Estimates of W-exchange contributions to Ξ_{cc} decays

Neelesh Sharma[*] and Rohit Dhir[†]

SRM Research Institute & Department of Physics, SRM University, Kattankulathur 603203, India

(Received 24 September 2017; published 20 December 2017)

Encouraged by the recent discovery of the Ξ_{cc} baryon, we investigate two-body nonleptonic weak decays of doubly charmed, Ξ_{cc}, baryons. We calculate the branching ratios for Cabibbo-Kobayashi-Maskawa–favored and–suppressed modes in factorization and pole model approaches. The preliminary estimates of nonfactorizable W-exchange contributions are obtained using the pole model. We find that the W-exchange contributions to Ξ_{cc} decays, being sizable, cannot be ignored.

I. INTRODUCTION

The resolution of the longstanding puzzle involving the Ξ_{cc} states has been much awaited since their first observations were reported by the SELEX Collaboration [1,2]. Most recently, the LHCb Collaboration announced the observation of a doubly charmed Ξ_{cc}^{++} baryon [3] found in the $\Lambda_c^+ K^- \pi^+ \pi^+$ mass spectrum. The mass of the observed state is determined as $M_{\Xi_{cc}^{++}} = 3621.40 \pm 0.72 \pm 0.27 \pm 0.14$ MeV, while the mass difference $M_{\Xi_{cc}^{++}} - M_{\Lambda_c^+} = 1334.94 \pm 0.72 \pm 0.27$ MeV. The new observation of a doubly heavy charm baryon has revamped the interest of heavy flavor physicists as being a good candidate with which to study the heavy-quark dynamics. Although the lifetime, $\tau_{\Xi_{cc}}$, has not yet been given experimentally, ample theoretical estimates exist in the literature that range from $\sim$50–670 fs [4–9]. Another interesting aspect of doubly heavy baryons is their spectroscopy [8–11]. In addition to the three quark dynamics, the doubly heavy baryons can be identified by the set of quantum numbers (J^P, S_d) in the diquark picture, where S_d is the spin of the heavy diquark. Thus, spins of the two heavy quarks are coupled to form the $(S_d = 1)$ symmetric spin configuration of a diquark $\{Q_1 Q_2\}$ and the $(S_d = 0)$ antisymmetric spin configuration of a diquark $[Q_1 Q_2]$. The general convention is to denote the antisymmetric state as a primed one, i.e., $|B'\rangle$, and the symmetric heavy-diquark state as an unprimed, $|B\rangle$, state. Also, the wave functions of the $|B\rangle$ and $|B'\rangle$ states are expected to mix [12–18]. However, in the present work we consider three quarks as a independent dynamical entities.

Theoretically, the mass spectra, magnetic moments, and radiative and semileptonic decays of the doubly charmed baryons have been the center of interest for the last decade [4–36]. On the contrary, the progress in the heavy-baryon nonleptonic weak decays has been very slow [37–46], although the recent experimental observations have revived the activities in nonleptonic decays of heavy baryons in the last few years [47–61]. Thus, we put our focus on the two-body nonleptonic weak decays of doubly charmed baryons. Very recently, weak decays of doubly heavy baryons were analyzed in SU(3) symmetry and in the quark-diquark picture using factorization and the light front approach [57,58]. In another interesting work, the analysis of factorizable $\Xi_{cc}^{++} \to \Sigma_c^{++} \bar{K}^{(*)0}$ decays was carried out using the covariant confined quark model (CCQM) [59]. The theoretical interpretation of the experimentally favored decay chain $\Xi_{cc}^{++} \to \Sigma_c^{++} (\to \Lambda_c^+ \pi^+) + \bar{K}^{*0} (\to K^- \pi^+)$ due to the dominant branching ratios of the daughter decays is first presented in Ref. [60]. In addition, the short-distance and long-distance (W-exchange) contributions to the decay channels of Ξ_{cc} baryons are calculated more systematically using factorization and final-state interaction (FSI) rescattering, respectively [60]. The branching ratios of nonleptonic decays of the doubly heavy baryons are predicted in the perturbative QCD (pQCD) [61].

Unlike meson decays, W-exchange contributions do not experience helicity and color suppression in heavy baryon decays [62–71]. Consequently, the W-exchange contributions are expected to be as important as the factorization. In fact, many of the observed charm baryon decays receive contributions solely from W-exchange diagrams. Therefore, in the present work, we give preliminary estimates of W-exchange (pole) contributions using the pole model. To obtain the factorization contributions, we use form factors based on the nonrelativistic quark model (NRQM) [72] and heavy quark effective theory (HQET) [73], which have worked reasonably well in explaining weak decays of charm baryons. Also, we use a more accurate approach [74] to include SU(4) symmetry-breaking effects in the evaluation of meson-baryon strong couplings. To calculate the pole amplitude (W-exchange contributions), we use nonrelativistic approximation [37,75] to evaluate the weak matrix element. It may be noted that for a first estimate of pole contributions we consider ground-state $\frac{1}{2}^+$–intermediate baryon pole terms only. Moreover, the $\frac{1}{2}^-$–intermediate pole terms are difficult

[*]nishu.vats@gmail.com
[†]dhir.rohit@gmail.com

(for being nontrivial) to calculate, and little is known about the strong coupling constants of the $\frac{1}{2}^-$–baryons involved. It has been shown in the past [70,76] that SU(4) symmetry breaking could be induced by the variation of a spatial baryon wave function overlap in weak decay amplitude. We find that pole contributions are significantly enhanced upon the inclusion of flavor-dependent effects via $|\psi(0)|^2$ variation; consequently, we get larger branching ratios for the decays involving W-exchange diagrams. Thus, a number of decays have sizable branching ratios that could be suitable for future experimental measurements at LHCb, CEPC Belle II, etc. The present paper is

organized as follows. In Sec. II, we give the Hamiltonian and decay rate formula. Section III deals with the evaluation of decay amplitudes. Numerical results and discussions are given in Sec. IV. We summarize our findings in the last section.

II. HAMILTONIAN AND DECAY RATE

The charm changing two-body nonleptonic decays of (doubly heavy) baryons, emitting pseudoscalar (P) meson, proceed through usual current $\otimes$ current effective weak Hamiltonian,

$$
\begin{aligned}
\mathcal{H}_W^{\text{eff}} = \frac{G_F}{\sqrt{2}} \{ & V_{ud}V_{cs}^*[c_1(\bar{u}d)_{V-A}(\bar{c}s)_{V-A} + c_2(\bar{s}d)_{V-A}(\bar{u}c)_{V-A}]_{(\Delta C = \Delta S = -1)} \\
& + V_{ud}V_{cd}^*[c_1\{(\bar{s}c)_{V-A}(\bar{u}s)_{V-A} - (\bar{d}c)_{V-A}(\bar{u}d)_{V-A}\} \\
& + c_2\{(\bar{u}c)_{V-A}(\bar{s}s)_{V-A} - (\bar{u}c)_{V-A}(\bar{d}d)_{V-A}\}]_{(\Delta C = -1, \Delta S = 0)} \\
& - V_{us}V_{cd}^*[c_1(\bar{d}c)_{V-A}(\bar{u}s)_{V-A} + c_2(\bar{u}c)_{V-A}(\bar{d}s)_{V-A}]_{(\Delta C = -\Delta S = -1)} \},
\end{aligned} \tag{1}
$$

where V_{ij} denote the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and $(\bar{q}_iq_j)_{V-A} \equiv \bar{q}_i\gamma_\mu(1 - \gamma_5)q_j$ denotes the weak V-A current. The Hamiltonian consists of CKM-favored ($\Delta C = \Delta S = -1$), CKM-suppressed ($\Delta C = -1$, $\Delta S = 0$), and CKM–doubly suppressed ($\Delta C = -\Delta S = -1$) decay modes. The QCD (Wilson) coefficients $c_1(\mu) = 1.2$, $c_2(\mu) = -0.51$ at $\mu \approx m_c^2$ in the large N_c limit are used in the analysis (for a review, see Ref. [77]). The coefficients c_1 and c_2 may be treated as free parameters for being affected by nonfactorizable contributions. In general, the transition amplitude can be expressed in terms of the reduced matrix element for the $B_i(\frac{1}{2}^+, p_i) \to B_f(\frac{1}{2}^+, p_f) + P_k(0^-, q)$ decay process,

$$
\begin{aligned}
\mathcal{A}(B_i \to B_fP) &\equiv \langle B_f(p_f)P_k(q)|\mathcal{H}_W^{\text{eff}}|B_i(p_i)\rangle \\
&= i\bar{u}_{B_f}(p_f)(A + B\gamma_5)u_{B_i}(p_i), \tag{2}
\end{aligned}
$$

where u_{B_i} represent Dirac spinors for initial and final ($\frac{1}{2}^+$) baryons B_i and B_f. A and B denote the parity-violating (PV) s-wave and parity-conserving (PC) p-wave amplitudes, respectively.

The decay rate formula for the $B_i \to B_fP$ process is given by

$$
\Gamma(B_i \to B_fP) = \frac{p_c}{8\pi}\frac{E_f + m_f}{m_i}\left[|A|^2 + \frac{E_f - m_f}{E_f + m_f}(|B|^2)\right]. \tag{3}
$$

Here, m_i and m_f are the masses of the initial- and final-state baryons. The magnitude of the 3-momentum p_c of the final-state particles in the rest frame of B_i is

$$
p_c = \frac{1}{2m_i}\sqrt{[m_i^2 - (m_f - m_P)^2][m_i^2 - (m_f + m_P)^2]},
$$

where m_P is the mass of emitted pseudoscalar meson, and

$$
E_f \pm m_f = \frac{(m_i \pm m_f)^2 - m_P^2}{2m_i}.
$$

The corresponding asymmetry parameter is given by

$$
\alpha = \frac{2\frac{p_c}{E_f + m_f}\text{Re}[A * B]}{\left(|A|^2 + \frac{p_c^2}{(E_f + m_f)^2}|B|^2\right)}. \tag{4}
$$

To estimate the decay rate and asymmetry parameters, we require numerical evaluation of calculating the amplitudes, A and B.

III. DECAY AMPLITUDES

The hadronic matrix element for the $B_i \to B_f + P_k$ process can receive dominant contributions from factorization and pole processes and thus can be given as

$$
\langle B_fP_k|H_W|B_i\rangle \equiv \mathcal{A}_{\text{Pole}} + \mathcal{A}_{\text{Fac}}, \tag{5}
$$

where $\mathcal{A}_{\text{Pole}}$ and $\mathcal{A}_{\text{Fac}}$ denote the pole and factorization amplitudes, respectively. The pole diagrams mainly involve the W-exchange process contributions that are evaluated using the pole model framework [63]. In the pole model, the weak and strong vertices are separated by the introduction of a set of intermediate states into the decay process. It may also be noted that factorization may be considered as a correction to pole contributions where t-channel pole process is equivalent to the tree-level

diagram, i.e., factorizable process. The contribution of both pole and factorization processes can be summed up in terms of s-wave (PV) and p-wave (PC) amplitudes. We wish to point out that we have ignored the relative strong phases involved in the decay amplitudes in our calculation for being difficult to estimate in the present scenario; however, such phases can contribute to some of the CP-violating asymmetries.

A. Pole amplitudes

The decay amplitude, $\mathcal{A}_{\text{Pole}}$, can be calculated from the reduced matrix element

$$\langle B_f|H|B_i\rangle = \bar{u}_{B_i}(A + \gamma_5 B)u_{B_f}, \qquad (6)$$

between two $\frac{1}{2}^+$–baryon states expressed in terms of PV and PC amplitudes, A and B, receptively. The baryonic decay in the pole model involves a hadronic intermediate state that first is produced in the strong process and then goes through a weak transition to the final baryon. Thus, A and B can simply be expressed in term of masses, strong couplings, and weak matrix elements. The pole amplitude consisting of contributions of s and u channels for positive-parity intermediate baryon ($J^P = \frac{1}{2}^+$) poles are denoted by A^{Pole} and B^{Pole},

$$A^{\text{Pole}} = \sum_n \left[\frac{g_{P_k}^{B_f B_n} b_{ni}}{m_i + m_n} + \frac{g_{P_k}^{B_n B_i} b_{fn}}{m_f + m_n}\right], \qquad (7)$$

$$B^{\text{Pole}} = -\sum_n \left[\frac{g_{P_k}^{B_f B_n} a_{ni}}{m_i - m_n} + \frac{g_{P_k}^{B_n B_i} a_{fn}}{m_f - m_n}\right], \qquad (8)$$

where g_k^{ij} are the strong meson-baryon coupling constants. The weak baryon-baryon matrix elements a_{ij} and b_{ij} are defined as

$$\langle B_i|H_W|B_j\rangle = \bar{u}_{B_i}(a_{ij} + \gamma_5 b_{ij})u_{B_j}. \qquad (9)$$

For a preliminary study, we will restrict ourself to the contributions from parity-conserving amplitudes for the following reasons:

(1) It is well known that the PV matrix element b_{ij} vanishes in the SU(3) flavor symmetry limit, i.e., $\langle B_f P_k|H_W^{PV}|B_i\rangle = 0$. Since for charmed baryon decays $b_{ij} \ll a_{ij}$, the contributions of $\frac{1}{2}^+$–pole terms are expected to be suppressed in s-wave amplitudes and dominant in p-wave amplitudes. Moreover, the presence of the sum of the baryon masses in the denominator further suppresses their contributions. Thus, consideration of PC terms only turns out to be a good approximation for heavy-baryon decays.

(2) Estimation of $\frac{1}{2}^-$–pole terms is a nontrivial task in the present scenario as it involves knowledge of strong coupling constants and weak metrics elements of $\frac{1}{2}^-$–baryons.

(3) Furthermore, it has been argued by Fayyazuddin and Riazuddin [37] that, in the leading nonrelativistic approximation, one can ignore $J^P = \frac{1}{2}^-, \frac{3}{2}^- \ldots$, and higher (orbital) resonances in order to connect them to the relevant ground-state (s-wave) wave function in the overlap integral to satisfy the normalization condition; thus, only the PC amplitude survives.

B. Weak transitions

The flavor symmetric and quark model weak Hamiltonian [41,67] involved in weak transitions for the quark-level process $q_i + q_j \rightarrow q_l + q_m$ is given by

$$H_W \cong V_{il}V_{jm}^* c_-(m_c)[\bar{B}^{[i,j]k} B_{[l,m]k} H_{[i,j]}^{[l,m]}]; \qquad (10)$$

here, $c_- = c_1 + c_2$, and the antisymmetrization among the indices is represented by the brackets, [,]. The spurion transforms like $H_{[2,4]}^{[1,3]}$. Equation (10) can be written in terms of the weak amplitude, a_W, for CKM-favored and CKM-suppressed modes:

$$H_W \cong a_W[\bar{B}^{[i,j]k} B_{[l,m]k} H_{[i,j]}^{[l,m]}]. \qquad (11)$$

As discussed in the literature [42,70,71], a rough estimate of a_W can be made based on symmetry arguments. However, SU(4) symmetry (being badly broken) ignores QCD enhancements due to hard gluon exchanges, contributing through c_-, at corresponding mass scales, that will affect the weak transition.

To calculate numerical values of pole terms, the weak matrix element $\langle B_f|H_W^{PC}|B_i\rangle$ can be treated in the leading nonrelativistic approximation [37]. Moreover, decays of doubly heavy baryons involve heavy-to-heavy transitions; thus, the use of nonrelativistic approximation suits the present analysis. Following the analysis of Riazuddin and Fayyazuddin [37], we obtained the weak transition amplitudes for the charm baryons as a *first approximation*,

$$\mathcal{M}^{PC} = \frac{G_F}{\sqrt{2}}V_{du}V_{cs}\sum_{i>j}(\gamma_i^-\alpha_j^+ + \alpha_i^+\gamma_j^-)(1 - \sigma_i \cdot \sigma_j), \quad (12)$$

where $\mathbf{S}_i = \sigma_i/2$ are Pauli spinors representing the spin of ith quark. The operators α_i^+ and γ_j^- convert $d \rightarrow u$ and $c \rightarrow s$, respectively [76]. The weak Hamiltonian can be obtained by using Fourier transformation of (12),

$$H_W^{PC} = \frac{G_F}{\sqrt{2}}V_{du}V_{cs}\sum_{i \neq j}\alpha_i^+\gamma_j^-(1 - \sigma_i \cdot \sigma_j)\delta^3(r), \qquad (13)$$

which gives the first estimate of the pole terms. The spatial baryon wave function overlap, $\delta^3(r) \equiv \langle \psi_f | \delta^3(r) | \psi_i \rangle$, is usually assumed to be flavor invariant such that

$$\langle \psi_f | \delta^3(r) | \psi_i \rangle_c \approx \langle \psi_f | \delta^3(r) | \psi_i \rangle_s. \tag{14}$$

The relation (14) connects nonleptonic charmed baryon decays with hyperon decays in SU(4) symmetry. The SU(4) being badly broken due to the large mass difference between s and c quarks should yield a larger mismatch between strange and charm baryon wave function overlaps. Several methods have been proposed in the literature to address this issue by the introduction of a correction factor based on different arguments (for a summary, see Ref. [71]). In the present analysis, we follow our previous work [76] by treating $|\psi(0)|^2$ (based of dimensionality argument) as a flavor-dependent quantity. It may be noted that a reliable estimate of the baryon ground-state wave function at the origin (at charm mass scale) can be obtained from, precisely known, experimental masses of baryons using hyperfine splitting, which in turn yields

$$\frac{m_{\Sigma_c} - m_{\Lambda_c}}{m_\Sigma - m_\Lambda} = \frac{\alpha_s(m_c)}{\alpha_s(m_s)} \frac{m_s(m_c - m_u) |\psi(0)|_c^2}{m_c(m_s - m_u) |\psi(0)|_s^2}. \tag{15}$$

Thus, we get

$$\frac{|\psi(0)|_c^2}{|\psi(0)|_s^2} \approx 2.1, \tag{16}$$

for $\frac{\alpha_s(m_c)}{\alpha_s(m_s)} \approx 0.53$ [70,76]. Thus, the variation of flavor-dependent baryon spatial wave function overlap would lead to a substantial correction in branching ratios of doubly heavy baryons. The numerical results are discussed in Sec. IV.

C. Strong coupling constants

In general, meson-baryon strong couplings are obtained from the SU(4)-invariant strong Hamiltonian. In the present work, we follow a relatively accurate method used by Khanna and Verma [74] to calculate the baryon-baryon-pseudoscalar (BB′P) couplings. We extend their analysis to include SU(4)-breaking effects by employing the null result of Coleman and Glashow for the tadpole-type symmetry breaking. The SU(4)-broken (SB) baryon-meson strong couplings are calculated by

$$g_P^{BB'}(\text{SB}) = \frac{M_B + M_B'}{2M_N} \left(\sqrt{\frac{8}{3}} \frac{m_s - m_u}{m_c - m_u} \right) g_P^{BB'}(\text{Sym}), \tag{17}$$

where $g_P^{BB'}(\text{Sym})$ is the value of SU(4) symmetric couplings [74,76]. Effects of symmetry breaking are such that

it should yield larger values of strong couplings as compared to symmetric ones due to mass dependence, consequently leading to larger pole contributions for heavy-baryon decays. The obtained absolute numerical values and expressions of relevant strong meson-baryon coupling constants are presented in Table I. The $g_P^{BB'}(\text{SB})$ are expressed in terms of $g_D (= 8.4)$ and $g_F (= 5.6)$ [41,78].

TABLE I. Expressions of strong-coupling constants and their absolute numerical values.

Strong couplings $g_P^{BB'}$		Absolute values $g_P^{BB'}$ (SB)
$g_D^{\Xi_c \Lambda}$	$\left(\frac{g_D}{\sqrt{2}} + \frac{g_F}{3\sqrt{2}} \right)$	3.30
$g_D^{\Xi_c' \Lambda}$	$\frac{\sqrt{3}}{\sqrt{2}}(g_D - g_F)$	1.60
$g_D^{\Xi_{cc}^{++} \Sigma_c}$	$-(g_D + g_F)$	11.00
$g_D^{\Xi_c \Sigma}$	$\left(\sqrt{3} g_D + \frac{g_F}{\sqrt{3}} \right)$	8.40
$g_D^{\Xi_c' \Sigma}$	$(-g_D + g_F)$	1.40
$g_D^{\Xi_{cc}^+ \Lambda_c^+}$	$\left(\sqrt{3} g_D - \frac{g_F}{\sqrt{3}} \right)$	8.60
$g_D^{\Xi_c \Sigma}$	$\left(\frac{\sqrt{3}}{\sqrt{2}} g_D + \frac{g_F}{\sqrt{6}} \right)$	5.90
$g_D^{\Xi_c' \Sigma}$	$\left(-\frac{g_D}{\sqrt{2}} + \frac{g_F}{\sqrt{2}} \right)$	0.01
$g_{D_s}^{\Xi_c \Xi}$	$-\sqrt{3} g_D - \frac{g_F}{\sqrt{3}}$	8.60
$g_{D_s}^{\Xi_c \Sigma}$	$(-g_D + g_F)$	1.40
$g_K^{\Xi_c \Lambda_c^+}$	$\left(\sqrt{2} g_D - 2\sqrt{2} \frac{g_F}{3} \right)$	16.70
$g_K^{\Xi_c' \Lambda_c^+}$	$\sqrt{2} \frac{g_F}{\sqrt{3}}$	11.80
$g_\pi^{\Xi_c \Xi_c}$	$\left(g_D - 2\frac{g_F}{3} \right)$	12.30
$g_\pi^{\Xi_c' \Xi_c}$	$-\frac{g_F}{\sqrt{3}}$	8.70
$g_\pi^{\Xi_{cc}^+ \Xi_{cc}^+}$	$(-g_D + g_F)$	10.80
$g_\eta^{\Xi_{cc}^+ \Xi_{cc}^+}$	$0.12 g_D + 0.08 g_F$	1.50
$g_\eta^{\Xi_c \Xi_c}$	$-0.96 g_F$	14.50
$g_\eta^{\Xi_c' \Xi_c}$	$0.80(g_D - g_F)$	8.40
$g_{\eta'}^{\Xi_c \Xi_c}$	$1.70 g_D - 1.1 g_F$	21.20
$g_{\eta'}^{\Xi_c' \Xi_c}$	$0.27 g_F$	4.00
$g_{\eta'}^{\Xi_{cc}^+ \Xi_{cc}^+}$	$0.60(g_D - g_F)$	6.90
$g_\pi^{\Xi_{cc}^+ \Xi_c}$	$-\sqrt{2} \frac{g_F}{3}$	12.30
$g_\pi^{\Xi_c \Xi_c}$	$\left(\sqrt{2} g_D - 2\sqrt{2} \frac{g_F}{3} \right)$	17.40
$g_K^{\Xi_c \Sigma_c^{++}}$	$2\frac{g_F}{\sqrt{3}}$	17.00
$g_\pi^{\Xi_c' \Xi_c'}$	g_D	23.10
$g_\eta^{\Xi_c \Xi_c'}$	$0.12 g_D$	2.80
$g_\eta^{\Xi_{cc}^+ \Xi_{cc}^+}$	$0.77 \left(g_D - g_F \right)$	8.40
$g_{\eta'}^{\Xi_c \Xi_c'}$	$1.7 g_D$	39.80
$g_\pi^{\Xi_c \Xi_c'}$	$-\frac{\sqrt{2}}{\sqrt{3}} g_F$	12.30
$g_\pi^{\Sigma_c \Lambda_c^+}$	$-\frac{2 g_F}{\sqrt{3}}$	16.33

(Table continued)

TABLE I. *(Continued)*

Strong couplings $g_P^{BB'}$		Absolute values $g_P^{BB'}$ (SB)
$g_\pi^{\Xi_c'\Xi_c'}$	$\sqrt{2}g_D$	32.60
$g_K^{\Xi_c\Omega_c^0}$	$-2\frac{g_F}{\sqrt{3}}$	17.80
$g_K^{\Omega_c^0\Xi_c'}$	$2g_D$	47.20
$g_{D_s}^{\Xi_{cc}^{++}\Xi_c}$	$\left(-\sqrt{3}g_D+\frac{g_F}{\sqrt{3}}\right)$	8.86
$g_{D_s}^{\Xi_{cc}^{++}\Xi_c'}$	$-(g_D+g_F)$	11.16
$g_K^{\Xi_{cc}^{++}\Omega_{cc}^+}$	$\sqrt{2}(g_D-g_F)$	15.46
$g_D^{\Sigma_c^{++}p}$	$\sqrt{2}(g_D-g_F)$	1.72
$g_D^{\Xi_{cc}^{++}\Lambda_c^+}$	$\left(-\sqrt{3}g_D+\frac{g_F}{\sqrt{3}}\right)$	8.60
$g_{D_s}^{\Sigma_c^{++}\Sigma}$	$\sqrt{2}(g_D-g_F)$	1.85
$g_{D_s}^{\Xi_{cc}^{++}\Xi_c'}$	$-(g_D+g_F)$	11.16
$g_\pi^{\Sigma_c^{++}\Lambda_c^+}$	$\frac{2g_F}{\sqrt{3}}$	16.33
$g_\pi^{\Xi_{cc}^{++}\Xi_{cc}^+}$	$\sqrt{2}(g_D-g_F)$	15.31
$g_K^{\Sigma_c^{++}\Xi_c}$	$\frac{2g_F}{\sqrt{3}}$	16.96
$g_\pi^{\Sigma_c^{++}\Sigma_c^{++}}$	$2g_D$	43.91
$g_\pi^{\Xi_{cc}^{++}\Xi_{cc}^{++}}$	(g_D-g_F)	10.82
$g_\eta^{\Sigma_c^{++}\Sigma_c^{++}}$	$1.55g_D$	33.98
$g_\eta^{\Xi_{cc}^{++}\Xi_{cc}^{++}}$	$0.77(g_D-g_F)$	8.38
$g_{\eta'}^{\Sigma_c^{++}\Sigma_c^{++}}$	$1.27g_D$	27.81
$g_{\eta'}^{\Xi_{cc}^{++}\Xi_{cc}^{++}}$	$0.63(g_D-g_F)$	6.86
$g_K^{\Sigma_c^{++}\Xi_c'}$	$2g_D$	45.02
$g_D^{\Lambda_c^+p(n)}$	$\left(\sqrt{3}g_D+\frac{g_F}{\sqrt{3}}\right)$	7.37
$g_D^{\Sigma_c p(n)}$	$(-g_D+g_F)$	1.22
$g_D^{\Xi_{cc}^+\Sigma_c}$	$-\sqrt{2}(g_D+g_F)$	15.47
$g_{D_s}^{\Lambda_c^+\Lambda}$	$\sqrt{2}\left(g_D+\frac{g_F}{3}\right)$	6.35
$g_{D_s}^{\Xi_c^+\Sigma_c}$	$\left(-\sqrt{3}g_D+\frac{g_F}{\sqrt{3}}\right)$	8.87
$g_{D_s}^{\Sigma_c\Sigma}$	$\sqrt{2}(g_D-g_F)$	1.86
$g_\eta^{\Lambda_c^+\Lambda_c^+}$	$1.55g_D-1.03g_F$	17.60
$g_{D_s}^{\Lambda_c^+\Sigma}$	0	0
$g_\pi^{\Lambda_c^+\Lambda_c^+}$	0	0
$g_\eta^{\Sigma_c\Lambda_c^+}$	0	0

D. Factorization

The factorizable decay amplitudes (ignoring the scale factors) can be expanded in terms of the following reduced matrix elements:

$$\mathcal{A}^{\mathrm{Fac}}(B_i \to B_f + P_k)$$
$$\equiv \langle P_k(q)|A_\mu|0\rangle\langle B_f(p_f)|V^\mu + A^\mu|B_i(p_i)\rangle. \quad (18)$$

The baryon-baryon matrix elements of the weak currents can be expressed in terms of form factors f_i and g_i (as functions of q^2) [62,63] as

$$\langle B_f(p_f)|V_\mu|B_i(p_i)\rangle$$
$$= \bar{u}_f(p_f)\left[f_1\gamma_\mu - \frac{f_2}{m_i}i\sigma_{\mu\nu}q^\nu + \frac{f_3}{m_i}q_\mu\right]u_i(p_i), \quad (19)$$

and

$$\langle B_f(p_f)|A_\mu|B(p_i)\rangle$$
$$= \bar{u}_f(p_f)\left[g_1\gamma_\mu\gamma_5 - \frac{g_2}{m_i}i\sigma_{\mu\nu}q^\nu\gamma_5 + \frac{g_3}{m_i}q_\mu\gamma_5\right]u_i(p_i). \quad (20)$$

The decay constant f_P of the emitted pseudoscalar meson, P_k, is defined as

$$\langle P_k(q)|A_\mu|0\rangle = if_P m_P. \quad (21)$$

The factorizable amplitudes could be simplified to

$$A_1^{\mathrm{Fac}} = -\frac{G_F}{\sqrt{2}}F_C f_P c_k[(m_i-m_f)f_1^{B_i,B_f}(m_P^2)],$$

$$B_1^{\mathrm{Fac}} = \frac{G_F}{\sqrt{2}}F_C f_P c_k[(m_i+m_f)g_1^{B_i,B_f}(m_P^2)],$$

where the factor F_C is a product of appropriate CKM factors and Clebsch-Gordan (CG) coefficients and c_k are corresponding QCD coefficients.

We use the NRQM [72] and the HQET [73] to calculate the baryon-baryon transition form factors f_i and g_i. In the NRQM calculations, the form factors are calculated in the Breit frame and include several corrections like the hard-gluon QCD contributions, the q^2 dependence of the form factors, and the wave-function mismatch. Later, in the heavy-quark sector, a $1/m_Q$ correction to the baryon-baryon transition form factors was introduced within the heavy-quark symmetry constraints using HQET. The obtained transition form factors are given in Table II.

We use the mixing scheme for η and η' mesons

$$\eta'(0.958) = \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})\cos\phi_P + (s\bar{s})\sin\phi_P,$$

$$\eta(0.547) = \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})\sin\phi_P - (s\bar{s})\cos\phi_P, \quad (22)$$

where $\phi_P = \theta_{\mathrm{ideal}} - \theta_P^{\mathrm{phy}}$ and $\theta_P^{\mathrm{phy}} = -15.4°$ [79]. The decay constants [79,80] relevant for the present analysis are given as

$$f_\pi = 131 \text{ MeV}, \qquad f_\eta = 133 \text{ MeV},$$
$$f_{\eta'} = 126 \text{ MeV}, \qquad f_K = 160 \text{ MeV},$$
$$f_D = 207.4 \text{ MeV} \quad \text{and} \quad f_{D_s} = 255 \text{ MeV}.$$

TABLE II. Ξ_{cc}^{++} and Ξ_{cc}^{+} transition form factors in NRQM [72] and HQET [73].

		Form factors	
Transitions	Models [72,73]	f_1	g_1
$\Xi_{cc}^{++} \to \Lambda_c^+$	NRQM	−0.35	−0.19
	HQET	−0.59	−0.27
$\Xi_{cc}^{++} \to \Sigma_c^{++}$	NRQM	−0.39	−0.96
	HQET	−0.54	−1.35
$\Xi_{cc}^{++} \to \Sigma_c^{+}$	NRQM	−0.27	−0.68
	HQET	−0.38	−0.95
$\Xi_{cc}^{++} \to \Xi_c^{+}$	NRQM	−0.57	−0.24
	HQET	−0.74	−0.29
$\Xi_{cc}^{++} \to \Xi_c'^{+}$	NRQM	−0.37	−0.78
	HQET	−0.43	−0.91
$\Xi_{cc}^{+} \to \Lambda_c^+$	NRQM	0.35	0.19
	HQET	0.59	0.27
$\Xi_{cc}^{+} \to \Sigma_c^{+}$	NRQM	−0.27	−0.68
	HQET	−0.38	−0.95
$\Xi_{cc}^{+} \to \Sigma_c^{0}$	NRQM	−0.39	−0.96
	HQET	−0.54	−1.35
$\Xi_{cc}^{+} \to \Xi_c^{0}$	NRQM	−0.57	−0.24
	HQET	−0.74	−0.29
$\Xi_{cc}^{+} \to \Xi_c'^{0}$	NRQM	−0.37	−0.78
	HQET	−0.43	−0.91

IV. NUMERICAL RESULTS AND DISCUSSIONS

The preliminary results for the various decay channels of Ξ_{cc} are obtained as a sum of the factorization and the pole contributions to different PV and PC amplitudes. As mentioned before, SU(4) symmetry breaking could be substantially large; thus, the use of exact SU(4) symmetry could be questioned. Therefore, we include SU(4)-breaking effects in evaluating strong coupling constants as well weak transitions. First, we evaluate the factorizable amplitudes using NRQM- and HQET-based form factors for CKM-favored, CKM-suppressed, and CKM–doubly suppressed modes as listed in columns 3 and 4 of Tables III–VII. The flavor-independent pole amplitudes are calculated by using

SU(4) broken strong coupling constants as shown in column 5 of Tables III–VII.

Later, we introduce the flavor-dependent effects in weak transition amplitudes through hyperfine splitting. The variation of the spatial baryon wave function overlap, $|\psi(0)|^2$, with flavor results in larger pole contributions. The numerical values' of flavor-dependent pole amplitudes for Ξ_{cc} decays in CKM-favored, CKM-suppressed, and CKM–doubly suppressed modes are given in column 6 of Tables III–VII. It can be clearly seen that the pole contributions are enhanced by a factor of ∼2 due to flavor-dependent effects caused by SU(4) breaking. Moreover, the increment in pole amplitudes could be viewed as variation of scale (charm to strange) by 2.

We wish to remark that a significant contribution to the parity-violating amplitudes may come from, $\frac{1}{2}^-$, the lowest-lying negative-parity excited baryons; however, the estimation of such terms is far from simple, as discussed in Refs. [63–66,71]. In addition, symmetry-based attempts have also been made to estimate their contributions for singly charmed baryons. Such attempts required sufficient experimental information on decays which is not available at present for doubly heavy Ξ_{cc} baryons. Therefore, we have only considered ground-state $\frac{1}{2}^+$–intermediate baryon pole terms as a first estimate of pole contributions. It may be noted that a large theoretical uncertainty in the lifetime of Ξ_{cc} states could be seen as another source of uncertainty in the results. We use $\tau_{\Xi_{cc}^{++}} = 300$ fs and $\tau_{\Xi_{cc}^{+}} = 100$ fs [58] to obtain the branching ratios in the present work.

After adding factorizable and pole contributions, we calculate the branching ratios and asymmetry parameters for two-body weak decays of doubly heavy Ξ_{cc} baryons for the flavor-independent and flavor-dependent cases. To emphasize the importance of the W-exchange contribution to Ξ_{cc} decays, we present our predictions for the branching ratios of Ξ_{cc} decays receiving contributions only from pole amplitudes in Tables VIII and IX. The prediction for branching ratios receiving contributions from both the factorization and pole or factorization only are given in Tables X–XII for CKM-favored, CKM-suppressed, and

TABLE III.   Decay amplitudes (in units of $\frac{G_F}{\sqrt{2}} V_{uq} V_{cq}^*$) for CKM-favored ($\Delta C = \Delta S = -1$) mode.

		Factorization[a]		Pole amplitude	
Decays	Models [72,73]	A^{Fac}	B^{Fac}	Flavor independent	Flavor dependent
$\Xi_{cc}^{++} \to \Sigma^+ D^+$	NRQM	0	0	0.101	0.212
	HQET	0	0		
$\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$	NRQM	0.110	−0.250	0.372	0.782
	HQET	0.142	−0.290		
$\Xi_{cc}^{++} \to \Sigma_c^{++} \bar{K}^0$	NRQM	−0.042	0.520	0	0
	HQET	−0.060	0.730		
$\Xi_{cc}^{++} \to \Xi_c'^+ \pi^+$	NRQM	0.064	−0.800	0	0
	HQET	0.076	−0.930		

[a]A and B represent PV and PC amplitudes, respectively.

TABLE IV.   Decay amplitudes (in units of $\frac{G_F}{\sqrt{2}} V_{uq} V_{cq}^*$) for the CKM-favored ($\Delta C = \Delta S = -1$) mode.

		Factorization		Pole amplitude	
Decays	Models [72,73]	A^{Fac}	B^{Fac}	Flavor independent	Flavor dependent
$\Xi_{cc}^+ \to \Lambda^0 D^+$	NRQM	0	0	0.082	0.172
	HQET	0	0		
$\Xi_{cc}^+ \to \Sigma^+ D^0$	NRQM	0	0	0.119	0.249
	HQET	0	0		
$\Xi_{cc}^+ \to \Sigma^0 D^+$	NRQM	0	0	0.156	0.327
	HQET	0	0		
$\Xi_{cc}^+ \to \Xi^0 D_s{}^+$	NRQM	0	0	-0.114	-0.239
	HQET	0	0		
$\Xi_{cc}^+ \to \Lambda_c^+ \bar{K}^0$	NRQM	0.043	-0.102	-0.407	-0.854
	HQET	0.072	-0.144		
$\Xi_{cc}^+ \to \Xi_c^+ \pi^0$	NRQM	0	0	-0.562	-1.179
	HQET	0	0		
$\Xi_{cc}^+ \to \Xi_c'^+ \pi^0$	NRQM	0	0	0.211	0.444
	HQET	0	0		
$\Xi_{cc}^+ \to \Xi_c^+ \eta$	NRQM	0	0	0.240	0.504
	HQET	0	0		
$\Xi_{cc}^+ \to \Xi_c'^+ \eta$	NRQM	0	0	0.353	0.741
	HQET	0	0		
$\Xi_{cc}^+ \to \Xi_c^+ \eta'$	NRQM	0	0	-0.349	-0.733
	HQET	0	0		
$\Xi_{cc}^+ \to \Xi_c'^+ \eta'$	NRQM	0	0	-0.097	-0.205
	HQET	0	0		
$\Xi_{cc}^+ \to \Xi_c^0 \pi^+$	NRQM	0.110	-0.250	-0.422	-0.887
	HQET	0.143	-0.290		
$\Xi_{cc}^+ \to \Xi_c'^0 \pi^+$	NRQM	0.064	-0.802	0.299	0.628
	HQET	0.080	-0.940		
$\Xi_{cc}^+ \to \Sigma_c^{++} K^-$	NRQM	0	0	-0.412	-0.866
	HQET	0	0		
$\Xi_{cc}^+ \to \Sigma_c^+ \bar{K}^0$	NRQM	-0.030	0.370	-0.291	-0.612
	HQET	-0.042	0.515		
$\Xi_{cc}^+ \to \Omega_c^0 K^+$	NRQM	0	0	0.433	0.909
	HQET	0	0		

TABLE V.   Decay amplitudes (in units of $\frac{G_F}{\sqrt{2}} V_{uq} V_{cq}^*$) for the CKM-suppressed ($\Delta C = -1$, $\Delta S = 0$) mode.

		Factorization		Pole amplitude	
Decays	Models [72,73]	A^{Fac}	B^{Fac}	Flavor independent	Flavor dependent
$\Xi_{cc}^{++} \to p D^+$	NRQM	0	0	0.087	0.182
	HQET	0	0		
$\Xi_{cc}^{++} \to \Sigma^+ D_s{}^+$	NRQM	0	0	0.099	0.207
	HQET	0	0		
$\Xi_{cc}^{++} \to \Lambda_c^+ \pi^+$	NRQM	0.078	-0.190	0.322	0.676
	HQET	0.131	-0.270		
$\Xi_{cc}^{++} \to \Xi_c^+ K^+$	NRQM	0.150	-0.320	0.354	0.743
	HQET	0.190	-0.380		
$\Xi_{cc}^{++} \to \Xi_c'^+ K^+$	NRQM	0.090	-1.060	0	0
	HQET	0.100	-1.230		
$\Xi_{cc}^{++} \to \Sigma_c^{++} \pi^0$	NRQM	0.022	-0.280	0	0
	HQET	0.030	-0.400		
$\Xi_{cc}^{++} \to \Sigma_c^{++} \eta$	NRQM	0.042	-0.530	0	0
	HQET	0.062	-0.730		
$\Xi_{cc}^{++} \to \Sigma_c^{++} \eta'$	NRQM	-0.017	0.170	0	0
	HQET	-0.023	0.230		
$\Xi_{cc}^{++} \to \Sigma_c^+ \pi^+$	NRQM	0.050	-0.690	0	0
	HQET	0.080	-0.960		

TABLE VI.   Decay amplitudes (in units of $\frac{G_F}{\sqrt{2}}V_{uq}V_{cq}^*$) for the CKM-suppressed ($\Delta C = -1, \Delta S = 0$) mode.

Decays	Models [72,73]	Factorization		Pole amplitude	
		A^{Fac}	B^{Fac}	Flavor independent	Flavor dependent
$\Xi_{cc}^+ \to pD^0$	NRQM	0	0	-0.111	-0.234
	HQET	0	0		
$\Xi_{cc}^+ \to nD^+$	NRQM	0	0	0.198	0.416
	HQET	0	0		
$\Xi_{cc}^+ \to \Lambda^0 D_s^+$	NRQM	0	0	0.056	0.117
	HQET	0	0		
$\Xi_{cc}^+ \to \Sigma^0 D_s^+$	NRQM	0	0	0.070	0.147
	HQET	0	0		
$\Xi_{cc}^+ \to \Lambda_c^+ \pi^0$	NRQM	-0.022	0.054	-0.228	-0.478
	HQET	-0.037	0.077		
$\Xi_{cc}^+ \to \Lambda_c^+ \eta$	NRQM	-0.044	0.010	0.194	0.407
	HQET	-0.074	0.144		
$\Xi_{cc}^+ \to \Lambda_c^+ \eta'$	NRQM	-0.018	0.034	-0.159	-0.333
	HQET	-0.028	0.046		
$\Xi_{cc}^+ \to \Xi_c^+ K^0$	NRQM	0	0	-0.710	-1.49
	HQET	0	0		
$\Xi_{cc}^+ \to \Xi_c'^+ K^0$	NRQM	0	0	-0.249	-0.523
	HQET	0	0		
$\Xi_{cc}^+ \to \Xi_c^0 K^+$	NRQM	0.150	-0.330	-0.352	-0.739
	HQET	0.190	-0.380		
$\Xi_{cc}^+ \to \Xi_c'^0 K^+$	NRQM	0.087	-1.060	-0.249	-0.523
	HQET	0.103	-1.230		
$\Xi_{cc}^+ \to \Sigma_c^{++} \pi^-$	NRQM	0	0	-0.343	-0.721
	HQET	0	0		
$\Xi_{cc}^+ \to \Sigma_c^+ \pi^0$	NRQM	0.015	-0.200	0.343	0.721
	HQET	0.022	-0.275		
$\Xi_{cc}^+ \to \Sigma_c^+ \eta$	NRQM	0.030	-0.370	0	0
	HQET	0.043	-0.520		
$\Xi_{cc}^+ \to \Sigma_c^+ \eta'$	NRQM	-0.011	0.122	0	0
	HQET	-0.016	0.171		
$\Xi_{cc}^+ \to \Sigma_c^0 \pi^+$	NRQM	0.076	-0.971	0.343	0.721
	HQET	0.110	-1.360		

TABLE VII.   Decay amplitudes (in units of $\frac{G_F}{\sqrt{2}}V_{uq}V_{cq}^*$) for the CKM–doubly suppressed ($\Delta C = -\Delta S = -1$) mode.

Decays	Models [72,73]	Factorization		Pole amplitude	
		A^{Fac}	B^{Fac}	Flavor independent	Flavor dependent
$\Xi_{cc}^{++} \to pD_s^+$	NRQM	0	0	0.085	0.178
	HQET	0	0		
$\Xi_{cc}^{++} \to \Lambda_c^+ K^+$	NRQM	0.110	-0.025	0.308	0.647
	HQET	0.180	-0.360		
$\Xi_{cc}^{++} \to \Sigma_c^{++} K^0$	NRQM	-0.042	0.520	0	0
	HQET	-0.059	0.730		
$\Xi_{cc}^{++} \to \Sigma_c^+ K^+$	NRQM	-0.004	0.045	0	0
	HQET	-0.005	0.063		
$\Xi_{cc}^+ \to nD_s^+$	NRQM	0	0	-0.085	-0.178
	HQET	0	0		
$\Xi_{cc}^+ \to \Lambda_c^+ K^0$	NRQM	0.043	-0.102	0.308	0.647
	HQET	0.072	-0.144		
$\Xi_{cc}^+ \to \Sigma_c^+ K^0$	NRQM	-0.030	0.370	0	0
	HQET	-0.042	0.515		
$\Xi_{cc}^+ \to \Sigma_c^0 K^+$	NRQM	0.104	-1.283	0	0
	HQET	0.150	-1.797		

TABLE VIII. Branching ratios for the CKM-favored ($\Delta C = \Delta S = -1$) mode with only pole contributions. The branching ratios for an arbitrary lifetime can be obtained by using $\left(\frac{\tau_{\Xi_{cc}^{++}}}{300}\right) \times \mathcal{B}(B_i \to B_f P)$ and $\left(\frac{\tau_{\Xi_{cc}^{+}}}{100}\right) \times \mathcal{B}(B_i \to B_f P)$.

	Branching ratios	
Decays	Flavor independent	Flavor dependent
$\Xi_{cc}^{++} \to \Sigma^+ D^+$	2.0×10^{-3}	8.9×10^{-3}
$\Xi_{cc}^{+} \to \Lambda^0 D^+$	5.3×10^{-4}	2.4×10^{-3}
$\Xi_{cc}^{+} \to \Sigma^+ D^0$	9.4×10^{-4}	4.2×10^{-3}
$\Xi_{cc}^{+} \to \Sigma^0 D^+$	1.6×10^{-3}	7.0×10^{-3}
$\Xi_{cc}^{+} \to \Xi^0 D_s^+$	4.1×10^{-4}	1.8×10^{-3}
$\Xi_{cc}^{+} \to \Xi_c^+ \pi^0$	1.1×10^{-2}	5.0×10^{-2}
$\Xi_{cc}^{+} \to \Xi_c'^+ \pi^0$	1.2×10^{-3}	5.4×10^{-3}
$\Xi_{cc}^{+} \to \Xi_c^+ \eta$	1.4×10^{-3}	6.4×10^{-3}
$\Xi_{cc}^{+} \to \Xi_c'^+ \eta$	2.2×10^{-3}	9.5×10^{-3}
$\Xi_{cc}^{+} \to \Xi_c^+ \eta'$	7.9×10^{-4}	3.5×10^{-3}
$\Xi_{cc}^{+} \to \Xi_c'^+ \eta'$	1.8×10^{-5}	8.1×10^{-5}
$\Xi_{cc}^{+} \to \Sigma_c^{++} K^-$	4.8×10^{-3}	2.1×10^{-2}
$\Xi_{cc}^{+} \to \Omega_c^0 K^+$	2.2×10^{-3}	1.0×10^{-2}

TABLE IX. Branching ratios for the CKM-suppressed ($\Delta C = -1, \Delta S = 0$) and CKM–doubly suppressed ($\Delta C = -\Delta S = -1$) modes with only pole contributions.

	Branching ratios	
Decays	Flavor independent	Flavor dependent
($\Delta C = -1, \Delta S = 0$)		
$\Xi_{cc}^{++} \to p D^+$	1.4×10^{-4}	6.0×10^{-4}
$\Xi_{cc}^{+} \to \Sigma^+ D_s^+$	7.7×10^{-5}	3.4×10^{-4}
$\Xi_{cc}^{+} \to p D^0$	7.6×10^{-5}	3.4×10^{-4}
$\Xi_{cc}^{+} \to n D^+$	2.4×10^{-4}	1.1×10^{-3}
$\Xi_{cc}^{+} \to \Lambda^0 D_s^+$	1.0×10^{-5}	4.5×10^{-5}
$\Xi_{cc}^{+} \to \Sigma^0 D_s^+$	1.3×10^{-5}	5.6×10^{-5}
$\Xi_{cc}^{+} \to \Xi_c^+ K^0$	7.0×10^{-4}	3.1×10^{-3}
$\Xi_{cc}^{+} \to \Xi_c'^+ K^0$	6.2×10^{-5}	2.7×10^{-3}
$\Xi_{cc}^{+} \to \Sigma_c^{++} \pi^-$	2.3×10^{-4}	1.0×10^{-3}
($\Delta C = -\Delta S = -1$)		
$\Xi_{cc}^{++} \to p D_s^+$	5.7×10^{-6}	2.5×10^{-5}
$\Xi_{cc}^{+} \to n D_s^+$	1.9×10^{-6}	8.4×10^{-6}

CKM–doubly suppressed, respectively. We draw the following observations:

(1) As expected, a large number of the Ξ_{cc} decay channels receive contributions from the W-exchange process. Upon comparison with factorizable contributions, we find that the pole amplitudes are not only equipollent but also are dominant in several decays.

(2) In the CKM-favored ($\Delta C = \Delta S = -1$) decay mode, most of the decays come from the pole diagrams alone, and only two of the decay channels come from factorization. The rest of the decays receive dominant pole contributions except for $\Xi_{cc}^{+} \to \Xi_c'^0 \pi^+$.

The order of the branching ratios for all the decays range from 10^{-2} to 10^{-5} for the flavor-independent case. While the inclusion of flavor-dependent effects enhances the pole contributions, consequently the branching ratios of dominant modes become $\mathcal{O}(10^{-1}) \sim \mathcal{O}(10^{-3})$.

(3) The pole and factorizable amplitudes can interfere constructively or destructively in decay modes with both, factorizable and pole, contributions. The pole and factorization amplitudes interfere constructively, in $\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$, $\Xi_{cc}^{+} \to \Xi_c'^0 \pi^+$, and $\Xi_{cc}^{+} \to \Sigma_c^+ \bar{K}^0$ decay channels; however, these interfere destructively in $\Xi_{cc}^{+} \to \Lambda_c^+ \bar{K}^0$ and $\Xi_{cc}^{+} \to \Xi_c^0 \pi^+$ decays.

TABLE X. Branching ratios for the CKM-favored ($\Delta C = \Delta S = -1$) mode including factorization and pole contributions.

		Branching ratios		Asymmetries (α)	
Decays	Models [72,73]	Flavor independent	Flavor dependent	Flavor independent	Flavor dependent
$\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$	NRQM	7.8×10^{-2}	15.1×10^{-2}	-0.997	-0.856
	HQET	10.9×10^{-2}	18.5×10^{-2}	-0.991	-0.942
$\Xi_{cc}^{++} \to \Sigma_c^{++} \bar{K}^0$	NRQM	2.8×10^{-2}	$\cdots$	-0.760	$\cdots$
	HQET	5.5×10^{-2}		-0.760	$\cdots$
$\Xi_{cc}^{++} \to \Xi_c'^+ \pi^+$	NRQM	6.4×10^{-2}	$\cdots$	-0.780	$\cdots$
	HQET	8.8×10^{-2}		-0.780	$\cdots$
$\Xi_{cc}^{+} \to \Lambda_c^+ \bar{K}^0$	NRQM	6.0×10^{-3}	2.7×10^{-2}	0.927	0.504
	HQET	8.3×10^{-3}	2.7×10^{-2}	0.964	0.785
$\Xi_{cc}^{+} \to \Xi_c^0 \pi^+$	NRQM	1.3×10^{-2}	2.7×10^{-2}	0.552	0.996
	HQET	2.1×10^{-2}	3.3×10^{-2}	0.341	0.972
$\Xi_{cc}^{+} \to \Xi_c'^0 \pi^+$	NRQM	3.3×10^{-2}	5.9×10^{-2}	-0.653	-0.502
	HQET	4.7×10^{-2}	7.2×10^{-2}	-0.647	-0.535
$\Xi_{cc}^{+} \to \Sigma_c^+ \bar{K}^0$	NRQM	1.3×10^{-2}	2.8×10^{-2}	-0.483	-0.336
	HQET	2.0×10^{-2}	3.8×10^{-2}	-0.543	-0.404

TABLE XI. Branching ratios for the CKM-suppressed ($\Delta C = -1, \Delta S = 0$) mode including factorization and pole contributions.

Decays	Models [72,73]	Branching ratios		Asymmetries (α)	
		Flavor independent	Flavor dependent	Flavor independent	Flavor dependent
$\Xi_{cc}^{++} \to \Lambda_c^+ \pi^+$	NRQM	3.2×10^{-3}	7.4×10^{-2}	-0.930	-0.690
	HQET	5.8×10^{-3}	1.0×10^{-2}	-1.000	-0.890
$\Xi_{cc}^{++} \to \Xi_c^+ K^+$	NRQM	5.1×10^{-3}	8.0×10^{-3}	-0.970	-0.980
	HQET	7.6×10^{-3}	1.1×10^{-2}	-0.920	-1.000
$\Xi_{cc}^{++} \to \Xi_c'^+ K^+$	NRQM	4.4×10^{-3}	$\cdots$	-0.850	$\cdots$
	HQET	6.0×10^{-3}		-0.850	$\cdots$
$\Xi_{cc}^{++} \to \Sigma_c^{++} \pi^0$	NRQM	5.3×10^{-4}	$\cdots$	-0.700	$\cdots$
	HQET	1.0×10^{-3}		-0.690	$\cdots$
$\Xi_{cc}^{++} \to \Sigma_c^{++} \eta$	NRQM	1.3×10^{-3}	$\cdots$	-0.780	$\cdots$
	HQET	2.7×10^{-3}		-0.780	$\cdots$
$\Xi_{cc}^{++} \to \Sigma_c^{++} \eta'$	NRQM	5.7×10^{-5}	$\cdots$	-0.980	$\cdots$
	HQET	1.1×10^{-4}		-0.980	$\cdots$
$\Xi_{cc}^{++} \to \Sigma_c^+ \pi^+$	NRQM	3.2×10^{-3}	$\cdots$	-0.690	$\cdots$
	HQET	6.3×10^{-3}		-0.690	$\cdots$
$\Xi_{cc}^+ \to \Lambda_c^+ \pi^0$	NRQM	2.5×10^{-4}	8.3×10^{-4}	-0.625	-0.360
	HQET	3.4×10^{-4}	9.5×10^{-4}	-0.840	-0.548
$\Xi_{cc}^+ \to \Lambda_c^+ \eta$	NRQM	1.2×10^{-4}	3.0×10^{-4}	0.734	0.935
	HQET	2.8×10^{-4}	4.2×10^{-4}	0.277	0.960
$\Xi_{cc}^+ \to \Lambda_c^+ \eta'$	NRQM	4.6×10^{-5}	1.4×10^{-4}	-0.829	-0.517
	HQET	7.0×10^{-5}	1.7×10^{-4}	-0.983	-0.747
$\Xi_{cc}^+ \to \Xi_c^0 K^+$	NRQM	1.1×10^{-3}	1.3×10^{-3}	0.061	0.778
	HQET	1.8×10^{-3}	2.0×10^{-3}	-0.052	0.578
$\Xi_{cc}^+ \to \Xi_c'^0 K^+$	NRQM	1.0×10^{-3}	6.2×10^{-4}	-0.950	-1.000
	HQET	1.4×10^{-3}	9.7×10^{-4}	-0.940	-1.000
$\Xi_{cc}^+ \to \Sigma_c^+ \pi^0$	NRQM	5.8×10^{-4}	1.7×10^{-3}	-0.290	-0.173
	HQET	7.7×10^{-4}	2.0×10^{-3}	-0.351	-0.222
$\Xi_{cc}^+ \to \Sigma_c^+ \eta$	NRQM	2.3×10^{-4}	$\cdots$	-0.780	$\cdots$
	HQET	4.5×10^{-4}		-0.780	$\cdots$
$\Xi_{cc}^+ \to \Sigma_c^+ \eta'$	NRQM	1.0×10^{-5}	$\cdots$	-0.980	$\cdots$
	HQET	1.9×10^{-5}		-0.980	$\cdots$
$\Xi_{cc}^+ \to \Sigma_c^0 \pi^+$	NRQM	3.7×10^{-3}	5.9×10^{-3}	-0.552	-0.443
	HQET	6.3×10^{-3}	9.1×10^{-3}	-0.589	-0.498

TABLE XII. Branching ratios for the CKM–doubly suppressed ($\Delta C = -\Delta S = -1$) mode including factorization and pole contributions.

Decays	Models [72,73]	Branching ratios		Asymmetries (α)	
		Flavor independent	Flavor dependent	Flavor independent	Flavor dependent
$\Xi_{cc}^{++} \to \Lambda_c^+ K^+$	NRQM	2.1×10^{-4}	3.8×10^{-4}	-1.000	-0.860
	HQET	4.2×10^{-4}	6.2×10^{-4}	-0.970	-1.000
$\Xi_{cc}^{++} \to \Sigma_c^{++} K^0$	NRQM	7.6×10^{-5}	$\cdots$	-0.760	$\cdots$
	HQET	1.5×10^{-4}		-0.760	$\cdots$
$\Xi_{cc}^{++} \to \Sigma_c^+ K^+$	NRQM	2.3×10^{-4}	$\cdots$	-0.760	$\cdots$
	HQET	4.6×10^{-4}		-0.760	$\cdots$
$\Xi_{cc}^+ \to \Lambda_c^+ K^0$	NRQM	2.5×10^{-5}	7.2×10^{-5}	-0.802	-0.510
	HQET	3.9×10^{-5}	8.9×10^{-5}	-0.964	-0.731
$\Xi_{cc}^+ \to \Sigma_c^+ K^0$	NRQM	1.3×10^{-5}	$\cdots$	-0.760	$\cdots$
	HQET	2.5×10^{-5}		-0.760	$\cdots$
$\Xi_{cc}^+ \to \Sigma_c^0 K^+$	NRQM	1.5×10^{-4}	$\cdots$	-0.760	$\cdots$
	HQET	3.0×10^{-4}		-0.760	$\cdots$

Because of flavor dependence, branching ratios of the most dominant modes $\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^+ \pi^+)$, $\mathcal{B}(\Xi_{cc}^+ \to \Xi_c'^0 \pi^+)$, and $\mathcal{B}(\Xi_{cc}^+ \to \Xi_c^+ \pi^0)$ are enhanced by an order of magnitude, and the last decay comes from W-exchange diagrams only. The large decay width of $\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$ decay makes it the best candidate to look out for in experimental searches.

(4) The factorization contributions obtained from NRQM and HQET differ, owing to the difference in form factors. The results based on HQET, in general, have larger values.

(5) In the CKM-suppressed ($\Delta C = -1, \Delta S = 0$) decay mode, most of the dominant decays receive contributions from both pole and decay amplitudes via their constructive inference. The flavor-dependent branching ratios of such decay channels are $\mathcal{O}(10^{-2}) \sim \mathcal{O}(10^{-3})$ with a few exceptions. However, the pole-only decays have branching ratios of $\mathcal{O}(10^{-3}) \sim \mathcal{O}(10^{-5})$. The most dominant decays in this mode are $\Xi_{cc}^{++} \to \Lambda_c^+ \pi^+$, $\Xi_{cc}^{++} \to \Xi_c^+ K^+$, and $\Xi_{cc}^+ \to \Sigma_c^0 \pi^+$.

(6) The decays, $\Xi_{cc}^+ \to \Xi_c^0 K^+$ and $\Xi_{cc}^+ \to \Xi_c'^0 K^+$, present an interesting case of destructive interference between pole and factorization terms. It is worth noting that in $\Xi_{cc}^+ \to \Xi_c^0 K^+$ decay pole and factorization contributions to PC amplitudes are roughly comparable, while the PC factorization amplitude in $\Xi_{cc}^+ \to \Xi_c'^0 K^+$ is predominant. Experimental searches for such decays will provide a useful test of the theory.

(7) The decay channels in the CKM–doubly suppressed ($\Delta C = \Delta S = -1$) modes have branching ratios $\mathcal{O}(10^{-4}) \sim \mathcal{O}(10^{-6})$. Only two of the decays attain contributions from the pole alone. The decays having both pole and factorization contributions have larger branching ratios. It is interesting to note that decays with factorization-only contributions have branching ratios comparable to the decays with pole-only contributions.

(8) We wish to point out that the flavor-dependent results enhance the contribution of pole terms roughly by a factor of 4, consequently giving larger branching ratios. Thus, results based on flavor dependence and flavor-independent analyses provide a useful domain for experimental searches.

To compare our results with other works, we present corresponding decay modes in Table XIII. We first compare our results with some of the very recent analyses of nonleptonic decays Ξ_{cc} baryons based on the factorization scheme [58,59]. Wang *et al.* [58] have given an analysis of weak decays of doubly heavy baryons in the quark-diquark picture using the light front approach. Their branching ratios for dominant CKM-favored modes $\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^{(\prime)+} \pi^+)$ and $\mathcal{B}(\Xi_{cc}^+ \to \Xi_c^{(\prime)0} \pi^+)$ are of the order of a few percent. The $\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c'^+ \pi^+)$ compares well with our

result with no pole contribution (owing to a zero CG coefficient of baryon-baryon weak coupling for W-exchange pole terms[1]). Despite the inclusion of dominant pole contributions and constructive interference between PC pole and factorization amplitudes, our result for the most dominant $\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^+ \pi^+)$ is comparable to their result, i.e., 7.24%. Thus, the major difference in results is due to the different form factors used in both the works. As mentioned before, $\Xi_{cc}^+ \to \Xi_c^0 \pi^+$ and $\Xi_{cc}^+ \to \Xi_c'^0 \pi^+$ represent peculiar cases of destructive and constructive interference between pole and factorization amplitudes, respectively. Therefore, the magnitude of the $\mathcal{B}(\Xi_{cc}^+ \to \Xi_c^0 \pi^+)$ in our case is smaller as compared to their branching 2.4% and vice versa for $\mathcal{B}(\Xi_{cc}^+ \to \Xi_c'^0 \pi^+)$. Similarly, for CKM-suppressed and CKM–doubly suppressed modes, the branching ratios are of the same order when compared with Ref. [58], i.e., $\mathcal{O}(10^{-3})$ and $\mathcal{O}(10^{-4})$, respectively. In general, our results for branching ratios including both pole and factorization amplitudes are larger than their values as expected. The decay $\Xi_{cc}^{++} \to \Sigma_c^{++} \bar{K}^{*0}$ is first figured as a four-body process in Ref. [60], which is predicted to be one of the most dominant modes. Gutsche *et al.* [59] have analyzed weak decay of Ξ_{cc}^{++} as decay chain $\Xi_{cc}^{++} \to \Sigma_c^{++}(\to \Lambda_c^+ \pi^+) + \bar{K}^{*0}(\to K^- \pi^+)$, which is expected to be experimentally favored due to the dominant branching ratios of the daughter decays. The $\Xi_{cc}^{++} \to \Sigma_c^{++} \bar{K}^{(*)0}$ decays are studied using the factorization scheme in CCQM. The obtained branching ratio, $\mathcal{B}(\Xi_{cc}^{++} \to \Sigma_c^{++} \bar{K}^0) = 1.5\%$ at 300 fs, is of the same order when compared with our result. Other than the factorization scheme, the nonperturbative long-distance (W-exchange) contributions to Ξ_{cc} decays have been calculated by Yu *et al.* [60]. The rescattering mechanism of FSIs, which has been ignored in the present work, is used to evaluate long-distance contributions. Authors have used the one-particle exchange method, where FSI is assumed to be dominated by the rescattering of intermediate states [81]. Thus, the amplitude is expressed in terms of strong coupling (of particles on mass shell) and the form factor (for exchanged baryons that are off mass shell). Here, also, the branching ratios in the case of CKM-favored and CKM-suppressed modes for factorizable decay channels (see Table XIII) are of the same order as compared to our results. However, their branching ratios for (pole-only) $\Xi_{cc}^+ \to \Sigma_c^{++} K^-$ and $\Xi_{cc}^{++} \to p D^+$ decays are smaller by an order of magnitude as compared to our results for the flavor-independent case. The difference in results may be attributed mainly to distinctive approaches. Although all the results compared here are based on different models/approaches, they agree at least on the order of magnitude of the doubly charmed

---

[1]The weak coupling $a_{\Xi_{cc}^+ \Xi_c^+}$ becomes zero following the operation of $(1 - \sigma_i \cdot \sigma_j)$ on the wave function using (13); for details, see Ref. [37].

TABLE XIII. Comparison of branching ratios with other works.[a]

Decays	Models [72,73]	Branching ratios		
		Flavor independent	Flavor dependent	Other works
($\Delta C = \Delta S = -1$)				
$\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$	NRQM	7.8×10^{-2}	15.1×10^{-2}	7.24×10^{-2} [58]
	HQET	10.9×10^{-2}	18.5×10^{-2}	3.4×10^{-2} [60]
$\Xi_{cc}^{++} \to \Xi_c'^+ \pi^+$	NRQM	6.4×10^{-2}	$\cdots$	5.08×10^{-2} [58]
	HQET	8.8×10^{-2}		
$\Xi_{cc}^+ \to \Xi_c^0 \pi^+$	NRQM	1.3×10^{-2}	2.7×10^{-2}	2.40×10^{-2}[58]
	HQET	2.1×10^{-2}	3.3×10^{-2}	1.2×10^{-2} [60]
$\Xi_{cc}^+ \to \Xi_c'^0 \pi^+$	NRQM	3.3×10^{-2}	5.9×10^{-2}	1.68×10^{-2} [58]
	HQET	4.7×10^{-2}	7.2×10^{-2}	
$\Xi_{cc}^+ \to \Sigma_c^{++} K^-$	Pole only	4.8×10^{-3}	2.1×10^{-2}	4.8×10^{-4}[60]
$\Xi_{cc}^+ \to \Lambda^0 D^+$	Pole only	5.3×10^{-4}	2.4×10^{-3}	2.4×10^{-4} [60]
($\Delta C = -1, \Delta S = 0$)				
$\Xi_{cc}^{++} \to \Lambda_c^+ \pi^+$	NRQM	3.2×10^{-3}	7.4×10^{-2}	4.09×10^{-3} [58]
	HQET	5.8×10^{-3}	1.0×10^{-2}	1.2×10^{-3} [60]
$\Xi_{cc}^{++} \to \Xi_c^+ K^+$	NRQM	5.1×10^{-3}	8.0×10^{-3}	6.06×10^{-3} [58]
	HQET	7.6×10^{-3}	1.1×10^{-2}	
$\Xi_{cc}^{++} \to \Xi_c'^+ K^+$	NRQM	4.4×10^{-3}	$\cdots$	3.48×10^{-3} [58]
	HQET	6.0×10^{-3}		
$\Xi_{cc}^{++} \to \Sigma_c^+ \pi^+$	NRQM	3.2×10^{-3}	7.4×10^{-2}	2.66×10^{-3} [58]
	HQET	6.0×10^{-3}		
$\Xi_{cc}^+ \to \Xi_c^0 K^+$	NRQM	1.1×10^{-3}	1.3×10^{-3}	2.00×10^{-3} [58]
	HQET	1.8×10^{-3}	2.0×10^{-3}	
$\Xi_{cc}^+ \to \Xi_c'^0 K^+$	NRQM	1.0×10^{-3}	6.2×10^{-4}	1.15×10^{-3} [58]
	HQET	1.4×10^{-3}	9.7×10^{-4}	
$\Xi_{cc}^+ \to \Sigma_c^0 \pi^+$	NRQM	3.7×10^{-3}	5.9×10^{-3}	1.77×10^{-3} [58]
	HQET	6.3×10^{-3}	9.1×10^{-3}	
$\Xi_{cc}^{++} \to p D^+$	Pole only	1.4×10^{-4}	6.0×10^{-4}	4.8×10^{-5}[60]
$\Xi_{cc}^+ \to p D^0$	Pole only	7.6×10^{-5}	3.4×10^{-4}	1.2×10^{-4}[60]
($\Delta C = -\Delta S = -1$)				
$\Xi_{cc}^{++} \to \Lambda_c^+ K^+$	NRQM	2.1×10^{-4}	3.8×10^{-4}	3.60×10^{-4} [58]
	HQET	4.2×10^{-4}	6.2×10^{-4}	
$\Xi_{cc}^{++} \to \Sigma_c^+ K^+$	NRQM	2.3×10^{-4}	$\cdots$	1.95×10^{-4} [58]
	HQET	4.6×10^{-4}		
$\Xi_{cc}^+ \to \Sigma_c^0 K^+$	NRQM	1.5×10^{-4}	$\cdots$	1.30×10^{-4} [58]
	HQET	3.0×10^{-4}		

[a]The branching ratios are compared for the lifetime $\frac{\tau_{\Xi_{cc}^{++}}}{\tau_{\Xi_{cc}^+}} = 3$, and thus, for Ref. [60], we have used $\mathcal{R}_\tau = 0.3$.

baryon decays. These results could be of great importance for experimentalists for future searches.

In the present work, we have ignored the CP asymmetries as they have not yet been established in charmed baryon decays. However, like heavy-flavor mesons decays, the heavy-baryon decays are also prone to CP violation. Even though it is well established that nonfactorizable diagrams like W-exchange/annihilation have a sizable impact on baryon decays, it would be a difficult task to establish CP violation in charmed baryon decays as the CP asymmetries originating from the Standard Model (SM) are very small or even zero [82,83]. Moreover, the production of three-body final states with relatively larger branching ratios and many CP observables will require a large amount of experimental data. On the other hand, CP asymmetries have already been probed in two-body Λ_b decays [78]. The theoretical investigation based on the pQCD approach [52] indicates the dominance of nonfactorizable contributions in addition to penguin amplitudes. Similar conclusions were made by theoretical estimates based on generalized factorization and symmetries [47,48,51,84]. Obviously, measurements of the CP asymmetries provide a good tool to probe interference between the SM and new physics.

V. SUMMARY

The understanding of heavy-baryon decays is a longstanding problem as there does not exist a reliable approach

for investigating the weak decays of heavy baryons as of yet. The dynamics of baryon decays, unlike meson decays, seems to get more complicated once they become heavier. Motivated by the recent observations, especially by LHCb, we have analyzed nonleptonic weak decays of doubly charmed baryons. The branching ratios of Ξ_{cc} decays for CKM-favored and -suppressed modes are calculated using the factorization and pole model approaches. In the factorization scheme, we have obtained the form factors, f_i and g_i, using the nonrelativistic quark model [72] and heavy quark effective theory [73]. The nonfactorizable W-exchange diagrams, involving $\frac{1}{2}^+-$ intermediate states, are calculated using the pole model approach. In the case of singly charmed baryon decays, it has been well established that the W-exchange contributions are comparable to factorization amplitudes. Therefore, the purpose of the present work is to give first estimates of W-exchange terms in doubly charmed Ξ_{cc} decays to get a more comprehensive picture. As mentioned before, there has been some recent analysis involving doubly heavy baryons based mostly on factorization contributions only. However, the importance of W-exchange terms has also been emphasized in such works. Furthermore, we include SU(4)-breaking effects in meson-baryon strong couplings as well as in weak amplitudes. The results for the two scenarios, namely, flavor independent and flavor dependent, have been presented. We summarize our observations as follows:

(1) We find that W-exchange amplitude contributes the majority of the Ξ_{cc} decays. In contrast to factorization contributions, the W-exchange contributions are not only comparable but also dominant in many decay channels. Thus, W-exchange contributions in Ξ_{cc} decays cannot be ignored.

(2) It is interesting to note that most of the CKM-favored decay channels receive contributions from W-exchange pole amplitudes only. The overall branching ratios in this mode range from $10^{-1} \sim 10^{-5}$, The $\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^+ \pi^+)$ is as high as $\mathcal{O}(10^{-1})$ in the flavor-dependent case. Several decays in this node have branching ratios of the order of a few percent, which could be of experimental interest.

(3) We have shown that the pole and factorization amplitudes, depending on their signs, can interfere constructively and destructively. An experimental search of these decays could prove to be a useful test of theoretical models.

(4) In CKM-suppressed and CKM–doubly suppressed modes, the dominant decays receive contributions from factorization as well as pole amplitudes, indicating the importance of W-exchange processes. The branching ratios of dominant decay channels in the CKM-suppressed mode are $\mathcal{O}(10^{-2}) \sim \mathcal{O}(10^{-3})$.

(5) The pole contributions are significantly enhanced due to the flavor-dependent factor. Thus, our results based on the NRQM and HQET picture alongside flavor-dependent W-exchange contributions provide a useful range to search for experimental evidence.

Experimental searches for heavy-baryon decays could help theorists understand the underlying dynamics of W-exchange processes in such decays. The importance of nonfactorizable contributions in *CP* asymmetries in heavy-baryon decays could prove to be a challenge to the theory as well as experiment. New measurements on of doubly heavy baryons are in future plans of several ongoing experiments at Fermilab and CERN. We hope that our results could prove to be useful in experimental searches for new modes.

[1] M. Mattson *et al.* (SELEX Collaboration), Phys. Rev. Lett. **89,** 112001 (2002).
[2] A. Ocherashvili *et al.* (SELEX Collaboration), Phys. Lett. B **628,** 18 (2005).
[3] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **119,** 112001 (2017).
[4] V. V. Kiselev, A. K. Likhoded, and A. I. Onishchenko, Phys. Rev. D **60,** 014007 (1999).
[5] V. V. Kiselev and A. K. Likhoded, Usp. Fiz. Nauk **172,** 497 (2002) [Phys. Usp. **45,** 455 (2002)].
[6] K. Anikeev *et al.*, arXiv:hep-ph/0201071.
[7] C. H. Chang, T. Li, X. Q. Li, and Y. M. Wang, Commun. Theor. Phys. **49,** 993 (2008).
[8] M. Karliner and J. L. Rosner, Phys. Rev. D **90,** 094007 (2014) and references therein.
[9] S. S. Gershtein, V. V. Kiselev, A. K. Likhoded, and A. I. Onishchenko, Phys. Rev. D **62,** 054021 (2000).

[10] D. Ebert, R. N. Faustov, V. O. Galkin, and A. P. Martynenko, Phys. Rev. D **66,** 014008 (2002).
[11] C. Albertus, E. Hernandez, J. Nieves, and J. M. Verde-Velasco, Eur. Phys. J. A **31,** 691 (2007).
[12] B. Patel, A. K. Rai, and P. C. Vinodkumar, J. Phys. G **35,** 065001 (2008); J. Phys. Conf. Ser. **110,** 122010 (2008).
[13] A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner, and V. E. Lyubovitskij, Phys. Rev. D **80,** 034025 (2009).
[14] T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij, and B. Oexl, Phys. Rev. D **81,** 114036 (2010).
[15] T. M. Aliev, K. Azizi, and M. Savci, Nucl. Phys. **A895,** 59 (2012).
[16] T. M. Aliev, K. Azizi, and M. Savci, J. Phys. G **40,** 065003 (2013).
[17] R. Dhir, C. S. Kim, and R. C. Verma, Phys. Rev. D **88,** 094002 (2013).

[18] A. Bernotas and V. Imonis, Phys. Rev. D 87, 074016 (2013).

[19] H. Y. Cheng, Front. Phys. 10, 101406 (2015).

[20] Q. F. Lü, K. L. Wang, L. Y. Xiao, and X. H. Zhong, Phys. Rev. D 96, 114006 (2017).

[21] A. V. Kiselev, A. V. Berezhnoy, and A. K. Likhoded, arXiv:1706.09181.

[22] H. Y. Cheng and C. W. Chiang, Phys. Rev. D 95, 094018 (2017).

[23] D. Ebert, R. N. Faustov, V. O. Galkin, and A. P. Martynenko, Phys. Rev. D 70, 014018 (2004); 77, 079903(E) (2008).

[24] C. Albertus, E. Hernandez, J. Nieves, and J. M. Verde-Velasco, Eur. Phys. J. A 32, 183 (2007); 36, 119(E) (2008).

[25] E. Hernandez, J. Nieves, and J. M. Verde-Velasco, Phys. Lett. B 663, 234 (2008).

[26] C. Albertus, E. Hernndez, and J. Nieves, Phys. Lett. B 704, 499 (2011).

[27] C. Albertus, E. Hernandez, and J. Nieves, Phys. Lett. B 683, 21 (2010).

[28] C. Albertus, E. Hernandez, and J. Nieves, Phys. Rev. D 85, 094035 (2012).

[29] J. M. Flynn and J. Nieves, Phys. Rev. D 76, 017502 (2007); 77, 099901(E) (2008).

[30] S. Meinel, Phys. Rev. Lett. 118, 082001 (2017).

[31] B. O. Kerbikov, arXiv:1707.04031.

[32] H. Y. Cheng, Int. J. Mod. Phys. A 24, 593 (2009).

[33] M. Karliner and J. L. Rosner, Phys. Rev. D 96, 033004 (2017).

[34] H. S. Li, L. Meng, Z. W. Liu, and S. L. Zhu, Phys. Rev. D 96, 076011 (2017).

[35] M. Karliner and J. L. Rosner, Phys. Rev. Lett. 119, 202001 (2017).

[36] H. X. Chen, Q. Mao, W. Chen, X. Liu, and S. L. Zhu, Phys. Rev. D 96, 031501 (2017).

[37] Fayyazuddin and Riazuddin, Phys. Rev. D 55, 255 (1997); 56, 531(E) (1997).

[38] Fayyazuddin and Riazuddin, Phys. Rev. D 58, 014016 (1998).

[39] H. Y. Cheng, Phys. Rev. D 56, 2799 (1997).

[40] M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij, and A. G. Rusetsky, Phys. Rev. D 57, 5632 (1998).

[41] S. Sinha, M. P. Khanna, and R. C. Verma, Phys. Rev. D 57, 4483 (1998).

[42] K. K. Sharma and R. C. Verma, Eur. Phys. J. C 7, 217 (1999).

[43] B. Guberina and H. Stefancic, Phys. Rev. D 65, 114004 (2002).

[44] V. E. Lyubovitskij, A. Faessler, T. Gutsche, M. A. Ivanov, and J. G. Korner, Prog. Part. Nucl. Phys. 50, 329 (2003).

[45] A. Datta, H. J. Lipkin, and P. J. O'Donnell, Phys. Rev. D 69, 094002 (2004).

[46] C. H. Chou, H. H. Shih, S. C. Lee, and H. n. Li, Phys. Rev. D 65, 074030 (2002).

[47] M. Gronau and J. L. Rosner, Phys. Rev. D 89, 037501 (2014).

[48] M. He, X. G. He, and G. N. Li, Phys. Rev. D 92, 036010 (2015).

[49] S. Wang, J. Huang, and G. Li, Chin. Phys. C 37, 063103 (2013).

[50] S. W. Wang, G. L. Sun, X. Q. Yang, and J. S. Huang, Eur. Phys. J. C 72, 1852 (2012).

[51] Y. K. Hsiao and C. Q. Geng, Phys. Rev. D 91, 116007 (2015).

[52] C. D. Lu, Y. M. Wang, H. Zou, A. Ali, and G. Kramer, Phys. Rev. D 80, 034011 (2009).

[53] H. W. Ke, X. Q. Li, and Z. T. Wei, Phys. Rev. D 77, 014020 (2008).

[54] H. W. Ke, X. H. Yuan, X. Q. Li, Z. T. Wei, and Y. X. Zhang, Phys. Rev. D 86, 114005 (2012).

[55] Fayyazuddin, Phys. Rev. D 95, 053008 (2017).

[56] Fayyazuddin and M. J. Aslam, Phys. Rev. D 95, 113002 (2017).

[57] W. Wang, Z. P. Xing, and J. Xu, Eur. Phys. J. C 77, 800 (2017).

[58] W. Wang, F. S. Yu, and Z. X. Zhao, Eur. Phys. J. C 77, 781 (2017).

[59] T. Gutsche, M. A. Ivanov, J. G. Krner, and V. E. Lyubovitskij, Phys. Rev. D 96, 054013 (2017).

[60] F. S. Yu, H. Y. Jiang, R. H. Li, C. D. L, W. Wang, and Z. X. Zhao, arXiv:1703.09086.

[61] R. H. Li, C. D. L, W. Wang, F. S. Yu, and Z. T. Zou, Phys. Lett. B 767, 232 (2017).

[62] J. G. Korner and M. Kramer, Z. Phys. C 55, 659 (1992).

[63] H. Y. Cheng and B. Tseng, Phys. Rev. D 46, 1042 (1992); 55, 1697(E) (1997).

[64] H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin, T. M. Yan, and H. L. Yu, Phys. Rev. D 46, 5060 (1992).

[65] H. Y. Cheng and B. Tseng, Phys. Rev. D 48, 4188 (1993) and references therein.

[66] Q. P. Xu and A. N. Kamal, Phys. Rev. D 46, 270 (1992).

[67] Q. P. Xu and A. N. Kamal, Phys. Rev. D 46, 3836 (1992).

[68] Q. P. Xu and A. N. Kamal, Phys. Rev. D 47, 2849 (1993).

[69] M. P. Khanna, Phys. Rev. D 49, 5921 (1994).

[70] T. Uppal, R. C. Verma, and M. P. Khanna, Phys. Rev. D 49, 3417 (1994).

[71] P. Zenczykowski, Phys. Rev. D 50, 402 (1994) and references therein.

[72] R. Perez-Marcial, R. Huerta, A. Garcia, and M. Avila-Aoki, Phys. Rev. D 40, 2955 (1989); 44, 2203(E) (1991).

[73] H. Y. Cheng and B. Tseng, Phys. Rev. D 53, 1457 (1996); 55, 1697(E) (1997).

[74] M. P. Khanna and R. C. Verma, Z. Phys. C 47, 275 (1990).

[75] Riazuddin and Fayyazuddin, Phys. Rev. D 18, 1578 (1978); 19, 1630(E) (1979).

[76] R. Dhir and C. S. Kim, Phys. Rev. D 91, 114008 (2015) and references therein.

[77] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).

[78] T. M. Aliev, A. Ozpineci, and M. Savci, Phys. Rev. D 64, 034001 (2001).

[79] C. Patrignani et al. (Particle Data Group Collaboration), Chin. Phys. C 40, 100001 (2016).

[80] H. Y. Cheng, C. K. Chua, and C. W. Hwang, Phys. Rev. D 69, 074025 (2004).

[81] S. L. Chen, X. H. Guo, X. Q. Li, and G. L. Wang, Commun. Theor. Phys. 40, 563 (2003) and references therein.

[82] I. I. Bigi, arXiv:1206.4554.

[83] I. I. Bigi, Front. Phys. 10, 240 (2015).

[84] X. G. He and G. N. Li, Phys. Lett. B 750, 82 (2015).