

Hadronic decays of $B \rightarrow a_1(1260)b_1(1235)$ in the perturbative QCD approach

Hao-Yang Jing and Xin Liu*

School of Physics and Electronic Engineering, Jiangsu Normal University, Xuzhou 221116, China

Zhen-Jun Xiao†

*Department of Physics and Institute of Theoretical Physics,**Nanjing Normal University, Nanjing 210023, China*

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We calculate the branching ratios and polarization fractions of the $B \rightarrow a_1 b_1$ decays in the perturbative QCD(pQCD) approach at leading order, where $a_1(b_1)$ stands for the axial-vector $a_1(1260)[b_1(1235)]$ state. By combining the phenomenological analyses with the perturbative calculations, we find the following results: (a) the large decay rates around 10^{-5} to 10^{-6} of the $B \rightarrow a_1 b_1$ decays dominated by the longitudinal polarization(except for the $B^+ \rightarrow b_1^+ a_1^0$ mode) are predicted and basically consistent with those in the QCD factorization(QCDF) within errors, which are expected to be tested by the Large Hadron Collider and Belle-II experiments. The large $B^0 \rightarrow a_1^0 b_1^0$ branching ratio could provide hints to help explore the mechanism of the color-suppressed decays. (b) the rather different QCD behaviors between the a_1 and b_1 mesons result in the destructive(constructive) contributions in the nonfactorizable spectator diagrams with $a_1(b_1)$ emission. Therefore, an interesting pattern of the branching ratios appears for the color-suppressed $B^0 \rightarrow a_1^0 a_1^0, a_1^0 b_1^0$, and $b_1^0 b_1^0$ modes in the pQCD approach, $\text{BR}(B^0 \rightarrow b_1^0 b_1^0) > \text{BR}(B^0 \rightarrow a_1^0 b_1^0) \gtrsim \text{BR}(B^0 \rightarrow a_1^0 a_1^0)$, which is different from $\text{BR}(B^0 \rightarrow b_1^0 b_1^0) \sim \text{BR}(B^0 \rightarrow a_1^0 b_1^0) \gtrsim \text{BR}(B^0 \rightarrow a_1^0 a_1^0)$ in the QCDF and would be verified at future experiments. (c) the large naive factorization breaking effects are observed in these $B \rightarrow a_1 b_1$ decays. Specifically, the large nonfactorizable spectator(weak annihilation) amplitudes contribute to the $B^0 \rightarrow b_1^+ a_1^- (B^+ \rightarrow a_1^+ b_1^- \text{ and } B^+ \rightarrow b_1^+ a_1^-)$ mode(s), which demand confirmations via the precise measurements. Furthermore, the different phenomenologies shown among $B \rightarrow a_1 b_1, B \rightarrow a_1 a_1$, and $B \rightarrow b_1 b_1$ decays are also expected to be tested stringently, which could shed light on the typical QCD dynamics involved in these modes, even further distinguish those two popular pQCD and QCDF approaches.

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It is well known that the nonleptonic B meson decays can provide highly important information to understand the physics within and/or beyond the standard model(SM). Specifically, they can help us to study the perturbative and nonperturbative quantum chromodynamics (QCD), search for the charge-parity (CP) violation to further find out its origin, determine the Cabibbo-Kobayashi-Maskawa (CKM) phases $\alpha(\phi_2), \beta(\phi_1)$, and $\gamma(\phi_3)$ in the unitary triangle, even identify the possible new physics hidden in the higher energy scale, etc. Moreover, one can also indirectly conjecture the inner structure of the hadrons involved in the final states through the precise measurements experimentally. The great efforts have been extensively contributed to the exclusive $B \rightarrow PP, PV$, and VV decays at both theoretical and experimental aspects in the past decades, for example, see Refs. [1–18], where P and V denote the S -wave pseudoscalar and vector states,

respectively. However, the known “puzzles,” for example, the large observed $B^0 \rightarrow \pi^0 \pi^0, B^0 \rightarrow \rho^0 \pi^0$, and $B \rightarrow K \eta'$ decay rates, the experimental inequality of the direct CP asymmetries between $B^\pm \rightarrow K^\pm \pi^0$ and $B^0 \rightarrow K^\pm \pi^\mp$ modes, the unknown mechanism of the polarization in the penguin-dominated $B \rightarrow VV$ processes etc., are still not elegantly resolved [17–19]. Therefore, a large variety of relevant B meson decay modes should be opened to help us get deep understanding complementarily.

Fortunately, two successful B -factory experiments, i.e., BABAR at SLAC and Belle at KEK, have measured many nonleptonic B meson decays into the final states containing p -wave light hadrons in the last decade [17,18]. Then the Large Hadron Collider-beauty(LHCb) experiments at CERN almost became the only apparatus to explore the physics of b quark in recent years. A large number of data related to nonleptonic B decays have been reported [17,18]. The forthcoming start of the upgraded Belle-II experiment will further improve the measurements. The Future Circular Collider and Circular Electron-Positron Collider are expected to give further chance for the studies on B

*Corresponding author.
liuxin@jsnu.edu.cn
†xiaozhenjun@nynu.edu.cn

meson decays [20]. Therefore, it is believed that the great supports coming from these current running and forthcoming experiments could dramatically promote our understanding of the nature.

In this work, we will study the nonleptonic charmless decays of $B \rightarrow a_1(1260)b_1(1235)$ in the SM. For the sake of simplicity, the abbreviation a_1 and b_1 will be used in the following content to denote the $a_1(1260)$ and $b_1(1235)$ mesons, respectively, unless otherwise stated. As we know, the considered processes contain the same components as the $B \rightarrow \pi\pi, \rho\pi, \rho\rho$ modes at the quark level. The latter decays have contributed to the determination and constraints on the CKM angle α [17]. Certainly, the $B \rightarrow a_1(b_1)\pi, a_1(b_1)\rho$, and $a_1(b_1)b_1(a_1)$ decays can also provide useful information to the angle α complementarily [21–27]. Particularly, because a_1 and b_1 behave differently from each other, these considered decays could provide opportunities for us to explore the interesting QCD dynamics. Furthermore, the $B \rightarrow a_1b_1$ decays with b_1 emission could provide more evidence for probing the naive factorization breaking effects [28] because the decay constant f_{b_1} vanishes owing to the charge conjugation invariance for the neutral b_1^0 state or the even G-parity validity in the isospin limit for the charged b_1^\pm states [24,29,30].

As stated in the naive factorization hypothesis [2], the hadronic matrix element of a B meson decay amplitude can be expressed by the factorizable emission amplitudes as a production of the decay constants and the transition form factors. Then, for example, the $B^0 \rightarrow b_1^+ a_1^-$ mode with b_1 emission almost receives no factorizable contributions due to the vanishing decay constant f_{b_1} and the branching ratio would approach to zero in the naive factorization. While, it is worth emphasizing that the corresponding decay rate predicted in the QCD factorization(QCDF) [14,31] by including the nonfactorizable spectator and annihilation contributions can reach $\mathcal{O}(10^{-6})$ [23], which is detectable at the current experiments. It means that these important contributions violate the naive factorization if this large decay rate would be confirmed by the related experiments. However, because of the unavoidable endpoint singularities, the nonfactorizable spectator amplitudes, as well as the annihilation ones, have to be determined by data fitting accompanied with large uncertainties in the framework of QCDF [15,31]. Luckily, the perturbative QCD(pQCD) approach [32,33], which bases on the framework of k_T factorization theorem, is appropriate to calculate the decay amplitudes with the nonfactorizable spectator and annihilation topologies. Since it keeps the transverse momentum k_T of the valence quark in the hadrons, then the resultant Sudakov factor $[e^{(-S)}]$ and threshold factor $[S_t(x)]$, which smear the endpoint singularities, make the pQCD approach more self-consistent. More details about this pQCD approach can be found in the review paper [34].

We will therefore study the branching ratios and polarization fractions of the considered $B \rightarrow a_1b_1$ decays in the pQCD approach, with which the nonfactorizable spectator and annihilation Feynman diagrams can be calculated perturbatively. It is worth stressing that the observations of the pure annihilation $B_s^0 \rightarrow \pi^+\pi^-$ and $B_d^0 \rightarrow K^+K^-$ decays performed by the CDF [35] and LHCb [36] collaborations have confirmed the pQCD calculations [7,37,38] of the annihilation type diagrams.¹ Moreover, both of a_1 and b_1 are axial-vector(A) states but with different quantum numbers $J^{PC} = 1^{+-}$ and 1^{++} correspondingly. It is believed that the $B \rightarrow AA$ decays could provide more information on the helicity structure of the decay mechanism because, like $B \rightarrow VV$ decays, they also contain three polarization states [23], which would be helpful to understand the famous “polarization puzzle” in a different way.

For the considered $B \rightarrow a_1b_1$ decays with $\bar{b} \rightarrow \bar{d}$ transition, the related weak effective Hamiltonian H_{eff} [41] can be written as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}^* V_{ud} [C_1(\mu) O_1^u(\mu) + C_2(\mu) O_2^u(\mu)] - V_{tb}^* V_{td} \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right\}, \quad (1)$$

with the Fermi constant $G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$, CKM matrix elements V , and Wilson coefficients $C_i(\mu)$ at the renormalization scale μ . The local four-quark operators $O_i (i = 1, \dots, 10)$ are written as

(1) current-current (tree) operators

$$\begin{aligned} O_1^u &= (\bar{d}_\alpha u_\beta)_{V-A} (\bar{u}_\beta b_\alpha)_{V-A}, \\ O_2^u &= (\bar{d}_\alpha u_\alpha)_{V-A} (\bar{u}_\beta b_\beta)_{V-A}; \end{aligned} \quad (2)$$

(2) QCD penguin operators

$$\begin{aligned} O_3 &= (\bar{d}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \\ O_4 &= (\bar{d}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A}, \\ O_5 &= (\bar{d}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \\ O_6 &= (\bar{d}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A}; \end{aligned} \quad (3)$$

¹Certainly, the soft-collinear effective theory(SCET) [39] has a different point of view on the calculations of the annihilation diagrams [40]. We believe that this discrepancy between the pQCD approach and SCET could be finally resolved through the precise measurements experimentally. Therefore, this conversation will be put aside in the present work.

(3) electroweak penguin operators

$$\begin{aligned}
O_7 &= \frac{3}{2} (\bar{d}_\alpha b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \\
O_8 &= \frac{3}{2} (\bar{d}_\alpha b_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A}, \\
O_9 &= \frac{3}{2} (\bar{d}_\alpha b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \\
O_{10} &= \frac{3}{2} (\bar{d}_\alpha b_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A}.
\end{aligned} \tag{4}$$

with the color indices α, β (not to be confused with the CKM angles) and the notations $(\bar{q}' q')_{V\pm A} = \bar{q}' \gamma_\mu (1 \pm \gamma_5) q'$. The index q' in the summation of the above operators runs through u, d, s, c , and b . We will use the leading order Wilson coefficients to keep the consistency since the calculations in this work are at leading order $[\mathcal{O}(\alpha_s)]$ of the pQCD approach. For the renormalization group evolution of the Wilson coefficients from higher scale to lower scale, we use the formulas as given in Ref. [32] directly.

Similar to the vector meson, the axial-vector one also has three kinds of polarizations, i.e., longitudinal (L), normal (N), and transverse (T), respectively. Therefore, analogous to the $B \rightarrow VV$ decays, the $B \rightarrow a_1 b_1$ decay amplitudes will be characterized by the polarization states of these axial-vector mesons. In terms of helicities, the decay amplitudes $\mathcal{M}^{(\sigma)}$ for $B \rightarrow a_1(P_2, \epsilon_2^*) b_1(P_3, \epsilon_3^*)$ decays can be generally described by

$$\begin{aligned}
\mathcal{M}^{(\sigma)} &= \epsilon_{2\mu}^*(\sigma) \epsilon_{3\nu}^*(\sigma) \left[a g^{\mu\nu} + \frac{b}{m_{a_1} m_{b_1}} P_1^\mu P_1^\nu \right. \\
&\quad \left. + i \frac{c}{m_{a_1} m_{b_1}} \epsilon^{\mu\nu\alpha\beta} P_{2\alpha} P_{3\beta} \right], \\
&\equiv m_B^2 \mathcal{M}_L + m_B^2 \mathcal{M}_N \epsilon_2^*(\sigma = T) \cdot \epsilon_3^*(\sigma = T) \\
&\quad + i \mathcal{M}_T \epsilon^{\alpha\beta\gamma\rho} \epsilon_{2\alpha}^*(\sigma) \epsilon_{3\beta}^*(\sigma) P_{2\gamma} P_{3\rho},
\end{aligned} \tag{5}$$

where the superscript σ denotes the helicity states of two mesons with $L(T)$ standing for the longitudinal (transverse) component and the definitions of the amplitudes \mathcal{M}_h ($h = L, N, T$) in terms of the Lorentz-invariant amplitudes a, b and c are

$$\begin{aligned}
m_B^2 \mathcal{M}_L &= a \epsilon_2^*(L) \cdot \epsilon_3^*(L) + \frac{b}{m_{a_1} m_{b_1}} \epsilon_2^*(L) \cdot P_3 \epsilon_3^*(L) \cdot P_2, \\
m_B^2 \mathcal{M}_N &= a, \\
m_B^2 \mathcal{M}_T &= \frac{c}{r_2 r_3},
\end{aligned} \tag{6}$$

with $\epsilon_{2(3)}$ and $P_{2(3)}$ denoting the polarization vector and momentum of the $a_1(b_1)$ state correspondingly. Here,

$r_{2(3)} \equiv m_{a_1(b_1)}/m_B$ with $m_{a_1(b_1)}$ and m_B , the masses of the light $a_1(b_1)$ and heavy B mesons, respectively. We will therefore analyze the helicity amplitudes $\mathcal{M}_L, \mathcal{M}_N, \mathcal{M}_T$ based on the pQCD approach. According to the helicity amplitudes (6), the transversity ones can be defined as

$$\begin{aligned}
\mathcal{A}_L &= \xi m_B^2 \mathcal{M}_L, & \mathcal{A}_\parallel &= \xi \sqrt{2} m_B^2 \mathcal{M}_N, \\
\mathcal{A}_\perp &= \xi r_2 r_3 \sqrt{2(r^2 - 1)} m_B^2 \mathcal{M}_T.
\end{aligned} \tag{7}$$

for the longitudinal, parallel, and perpendicular polarizations, respectively, where the ratio $r = P_2 \cdot P_3 / (m_B^2 r_2 r_3)$ and the normalization factor $\xi = \sqrt{G_F^2 |\mathbf{P}_c| / (16\pi m_B^2 \Gamma)}$ with the decay width $\Gamma = \frac{G_F^2 |\mathbf{P}_c|}{16\pi m_B^2} \sum_\sigma \mathcal{M}^{(\sigma)\dagger} \mathcal{M}^{(\sigma)}$ and the momentum of either of the outgoing axial-vector mesons $|\mathbf{P}_c| \equiv |\mathbf{P}_{2z}| = |\mathbf{P}_{3z}|$. These amplitudes satisfy the following relation,

$$|\mathcal{A}_L|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2 = 1. \tag{8}$$

As illustrated in Fig. 1, analogous to the $B \rightarrow a_1 a_1$ and $b_1 b_1$ decays [22], there are 8 types of diagrams contributing to the $B \rightarrow a_1 b_1$ decays at the lowest order in the pQCD approach. Because the amplitudes for the Feynman diagrams of the $B \rightarrow AA$ decays have been analyzed explicitly in Ref. [22], then the $B \rightarrow a_1 b_1$ decay amplitudes can be easily obtained from the Eqs. (25)-(60) by appropriate replacements correspondingly:

- (1) When the $a_1(b_1)$ state flies(recoils) along with the $+z(-z)$ direction in the B meson rest frame, the above mentioned Eqs. (25)–(60) [22] will describe the $B \rightarrow a_1 b_1$ decays with $B \rightarrow b_1$ transition, in which the related $B \rightarrow b_1$ form factor can be factored out. The Feynman decay amplitudes will be expressed with F^h and M^h ;
- (2) When the $b_1(a_1)$ state flies(recoils) along with the $+z(-z)$ direction in the B meson rest frame, the above mentioned Eqs. (25)–(60) [22] will describe the $B \rightarrow a_1 b_1$ decays with $B \rightarrow a_1$ transition, in which the related $B \rightarrow a_1$ form factor can also be extracted out. The Feynman decay amplitudes will be presented with $F^{h'}$ and $M^{h'}$.

Hence, for simplicity, we will not present the factorization formulas for these $B \rightarrow a_1 b_1$ modes again in this work. The interested readers can refer to Ref. [22] for details. By combining various contributions from the relevant Feynman diagrams together, the decay amplitudes of the $B \rightarrow a_1 b_1$ decays can then be collected straightforwardly with three polarizations $h = L, N, T$ as follows:

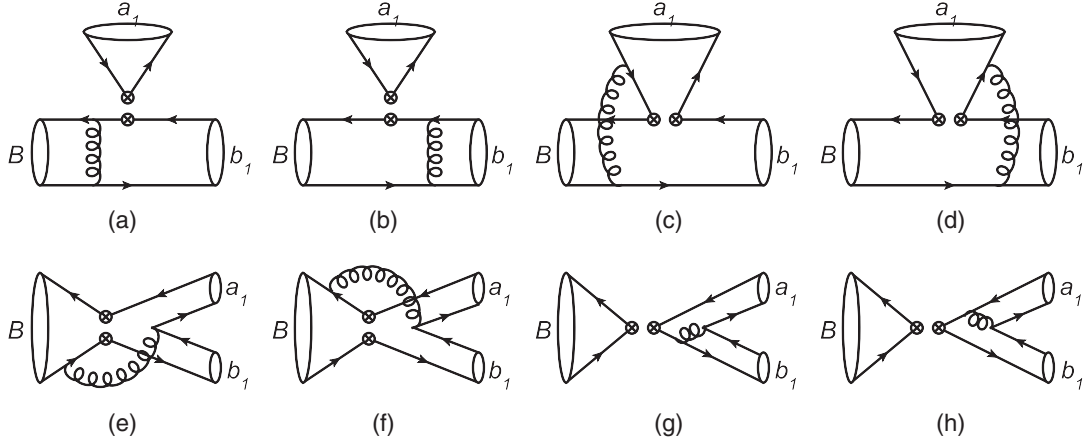


FIG. 1. Typical Feynman diagrams for $B \rightarrow a_1 b_1$ decays at leading order in the pQCD approach. By exchanging the position of the a_1 and b_1 mesons, one will obtain another eight Feynman diagrams that possibly contribute to the considered $B \rightarrow a_1 b_1$ modes.

$$\begin{aligned}
 \mathcal{M}_h(B^0 \rightarrow a_1^+ b_1^-) = & \xi_u [a_1 F_{fs}^h + C_1 M_{nfs}^h + C_2 M_{nfa}^h + a_2 f_B F_{fa}^h] - \xi_t \left[(a_4 + a_{10}) F_{fs}^h + (C_3 + C_9) M_{nfs}^h \right. \\
 & + (C_5 + C_7) M_{nfs}^{h,P_1} + \left(C_3 + C_4 - \frac{1}{2} (C_9 + C_{10}) \right) M_{nfa}^h + (C_4 + C_{10}) M_{nfa}^{h,P_1} \\
 & + \left(C_5 - \frac{1}{2} C_7 \right) M_{nfa}^{h,P_1} + \left(C_6 - \frac{1}{2} C_8 \right) M_{nfa}^{h,P_2} + \left(a_3 + a_4 + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}) \right) f_B F_{fa}^h \\
 & \left. + (C_6 + C_8) M_{nfa}^{h,P_2} + (a_3 + a_5 + a_7 + a_9) f_B F_{fa}^h + \left(a_6 - \frac{1}{2} a_8 \right) f_B F_{fa}^{h,P_2} \right], \quad (9)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{M}_h(B^0 \rightarrow b_1^+ a_1^-) = & \xi_u [a_1 F_{fs}^h + C_1 M_{nfs}^h + C_2 M_{nfa}^h + a_2 f_B F_{fa}^h] - \xi_t \left[(a_4 + a_{10}) F_{fs}^h + (C_3 + C_9) M_{nfs}^h \right. \\
 & + (C_5 + C_7) M_{nfs}^{h,P_1} + \left(C_3 + C_4 - \frac{1}{2} (C_9 + C_{10}) \right) M_{nfa}^h + (C_4 + C_{10}) M_{nfa}^{h,P_1} \\
 & + \left(C_5 - \frac{1}{2} C_7 \right) M_{nfa}^{h,P_1} + \left(C_6 - \frac{1}{2} C_8 \right) M_{nfa}^{h,P_2} + \left(a_3 + a_4 + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}) \right) f_B F_{fa}^h \\
 & \left. + (C_6 + C_8) M_{nfa}^{h,P_2} + (a_3 + a_5 + a_7 + a_9) f_B F_{fa}^h + \left(a_6 - \frac{1}{2} a_8 \right) f_B F_{fa}^{h,P_2} \right], \quad (10)
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{2} \mathcal{M}_h(B^+ \rightarrow a_1^+ b_1^0) = & \xi_u [a_1 (F_{fs}^h - f_B F_{fa}^h + f_B F_{fa}^h) + a_2 F_{fs}^h + C_1 (M_{nfs}^h + M_{nfa}^h - M_{nfa}^h) - C_2 M_{nfs}^h] \\
 & - \xi_t \left[\left(\frac{5}{3} C_9 + C_{10} - \frac{1}{2} a_8 - a_4 \right) F_{fs}^h + (a_4 + a_{10}) F_{fs}^h + \left(\frac{1}{2} a_9 - C_3 \right) M_{nfs}^h \right. \\
 & + \left(\frac{1}{2} C_7 - C_5 \right) M_{nfs}^{h,P_1} + \frac{3}{2} C_8 M_{nfs}^{h,P_2} + (C_3 + C_9) M_{nfs}^h + (C_5 + C_7) M_{nfs}^{h,P_1} \\
 & + (C_3 + C_9) (M_{nfa}^{h,P_1} - M_{nfa}^{h,P_1}) + (a_4 + a_{10}) (f_B F_{fa}^h - f_B F_{fa}^h) \\
 & \left. + (a_6 + a_8) (f_B F_{fa}^{h,P_2} - f_B F_{fa}^{h,P_2}) \right], \quad (11)
 \end{aligned}$$

$$\begin{aligned}
\sqrt{2}\mathcal{M}_h(B^+ \rightarrow b_1^+ a_1^0) = & \xi_u [a_1 (F_{fs}^{fh} - f_B F_{fa}^{fh} + f_B F_{fs}^{fh}) + a_2 F_{fs}^{fh} + C_1 (M_{nfs}^{fh} + M_{nfa}^{fh} - M_{nfa}^{fh}) - C_2 M_{nfs}^{fh}] \\
& - \xi_t \left[\left(\frac{5}{3} C_9 + C_{10} - \frac{1}{2} a_8 - a_4 \right) F_{fs}^{fh} + (a_4 + a_{10}) F_{fs}^{fh} + \left(\frac{1}{2} a_9 - C_3 \right) M_{nfs}^{fh} \right. \\
& + \left(\frac{1}{2} C_7 - C_5 \right) M_{nfs}^{h,P_1} + \frac{3}{2} C_8 M_{nfs}^{h,P_2} + (C_3 + C_9) M_{nfs}^{fh} + (C_5 + C_7) M_{nfs}^{h,P_1} \\
& + (C_3 + C_9) (M_{nfa}^{h,P_1} - M_{nfa}^{h,P_1}) + (a_4 + a_{10}) (f_B F_{fa}^{fh} - f_B F_{fa}^{fh}) \\
& \left. + (a_6 + a_8) (f_B F_{fa}^{h,P_2} - f_B F_{fa}^{h,P_2}) \right], \tag{12}
\end{aligned}$$

$$\begin{aligned}
2\mathcal{M}_h(B^0 \rightarrow a_1^0 b_1^0) = & \xi_u [-a_2 (F_{fs}^{fh} + F_{fs}^{fh} - f_B F_{fa}^{fh} - f_B F_{fa}^{fh}) - C_2 (M_{nfa}^{fh} + M_{nfa}^{fh} - M_{nfs}^{fh} - M_{nfs}^{fh})] \\
& - \xi_t \left[\left(a_4 - \frac{1}{2} (3a_7 + 3a_9 + a_{10}) \right) (F_{fs}^{fh} + F_{fs}^{fh}) - \left(C_5 - \frac{1}{2} C_7 \right) (M_{nfs}^{h,P_1} + M_{nfs}^{h,P_1}) \right. \\
& + \left(C_3 - \frac{1}{2} (C_9 + 3C_{10}) \right) (M_{nfs}^{fh} + M_{nfs}^{fh}) + \left(C_3 + 2C_4 - \frac{1}{2} (C_9 - C_{10}) \right) (M_{nfa}^{fh} + M_{nfa}^{fh}) \\
& - \frac{3}{2} C_8 (M_{nfs}^{h,P_2} + M_{nfs}^{h,P_2}) + \left(2a_3 + a_4 + 2a_5 + \frac{1}{2} (a_7 - a_9 + a_{10}) \right) (f_B F_{fa}^{fh} + f_B F_{fa}^{fh}) \\
& + \left(C_5 - \frac{1}{2} C_7 \right) (M_{nfa}^{h,P_1} + M_{nfa}^{h,P_1}) + \left(2C_6 + \frac{1}{2} C_8 \right) (M_{nfa}^{h,P_2} + M_{nfa}^{h,P_2}) \\
& \left. + \left(a_6 - \frac{1}{2} a_8 \right) (f_B F_{fa}^{h,P_2} + f_B F_{fa}^{h,P_2}) \right]. \tag{13}
\end{aligned}$$

where ξ_u and ξ_t stand for the products of CKM matrix elements $V_{ub}^* V_{ud}$ and $V_{tb}^* V_{td}$, respectively. The standard combinations a_i of Wilson coefficients are defined as follows,

$$\begin{aligned}
a_1 &= C_2 + \frac{C_1}{3}, & a_2 &= C_1 + \frac{C_2}{3}, \\
a_i &= C_i + \frac{C_{i\pm 1}}{3} \quad (i = 3 - 10). \tag{14}
\end{aligned}$$

where $C_2 \sim \mathcal{O}(1)$ and the upper(lower) sign applies when i is odd(even).

Now, we will turn to the numerical evaluations of the branching ratios and polarization fractions of the considered $B \rightarrow a_1 b_1$ decays in the pQCD approach. The essential comments on the input parameters are given as follows:

- (1) For the heavy B mesons and light axial-vector a_1 and b_1 states, the same hadron wave functions and distribution amplitudes including Gegenbauer moments are adopted as those in Ref. [22]. And the same QCD scale, masses of hadrons, and decay constants are also utilized. The B^0 meson lifetime is updated as 1.52 ps [17].
- (2) As for the CKM matrix elements, we adopt the Wolfenstein parametrization at leading order and the newly updated parameters $A = 0.811$, $\lambda = 0.22506$, $\bar{\rho} = 0.124$, and $\bar{\eta} = 0.356$ [17].

The theoretical predictions for $B \rightarrow a_1 b_1$ decays evaluated in the pQCD approach, together with the results in the QCDF approach, have been grouped in the Tables I–III, in which the first error is induced by the uncertainties of the shape parameter $\omega_B = 0.40 \pm 0.04$ GeV in the B meson wave function, the second error arises from the combination

TABLE I. Branching ratios and polarization fractions of the $B^+ \rightarrow a_1^+ b_1^0$, $B^+ \rightarrow b_1^+ a_1^0$, and $B^0 \rightarrow a_1^0 b_1^0$ decays in the pQCD approach (This work). For comparison, we also quote the related results predicted in the QCDF approach [23].

Decay Channels		$B^+ \rightarrow a_1^+ b_1^0$		$B^+ \rightarrow b_1^+ a_1^0$		$B^0 \rightarrow a_1^0 b_1^0$	
Parameter	Definition	This work	QCDF	This work	QCDF	This work	QCDF
BR(10^{-6})	$\Gamma/\Gamma_{\text{total}}$	$9.0^{+1.5+5.2+0.8}_{-1.3-3.8-0.7}$	$37.8^{+23.9+11.4}_{-15.3-5.3}$	$4.2^{+0.4+2.0+0.4}_{-0.3-1.4-0.3}$	$1.0^{+1.6+6.2}_{-0.5-0.1}$	$3.3^{+0.6+1.8+0.3}_{-0.5-1.5-0.3}$	$3.8^{+6.2+2.6}_{-2.3-0.5}$
f_L	$ \mathcal{A}_L ^2$	$0.62^{+0.01+0.01+0.00}_{-0.03-0.04-0.00}$	$0.92^{+0.02}_{-0.24}$	$0.28^{+0.00+0.02+0.00}_{-0.01-0.04-0.01}$	$0.73^{+0.12}_{-0.82}$	$0.63^{+0.00+0.06+0.00}_{-0.01-0.10-0.01}$	$0.98^{+0.01}_{-0.31}$
$f_{ }$	$ \mathcal{A}_{ } ^2$	$0.10^{+0.00+0.05+0.00}_{-0.00-0.02-0.00}$		$0.16^{+0.01+0.03+0.00}_{-0.02-0.04-0.02}$		$0.17^{+0.00+0.04+0.01}_{-0.00-0.00-0.00}$	
f_{\perp}	$ \mathcal{A}_{\perp} ^2$	$0.28^{+0.03+0.06+0.00}_{-0.02-0.03-0.00}$		$0.57^{+0.02+0.04+0.01}_{-0.02-0.07-0.00}$		$0.20^{+0.00+0.09+0.00}_{-0.00-0.07-0.00}$	

TABLE II. Same as Table I but for the $B^0 \rightarrow a_1^+ b_1^-$ and $B^0 \rightarrow b_1^+ a_1^-$ decays.

Decay Channels		$B^0 \rightarrow a_1^+ b_1^-$		$B^0 \rightarrow b_1^+ a_1^-$	
Parameter	Definition	This work	QCDF	This work	QCDF
BR(10^{-6})	$\Gamma/\Gamma_{\text{total}}$	$73.6^{+23.4+12.8+6.5}_{-17.0-12.1-6.0}$	$41.3^{+20.7+16.6}_{-18.2-3.4}$	$3.7^{+0.6+2.0+0.2}_{-0.5-1.6-0.2}$	$0.8^{+1.1+3.6}_{-0.4-0.1}$
f_L	$ \mathcal{A}_L ^2$	$0.94^{+0.00+0.00+0.00}_{-0.01-0.03-0.01}$	$0.90^{+0.02}_{-0.05}$	$0.96^{+0.01+0.01+0.00}_{-0.01-0.03-0.01}$	$0.98^{+0.00}_{-0.80}$
$f_{ }$	$ \mathcal{A}_{ } ^2$	$0.04^{+0.00+0.00+0.00}_{-0.00-0.02-0.00}$		$0.02^{+0.01+0.02+0.00}_{-0.00-0.01-0.00}$	
f_{\perp}	$ \mathcal{A}_{\perp} ^2$	$0.03^{+0.00+0.00+0.00}_{-0.00-0.02-0.00}$		$0.02^{+0.01+0.02+0.00}_{-0.00-0.01-0.00}$	

TABLE III. Same as Table I but for the $B^0/\bar{B}^0 \rightarrow a_1^+ b_1^-$, $B^0/\bar{B}^0 \rightarrow b_1^+ a_1^-$, $B^0 \rightarrow a_1^+ b_1^- + b_1^+ a_1^-$ decays.

Decay Channels		$B^0/\bar{B}^0 \rightarrow a_1^+ b_1^-$	$B^0/\bar{B}^0 \rightarrow b_1^+ a_1^-$	$B^0 \rightarrow a_1^+ b_1^- + b_1^+ a_1^-$
Parameter	Definition	This work	This work	This work
BR(10^{-6})	$\Gamma/\Gamma_{\text{total}}$	$91.1^{+29.1+20.7+9.1}_{-21.2-18.9-8.6}$	$44.2^{+18.0+8.1+3.6}_{-12.5-7.6-3.2}$	$85.8^{+24.3+19.3+6.3}_{-17.8-17.5-5.8}$
f_L	$ \mathcal{A}_L ^2$	$0.91^{+0.01+0.05+0.01}_{-0.00-0.02-0.00}$	$0.81^{+0.02+0.07+0.01}_{-0.02-0.06-0.00}$	$0.91^{+0.00+0.03+0.00}_{-0.00-0.01-0.00}$
$f_{ }$	$ \mathcal{A}_{ } ^2$	$0.05^{+0.00+0.02+0.00}_{-0.00-0.02-0.00}$	$0.11^{+0.01+0.03+0.00}_{-0.01-0.05-0.00}$	$0.05^{+0.00+0.01+0.00}_{-0.00-0.01-0.00}$
f_{\perp}	$ \mathcal{A}_{\perp} ^2$	$0.04^{+0.00+0.00+0.00}_{-0.01-0.03-0.01}$	$0.08^{+0.01+0.02+0.00}_{-0.01-0.02-0.00}$	$0.04^{+0.00+0.00+0.00}_{-0.00-0.01-0.00}$

of the uncertainties of Gegenbauer moments $a_{2,a_1}^{||}, a_{1,b_1}^{||}$, a_{1,a_1}^{\perp} , and a_{2,b_1}^{\perp} in the distribution amplitudes of a_1 and b_1 mesons, and the last error is also the combined uncertainty from the CKM matrix elements: $\bar{\rho} = 0.124^{+0.019}_{-0.018}$ and $\bar{\eta} = 0.356^{+0.011}_{-0.011}$ [17]. It is easily seen that the theoretical predictions suffer from large uncertainties that mainly induced by the parameters describing the nonperturbative hadron dynamics. It is therefore expected that the predictions given in the pQCD approach could be improved greatly with the well-constrained inputs based on the nonperturbative QCD, e.g., lattice QCD, calculations with high precision and/or the future precise measurements experimentally.

I. BRANCHING RATIOS

We first analyze the branching ratios of the $B \rightarrow a_1 b_1$ decays according to the numerical results obtained in the pQCD approach. And furthermore, since these considered modes have been studied in another popular QCDF approach, we also quote the related predictions to make an essential comparison and discussion, which could be helpful to further discriminate these two rather different tools through the future precise measurements.

As presented in Tables I–II, the pQCD predictions for the branching ratios of the classified five modes² are from 10^{-6} to 10^{-5} , explicitly,

²It should be stressed that the final states in the former $B^+ \rightarrow a_1^+ b_1^0$, $B^+ \rightarrow b_1^+ a_1^0$, and $B^0 \rightarrow a_1^0 b_1^0$ modes are the CP eigenstates, while those in the latter $B^0 \rightarrow a_1^+ b_1^-$ and $B^0 \rightarrow b_1^+ a_1^-$ ones are not, which therefore result in the branching ratios with and without the CP -averaged final states as presented in Tables I and II, respectively.

$$\left. \begin{aligned} \text{BR}(B^+ \rightarrow a_1^+ b_1^0) &= 9.0^{+5.5}_{-4.0} \times 10^{-6}, \\ \text{BR}(B^+ \rightarrow b_1^+ a_1^0) &= 4.2^{+2.1}_{-1.5} \times 10^{-6}, \\ \text{BR}(B^0 \rightarrow a_1^0 b_1^0) &= 3.3^{+1.9}_{-1.6} \times 10^{-6}, \\ \text{BR}(B^0 \rightarrow a_1^+ b_1^-) &= 73.6^{+27.5}_{-21.7} \times 10^{-6}, \\ \text{BR}(B^0 \rightarrow b_1^+ a_1^-) &= 3.7^{+2.1}_{-1.7} \times 10^{-6}, \end{aligned} \right\} \text{(In pQCD)} \quad (15)$$

which are generally consistent with those estimated in the QCDF approach, namely,

$$\left. \begin{aligned} \text{BR}(B^+ \rightarrow a_1^+ b_1^0) &= 37.8^{+26.5}_{-16.2} \times 10^{-6}, \\ \text{BR}(B^+ \rightarrow b_1^+ a_1^0) &= 1.0^{+6.4}_{-0.5} \times 10^{-6}, \\ \text{BR}(B^0 \rightarrow a_1^0 b_1^0) &= 3.8^{+6.7}_{-2.4} \times 10^{-6}, \\ \text{BR}(B^0 \rightarrow a_1^+ b_1^-) &= 41.3^{+26.5}_{-18.5} \times 10^{-6}, \\ \text{BR}(B^0 \rightarrow b_1^+ a_1^-) &= 0.8^{+3.8}_{-0.4} \times 10^{-6}. \end{aligned} \right\} \text{(In QCDF)} \quad (16)$$

within still large theoretical errors. Notice that various errors here have been added in quadrature. All these predictions of the $B \rightarrow a_1 b_1$ decay rates with both QCDF and pQCD approaches are expected to be accessed by the current LHCb and the forthcoming Belle-II experiments.

As discussed in Refs. [24,29,30] with QCD sum rule method, relative to the vector ρ meson, the two axial-vector a_1 and b_1 states exhibit rather different hadron dynamics, namely, the former(latter) is similar(contrary) to ρ with (anti) symmetric leading-twist distribution amplitude dominated by the longitudinal(transverse) polarization. Therefore, the involved QCD dynamics in the $B \rightarrow a_1 b_1$ decays should be different from that in the $B \rightarrow a_1 a_1$ and $B \rightarrow b_1 b_1$

processes, while similar to that in the $B \rightarrow b_1 \rho$ modes. The $B \rightarrow a_1 a_1, b_1 b_1$ and $b_1 \rho$ channels have been investigated in the QCDF [23] and pQCD [22,25] approaches.

Some remarks on the branching ratios of the $B \rightarrow a_1 b_1$ decays are in order as follows:

- (a) For the $B^+ \rightarrow a_1^+ b_1^0$ and $B^+ \rightarrow b_1^+ a_1^0$ decays, the branching ratios predicted in the pQCD approach show different phenomena to those in the QCDF approach, though the similar pattern of $\text{BR}(B^+ \rightarrow a_1^+ b_1^0) > \text{BR}(B^+ \rightarrow b_1^+ a_1^0)$ has been gotten in terms of the central values. One can clearly see from Eqs. (15) and (16) that $\text{BR}(B^+ \rightarrow a_1^+ b_1^0)_{\text{pQCD}} \sim \text{BR}(B^+ \rightarrow b_1^+ a_1^0)_{\text{pQCD}}$ while $\text{BR}(B^+ \rightarrow a_1^+ b_1^0)_{\text{QCDF}} > \text{BR}(B^+ \rightarrow b_1^+ a_1^0)_{\text{QCDF}}$ within errors. The underlying reason is that the weak annihilation contributions play an important role in these two decays, which can be seen explicitly from the pQCD results of the decay amplitudes given in the Table IV with different topologies.

Different from the $B^+ \rightarrow \rho^+ \rho^0, a_1^+ a_1^0$, and $b_1^+ b_1^0$ decays, the large annihilation contributions appear in the $B^+ \rightarrow a_1^+ b_1^0$ and $b_1^+ a_1^0$ ones. Based on the assumption of the isospin symmetry, the final states such as $\rho^+ \rho^0, a_1^+ a_1^0$, and $b_1^+ b_1^0$ are the identical bosons, which, because of Bose-Einstein statistics, consequently lead to the exact cancellation between the amplitudes induced by the $u\bar{u}$ and $d\bar{d}$ components of the neutral state in the annihilation diagrams. However, the a_1 and b_1 states are not the identical

particles with different quantum numbers. The rather different QCD behaviors between the a_1 and b_1 mesons further result in the largely nonzero annihilation decay amplitudes associated with the $a_1^+ b_1^0$ and $b_1^+ a_1^0$ final states, respectively.

These two $B^+ \rightarrow a_1^+ b_1^0$ and $B^+ \rightarrow b_1^+ a_1^0$ decays with large decay rates [$\mathcal{O}(10^{-6})$] are expected to be tested by the LHCb and Belle-II experiments, which could, on one hand, confirm the reliability of the perturbative calculations in the framework of pQCD or QCDF; on the other hand, provide more evidences to distinguish the validity of the treatments in calculating the annihilation diagrams between the pQCD approach and SCET, even to further understand the annihilation decay mechanism in the B meson decays.

- (b) Analogous to $B^0 \rightarrow \rho^0 \rho^0, a_1^0 a_1^0$, and $b_1^0 b_1^0$ decays, the $B^0 \rightarrow a_1^0 b_1^0$ channel is also dominated by the color-suppressed tree amplitude. But, different from the small $\text{BR}(B^0 \rightarrow \rho^0 \rho^0) \sim 0.3 \times 10^{-6}$ at leading order in the pQCD approach [42], the $B^0 \rightarrow a_1^0 b_1^0$ decay rate is about ten times larger, which is slightly larger than the $B^0 \rightarrow a_1^0 a_1^0$ one while almost one order less than the $B^0 \rightarrow b_1^0 b_1^0$ one in the pQCD approach [22]. It is interesting to note that this phenomenon, i.e., $\text{BR}(B^0 \rightarrow a_1^0 a_1^0) < \text{BR}(B^0 \rightarrow a_1^0 b_1^0) < \text{BR}(B^0 \rightarrow b_1^0 b_1^0)$, is attributed to the rather different QCD behaviors between the a_1 and b_1 mesons. Because of the extremely small Wilson coefficient a_2 or vanished decay constant

TABLE IV. The decay amplitudes (in unit of 10^{-3} GeV^3) of the $B^+ \rightarrow a_1^+ b_1^0$, $B^+ \rightarrow b_1^+ a_1^0$, and $B^0 \rightarrow a_1^0 b_1^0$ channels with three polarizations, where only the central values are quoted for clarification.

Channel Decay Amplitudes	$B^+ \rightarrow a_1^+ b_1^0$							
	\mathcal{A}_{fs}^T	\mathcal{A}_{fs}^P	\mathcal{A}_{nfs}^T	\mathcal{A}_{nfs}^P	\mathcal{A}_{nfa}^T	\mathcal{A}_{nfa}^P	\mathcal{A}_{fa}^T	\mathcal{A}_{fa}^P
L	$0.52 + i1.50$	$-0.13 + i0.05$	$2.29 - i0.97$	$0.05 + i0.12$	$0.37 - i0.47$	$0.04 + i0.02$	$0.01 - i0.01$	$-0.51 + i0.31$
N	$0.31 + i0.89$	$-0.09 + i0.04$	$-0.59 - i0.02$	$-0.01 - i0.05$	$0.02 - i0.02$	~ 0.00	~ 0.00	$0.08 + i0.27$
T	$-0.16 - i0.47$	$0.07 - i0.03$	$0.23 - i0.31$	$0.02 + i0.06$	~ 0.00	~ 0.00	$-0.25 - i2.69$	$0.46 + i0.49$
Channel Decay Amplitudes	$B^+ \rightarrow b_1^+ a_1^0$							
	\mathcal{A}_{fs}^T	\mathcal{A}_{fs}^P	\mathcal{A}_{nfs}^T	\mathcal{A}_{nfs}^P	\mathcal{A}_{nfa}^T	\mathcal{A}_{nfa}^P	\mathcal{A}_{fa}^T	\mathcal{A}_{fa}^P
L	$-0.03 - i0.09$	$-0.05 + i0.02$	$-0.62 + i0.49$	$-0.03 - i0.06$	$-0.36 + i0.47$	$-0.04 - i0.02$	$-0.04 + i0.02$	$0.50 - i0.32$
N	$-0.05 - i0.14$	$-0.03 + i0.01$	$0.66 - i0.33$	$0.01 + i0.05$	$-0.02 + i0.02$	~ 0.00	~ 0.00	$-0.09 - i0.28$
T	$0.08 + i0.23$	$0.01 - i0.01$	$-1.63 + i0.16$	$-0.06 - i0.07$	~ 0.00	~ 0.00	$0.25 + i2.69$	$-0.46 - i0.49$
Channel Decay Amplitudes	$B^0 \rightarrow a_1^0 b_1^0$							
	\mathcal{A}_{fs}^T	\mathcal{A}_{fs}^P	\mathcal{A}_{nfs}^T	\mathcal{A}_{nfs}^P	\mathcal{A}_{nfa}^T	\mathcal{A}_{nfa}^P	\mathcal{A}_{fa}^T	\mathcal{A}_{fa}^P
L	$0.02 + i0.06$	$-0.06 + i0.02$	$-2.13 + i0.50$	$-0.02 - i0.12$	$0.25 - i0.33$	$0.04 - i0.27$	$-0.11 - i0.07$	$-0.07 + i0.05$
N	$0.03 + i0.09$	$-0.04 + i0.02$	$-0.13 + i0.45$	$-0.03 - i0.01$	~ 0.00	~ 0.00	$0.06 + i0.18$	$0.15 - i0.05$
T	$-0.06 - i0.17$	$0.05 - i0.02$	$1.95 + i0.17$	$-0.00 + i0.13$	$-0.10 + i0.12$	$0.01 + i0.01$	~ 0.00	~ 0.00

$f_{b_1^0}$, then the $B^0 \rightarrow a_1^0 b_1^0$ decay amplitude will be determined by the nonfactorizable spectator and annihilation diagrams. But, due to the great cancelation of the annihilation contributions, as can be seen in Table IV, the nonfactorizable spectator amplitudes dominate the $B^0 \rightarrow a_1^0 b_1^0$ process. The underlying reason is that the destructive(constructive) interferences between the diagrams (c) and (d) in Fig. 1 exhibit for the $a_1(b_1)$ emission associated with the (anti)symmetric distribution amplitudes. Moreover, the $B^0 \rightarrow a_1^0 a_1^0, a_1^0 b_1^0$, and $b_1^0 b_1^0$ decay rates have also been studied in the QCDF approach, which presented a different pattern, i.e., $\text{BR}(B^0 \rightarrow a_1^0 a_1^0) \lesssim \text{BR}(B^0 \rightarrow a_1^0 b_1^0) \sim \text{BR}(B^0 \rightarrow b_1^0 b_1^0)$ [23]. These two different patterns among the branching ratios of the $B^0 \rightarrow a_1^0 a_1^0, a_1^0 b_1^0$, and $b_1^0 b_1^0$ decays in the pQCD and QCDF approaches would be tested by the near future experiments due to their sizable values.

- (c) It is of great interest to note that the $B^0 \rightarrow a_1^+ b_1^-$ and $B^0 \rightarrow b_1^+ a_1^-$ decays are dominated by the factorizable emission contributions and nonfactorizable spectator amplitudes correspondingly. Furthermore, for the former decay, with the decay constant $f_{a_1} = 0.238$ GeV, a bit larger than that of the ρ meson, meanwhile, with the form factor $V_0^{B \rightarrow b_1} > V_0^{B \rightarrow a_1}$, then the pattern $\text{BR}(B^0 \rightarrow a_1^+ b_1^-) > \text{BR}(B^0 \rightarrow a_1^+ a_1^-) > \text{BR}(B^0 \rightarrow \rho^+ \rho^-)$ would be observed naturally. But, for the latter mode, i.e., $B^0 \rightarrow b_1^+ a_1^-$, with b_1^+ emission,

because of the extremely suppressed decay constant $f_{b_1} \sim 0.0028$ GeV, the factorizable emission diagrams give nearly zero contributions, which means that the related decay amplitude might be induced by the nonfactorizable spectator and weak annihilation diagrams if it could be detected in the future. In fact, it is hopeful to be measured at LHCb and/or Belle-II experiments in the near future in light of its large decay rate about 10^{-6} in the pQCD approach. Indeed, because of the antisymmetric property of the b_1 meson twist-2 distribution amplitude, then the constructive interferences between the diagrams Fig. 1(c) and 1(d) lead to a dominant contribution to the $B^0 \rightarrow b_1^+ a_1^-$ mode, which can be seen from the values of the decay amplitudes shown in the Table V. As aforementioned, the nonfactorizable spectator and annihilation amplitudes in the QCDF approach have to be fitted by the precision measurements due to the endpoint singularities occurring in the collinear factorization theorem. Therefore, this channel could act as one of the important roles to identify the naive factorization breaking effects and distinguish the different factorization approaches simultaneously.

- (d) It should be stressed that the branching ratios shown in Table II are not the CP -averaged ones. Actually, the analyses of the $B^0 \rightarrow a_1^\pm b_1^\mp$ modes are complicated because the involved final states are not the CP eigenstates. Both B^0 and \bar{B}^0 mesons can decay into the same final states simultaneously, i.e., $B^0/\bar{B}^0 \rightarrow a_1^+ b_1^-$ and $B^0/\bar{B}^0 \rightarrow b_1^+ a_1^-$. Due to $B^0 - \bar{B}^0$ mixing, it

TABLE V. The decay amplitudes (in unit of 10^{-3} GeV^3) of the $B^0 \rightarrow a_1^+ b_1^-$, $B^0 \rightarrow b_1^+ a_1^-$, and $B^0 \rightarrow a_1^+ b_1^- + b_1^+ a_1^-$ channels with three polarizations, where only the central values are quoted for clarification.

Channel Decay Amplitudes	$B^0 \rightarrow a_1^+ b_1^-$							
	\mathcal{A}_{fs}^T	\mathcal{A}_{fs}^P	\mathcal{A}_{nfs}^T	\mathcal{A}_{nfs}^P	\mathcal{A}_{nfa}^T	\mathcal{A}_{nfa}^P	\mathcal{A}_{fa}^T	\mathcal{A}_{fa}^P
L	$3.84 + i11.03$	$-1.06 + i0.44$	$-0.32 - i0.13$	$0.01 - i0.03$	$0.84 - i1.16$	$-0.05 - i0.27$	$-0.02 - i0.04$	$-0.60 - i0.21$
N	$0.53 + i1.53$	$-0.15 + i0.06$	$-0.04 + i0.30$	$-0.03 + i0.00$	$0.02 + i0.00$	$-0.00 + i0.01$	~ 0.00	$0.29 + i0.21$
T	$1.00 + i2.87$	$-0.29 + i0.12$	$-0.06 + i0.65$	$-0.06 - i0.01$	$0.01 - i0.00$	$-0.01 - i0.01$	$-0.01 - i0.00$	$0.58 + i0.42$
Channel Decay Amplitudes	$B^0 \rightarrow b_1^+ a_1^-$							
	\mathcal{A}_{fs}^T	\mathcal{A}_{fs}^P	\mathcal{A}_{nfs}^T	\mathcal{A}_{nfs}^P	\mathcal{A}_{nfa}^T	\mathcal{A}_{nfa}^P	\mathcal{A}_{fa}^T	\mathcal{A}_{fa}^P
L	$-0.01 - i0.02$	~ 0.00	$-1.57 + i0.43$	$-0.06 - i0.10$	$-0.31 + i0.38$	$-0.03 - i0.30$	$-0.02 - i0.04$	$0.57 + i0.23$
N	~ 0.00	~ 0.00	$-0.06 - i0.05$	~ 0.00	$-0.02 - i0.00$	$-0.01 + i0.01$	~ 0.00	$-0.30 - i0.21$
T	$-0.00 - i0.01$	~ 0.00	$-0.10 - i0.08$	~ 0.00	$0.10 - i0.00$	$-0.12 - i0.22$	$0.01 + i0.00$	$-0.58 - i0.42$
Channel Decay Amplitudes	$B^0 \rightarrow a_1^+ b_1^- + b_1^+ a_1^-$							
	\mathcal{A}_{fs}^T	\mathcal{A}_{fs}^P	\mathcal{A}_{nfs}^T	\mathcal{A}_{nfs}^P	\mathcal{A}_{nfa}^T	\mathcal{A}_{nfa}^P	\mathcal{A}_{fa}^T	\mathcal{A}_{fa}^P
L	$3.83 + i11.01$	$-1.06 + i0.44$	$-1.89 + i0.30$	$-0.05 - i0.13$	$0.53 - i0.78$	$-0.08 - i0.57$	$-0.04 - i0.08$	$-0.03 + i0.02$
N	$0.53 + i1.53$	$-0.15 + i0.06$	$-0.10 + i0.25$	$-0.03 + i0.00$	~ 0.00	$-0.01 + i0.02$	~ 0.00	$-0.01 - i0.00$
T	$1.00 + i2.86$	$-0.29 + i0.12$	$-0.16 + i0.57$	$-0.06 - i0.01$	$0.11 - i0.00$	$-0.13 - i0.23$	~ 0.00	~ 0.00

is very difficult to distinguish B^0 from \bar{B}^0 . However, it is easy to identify the charged final states in the considered decays. We therefore sum up $B^0/\bar{B}^0 \rightarrow a_1^+ b_1^-$ as one channel and $B^0/\bar{B}^0 \rightarrow b_1^+ a_1^-$ as another. Meanwhile, following the convention adopted by the experimental measurements [17], we also define the CP -averaged channel as $B^0 \rightarrow a_1^+ b_1^- + b_1^+ a_1^-$. The numerical results for the branching ratios of these newly defined channels are collected in the Table III, specifically,

$$\text{BR}(B^0/\bar{B}^0 \rightarrow a_1^+ b_1^-) = 91.1_{-29.7}^{+36.9} \times 10^{-6}, \quad (17)$$

$$\text{BR}(B^0/\bar{B}^0 \rightarrow b_1^+ a_1^-) = 44.2_{-15.0}^{+20.1} \times 10^{-6}, \quad (18)$$

$$\text{BR}(B^0 \rightarrow a_1^+ b_1^- + b_1^+ a_1^-) = 85.8_{-25.6}^{+31.7} \times 10^{-6}; \quad (19)$$

Although the above-mentioned three channels are not discussed in the QCDF approach, the values predicted in the pQCD approach are such large that can be easily accessed at the current LHCb and forthcoming Belle-II experiments. The near future confirmations would help us to further explore the CP violation, the CKM unitary angle α , and so on in these interesting channels.

- (e) From the results presented in the Tables I–III, one can find that the predicted branching ratios suffer from large theoretical uncertainties from the not well-constrained meson wave functions in the considered decay modes. To date, most of the $B \rightarrow AA$ decays are not measured yet, except for the $B^0 \rightarrow a_1^+ a_1^-$ one observed by the BABAR Collaboration [43]. Therefore, we will define some ratios among the branching ratios predicted in the pQCD approach by adopting the $B^0 \rightarrow a_1^+ a_1^-$ decay rate as the normalized one. Therefore, the related ratios are provided for experimental detection in the (near) future as follows:

$$R_1 \equiv \frac{\text{BR}(B^0 \rightarrow a_1^+ b_1^- + b_1^+ a_1^-)}{\text{BR}(B^0 \rightarrow a_1^+ a_1^-)} \approx 1.57_{-0.08-0.32-0.05}^{+0.23+0.60+0.02}, \quad (20)$$

$$\begin{aligned} R_2 &\equiv \frac{\text{BR}(B^+ \rightarrow a_1^+ b_1^0)}{\text{BR}(B^0 \rightarrow a_1^+ a_1^-)} \approx 0.17_{-0.03-0.00-0.00}^{+0.03+0.00+0.01}, \\ R_3 &\equiv \frac{\text{BR}(B^+ \rightarrow b_1^+ a_1^0)}{\text{BR}(B^0 \rightarrow a_1^+ a_1^-)} \approx 0.08_{-0.02-0.01-0.00}^{+0.02+0.01+0.00}, \end{aligned} \quad (21)$$

$$R_4 \equiv \frac{\text{BR}(B^0 \rightarrow a_1^0 b_1^0)}{\text{BR}(B^0 \rightarrow a_1^+ a_1^-)} \approx 0.06_{-0.01-0.00-0.00}^{+0.01+0.00+0.00}. \quad (22)$$

Moreover, we also define several ratios among the branching ratios themselves of the $B \rightarrow a_1 b_1$ decays in this work as follows:

$$\begin{aligned} R_5 &\equiv \frac{\text{BR}(B^0 \rightarrow b_1^+ a_1^-)}{\text{BR}(B^0 \rightarrow a_1^+ b_1^-)} \approx 0.05_{-0.01-0.02-0.00}^{+0.01+0.02+0.00}, \\ R_6 &\equiv \frac{\text{BR}(B^+ \rightarrow b_1^+ a_1^0)}{\text{BR}(B^+ \rightarrow a_1^+ b_1^0)} \approx 0.46_{-0.02-0.02-0.00}^{+0.04+0.07+0.02}, \end{aligned} \quad (23)$$

$$\begin{aligned} R_7 &\equiv \frac{\text{BR}(B^0 \rightarrow a_1^0 b_1^0)}{\text{BR}(B^+ \rightarrow a_1^+ b_1^0)} \approx 0.37_{-0.01-0.03-0.01}^{+0.01+0.00+0.00}, \\ R_8 &\equiv \frac{\text{BR}(B^0 \rightarrow a_1^0 b_1^0)}{\text{BR}(B^+ \rightarrow b_1^+ a_1^0)} \approx 0.79_{-0.07-0.15-0.03}^{+0.06+0.03+0.00}, \end{aligned} \quad (24)$$

$$R_9 \equiv \frac{\text{BR}(B^+ \rightarrow a_1^+ b_1^0)}{\text{BR}(B^0 \rightarrow a_1^+ b_1^- + b_1^+ a_1^-)} \approx 0.11_{-0.00-0.02-0.01}^{+0.01+0.03+0.00}, \quad (25)$$

$$R_{10} \equiv \frac{\text{BR}(B^+ \rightarrow b_1^+ a_1^0)}{\text{BR}(B^0 \rightarrow a_1^+ b_1^- + b_1^+ a_1^-)} \approx 0.05_{-0.01-0.01-0.00}^{+0.01+0.01+0.00}, \quad (26)$$

$$R_{11} \equiv \frac{\text{BR}(B^0 \rightarrow a_1^0 b_1^0)}{\text{BR}(B^0 \rightarrow a_1^+ b_1^- + b_1^+ a_1^-)} \approx 0.04_{-0.00-0.01-0.00}^{+0.00+0.01+0.00}. \quad (27)$$

In the above ratios, the large uncertainties induced by the nonperturbative inputs could be canceled to a great extent, which are expected to be measured in the future.

II. POLARIZATION FRACTIONS

We now turn to the analyses of the polarization fractions. Usually, the observables such as polarization fractions are presented by employing the transversity amplitudes. Then, based on the Eqs. (7) and (8), the longitudinal polarization fraction can be defined as

$$f_L \equiv \frac{|\mathcal{A}_L|^2}{|\mathcal{A}_L|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2} = |\mathcal{A}_L|^2; \quad (28)$$

The other two polarization fractions f_\parallel and f_\perp can be easily obtained with similar definition to that shown in Eq. (28). One often use another convention f_T , relative to f_L , to denote the transverse polarization fraction as,

$$f_T \equiv f_\parallel + f_\perp = 1 - f_L; \quad (29)$$

The polarization fractions predicted in both of the pQCD and QCDF approaches have been collected in the Tables I–II. The longitudinal and transverse polarization fractions can be read as follows:

$$\left. \begin{aligned} f_L(B^+ \rightarrow a_1^+ b_1^0) &= 0.62_{-0.05}^{+0.01}, & f_T(B^+ \rightarrow a_1^+ b_1^0) &= 0.38_{-0.04}^{+0.08}, \\ f_L(B^+ \rightarrow b_1^+ a_1^0) &= 0.28_{-0.04}^{+0.02}, & f_T(B^+ \rightarrow b_1^+ a_1^0) &= 0.72_{-0.09}^{+0.06}, \\ f_L(B^0 \rightarrow a_1^0 b_1^0) &= 0.63_{-0.10}^{+0.06}, & f_T(B^0 \rightarrow a_1^0 b_1^0) &= 0.37_{-0.07}^{+0.10}, \\ f_L(B^0 \rightarrow a_1^+ b_1^-) &= 0.94_{-0.03}^{+0.00}, & f_T(B^0 \rightarrow a_1^+ b_1^-) &= 0.07_{-0.03}^{+0.00}, \\ f_L(B^0 \rightarrow b_1^+ a_1^-) &= 0.96_{-0.03}^{+0.01}, & f_T(B^0 \rightarrow b_1^+ a_1^-) &= 0.04_{-0.01}^{+0.03}, \end{aligned} \right\} \quad (\text{In pQCD}) \quad (30)$$

$$\left. \begin{aligned} f_L(B^+ \rightarrow a_1^+ b_1^0) &= 0.92_{-0.24}^{+0.02}, \\ f_L(B^+ \rightarrow b_1^+ a_1^0) &= 0.73_{-0.82}^{+0.12}, \\ f_L(B^0 \rightarrow a_1^0 b_1^0) &= 0.98_{-0.31}^{+0.01}, \\ f_L(B^0 \rightarrow a_1^+ b_1^-) &= 0.90_{-0.05}^{+0.02}, \\ f_L(B^0 \rightarrow b_1^+ a_1^-) &= 0.98_{-0.80}^{+0.00}, \end{aligned} \right\} \quad (\text{In QCDF}) \quad (31)$$

and

$$\left. \begin{aligned} f_L(B^0/\bar{B}^0 \rightarrow a_1^+ b_1^-) &= 0.91_{-0.02}^{+0.05}, & f_T(B^0/\bar{B}^0 \rightarrow a_1^+ b_1^-) &= 0.09_{-0.04}^{+0.02}, \\ f_L(B^0/\bar{B}^0 \rightarrow b_1^+ a_1^-) &= 0.81_{-0.06}^{+0.07}, & f_T(B^0/\bar{B}^0 \rightarrow b_1^+ a_1^-) &= 0.19_{-0.06}^{+0.04}, \\ f_L(B^0 \rightarrow a_1^+ b_1^- + b_1^+ a_1^-) &= 0.91_{-0.01}^{+0.03}, & f_T(B^0 \rightarrow a_1^+ b_1^- + b_1^+ a_1^-) &= 0.09_{-0.01}^{+0.01}, \end{aligned} \right\} \quad (\text{In pQCD}) \quad (32)$$

n which various errors have been added in quadrature. These predictions in both pQCD and QCDF approaches need tests by the related experiments in the future. In light of these numerical results, generally speaking, the considered $B \rightarrow a_1 b_1$ decays are dominated by the longitudinal polarization contributions in the pQCD approach, except for the $B^+ \rightarrow b_1^+ a_1^0$ mode with $f_L \sim (24\% - 30\%)$. It is very interesting to note that the longitudinal polarization fraction f_L of the $B^+ \rightarrow b_1^+ a_1^0$ decay was estimated in the QCDF approach with quite large uncertainties, which can possibly lead to a domination of the transverse polarization amplitudes.

According to the decay amplitudes from every topology of the $B \rightarrow a_1 b_1$ decays as shown in the Tables IV–V, the clarifications on those polarization fractions in the pQCD approach are in more detail as follows:

(a) For the $B^+ \rightarrow a_1^+ b_1^0$ and $B^+ \rightarrow b_1^+ a_1^0$ decays, different from the $B^+ \rightarrow a_1^+ a_1^0$ and $B^+ \rightarrow b_1^+ b_1^0$ ones, the largely nonvanishing transverse amplitudes contribute significantly from the factorizable annihilation topology. Meanwhile, at the longitudinal polarization, due to the antisymmetric leading twist distribution amplitude of the emitted b_1 meson, the nonfactorizable spectator diagrams as shown in Fig. 1(c) and 1(d) can interfere with each other constructively accompanied with a large and positive Wilson coefficient C_2 for the $B^+ \rightarrow a_1^+ b_1^0$ mode while with a much smaller and negative Wilson coefficient C_1 for the $B^+ \rightarrow b_1^+ a_1^0$ one. Consequently, the further constructive interferences between the factorizable emission and nonfactorizable spectator amplitudes result in the slightly dominant longitudinal contribution to the $B^+ \rightarrow a_1^+ b_1^0$ decay.

(b) As we know, the $B^0 \rightarrow \rho^0 \rho^0$ mode has a small longitudinal polarization fraction in the pQCD approach at leading order [42,44]. Phenomenologically, this is attributed to the significant cancellation at the longitudinal polarization between the factorizable emission and nonfactorizable spectator decay amplitudes, which result in the well-known color-suppressed tree amplitude C , quite small in magnitude. Because the behavior of a_1 meson is similar to that of the ρ meson, so the polarization fractions of $B^0 \rightarrow a_1^0 a_1^0$ decay [22] is also analogous to those of the $B^0 \rightarrow \rho^0 \rho^0$ one. In other words, the large transverse decay amplitudes still exist. While, for the $B^0 \rightarrow a_1^0 b_1^0$ channel, the aforementioned enhancement of the nonfactorizable spectator amplitudes associated with the b_1 emission governs the longitudinal helicity amplitude and finally results in the different polarization fractions to those of the $B^0 \rightarrow \rho^0 \rho^0$ and $a_1^0 a_1^0$ decays. Therefore, one can observe an interesting relation of the longitudinal polarization fractions in the pQCD approach at leading order, that is, $f_L(B^0 \rightarrow a_1^0 a_1^0) < f_L(B^0 \rightarrow a_1^0 b_1^0) < f_L(B^0 \rightarrow b_1^0 b_1^0)$, whose confirmation would provide more information to explore the least understood quantity [45], namely, the color-suppressed tree amplitude C , in the B physics.

(c) As shown in the Table V, both of the $B^+ \rightarrow a_1^+ b_1^-$ and $B^+ \rightarrow b_1^+ a_1^-$ decays are highly dominated by the longitudinal polarization amplitudes but with different sources. The former decay has a large color-allowed tree amplitude mainly arising from the factorizable emission diagrams with Wilson coefficient a_1 (not to

be confused with the abbreviation a_1 for the $a_1(1260)$ state). However, the latter one has a bit smaller tree amplitude induced by the nonfactorizable spectator diagrams with Wilson coefficient C_1 . Therefore, the $B^0 \rightarrow a_1^+ b_1^- + b_1^+ a_1^-$ decay with CP eigenstate is certainly dominated by the longitudinal polarization amplitude, which gives a large fraction around 90%.

III. NAIVE FACTORIZATION BREAKING EFFECTS: NONFACTORIZABLE SPECTATOR AND/OR WEAK ANNIHILATION CONTRIBUTIONS

Now, we will discuss the naive factorization breaking effects, that is, the nonfactorizable spectator and/or weak annihilation diagrams contribute to the above mentioned observables in the $B \rightarrow a_1 b_1$ decays.

It is well known that the naive factorization hypothesis has been successfully applied into various decay modes of heavy mesons and, particularly, the obtained branching ratios for the color-allowed processes governed by the factorizable contributions agree well with the data generally. However, for the modes belonging to the color-suppressed category [46] such as $B \rightarrow J/\psi K^{(*)}$ (e.g., see [47–50]), $B^0 \rightarrow \pi^0 \pi^0$ (e.g., see [45, 51–54]), etc., the decay rates estimated in the naive factorization are always too small to be compared with the measurements due to the nearly vanishing Wilson coefficient $a_2 \sim 0$. Then the nonfactorizable spectator even weak annihilation amplitudes should be included to clarify the experimental measurements, although they are usually considered as higher order (or power) corrections contributing less in the naive factorization.

In order to simply investigate the naive factorization breaking effects, we here just explore the branching ratios and longitudinal polarization fractions in the considered modes when the nonfactorizable spectator and/or annihilation contributions are turned off. For the sake of simplicity, only the central values of the related observables are quoted here for clarifications.

- (1) When we neglect the contributions from the weak annihilation diagrams, the decay rates and polarization fractions will become

$$\begin{aligned} \text{BR}(B^+ \rightarrow a_1^+ b_1^0) &\approx 5.3 \times 10^{-6}, \\ f_L(B^+ \rightarrow a_1^+ b_1^0) &\approx 0.81; \end{aligned} \quad (33)$$

$$\begin{aligned} \text{BR}(B^+ \rightarrow b_1^+ a_1^0) &\approx 1.5 \times 10^{-6}, \\ f_L(B^+ \rightarrow b_1^+ a_1^0) &\approx 0.21; \end{aligned} \quad (34)$$

$$\begin{aligned} \text{BR}(B^0 \rightarrow a_1^0 b_1^0) &\approx 3.7 \times 10^{-6}, \\ f_L(B^0 \rightarrow a_1^0 b_1^0) &\approx 0.71; \end{aligned} \quad (35)$$

$$\begin{aligned} \text{BR}(B^0 \rightarrow a_1^+ b_1^-) &\approx 75.5 \times 10^{-6}, \\ f_L(B^0 \rightarrow a_1^+ b_1^-) &\approx 0.91; \end{aligned} \quad (36)$$

$$\begin{aligned} \text{BR}(B^0 \rightarrow b_1^+ a_1^-) &\approx 1.4 \times 10^{-6}, \\ f_L(B^0 \rightarrow b_1^+ a_1^-) &\approx 0.99. \end{aligned} \quad (37)$$

One can observe that the weak annihilation amplitudes contribute constructively to the decay rates of the $B^+ \rightarrow a_1^+ b_1^0$, $B^+ \rightarrow b_1^+ a_1^0$, and $B^0 \rightarrow b_1^+ a_1^-$ modes around 41%, 64%, and 58%, respectively, however, destructively to those of the $B^0 \rightarrow a_1^0 b_1^0$ and $B^0 \rightarrow a_1^+ b_1^-$ ones about 12% and 3%, respectively. Moreover, the weak annihilation contributions, in particular, the large factorizable annihilation amplitudes, decrease the longitudinal polarization fraction nearly 31% of the $B^+ \rightarrow a_1^+ b_1^0$ decay while increase that about 25% of the $B^+ \rightarrow b_1^+ a_1^0$ one. And an enhancement to the transverse polarization fraction of the $B^0 \rightarrow a_1^0 b_1^0$ channel around 12% can be easily seen because of a bit large nonfactorizable annihilation contributions. The polarization fractions only vary with 0.03 for the $B^0 \rightarrow a_1^+ b_1^-$ and $B^0 \rightarrow b_1^+ a_1^-$ decays with neglecting the annihilation amplitudes since these two modes are governed by the factorizable emission and nonfactorizable spectator diagrams correspondingly. Nevertheless, one can still observe the significant naive factorization breaking effects in the $B^+ \rightarrow a_1^+ b_1^0$, $B^+ \rightarrow b_1^+ a_1^0$, and $B^0 \rightarrow b_1^+ a_1^-$ decays induced by the annihilation diagrams, though which usually are regarded as being negligible due to its power suppression.

- (2) Without the nonfactorizable spectator and weak annihilation contributions, then the branching ratios and the polarization fractions will become

$$\begin{aligned} \text{BR}(B^+ \rightarrow a_1^+ b_1^0) &\approx 2.3 \times 10^{-6}, \\ f_L(B^+ \rightarrow a_1^+ b_1^0) &\approx 0.58; \end{aligned} \quad (38)$$

$$\begin{aligned} \text{BR}(B^+ \rightarrow b_1^+ a_1^0) &\approx 4.2 \times 10^{-8}, \\ f_L(B^+ \rightarrow b_1^+ a_1^0) &\approx 0.16; \end{aligned} \quad (39)$$

$$\begin{aligned} \text{BR}(B^0 \rightarrow a_1^0 b_1^0) &\approx 2.2 \times 10^{-8}, \\ f_L(B^0 \rightarrow a_1^0 b_1^0) &\approx 0.16; \end{aligned} \quad (40)$$

$$\begin{aligned} \text{BR}(B^0 \rightarrow a_1^+ b_1^-) &\approx 76.4 \times 10^{-6}, \\ f_L(B^0 \rightarrow a_1^+ b_1^-) &\approx 0.94; \end{aligned} \quad (41)$$

$$\begin{aligned} \text{BR}(B^0 \rightarrow b_1^+ a_1^-) &\approx 2.4 \times 10^{-10}, \\ f_L(B^0 \rightarrow b_1^+ a_1^-) &\approx 0.80. \end{aligned} \quad (42)$$

Relative to the naive factorization, when the so-called factorization breaking terms are removed, then the considered $B \rightarrow a_1 b_1$ decays show different phenomena in light of the branching ratios: the numerical results of $\text{BR}(B^+ \rightarrow b_1^+ a_1^0)$, $\text{BR}(B^0 \rightarrow a_1^0 b_1^0)$, and $\text{BR}(B^0 \rightarrow b_1^+ a_1^-)$ change from 10^{-6} to 10^{-8} ,

even 10^{-10} , which indicate evidently that these modes are governed by the naive factorization breaking effects. Therefore, it is proposed that these processes could be detected by the relevant experiments in the (near) future to verify those phenomenologies induced by the naive factorization breaking effects. Of course, the $B^0 \rightarrow a_1^+ b_1^-$ mode is also an ideal candidate with a much large decay rate to test the naive factorization due to its extreme dominance of the factorizable emission diagrams.

Finally, frankly speaking, the theoretical predictions in both of the pQCD and QCDF approaches still have large uncertainties arising from various sources. In terms of the pQCD approach, the theoretical errors mainly come from the not well-constrained input parameters involved in the hadron distribution amplitudes such as the shape parameter ω_B of heavy B meson and the Gegenbauer moments $a_i^{\parallel,\perp}$ of light axial-vector a_1 and b_1 states. Therefore, the great efforts from nonperturbative QCD aspects such as QCD sum rule and/or Lattice QCD methods, as well as from the experimental aspects, are eagerly desired to effectively reduce the errors of these important inputs. Certainly, any progress of the hadron dynamics would improve the precision of the predictions more or less in the pQCD approach.

In summary, because of the dramatically small or vanishing decay constant f_{b_1} of the light axial-vector b_1 state, the naive factorization would provide an extremely small or nearly zero branching ratios, for example, the $B^0 \rightarrow b_1^+ a_1^-$ mode. However, as indicated from data, many processes may have large branching ratios since the large naive factorization breaking effects such as nonfactorizable spectator and/or annihilation contributions could exist. Therefore, we should go beyond the naive factorization to explore those possibly large factorization breaking effects.

We investigated the branching ratios and polarization fractions of the charmless hadronic $B \rightarrow a_1 b_1$ decays by

employing the pQCD approach based on the k_T factorization theorem, with which we perturbatively calculated the factorizable emission, nonfactorizable spectator, and weak annihilation diagrams. The predicted branching ratios as large as 10^{-5} – 10^{-6} in the pQCD approach are in general consistency with those estimated in the QCDF approach within still large theoretical errors. Due to the antisymmetric behavior of the b_1 meson leading twist distribution amplitude, the nonfactorizable spectator contributions with b_1 emission can change from destruction into construction, which provide a large naive factorization breaking term and further enhance the decay amplitudes significantly. The predicted polarization fractions in the pQCD approach are also consistent with those given in the QCDF approach.

The detailed analyses show that the pQCD predictions of the considered $B \rightarrow a_1 b_1$ decays could provide more evidences to test the SM, explore the helicity structure with polarizations, constrain the parameters from the hadron wave functions, and so forth. The large $B^0 \rightarrow a_1^0 a_1^0, a_1^0 b_1^0$, and $b_1^0 b_1^0$ decay rates would provide an opportunity to make further constraints to the CKM unitary angles and understandings of the decay mechanism of the color-suppressed modes. Certainly, it is worth stressing that we firstly consider the short-distance contributions at leading order in the evaluations of the hadronic matrix element of the $B \rightarrow a_1 b_1$ decays. The effects of final state interaction might play an important role in these considered processes as they should. However, it is beyond the scope of the present work and will be studied elsewhere.

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