

Possible roles of Peccei-Quinn symmetry in an effective low energy model

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The strong CP problem is known to be solved by imposing Peccei-Quinn (PQ) symmetry. However, the domain wall problem caused by the spontaneous breaking of its remnant discrete subgroup could make models invalid in many cases. We propose a model in which the PQ charge is assigned quarks so as to escape this problem without introducing any extra colored fermions. In the low energy effective model resulting after the PQ symmetry breaking, both the quark mass hierarchy and the CKM mixing could be explained through Froggatt-Nielsen mechanism. If the model is combined with the lepton sector supplemented by an inert doublet scalar and right-handed neutrinos, the effective model reduces to the scotogenic neutrino mass model in which both the origin of neutrino masses and dark matter are closely related. The strong CP problem could be related to the quark mass hierarchy, neutrino masses, and dark matter through the PQ symmetry.

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I. INTRODUCTION

The strong CP problem is one of the serious problems in the standard model (SM), which is suggested by the experimental bound of the electric dipole moment of a neutron [1]. Invisible axion models based on the Peccei-Quinn (PQ) symmetry $U(1)_{PQ}$ are known to give a simple and interesting solution to it [2]. Since the models predict the existence of a pseudoscalar called axion [3], which has very small mass and extremely weak interaction with matter, the scenario could be examined through its search. However, domain wall problem makes models invalid in many cases [4]. It is caused by the spontaneous breaking of a discrete symmetry which remains as an exact one through the explicit breaking of $U(1)_{PQ}$ due to QCD anomaly.

A well-known simple model without the domain wall problem is the Kim-Shifman-Vainstein-Zakharov (KSVZ) model [5]. This model has a pair of extra color triplet fermions (D_L, D_R) with the PQ charge and a Yukawa coupling $S\bar{D}_L D_R$ with a singlet scalar S . Although domain walls bounded by the string due to the $U(1)_{PQ}$ breaking are generated in this model, the domain wall problem is not caused because they are not topologically stable [6]. Moreover, after the spontaneous breaking of $U(1)_{PQ}$, the model could have an effective discrete symmetry which could be violated only through the QCD anomaly depending on the charge assignment to the fields. This effective symmetry could play an interesting role in low energy phenomena.¹ In this paper, we consider this kind of possibility for the PQ symmetry without introducing extra colored fermions. We require the model to be free from the domain wall problem and have the above

mentioned remnant effective symmetry which could be responsible in the low energy phenomena.

We start our discussion with examining the Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) model [8] as a typical example. It has a singlet scalar S and two doublet Higgs scalars H_u and H_d . These doublet scalars have weak hypercharge with reverse sign so as to couple with the up- and down- quarks, respectively. Yukawa couplings and scalar potential in this model are fixed as [8]

$$\begin{aligned}
 -\mathcal{L}_{PQ}^y &= \sum_{i,j=1}^3 (y_{ij}^u H_u \bar{q}_{L_i} u_{R_j} + y_{ij}^d H_d \bar{q}_{L_i} d_{R_j} \\
 &\quad + y_{ij}^e H_d \bar{\ell}_{L_i} e_{R_j} + \text{H.c.}), \\
 V_{PQ} &\supset \frac{\kappa}{2} (S^2 [H_u H_d] + \text{H.c.}) + \dots, \quad (1)
 \end{aligned}$$

where $[\psi\phi]$ in V_{PQ} stands for the abbreviation of $\epsilon_{ij}\psi_i\phi_j$. This model could have a global $U(1)_{PQ}$ symmetry which is broken through the QCD anomaly. If we represent its charge of the field f as X_f , the $U(1)_{PQ}$ invariance of quark Yukawa couplings in \mathcal{L}_{PQ}^y requires us to satisfy

$$H_u - X_{q_i} + X_{u_j} = 0, \quad H_d - X_{q_i} + X_{d_j} = 0. \quad (2)$$

On the other hand, if the quarks are transformed for this $U(1)_{PQ}$ as

$$q_{L_i} \rightarrow e^{iX_{q_i}\alpha} q_{L_i}, \quad u_{R_i} \rightarrow e^{iX_{u_i}\alpha} u_{R_i}, \quad d_{R_i} \rightarrow e^{iX_{d_i}\alpha} d_{R_i}, \quad (3)$$

the QCD parameter θ_{QCD} is shifted as a result of anomaly as

$$\begin{aligned}
 \theta_{\text{QCD}} &\rightarrow \theta_{\text{QCD}} + \frac{1}{2} \sum_{i=1}^3 (X_{u_i} + X_{d_i} - 2X_{q_i})\alpha \\
 &= \theta_{\text{QCD}} - \frac{3}{2} (X_{H_u} + X_{H_d})\alpha, \quad (4)
 \end{aligned}$$

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¹There are several articles which study a phenomenological role of the remnant discrete symmetry of $U(1)_{PQ}$ [7].

where Eq. (2) is used. Since this $U(1)_{PQ}$ is assumed to have the QCD anomaly, $X_{H_u} + X_{H_d} \neq 0$ should be satisfied. In this context, a term explicitly shown in V_{PQ} is essential for this model since it requires that the S should have $U(1)_{PQ}$ charge $2X_S = -X_{H_u} - X_{H_d}$. As a result, the $U(1)_{PQ}$ is spontaneously broken through the vacuum expectation value (VEV) of S , and the pseudo-Nambu-Goldstone boson (axion) associated to this breaking could solve the strong CP problem [3]. If the axion decay constant f_a satisfies $10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}$, any cosmological and astrophysical problems are known not to be caused except for the domain wall problem [9].

Since θ_{QCD} has a period 2π in Eq. (4), α should be written as $\alpha = 2\pi \frac{k}{N}$ where $k = 1, \dots, N (\equiv 3X_S)$ for X_S , which is normalized to be an integer. The model is found to have a Z_N symmetry, which is a subgroup of $U(1)_{PQ}$ and corresponds to the symmetry among N degenerate vacua [4].² Since topologically stable domain walls are generated among these vacua for $N > 1$ and they overclose the universe, such models are ruled out cosmologically. In order to escape this situation, one may consider the model with $N = 1$ or the introduction of a suitable explicit breaking of Z_N which resolves the degeneracy among vacua for $N > 1$ [4,10].

Here we consider a PQ charge assignment which realizes both $N = 1$ and the existence of an effective Z_2 symmetry starting from the DFSZ model. For that purpose, we may consider a possibility that only a part of quarks has the PQ charge by introducing a Higgs doublet with no PQ charge. In such a case, we may find $N = X_S$ from the above discussion. On the other hand, the existence of the effective Z_2 symmetry requires $|X_S| = 2$ for the PQ charge normalized as an integer. This suggests that it seems to be difficult to solve the domain wall problem imposing the existence of an effective remnant symmetry Z_2 without introducing extra colored fermions as long as the doublet Higgs is assumed to have the PQ charge.

From this view point, in the following part, we consider the PQ charge assignment in the SM framework supplemented only by a singlet scalar S . First, we assign the PQ charge to the quarks so as not to cause the domain wall problem, that is, to realize $N = 1$ where N is calculated as $N = \frac{1}{2} \sum_f (X_{f_R} - X_{f_L})$. Using this PQ charge assignment, we study quark mass matrices. Next, in order to extend the model to the lepton sector, we introduce an extra doublet scalar η and three right-handed neutrinos N_i . After the PQ symmetry breaking at a high energy scale such as $10^9 \text{ GeV} \lesssim \langle S \rangle \lesssim 10^{12} \text{ GeV}$, the effective model is shown to be reduced to the scotogenic neutrino mass model proposed by Ma [11]. The resulting model has no strong CP problem, and has favorable quark mass hierarchy and

TABLE I. A summary of $U(1)_{PQ}$ charge of quarks. Other field contents of the SM have no PQ charges. Parity for the remnant effective Z_2 is also shown.

	q_{L_1}	q_{L_2}	q_{L_3}	u_{R_1}	u_{R_2}	u_{R_3}	d_{R_1}	d_{R_2}	d_{R_3}	S
X_f	-4	-2	0	4	2	0	-10	-8	2	-2
Z_2	+	+	+	+	+	+	+	+	+	+

CKM mixing in addition to several features of the original scotogenic model such as the explanation of neutrino mass generation, leptogenesis and the existence of dark matter [12–15].

We modify the DSFZ model without introducing new colored fermions but changing the PQ charge assignment to the quarks as shown in Table I. As in the SM, the model has only one Higgs doublet ϕ which has no PQ charge. The singlet scalar S has the PQ charge. Since ϕ has no PQ charge, quark Yukawa couplings are allowed only for the top quark as a renormalizable term. However, if we take account of nonrenormalizable Yukawa couplings containing the singlet scalar S , the following couplings are found to be allowed,

$$\begin{aligned}
-\mathcal{L}_y = & \sum_{i=1}^3 \left[\sum_{j=1}^3 y_{ij}^u \left(\frac{S}{M_*} \right)^{\frac{1}{2}(X_{u_{R_j}} - X_{q_{L_i}})} \bar{q}_{L_i} \phi u_{R_j} \right. \\
& + \sum_{j=1}^2 y_{ij}^d \left(\frac{S^*}{M_*} \right)^{\frac{1}{2}(X_{d_{R_j}} - X_{q_{L_i}})} \bar{q}_{L_i} \tilde{\phi} d_{R_j} \\
& \left. + y_{i3}^d \left(\frac{S}{M_*} \right)^{\frac{1}{2}(X_{d_{R_3}} - X_{q_{L_i}})} \bar{q}_{L_i} \tilde{\phi} d_{R_3} + \text{H.c.} \right], \quad (5)
\end{aligned}$$

where $\tilde{\phi} = i\tau_2 \phi^*$ and M_* is a cutoff scale of the model. In this case, Eq. (4) is easily found to be written as

$$\theta_{\text{QCD}} \rightarrow \theta_{\text{QCD}} + \alpha. \quad (6)$$

This shows that the model has no degenerate vacua ($N = 1$) and then no domain wall problem exists.

After the PQ symmetry breaking, the above nonrenormalizable Yukawa terms induce a suppression factor to each Yukawa coupling which is determined by the PQ charge just like Froggatt-Nielsen mechanism [16].³ If we take account of the PQ charge in Table I, the quark mass matrices defined by $\bar{u}_L \mathcal{M}_u u_R$ and $\bar{d}_L \mathcal{M}_d d_R$ can be written as

$$\begin{aligned}
\mathcal{M}_u = & \begin{pmatrix} y_{11}^u \epsilon^4 & y_{12}^u \epsilon^3 & y_{13}^u \epsilon^2 \\ y_{21}^u \epsilon^3 & y_{22}^u \epsilon^2 & y_{23}^u \epsilon \\ y_{31}^u \epsilon^2 & y_{32}^u \epsilon & y_{33}^u \end{pmatrix} \langle \phi \rangle, \\
\mathcal{M}_d = & \begin{pmatrix} y_{11}^d \epsilon^3 & y_{12}^d \epsilon^2 & y_{13}^d \epsilon^3 \\ y_{21}^d \epsilon^4 & y_{22}^d \epsilon^3 & y_{23}^d \epsilon^2 \\ y_{31}^d \epsilon^5 & y_{32}^d \epsilon^4 & y_{33}^d \epsilon \end{pmatrix} \langle \phi \rangle, \quad (7)
\end{aligned}$$

²The axion decay constant f_a is connected to the PQ symmetry breaking scale $\langle S \rangle$ by $f_a = \langle S \rangle / N$.

³The possibility to identify the PQ symmetry with the Froggatt-Nielsen symmetry has already been discussed in [17].

where $\epsilon \equiv \frac{\langle S \rangle}{M_*}$ is used. In order to examine the features of these mass matrices, we take an assumption for simplification such as

$$\begin{aligned} y_{11}^u &= y_{23}^u = y_{32}^u = y_{33}^u = 1, & y_{13}^u &= y_{22}^u = y_{31}^u = 0.1, \\ y_{12}^u &= y_{21}^u = 0.7, & y_{21}^d &= y_{22}^d = y_{31}^d = y_{32}^d = 1, \\ y_{11}^d &= y_{13}^d = y_{23}^d = 0.1, & y_{12}^d &= 0.022, & y_{33}^d &= 0.3. \end{aligned} \quad (8)$$

If we assume $\epsilon = 0.08$ in these matrices, we can obtain the quark mass eigenvalues and the Cabibbo-Kobayashi-Maskawa (CKM) model matrix as follows,

$$\begin{aligned} m_u &= 2.6 \text{ MeV}, & m_c &= 1.1 \text{ GeV}, & m_t &= 174 \text{ GeV}, \\ m_d &= 6.7 \text{ MeV}, & m_s &= 92 \text{ MeV}, & m_b &= 4.2 \text{ GeV}, \end{aligned} \quad (9)$$

and

$$V_{\text{CKM}} = \begin{pmatrix} 0.974 & -0.226 & -0.00515 \\ 0.226 & 0.974 & -0.0184 \\ 0.00918 & 0.0168 & 0.9998 \end{pmatrix}. \quad (10)$$

Although the number of free parameters are restricted to 8 in the present case and we do not make severe tuning for them as shown in Eq. (8), these results seem to be rather good [18].

Here we order some remarks related to the present PQ charge assignment. Since the PQ charge is assigned to the quarks in the flavor dependent way, the axion-quarks couplings are not diagonal to induce the flavor changing neutral current processes such as $K^\pm \rightarrow \pi^\pm a$ and then the PQ breaking scale $\langle S \rangle$ is constrained by them. This problem is studied in [17]. If we follow their analysis, the PQ breaking scale $\langle S \rangle$ is found to have to satisfy $\langle S \rangle \gtrsim 8 \times 10^{10}$ GeV in the present scenario. It gives a stronger constraint on $\langle S \rangle$ than the astrophysical one. Apart from the above interesting feature, it is a crucial problem how this axion model could be distinguished from others based on different PQ charge assignments. On this point, we should note that the axion nature determined by the PQ charge assignment might be experimentally examined through the axion-photon coupling [19,20]. The present model predicts it as

$$g_{a\gamma\gamma} = \frac{m_a}{\text{eV}} \frac{2.0}{10^{10} \text{ GeV}} \times 1.75. \quad (11)$$

It is also useful to note that this relation is not affected if the fourth generation quarks exist in this model. This is because the domain wall free requirement imposes their PQ charge should be vectorlike.

In order to apply this model to the lepton sector, we combine it with the scotogenic neutrino mass model. For this purpose, we introduce a new doublet scalar η and three right-handed neutrinos N_i and assign the PQ charge them as -1 and 1 , respectively. The leptons in the SM are assumed to have no PQ charge. This charge assignment shows that only the η and N_i have odd parity of the Z_2 symmetry which remains as an effective symmetry of the model. Although it is broken by the QCD anomaly, this breaking does not affect the lepton sector. Thus, it could play the same role as the Z_2 in the scotogenic model.

Yukawa couplings in the lepton sector and scalar potential are represented as [21]

$$\begin{aligned} -\mathcal{L}_y^\ell &= \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^3 h_{\alpha i} \bar{\ell}_\alpha \eta N_i + \sum_{i=1}^3 y_i S \bar{N}_i^c N_i + \text{H.c.}, \\ V &= m_S^2 S^\dagger S + \kappa_1 (S^\dagger S)^2 + \kappa_2 (S^\dagger S) (\phi^\dagger \phi) + \kappa_3 (S^\dagger S) (\eta^\dagger \eta) \\ &\quad + m_\phi^2 \phi^\dagger \phi + m_\eta^2 \eta^\dagger \eta + \lambda_1 (\phi^\dagger \phi)^2 + \lambda_2 (\eta^\dagger \eta)^2 \\ &\quad + \lambda_3 (\phi^\dagger \phi) (\eta^\dagger \eta) + \lambda_4 (\phi^\dagger \eta) (\eta^\dagger \phi) \\ &\quad + \frac{\lambda_5}{2} \left[\frac{S}{M_*} (\eta^\dagger \phi)^2 + \text{H.c.} \right], \end{aligned} \quad (12)$$

where we add a dimension 5 term to V as the lowest order one. We find that Eqs. (5) and (12) present the most general Yukawa couplings and scalar potential which contains the lowest order terms invariant under the assumed symmetry.

After the symmetry breaking due to $\langle S \rangle \neq 0$, N_i , and S are found to get masses such as $M_i = y_i \langle S \rangle$ and $M_S^2 = 4\kappa_1 \langle S \rangle^2$, respectively. The effective model at the scale below M_S could be obtained by integrating out S . If we do it by using the equation of motion for S , we can obtain the corresponding low energy effective model. Its scalar potential composed of the light scalars can be written as [21]

$$\begin{aligned} V_{\text{eff}} &= \tilde{m}_\phi^2 (\phi^\dagger \phi) + \tilde{m}_\eta^2 (\eta^\dagger \eta) + \tilde{\lambda}_1 (\phi^\dagger \phi)^2 + \tilde{\lambda}_2 (\eta^\dagger \eta)^2 \\ &\quad + \tilde{\lambda}_3 (\phi^\dagger \phi) (\eta^\dagger \eta) + \lambda_4 (\phi^\dagger \eta) (\eta^\dagger \phi) \\ &\quad + \frac{\tilde{\lambda}_5}{2} [(\eta^\dagger \phi)^2 + \text{H.c.}], \end{aligned} \quad (13)$$

where we use the shifted parameters which are defined as

$$\begin{aligned} \tilde{\lambda}_1 &= \lambda_1 - \frac{\kappa_2^2}{4\kappa_1}, & \tilde{\lambda}_2 &= \lambda_2 - \frac{\kappa_3^2}{4\kappa_1}, & \tilde{\lambda}_3 &= \lambda_3 - \frac{\kappa_2 \kappa_3}{2\kappa_1}, \\ \tilde{\lambda}_5 &= \lambda_5 \frac{\langle S \rangle}{M_*}, & \tilde{m}_\phi^2 &= m_\phi^2 + \kappa_2 \langle S \rangle^2, & \tilde{m}_\eta^2 &= m_\eta^2 + \kappa_3 \langle S \rangle^2. \end{aligned} \quad (14)$$

This effective model obtained after the spontaneous breaking of $U(1)_{PQ}$ is just the original scotogenic model with a Z_2 symmetry [11], which connects the neutrino mass generation with the DM existence. In the present case, the

right-handed neutrinos do not have their masses in a TeV region but they are considered to be much heavier. The coupling $\tilde{\lambda}_5$ which is crucial for the one-loop neutrino mass generation is derived from a nonrenormalizable term as a result of the $U(1)_{PQ}$ breaking. The model also contains the inert doublet scalar η whose lightest component can be DM since it has odd parity of the remnant Z_2 . It has charged components η^\pm and two neutral components $\eta_{R,I}$. Their mass eigenvalues can be expressed as

$$M_{\eta^\pm}^2 = \tilde{m}_\eta^2 + \tilde{\lambda}_3 \langle \phi \rangle^2, \quad M_{\eta_{R,I}}^2 = \tilde{m}_\eta^2 + (\tilde{\lambda}_3 + \lambda_4 \pm \tilde{\lambda}_5) \langle \phi \rangle^2. \quad (15)$$

We suppose $\tilde{m}_\eta = O(1)$ TeV although it requires fine tuning because of $|\langle S \rangle| \gg |\langle \phi \rangle|$.

Phenomenology and the related analysis in the lepton sector is almost the same as the one given in [21] where the extension of the KSVZ model is studied. However, in order to make the paper self-contained, we briefly review important points on neutrino mass generation, leptogenesis, and DM relic abundance and also add some analyses which are changed from the ones given in [21] here.

A. Neutrino mass generation

The model contains the heavy right-handed neutrinos, whose Yukawa couplings with the doublet leptons ℓ_α and the inert doublet scalar η are shown in the above \mathcal{L}_y^ℓ . However, neutrino masses are not generated at tree-level since η is assumed to have no VEV. They can be generated radiatively through the one-loop diagram shown in Fig. 1 since both the right-handed neutrino masses and the mass difference between η_R and η_I are induced after the $U(1)_{PQ}$ breaking. The latter is generated through the $\tilde{\lambda}_5 (\eta^\dagger \phi)^2$ as easily found from Eq. (15). If we note that $M_{\eta_{R,I}}^2 \gg |M_{\eta_R}^2 - M_{\eta_I}^2|$ is satisfied in the present model, the neutrino mass formula can be approximately expressed as

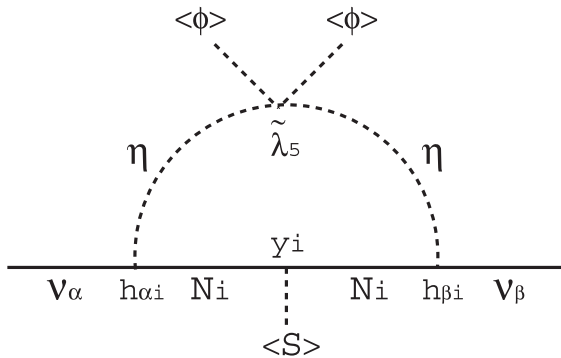


FIG. 1. An effective one-loop diagram which generates neutrino masses through the PQ symmetry breaking.

$$\mathcal{M}_{\alpha\beta} = \sum_i h_{\alpha i} h_{\beta i} \Lambda_i, \quad \Lambda_i \simeq \frac{\tilde{\lambda}_5 \langle \phi \rangle^2}{8\pi^2 M_i} \ln \frac{M_i^2}{\tilde{M}_\eta^2}, \quad (16)$$

where $\tilde{M}_\eta^2 = \tilde{m}_\eta^2 + (\tilde{\lambda}_3 + \lambda_4) \langle \phi \rangle^2$.

For simplicity, we assume the flavor structure of neutrino Yukawa couplings $h_{\alpha i}$ which induces the tri-bimaximal mixing. It is considered to be rather good approximation for the lepton mixing for the purpose of the present study. It can be realized for [13]

$$h_{ej} = 0, \quad h_{\mu j} = h_{\tau j} \equiv h_j (j = 1, 2); \\ h_{e3} = h_{\mu 3} = -h_{\tau 3} \equiv h_3, \quad (17)$$

where the charged lepton mass matrix is assumed to be diagonal. In that case, the mass eigenvalues are estimated as

$$m_1 = 0, \quad m_2 = 3|h_3|^2 \Lambda_3, \\ m_3 = 2[|h_1|^4 \Lambda_1^2 + |h_2|^4 \Lambda_2^2 \\ + 2|h_1|^2 |h_2|^2 \Lambda_1 \Lambda_2 \cos 2(\theta_1 - \theta_2)]^{1/2}, \quad (18)$$

where $\theta_j = \arg(h_j)$. We find that the squared mass differences required by the neutrino oscillation data could be derived [18] if the parameters relevant to the neutrino masses are fixed as

$$M_1 = 10^8 \text{ GeV}, \quad M_2 = 4 \times 10^8 \text{ GeV}, \quad M_3 = 10^9 \text{ GeV}, \\ |h_1| = 10^{-4.5}, \quad |h_2| \simeq 7.2 \times 10^{-4} \tilde{\lambda}_5^{-0.5}, \\ |h_3| \simeq 3.1 \times 10^{-4} \tilde{\lambda}_5^{-0.5}, \quad (19)$$

for $\tilde{m}_\eta = 1$ TeV, for example.

B. Leptogenesis

The mass formula (18) and the condition (19) suggest that the requirement for neutrino masses could be satisfied by using two right-handed neutrinos only. Thus, both mass and neutrino Yukawa couplings of a remaining right-handed neutrino are free from the neutrino oscillation data if its contribution to the neutrino mass is negligible. Such a situation is found to be realized for $|h_1|^2 \Lambda_1 \ll |h_2|^2 \Lambda_2$ in Eq. (18). It is suitable for thermal leptogenesis [22] since a sufficiently small neutrino Yukawa coupling h_1 makes the out-of-equilibrium decay of N_1 efficient.

Using the values of parameters shown in Eq. (19), we find the required baryon number asymmetry could be generated for $M_1 \gtrsim 10^8$ GeV through the decay of the thermal N_1 by solving the Boltzmann equation. An interesting point is that the lightest right-handed neutrino mass could be smaller than the Davidson-Ibarra bound [23] in the ordinary thermal leptogenesis. We have $Y_B (\equiv \frac{n_B}{s}) > 8.3 \times 10^{-11}$ for a maximal CP phase in the CP violation parameter ϵ and the parameter set given in (19) if $\tilde{\lambda}_5$ takes a value in the range $8 \times 10^{-4} \lesssim \tilde{\lambda}_5 \lesssim 2 \times 10^{-2}$ [21]. An assumed value $\epsilon = 0.08$ in Eqs. (9) and (10) requires λ_5

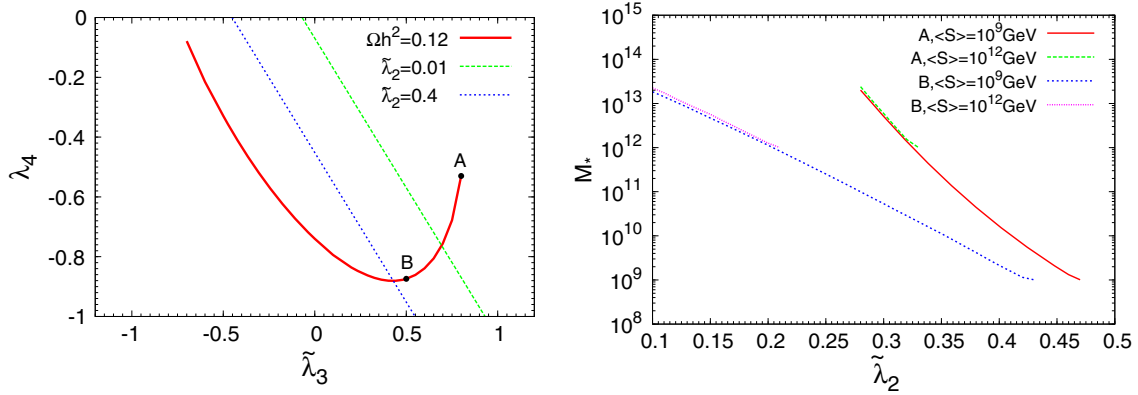


FIG. 2. Left panel: Points plotted by a red solid line in the $(\tilde{\lambda}_3, \lambda_4)$ plane can realize the required DM relic abundance $\Omega h^2 = 0.12$. The last condition in Eq. (21) is satisfied at a region above a straight line which represents $\tilde{\lambda}_3 + \lambda_4 = |\tilde{\lambda}_5| - 2\sqrt{\tilde{\lambda}_1 \tilde{\lambda}_2}$ for a fixed $\tilde{\lambda}_2$. Right panel: Cutoff scale M_* as a function of $\tilde{\lambda}_2$ which is fixed as a value at M_Z . These are plotted for two points A (0.80, -0.530) and B (0.50, -0.874) which are marked by the black bulbs in the left panel.

to be in the range $0.01 \lesssim \lambda_5 \lesssim 0.25$. It seems to be a natural value as a coefficient of the nonrenormalizable term and also to be a consistent value in comparison with the values used in Eq. (8).

C. DM relic abundance

As is well known, the axion can be a good DM candidate. Its energy density in the present universe is estimated as [10]

$$\Omega_a h^2 = 2 \times 10^4 \left(\frac{\langle S \rangle}{10^{16} \text{ GeV}} \right)^{7/6} \langle \theta_i^2 \rangle, \quad (20)$$

where $\langle \theta_i \rangle$ stands for the initial axion misalignment. The axion contribution to the DM abundance crucially depends on $\langle \theta_i \rangle$. We find that it could be too small to give the required value $\Omega_{\text{DM}} h^2 = 0.12$ [18] for $\langle S \rangle < 10^{11}$ GeV even if we assume $\langle \theta_i \rangle = O(1)$. However, the model has a unique Z_2 odd field η among the weak scale fields. The lightest neutral component η_R could be a good DM candidate in case of $\lambda_4, \tilde{\lambda}_5 < 0$. In fact, since \tilde{m}_η is assumed to be of $O(1)$ TeV in this model, the mass of each component of η could be degenerate sufficiently for wide range values of $\tilde{\lambda}_3$ and λ_4 as found from Eq. (15). This makes the coannihilation among the components of η effective enough to reduce the η_R abundance [15,24]. Thus, if the couplings $\tilde{\lambda}_3$ and λ_4 take suitable values, the relic abundance of η_R can be tuned to the required value. In the left panel of Fig. 2, such points in the $(\tilde{\lambda}_3, \lambda_4)$ plane are plotted by a red solid line. The figure shows that $\tilde{\lambda}_3$ and/or $|\lambda_4|$ are required to take rather large values for realization of the DM abundance.⁴

⁴The axion could be the dominant component of DM for $\langle S \rangle > 10^{11}$ GeV and $\langle \theta_i \rangle = O(1)$. In that case, we need to consider larger $|\lambda_{3,4}|$ to reduce the η_R abundance. In this paper, we focus our study on the η_R dominated DM case.

A problem related to these values is their influence to the vacuum stability and the perturbativity of the model. It can be examined by solving the renormalization group equations (RGEs) for the quartic scalar couplings. Since the model has no extra colored fermion and the running of gauge couplings changes from the one found in [21], we should take account of it in the present analysis. Here we note that vacuum stability conditions in this model are given at a scale below M_S as [25]

$$\begin{aligned} \tilde{\lambda}_1 > 0, \quad \tilde{\lambda}_2 > 0, \quad \tilde{\lambda}_3 > -2\sqrt{\tilde{\lambda}_1 \tilde{\lambda}_2}, \\ \tilde{\lambda}_3 + \lambda_4 - |\tilde{\lambda}_5| > -2\sqrt{\tilde{\lambda}_1 \tilde{\lambda}_2}. \end{aligned} \quad (21)$$

At a scale above M_S , in addition to the same conditions for $\lambda_{1,2,3}$ as Eq. (21) except for the last one, new conditions

$$\kappa_1 > 0, \quad \kappa_2 > -2\sqrt{\lambda_1 \kappa_1}, \quad \kappa_3 > -2\sqrt{\lambda_2 \kappa_1}, \quad (22)$$

should be satisfied. The couplings in both regions are connected through Eq. (14).

If we use $\tilde{\lambda}_1$ determined by the observed Higgs mass, we can find an a region in the $(\tilde{\lambda}_3, \lambda_4)$ plane allowed by the stability for a fixed $\tilde{\lambda}_2$ by applying the last condition in Eq. (21). Here we should remind the reader that $\tilde{\lambda}_5$ is restricted to small values through the leptogenesis. In the left panel of Fig. 2, we plotted lines $\tilde{\lambda}_3 + \lambda_4 - |\tilde{\lambda}_5| = -2\sqrt{\tilde{\lambda}_1 \tilde{\lambda}_2}$ for $\tilde{\lambda}_2 = 0.01$ and 0.4 . Points above the line corresponding to each $\tilde{\lambda}_2$ satisfy the stability condition at the weak scale. From this figure, we can find values of $\tilde{\lambda}_3$ and λ_4 which are used as initial values at the weak scale for the RGEs study. Large values required for $\tilde{\lambda}_3$ and/or $|\lambda_4|$ are expected to improve the situation for the vacuum stability

up to M_S compared with the SM since they give positive contributions to the β -function for the quartic coupling $\tilde{\lambda}_1$. On the other hand, their large contributions to the β -function of the scalar quartic couplings $\tilde{\lambda}_i$ tend to make the perturbativity of the model break at a scale below $\langle S \rangle$. We identify a perturbativity violating scale with the cutoff scale M_* of the model so that M_* is fixed at a scale where $\lambda_i(M_*) > 4\pi$ happens for any λ_i . The validity of the present scenario is guaranteed only for $\langle S \rangle < M_*$.

We estimate M_* by using one-loop RGEs and checking the vacuum stability at each scale. Since the PQ symmetry breaking scale is constrained through the axion physics, M_* should be in the range $1.25 \times 10^{10} \text{ GeV} \lesssim M_* \lesssim 1.25 \times 10^{12} \text{ GeV}$ for $\epsilon = 0.08$ which is used in this model. In the right panel of Fig. 2, M_* is plotted as a function of $\tilde{\lambda}_2$ for two points A and B in the $(\tilde{\lambda}_3, \lambda_4)$ plane where the required relic abundance is realized. In this study, $M_s = \langle S \rangle$ is assumed and quartic couplings κ_i are fixed as $\kappa_1 = \frac{M_s^2}{4\langle S \rangle^2}$ and $\kappa_{2,3} = 0.1$ at M_s .⁵ A lower endpoint in each line corresponds to $M_* = \langle S \rangle$. Upper endpoints found in lines for the case A stand for the value of $\tilde{\lambda}_2$ where the vacuum stability is violated before reaching M_* . This figure shows that M_* could take a consistent value with ϵ used here and $\langle S \rangle$ in the range imposed by the axion physics as long as $\tilde{\lambda}_2$ takes a suitable value.

Finally, we summarize the paper. We have proposed an invisible axion model free from the domain wall problem by taking a novel PQ charge assignment to the quarks.

⁵It is useful to note that larger values of $\kappa_{2,3}$ make M_* smaller.

The PQ charge in the quark sector can play a role of $U(1)$ charge in the Froggatt-Nielsen mechanism so that both the favorable quark mass eigenvalues and the CKM matrix are obtained. An extension of the model to the lepton sector can be done by introducing an extra inert doublet scalar η and three right-handed neutrinos N_i as new ingredients. After the PQ symmetry breaking, the effective low energy model can be identified with the scotogenic model for neutrino masses. It has an effective Z_2 symmetry as a remnant of the $U(1)_{PQ}$ symmetry, which is violated only through the QCD anomaly. Under this Z_2 symmetry, only these new ones η and N_i have its odd parity and the lightest one could be a good DM candidate just as the scotogenic neutrino mass model. In the similar way as it, the neutrino masses are generated at one-loop level and the DM abundance can be explained by the relic neutral component of η .

The baryon number asymmetry could be generated through the out-of-equilibrium decay of the lightest right-handed neutrino just as the ordinary thermal leptogenesis in the tree-level seesaw model. However, the mass bound of the lightest right-handed neutrino could be relaxed. This simple extension can relate the strong CP problem to the quark mass hierarchy and also the origin of neutrino masses and DM. The model might suggest a hint for a new way to construct models beyond the SM.

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