What are the correct $\rho^0(770)$ meson mass and width values?

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The accuracy of the Gounaris-Sakurai pion electromagnetic form factor model at the elastic region, in which just the $\rho^0(770)$ resonance appears, is investigated by the particular analysis of the most accurate P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data, obtained by the Garcia-Martin-Kamiński-Peláez-Yndurain approach, and by an application of the Unitary&Analytic pion electromagnetic structure model to a description of the newest precise data on the $e^+e^- \to \pi^+\pi^-$ process.

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I. INTRODUCTION

In recent years, experimental measurements of the total cross section of the process $e^+e^- \to \pi^+\pi^-$ [1,2] have captured much attention. By the initial state radiation method, very precise information on the pion electromagnetic (EM) form factor (FF) $F_\pi^{EM,I=1}(t)$ has been obtained, measuring total cross section

$$\sigma_{\rm tot}(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2(0)}{3t}\beta_\pi^3(t)\left|F_\pi^{EM,I=1}(t) + Re^{i\phi}\frac{m_\omega^2}{m_\omega^2 - t - im_\omega\Gamma_\omega}\right|^2 \quad (1)$$

with the pion velocity $\beta_\pi(t) = \sqrt{1 - \frac{4m_\pi^2}{t}}$, the $\rho - \omega$ interference phase ϕ to be expressed as $\phi = arctg\frac{m_\rho\Gamma_\rho}{m_\rho^2 - m_\omega^2}$, and the amplitude R as a free parameter.

On the other hand, the most accurate up-to-now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data at the elastic region with theoretical errors, to be generated from the existing inaccurate experimental information by the Garcia-Martin-Kamiński-Peláez-Yndurain (GKPY) Roy-like equations with an imposed crossing symmetry condition [3], appeared (see Table I and Fig. 1).

Further, by using both these experimental sets of data, the pion EM FF Gounaris-Sakurai (G.-S.) model [4], which is constructed from the P-wave isovector $\pi\pi$ scattering phase shift at the elastic region, to be given by a generalized effective-range formula of the Chew-Mandelstam type [5], is demonstrated to not be accurate enough to generate the $\rho^0(770)$ meson mass and width values presented in Ref. [6].

For a determination of the m_ρ and Γ_ρ, also the Unitary&Analytic (U&A) model of the pion EM FF is constructed and reflects (unlike the G.-S. model) all known pion EM FF properties, including the $\rho^0(770)$ resonance contribution in the form of a complex conjugate pair of poles on the second Riemann sheet. So, the model represents a well-matched unification of these pole contributions with the continua to be represented by the cuts on the positive real axis in the t-complex plane. However, an application of the U&A model to a description of $e^+e^- \to \pi^+\pi^-$ data gives the $\rho^0(770)$ meson parameters different from the parameters given in Ref. [6]. Nevertheless, these values are independently confirmed in the framework of the worked-out fully solvable mathematical scheme [7,8], in which the most accurate $\delta_1^1(t)$ phase shift data up to 1 GeV^2 (see Fig. 1), to be optimally described with $\chi^2/ndf = 0.0244$ by a model-independent expression, have been used.

On the other hand, if the same most accurate $\delta_1^1(t)$ phase shift data up to 1 GeV^2 are described by the generalized effective-range formula of the Chew-Mandelstam type, which is declared to be the basis of the G.-S. model, despite of its two free parameters, the optimal description is achieved only with $\chi^2/ndf = 2.4499$, and moreover, the obtained m_ρ and Γ_ρ are different from values evaluated by the original G.-S. model [4] and also from values evaluated by the U&A model.

This problem can be resolved by a comparison of χ^2/ndf values in descriptions of the most accurate up-to-now $\delta_a^1(t)$ representations as well as by a comparison of obtained parameter from $\delta_1^1(t)$. With those specified by explicit forms of the pion EM FF models. Such analysis clearly make evident that the correct $\rho^0(770)$ mass and width values seem to be those which are determined by the application of the pion EM FF U&A model to the description of the $e^+e^- \to \pi^+\pi^-$ data.

TABLE I. Numerical values of the most accurate up-to-now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data with theoretical errors, to be generated from the existing inaccurate experimental information by the GKPY Roy-like equations with an imposed crossing symmetry condition [3].

t (GeV^2)	$\delta_1^1(t)$	$\Delta\delta_1^1(t)$	t (GeV^2)	$\delta_1^1(t)$	$\Delta\delta_1^1(t)$	t (GeV^2)	$\delta_1^1(t)$	$\Delta\delta_1^1(t)$	t (GeV^2)	$\delta_1^1(t)$	$\Delta\delta_1^1(t)$
0.0841	0.0091	0.0935	0.2209	4.3756	0.2061	0.4225	23.1183	0.7699	0.6889	124.1276	1.5944
0.0900	0.0724	0.0968	0.2304	4.8350	0.2024	0.4356	25.5826	0.9700	0.7056	128.1464	2.6146
0.0961	0.1678	0.1020	0.2401	5.3283	0.1985	0.4489	28.3899	1.1605	0.7225	131.6519	4.1306
0.1024	0.2873	0.0883	0.2500	5.8586	0.1953	0.4624	31.6006	1.3289	0.7396	134.7173	1.9867
0.1089	0.4273	0.0843	0.2601	6.4298	0.1935	0.4761	35.2838	1.4636	0.7569	137.4084	1.9692
0.1156	0.5862	0.0880	0.2704	7.0459	0.1936	0.4900	39.5146	1.5555	0.7744	139.7824	1.8914
0.1225	0.7629	0.0975	0.2809	7.7121	0.1958	0.5041	44.3680	1.5975	0.7921	141.8876	1.7700
0.1296	0.9570	0.1109	0.2916	8.4340	0.1997	0.5184	49.9070	1.5855	0.8100	143.7650	1.6216
0.1369	1.1684	0.1267	0.3025	9.2181	0.2045	0.5329	56.1628	1.5188	0.8281	145.4486	1.4629
0.1444	1.3972	0.1433	0.3136	10.0724	0.2082	0.5476	63.1077	1.4010	0.8464	146.9669	1.3105
0.1521	1.6440	0.1596	0.3249	11.0057	0.2078	0.5625	70.6299	1.2401	0.8649	148.3441	1.1811
0.1600	1.9091	0.1744	0.3364	12.0287	0.1991	0.5776	78.5227	1.0498	0.8836	149.6003	1.0914
0.1681	2.1936	0.1871	0.3481	13.1539	0.1846	0.5929	86.5064	0.8501	0.9025	150.7528	1.0581
0.1764	2.4983	0.1972	0.3600	14.3962	0.1333	0.6084	94.2821	0.6680	0.9216	151.8164	1.0978
0.1849	2.8246	0.2043	0.3721	15.7732	0.1617	0.6241	101.5948	0.5388	0.9409	152.8043	1.2271
0.1936	3.1736	0.2084	0.3844	17.3061	0.2549	0.6400	108.2761	0.5065	0.9604	153.7282	1.2124
0.2025	3.5472	0.2098	0.3969	19.0200	0.3975	0.6561	114.2509	0.6257	0.9801	154.5991	1.2247
0.2116	3.9471	0.2088	0.4096	20.9453	0.5742	0.6724	119.5190	0.9622	1.0000	155.3724	1.2371

II. $\rho^0(770)$ MESON PARAMETER VALUES DETERMINED BY GOUNARIS-SAKURAI AND UNITARY&ANALYTIC PION EM FF MODELS

The extensively quoted pion EM FF G.-S. model [4] has been constructed by assuming that for a wide energy of the elastic region up to $t = 1$ GeV^2 the P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ satisfies a two parametric effective-range formula of the Chew-Mandelstam type [5]

$$\frac{q^3}{\sqrt{t}}\mathrm{cotg}\delta_1^1(t) = a + bq^2 + q^2 h(t), \qquad (2)$$

with

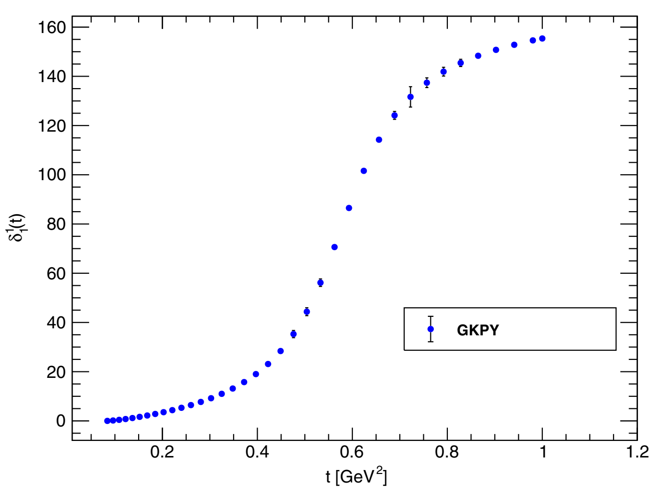

FIG. 1. The most accurate up-to-now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data with theoretical errors.

$$h(t) = \frac{2}{\pi}\frac{q}{\sqrt{t}} ln\left(\frac{\sqrt{t}+2q}{2m_\pi}\right) \qquad (3)$$

and

$$q = [(t - 4m_\pi^2)/4]^{1/2} \qquad (4)$$

to be absolute value of the pion three-momentum in the c.m. system.

Then, one can prove by means of the so-called N/D method that

$$F_\pi^{GS}(t) = D(t)^{-1}, \qquad (5)$$

where

$$D(t) = \frac{q^3}{\sqrt{t}}\mathrm{cotg}\delta_1^1(t) - i\frac{q^3}{\sqrt{t}}. \qquad (6)$$

On this stage also, $F_\pi^{GS}(t)$ contains two unknown constants a and b, and there is no dependence on the $\rho^0(770)$ mass and width until now. However, taking into account the fact that the phase $\delta_1^1(t)$ is crossing the value $\pi/2$ in which the $\rho^0(770)$ resonance appears in the $\pi\pi$ scattering amplitude, one can introduce the first natural condition

$$\mathrm{cotg}\delta_1^1(t)|_{t=m_\rho^2} = 0 \qquad (7)$$

for the relation (2).

Another condition can be derived from the Breit-Wigner form

$$F_\pi^{BW}(t) = \frac{m_\rho^2}{m_\rho^2 - t - im_\rho\Gamma_\rho}, \quad (8)$$

describing quite well a characteristic bump corresponding to the elastic $\rho^0(770)$ resonance. On the base of (8), the relations

$$tg\delta_1^1(t) = \frac{m_\rho\Gamma_\rho}{m_\rho^2 - t} \Rightarrow \delta_1^1(t) = \mathrm{arctg}\frac{m_\rho\Gamma_\rho}{m_\rho^2 - t} \quad (9)$$

can be written, and from there, the second natural condition

$$\left.\frac{d\delta_1^1(t)}{dt}\right|_{t=m_\rho^2} = \frac{1}{m_\rho\Gamma_\rho} \quad (10)$$

is found for the relation (2). The relations (7) and (10) give two equations for two unknown constants a and b in (2), and their solutions

$$a = \frac{4q_\rho^5}{m_\rho^2\Gamma_\rho} + 4q_\rho^4 h'(m_\rho^2) \quad (11)$$

$$b = -\frac{4q_\rho^3}{m_\rho^2\Gamma_\rho} - 4q_\rho^2 h'(m_\rho^2) - h(m_\rho^2) \quad (12)$$

are expressing $F_\pi^{GS}(t)$ through m_ρ and Γ_ρ as follows:

$$F_\pi^{GS}(t) = \frac{m_\rho^2 + m_\rho\Gamma_\rho\left(\frac{3}{\pi}\frac{m_\pi^2}{q_\rho^2} ln\left(\frac{m_\rho+2q_\rho}{2m_\pi}\right) + \frac{m_\rho}{2\pi q_\rho} - \frac{m_\pi^2 m_\rho}{\pi q_\rho^3}\right)}{(m_\rho^2 - t) + \Gamma_\rho\left(\frac{m_\rho^2}{q_\rho^3}\right)(q^2(h(t) - h(m_\rho^2)) + q_\rho^2 h'(m_\rho^2)(m_\rho^2 - t)) - im_\rho\Gamma_\rho\left(\frac{q}{q_\rho}\right)^3\frac{m_\rho}{\sqrt{t}}}. \quad (13)$$

The application of the formula (13) to an optimal description of the unified BESIII-*BABAR* data at the elastic region (see Fig. 2) with $\chi^2/ndf = 40.6341$ gives parameters

$$m_\rho = (775.73 \pm 0.10)\ \mathrm{MeV}$$
$$\Gamma_\rho = (126.51 \pm 0.13)\ \mathrm{MeV} \quad (14)$$

different (especially the width) from the parameter values in Ref. [6].

Now, the $\rho^0(770)$ meson parameters will be determined by an application of the U&A model of the pion EM FF to an optimal description of the same data on the total cross section of the $e^+e^- \to \pi^+\pi^-$ process.

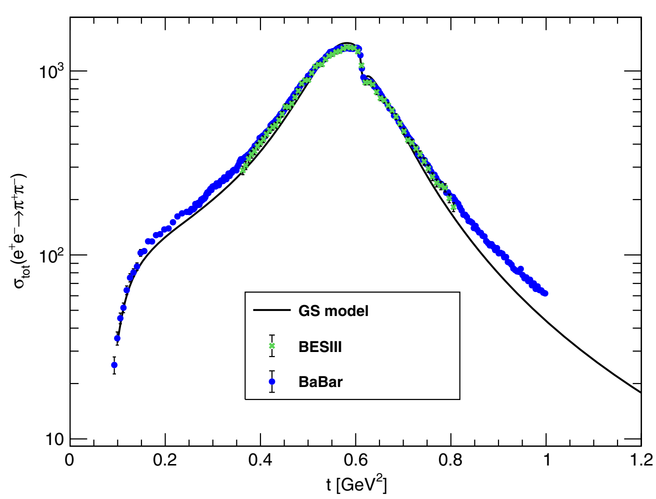

FIG. 2. Optimal description of the unified BESIII-*BABAR* data on $\sigma_{\mathrm{tot}}(e^+e^- \to \pi^+\pi^-)$ at the elastic region by the pion EM FF G.-S. model.

Here, the pion EM FF model contains the right-hand unitary cut in the t plane and also a contribution of the left-hand cut from the second Riemann sheet. To obtain its explicit form, one starts from the standard VMD representation for the ρ meson,

$$F_\pi^{(VMD)}(t) = \frac{m_\rho^2(f_{\rho\pi\pi}/f_\rho)}{m_\rho^2 - t}, \quad (15)$$

where $f_{\rho\pi\pi}$ is the $\rho^0(770)$ meson two-pion decay constant and f_ρ is the universal ρ meson constant determining the ρ meson lepton width.

If we take into account the relation for total energy squared in the c.m. system

$$t = 4(q^2 + 1), \quad (16)$$

which is inverse to (4) ($m_\pi = 1$), and, further, denote the normalization point $t = 0$ at the q plane by $q_N = i$ and the position of the ρ meson by $t_{\rho 0}$ (the 0 means $\Gamma_\rho = 0$), while

$$m_\rho^2 = 4(q_{\rho 0}^2 + 1), \quad (17)$$

then the expression (15) can be transformed into the form

$$F_\pi^{(VMD)}(q) = \frac{(q_N - q_{\rho 0})(q_N - q_{\bar\rho 0})}{(q - q_{\rho 0})(q - q_{\bar\rho 0})}(f_{\rho\pi\pi/f_\rho}), \quad (18)$$

with $q_{\bar\rho 0} = -q_{\rho 0}$.

At this moment, we are in a competent position to introduce $\Gamma \neq 0$ into (18) by the relation $t_\rho = (m_\rho - i\Gamma/2)^2$, leading to the interchange

$$q_{\rho 0} \Rightarrow q_\rho = [[(m_\rho - i\Gamma_\rho/2)^2 - 4]/4]^{\frac{1}{2}}, \tag{19}$$

and, as a consequence of the latter, the pole corresponding to the ρ meson is shifted from the real axis into the complex region of the second sheet of the Riemann surface. As a result, the U&A pion EM FF model

$$F_\pi^{EM,I=1}(q) = \frac{(q_N - q_\rho)(q_N - q_{\bar\rho})}{(q - q_\rho)(q - q_{\bar\rho})}(f_{\rho\pi\pi}/f_\rho) \tag{20}$$

is obtained, which retains the normalization and asymptotic behavior of the VMD model.

However, the form (20) does not contain a contribution of the left-hand cut, which has been shown to be important [9,10] in a description of the pion EM FF data.

For its incorporation into (20) the well-known properties of Pade-type approximations to simulate the cut contribution by a set of alternating poles and zeros on his position are used, and we multiply the right-hand side of (20) by a normalized factor consisting of one pole and one zero, which have shown to be sufficient to substitute for the left-hand cut contribution to the pion EM FF behavior. Then, finally, the pion EM FF U&A model at the elastic region is

$$F_\pi^{EM,I=1}(q) = \frac{(q - q_Z)(q_N - q_P)}{(q - q_P)(q_N - q_Z)}\frac{(q_N - q_\rho)(q_N - q_{\bar\rho})}{(q - q_\rho)(q - q_{\bar\rho})} \times (f_{\rho\pi\pi}/f_\rho). \tag{21}$$

The optimal description of the recent data [1,2] on the total cross section of the $e^+e^- \to \pi^+\pi^-$ process at the region of the ρ meson resonance by (21) (see Fig. 3), achieved with $\chi^2/ndf = 1.5443$, gives the ρ meson mass and width

$$m_\rho = (763.026 \pm 0.136)\ \text{MeV}$$
$$\Gamma_\rho = (144.800 \pm 0.233)\ \text{MeV}, \tag{22}$$

which differ from (14) and also from the values in Ref. [6].

Which of the values (14) or (22) is more likely correct? For the solution of this problem, in the next section, the most accurate up-to-now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data at the elastic region with theoretical errors, to be generated from the existing inaccurate experimental information by the GKPY Roy-like equations with an imposed crossing symmetry condition [3], are exploited.

III. $\rho^0(770)$ MESON MASS AND WIDTH VALUES DETERMINED FROM $\delta_1^1(t)$ DATA

The $\rho^0(770)$ meson parameters can be found also by exploiting (independently from the data on the total cross section of the $e^+e^- \to \pi^+\pi^-$ process) the data on $\delta_1^1(t)$.

First, with this aim, the effective-range formula of the Chew-Mandelstam type (2), which appears to be the basis of the pion EM FF G.-S. model, will be used. The effective range formula contains two unknown parameters, a and b. The numerical values

$$a = 0.2860 \pm 0.0011$$
$$b = -2.7025 \pm 0.0089 \tag{23}$$

are evaluated in a fitting procedure of the data in Table I (see Fig. 4) with $\chi^2/ndf = 2.4499$. Substitution of this numbers into (11) and (12) two nonlinear equations for m_ρ and Γ_ρ are obtained. Numerical solutions of these equations (the nonlinear system of equations was solved by multidimensional root-finding algorithm with the hybrid version based on GSL [11]) give the rho meson mass and width values

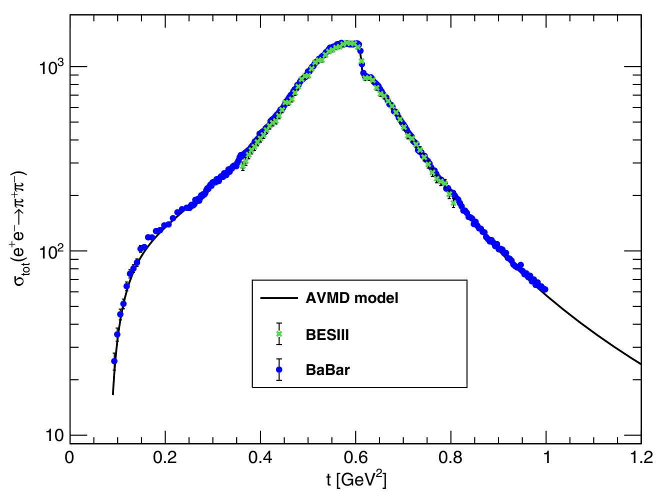

FIG. 3. Optimal description of the unified BESIII-*BABAR* data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ at the elastic region by the pion EM FF U&A model.

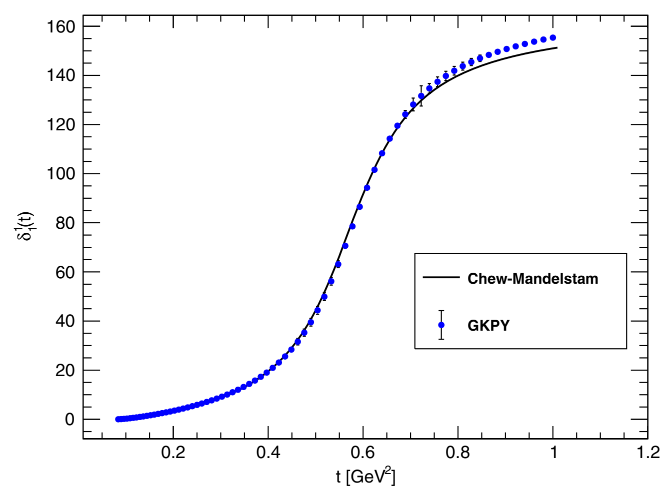

FIG. 4. Optimal description of the most accurate up-to-now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data with the effective-range formula of the Chew-Mandelstam type (2).

$$m_\rho = (772.42 \pm 0.03)\ \text{MeV}$$
$$\Gamma_\rho = (153.85 \pm 0.11)\ \text{MeV}, \quad (24)$$

which are diametrally different from (14).

Now, an investigation of the same most accurate P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data, which are totally independent from the data on $\sigma_{\text{tot}}(e^+e^- \to \pi^+\pi^-)$, by means of the fully solvable mathematical scheme elaborated in Refs. [7,8], is carried out.

Starting from the analytic properties, consisting of a cut on the positive real axis at the complex t plane, and the asymptotic behavior of $F_\pi^{EM,I=1}(t)$, the Cauchy formula with one subtraction is written, which in combination with the elastic unitarity condition leads to the phase representation

$$F_\pi^{EM,I=1}(t) = P_n(t) \exp\left[\frac{t}{\pi}\int_4^\infty \frac{\delta_1^1(t')}{t'(t'-t)}dt'\right]. \quad (25)$$

As the branch point $t = 4m_\pi^2$, generating the cut, is a square-root type, the transformation (4) maps the two-sheeted Riemann surface of $F_\pi^{EM,I=1}(t)$ into one q plane and if one considers only the elastic region, the pion EM FF has in it only poles and zeros. The latter leads to a model-independent phase shift $\delta_1^1(t)$ representation,

$$\delta_1^1(q) = arctg\frac{A_3q^3 + A_5q^5 + \cdots}{1 + A_2q^2 + A_4q^4 + \cdots}, \quad (26)$$

where $A_1 \equiv 0$ in order to secure the threshold behavior of $\delta_1^1(q)$. An optimal description of the GKPY phase shift $\delta_1^1(q)$ data is achieved (see Fig. 5) with $\chi^2/ndf = 0.0244$ and four nonzero coefficients A_2, A_3, A_4, and A_5.

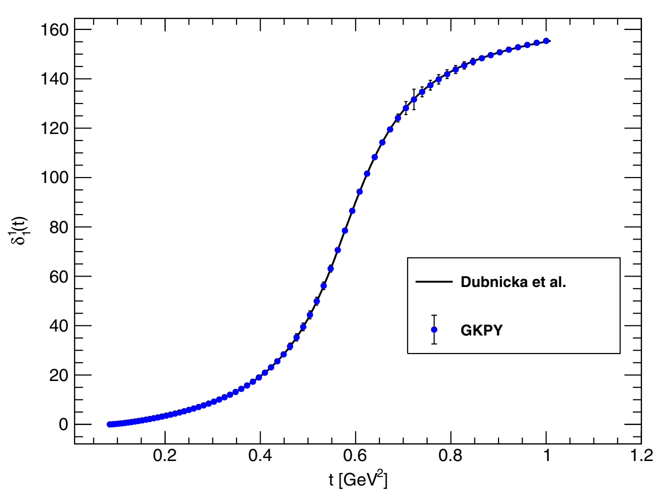

FIG. 5. Optimal description of the most accurate up-to-now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data with model-independent parametrization (26).

Substituting an equivalent form of the phase shift

$$\delta_1^1(q) = \frac{1}{2i} ln \frac{(1 + A_2q^2 + A_4q^4) + i(A_3q^3 + A_5q^5)}{(1 + A_2q^2 + A_4q^4) - i(A_3q^3 + A_5q^5)} \quad (27)$$

into (25), one finds $F_\pi^{EM,I=1}(q)$ in the form of a rational function in the denominator of which two conjugate, according to the negative imaginary axis, ρ meson poles appear, from which the mass and width

$$m_\rho = (763.56 \pm 0.51)\ \text{MeV}$$
$$\Gamma_\rho = (143.09 \pm 0.82)\ \text{MeV} \quad (28)$$

are determined to be very like (22).

By a comparison of χ^2/ndf values in obtaining of the ρ^0 meson parameters (14), (22), (24) and (28) we come to the conclusion that the correct ρ^0 meson parameters are most likely given by the averaged values (22) and (28).

IV. GENERALIZATION OF GOUNARIS-SAKURAI AND UNITARY&ANALYTIC MODELS TO EXCITED STATES OF THE ρ MESON

With the appearance of the data on the total cross section of the $e^+e^- \to \pi^+\pi^-$ process also at the region of the excited states $\rho'(1450)$ and $\rho''(1700)$ of the ρ meson, the pion EM FF G.-S. model was generalized in various forms [2,12,13]. However, such generalization is without any deeper physical background, as the original G.-S. model for the ρ^0 meson contribution has been constructed from the P-wave isoscalar $\pi\pi$ scattering phase shift given by the generalized effective-range formula of the Chew-Mandelstam type (2), which is evidently valid only in the elastic region, and as a result, the G.-S. model more or less reproducing the ρ^0 meson region cannot be expected to describe equally well the $\rho'(1450)$ and $\rho''(1700)$ at the deep inelastic region.

We have applied the generalization of [2]

$$F_\pi(t) = \frac{1}{1+\beta+\gamma}\left[F^{GS}_{\rho(770)}(t).\left(1 + \delta\frac{t}{m_\omega^2}BW_\omega(t)\right) + \beta F^{GS}_{\rho(1450)}(t) + \gamma F^{GS}_{\rho(1700)}(t)\right] \quad (29)$$

to an optimal description of the unified BESIII + *BABAR* data (see Fig. 6) with $\chi^2/ndf = 0.981$, and obtained parameters are presented in Table II. However, with regard to our earlier discussion, one cannot be confident in the determined parameters of all three vector mesons simultaneously. Despite this fact, just the parameter values of the $\rho(770)$ meson are presented in Ref. [6], though they are completely different from (14), which have been

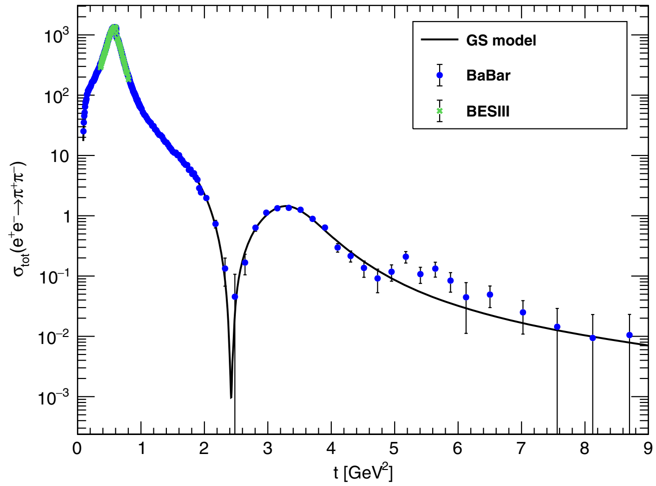

FIG. 6. Optimal description of the unified BESIII-*BABAR* complete data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ by the generalized pion EM FF G.-S. model.

determined by the original pion EM FF G.-S. model (13) to be valid only at the elastic region.

A totally different situation is in a generalization of the U&A pion EM FF model. Here, the contribution of all three vector mesons is at an equal level. Only now, the effective inelastic threshold, which is left as a free parameter of the model, has to be taken into account explicitly. Therefore, instead of the q variable, the W variable

TABLE II. The values of ρ meson parameters obtained from fits of BESIII + *BABAR* data [1,2] on the total cross section of the $e^+e^- \to \pi^+\pi^-$ process with Gounaris-Sakurai and Unitary& Analytic pion EM FF models to be compared to PDG values.

Parameter	PDG value (MeV)	Gounaris-Sakurai (MeV)	Unitary & Analytic (MeV)
m_ρ	775.26 ± 0.25	774.81 ± 0.01	763.88 ± 0.04
$m_{\rho'}$	1465.00 ± 25.00	1497.70 ± 1.07	1326.35 ± 3.46
$m_{\rho''}$	1720.00 ± 20.00	1848.40 ± 0.09	1770.54 ± 5.49
Γ_ρ	149.10 ± 0.80	149.22 ± 0.01	144.28 ± 0.01
$\Gamma_{\rho'}$	400.00 ± 60.00	442.15 ± 0.54	324.13 ± 12.01
$\Gamma_{\rho''}$	250.00 ± 100.00	322.48 ± 0.69	268.98 ± 11.40
χ^2/ndf		0.981 [14 parameters]	1.842 [11 parameters]

$$W(t) = i\frac{\sqrt{(\frac{t_{in}-t_0}{t_0})^{1/2} + (\frac{t-t_0}{t_0})^{1/2}} - \sqrt{(\frac{t_{in}-t_0}{t_0})^{1/2} - (\frac{t-t_0}{t_0})^{1/2}}}{\sqrt{(\frac{t_{in}-t_0}{t_0})^{1/2} + (\frac{t-t_0}{t_0})^{1/2}} + \sqrt{(\frac{t_{in}-t_0}{t_0})^{1/2} - (\frac{t-t_0}{t_0})^{1/2}}} \tag{30}$$

is now considered in a construction of the pion EM FF U&A model, which is mapping the four-sheeted Riemann surface in the t variable into one W plane.

Then, the pion EM FF model, considering contributions of all three vector meson resonances, takes the form [14]

$$\begin{aligned} F_\pi^{EM,I=1}[W(t)] = \left(\frac{1-W^2}{1-W_N}\right)^2 \frac{(W-W_Z)(W_N-W_P)}{(W_N-W_Z)(W-W_P)}. \\ \cdot\left[\frac{(W_N-W_\rho)(W_N-W_\rho^*)(W_N-1/W_\rho)(W_N-1/W_\rho^*)}{(W-W_\rho)(W-W_\rho^*)(W-1/W_\rho)(W-1/W_\rho^*)}(f_{\rho\pi\pi}/f_\rho) \right. \\ \left. + \sum_{v=\rho',\rho''} \frac{(W_N-W_v)(W_N-W_v^*)(W_N+W_v)(W_N+W_v^*)}{(W-W_v)(W-W_v^*)(W+W_v)(W+W_v^*)}(f_{v\pi\pi}/f_v)\right], \end{aligned} \tag{31}$$

with

$$(f_{\rho'\pi\pi}/f_{\rho'}) = -\frac{\frac{N_{\rho''}}{|W_{\rho''}|^4}}{\frac{N_{\rho'}}{|W_{\rho'}|^4} - \frac{N_{\rho''}}{|W_{\rho''}|^4}} + \frac{\frac{N_{\rho''}}{|W_{\rho''}|^4} - (1 + 2\frac{W_Z W_P}{W_Z - W_P}.\mathrm{Re}[W_\rho(1+|W_\rho|^{-2})])N_\rho}{\frac{N_{\rho'}}{|W_{\rho'}|^4} - \frac{N_{\rho''}}{|W_{\rho''}|^4}}(f_{\rho\pi\pi}/f_\rho), \tag{32}$$

$$(f_{\rho''\pi\pi}/f_{\rho''}) = 1 + \frac{\frac{N_{\rho''}}{|W_{\rho''}|^4}}{\frac{N_{\rho'}}{|W_{\rho'}|^4} - \frac{N_{\rho''}}{|W_{\rho''}|^4}} - \left[\frac{\frac{N_{\rho''}}{|W_{\rho''}|^4} - (1 + 2\frac{W_Z W_P}{W_Z - W_P}.\mathrm{Re}[W_\rho(1+|W_\rho|^{-2})])N_\rho}{\frac{N_{\rho'}}{|W_{\rho'}|^4} - \frac{N_{\rho''}}{|W_{\rho''}|^4}} - 1\right](f_{\rho\pi\pi}/f_\rho), \tag{33}$$

and

$$N_\rho = (W_N - W_\rho)(W_N - W_\rho^*)(W_N - 1/W_\rho)(W_N - 1/W_\rho^*), \tag{34}$$

$$N_v = (W_N - W_v)(W_N - W_v^*)(W_N + W_v)(W_N + W_v^*). \tag{35}$$

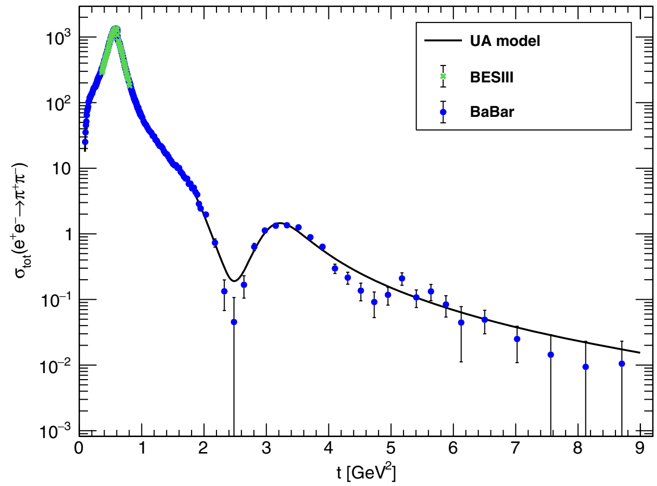

FIG. 7. Optimal description of the unified BESIII-*BABAR* complete data on $\sigma_{\text{tot}}(e^+e^- \to \pi^+\pi^-)$ by the generalized pion EM FF U&A model.

We have applied this generalized pion EM FF U&A model to an optimal description of the complete BESIII + *BABAR* data (see Fig. 7) with $\chi^2/ndf = 1.844$ and obtained all three pairs of vector meson parameters as they are also presented in Table II.

V. CONCLUSION

The accuracy of the original pion EM FF Gounaris-Sakurai model in a determination of the elastic $\rho(770)$ meson resonance parameters has been investigated. For this aim, totally independent experimental information on the P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ and on the total cross section of the $e^+e^- \to \pi^+\pi^-$ process was exploited.

Just by a comparison of χ^2/ndf values in obtaining of the ρ^0 meson parameters (14), (22), (24) and (28) we come to the conclusion that the correct ρ^0 meson parameters are most likely given by the averaged values (22) and (28).

We conjecture that the average value of (22), (28) and the $\rho(770)$ parameters in third column of the Table II, i.e. resultant values $m_\rho = 763.49 \pm 0.53$ MeV, $\Gamma_\rho = 144.06 \pm 0.85$ MeV, are the most suitable for considerations in the next corrections of the $\rho^0(770)$ meson parameters in the Review of Particle Physics [6].

We would like to draw an attention to the papers [15,16], in which similar results for the $\rho^0(770)$ meson parameters in the framework of the resonance chiral perturbation theory and in the framework of the chiral effective field theory with vector mesons included as dynamical degrees of freedom, respectively, have been found.

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