NLO QCD corrections to triply heavy baryon fragmentation function considering the effect of nonperturbative dynamics of baryon bound states

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Using a quark-diquark model of baryons and by working in the framework of perturbative quantum chromodynamics we present, for the first time, the next-to-leading-order (NLO) corrections to the fragmentation process of a heavy quark into a ground state triply heavy baryon. For a complete analysis, we also study the effect of nonperturbative dynamic of baryon bound state on the extracted fragmentation functions considering a typical wave function which is different from the Dirac-delta function that was applied in all previous works. Our results show that the NLO corrections are significant, while the effect of the baryonic wave function is of minor importance.

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I. INTRODUCTION

The quark model of particle physics predicts heavy flavor baryons containing single, doubly, or triply heavy charm or bottom quarks. The first state containing one heavy flavor are interesting cases because of the fact that they carry the spin of original heavy flavor [1]. Their production has been studied theoretically [2] and experimentally [3], so that all heavy baryons with single charm quark have been yet discovered in the experiments. Some of heavy baryons with single bottom quark, like Λ_b baryon with the quark structure $|udb\rangle$ and spin-$1/2$ as well as $\Lambda_b^\star$ baryon with the spin-$3/2$, have been observed in the experiments (for a review see [4]).

Baryons with two heavy flavors, such as Ξ_{cc}, Ξ_{bb}, and Ξ_{bc}, are treated within the approximate quark-diquark model [5]. The light degree of freedom within these states does not allow full perturbative calculations for their production. At present, among all possible doubly heavy baryon states only the spin-$1/2$ Ξ_{cc}^+ charmed baryon have experimentally been observed by the SELEX Collaboration [6–8]. From the theoretical side, there are a lot of works in the literature devoted to the spectroscopy and decay properties of the heavy baryons containing single heavy quark. There are also dozens of works dedicated to the study of the properties of the doubly heavy baryons.

The third category of baryons are the color-singlet bound states of three heavy flavors, i.e. Ω_{bbb}, Ω_{bbc}, Ω_{bcc}, and Ω_{ccc}. Triply heavy baryons would reveal a pure baryonic spectrum without valence light-quark complications and provide valuable insight into the quark confinement mechanism. Their masses have been studied in framework of the various approaches such as effective field theory, lattice QCD, bag model, various quark models, variational approach, hyper central model, potential model and Regge trajectory ansatz [9–14]. Information on the masses of these baryons can play essential role in understanding the heavy quark dynamics and they might yield sharp tests for QCD. Actually, the triply heavy baryons are very interesting hadrons to be explored for they provide particular information about strong interactions, hadron structures, and weak decays of heavy baryons. Although, they have not been observed yet but their relevance has been emphasized since long ago [15]. However, there are limited numbers of works devoted to the investigation of the properties of the triply heavy baryons. With no experimental information available on these systems, previous studies have concentrated on their spectrum. In Ref. [16] a possibility for discovery of the triply charmed Ω_{ccc} baryons was discussed. In [17], production of Ω_{ccc} state in e^+e^- annihilation is studied and in [18] it is studied the Ω_{ccc} production via fragmentation at LHC. In [19], the S-wave hadronic production cross sections of the Ω_{bcc} and Ω_{ccc} at the CERN LHC is studied in a direct production mechanism, theoretically. They have concluded that it is quite promising to discover them at the LHC. With this in mind, study of their properties beyond spectroscopy seems timely. The experimental attempts, especially at the CERN LHCb, have still been continuing to complete the remaining members of the heavy baryons predicted by the quark model.

It is well known that the dominant production mechanism for heavy baryons at high transverse momentum is fragmentation [20]. The fragmentation refers to the process of a parton with high transverse momentum which subsequently decays to form a jet including the expected hadron. The specific importance of fragmentation functions (FFs) is for their model independent predictions of the cross sections at colliders. In [21], in a direct fragmentation process we obtained an analytical result for the fragmentation of a heavy quark into a ground state triply heavy baryon at lowest order of perturbative QCD. In the present

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work, in a two-step fragmentation process based on the quark-diquark model we extend it to next-to-leading order (NLO) in α_s. This is the first attempt to include the one-loop corrections to the heavy baryon FFs and our results show that the NLO corrections are significant. To complete our analysis, we also study the effect of baryon wave function on the extracted FFs; a topic with a little attention in the literature. The baryon wave function contains the bound state nonperturbative dynamic of produced baryon. In all manuscripts the wave function of bound state was estimated by a delta function which simplifies the calculations, for example see Refs. [22–29]. By this estimation, the Fermi motion of constituents is ignored. Here, we will consider the real aspects of the baryon bound state and show how this effect changes the FF of baryons. Considering both effects due to the QCD one-loop corrections and the changes due to including a more realistic baryon wave function, we present our numerical results for the heavy baryon fragmentation functions. Since, there are no experimental data for triply heavy baryons then, at the present, it is not possible to compare our theoretical results to the real world.

The plan of paper goes as follows. In Sec. II, we describe the quark-diquark model briefly, and in Sec. III this model is applied to calculate the FF of heavy baryons at lowest-order of perturbative QCD. In Sec. IV, in a different approach and on the basis of quark-diquark model we calculate the baryon FF in a two-step fragmentation at LO. In this section, we also consider two corrections to the LO results: NLO QCD corrections and the effect of baryon wave function. Our conclusion is summarized in Sec. V.

II. QUARK-DIQUARK MODEL

The FFs are related to the low-energy part of the hadroproduction processes and form the nonperturbative aspect of QCD. Fortunately, it was found that these functions for heavy hadron productions are analytically calculable by virtue of perturbative QCD with limited phenomenological parameters. Triply heavy baryons have simple internal structures and the perturbative QCD approximations to their FFs are well-defined in the non-relativistic QCD factorization framework [30,31]. However, the FFs calculated in such approach are lengthy and are not favored from experimental point of view. To cast these functions into brief expressions one possible and valuable way is to approximate the production of two heavy quark pairs in the perturbative picture by a pair of heavy diquark: quark-diquark model. The physical principle behind this idea is the unification of any two quarks to form a colored quasi-bound state of diquark. This choice allows first of all to explore the possibility of employing the heavy flavor diquarks in this situation and secondly to obtain rather brief expressions for the FFs. The concept of diquark is useful phenomenological model which is introduced by Gell-Mann [32] and has proven to be very helpful in the phenomenology of the strong interactions. The quark-diquark idea is used extensively in hadron physics where nucleons have received special attention, e.g. see Refs. [33–36]. Its application has also been extended to treat the production and decay of baryons involving two or three heavy flavors.

The composite nature of the diquark is taken into account by diquark form factors (phenomenological n-point vertex functions). Since a pointlike diquark has the quantum numbers of two-quark system, a diquark in its ground state might be a scalar or a pseudovector. Note that, due to the Fermi statistics the ground state diquark composed of different flavors is a scalar spin-0 state, while a diquark with identical constituents should be considered in a vector spin-1 state. In the quark-diquark model, ground state heavy baryons are composed of a heavy quark and a heavy diquark system with spin-0 or spin-1, moving in a S-wave state. All possible heavy baryons along with their corresponding production processes are shown in Table I.

For a scalar diquark, a single form factor is involved. However for a pseudovector diquark, several form factors will be relevant and usually the calculation of FFs leads to numerical evaluation of the properties of the fragmentation production [2]. Therefore, in this section we consider a scalar diquark in our calculation for the fragmentation of heavy quark into the spin-$1/2$ triply heavy baryons. In other words, we restrict ourselves to the splitting processes $c \to \Omega_{bcc}$ and $b \to \Omega_{bbc}$ where the structure of scalar diquark is bc, see Table I.

The color form factor of a heavy diquark is unknown and one has to assume a certain dependence on the momentum transfer squared regarding the formation of a diquark. Parameterization of diquark form factors is obtained from the requirement that, asymptotically the diquark model evolves into the pure quark model. Since there is no direct information about chromomagnetic form factors, then we may expect that the ordinary electromagnetic form factors will have the same functional form as their QCD counterparts. This is due to the fact that the source of both form factors is the matrix elements of a conserved vector operator. Here, the vector operator is the color octet gluon field so the parameterization of the diquark form factors may be inferred

TABLE I. Ground state baryons with corresponding possible spin states when considered to be formed in the quark-diquark model. The relevant diquark spin states are also shown.

Baryon production process	Diquark spin	Baryon spin
$c \to \Omega_{ccc}, \Omega^\star_{ccc}$	1	$\frac{1}{2}, \frac{3}{2}$
$b \to \Omega_{bbb}, \Omega^\star_{bbb}$	1	$\frac{1}{2}, \frac{3}{2}$
$b \to \Omega_{bbc}$	0	$\frac{1}{2}$
$c \to \Omega_{bbc}, \Omega^\star_{bbc}$	1	$\frac{1}{2}, \frac{3}{2}$
$c \to \Omega_{bcc}$	0	$\frac{1}{2}$
$b \to \Omega_{bcc}, \Omega^\star_{bcc}$	1	$\frac{1}{2}, \frac{3}{2}$

from quark-diquark models of the nucleons. In this view, the form factor $F_S(q^2) = q_S^2/(q_S^2 - q^2)$ is used as a relevant form factor in the spacelike region [2] where q is the momentum transfer. This scalar diquark form factor is assumed to have a simple pole, with pole position at q_S above 1 GeV so that the value of q_S is very different for a vector diquark. The form factor mentioned works nicely with the light quarks. In the case of heavy diquarks, the following expression

$$F_S(q^2) = \frac{q_S^2}{q^2}, \tag{1}$$

is more useful [37]. We apply this form in our calculation. The parameter q_S is a free parameter so has a crucial role in determination of the fragmentation probabilities. We will comment more about its values when our numerical results are presented.

III. HEAVY QUARK FF IN QUARK-DIQUARK MODEL

The fragmentation mechanism is characterized by the function $D_Q^H(z,\mu)$ which refers to the probability for a parton Q at the fragmentation scale μ to fragment into a hadron H carrying away a fraction $z = (E + p_\parallel)_H/(E + p_\parallel)_Q$ of its momentum.

There are several different approaches to calculate the FFs, so once they are given at the initial scale μ_0, their μ evolution is determined by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) renormalization group equations [38]. In [39,40], we applied the phenomenological approaches based on the experimental data to extract the FF of light mesons.

The first theoretical attempt to explain the production procedure of mesons containing a heavy flavor quark was made by Bjorken [41]. The perturbative QCD approach was followed by Suzuki [42], Ji and Amiri [43] by considering more elaborate models. While, Suzuki calculates the heavy quark FFs using a Feynman diagram similar to that in Fig. 1, Amiri and Ji compute their FFs in e^+e^- annihilation in the same order of perturbative QCD. The Suzuki model includes most of the kinematical and dynamical properties of the splitting process and gives us a detailed insight about the hadronization process. In this approach, Suzuki calculates the FF of a heavy quark using the convenient Feynman diagrams for the parton level of the process and also by considering the wave function of heavy meson/baryon which contains the nonperturbative dynamic of hadroproduction process. In fact, the Suzuki model mixes a perturbative picture with nonperturbative dynamics of fragmentation and not only predicts the z-dependence of the FFs, but also their dependence on the transverse momentum of the hadron relative to the jet direction.

Here, using the Suzuki model and applying the quark-diquark idea we focus on the triply heavy baryon FFs by considering two following significant points:

(1) In the quark-diquark model, two quarks can be coupled in either a color sextet or a color antitriplet state. Furthermore, it is the diquark in a color antitriplet state that can couple with a quark to form a color-singlet baryon. In a quark-quark system (diquark state), single gluon exchange leads to an interaction that is attractive for diquark in a color antitriplet configuration. Here, we just consider the antitriplet configuration of quark-quark states.

(2) Consistent with our assumption, the constituents are heavy enough so that the binding is relatively weak, i.e., the quark and diquark are both on-shell and the binding energy is negligibly small. This is expected to be true for constituents with masses well above Λ_{QCD}.

To obtain the FF of a heavy quark Q splitting into a triply heavy baryon Ω in the lowest order perturbative regime we consider the Feynman diagram shown in Fig. 1. According to this diagram, a heavy quark Q fragments into a heavy baryon through the mission of a time-like off-shell gluon, that in turn splits into a scalar heavy flavor diquark pair. The gluon must have enough energy, at least twice the diquark mass, since both the diquark D and antidiquark $\bar{D}$ are considered to be on or near their mass-shell. Finally, the produced diquark combines with the heavy quark Q to form the heavy baryon Ω. We assume that the heavy baryon moves in the $\hat{z}$-direction (fragmentation axes) so that, due to the momentum conservation the transverse momentum of the final state jet will be identical to the transverse momentum of the initial heavy quark. Considering Fig. 1, we set the relevant four-momenta as

$$\begin{aligned} p'_\mu &= [p'_0, \vec{k}_T, p'_L] \qquad p_\mu = [p_0, \vec{0}, p_L] \\ k'_\mu &= [k'_0, \vec{k}_T, k'_L] \qquad k_\mu = [k_0, \vec{0}, k_L] \end{aligned} \tag{2}$$

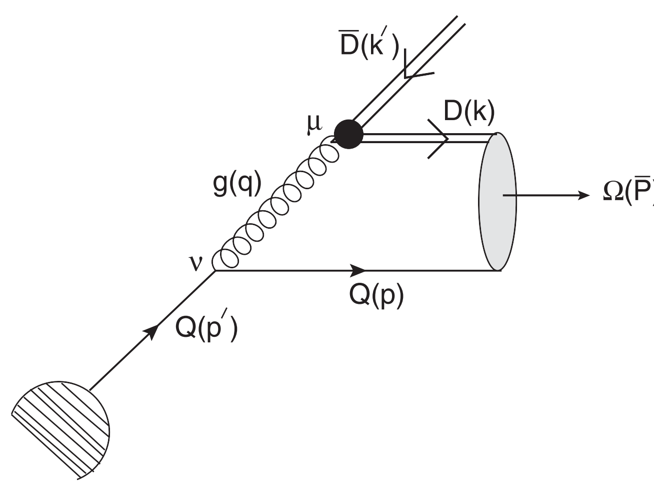

FIG. 1. Leading order Feynman diagram contributing to the fragmentation of a heavy quark into a triply heavy baryon Ω in a quark-diquark model.

and, also $\bar{P}_\mu = [\bar{P}_0, \vec{0}, \bar{P}_L]$ where $\bar{P}$ refers to the four-momentum of the produced baryon so that $\bar{P}_L = p_L + k_L$. For the later uses, we also define the momentum fractions $x_1 = (p_0 + p_L)/(\bar{P}_0 + \bar{P}_L)$ and $x_2 = (k_0 + k_L)/(\bar{P}_0 + \bar{P}_L)$ carried by the constituents of outgoing baryon, i.e., the heavy quark and diquark. Due to the momentum conservation one has $x_1 + x_2 = 1$. Following Ref. [22], we adopt the infinite momentum frame where the fragmentation parameter in the usual light-cone form $z = (E + p_\parallel)_\Omega / (E + p_\parallel)_Q$ is reduced to the more popular form $z = E_\Omega / E_Q = \bar{P}_0 / p'_0$ which ranges as $0 \le z \le 1$. The momentum fractions x_1 and x_2 are also simplified as

$$x_1 = \frac{p_0}{\bar{P}_0}, \qquad x_2 = \frac{k_0}{\bar{P}_0}, \tag{3}$$

which refer to the baryon energy fractions carried by the constituents. Considering the definition of fragmentation parameter z, we also may write the energy of components in terms of the energy of the initial heavy quark p'_0 as: $p_0 = x_1 z p'_0$, $k_0 = x_2 z p'_0$ and $k'_0 = (1 - z) p'_0$.

To obtain the hard scattering amplitude related to the process shown in Fig. 1, we consider the currents produced by the diquark pair and the heavy quark. In covariant coupling of a diquark to a gluon, the coupling is assumed to have a simple structure as: $+ i g_s t^a q^\mu$ [44], where $g_s = \sqrt{4\pi\alpha_s}$ and $t^a = \lambda^a/2$ is the Gell-Mann color matrix. To complete the current, one needs a color charge strength, along with a possible form factor F_S [2]. The momentum space color octet current reads $j_S^\mu \propto g_s F_S(q^2) q^\mu e^{-iq\cdot x}$ where F_S is given in (1) and $q = k + k'$ so k and k' are the scalar diquark pair four-momenta shown in Fig. 1. The current due to the initial heavy quark is expressed as: $j_Q^\nu \propto g_s[\bar{u}(p')\gamma^\nu u(p)] e^{-i(p-p')\cdot x}$. At lowest order perturbation theory, the hard scattering amplitude for the process shown in Fig. 1 is written as

$$T_H = -i \int j_S^\mu \left(\frac{g_{\mu\nu}}{q^2}\right) j_Q^\nu d^4x, \tag{4}$$

which leads to the following hard scattering amplitude

$$T_H = -\frac{g_s^2(\mu) C_F F_S(q^2)}{2q^2 D_0 \sqrt{2\bar{P}_0 p'_0 k'_0}} q^\mu [\bar{u}(p)\gamma_\mu u(p')], \tag{5}$$

where, $\mu = 2m_D$ is the renormalization scale of QCD coupling constant, $C_F = 4/(3\sqrt{3})$ is the color factor [2] and $D_0 = \bar{P}_0 + k'_0 - p'_0$ is the energy denominator which is produced through the energy integration.

According to the definition of the FF in the Suzuki model [22,42], this function is obtained by performing the phase space integration over the scattering amplitude squared. Then, the FF for the production of S-wave baryon Ω from the splitting of the heavy quark Q, reads

$$D_Q^\Omega(z, \mu_0) = \frac{1}{1 + 2s_Q} \sum \int d^3\vec{\bar{P}} d^3\vec{k'} |T_M|^2 \delta^3(\vec{\bar{P}} + \vec{k'} - \vec{p'}), \tag{6}$$

where, T_M is the probability amplitude, s_Q refers to the spin of the fragmenting quark and the summation is going over the spins and colors. Four-momenta are as labeled in Fig. 1. To impose the long-distance effects, the probability amplitude T_M at large momentum transfer factorizes into the convolution of a hard scattering amplitude T_H (5) and the process-independent distribution amplitude Φ_M [45,46], i.e.,

$$T_M(\bar{P}, k', p') = \int [dx_i] T_H(\bar{P}, k', p', x_i) \Phi_M(x_i, w^2), \tag{7}$$

where $[dx_i] = dx_1 dx_2 \delta(1 - x_1 - x_2)$. The fractions x_1 and x_2 are defined in (3).

The scheme applied to describe the probability amplitude T_M as the convolution presented in (7) is convenient to absorb the soft behavior of the bound state into the hard scattering amplitude T_H [45]. In (7), the short-distance coefficient T_H is calculated perturbatively from quark-gluon subprocesses and expanded as a power series of α_s at the energy scale m_Q or higher, and the long-distance distribution amplitude Φ_M contains the bound state non-perturbative dynamic of the outgoing baryon. In fact, the Φ_M refers to the probability amplitude for a quark-diquark pair to evolve into a particular bound state. In the quark-diquark model, at the lowest order perturbative QCD, a diquark in a baryon interacts like an antiquark in a meson because both belong to the $\bar{3}$ representation of color SU(3) [47]. In other words, a baryon bound state is approximated as a quasimeson bound state with the constituents including a quark and pointlike diquark.

Following the Lepage-Brodsky's approach [48], if we ignore the Fermi motion of the constituents and let them to fly together in parallel within the bound state, then the distribution amplitude is approximated by a Dirac-delta function as

$$\Phi_M \approx \frac{f_M}{2\sqrt{3}} \delta\left(x_1 - \frac{m_Q}{M}\right), \tag{8}$$

where f_M is the baryon decay constant so we choose the value of $f_M \approx 0.25$ GeV [28] for all triply heavy baryons. The assumption (8) was applied in Refs. [22–29] and many other papers. We also follow this assumption and derive an analytical form for the $Q \to \Omega$ FF.

Using Eqs. (5), (7), and (8), and by performing the necessary integrations one has

$$T_M = -\frac{g_s^2 C_F f_M F_S(q^2)}{4q^2 D_0 \sqrt{6\bar{P}_0 p'_0 k'_0}} q^\mu [\bar{u}(p)\gamma_\mu u(p')]. \tag{9}$$

Putting the above result in (6), the FF reads

$$D_Q^\Omega = A^2 \int \frac{d^3\vec{k}' d^3\vec{\bar{P}}}{\bar{P}_0 p'_0 k'_0} \frac{\tilde{\Pi}}{D_0^2 (k+k')^4} \delta^3(\vec{\bar{P}} + \vec{k}' - \vec{p}'), \quad (10)$$

where A is proportional to $(\pi\alpha_s f_M F_S C_F/\sqrt{6})^2$, and

$$\tilde{\Pi} = (m_Q^2 - p\cdot p')(m_D^2 + k\cdot k') + (p\cdot k + p\cdot k')(k\cdot p' + k'\cdot p'). \quad (11)$$

To perform the phase space integrations, we consider the following integral

$$\int \frac{d^3\vec{\bar{P}}\delta^3(\vec{\bar{P}} + \vec{k}' - \vec{p}')}{\bar{P}_0 p'_0 D_0^2} = \frac{z}{(M^2 - m_Q^2 - m_D^2 + 2p'\cdot k')^2}, \quad (12)$$

and in (10), instead of integration over the transverse momentum we simply replace the integration variable by its average value. Therefore, one has

$$\int d^3\vec{k}' f(z, k_T^2) \approx m_D^2 k'_0 f(z, \langle k_T^2 \rangle), \quad (13)$$

where $\langle k_T^2 \rangle$ is a free parameter which can be specified phenomenologically.

According to the approximation (8), it is assumed that the contribution of each constituent from the baryon energy is proportional to its mass, i.e., $x_1 = m_Q/M$ and $x_2 = m_D/M$. This simplifying assumption makes the dot products of the relevant four-momenta so simple, for example: $k\cdot p = m_Q m_D$, $2p\cdot p' = m_Q z(m_Q^2 + k_T^2)/M + (m_Q M)/z$, $2k\cdot p' = m_D z(m_Q^2 + k_T^2)/M + (m_D M)/z$, etc.

Putting all in (10), one obtains the following simple form for the D_Q^Ω-FF

$$D_Q^\Omega(z, \mu_0) = N \frac{z^3 \langle k_T^2 \rangle + z(m_D + m_Q(1-z))^2}{(1-z)F(z, \langle k_T^2 \rangle)}, \quad (14)$$

where,

$$F(z, \langle k_T^2 \rangle) = \left[2m_D + \frac{M(1-z)}{z} + \frac{z(\langle k_T^2 \rangle + m_D^2)}{M(1-z)}\right]^3 \times \left[M^2 - z m_Q^2 + \frac{z}{1-z}(m_D^2 + z\langle k_T^2 \rangle)\right]^2. \quad (15)$$

Here, N is proportional to $(2\sqrt{2m_Q} q_S^2 \pi C_F \alpha_s(2m_D) f_M)^2$ but it is determined through the normalization condition $\int_0^1 D_Q^\Omega(z, \mu_0) dz = 1$ [22,43].

In the Suzuki model, the FFs depend on both the fragmentation parameter z and the phenomenological parameter $\langle k_T^2 \rangle$. The z-dependence of the FFs is not yet calculable at each desired scale. However, once they are computed at some initial fragmentation scale μ_0, their μ evolution is specified by the DGLAP equations. Here, we set the initial scale to $\mu_0 = m_Q + m_D$ which is the minimum value of the invariant mass of the fragmenting quark. Therefore, Eq. (14) should be regarded as a model for the heavy quark fragmentation at the initial scale μ_0.

To present our numerical analysis, from Ref. [49] we adopt the input parameters as $m_c = 1.67$ GeV and $m_b = 4.78$ GeV. Application of a scalar diquark brings in the parameter q_S which appears in the diquark form factor (1). In [27], it is pointed out that for the creation of a heavy diquark-antidiquark pair the values of q_S is 5 GeV, when the heavy quark is a c-quark, and 7 GeV when it is a b-quark. Further future theoretical and experimental studies on heavy diquarks will provide more precise values for q_S.

It is worth mentioning here that the average transverse momentum $\langle k_T^2 \rangle$ is not a constant but a function of the fragmentation parameter z. In [50], by considering the z-dependence of this quantity, it is shown that the choice of $\langle k_T^2 \rangle = 1$ GeV^2 is an extreme value for this quantity. Nevertheless, in Fig. 2 we studied the $\langle k_T^2 \rangle$-dependence of $D_c^{\Omega(ccb)}$-FF considering four different values for the average transverse momentum, i.e., $\langle k_T^2 \rangle = (0.5)^2, 1^2, 2^2$, and 3^2 GeV^2. Any higher value of $\langle k_T^2 \rangle$ shifts the FF toward the lower values of z which is not compatible with the behavior of heavy baryon FFs. In this work, we set the average transverse momentum as $\langle k_T^2 \rangle = 1$ GeV^2. In Figs. 3 and 4, we showed our theoretical predictions for the fragmentation of $c \to \Omega_{ccb}$ and $b \to \Omega_{bbc}$ (dashed lines) considering the scalar bc-diquark which is created by a hard gluon. The value of the normalization constant N appeared in (14) is $N = 1.62 \times 10^{-6}$ for the FF of $c \to \Omega_{ccb}$ and $N = 3.2 \times 10^{-6}$ for the $b \to \Omega_{bbc}$ FF.

Our results are also compared with our previous ones (solid lines) [21], obtained through a direct fragmentation.

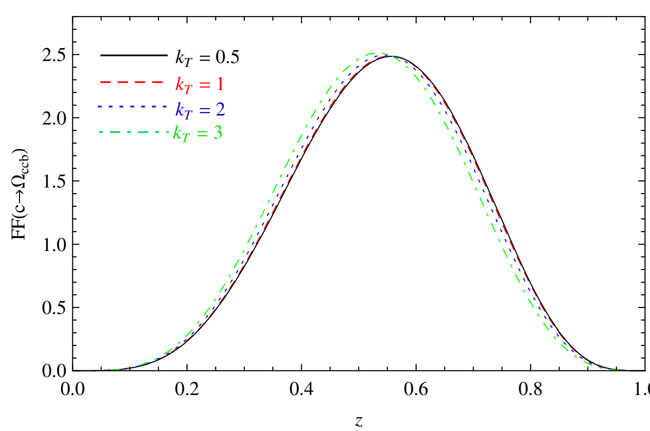

FIG. 2. The $D_c^{\Omega(ccb)}$-FF considering four different values for the average transverse momentum, i.e., $\langle k_T^2 \rangle = (0.5)^2, 1^2, 2^2$, and 3^2 GeV^2.

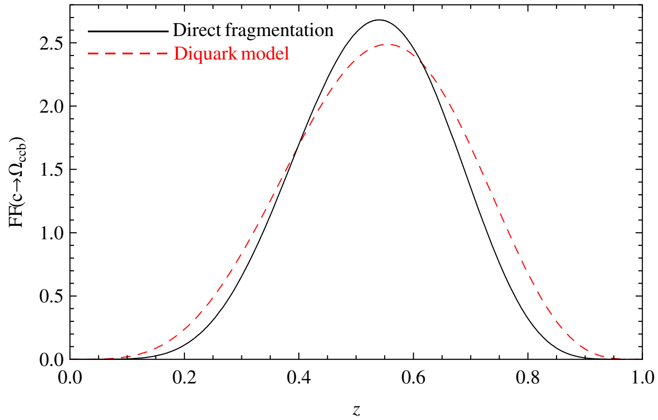

FIG. 3. The $D_c^{\Omega(ccb)}$-FF at the initial scale $\mu_0 = m_b + 2m_c$ as a function of z in the quark-diquark model (dashed line) compared with the one calculated through direct fragmentation (solid line) [21].

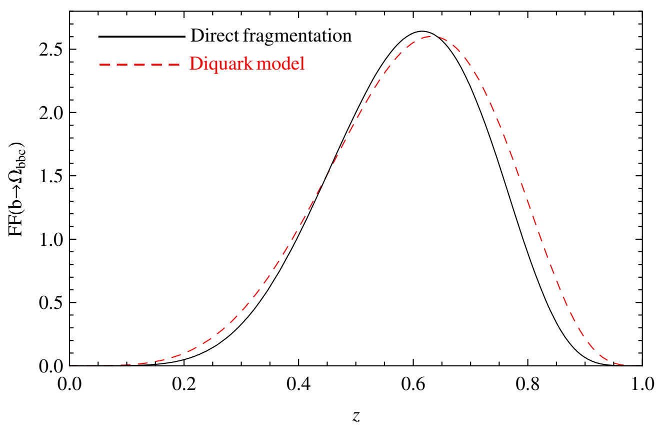

FIG. 4. As in Fig. 3, but for the $b \to \Omega_{bbc}$ FF. Here, the initial scale of FF is $\mu_0 = m_c + 2m_b$.

In [21], independent of diquark model, to extract the triply heavy baryon FFs we applied the Suzuki model considering all lowest-order Feynman diagrams contributing to the direct fragmentation of a heavy quark Q into a triply heavy baryon $\Omega_{Q_1Q_2Q_3}$.

As is seen from Figs. 3 and 4, there are good consistencies between both results, and this proves the possibility of employing the idea of heavy flavor diquarks in this situation. The advantage of diquark approach is to cast the heavy baryon FF into a brief expression (14), unlike the result obtained in the direct fragmentation approach which was lengthy and inappropriate from experimental point of view.

IV. HEAVY QUARK FF IN A TWO-STEP FRAGMENTATION

In this section, we introduce a completely different approach to obtain a brief expression for the heavy quark FFs: two-step fragmentation or indirect fragmentation. In this approach, one can analyze the two stage fragmentation of a heavy quark into a scalar diquark followed by the fragmentation of such a scalar diquark into a triply heavy baryon. The physical principle behind this idea is the unification of any two quarks to form a colored quasi-bound state of diquark and the splitting of the formed diquark into the outgoing baryon. This approach is so convenient to extend our previous result [21] to the next-to-leading order in α_s. The present work is a first attempt to consider the NLO QCD corrections to the fragmentation of a heavy quark into a three heavy flavor baryon.

In the following, we describe our strategy to obtain the FFs at NLO by working in the framework of diquark model. In the two-step fragmentation approach, the fragmentation of a heavy quark Q into a heavy baryon Ω factorizes into the short- and long-distance contributions. According to this approach, two quarks are approximated as a single particle (pointlike diquark $\mathcal{D}$) which can be produced directly in the hard interaction of quarks and gluons at short distances. Therefore, the formation of a diquark can be described in the pQCD theory. In the following, the resultant diquark $\mathcal{D}$ fragments into a baryon Ω in a nonperturbative manner. Actually, attaching a quark to a fast moving heavy diquark $\mathcal{D}$ for baryon production decelerates the heavy diquark in the fragmentation process only slightly. Motivated by kinematical considerations and the QCD factorization theorem [51] and also regarding the work of [5], the baryon FF is expressed as

$$D_Q^{\Omega}(z, \mu_0) = \int_z^1 \frac{dy}{y} D_Q^{\mathcal{D}}\left(\frac{z}{y}\right) D_{\mathcal{D}}^{\Omega}(y). \tag{16}$$

Such a two stage process for production of a triply heavy baryon, where a heavy diquark $\mathcal{D}$ intermediates the initial heavy quark and the final state baryon, would be interesting in the LHC and future colliders.

As was mentioned, in our new approach the fragmentation process is treated in two steps, namely formation of a heavy diquark and subsequent fragmentation of a diquark into a heavy baryon. On the one hand, the formation of a colored heavy diquark system is occurred in a hard process which is calculable in pQCD and on the other hand, the fragmentation process of a heavy diquark into a baryon is treated nonperturbatively. The second stage splitting is called a soft fragmentation where a small value of transverse momentum is involved. At lowest order perturbative QCD, a diquark in a baryon interacts like an antiquark in a meson [47]. Therefore, the nonperturbative soft fragmentation of a colored diquark $\mathcal{D}$ into a baryon Ω, i.e. $D_{\mathcal{D}}^{\Omega}$ in (16), is identical to the fragmentation of an antiquark $\bar{Q}$ into a meson with the constituents $\bar{Q}Q''$ [2,37]. This is a situation where a successful phenomenological model such as the Peterson model [52] is valuable to be used for describing this soft splitting process. According to the Peterson model, which is based on the standard quantum-mechanical parton-model, it is postulated that the dominant part of the fragmentation amplitude of a fast moving heavy antiquark $\bar{Q}$ into a heavy meson $\mathcal{M}(\bar{Q}Q'')$ is determined by the value of the energy transfer (the energy denominator): $\Delta E = E_{\mathcal{M}} + E_{Q''} - E_{\bar{Q}}$, where $E_{\mathcal{M}}$ refers to the energy of produced meson. In this model, the FF of a heavy quark (antiquark) is described by

$$D_{\bar{Q}}^{\mathcal{M}}(z) = \frac{N}{z}\left[1 - \frac{1}{z} - \frac{\epsilon_Q}{1-z}\right]^{-2}, \tag{17}$$

where $\epsilon_Q = m_{\bar{Q}}^2/(m_Q + m_{\bar{Q}})^2$ and N is the normalization constant which is fixed by summing over all hadrons containing $\bar{Q}$, i.e., $\sum \int D_{\bar{Q}}^{\mathcal{M}}(z)dz = 1$ [52].

In (16), the first stage fragmentation including the formation of a colored heavy diquark through the splitting of an initial heavy quark, i.e., $D_Q^{\mathcal{D}}$, is occurred in a hard subprocess which is calculable in perturbative QCD. In the next section, we will calculate this contribution at the LO and NLO pQCD so by their convolution with the Peterson model, according to the convolution relation (16), we will present our predictions for the heavy baryon FFs. It should be noted that, to obtain a full NLO QCD correction the nonperturbative fragmentation $\mathcal{D} \to \Omega$ should be also computed at NLO. Since this part of process is related to a soft splitting then must be evaluated phenomenologically in higher orders. For phenomenological determination of a soft fragmentation one needs to have experimental data. In this work we approximately use the analytical model of Peterson for describing the soft splitting of a diquark into the Ω-baryon which is well known as a successful phenomenological model at LO.

A. Heavy baryon FF at NLO

In [50], using the Suzuki model we calculated the perturbative QCD FF for a heavy quark to fragment into a S-wave heavy meson at NLO. As an example, we studied the LO and NLO FFs for a charm quark to split into the S-wave D^0/D^+-mesons and compared our analytic results with experimental data from BELLE [53] and CLEO [54]. We found that the NLO corrections change the theoretical results at lowest-order and make good agreement with experimental data. We also compared our analytical results with a well-known phenomenological model (Bowler model [55]) extracted in [56]. There was also good consistency between both results.

In order for next usage, here, we briefly review the perturbative FF formalism to calculate the fragmentation of a heavy quark Q into a heavy colorless meson $\mathcal{M}(Q\bar{q})$ at lowest-order of perturbative QCD. Here, we also neglect the relative motion of the constituent quarks Q and $\bar{q}$, therefore we assume, for simplicity, that the Q and $\bar{q}$ are emitted collinearly with each other and they move along the 3-axes (fragmentation axes). By this assumption, a meson is replaced by its collinear constituents. This unrealistic assumption will be improved later, see subsection IV B. In the Suzuki model, to obtain the $D_Q^{\mathcal{M}}(z,\mu)$-FF in the lowest order perturbative regime we consider the fragmentation of a heavy quark Q into a heavy meson $\mathcal{M}(Q\bar{q})$ through emission of a gluon by the heavy quark, which splits into a heavy quark pair $q\bar{q}$. The gluon must have enough energy, at least twice the q-quark mass, since the heavy quark pair produced are considered to be on their mass shell at the first stage. The strong coupling constant at the scale $\mu = m_Q + m_{\bar{q}}$ is small enough so it justifies the validity of the perturbative approach.

In the Suzuki model, the $D_Q^{\mathcal{M}}(z,\mu)$-FF is obtained by performing the phase space integration over the squared scattering amplitude of the fragmentation process, see Eq. (6). As before, to compute the hard scattering amplitude T_H related to the hard process $Q \to \mathcal{M}(Q\bar{q}) + q$, we consider the currents produced by the quark pair $q\bar{q}$ and the initial heavy quark Q. If one labels the four-momenta as $Q(p') \to Q(p)\bar{q}(k) + q(k')$, the currents read

$$\begin{aligned} j_1^\nu &\propto g_s[\bar{u}(p)\gamma^\nu u(p')]e^{-i(p-p')\cdot x}, \\ j_2^\mu &\propto g_s[\bar{u}(k')\gamma^\mu v(k)]e^{-i(k+k')\cdot x}, \end{aligned} \tag{18}$$

where, $g_s\gamma^\nu$ is due to the coupling of a gluon to a quark.

At the lowest order perturbation theory, the hard scattering amplitude T_H is written as

$$T_H = -i\int j_1^\mu\left(\frac{g_{\mu\nu}}{q^2}\right)j_2^\nu d^4x, \tag{19}$$

where $q = k + k'$ is the four-momentum of intermediate gluon. This leads to the hard scattering amplitude as

$$T_H = -\frac{g_s^2(\mu)m_q m_Q C_F}{2q^2 D_0\sqrt{2p_0p_0'k_0k_0'}}[\bar{u}(k')\gamma^\mu v(k)][\bar{u}(p)\gamma_\mu u(p')], \tag{20}$$

where, $\mu = m_Q + m_{\bar{q}}$ is the renormalization scale of strong coupling constant, $C_F = 4/3$ is the color factor and $D_0 = p_0 + k_0' + k_0 - p_0'$ is the energy denominator.

As in (7), the amplitude for production of a heavy meson is obtained by the convolution of short-distance amplitude T_H and the long-distance distribution amplitude $\Phi_{\mathcal{M}}$. Adopting the Lepage-Brodsky's approach we apply the distribution amplitude presented in (8) where the wave function of bound state is approximated by a delta function. With this approximation, we are assuming that the contribution of each constituent quark from the energy of meson $\mathcal{M}(Q\bar{q})$ is proportional to its mass, i.e. $x_1 = m_Q/M$ and $x_2 = m_{\bar{q}}/M$.

In the Suzuki model, the heavy quark FF in the splitting process $Q(p') \to \mathcal{M}(\bar{P}) + q(k')$ is defined as

$$D_Q^{\mathcal{M}}(z,\mu_0) = \sum \int d^3\vec{\bar{P}} d^3\vec{k}' \frac{|T_M|^2}{1+2s_Q} \delta^3(\vec{\bar{P}} + \vec{k}' - \vec{p}'), \tag{21}$$

where, s_Q is the spin of fragmenting heavy quark and the average/sum goes over the spin of initial/final state particles. Considering Eqs. (7) and (8), one has $T_{\mathcal{M}}(p,k,k',p') = f_{\mathcal{M}} T_H(p,k,k',p',x_i = m_i/M)/(2\sqrt{3})$ where the hard scattering amplitude T_H is defined in (20). The short-distance coefficient T_H is calculated perturbatively from quark-gluon subprocesses and is expanded as a power series of α_s at the energy scale μ or higher.

Putting all together, one finds the heavy quark FF as

$$D_Q^{\mathcal{M}}(z,\mu_0) = \frac{8}{3}(f_{\mathcal{M}}\pi\alpha_s m_{\bar{q}} m_Q C_F)^2 \int \frac{d^3\vec{\bar{P}} d^3\vec{k}'}{p_0' k_0' \bar{P}_0 D_0^2} \times \{m_q^2(p\cdot p') + (p\cdot k)(p'\cdot k') + (p\cdot k')(k\cdot p') - m_Q^2(k\cdot k') - 2m_q^2 m_Q^2\}\delta^3(\vec{\bar{P}} + \vec{k}' - \vec{p}'). \tag{22}$$

Finally, to obtain an explicit form of the heavy quark FF we perform the phase space integrations as follows

$$\int \frac{d^3\vec{\bar{P}}\delta^3(\vec{\bar{P}} + \vec{k}' - \vec{p}')}{p_0' \bar{P}_0 D_0^2} = \frac{z}{(M^2 - m_Q^2 - m_q^2 + 2p'\cdot k')^2}, \tag{23}$$

and, as in (13), instead of integrating over transverse momentum we replace the integration variable by its average value, i.e.,

$$\int d^3\vec{k}' g(z,k_T^2) \approx m_q^2 k_0' g(z, \langle k_T^2\rangle). \tag{24}$$

Putting all in (22) and by considering the dot product of four-momenta, we obtain the LO FF of a heavy quark as

$$D_Q^{\mathcal{M}}(z,\mu_0) = \frac{Nz^3(1-z)^3}{F(z,\langle k_T^2\rangle)} \Big\{ 4m_q m_Q^3 z^2 + 8Mm_Q^3 z(1-z) + 4\frac{m_Q^3}{m_q}(3M^2(1-z)^2 + z^2\langle k_T^2\rangle) + 32M^2 m_Q^2(z-1) + 8Mm_q m_Q^2 z - \frac{8Mm_Q^2}{zm_q}(M^2(1-z)^2 + z^2\langle k_T^2\rangle) - 8Mm_Q\frac{z-1}{z}(M^2 + z^2\langle k_T^2\rangle) + 4m_q m_Q(3M^2 + z^2\langle k_T^2\rangle) + \frac{4m_Q}{z^2 m_q}([M^2 + z^2\langle k_T^2\rangle]^2 + z^3\langle k_T^2\rangle[M^2(3z-2) + zM^2(z-2)]) \Big\}, \tag{25}$$

where,

$$F(z,\langle k_T^2\rangle) = [(z-1)(M^2 - zm_Q^2) - z(m_q^2 + z\langle k_T^2\rangle)]^2 \times [z^2\langle k_T^2\rangle + (M(z-1) - zm_q)^2]^2. \tag{26}$$

In the above relation, the coefficient N is proportional to $(\pi m_Q m_{\bar{q}} C_F \alpha_s f_{\mathcal{M}}/\sqrt{6})^2$ but is related to the normalization condition [22,43].

Now, considering the convolution (16) and by having the Peterson model (17) for the $D_{\mathcal{D}}^{\Omega}$-FF, and Eq. (25) for the $D_Q^{\mathcal{D}}$-FF, we are in a situation to present our predictions for the heavy baryon FFs at LO. As before, we set $\langle k_T^2\rangle = 1\ \mathrm{GeV}^2$. In Figs. 5 and 6, we showed our theoretical predictions for the fragmentations of $c \to \Omega_{ccb}$ and $b \to \Omega_{bbc}$ at the lowest order of pQCD in two-step fragmentation (dashed lines) and compared them with the results obtained through the direct fragmentation approach (solid lines) [21]. As is seen, there are good consistencies between both results.

In [50], using the direct fragmentation approach in the pQCD regime we computed the $D_Q^{\mathcal{M}}$-FF at NLO, and showed that the theoretical results are in good agreement with the experimental data when the one-loop QCD corrections are involved. Now, considering the convolution relation (16) and by having the NLO $D_Q^{\mathcal{D}}$-FF ($\equiv D_Q^{\mathcal{M}}$-FF) from [50], it is possible to obtain the triply heavy baryon FF at NLO. Note that, we still use the Peterson model (17) for the nonperturbative part of the fragmentation process, i.e. the fragmentation of heavy diquark into the observed baryon. In other words, we use the Peterson model (17) for the $D_{\mathcal{D}}^{\Omega}$-FF.

In Figs. 7 and 8, using the indirect fragmentation approach (or two-step fragmentation approach) we showed our theoretical predictions for the $D_c^{\Omega_{ccc}}$- and $D_b^{\Omega_{bbb}}$-FFs at the NLO (solid lines) and they are compared with the LO ones (dashed lines). As is seen, the NLO contributions are considerable and change the results obtained at LO, too much. The peak positions of the FFs are shifted towards higher-z values and the maximum value of FFs increases, significantly.

B. The effect of bound state wave function

Our calculations for the FFs are based on the Suzuki model where the heavy quark FFs are evaluated by considering some convenient Feynman diagrams for the

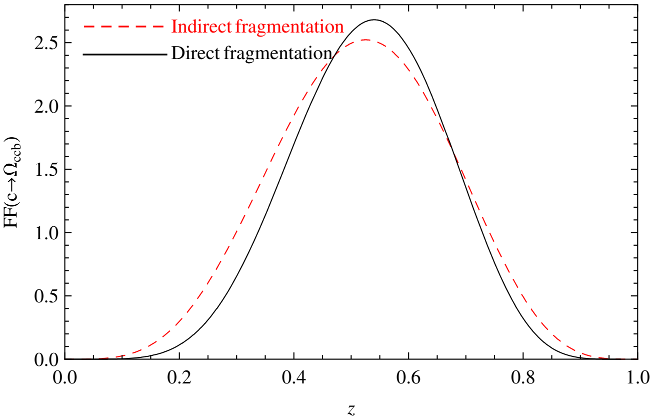

FIG. 5. The $D_c^{\Omega(ccb)}$-FF as a function of z in the two-step fragmentation (dashed line) compared with the one calculated through the direct fragmentation (solid line) [21].

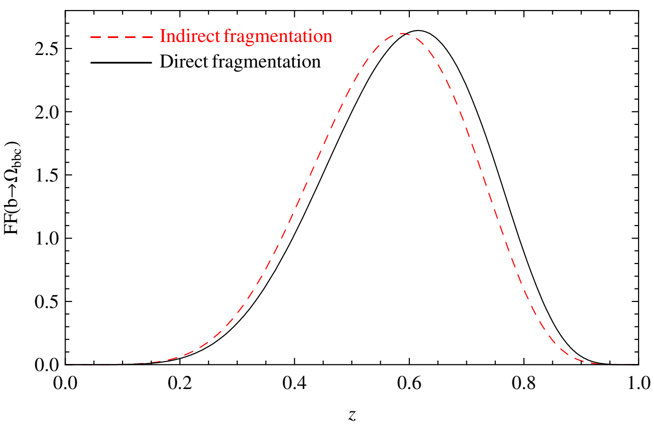

FIG. 6. As in Fig. 5, but for the $D_b^{\Omega(bbc)}$-FF at the initial scale $\mu_0 = m_c + 2m_b$.

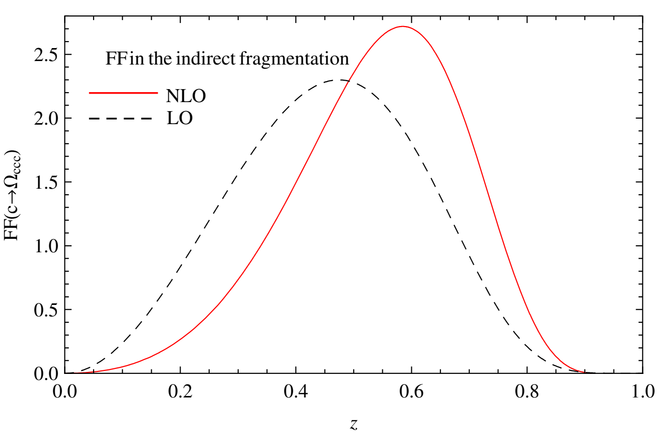

FIG. 7. The FF of $c \to \Omega_{ccc}$ at LO (dashed line) and NLO (solid line) in the indirect fragmentation. The initial scale is set as $\mu_0 = 3m_c$.

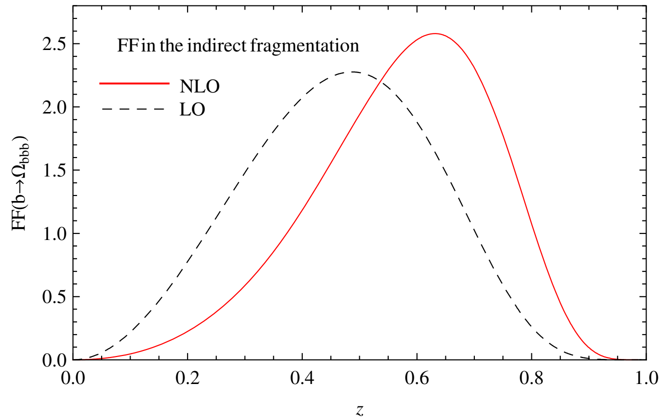

FIG. 8. As in Fig. 7, but for the FF of $b \to \Omega_{bbb}$. The initial scale is set as $\mu_0 = 3m_b$.

parton level of the process, and the wave function of bound state which contains the nonperturbative dynamic of the bound state. In the previous discussions, ignoring the Fermi motion of constituents we estimated the meson/baryon wave function to a delta function form, see Eq. (8). This assumption was also applied in all previous works, see for example Refs. [22–29]. According to this assumption, the constituents are let to fly together in parallel within the bound state. In fact, by this estimation the heavy baryon is replaced by collinear constituents.

In (8), the long-distance distribution amplitude $\Phi_{\mathcal{M}}$ refers to the probability amplitude for a $Q\bar{q}$-pair to evolve into a particular heavy state and contains the nonperturbative dynamic of the bound state. The distribution amplitude $\Phi_{\mathcal{M}}$ is related to the valence wave function $\Psi_{\mathcal{M}}$ as [46]

$$\Phi_{\mathcal{M}}(x_i, w^2) = \frac{1}{2(2\pi)^3} \int \delta(\vec{k}_{\perp 1} + \vec{k}_{\perp 2}) d^2\vec{k}_{\perp 1} d^2\vec{k}_{\perp 2} \times \Psi_{\mathcal{M}}(x_i, \vec{k}_{\perp i}) \Theta(k_{\perp i}^2 < w^2), \quad (27)$$

where $\Theta(x)$ is the Heaviside step function, $\vec{k}_{\perp i}$ stands for the transverse momentum of constituents and the momentum fractions $x_1 = p_0/\bar{P}_0$ and $x_2 = k_0/\bar{P}_0$ are defined as before. In the case of heavy bound states, we assume that the constituents are sufficiently nonrelativistic so that the gluon emission can be neglected. The constituent distributions might be controlled by a simple two-body wave function as [45]

$$\Psi_{\mathcal{M}}(x_i, \vec{k}_{\perp i}) = \frac{\sqrt{128\pi b^3 (m_1 + m_2)}}{x_1^2 x_2^2 [M^2 - \frac{m_1^2 + k_{\perp 1}^2}{x_1} - \frac{m_2^2 + k_{\perp 2}^2}{x_2}]^2}, \quad (28)$$

where b and M are the binding energy and the mass of two-body bound state, respectively. Both in the equal-mass case $m_1 = m_2$ and in the unequal-mass case

$m_1 \ll m_2$, it can be shown that the above equation is the solution of the Schrödinger equation with a Coulomb potential, which is the nonrelativistic limit of the Bethe-Salpeter equation with the QCD kernel, see [45].

Putting (28) in (27) and by working in the infinite momentum frame we integrate over $\vec{k}_{\perp i}(0 \leq k_{\perp i}^2 \leq \infty)$, so it yields

$$\Phi_{\mathcal{M}} = \frac{1}{2(2\pi)^3} \frac{(128\pi b^3 M)^{\frac{1}{2}}}{x_1(1-x_1)M^2 - (1-x_1)m_1^2 - x_1 m_2^2}, \quad (29)$$

where the constraint $x_2 + x_1 = 1$ is imposed. After simplifying, one has

$$\Phi_{\mathcal{M}}(x_i, Q^2) = \frac{f_{\mathcal{M}}}{2\sqrt{3}} \frac{1}{M(x_1 - \frac{m_1}{M})^2}, \quad (30)$$

where $f_{\mathcal{M}} = \sqrt{6b^3/(\pi M)}$ is the meson decay constant. As is seen, Eq. (30) grows rapidly at $x_1 = m_1/M$. In this regard, in all previous works authors have estimated the distribution amplitude (30) by a delta function (8). In fact, by this choice, authors have assumed that the contribution of each constituent from the meson energy is proportional to its mass so that this assumption makes the calculations so simple.

To incorporate more realistic aspect of the nonperturbative dynamic of the bound state into the baryon FFs, we apply the distribution amplitude (30). We start with Eq. (7) and write the probability amplitude $T_{\mathcal{M}}$ as

$$T_{\mathcal{M}} = \frac{f_{\mathcal{M}}}{2\sqrt{3}M} \int \frac{dx_1}{(x_1 - \frac{m_q}{M})^2} T_H(\bar{P}, x_2 = 1 - x_1). \quad (31)$$

In conclusion, considering Eqs. (21) and (31), the initial scale meson FF reads

$$D_Q^{\mathcal{M}}(z, \mu_0) = R^2 \int \frac{dx_1}{(x_1 - \frac{m_q}{M})^2} \left\{ \int \frac{dx_2}{(x_2 - \frac{m_Q}{M})^2} \times \int \frac{d^3\vec{\bar{P}} d^3\vec{k'}}{\bar{P}_0 p'_0 k'_0} \frac{L^{\mu\nu} L_{\mu\nu}}{D_0^2 q^4} \delta^3(\vec{\bar{P}} + \vec{k'} - \vec{p'}) \right\}, \quad (32)$$

where $L^{\mu\nu}$ is the leptonic tensor which is calculated perturbatively from quark-gluon subprocesses. Here, $R = (m_q m_Q f_{\mathcal{M}})/(4\sqrt{6}\pi^2 M)$ but is related to the normalization condition as before. The integrations over the phase space are as in Eqs. (23) and (24).

Note that, in the presence of more realistic form of the meson wave function the NLO QCD calculations would be so complicated. Due to the lengthy expression obtained from (32), we just present our numerical results.

Now, considering the convolution (16) and by having the Peterson model (17) for the $D_{\mathcal{D}}^{\Omega}$-FF, and Eq. (32) for the $D_Q^{\mathcal{D}}$-FF, we can present our predictions for the heavy baryon FFs. Applying the numerical inputs as before, in Fig. 9 the effect of meson wave function on the Ω_{ccb}-FF in splitting of a c-quark is shown at the lowest-order of perturbative QCD. In this calculation, the product of leptonic tensors $L^{\mu\nu}L_{\mu\nu}$ in (32) is as in (22). Here, this effect is studied once by considering the nonrelativistic wave function (30) for the meson bound state (solid line) and once again by considering a delta function form (8) for the meson wave function (dashed line). In Fig. 10, we studied the effect of wave function on the $D_c^{\Omega(ccc)}$-FF at the LO (dot-dashes) and NLO (dots) QCD approximations. For the NLO calculations, the product of leptonic tensors $L^{\mu\nu}L_{\mu\nu}$ (32) is given in [50]. For a more quantitative comparison, we have also presented our results at LO (dashed line) and NLO (solid line) when a delta function form is used for the bound state wave

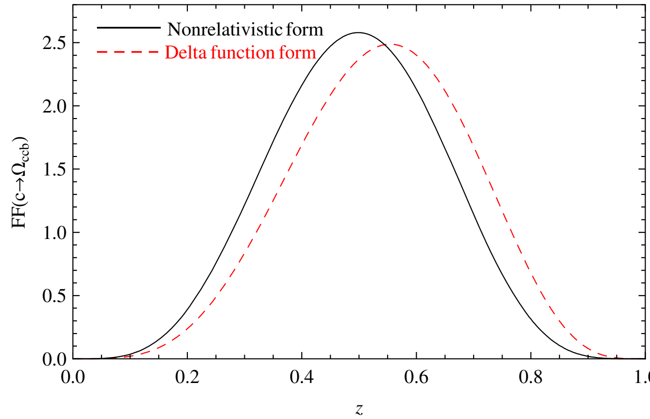

FIG. 9. The $D_c^{\Omega(ccb)}$-FF at LO, once considering a delta function form (8) for the meson wave function (dashed line) and once again with a nonrelativistic form (30) for the wave function (solid line).

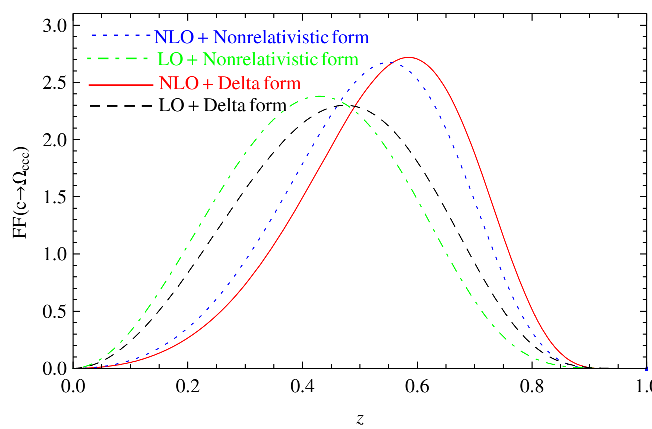

FIG. 10. The FF of $c \to \Omega_{ccc}$ considering both the NLO corrections and the effect of meson wave function (dotted line). For a quantitative comparison, we also plotted the $D_c^{\Omega(ccc)}$-FF at LO (dashed line) and NLO (solid line) in the presence of delta function form for the meson wave function. The effect of meson wave function is studied at LO (dot-dashes), too.

function. As is seen, the changes due to considering the nonrelativistic wave function are less than the NLO corrections, as is expected. Note that, unlike the effect of nonrelativistic meson wave function the NLO corrections shift the peak position of the FFs towards higher values of z.

V. SUMMARY AND CONCLUSION

The study of heavy baryons is a subject of interest for particle physicists, because they provide particular information for the strong interactions, hadron structures, and weak decays of heavy baryons. In our previous work [21], we analyzed the direct fragmentation of triply heavy baryons in the fully perturbative QCD regime to the lowest-order in α_s. Here, in the first step, we applied the pQCD approach considering the quark-diquark model and calculated the heavy baryon FFs at LO and then compared them with the ones extracted through the direct fragmentation [21]. Good agreement occurred between both results ensures the correctness of the idea of diquark in this situation. Actually, the quark-diquark model is used to obtain a brief and feasible expression for the FFs from experimental point of view. In the following, considering the diquark model we studied the FF of heavy baryons in a two-step fragmentation approach (indirect approach) and then compared them with the ones obtained in a direct fragmentation approach. We also found good consistency between both approaches. Relying on the indirect approach, and by having the meson FF at NLO [50] we calculated, for the first time, the heavy baryon FF up to next-to-leading order of QCD coupling constant and showed that the NLO corrections change the results, considerably. To extract our results, following the Lepage-Brodsky's approach [48], we applied a Dirac-delta function for the bound state wave function as was applied in all previous works. This choice corresponds to neglecting the relative motion of the constituent quarks. The underlying link between hadronic phenomena in QCD at long- and short-distances is the hadronic wave function. In fact, the nonperturbative aspect of the hadroproduction processes is emerged in the bound state of the hadron which is described by the wave function $\Phi_{\mathcal{M}}$. We also applied, for the first time, a typical nonrelativistic wave function which includes more realistic aspect of nonperturbative dynamics of bound state and is completely different from the delta function form, used in all previous works. We found that, taking this effect into account leads to a changed result which is considerable. As a final result, we considered both the NLO QCD corrections and the effect of nonrelativistic wave function and showed that these effects are significant.

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