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Lepton mass effects and angular observables in $\Lambda_b \to \Lambda(\to p\pi)\ell^+\ell^-$

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The flavor-changing rare decay $B \to K^*(\to K\pi)\ell^+\ell^-$ is one of the most studied modes due to its sensitivity to physics beyond the standard model, and several discrepancies have come to light among the plethora of observables that are measured. In this paper, we revisit the analogous baryonic decay mode $\Lambda_b \to \Lambda(\to p\pi)\ell^+\ell^-$, and we present a complete set of ten angular observables that can be measured using this decay mode. Our calculations are done while retaining the finite lepton mass so that the signal of lepton non-universality observed in $B \to K^*\ell^+\ell^-$ can be corroborated by the corresponding baryonic decay mode. We show that due to the parity-violating nature of the subsequent $\Lambda \to p\pi$ decay, there exists at least one angular asymmetry that is nonvanishing in the large-recoil limit, unlike the case in $B \to K^*\ell^+\ell^-$ decay mode, making it particularly sensitive to new physics that violates lepton flavor universality.

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I. INTRODUCTION

It is well known that the rare decay $B \to K^*\ell^+\ell^-$ involves a $b \to s$ flavor-changing loop-induced transition at the quark level, making it sensitive to physics beyond the standard model (SM) [1–14]. The nature of this decay provides one with a significant number of observables, many of which have been recently measured [15,16] to a great deal of accuracy. There are several discrepancies observed when compared to the SM predictions; among these, $R_{K^{(*)}}$, the ratio of the differential decay rate $d(B \to K^{(*)}\ell^+\ell^-)/dq^2$, for $\ell = \mu$ and e, has generated a great deal of interest. The deviation of $R_{K^{(*)}}$ from the expected value in the SM implies a challenge to the idea of lepton universality [17] within the SM and points towards possible evidence of new physics (NP). Naturally, the question arises whether we can observe a similar deviation in other decay modes that capture this non-universal behavior of the leptons. This will go a long way in establishing lepton non-universality on firm footing. Here we reexamine the analogous baryonic decay of Λ_b to Λ and a lepton-antilepton pair, where the Λ baryon further decays to a proton p^+ and a pion π^-, as already discussed by various authors [18–46]. The underlying quark-level $b \to s\ell^+\ell^-$ transition for $\Lambda_b \to \Lambda\ell^+\ell^-$ decay is the same as in the well-studied $B \to K^{(*)}\ell^+\ell^-$ decay, making it an ideal candidate to study in depth.

Before we study the consequences of lepton non-universality in baryonic decay $\Lambda_b \to \Lambda(\to p\pi)\ell^+\ell^-$, we recall that $R_{K^{(*)}}$ is defined [47] within a given range of the dilepton mass squared, $q^2_{\rm min}$ to $q^2_{\rm max}$, as

$$R_{K^{(*)}} = \frac{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{d\Gamma(B\to K^{(*)}\mu^+\mu^-)}{dq^2} dq^2}{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{d\Gamma(B\to K^{(*)}e^+e^-)}{dq^2} dq^2}. \tag{1}$$

The measured R_K and R_{K^*} lie systematically below the SM expectations [48,49]:

$$R_K(q^2 \in [1:6]\ {\rm GeV}^2) = 0.745^{+0.090}_{-0.074} \pm 0.036,$$
$$R_{K^*}(q^2 \in [0.045:1.1]\ {\rm GeV}^2) = 0.660^{+0.110}_{-0.070} \pm 0.024,$$
$$R_{K^*}(q^2 \in [1.1:6]\ {\rm GeV}^2) = 0.685^{+0.113}_{-0.069} \pm 0.047.$$

In the SM, both R_K and R_{K^*} are predicted to be virtually indistinguishable from unity [50] for $(q^2 \in [1:6]\ {\rm GeV}^2)$, whereas $R_{K^*} \sim 0.9$ for $q^2 \in [0.045:1.1]\ {\rm GeV}^2$, owing to a finite m_μ. The measurements correspond to 2.6σ, 2.1σ, and 2.4σ shortfalls from the SM expectations, respectively.

It is obvious that an observable R_Λ can be proposed in the same spirit as $R_{K^{(*)}}$ for the corresponding baryonic decay $\Lambda_b \to \Lambda\ell^+\ell^-$ as

$$R_\Lambda = \frac{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{d\Gamma(\Lambda_b\to\Lambda\mu^+\mu^-)}{dq^2} dq^2}{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{d\Gamma(\Lambda_b\to\Lambda e^+e^-)}{dq^2} dq^2}. \tag{2}$$

One should expect the lepton mass effect to play a significant role on R_Λ in the low-q^2 region, just as in the case of $R_{K^{(*)}}$. The discrepancy between the SM expectation and the experimentally observed value of $R_{K^{(*)}}$ was largest in the low-q^2 region. The observation of a similar discrepancy in R_Λ is therefore necessary to substantiate the idea of lepton non-universality in FCNC processes, since such an observation cannot be restricted to the celebrated $B \to K^{(*)}\ell^+\ell^-$ alone. In order to disentangle the new

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physics contribution that may manifest as lepton nonuniversality, one must take into account the SM contribution to R_Λ including the effect of finite lepton mass [51,52]. We therefore derive the expression for R_Λ without any approximation.

Another salient feature of the $\Lambda_b \to \Lambda(\to p^+\pi^-)\ell^+\ell^-$ decay is the wealth of information carried by the angular observables expressed in terms of the angular asymmetries, of which the forward-backward asymmetry in the hadron angle θ_Λ is of particular interest. We show that due to the parity-violating nature of the $\Lambda \to p\pi$ decay, angular asymmetries are nonvanishing in the large-recoil or low-q^2 limit, unlike the case in $B \to K^*(\to K\pi)\ell^+\ell^-$, making it particularly sensitive to new physics that violates lepton flavor universality. This follows since the ratios of hadronic forward-backward asymmetry for $\ell = \mu$ and $\ell = e$, in the low-q^2 region, are ratios of two finite quantities for $\Lambda_b \to \Lambda(\to p^+\pi^-)\ell^+\ell^-$ decay. It may be recollected that all asymmetries for $B \to K^*\ell^+\ell^-$ decay mode vanish in the low-q^2 region [11].

Our paper is arranged in the following way: In Sec. II, we derive the complete angular distribution consisting of ten angular observables retaining all the helicities and lepton mass. Section III is devoted to the calculation of hadronic helicity amplitudes in terms of known parameters—namely, the Wilson coefficients and form factors. In Sec. IV, the decay rate and angular asymmetries are written in terms of the helicity amplitudes. We also define observables that are free from hadronic uncertainties. Finally, we conclude how these observables can play an important role in pinning down lepton universality violating new physics.

II. THE DECAY OF $\Lambda_b \to \Lambda(\to p^+\pi^-)\ell^+\ell^-$

The process $\Lambda_b \to \Lambda(\to p\pi^-) + j_{\rm eff}(\to \ell^+\ell^-)$ can be thought of as a sequential decay, where it is assumed that the daughter Λ baryon is on shell and subsequently decays resonantly. This enables one to write down a joint angular decay distribution [53–56] which is described fully by four independent kinematic variables: the dilepton invariant mass squared q^2, the polar angles θ_l, θ_Λ, and the azimuthal angle ϕ, defined by the decay products in their respective center-of-mass (c.m.) frames. At this point, we would like to clarify that in our convention we have chosen θ_l to be the angle between the lepton (ℓ^-) and the flight direction of the $j_{\rm eff}$ system, θ_Λ to be the angle between the nucleon (p) and the Λ flight direction, and ϕ to be the angle between the two decay planes. The angular distribution involves the helicity amplitudes $H_{\lambda_\Lambda,\lambda_j}(J)$ for the decay $\Lambda_b \to \Lambda + j_{\rm eff}$, $h^a_{\lambda_1,\lambda_2}$ for the decay $j_{\rm eff} \to \ell^+\ell^-$, and $h_{\lambda_p,0}$ for the decay $\lambda \to p + \pi^-$. The joint angular distribution for a unpolarized Λ_b decay is given by

$$\begin{aligned} K(q^2,\theta_l,\theta_\Lambda,\phi) = &\sum_{J,J',M_i=\pm\frac{1}{2},M_i'=\frac{1}{2},\lambda_j,\lambda_j',a,a',\lambda_\Lambda,\lambda_\Lambda',\lambda_p,\lambda_1,\lambda_2} H^a_{\lambda_\Lambda,\lambda_j}(J) H^{a'*}_{\lambda_{\Lambda'},\lambda_j'}(J')\rho_{M_i,M_i'} \\ &\times \mathcal{D}^{\frac{1}{2}}_{M_i,\lambda_\Lambda-\lambda_j}(0,0,0)\mathcal{D}^{*\frac{1}{2}}_{M_i',\lambda_\Lambda'-\lambda_j'}(0,0,0)\delta_{JJ'}h^a_{\lambda_1,\lambda_2}(J)h^{a'*}_{\lambda_1,\lambda_2}(J') \\ &\times \mathcal{D}^J_{\lambda_j,\lambda_1-\lambda_2}(0,\theta_l,0)\mathcal{D}^{*J'}_{\lambda_j',\lambda_1-\lambda_2}(0,\theta_l,0)h_{\lambda_p,0}h^*_{\lambda_p,0} \\ &\times \mathcal{D}^{\frac{1}{2}}_{\lambda_\Lambda,\lambda_p}(-\phi,\theta_\Lambda,\phi)\mathcal{D}^{*\frac{1}{2}}_{\lambda_{\Lambda'},\lambda_p}(-\phi,\theta_\Lambda,\phi). \end{aligned} \tag{3}$$

The polarization density matrix of Λ_b, $\rho_{M_i,M_i'}$ in Eq. (3) is a Hermitian 2×2 matrix, with $\mathrm{Tr}(\rho) = 1$. ρ_{++} and ρ_{--} represent the probabilities that the initial state Λ_b has $M_i = \frac{1}{2}$ and $M_i = -\frac{1}{2}$, respectively. For an unpolarized sample of the Λ_b baryon, $\rho_{M_i,M_i'} = \frac{1}{2}\delta_{M_i,M_i'}$. In the rest frame of the Λ_b baryon, the daughter Λ baryon and $j_{\rm eff}$ fly back to back, and without loss of generality it can be assumed that the motion of Λ and $j_{\rm eff}$ is along the z axis. This reduces the first two Wigner's $\mathcal{D}$ functions to Kronecker delta functions $\delta_{M_i,\lambda_\Lambda-\lambda_j}$ and $\delta_{M_i',\lambda_\Lambda'-\lambda_j'}$, respectively, where M_i, $M_i' = \pm\frac{1}{2}$. After summing over M_i, M_i', we are left with a Kronecker delta $\delta_{\lambda_\Lambda'-\lambda_j',\lambda_\Lambda-\lambda_j}$ which signifies the fact that we considered the decay of an unpolarized Λ_b. We also observe that $|\lambda_\Lambda' - \lambda_j'| = |\lambda_\Lambda - \lambda_j| = \frac{1}{2}$, as the initial Λ_b is spin $1/2$. This condition further restricts the values that λ_Λ, λ_j can take, and that fact has been already taken into account while calculating $K(q^2,\theta_l,\theta_\Lambda,\phi)$. The choices of possible values for λ_Λ and λ_j are depicted in Table I.

The hadronic helicity amplitudes $H^a_{\lambda_\Lambda,\lambda_j}(J)$ contain all the information of the $\Lambda_b \to \Lambda + j_{\rm eff}$ transition in terms of the relevant form factors and Wilson coefficients

TABLE I. The possible values of λ_Λ and λ_j.

λ_Λ	λ_j	M_i
1/2	1	−1/2
1/2	0	1/2
−1/2	−1	1/2
−1/2	0	−1/2

parametrizing the underlying $b \to s\ell^+\ell^-$ process, explained in detail in Sec. III. In the case of a spin-$\frac{1}{2}$ Λ_b baryon decaying to an intermediate on-shell spin-$\frac{1}{2}$ Λ baryon, there are four hadronic helicitiy amplitudes $H^a_{\lambda_\Lambda,\lambda_j}(J)$, where the index "$a$" denotes whether the hadronic helicity amplitudes multiply the lepton vector current ($a = 1$) or the axial vector current ($a = 2$). The label (J) takes the values ($J = 0$) with $\lambda_j = t$ and ($J = 1$) with $\lambda_j = \pm 1, 0$ for the scalar and vector parts of the effective current $j_{\rm eff}$, respectively.

Let us also discuss here the helicity amplitudes $h^a_{\lambda_j;\lambda_1,\lambda_2}$ appearing in Eq. (3) describing the process $j_{\rm eff} \to \ell^+\ell^-$, where $\lambda_j = \lambda_1 - \lambda_2$. Explicitly,

$$a = 1(\mathrm{V}): h^1_{\lambda_j;\lambda_1,\lambda_2}(J) = \bar{u}_1(\lambda_1)\gamma_\mu v_2(\lambda_2)\epsilon^\mu(\lambda_j),$$
$$a = 2(\mathrm{A}): h^2_{\lambda_j;\lambda_1,\lambda_2}(J) = \bar{u}_1(\lambda_1)\gamma_\mu\gamma_5 v_2(\lambda_2)\epsilon^\mu(\lambda_j). \quad (4)$$

These helicity amplitudes are evaluated in the ($\ell^+\ell^-$) c.m. frame with ℓ^- defined in the $-z$ direction. The label (J) is the same as defined previously and takes the values ($J = 0$) with $\lambda_j = 0(t)$ and ($J = 1$) with $\lambda_j = \pm 1, 0$ for the scalar and vector parts of the effective current $j_{\rm eff}$, respectively. The leptonic helicity amplitudes are calculated and given below:

$$\begin{aligned}
h^1_{t;\frac{1}{2},\frac{1}{2}}(J = 0) &= 0,\\
h^2_{t;\frac{1}{2},\frac{1}{2}}(J = 0) &= 2m_\ell,\\
h^1_{0;\frac{1}{2},\frac{1}{2}}(J = 1) &= 2m_\ell,\\
h^2_{0;\frac{1}{2},\frac{1}{2}}(J = 1) &= 0,\\
h^1_{1;\frac{1}{2},-\frac{1}{2}}(J = 1) &= -\sqrt{2q^2},\\
h^2_{1;\frac{1}{2},-\frac{1}{2}}(J = 1) &= \sqrt{2q^2}v,
\end{aligned} \quad (5)$$

where $v = \sqrt{1 - 4m_\ell^2/q^2}$ is the velocity of the lepton in the ($\ell^+\ell^-$) c.m. frame, m_ℓ being the lepton mass. As the leptonic current is either purely vector or axial vector in nature, they have definite parity properties, which are given by

$$h^1_{-\lambda_j;-\lambda_1,-\lambda_2} = h^1_{\lambda_j;\lambda_1,\lambda_2}, \quad (6)$$

$$h^2_{-\lambda_j;-\lambda_1,-\lambda_2} = -h^2_{\lambda_j;\lambda_1,\lambda_2}. \quad (7)$$

Finally, we move on to the helicity amplitudes $h_{\lambda_p,0}$ describing the decay $\Lambda \to p\pi^-$. We note that this decay is in itself parity nonconserving in addition to the main decay of $\Lambda_b \to \Lambda + j_{\rm eff}$. This is in contrast to the well-studied mesonic analogue of $B \to K^*\ell^+\ell^-$, where the K^* meson subsequently decays to $K\pi$, conserving parity. Also, there is only one helicity amplitude for $K^* \to K\pi$, compared to two helicity amplitudes, as is the case for the $\Lambda \to p\pi^-$ decay. The parity-nonconserving nature of the $\Lambda \to p\pi^-$ decay leads to the forward-backward asymmetry on the hadron side (angular asymmetry in θ_Λ) as well as double asymmetries (angular asymmetry in θ_Λ and θ_l), in addition to the lepton side (angular asymmetry in θ_l).

A. Full angular distribution

$$\begin{aligned}
&K(q^2, \theta_l, \theta_\Lambda, \phi)\\
&= (K_{1ss}\sin^2\theta_\ell + K_{1cc}\cos^2\theta_\ell + K_{1c}\cos\theta_\ell)\\
&\quad + (K_{2ss}\sin^2\theta_\ell + K_{2cc}\cos^2\theta_\ell + K_{2c}\cos\theta_\ell)\cos\theta_\Lambda\\
&\quad + (K_{3sc}\sin\theta_\ell\cos\theta_\ell + K_{3s}\sin\theta_\ell)\sin\theta_\Lambda\sin\phi\\
&\quad + (K_{4sc}\sin\theta_\ell\cos\theta_\ell + K_{4s}\sin\theta_\ell)\sin\theta_\Lambda\cos\phi.
\end{aligned} \quad (8)$$

Here $K_{1ss} \ldots K_{4s}$ are the angular observables, and they are functions of q^2 and m_ℓ [12,20]. We cast this angular distribution in terms of orthogonal Legendre functions, which is advantageous, as the angular observables are uncorrelated to each other. We then provide a set of relations between $K_{1ss} \ldots K_{4s}$ and the new uncorrelated angular observables $I_1 \ldots I_{10}$ given below:

$$\begin{aligned}
&K_{1ss} = I_1 - \frac{I_2}{2}, \qquad K_{1cc} = I_1 + I_2,\\
&K_{2ss} = I_4 - \frac{I_5}{2}, \qquad K_{2cc} = I_4 + I_5,\\
&K_{1c} = I_3, \qquad K_{2c} = I_6, \qquad K_{4sc} = I_7,\\
&K_{4s} = I_8, \qquad K_{3sc} = I_9, \qquad K_{3s} = I_{10}.
\end{aligned} \quad (9)$$

The expressions for $I_1 \ldots I_{10}$ are derived in terms of the transversality amplitudes in Sec. III and are presented in Table II.

In Eq. (8), a full angular analysis is presented, from which the complete set of q^2-dependent observables are extracted. Once a good deal of statistics are available in the future, it is expected that full reconstruction of the angular observables will be possible. For the sake of completeness here, we also provide angular observables made of partially integrated distributions. Starting from full angular distribution [Eq. (8)], the three uniangular distributions can be obtained:

$$\frac{d^2\Gamma}{dq^2 d\phi} = \frac{1}{4}(16I_1 + \pi^2 I_8\cos\phi + \pi^2 I_{10}\sin\phi), \quad (10)$$

$$\frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} = -\pi(4I_1 + I_2(1 + 3\cos 2\theta_\ell) + 4I_3\cos\theta_\ell), \quad (11)$$

TABLE II. Angular observables expressed in terms of transversity amplitudes defined in Sec. III [see Eqs. (24)–(29)]. τ and β are the total decay rate and the forward-backward asymmetry of the subsequent hadronic decay of Λ to $p\pi$, respectively.

Label	Angular term	Transversity amplitude
I_1	Const.	$\tau q^2[\frac{2}{3}(1-\frac{m_\ell^2}{q^2})\{\lvert A^L_{\parallel,0}\rvert^2+\lvert A^L_{\parallel,1}\rvert^2+\lvert A^L_{\perp,0}\rvert^2+\lvert A^L_{\perp,1}\rvert^2+(L\leftrightarrow R)\}$ $+\frac{4m_\ell^2}{q^2}\mathrm{Re}(A^{*R}_{\parallel,0}A^L_{\parallel,0}+A^{*R}_{\parallel,1}A^L_{\parallel,1}+A^{*R}_{\perp,0}A^L_{\perp,0}+A^{*R}_{\perp,1}A^L_{\perp,1})$ $+\frac{2m_\ell^2}{q^2}(\lvert A_{t,\perp}\rvert^2+\lvert A_{t,\parallel}\rvert^2)]$
I_2	$P_2(\cos\theta_l)=\frac{1}{2}(3\cos\theta_l^2-1)$	$\frac{q^2\tau}{3}(1-\frac{4m_\ell^2}{q^2})[\lvert A^L_{\parallel,1}\rvert^2+\lvert A^L_{\perp,1}\rvert^2-2(\lvert A^L_{\parallel,0}\rvert^2+\lvert A^L_{\perp,0}\rvert^2)+(L\leftrightarrow R)]$
I_3	$P_1(\cos\theta_l)=\cos\theta_l$	$-2q^2\tau v\mathrm{Re}[A^{*L}_{\perp,1}A^L_{\parallel,1}-(L\leftrightarrow R)]$
I_4	$P_1(\cos\theta_\Lambda)=\cos\theta_\Lambda$	$\frac{4}{3}q^2\beta[(1-\frac{m_\ell^2}{q^2})\mathrm{Re}\{A^{*L}_{\perp,0}A^L_{\parallel,0}+A^{*L}_{\perp,1}A^L_{\parallel,1}+(L\leftrightarrow R)\}$ $+\frac{3m_\ell^2}{q^2}\{\mathrm{Re}(A^{*R}_{\perp,0}A^L_{\parallel,0}+A^{*L}_{\perp,0}A^R_{\parallel,0}+A^{*R}_{\perp,1}A^L_{\parallel,1}$ $+A^{*L}_{\perp,1}A^R_{\parallel,1})\}+\frac{3m_\ell^2}{q^2}\mathrm{Re}[A^*_{t,\parallel}A_{t,\perp}]]$
I_5	$P_2(\cos\theta_l)P_1(\cos\theta_\Lambda)=\frac{1}{2}(3\cos\theta_l^2-1)\cos\theta_\Lambda$	$\frac{2}{3}q^2\beta(1-\frac{4m_\ell^2}{q^2})\mathrm{Re}[A^{*L}_{\perp,1}A^L_{\parallel,1}-2A^{*L}_{\perp,0}A^L_{\parallel,0}+(L\leftrightarrow R)]$
I_6	$P_1(\cos\theta_l)P_1(\cos\theta_\Lambda)=\cos\theta_l\cos\theta_\Lambda$	$-q^2 v\beta[\lvert A^L_{\parallel,1}\rvert^2+\lvert A^L_{\perp,1}\rvert^2-(L\leftrightarrow R)]$
I_7	$P_1(\cos\theta_l)\sin\theta_l\sin\theta_\Lambda\cos\phi=\cos\theta_l\sin\theta_l\sin\theta_\Lambda\cos\phi$	$\sqrt{2}q^2\beta(1-\frac{4m_\ell^2}{q^2})\mathrm{Re}[(A^{*L}_{\perp,1}A^L_{\parallel,0}-A^{*L}_{\parallel,1}A^L_{\perp,0})+(L\leftrightarrow R)]$
I_8	$\sin\theta_l\sin\theta_\Lambda\cos\phi$	$\sqrt{2}q^2 v\beta\mathrm{Re}[(A^{*L}_{\perp,1}A^L_{\perp,0}-A^{*L}_{\parallel,1}A^L_{\parallel,0})-(L\leftrightarrow R)]$
I_9	$P_1(\cos\theta_l)\sin\theta_l\sin\theta_\Lambda\sin\phi=\cos\theta_l\sin\theta_l\sin\theta_\Lambda\sin\phi$	$-\sqrt{2}q^2\beta(1-\frac{4m_\ell^2}{q^2})\mathrm{Im}[(A^{*L}_{\perp,1}A^L_{\perp,0}-A^{*L}_{\parallel,1}A^L_{\parallel,0})+(L\leftrightarrow R)]$
I_{10}	$\sin\theta_l\sin\theta_\Lambda\sin\phi$	$-\sqrt{2}q^2 v\beta\mathrm{Im}[(A^{*L}_{\perp,1}A^L_{\parallel,0}-A^{*L}_{\parallel,1}A^L_{\perp,0})-(L\leftrightarrow R)]$

$$\frac{d^2\Gamma}{dq^2 d\cos\theta_\Lambda}=-4\pi(I_1+I_4\cos\theta_\Lambda). \tag{12}$$

III. HADRONIC HELICITY AMPLITUDES

In Eq. (8), we have obtained the angular distribution of $\Lambda_b(\frac{1}{2})\to\Lambda(\frac{1}{2})+J(0,1)$, where the Λ further decays to $p\pi^-$. Before calculating the helicity amplitudes of the primary decay, let us go through the details of the subsequent hadronic decay briefly. An on-shell spin-$\frac{1}{2}$ Λ baryon (uds) decays into an on-shell proton p (uud) and a pion π^- via a parity nonconserving weak decay that involves two hadronic couplings a and b. The matrix element for this decay can be written in the following way:

$$\begin{aligned}&\langle p(k_1)\pi(k_2)\lvert(\bar{d}\gamma^\mu P_L u)(\bar{u}\gamma_\mu P_L s)\rvert\Lambda(k)\rangle\\&\quad=\bar{u}(k_1)[(a+b\gamma^5)]u(k).\end{aligned} \tag{13}$$

We also note that the helicity amplitudes $h_{\lambda_p,0}$ defined in Eq. (3) describe the same decay $\Lambda\to p^+\pi^-$. Moreover, it is clear that there are only two helicity amplitudes, as λ_p takes values $\pm\frac{1}{2}$. From a separate measurement of the $\Lambda\to p^+\pi^-$ decay width and the polarization asymmetry, these two helicity amplitudes can be inferred, which is equivalent to the extraction of the two hadronic couplings a and b.

To calculate the hadronic helicity amplitudes of the primary decay which in turn can be related to the invariant form factors, we start with the Hamiltonian for the decay described in Ref. [57].

The matrix element for the decay $\Lambda_b\to\Lambda\bar{\ell}\ell$ is defined by

$$\begin{aligned}\mathcal{M}(\Lambda_b\to\Lambda\bar{\ell}\ell)=&\frac{G_F}{\sqrt{2}}\frac{\alpha\lambda_t}{2\pi}[C_9^{\mathrm{eff}}\langle\Lambda\lvert\bar{s}\gamma^\mu(1-\gamma^5)b\rvert\Lambda_b\rangle\bar{\ell}\gamma_\mu\ell\\&+C_{10}\langle\Lambda\lvert\bar{s}\gamma^\mu(1-\gamma^5)b\rvert\Lambda_b\rangle\bar{\ell}\gamma_\mu\gamma_5\ell\\&-\frac{2m_b}{q^2}C_7^{\mathrm{eff}}\langle\Lambda\lvert\bar{s}i\sigma^{\mu q}(1+\gamma^5)b\rvert\Lambda_b\rangle\bar{\ell}\gamma_\mu\ell],\end{aligned} \tag{14}$$

where C_i's are the Wilson coefficients, $\lambda_t\equiv V_{ts}^*V_{tb}$, and m_b is the b-quark mass. For this paper, all the values of the Wilson coefficients have been taken from Ref. [19]. The hadronic matrix elements written in terms of dimensionless form factors are

$$\langle\Lambda(k)|\bar{s}\gamma^\mu b|\Lambda_b(p)\rangle = \bar{u}_2(p_2)[f_1^V(q^2)\gamma^\mu - f_2^V(q^2)i\sigma^{\mu q}/m_{\Lambda_b} + f_3^V(q^2)q^\mu/m_{\Lambda_b}]u_1(p_1),$$
$$\langle\Lambda(k)|\bar{s}\gamma^\mu\gamma^5 b|\Lambda_b(p)\rangle = \bar{u}_2(p_2)[f_1^A(q^2)\gamma^\mu - f_2^A(q^2)i\sigma^{\mu q}/m_{\Lambda_b} + f_3^A(q^2)q^\mu/m_{\Lambda_b}]\gamma^5 u_1(p_1),$$
$$\langle\Lambda(k)|\bar{s}i\sigma^{\mu q}/m_{\Lambda_b} b|\Lambda_b(p)\rangle = \bar{u}_2(p_2)[f_1^{TV}(q^2)(\gamma^\mu q^2 - q^\mu \not{q})/m_{\Lambda_b}^2 - f_2^{TV}(q^2)i\sigma^{\mu q}/m_{\Lambda_b}]u_1(p_1),$$
$$\langle\Lambda(k)|\bar{s}i\sigma^{\mu q}\gamma^5/m_{\Lambda_b} b|\Lambda_b(p)\rangle = \bar{u}_2(p_2)[f_1^{TA}(q^2)(\gamma^\mu q^2 - q^\mu \not{q})/m_{\Lambda_b}^2 - f_2^{TA}(q^2)i\sigma^{\mu q}/m_{\Lambda_b}]\gamma^5 u_1(p_1), \quad (15)$$

where $\sigma^{\mu\nu}q_\nu = \sigma^{\mu q}$ is used through out as a shorthand notation.

The helicity amplitudes $H^a_{\lambda_2,\lambda_j}$ are expressed by the following relation:

$$H^a_{\lambda_\Lambda,\lambda_j} = M^a_\mu(\lambda_\Lambda)\epsilon^{*\mu}(\lambda_j), \quad (16)$$

where M^a_μ are the hadronic matrix elements defined in Eq. (15). As before, the labels λ_j and λ_Λ denote the helicities of the effective current and daughter baryon, respectively. We shall work in the rest frame of the parent baryon Λ_b, where the daughter baryon Λ is moving in the positive z direction, and the effective current is moving along the negative z axis. The relevant momenta that describe the motion of particles in this frame are given below:

$$p^\mu = (m_{\Lambda_b},0,0,0), \qquad k^\mu = (E_2,0,0,p_2),$$
$$q^\mu = (q_0,0,0,-p_2),$$

where $q_0 = \frac{1}{2m_{\Lambda_b}}(m^2_{\Lambda_b} - m^2_\Lambda + q^2)$ and $E_2 = (m_{\Lambda_b} - q_0) = (m^2_{\Lambda_b} + m^2_\Lambda - q^2)/2m_{\Lambda_b}$. The helicity of the particles is fixed by angular momentum relation through the equation $M_i = \lambda_\Lambda - \lambda_j$. The $J = \frac{1}{2}$ baryon helicity spinors are given by

$$\bar{u}_2\left(\vec{k} = p_2\hat{z}, \pm\frac{1}{2}\right) = \sqrt{E_2 + m_\Lambda}\left(\chi^\dagger_\pm \;\; \frac{\mp|p_2|}{E_2+m_\Lambda}\chi^\dagger_\pm\right), \quad (17)$$

$$u_1\left(\vec{p} = 0, \pm\frac{1}{2}\right) = \sqrt{2m_{\Lambda_b}}\begin{pmatrix}\chi_\pm \\ 0\end{pmatrix}, \quad (18)$$

where $\chi_+ = \binom{1}{0}$ and $\chi_- = \binom{0}{1}$ are two-component Pauli spinors.

The polarization vectors of the effective current $J_{\rm eff}$ moving along the negative z axis look like

$$\epsilon^\mu(t) = \frac{1}{\sqrt{q^2}}(q_0,0,0,-p_2),$$
$$\epsilon^\mu(\pm) = \frac{1}{\sqrt{2}}(0,\pm1,-i,0),$$
$$\epsilon^\mu(0) = \frac{1}{\sqrt{q^2}}(p_2,0,0,-q_0). \quad (19)$$

They satisfy the $q_\mu\epsilon^\mu = 0$ equation, q_μ being the momentum four-vector of the effective current. We also note that hadronic helicity can be expressed as

$$H^a_{\lambda_\Lambda,\lambda_j} = H^{Va}_{\lambda_\Lambda,\lambda_j} - H^{Aa}_{\lambda_\Lambda,\lambda_j}, \quad (20)$$

where $H^{Va}_{\lambda_\Lambda,\lambda_j}$ and $H^{Aa}_{\lambda_\Lambda,\lambda_j}$ are the vector and axial-vector parts of the helicity amplitudes, respectively. $H^{Va}_{\lambda_\Lambda,\lambda_j}$, $H^{Aa}_{\lambda_\Lambda,\lambda_j}$ have definite parity properties:

$$H^{Va}_{-\lambda_\Lambda,-\lambda_j} = H^{Va}_{\lambda_\Lambda,\lambda_j}, \qquad H^{Aa}_{-\lambda_\Lambda,-\lambda_j} = -H^{Aa}_{\lambda_\Lambda,\lambda_j}. \quad (21)$$

Different hadronic helicity amplitudes that take part in the decay are presented below:

$$H^{Va}_{\frac{1}{2}t} = \sqrt{\frac{Q_+}{q^2}}\left(M_- F^{Va}_1 + \frac{q^2}{m_{\Lambda_b}}F^{Va}_3\right),$$
$$H^{Va}_{\frac{1}{2}1} = \sqrt{2Q_-}\left(F^{Va}_1 + \frac{M_+}{m_{\Lambda_b}}F^{Va}_2\right),$$
$$H^{Va}_{\frac{1}{2}0} = \sqrt{\frac{Q_-}{q^2}}\left(M_+ F^{Va}_1 + \frac{q^2}{m_{\Lambda_b}}F^{Va}_2\right),$$
$$H^{Aa}_{\frac{1}{2}t} = \sqrt{\frac{Q_-}{q^2}}\left(M_+ F^{Aa}_1 - \frac{q^2}{m_{\Lambda_b}}F^{Aa}_3\right),$$
$$H^{Aa}_{\frac{1}{2}1} = \sqrt{2Q_+}\left(F^{Aa}_1 - \frac{M_-}{m_{\Lambda_b}}F^{Aa}_2\right),$$
$$H^{Aa}_{\frac{1}{2}0} = \sqrt{\frac{Q_+}{q^2}}\left(M_- F^{Aa}_1 - \frac{q^2}{m_{\Lambda_b}}F^{Aa}_2\right), \quad (22)$$

where $M_\pm = m_{\Lambda_b} \pm m_\Lambda$, $Q_\pm = M^2_\pm - q^2$, with a being the leptonic current index ($a = 1$, vector current; $a = 2$, axial-vector current). The redefined form factors F^{Va}_i, F^{Aa}_i involve linear combinations of the form factors f^V_i, f^A_i, as well as the Wilson coefficients:

$$F^{V1}_1 = C^{\rm eff}_9 f^V_1 - \frac{2m_b}{m_{\Lambda_b}}C^{\rm eff}_7 f^{TV}_1,$$
$$F^{V1}_2 = C^{\rm eff}_9 f^V_2 - \frac{2m_b m_{\Lambda_b}}{q^2}C^{\rm eff}_7 f^{TV}_2,$$
$$F^{V1}_3 = C^{\rm eff}_9 f^V_3 + \frac{2m_b M_-}{q^2}C^{\rm eff}_7 f^{TV}_1,$$
$$F^{A1}_1 = C^{\rm eff}_9 f^A_1 + \frac{2m_b}{m_{\Lambda_b}}C^{\rm eff}_7 f^{TA}_1,$$
$$F^{A1}_2 = C^{\rm eff}_9 f^A_2 + \frac{2m_b m_{\Lambda_b}}{q^2}C^{\rm eff}_7 f^{TA}_2,$$
$$F^{A1}_3 = C^{\rm eff}_9 f^A_3 + \frac{2m_b M_+}{q^2}C^{\rm eff}_7 f^{TA}_1,$$

and

$$F_i^{V2} = C_{10} f_i^V,$$
$$F_i^{A2} = C_{10} f_i^A. \quad (23)$$

We switch to transversity amplitude, defined as

$$A_{\parallel,0}^{L(R)} = H_{\frac{1}{2},0}^{Va=1} \mp H_{\frac{1}{2},0}^{Va=2}, \quad (24)$$

$$A_{\perp,0}^{L(R)} = H_{\frac{1}{2},0}^{Aa=1} \mp H_{\frac{1}{2},0}^{Aa=2}, \quad (25)$$

$$A_{\parallel,1}^{L(R)} = H_{\frac{1}{2},1}^{Va=1} \mp H_{\frac{1}{2},1}^{Va=2}, \quad (26)$$

$$A_{\perp,1}^{L(R)} = H_{\frac{1}{2},1}^{Aa=1} \mp H_{\frac{1}{2},1}^{Aa=2}, \quad (27)$$

$$A_{\parallel,t} = H_{-\frac{1}{2},t}^{a=2} + H_{\frac{1}{2},t}^{a=2}, \quad (28)$$

$$A_{\perp,t} = H_{-\frac{1}{2},t}^{a=2} - H_{\frac{1}{2},t}^{a=2}. \quad (29)$$

The superscript $L(R)$ on $A_{\perp(\parallel)}$ denotes that the transversity amplitudes are multiplied by a left-handed (right-handed) lepton current. There are two additional transversity amplitudes that are relevant to the decay if the $j_{\rm eff}$ is virtual, corresponding to the $J=0$ contribution. These two amplitudes, $A_{\parallel,t}$ and $A_{\perp,t}$, do not have separate left-handed or right-handed parts, as the timelike polarization of $j_{\rm eff}$ couples only to the axial-vector part of the lepton current [5], a fact highlighted by Eq. (5). Moreover, the A_t contribution vanishes in the limit of massless leptons. The decay $\Lambda_b \to \Lambda(\to p\pi)\ell^+\ell^-$ is completely described by these transversity amplitudes, which include all contributions from the standard-model effective operators.

IV. TOTAL DECAY RATE AND ANGULAR OBSERVABLES

The total differential decay rate can be extracted in terms of the constant piece appearing in the angular distribution once we include the parameters in the effective Hamiltonian and the relevant phase space factors, i.e.

$$\begin{aligned}\frac{d\Gamma}{dq^2} &\equiv \frac{d\Gamma(\Lambda_b \to (\Lambda \to p\pi)\ell^+\ell^-)}{dq^2} \\ &= \mathrm{Br}(\Lambda \to p\pi^-) \times \frac{1}{2}\frac{1}{(2\pi)^3}\frac{|\mathbf{p_2}|q^2 v}{16 m_{\Lambda_b}^2}\left(\frac{G_F \alpha \lambda_t}{2\pi}\right)^2 \\ &\quad \times \left[\frac{2}{3}\left(1-\frac{m_\ell^2}{q^2}\right)\{|A_{\parallel,0}^L|^2 + |A_{\parallel,1}^L|^2 + |A_{\perp,0}^L|^2 + |A_{\perp,1}^L|^2 + (L \leftrightarrow R)\}\right. \\ &\quad \left. + \frac{4m_\ell^2}{q^2}\mathrm{Re}(A_{\parallel,0}^{*R}A_{\parallel,0}^L + A_{\parallel,1}^{*R}A_{\parallel,1}^L + A_{\perp,0}^{*R}A_{\perp,0}^L + A_{\perp,1}^{*R}A_{\perp,1}^L) + \frac{2m_\ell^2}{q^2}(|A_{t,\perp}|^2 + |A_{t,\parallel}|^2)\right], \end{aligned} \quad (30)$$

where α is the fine structure constant, G_F is the Fermi coupling constant, $\lambda_t = V_{ts}^\dagger V_{tb}$ is the product of CKM matrix elements relevant for the underlying quark level transition, and $|\mathbf{p_2}| = \lambda^{1/2}(m_{\Lambda_b}^2, m_\Lambda^2, q^2)/2m_{\Lambda_b}$ is the momentum of the Λ baryon in the Λ_b rest frame where $\lambda(m_{\Lambda_b}^2, m_\Lambda^2, q^2)$ is the Källén function. The $1/2$ factor appearing in the definition of the differential decay rate takes into account the decay of the unpolarized spin-$1/2$ initial-state Λ_b baryon. We note that there is an additional timelike contribution to the differential decay rate that becomes important for nonzero lepton masses $m_\ell \neq 0$, especially in the low-q^2 region.

The decay rates for $\Lambda_b \to \Lambda e^+e^-$ and $\Lambda_b \to \Lambda\mu^+\mu^-$ can be readily calculated once the values of relevant form factors are fixed. Our interest lies primarily in the low-q^2 (high recoil) region, i.e. $q^2 = 0.04\ \mathrm{GeV}^2$–$6\ \mathrm{GeV}^2$, and it has been emphasized in Refs. [58–62] that heavy quark symmetry is not reliable at low q^2. Heavy quark symmetry is expected to break down as one deviates from the zero-recoil point [58,59]. We therefore follow the approach

TABLE III. Parameters for the form factors as a function of q^2, $f(t) = f(0)/(1 - at + bt^2)$, $t = q^2/m_{\Lambda_b}^2$ for the $\Lambda_b \to \Lambda$ transition as given in Ref. [19].

Parameter	f_1^V	f_2^V	f_3^V	f_1^A	f_2^A	f_3^A	f_1^{TV}	f_2^{TV}	f_1^{TA}	f_2^{TA}
$f(0)$	0.107	0.043	0.003	0.104	0.003	−0.052	−0.043	−0.105	0.003	−0.105
a	2.271	2.411	2.815	2.232	2.955	2.437	2.411	0.072	2.955	2.233
b	1.367	1.531	2.041	1.328	3.620	1.559	1.531	0.001	3.620	1.328

of Ref. [19], which uses the covariant quark model (CQM) to calculate the required form factors at low q^2. We quote the values of those form factors [19] in Table III and calculate $R(\Lambda)$, the ratio of decay rates for $\Lambda_b \to \Lambda\mu^+\mu^-$ and $\Lambda_b \to \Lambda e^+e^-$. We also use soft collinear effective theory (SCET) and compare the two estimates obtained for the decay rate. A heavy-to-light transition of Λ_b to Λ in the large-recoil (low-q^2) limit is simplified as the number of independent form factors reduces to one. SCET is valid [62,63] in this energy range, as the energy of the daughter Λ is larger than its mass, and one can use $\lambda = \sqrt{\frac{m_\Lambda}{m_b}}$ as an expansion parameter which is small. In such a picture, the hadronic matrix elements in Eq. (15) for $\Lambda_b \to \Lambda$ decay can be parametrized in the following way:

$$\langle \Lambda(p_2)|\bar{s}\Gamma b|\Lambda_b(p)\rangle \simeq \xi_\lambda(E_2)\bar{u}_\Lambda(p_2)\Gamma u_{\Lambda_b}(p) + \mathcal{O}(\lambda^2\xi_\lambda).$$

To start with, let us highlight the relations between different form factors used in Eq. (15) in the low-q^2 limit,

$$f_1^V \approx f_1^A \approx -f_2^{TV} \approx -f_2^{TA} = \xi_\lambda, \tag{31}$$

$$f_2^V \approx f_3^V \approx f_2^A \approx f_3^A \approx f_1^{TV} \approx f_1^{TA} \approx 0, \tag{32}$$

where ξ_λ is the single parameter all nonzero form factors depend on in the limit of small q^2 [63,64]. In Fig. 1, we have plotted $R(\Lambda)$. We have gone further and also probed the reliability of the R_Λ value by randomly adding $\pm 30\%$ error to each form-factor estimate in the covariant quark model and by generating 10^4 points to evaluate the ratio. It is thus concluded that R_Λ is reliably predicted in the low-q^2 (high-recoil) region of $q^2 = 0.04\ \mathrm{GeV}^2$–$6\ \mathrm{GeV}^2$. The contributions to R_Λ from long-distance effects will be discussed later.

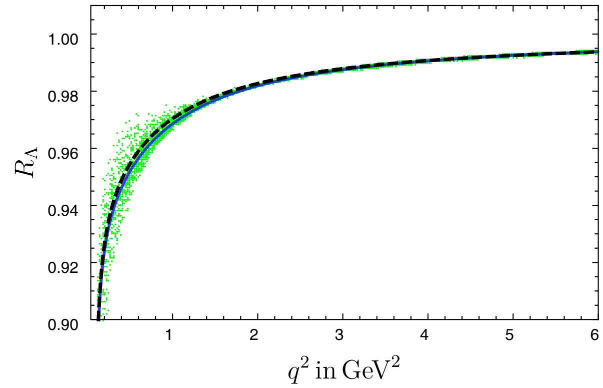

FIG. 1. The q^2-dependence of R_Λ. The blue solid line is found using the values of the form factors in the covariant quark model given in Table III. This line almost coincides with the estimate using SCET form factors plotted as a black dotted line. The green band represents the possible values of R_Λ obtained by randomly generating 10^4 points, corresponding to $\pm 30\%$ error in each of the covariant quark model form-factor estimates.

A. Angular observables

In this section, we list the angular asymmetries that allow us to extract the angular coefficients $I_2 \ldots I_{10}$ and contribute to nine of the ten observables, with I_1 being the total differential decay rate. These asymmetries result from orthogonal angular distribution and are thus independent observables.

$$A_2 = \frac{[\int_{-1}^{-\frac{1}{2}} - \int_{-\frac{1}{2}}^{\frac{1}{2}} + \int_{\frac{1}{2}}^{1}] d\cos\theta_l \int_{-1}^{1} d\cos\theta_\Lambda \int_0^{2\pi} d\phi \frac{1}{3\pi} \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}{\int_{-1}^{1} d\cos\theta_l \int_{-1}^{1} d\cos\theta_\Lambda \int_0^{2\pi} d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}, \tag{33}$$

$$A_3 = \frac{[-\int_{-1}^{0} + \int_{0}^{1}] d\cos\theta_l \int_{-1}^{1} d\cos\theta_\Lambda \int_0^{2\pi} d\phi \frac{1}{4\pi} \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}{\int_{-1}^{1} d\cos\theta_l \int_{-1}^{1} d\cos\theta_\Lambda \int_0^{2\pi} d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}, \tag{34}$$

$$A_4 = \frac{\int_{-1}^{1} d\cos\theta_l [-\int_{-1}^{0} + \int_{0}^{1}] d\cos\theta_\Lambda \int_0^{2\pi} d\phi \frac{1}{4\pi} \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}{\int_{-1}^{1} d\cos\theta_l \int_{-1}^{1} d\cos\theta_\Lambda \int_0^{2\pi} d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}, \tag{35}$$

$$A_5 = \frac{4}{3} \frac{[-\int_{-1}^{-\frac{1}{2}} + \int_{-\frac{1}{2}}^{\frac{1}{2}} - \int_{\frac{1}{2}}^{1}] d\cos\theta_l [\int_{-1}^{0} - \int_{0}^{1}] d\cos\theta_\Lambda \int_0^{2\pi} d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}{\int_{-1}^{1} d\cos\theta_l \int_{-1}^{1} d\cos\theta_\Lambda \int_0^{2\pi} d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}, \tag{36}$$

$$A_6 = \frac{[\int_{-1}^{0} - \int_{0}^{1}] d\cos\theta_l [\int_{-1}^{0} - \int_{0}^{1}] d\cos\theta_\Lambda \int_0^{2\pi} d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}{\int_{-1}^{1} d\cos\theta_l \int_{-1}^{1} d\cos\theta_\Lambda \int_0^{2\pi} d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}, \tag{37}$$

$$A_7 = -\frac{3}{4}\frac{[\int_{-1}^{0} - \int_{0}^{1}]d\cos\theta_l \int_{-1}^{1} d\cos\theta_\Lambda [\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}]d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}{\int_{-1}^{1} d\cos\theta_l \int_{-1}^{1} d\cos\theta_\Lambda \int_0^{2\pi} d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}, \tag{38}$$

$$A_8 = \frac{1}{\pi^2}\frac{\int_{-1}^{1} d\cos\theta_l \int_{-1}^{1} d\cos\theta_\Lambda [-\int_{-\pi}^{\frac{-\pi}{2}} + \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} - \int_{\frac{\pi}{2}}^{\pi}]d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}{\int_{-1}^{1} d\cos\theta_l \int_{-1}^{1} d\cos\theta_\Lambda \int_0^{2\pi} d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}, \tag{39}$$

$$A_9 = -\frac{3}{4}\frac{[\int_{-1}^{0} - \int_{0}^{1}]d\cos\theta_l \int_{-1}^{1} d\cos\theta_\Lambda [\int_{0}^{\pi} - \int_{\pi}^{2\pi}]d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}{\int_{-1}^{1} d\cos\theta_l \int_{-1}^{1} d\cos\theta_\Lambda \int_0^{2\pi} d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}, \tag{40}$$

$$A_{10} = \frac{1}{\pi^2}\frac{\int_{-1}^{1} d\cos\theta_l \int_{-1}^{1} d\cos\theta_\Lambda [\int_{-\pi}^{0} + \int_{0}^{\pi}]d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}{\int_{-1}^{1} d\cos\theta_l \int_{-1}^{1} d\cos\theta_\Lambda \int_0^{2\pi} d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_\Lambda d\cos\theta_l d\phi}}. \tag{41}$$

Note that A_9 and A_{10} are nonzero only if the amplitudes have imaginary contributions. These are expected to be extremely tiny in the SM. The asymmetries A_2, A_5, A_7, and A_8 are not simple forward-backward asymmetries. We note that A_3, A_4 are forward-backward asymmetries in the leptonic angle θ_l and the hadronic angle θ_Λ, respectively. There is also a double asymmetry involving θ_l and θ_Λ given by A_6. We provide an expression for each of these quantities in terms of known parameters.

$$A^l_{FB} = \frac{-3v\mathrm{Re}[A^{*L}_{\perp,1}A^L_{\parallel,1} - (L\leftrightarrow R)]q^2}{4I_1}, \tag{42}$$

$$A^h_{FB} = \frac{\alpha_\Lambda[(1-\frac{m_\ell^2}{q^2})\mathrm{Re}\{A^{*L}_{\perp,0}A^L_{\parallel,0} + A^{*L}_{\perp,1}A^L_{\parallel,1} + (L\leftrightarrow R)\} + \frac{3m_\ell^2}{q^2}\mathrm{Re}\{A^*_{t,\parallel}A_{t,\perp}\} + \frac{3m_\ell^2}{q^2}\{\mathrm{Re}(A^{*R}_{\perp,0}A^L_{\parallel,0} + A^{*L}_{\perp,0}A^R_{\parallel,0} + A^{*R}_{\perp,1}A^L_{\parallel,1} + A^{*L}_{\perp,1}A^R_{\parallel,1})\}]q^2}{2I_1}, \tag{43}$$

$$A^{hl}_{FB} = \frac{-3v\alpha_\Lambda[|A^L_{\parallel,1}|^2 + |A^L_{\perp,1}|^2 - (L\leftrightarrow R)]q^2}{8I_1}. \tag{44}$$

A^l_{FB}, A^h_{FB}, A^{hl}_{FB} are the lepton-side forward-backward asymmetry, hadron-side forward-backward asymmetry, and double forward-backward asymmetry, respectively. The parameter α_Λ is the asymmetry parameter of the decay $\Lambda \to p^+\pi^-$, which is defined as

$$\alpha_\Lambda = \frac{\beta}{\tau} = \frac{|h_{-\frac{1}{2},0}|^2 - |h_{\frac{1}{2},0}|^2}{|h_{-\frac{1}{2},0}|^2 + |h_{\frac{1}{2},0}|^2}. \tag{45}$$

Note that the convention used by us is the same as in Ref. [19] up to an overall negative sign. The asymmetry parameter has been measured to be $\alpha_\Lambda = 0.642 \pm 0.013$ [65]. As mentioned already, this is in contrast to the mesonic counterpart $B \to K^*(\to K\pi)\ell^+\ell^-$, where the subsequent $K^* \to K\pi$ decay is parity conserving, and thus no forward-backward asymmetry in the hadronic angle θ_{K^*} is observed. While we are discussing A^h_{FB}, we would also like to point out that it is sensitive to the timelike polarization of j_{eff}, as the presence of A_t can be seen in Eq. (43). If we are to restrict ourselves to the SM effective operators, the transversity amplitude A_t involves the Wilson coefficient C_{10} only. A_t receives an additional contribution in the presence of pseudoscalar operators of the form $(\bar{s}\gamma_5 b)(l\gamma_5 l)$, as shown in Ref. [5]. Thus, A^h_{FB} provides an independent test of pseudoscalar currents that are not present in the SM. The lepton forward-backward asymmetry is given by A^l_{FB}, which depends on the real part of the interference between two amplitudes $A_{\perp,1}$ and $A_{\parallel,1}$. The presence of the factor v suggests that lepton forward-backward asymmetry vanishes as $q^2 \to 4m_l^2$. The double forward-backward asymmetry is given by A^{hl}_{FB}. It is clear that A^{hl}_{FB} vanishes when either asymmetry parameter $\alpha_\Lambda = 0$ or $q^2 \to 4m_l^2$.

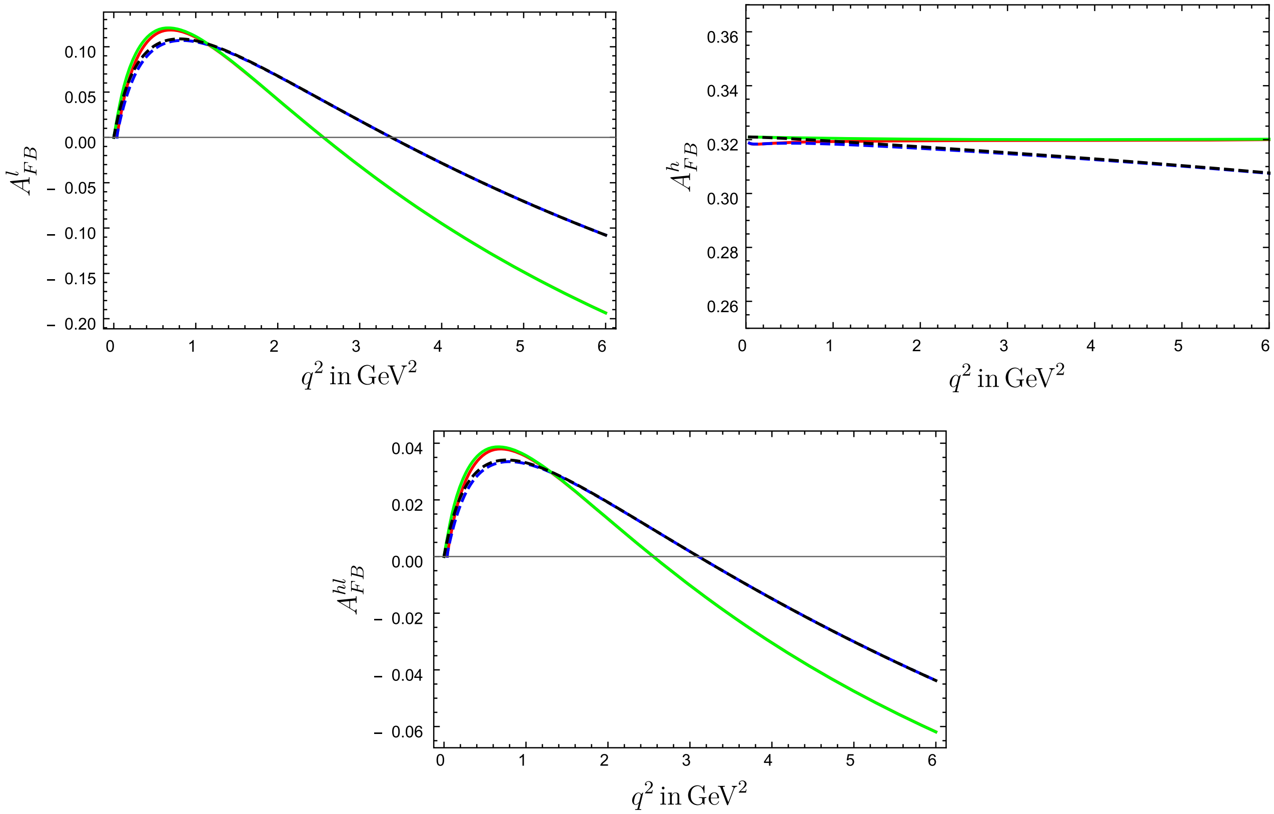

FIG. 2. The q^2 dependence of the forward-backward asymmetries A^l_{FB}, A^h_{FB}, A^{hl}_{FB} for the electron and muon are presented. For muons, the q^2 dependence of the asymmetries are given by the solid red line, obtained using covariant quark model form factors, and the blue dotted line is for SCET form factors. For electrons, the solid green line represents the q^2 dependence using covariant quark model form factors, whereas the black dotted line represents the q^2 dependence due to SCET form factors.

An interesting feature of the three forward-backward observables A^l_{FB}, A^h_{FB}, and A^{hl}_{FB} is their characteristic q^2 dependence. More precisely, within the SM, one finds that both A^l_{FB} and A^{hl}_{FB} cross zero, in contrast to A^h_{FB}, which does not. Moreover, to the leading order, the zero-crossing points are same for A^l_{FB} and A^{hl}_{FB}. The q_0^2 value only depends on ratios of Wilson coefficients, a result well known from other exclusive and inclusive $b \to s\ell^+\ell^-$ decay, and to leading order,

$$q_0^2 \approx -\frac{2m_b m_{\Lambda_b} C_7}{C_9}. \tag{46}$$

If we relax the assumption that Wilson coefficients are flavor blind and allow for the possibility of $C_9^\mu \neq C_9^e$, then the zero-crossing point will be different for muons and electrons. Thus, by observing q_0^2, the zero-crossing values for observables like A^e_{FB} and A^μ_{FB}, one can extract vital information about the underlying flavor structure of the theory. There are also other theoretically clean observables having zero-crossing points in the large-q^2 region, as emphasized in Ref. [22]. Thus, a careful study of the zero-crossing points of these observables is necessary over the whole q^2 range to disentangle any genuine new physics contribution, which is expected to be q^2 independent, from any unaccounted hadronic effects.

We construct ratios of A^μ_{FB}, $A^h_{FB,\mu}$, $A^{h\mu}_{FB}$ to the corresponding quantities for the electron, i.e. A^e_{FB}, $A^h_{FB,e}$, A^{he}_{FB}. These ratios are defined in a similar vein as done in Refs. [66–69]:

$$R_3 = R_{A^l_{FB}} = \frac{A^\mu_{FB}}{A^e_{FB}}, \tag{47}$$

$$R_4 = R_{A^h_{FB}} = \frac{A^h_{FB,\mu}}{A^h_{FB,e}}, \tag{48}$$

$$R_6 = R_{A^{hl}_{FB}} = \frac{A^{hl}_{FB,\mu}}{A^{hl}_{FB,e}}. \tag{49}$$

We additionally provide here ratios of other angular observables for muons—namely A_2^μ, A_5^μ, A_7^μ, A_8^μ—to the corresponding quantities for the electrons:

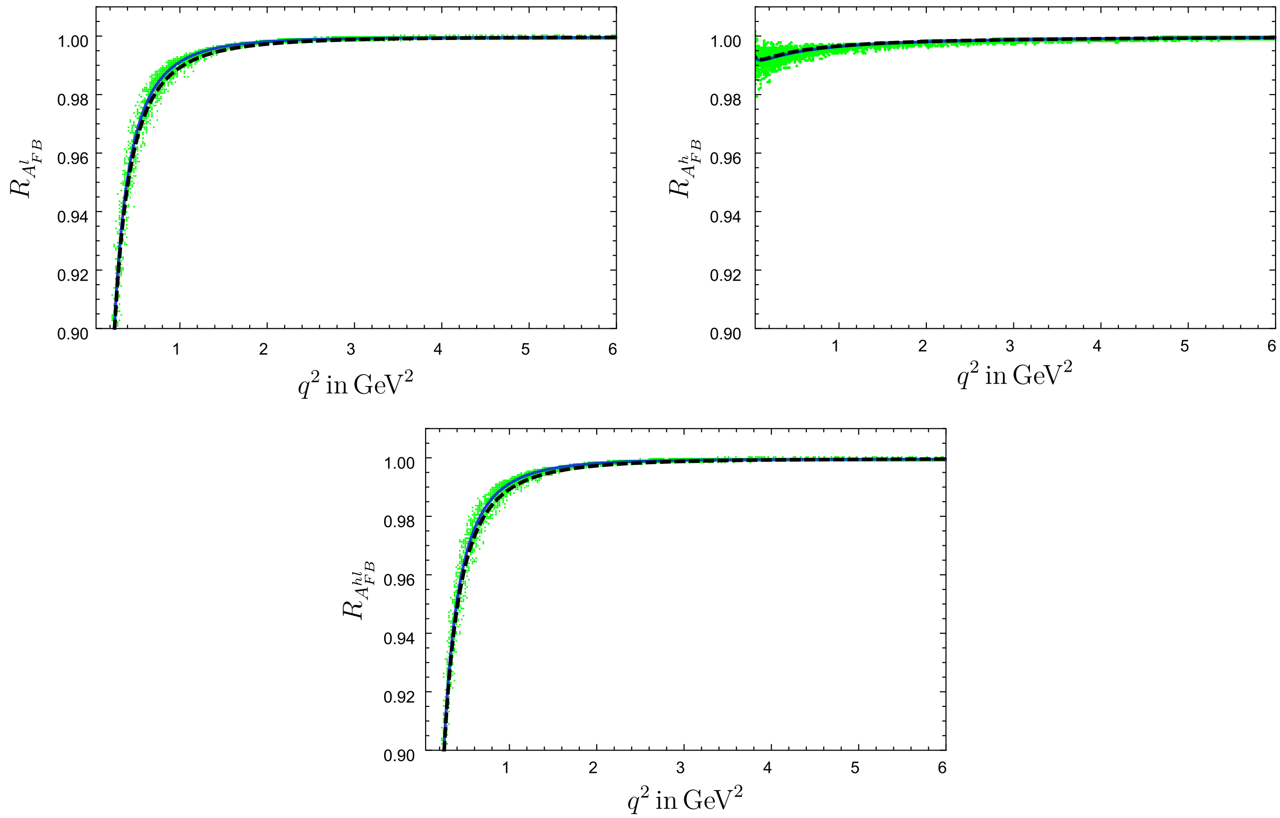

FIG. 3. The q^2 dependence for the ratios of forward-backward asymmetries $R_{A^l_{FB}}$, $R_{A^h_{FB}}$, and $R_{A^{hl}_{FB}}$ [see Eqs. (47)–(49)]. The blue solid line is found using the values of form factors in the covariant quark model given in Table III. This line almost coincides with the estimate using SCET form factors, represented as the black dotted line. Note that, while the two asymmetries $A^h_{FB,\mu}$ and $A^h_{FB,e}$ are form-factor dependent, the ratio $R_{A^h_{FB}}$ is independent of the choice of form factors (CQM or SCET). The green band represents the possible values of R_Λ obtained by randomly generating 10^4 points corresponding to $\pm 30\%$ error in each of the covariant quark model form-factor estimates.

$$R_2 = \frac{A_2^\mu}{A_2^e}, \tag{50}$$

$$R_5 = \frac{A_5^\mu}{A_5^e}, \tag{51}$$

$$R_7 = \frac{A_7^\mu}{A_7^e}, \tag{52}$$

$$R_8 = \frac{A_8^\mu}{A_8^e}. \tag{53}$$

A natural question that arises is to what extent the ratios defined in Eqs. (42)–(44) deviate from the case where individual form factors are only known to a certain accuracy. In SCET, the helicity amplitudes defined in Eq. (22) are expressible in terms of the parameter ξ_λ. This simplification leads to ξ_λ getting factored out, and it cancels when ratios like A^l_{FB}, A^h_{FB}, A^{hl}_{FB} are defined. In Fig. 2, we plot A^l_{FB}, A^h_{FB}, A^{hl}_{FB} for the cases of $\ell = e$ and $\ell = \mu$ separately in the low-q^2 region. There is, however, a dependence on form factor in the observable of special interest—the hadronic forward-backward asymmetry $A^h_{FB,l}$, as can be seen in Fig. 2. Fortunately, it turns out that the dependence on the choice of form factors cancels out in the ratios of asymmetries defined as $R_{A^l_{FB}}$, $R_{A^h_{FB}}$, and $R_{A^{hl}_{FB}}$. Nevertheless, if the measurements of these ratios differ from the predicted ones, one may question the accuracy of form factors which are only calculated based on a model. In order to ascertain the sensitivity of these ratios due to inaccuracies in the form factors, we randomly add $\pm 30\%$ error to each form-factor estimate in the covariant quark model and generate 10^4 points to evaluate the ratios $R_{A^l_{FB}}$, $R_{A^h_{FB}}$, and $R_{A^{hl}_{FB}}$. In Fig. 3, we have plotted these ratios $R_{A^l_{FB}}$, $R_{A^l_{FB}}$, and $R_{A^{hl}_{FB}}$. In contrast to $R_{A^l_{FB}}$ and $R_{A^{hl}_{FB}}$, $R_{A^h_{FB}}$ is a ratio of two nonvanishing asymmetries at low q^2; hence, it is likely to be more accurately measured. The ratio of the remaining angular observables defined in Eqn. (50)–(53) are plotted in Fig. 4. We conclude this section by providing a numerical estimate of the ratios R_Λ, $R_{A^l_{FB}}$, $R_{A^h_{FB}}$ in Table IV for two q^2-integrated bins. All the ratios show a remarkable insensitivity to the form-factor uncertainties.

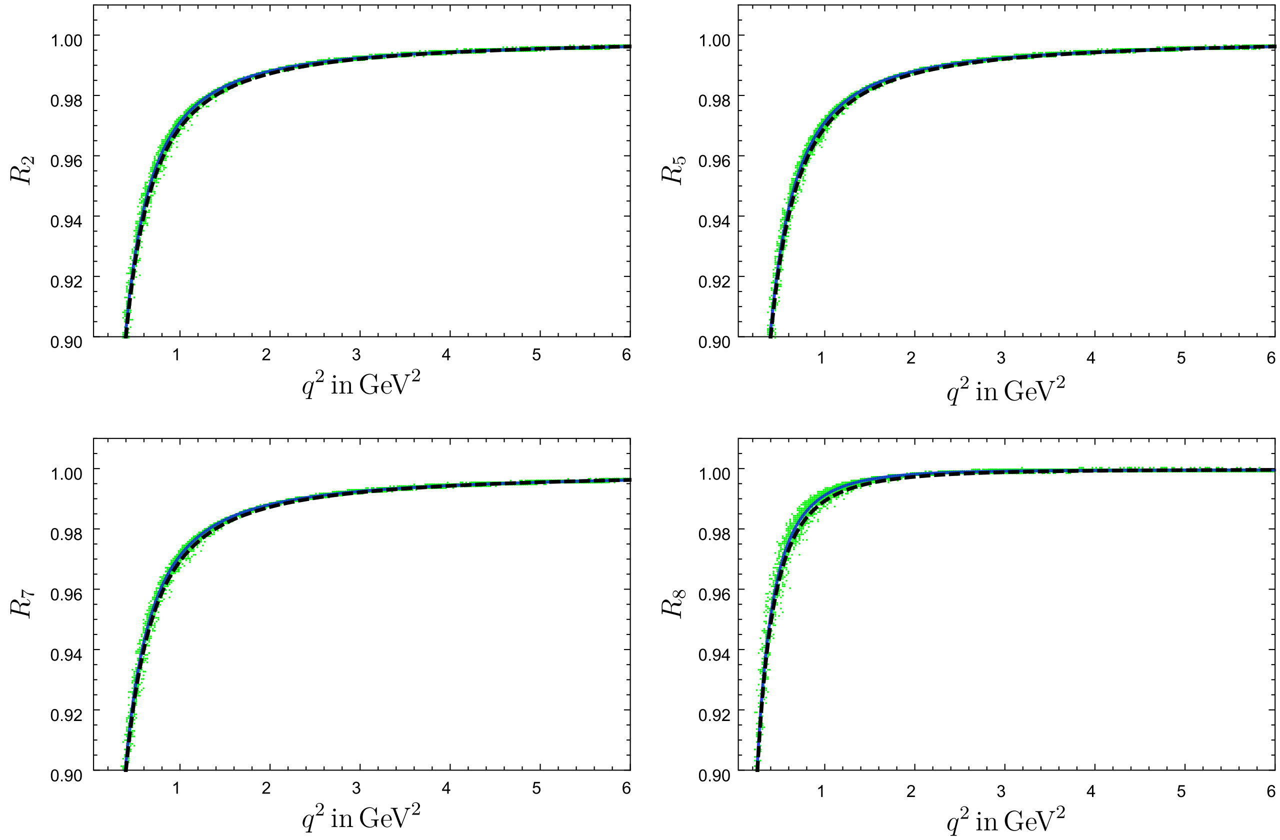

FIG. 4. The q^2 dependence of the ratios of other observables R_2, R_5, R_7, R_8 that are not simple forward-backward asymmetries [see Eqs. (50)–(53)]. The color code is the same as in Fig. 3.

In the above analysis, we have not considered the contribution from long-distance effects. However, this is unlikely to affect our conclusions based on a recent study [70] of long-distance effects in the analogous mode $B \to K^* \mu^+ \mu^-$. It has been shown in Ref. [70] that R_{K^*} is insensitive to long-distance effects within the realm of the SM.

V. CONCLUSIONS

The baryonic decay mode $\Lambda_b \to \Lambda \ell^+ \ell^-$ is similar at the quark level to the much studied mesonic decay mode $B \to K^* \ell^+ \ell^-$ and is hence also expected to provide a plethora of observables that can be used to probe NP and better understand the hadronic effects accompanying the weak decay. While this mode has been a subject of several studies, we have reexamined the decay mode with focus on aspects that have not been studied in detail earlier. We have derived the angular distribution without any approximations. In particular, we retain the finite lepton mass effects and the two timelike amplitudes. These contributions play a significant role in estimating accurately the size of lepton non-universality that may show up in the mode within SM. We estimate R_Λ, which is defined in a manner identical to $R_{K^{(*)}}$ [see Eqs. (1) and (2)]. The nonzero lepton mass effects become increasingly important in the low-q^2 region, where the discrepancy between SM expectation and the experimentally observed value of $R_{K^{(*)}}$ is largest. The observation of a similar discrepancy in R_Λ is therefore necessary to substantiate the idea of lepton non-universality in FCNC processes, since such observations cannot be restricted to the $B \to K^{(*)} \ell^+ \ell^-$ alone. A discrepancy between the estimates presented here and upcoming measurements at LHCb would establish the existence of non-universality in interactions involving fermions on firm footing.

The angular distribution of the decay products in $\Lambda_b \to \Lambda(\to p^+ \pi^-) \ell^+ \ell^-$ decay provides a wealth of information on the nature of decay. This is characterized by the angular observables expressed in terms of helicity amplitudes. We have presented a complete set of ten angular observables that can be measured using this decay mode. We study in detail the three forward-backward asymmetries A^l_{FB}, A^h_{FB}, and A^{hl}_{FB}. It may be noted that no hadron angle forward-backward asymmetry exists for the mode $B \to K^* \ell^+ \ell^-$. The asymmetry A^h_{FB} is found to be especially interesting, since it is nonvanishing in the large-recoil limit, unlike the case with the $B \to K^* \ell^+ \ell^-$ decay mode, where all asymmetries vanish in the low-q^2 limit. This is a consequence of the parity-violating nature of the subsequent $\Lambda \to p\pi$ decay. The nonvanishing asymmetry is particularly sensitive

TABLE IV. The binned values of the observables R_Λ, $R_{A^l_{FB}}$, $R_{A^h_{FB}}$. Only $R_{A^h_{FB}}$ for $q^2 \sim 1$–6 GeV^2 does not show a Gaussian behavior, as it is peaked towards unity.

	Bin 1	Bin 2
Binned ratios	$q^2 \sim 0.045$–1 GeV^2	$q^2 \sim 1$–6 GeV^2
R_Λ	0.907 ± 0.003	0.9885 ± 0.0002
$R_{A^l_{FB}}$	0.9469 ± 0.0007	0.998 ± 0.196
$R_{A^h_{FB}}$	0.993 ± 0.001	>0.9973 (0.999% C.L.)

to new physics that violates lepton flavor universality, since it involves comparing two finite quantities for the cases of $\ell = \mu$ and $\ell = e$, respectively. It may be noted that all the asymmetries in $B \to K^*\ell^+\ell^-$ vanish in the low-q^2 limit, and as such result in comparisons between two vanishing quantities.

We numerically estimate the three asymmetries A^l_{FB}, A^h_{FB}, and A^{hl}_{FB} and the ratios R_Λ, $R_{A^l_{FB}}$, $R_{A^h_{FB}}$, $R_{A^{hl}_{FB}}$. In order to ascertain that our results are not very sensitive to the choice of form factors, we use two different approaches to form factors. We have used both the covariant quark model and soft collinear effective theory to calculate all the observables. We find that all the above mentioned ratios are remarkably insensitive to the choice of factors, as can be seen from Figs. 1 and 3. In order to probe the reliability of the values estimated for these ratios, we have randomly added ±30% error to each form-factor estimate in the covariant quark model to evaluate these ratios as well. In contrast to $R_{A^l_{FB}}$ and $R_{A^{hl}_{FB}}$, $R_{A^h_{FB}}$ is a ratio of two nonvanishing asymmetries at low q^2; hence, it is likely to be more accurately measured. We have numerically estimated the ratios R_Λ, $R_{A^l_{FB}}$, $R_{A^h_{FB}}$ in two q^2-integrated bins. All the ratios show a remarkable insensitivity to the form-factor uncertainties. Since these ratios are expected to be insensitive [70] to long-distance contributions, we conclude that R_Λ and $R_{A^h_{FB}}$ are both experimentally and theoretically reliable observables to test new physics beyond the standard model.

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