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Positivity bound on the imaginary part of the right-chiral tensor coupling g_R in polarized top quark decay

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We derive a positivity bound on the right-chiral tensor coupling $\mathrm{Im}g_R$ in polarized top quark decay by analyzing the angular decay distribution of the three-body polarized top quark decay $t(\uparrow) \to b + \ell^+ + \nu_\ell$ in next-to-leading order QCD. We obtain the bound $-0.0420 \le \mathrm{Im}g_R \le 0.0420$.

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The general matrix element for the decay $t \to b + W^+$ including the leading order (LO) standard model (SM) contribution is usually written as (see, e.g., Ref. [1])

$$M_{tbW^+} = -\frac{g_W}{\sqrt{2}}\varepsilon^{\mu *}\bar{u}_b\bigg[(V^*_{tb} + f_L)\gamma_\mu P_L + f_R\gamma_\mu P_R + \frac{i\sigma_{\mu\nu}q^\nu}{m_W}(g_L P_L + g_R P_R)\bigg], \quad (1)$$

where $P_{L,R} = (1 \mp \gamma_5)/2$. The SM structure of the tbW^+ vertex is obtained by dropping all terms except for the contribution proportional to $V^*_{tb} \sim 1$.

The angular decay distribution for polarized top quark decay $t(\uparrow) \to b + \ell^+ + \nu_\ell$ in the top quark rest frame is given by

$$\frac{d\Gamma}{d\cos\theta d\phi} = A + BP_t\cos\theta_P + CP_t\sin\theta_P\cos\phi + DP_t\sin\theta_P\sin\phi \quad (2)$$

which corresponds to the decay distribution introduced in Refs. [2,3] augmented by the last T-odd term. At LO of the SM one has $A = B$ and $C = D = 0$. The second azimuthal term proportional to D corresponds to a T-odd contribution. This can be seen by rewriting the angular factor as a triple product according to

$$\sin\theta_P\sin\phi = \hat{p}_\ell\cdot(\hat{p}_b\times\hat{s}_t) \quad (3)$$

where (see Fig. 1)

$$\hat{p}_\ell = (0,0,1) \qquad \hat{p}_b = (\sin\theta_b, 0, \cos\theta_b)$$
$$\hat{s}_t = (\sin\theta_P\cos\phi, \cos\theta_P\sin\phi, \cos\theta_P). \quad (4)$$

Let us repeat the arguments presented in Ref. [3] that led us to the conclusion that the equality $A = B$ already implies the vanishing of the T-even azimuthal contribution C confirming the LO result $C = 0$. Consider Eq. (2) for $\phi = 0$, $P_t = 1$, and factor out the unpolarized rate term A. Assume first that C/A is positive and expand the trigonometric functions around $\theta_P = \pi$ for positive values of δ, i.e., $\cos(\pi+\delta) \approx -1 + \frac{1}{2}\delta^2$ and $\sin(\pi+\delta) \approx -\delta$. The differential rate is then proportional to

$$1 + \cos\theta_P + \frac{C}{A}\sin\theta_P \approx \frac{1}{2}\delta^2 - \frac{C}{A}\delta = \frac{1}{2}\delta\left(\delta - \frac{2C}{A}\right). \quad (5)$$

The differential rate can be seen to be negative for δ in the interval $[0, 2C/A]$. The interval can be shrunk to zero by setting $2C/A = 0$, i.e., by setting $C = 0$. If C/A is assumed to be negative, one has to expand the trigonometric functions around $\theta_P = \pi$ for negative values of δ, leading to the interval $[2C/A, 0]$.

The same chain of arguments, but this time with $\phi = \pi/2$, leads to the LO positivity constraint for the T-odd structure, $D = 0$.

At next-to-leading order (NLO) of QCD one no longer has $A = B$. However, the relative difference $(A - B)/A$ is quite small which, as we will see, in turn implies useful positivity constraints for the T-odd structure coefficient D. As concerns the T-even azimuthal structure, the NLO corrections to the LO result $C = 0$ are so small

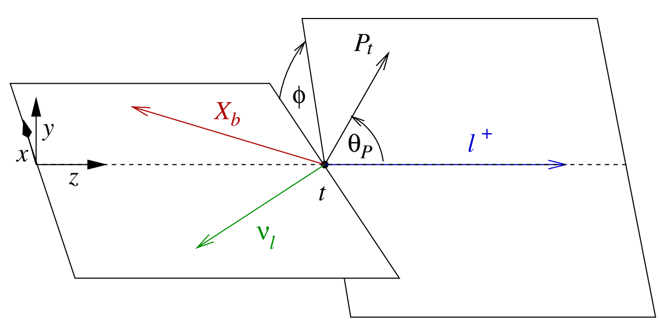

FIG. 1. Definitions of polar and azimuthal angles for the process $t \to b + W^+(\to \ell^+ + \nu_\ell)$

that the positivity of the differential rate is not endangered [3].

We now derive the NLO positivity constraint for the T-odd structure coefficient D. We shall work in the approximation $m_b = 0$, which implies that the coupling terms f_R and g_L in Eq. (1) are zero. The NLO forms of the integrated $m_b = 0$ rates are listed in Refs. [4–6]. They read

$$\frac{d\Gamma}{d\cos\theta d\phi} = (A^{(0)} + A^{(1)})\Bigg(1 + \frac{A^{(0)} + B^{(1)}}{A^{(0)} + A^{(1)}} P_t \cos\theta_P + \frac{C}{A^{(0)} + A^{(1)}} P_t \sin\theta_P \cos\phi + \frac{D}{A^{(0)} + A^{(1)}} P_t \sin\theta_P \sin\phi\Bigg), \tag{6}$$

where

$$\begin{aligned} \frac{A^{(1)}}{A^{(0)}} &= \frac{\alpha_s C_F}{4\pi} \frac{1}{(1-x^2)^2(1+2x^2)} \Big((1-x^2)(5+9x^2-6x^4) - 2(1-x^2)^2(1+2x^2)\left[\frac{2\pi^2}{3} + 4\ln(1-x^2)\ln x + 4\mathrm{Li}_2(x^2)\right] \\ &\quad - 8x^2(1+x^2)(1-2x^2)\ln x - 2(1-x^2)^2(5+4x^2)\ln(1-x^2)\Big) \\ &= -0.0846955, \end{aligned} \tag{7}$$

and

$$\begin{aligned} \frac{B^{(1)}}{A^{(0)}} &= \frac{\alpha_s C_F}{4\pi} \frac{1}{(1-x^2)^2(1+2x^2)} \Big(-(1-x^2)(15-x^2+2x^4) + (1-x^2)(1+x^2+4x^4)\frac{2\pi^2}{3} - 2(1-x^2)^2(5+4x^2)\ln(1-x^2) \\ &\quad - 16x^2(2+x^2-x^4)\ln x - 16(1-x^2)(2+2x^2-x^4)\ln x \ln(1-x^2) - 4(1-x^2)(5+5x^2-4x^4)\mathrm{Li}_2(x^2)\Big) \\ &= -0.0863048, \end{aligned} \tag{8}$$

where $x = m_W/m_t$. Here we have also listed the numerical values for the two ratios using $\alpha_s(m_t) = 0.1062$, $m_t = 173.21$ GeV, and $m_W = 80.385$ GeV [7]. The ratio expressions $A^{(1)}/A^{(0)}$ and $B^{(1)}/A^{(0)}$ have been rechecked in Ref. [3]. Reference [3] also contains results on the azimuthal rate coefficient C. This coefficient, however, will be of no concern in the derivation of the positivity bounds for the T-odd rate coefficient D. In fact, setting $\phi = \pi/2$ will eliminate the contribution of C. This will be our choice.

Next we must determine the contribution of the imaginary part of the coupling factor g_R to the T-odd azimuthal rate term D. The relevant contribution arises from the interference of the coupling factor g_R with the Born term contribution. It is for this reason that there is no $\mathrm{Im} f_L$ contribution to the T-odd rate coefficient D since the coupling term is self-interfering. After some algebra one finds

$$\frac{D}{A^{(0)}} = \frac{3\pi(1-x^2)}{4(1+2x^2)} \mathrm{Im} g_R \tag{9}$$

where we have only kept the contribution linear in $\mathrm{Im} g_R$. Further, we assume $\mathrm{Im} g_R$ to be positive and set $P_t = 1$. We expand around $\theta_P = \pi$ for small positive values of δ which gives $\cos(\pi+\delta) = -1 + \frac{1}{2}\delta^2$ and $\sin(\pi+\delta) = -\delta$ to obtain

$$\begin{aligned} W(\theta_P) &\sim 1 + (1-\Delta)\cos\theta_P + \frac{D}{A^{(0)} + A^{(1)}}\sin\theta_P \\ &= \Delta - \frac{D\delta}{A^{(0)} + A^{(1)}} + \frac{1-\Delta}{2}\delta^2, \end{aligned} \tag{10}$$

where we have defined the small quantity

$$\Delta = \frac{A^{\mathrm{NLO}} - B^{\mathrm{NLO}}}{A^{\mathrm{NLO}}} = \frac{A^{(1)} - B^{(1)}}{A^{(0)} + A^{(1)}} = \frac{A^{(1)} - B^{(1)}}{A^{(0)}(1 + A^{(1)}/A^{(0)})}, \tag{11}$$

keeping in mind that $A^{(0)} = B^{(0)}$. Numerically one has $\Delta = 0.001758$ where the small difference to the numerical results in Ref. [3] results from having used updated values $m_W = 80.385$ GeV and $m_t = 173.21$ GeV [7].

The rate proportional to $W(\theta_P)$ in Eq. (10) becomes negative if the contribution proportional to $\mathrm{Im} g_R$ becomes larger than the remaining terms. However, this is no longer the case if the quadratic Eq. (10) in δ has no real-valued zeros. The pertinent condition for the discriminant reads

$$\frac{3\pi(1-x^2)}{4(1+2x^2)} |\mathrm{Im} g_R| \le \sqrt{2\Delta(1-\Delta)}\left(1 + \frac{A^{(1)}}{A^{(0)}}\right). \tag{12}$$

Numerically one obtains

$$\mathrm{Im} g_R \in [-0.0420, 0.0420]. \tag{13}$$

The angle δ_0 for which the quadratic form (10) becomes degenerate can be calculated to be $\delta_0 = \sqrt{2\Delta/(1-\Delta)} = \pm 0.0593$. The corrections to the expansion of $\cos(\pi \pm \delta)$ and $\sin(\pi \pm \delta)$ are of the order $O(\delta^2 = 0.00352)$ and, therefore, quite small.

In Ref. [8] we have calculated the SM absorptive electroweak contributions to $\mathrm{Im}g_R$ with the result $\mathrm{Im}g_R = -2.175 \times 10^{-3}$ (see also Refs. [9,10]). This value is easily accommodated in the positivity bound (13).

The ATLAS Collaboration has recently published the bound [11]

$$\mathrm{Im}g_R \in [-0.18, 0.06] \tag{14}$$

based on the analysis of sequential polarized two-body top quark decays $t(\uparrow) \to b + W^+(\to \ell^+ + \nu)$. A somewhat tighter bound has been published in Ref. [12] using also sequential polarized two-body top quark decays. The bound reads

$$\mathrm{Im}\left(\frac{g_R}{f_L}\right) \in [-0.07, 0.06] \tag{15}$$

which we translate into a bound on $\mathrm{Im}g_R$ by substituting the LO result $f_L = 1$ in Eq. (15). Both bounds are not far away from the positivity bound on $\mathrm{Im}g_R$ derived in this paper. Using the same chain of arguments one can establish the corresponding bound for $m_b \neq 0$. Using NLO $m_b \neq 0$ results from Ref. [6] on the unpolarized and polarized rate functions $A^{(1)}$ and $B^{(1)}$ we find that for $m_b = 4.8$ GeV the bound is marginally strengthened to $\mathrm{Im}g_R \in [-0.0418, 0.0418]$. The condition for obtaining this bound reads

$$\frac{3\pi(1-x^2+y^2)\sqrt{\lambda}}{4(\lambda+3x^2(1-x^2+y^2))}|\mathrm{Im}g_R| \leq \sqrt{2\Delta(1-\Delta)}\left(1+\frac{A^{(1)}}{A^{(0)}}\right), \tag{16}$$

where $y = m_b/m_t$ and $\lambda = \lambda(1, x^2, y^2) = 1 + x^4 + y^4 - 2x^2 - 2y^2 - 2x^2y^2$ is the Källén function. $A^{(0)}$, $A^{(1)}$ and the small quantity Δ are evaluated for $m_b \neq 0$.

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