and OOE models provides a strong attraction to generate bound states.

Finally, let us give a brief conclusion, where we show the results with the two form factors \mathcal{F}_M ($\Lambda \sim 1$ GeV) in Table IV and those with \mathcal{F} ($\Lambda \sim 0.5$ GeV) in Table V. To be seen shortly, these results are qualitatively similar but have some differences quantitatively. The latter indicates uncertainties of the present model calculations. Nevertheless, we can predict several possible candidates for molecular states, S-wave $\Lambda_c D$, $\Lambda_c \Lambda_c$, and $\Lambda_c \bar{\Lambda}_c$ states.

IV. EXTENSION

A. Mass dependence

In addition to the effective potentials, the mass in the kinetic term is another important input for the discussion of bound states. In fact, in the heavy-quark limit ($M \rightarrow \infty$) as the kinetic energy $p^2/2M$ vanishes, hadrons will be more easily bound. In the following, we study the reduced mass dependence of the molecular systems.

The upper panel of Fig. 2 shows the binding energies of the $\Lambda_c \bar{D}$ -like (dashed line) and $\Lambda_c D$ -like (solid line) states, where the reduced mass of the two hadrons is varied as in the horizontal axis. The vertical dotted lines correspond to the reduced masses of the two hadrons, as indicated in the figure. The solid line stands for the binding energies of the $\Lambda_c D$ -like state; it starts to appear when the reduced mass becomes larger than ~ 0.75 GeV, and as expected, the binding energy increases as the reduced mass is increased. For the $\Lambda_c \bar{D}$ -like state, the OOE potential is repulsive,

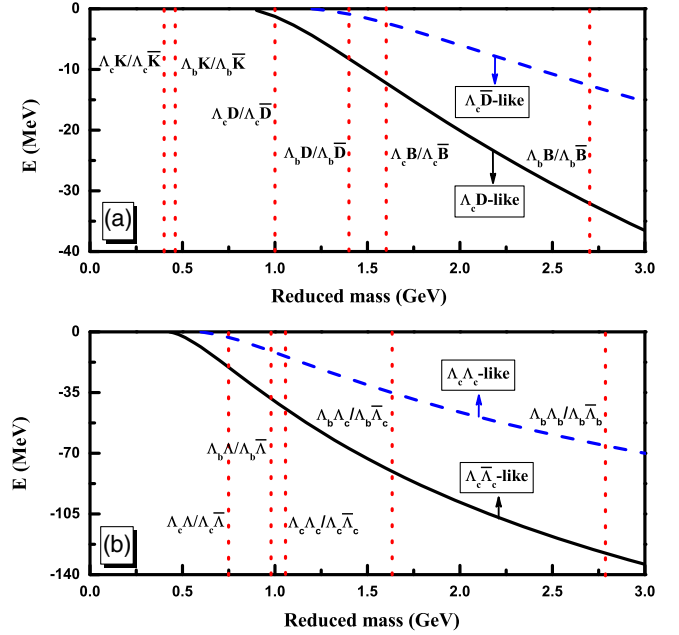


FIG. 2. Binding energies as functions of the reduced mass. Here, $\Lambda = 1.00$ GeV, $g_\sigma^q = 3.65$, and $g_\omega^q = 3.00$. The dotted lines stand for the reduced masses for several hadron-hadron systems as indicated.

resulting in less attractive potential than for the $\Lambda_c D$ -like state. Thus, the system allows weaker binding as the dashed line shows. The lower panel shows similar results for the $\Lambda_c \Lambda_c$ - and $\Lambda_c \bar{\Lambda}_c$ -like states. Because these systems have a larger attraction as proportional to the number of the light quarks as compared with the $\Lambda_c D$ and $\Lambda_c \bar{D}$ ones, larger binding energies are obtained.

When the reduced mass is sufficiently heavy, as in the charm and bottom regions, but not in the strange regions, binding energies for $\Lambda_c D$ and $\Lambda_c \bar{D}$ systems reach several to several tens MeV. Thus, the heavy flavors of charm and bottom are important in stabilizing pentaquark hadronic molecules. For the $\Lambda_c \Lambda_c$ ($\bar{\Lambda}_c$)-like systems, with stronger OSE and OOE interactions, more bound solutions are obtained even in the strangelike section. Therefore, the heavy dibaryon molecules can be more stable than the heavy pentaquark molecules. These results suggest that searching for the heavy dibaryon molecules is very promising in experiments.

B. $\Lambda_c N$ and $\Lambda_c \bar{N}$ systems

In this subsection, let us study $\Lambda_c N$ (\bar{N}) systems. In fact, the $\Lambda_c N$ (\bar{N}) interactions have been investigated [30–32]. In particular, a very shallow bound state was found for the S-wave $\Lambda_c N$ system [33]. According to Eqs. (7) and (8), the total effective potentials for $\Lambda_c N$, and $\Lambda_c \bar{N}$ systems are written as

$$\mathcal{V}_{\Lambda_c N}(r) = -6g_\sigma^2 Y(\Lambda, m_\sigma, r) + 6g_\omega^2 Y(\Lambda, m_\omega, r), \quad (14)$$

TABLE VI. Bound solutions for the S-wave $\Lambda_c N$, $\Lambda_c \bar{N}$, $\Lambda_b N$, and $\Lambda_b \bar{N}$ systems. Here, the parameters are taken as $g_\sigma^q = 3.65$, $g_\omega^q = 3.00$. The units for cutoff Λ , binding energy E , and root-mean-square radius r_{RMS} are GeV, MeV, and femtometer, respectively.

	$\Lambda_c N$		$\Lambda_c \bar{N}$	
Λ	0.9	1.0	0.9	1.0
E	-4.28	-17.84	-11.92	-59.97
r_{RMS}	2.25	1.28	1.50	0.82
	$\Lambda_b N$		$\Lambda_b \bar{N}$	
Λ	0.9	1.0	0.9	1.0
E	-10.38	-29.84	-21.57	-82.28
r_{RMS}	1.47	0.99	1.12	0.70

$$\mathcal{V}_{\Lambda_c \bar{N}}(r) = -6g_\sigma^2 Y(\Lambda, m_\sigma, r) - 6g_\omega^2 Y(\Lambda, m_\omega, r). \quad (15)$$

The bound solutions for the S-wave $\Lambda_{c,b} N(\bar{N})$ systems are summarized in Table VI. When we take the cutoff around 1 GeV, their binding energy reaches a few to several tens MeV, and their root-mean-square radii are around 1 fm. This means that they also can be possible molecular candidates. For $\Lambda_c N$ and $\Lambda_b N$ states, if they form bound states, they are stable under the strong interaction, while the $\Lambda_c \bar{N}$ and $\Lambda_b \bar{N}$ states can decay to charmed/antibottomed meson and the light mesons, like $D\pi\pi$ and $\bar{B}\pi\pi$.

V. CONCLUSION AND DISCUSSION

Stimulated by the observation of $X/Y/Z/P_c$ states near threshold, the study of the hadronic molecular picture becomes more and more essential. For the study of molecular states, it is essentially important to describe the interaction between the hadrons of the molecular system. For this purpose, currently, the most available approach is the one-boson-exchange model based on the knowledge of light quark and boson interactions, which is applied to the system of open heavy hadrons containing light quarks. In this work, we systematically study the properties of the interaction from the one- σ/ω -exchange model. The $\Lambda_c D(\bar{D})$, $\Lambda_c \Lambda_c(\bar{\Lambda}_c)$ systems have been taken into consideration. Meanwhile, all the parameters are estimated by the quark model.

In fact, the intermediate- and short-range interactions between hadrons are from a many pions exchange process. Here, σ and ω exchanges are adopted to approximately replace two and three pion exchanges interactions, respectively. Compared to a pion, σ meson is of uncertain mass and wide width, which affects the strength of the σ -exchange interaction. In the limit of small momentum transfer, the effective potential from σ exchange is proportional to the term of g_σ^2/m_σ^2 . Therefore, the uncertainty in the mass and wide width may be absorbed into the redefinition of the coupling constant. In nuclear physics, the mass of σ is often taken as a fixed input parameter to fit the phase shift of nucleon-nucleon interaction [34,35].

By working out suitable coupling constants and form factors, we find that in many cases the sum of OSE and OOE models becomes attractive, where the OSE plays the dominant role. The OSE force is always attractive and the dominant. Whereas, for the OOE force, it is repulsive when there exists $q - q (q = u, d)$ or $\bar{q} - \bar{q}$ combinations in the two hadrons system. The OSE and OOE interaction strength depends on the number of $q - q$, $\bar{q} - \bar{q}$, and $q - \bar{q}$ combinations. With reasonable inputs for the cutoff parameter for the form factor, we find that the interaction of the OSE and OOE models provides attraction, which may form the heavy molecular states, like the S-wave $\Lambda_c D$, $\Lambda_c \Lambda_c$, and $\Lambda_c \bar{\Lambda}_c$ states.

As a by-product, we also discuss the mass dependence of the binding energy. We have explicitly shown that heavier systems are more likely to accommodate various molecular states such as S-wave $\Lambda_b D(\bar{D})$, $\Lambda_c B(\bar{B})$, $\Lambda_b B(\bar{B})$, $\Lambda_c \Lambda(\bar{\Lambda})$, $\Lambda_b \Lambda(\bar{\Lambda})$, $\Lambda_b \Lambda_c(\bar{\Lambda}_c)$, and $\Lambda_b \Lambda_b(\bar{\Lambda}_b)$ states. Finally, the interaction between a heavy baryon $\Lambda_{c,b}$ and one nucleon has been investigated. In our calculation, there can exist S-wave $\Lambda_c N(\bar{N})$ and $\Lambda_b N(\bar{N})$ molecular states.

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