PHYSICAL REVIEW D **96,** 106018 (2017)

Holographic heavy baryons in the Witten-Sakai-Sugimoto model with the D0-D4 background

Si-wen Li*

Department of Physics, Center for Field Theory and Particle Physics, Fudan University, Shanghai 200433, China

(Received 2 August 2017; published 28 November 2017)

We extend the holographic analysis of the light-baryon spectrum by Cai *et al.* [Phys. Rev. D **90**, 106001 (2014)] to the case involving the heavy flavors. With the construction of the Witten-Sakai-Sugimoto model in the D0-D4 background, we use the mechanism proposed by Liu and Zahed by including two light-flavor and one heavy-flavor brane, to describe the heavy-light baryons as heavy mesons bound to a flavor instanton. The background geometry of this model corresponds to an excited state in the dual field theory with a nonzero glue condensate $\langle \mathrm{Tr}\mathcal{F} \wedge \mathcal{F} \rangle = 8\pi^2 N_c \tilde{\kappa}$ (or equivalently a nonzero θ angle), which is proportional to the number density of the D0-brane charge. In the strong-coupling limit, this model shows that the heavy meson is always bound in the form of the zero mode of the flavor instanton in the fundamental representation. We systematically study the quantization of the effective Lagrangian of heavy-light baryons by employing the soliton picture, and derive the mass spectrum of heavy-light baryons in the situation with single- and double-heavy baryons. We find that the difference in the mass spectrum becomes smaller if the density of the D0-brane charge increases, and the stability constraint of the heavy-light baryons is $1 < b < 3$. This indicates that a baryon cannot stably exist for a sufficiently large D0 charge density, which is in agreement with the conclusions in the previous study of this model.

DOI: 10.1103/PhysRevD.96.106018

I. INTRODUCTION

The heavy quarks (c, b, t) are characterized by the heavy-quark symmetry in QCD [1], while the light quarks (u, d, s) are dominated by the spontaneous breaking of chiral symmetry. As measured in Refs. [2,3], the chiral doubling in heavy-light mesons [4–7] combines these symmetries. Recently, a flurry of experiments have reported that many multiquark exotics are incommensurate with quarkonia [8–14]. Accordingly, some new phenomena involving heavy-light multiquark states are strongly supported by these experimental results. On the other hand, the spontaneous parity violation in QCD has also been discussed in the context of the RHIC [15–19]. It is well known that P or CP violation is usually described by a nonzero θ angle in the action of such theories. So when deconfinement happens in QCD, a metastable state with a nonzero θ angle may be produced in the hot and dense situation in the RHIC; then, a bubble that forms with odd P or CP parity would soon decay into the true vacuum. For example, the chiral magnetic effect was proposed as a test of such phenomena [20–22]. Thus, it is theoretically interesting to study the θ dependence of some observables in QCD or in the gauge theory, e.g., the θ dependence of the spectrum of the glueball [23] or the phase diagram [24,25], and the θ dependence in the large-N_c limit [26] (one can also review the details of the θ dependence in Ref. [27]).

The holographic construction by the gauge/gravity duality offers a framework to investigate the aspects of the strongly coupled quantum field theory [28,29] since QCD at the low-energy scale is nonperturbative. Using Witten's D4-brane construction [30], a concrete model was proposed by Sakai and Sugimoto [31,32] which contains almost all of the necessary ingredients for QCD, e.g., baryons [33,34], quark matter, chiral/deconfinement phase transitions [35–38], and the glueball spectrum and interaction [39–43]. Specifically, flavors are introduced into this model by a stack of N_f pairs of suitable D8-/$\overline{\mathrm{D8}}$-branes as probes embedded in the N_c D4-brane background. The chiral quarks are in the fundamental representation of the color and flavor groups which come from the massless spectrum of the open strings stretched between the color and flavor (D8/$\overline{\mathrm{D8}}$) branes. Since the flavor branes are connected, it provides a geometrical description of the spontaneous breaking of the chiral symmetry. The baryon in this model is the D4-branes[1] warped on S^4, which is called the "baryon vertex" and has been recognized as the instanton configuration of the gauge field on the worldvolume of the flavor branes [44]. In particular, the θ angle in the dual theory is holographically realized as the instantonic D-brane (D-instanton) in the construction of the string theory [45]. Hence, adding the D-instanton (D0-branes) to the background of the Witten-Sakai-Sugimoto model

*siwenli@fudan.edu.cn

[1] We will use "D4′-brane" to denote the baryon vertex in order to distinguish the N_c D4-branes in the following sections.

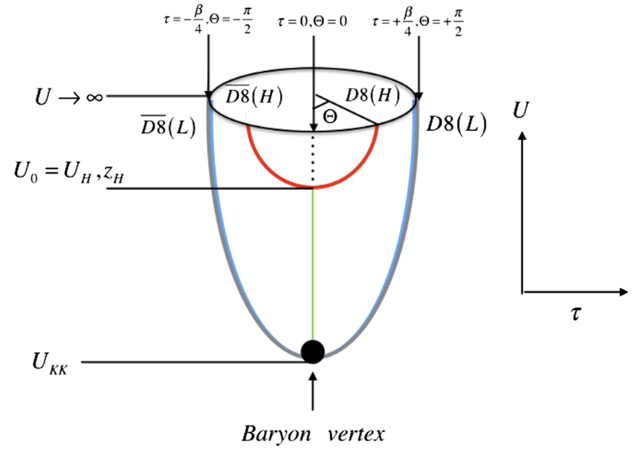

FIG. 1. Various brane configurations in the τ-U plane, where τ is compactified on S^1. The bubble background (cigar) is produced by N_c D4-branes with N_0 smeared D0-branes. The $N_f = 2$ light-flavor D8-/$\overline{\text{D8}}$-branes (L) living at the antipodal position of the cigar are represented by the blue line. A pair of heavy-flavor D8-/$\overline{\text{D8}}$-branes (H) is separated from the light-flavor branes, which is represented by the red line. The massive state is produced by the string stretched between the light- and heavy-flavor branes (HL-string), which is represented by the green line in this figure.

involves the θ dependence in the dual field theory [46,47]. The systematical study of the Witten-Sakai-Sugimoto model in the D0-D4 brane background (i.e., the D0-D4/D8 brane system) in Refs. [48,49] provided a way to holographically investigate the θ dependence of QCD or Yang-Mills theory [50–53].[2]

Since the baryon spectrum with light flavors was reviewed in Refs. [50,51,54,55] using the Witten-Sakai-Sugimoto model with the D0-D4 brane background, naturally the purpose of this paper is to extend the analysis to involve the heavy flavors. As mentioned, the fundamental quarks in this model are represented as the fermion states created by the open strings stretched between the N_c D4-branes and N_f coincident D8-/$\overline{\text{D8}}$-branes without length, which are therefore massless states [31,58]. So the fundamental quarks in this model can be called "light quarks" and the N_f coincident flavor branes are called "light-flavor branes." In order to address the chiral and heavy-quark symmetries using holographic duality, we follow the mechanism proposed in Refs. [59–61], that is, we consider one pair of flavor branes as a probe (called a "heavy-flavor brane") which is separated from the other N_f coincident flavor branes, as shown in Fig. 1. The string stretched between the heavy- and light-flavor branes (called the "HL-string") produces massive multiplets. Hence, the heavy-light mesons correspond to the low-energy modes of these strings, which can be approximated by the local vector fields in the bifundamental representation in the vicinity of the light-flavor branes. With a nonzero vacuum expectation value (VEV), the heavy-light fields are massive and their mass comes from the moduli spanned by the dilaton fields in the action. This setup allows to describe the radial spectra of heavy-light multiplets, their pertinent vector and axial correlations, and so on. The baryons with heavy flavor should have the form of instanton configurations in the worldvolume theory of D8-/$\overline{\text{D8}}$-branes bound to heavy-light vector mesons. This method will also allow us to holographically develop the bound-state approach in the context of the Skyrme model (e.g., Ref. [62]) by including the θ dependence.

The outline of this paper is as follows. In Sec. II, we briefly review the D0-D4 background and its dual field theory. In Sec. III, we outline the geometrical setup and derive the heavy-light effective action through the Dirac-Born-Infield (DBI) and Chern-Simons (CS) actions of the D-branes. It shows the heavy-meson interactions with the instanton on the flavor branes. In Sec. IV, we show the effective action in the double limit, and that a spin-1 vector meson bound to the bulk instanton is transmuted to spin-1/2. In Sec. V, we employ the quantization and show how to include the heavy flavors in the spectrum (as in previous works [50,51,54,55]) with a finite D0-brane charge or θ angle. The derivation of the baryon spectra with single- and double-heavy quarks is explicitly shown in this section. Section VI is the summary. In the Appendix, we briefly summarize the essential steps for the quantization of the light meson moduli without the heavy-flavor branes, which has already been studied in Ref. [50].

II. THE D0-D4 BACKGROUND AND THE DUAL FIELD THEORY

In this section, we will review the D0-D4 background and its dual field theory by following Ref. [48]. In the Einstein frame, the solution of a D4-brane with a smeared D0-brane in IIA supergravity is given as

$$ds^2 = H_4^{-\frac{3}{8}}[-H_0^{\frac{7}{8}} f(U)d\tau^2 + H_0^{\frac{1}{8}}\delta_{\mu\nu}dx^\mu dx^\nu] + H_4^{\frac{5}{8}}H_0^{\frac{1}{8}}\left[\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right]. \tag{2.1}$$

The direction τ is compactified on a cycle with the period β. The dilaton, and Ramond-Ramond 2-form and 4-form are given as

$$e^{-\Phi} = g_s^{-1}\left(\frac{H_4}{H_0^3}\right)^{\frac{1}{4}}, \qquad f_2 = \frac{(2\pi l_s)^7 g_s N_0}{\omega_4 V_4}\frac{1}{U^4 H_0^2}dU \wedge d\tau,$$
$$f_4 = \frac{(2\pi l_s)^3 N_c g_s}{\omega_4}\epsilon_4, \tag{2.2}$$

[2] See also Refs. [54,55] for a similar approach, the applications in Refs. [56,57] in hydrodynamics, or the model in Refs. [48,49].

where

$$H_4 = 1 + \frac{U_{Q4}^3}{U^3}, \qquad H_0 = 1 + \frac{U_{Q0}^3}{U^3},$$
$$f(U) = 1 - \frac{U_{KK}^3}{U^3},$$
$$U_{Q0}^3 = \frac{1}{2}\left(-U_{KK}^3 + \sqrt{U_{KK}^6 + ((2\pi)^5 l_s^7 g_s \tilde{\kappa} N_c)^2}\right),$$
$$U_{Q4}^3 = \frac{1}{2}\left(-U_{KK}^3 + \sqrt{U_{KK}^6 + ((2\pi)^5 l_s^7 g_s N_c)^2}\right). \qquad (2.3)$$

We have used $d\Omega_4$, ϵ_4, and $\omega_4 = 8\pi^2/3$ to represent the line element, the volume form, and the volume of a unit S^4. U_{KK} is the horizon position of the radius coordinate and V_4 is the volume of the D4-branes. The number of D4- and D0-branes are denoted by N_c and N_0, respectively. D0-branes are smeared in the directions x^0, x^1, x^2, and x^3, so the number density of the D0-branes can be represented by N_0/V_4. In order to take account of the backreaction from the D0-branes (as in Ref. [47]), we also require that N_0 is of the order of N_c. In the large-N_c limit, $\tilde{\kappa}$ would be of the order of $\mathcal{O}(1)$, which is defined as $\tilde{\kappa} = N_0/(N_c V_4)$.

In the string frame, making a double Wick rotation and taking the field limit, i.e., $\alpha' \to 0$ with fixed U/α' and U_{KK}/α', we obtain the D0-D4 bubble geometry and the metric becomes

$$ds^2 = \left(\frac{U}{R}\right)^{3/2}[H_0^{1/2}\eta_{\mu\nu}dx^\mu dx^\nu + H_0^{-1/2}f(U)d\tau^2] + H_0^{1/2}\left(\frac{R}{U}\right)^{3/2}\left[\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right]. \qquad (2.4)$$

The dilaton becomes

$$e^\Phi = g_s\left(\frac{U}{R}\right)^{3/4} H_0^{3/4}, \qquad (2.5)$$

where $R^3 = \pi g_s l_s^3 N_c$ is the limit of U_{Q4}^3. Here l_s is the length of the string and $\alpha' = l_s^2$. In the bubble geometry (2.4), the spacetime ends at $U = U_{KK}$. In order to avoid the conical singularity at U_{KK}, the period β of τ must satisfy

$$\beta = \frac{4\pi}{3}U_{KK}^{-1/2}R^{3/2}b^{1/2}, \qquad b \equiv H_0(U_{KK}). \qquad (2.6)$$

In the low-energy effective description, the dual theory is a five-dimensional $U(N_c)$ Yang-Mills (YM) theory which lives inside the worldvolume of the D4-brane. Since one direction of the D4-branes is compactified on a cycle τ, the four-dimensional Yang-Mills coupling could be obtained as in Ref. [30], i.e., relating the D4-brane tension and the five-dimensional Yang-Mills coupling constant g_5, and then analyzing the relation of the five-dimensionally compactified theory and four dimensions in the τ direction. Thus the resultant four-dimensional Yang-Mills coupling is

$$g_{\rm YM}^2 = \frac{g_5^2}{\beta} = \frac{4\pi^2 g_s l_s}{\beta}, \qquad (2.7)$$

and b and R^3 can be evaluated as

$$b = \frac{1}{2}[1 + (1 + C\beta^2)^{1/2}],$$
$$C \equiv (2\pi l_s^2)^6 \lambda^2 \tilde{\kappa}/U_{KK}^6,$$
$$R^3 = \frac{\beta\lambda l_s^2}{4\pi}, \qquad (2.8)$$

where the ’t Hooft coupling λ is defined as $\lambda = g_{\rm YM}^2 N_c$. The Kaluza-Klein (KK) modes can be introduced by defining a mass scale $M_{KK} = 2\pi/\beta$. The fermion and scalar become massive at the KK mass scale since the antiperiodic condition is imposed on the fermions [31]. Therefore, the massless modes of the open string dominate the dynamics in the low-energy theory which is described by a pure Yang-Mills theory. According to Eqs. (2.6) and (2.8), we have the following relations:

$$\beta = \frac{4\pi\lambda l_s^2}{9U_{KK}}b, \qquad M_{KK} = \frac{9U_{KK}}{2\lambda l_s^2 b}. \qquad (2.9)$$

Because $b \geq 1$ and $U_{KK} \geq 2\lambda l_s^2 M_{KK}/9$, β can be solved by using Eqs. (2.8) and (2.9) as

$$\beta = \frac{4\pi\lambda l_s^2}{9U_{KK}}\frac{1}{1 - \frac{(2\pi l_s^2)^8}{81U_{KK}^8}\lambda^4\tilde{\kappa}^2}, \qquad b = \frac{1}{1 - \frac{(2\pi l_s^2)^8}{81U_{KK}^8}\lambda^4\tilde{\kappa}^2}. \qquad (2.10)$$

Let us consider the effective action of a D4-brane with the smeared D0-branes in the background, which takes the following form:

$$S_{D_4} = -\mu_4 {\rm Tr}\int d^4x d\tau e^{-\phi}\sqrt{-\det(G + 2\pi\alpha'\mathcal{F})} + \mu_4\int C_5 + \frac{1}{2}(2\pi\alpha')^2\mu_4\int C_1\wedge\mathcal{F}\wedge\mathcal{F}, \qquad (2.11)$$

where $\mu_4 = (2\pi)^{-4}l_s^{-5}$, $\phi = \Phi - \Phi_0$, $e^{\Phi_0} = g_s$, and G is the induced metric on the worldvolume of D4-branes. $\mathcal{F}$ is the gauge field strength on the D4-brane. C_5 and C_1 are the Ramond-Ramond 5-form and 1-form, respectively, and their field strengths are given in Eq. (2.2). The Yang-Mills action can be obtained from the leading-order expansion with respect to small $\mathcal{F}$ of the first term in Eq. (2.11) (i.e., the DBI action). In the bubble D0-D4 solution, we have $C_1 \sim \theta d\tau$ in Eq. (2.2); thus, D0-branes are actually D-instantons (as shown in Table I) and the last term in Eq. (2.12) can be integrated as

TABLE I. The brane configurations: "=" denotes the smeared directions, while "–" denotes the worldvolume directions.

	0	1	2	3	4(τ)	5(U)	6	7	8	9
Smeared D0-branes	=	=	=	=	–					
N_c D4-branes	–	–	–	–	–					
N_f D8-/$\overline{\mathrm{D8}}$-branes	–	–	–	–		–	–	–	–	–
Baryon vertex D4′-branes	–						–	–	–	–

$$\int_{S^1_\tau} C_1 \sim \theta \sim \tilde{\kappa}, \qquad \int_{S^1_\tau\times\mathbb{R}^4} C_1\wedge\mathcal{F}\wedge\mathcal{F}\sim\theta\int_{\mathbb{R}^4}\mathcal{F}\wedge\mathcal{F}. \tag{2.12}$$

So the free parameter $\tilde{\kappa}$ (related to the θ angle in the dual field theory) has been introduced into the Witten-Sakai-Sugimoto model by this string theory background; however, this background is not dual to the vacuum state of the gauge field theory. Similarly as in Ref. [45], in the dual field theory, some excited states with a constant homogeneous field strength background may be described in the D0-D4 model. The expectation value of $\mathrm{Tr}\mathcal{F}\wedge\mathcal{F}$ can be evaluated as $\langle \mathrm{Tr}\mathcal{F}\wedge\mathcal{F}\rangle = 8\pi^2 N_c\tilde{\kappa}$ [48,50]. Then, the deformed relations in the presence of the D0-branes of the variables in QCD are given as follows:

$$R^3 = \frac{\lambda l_s^2}{2M_{KK}}, \qquad g_s = \frac{\lambda}{2\pi M_{KK} N_c l_s},$$
$$U_{KK} = \frac{2}{9} M_{KK}\lambda l_s^2 b. \tag{2.13}$$

III. HOLOGRAPHIC EFFECTIVE ACTION FOR HEAVY-LIGHT INTERACTION

A. D-brane setup

The chiral symmetry $U_R(N_f)\times U_L(N_f)$ can be introduced into the D0-D4 system by adding a stack of N_f probe D8–anti D8 (D8/$\overline{\mathrm{D8}}$) branes to the background, which are called flavor branes. The spontaneous breaking of $U_R(N_f)\times U_L(N_f)$ symmetry to $U_V(N_f)$ in the dual field theory can be geometrically understood as the distant D8-/$\overline{\mathrm{D8}}$-branes combining near the bottom of the bubble at $U = U_{KK}$ (as shown by the blue lines in Fig. 1). This can be verified by the appearance of massless Goldstones [63]. The brane configurations are illustrated in Table I.

The induced metric on the probe D8-/$\overline{\mathrm{D8}}$-branes is

$$ds^2_{\mathrm{D8}/\overline{\mathrm{D8}}} = \left(\frac{U}{R}\right)^{3/2} H_0^{-1/2}\left[f(U) + \left(\frac{R}{U}\right)^3 \frac{H_0}{f(U)} U'^2\right] d\tau^2 + \left(\frac{U}{R}\right)^{3/2} H_0^{1/2}\eta_{\mu\nu}dx^\mu dx^\nu + H_0^{1/2}\left(\frac{R}{U}\right)^{3/2} \times U^2 d\Omega_4^2, \tag{3.1}$$

where U' is the derivative with respect to τ. The action of the D8-/$\overline{\mathrm{D8}}$-branes can be obtained as

$$S_{\mathrm{D8}/\overline{\mathrm{D8}}} \propto \int d^4x dU H_0(U) U^4\left[f(U) + \left(\frac{R}{U}\right)^3 \frac{H_0}{f(U)} U'^2\right]^{1/2}. \tag{3.2}$$

Then, the equation of motion for $U(\tau)$ can be derived as

$$\frac{d}{d\tau}\left(\frac{H_0(U)U^4 f(U)}{[f(U) + (\frac{R}{U})^3 \frac{H_0}{f(U)} U'^2]^{1/2}}\right) = 0, \tag{3.3}$$

which can be interpreted as the conservation of energy. With the initial conditions $U(\tau = 0) = U_0$ and $U'(\tau = 0) = 0$, the generic formula of the embedding function $\tau(U)$ can be solved as

$$\tau(U) = E(U_0)\int_{U_0}^{U} dU \frac{H_0^{1/2}(U)(\frac{R}{U})^{3/2}}{f(U)[H_0^2(U)U^8 f(U) - E^2(U_0)]^{1/2}}, \tag{3.4}$$

where $E(U_0) = H_0(U_0)U_0^4 f^{1/2}(U_0)$ and U_0 denotes the connected position of the D8-/$\overline{\mathrm{D8}}$-branes. Following Refs. [31,48], we introduce the new coordinates (r, Θ) and (y, z), which satisfy

$$y = r\cos\Theta, \qquad z = r\sin\Theta, \qquad U^3 = U_{KK}^3 + U_{KK} r^2,$$
$$\Theta = \frac{2\pi}{\beta}\tau = \frac{3}{2}\frac{U_{KK}^{1/2}}{R^{3/2}H_0^{1/2}(U_{KK})}. \tag{3.5}$$

In this manuscript, we will consider the following configuration for the various flavor branes: the light-flavor branes live at the antipodal position (as in Refs. [31,48,49]), which means that they (the D8-/$\overline{\mathrm{D8}}$-branes) are embedded at $\Theta = \pm\frac{1}{2}\pi$, respectively, i.e., $y = 0$. The embedding function of the light-flavor branes is $\tau_L(U) = \frac{1}{4}\beta$, so that we have $U^3 = U_{KK}^3 + U_{KK} z^2$ on the light D8-/$\overline{\mathrm{D8}}$-branes.[3] Therefore, the induced metric on them becomes

$$ds^2_{\mathrm{Light-D8}/\overline{\mathrm{D8}}} = H_0^{1/2}\left(\frac{U}{R}\right)^{3/2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{4}{9}\frac{U_{KK}}{U}\left(\frac{U}{R}\right)^{3/2} H_0^{1/2} dz^2 + H_0^{1/2}\left(\frac{R}{U}\right)^{3/2} U^2 d\Omega_4^2. \tag{3.6}$$

For the heavy-flavor branes, we have to choose another solution as $\tau_H(U)$ from Eq. (3.4) with $U_0 = U_H \neq U_{KK}$ since they must live at the nonantipodal position of the

[3]With suitable boundary conditions, $\tau_L(U) = \frac{1}{4}\beta$ is indeed a solution of Eq. (3.3), as discussed in Refs. [31,48,49].

background.[4] Thus, the heavy-flavor brane is separated from the light-flavor branes with a finite separation at $\tau_H(U_0) = 0$, as shown in Fig. 1. With the approach presented in Refs. [44,50,64], the light baryon spectrum in the D0-D4/D8 system was studied in Refs. [50,51,54,55]. In this paper, we extend our previous work by following Refs. [59–61] to study the heavy-light interaction and baryon spectrum with heavy flavors in the D0-D4/D8 system. Thus, we consider $N_f = 2$ light-flavor D8-/$\overline{\text{D8}}$-branes and one pair of heavy flavor branes as a probe in the bubble D0-D4 geometry (2.4) that spontaneously breaks chiral symmetry. The massive states on the light D8-/$\overline{\text{D8}}$-branes are produced by the heavy-light (HL) strings connecting heavy-light branes.

B. Yang-Mills and Chern-Simons action of the flavor branes

Since the baryon vertex lives inside the light-flavor branes, the concern of this section is to study the effective dynamics of the baryons or mesons on the light-flavor branes involving the heavy-light interaction. The lowest modes of the open string stretched between the heavy and light branes are attached to the baryon vertex, as shown in Fig. 1. In our D0-D4/D8 system, these string modes consist of longitudinal modes Φ_a and transverse modes Ψ near the light brane worldvolume. These fields acquire a nonzero VEV at finite brane separation, which introduces the mass to the vector field [65]. These fields are always called "bilocal"; however, we will approximate them near the light-flavor branes by local vector fields, and hence they are described by the standard DBI action. Thus, this construction is distinct from the approaches presented in Refs. [66–72].

Keeping these points in mind, let us consider the action of the light-flavor branes. For the D8-branes, the generic expansion of the DBI action at the leading order can be written as

$$S_{\rm DBI}^{\rm D8/\overline{D8}} = -\frac{T_8(2\pi\alpha')^2}{4}\int d^9\xi\sqrt{-\det G}e^{-\Phi} \times \mathrm{Tr}\{\mathcal{F}_{ab}\mathcal{F}^{ab} - 2D_a\varphi^I D_a\varphi^J + [\varphi^I,\varphi^J]^2\}, \tag{3.7}$$

where φ^I is the transverse mode of the flavor branes and the indices a and b run over the flavor brane. Notice that only one coordinate is transverse to the D8-brane, and thus we define $\varphi^I \equiv \Psi$ to omit the index. The scalar field Ψ is traceless in adjoint representation, in addition to the adjoint gauge field $\mathcal{A}_a$. Since the one pair of heavy-flavor branes is separated from the $N_f = 2$ light-flavor branes with a string stretched between them, in string theory the worldvolume field can be combined in a superconnection. For the gauge field, we can use the following matrix-valued 1-form:

$$\boldsymbol{\mathcal{A}}_a = \begin{pmatrix} \mathcal{A}_a & \Phi_a \\ -\Phi_a^\dagger & 0 \end{pmatrix}, \tag{3.8}$$

where $\boldsymbol{\mathcal{A}}_a$ is $(N_f+1)\times(N_f+1)$ matrix-valued while Ψ and $\mathcal{A}_a$ are $N_f \times N_f$ valued. If all of the flavor branes are coincident, the Φ_a multiplet is massless; otherwise, Φ_a could be a massive field. The corresponding gauge field strength of Eq. (3.8) is

$$\boldsymbol{\mathcal{F}}_{ab} = \begin{pmatrix} \mathcal{F}_{ab} - \Phi_{[a}\Phi_{b]}^\dagger & \partial_{[a}\Phi_{b]} + \mathcal{A}_{[a}\Phi_{b]} \\ \partial_{[a}\Phi_{b]}^\dagger + \Phi_{[a}^\dagger\mathcal{A}_{b]} & -\Phi_{[a}^\dagger\Phi_{b]} \end{pmatrix}. \tag{3.9}$$

Inserting the induced metric (3.6) with Eq. (2.5) into Eq. (3.7), we can write the DBI action in two parts,

$$S_{\rm DBI}^{\rm D8/\overline{D8}} = S_{\rm YM}^{\rm D8/\overline{D8}} + S_\Psi. \tag{3.10}$$

The Yang-Mills part is calculated as

$$S_{\rm YM} = -2\tilde{T}U_{KK}^{-1}\int d^4xdzH_0^{1/2} \times \mathrm{Tr}\left[\frac{1}{4}\frac{R^3}{U}\boldsymbol{\mathcal{F}}_{\mu\nu}\boldsymbol{\mathcal{F}}^{\mu\nu} + \frac{9}{8}\frac{U^3}{U_{KK}}\boldsymbol{\mathcal{F}}_{\mu z}\boldsymbol{\mathcal{F}}^{\mu z}\right], \tag{3.11}$$

where μ, ν run over 0,1,2,3, and

$$\tilde{T} = \frac{(2\pi\alpha')^2}{3g_s}T_8\omega_4U_{KK}^{3/2}R^{3/2} = \frac{M_{KK}^2\lambda N_c b^{3/2}}{486\pi^3}. \tag{3.12}$$

In order to work with the dimensionless variables, we introduce the replacements $z \to zU_{KK}$, $x^\mu \to x^\mu/M_{KK}$, $\boldsymbol{\mathcal{A}}_z \to \boldsymbol{\mathcal{A}}_z/U_{KK}$, and $\boldsymbol{\mathcal{A}}_\mu \to \boldsymbol{\mathcal{A}}_\mu M_{KK}$.[5] Then, Eq. (3.11) takes the following form:

$$\begin{aligned} S_{\rm YM}^{\rm D8/\overline{D8}} &= -\tilde{T}M_{KK}^{-2}\frac{9}{4b}\int d^4xdzH_0^{1/2}(U) \times \mathrm{Tr}\left[\frac{1}{2}\frac{U_{KK}}{U}\boldsymbol{\mathcal{F}}_{\mu\nu}^2 + \frac{U^3}{U_{KK}^3}b\boldsymbol{\mathcal{F}}_{\mu z}^2\right], \\ &= -a\lambda N_c b^{1/2}\int d^4xdzH_0^{1/2}(U) \times \mathrm{Tr}\left[\frac{1}{2}K(z)^{-1/3}\eta^{\mu\nu}\eta^{\rho\sigma}\boldsymbol{\mathcal{F}}_{\mu\rho}\boldsymbol{\mathcal{F}}_{\nu\sigma} + K(z)b\eta^{\mu\nu}\boldsymbol{\mathcal{F}}_{\mu z}\boldsymbol{\mathcal{F}}_{\nu z}\right], \end{aligned} \tag{3.13}$$

[4]Since we are going to discuss the limit U_H, $z_H \to \infty$ in the next section, an analytical solution for the embedding function of the heavy-flavor brane could be $\tau_H(U) = -\frac{2}{9}\left(\frac{R}{U}\right)^{3/2}\frac{U_H}{U^3}\,{}_2F_1\left(\frac{1}{2},\frac{9}{16},\frac{25}{16},\frac{U_H^8}{U^8}\right) + \frac{2\sqrt{\pi}}{9}\frac{R^{3/2}}{U_H^{1/2}}\frac{\Gamma(\frac{25}{16})}{\Gamma(\frac{17}{16})}$, where ${}_2F_1$ is the hypergeometric function. In this limit, the integral region on the heavy-flavor brane is $U > U_H \to \infty$, so we have $f, H_0 \sim 1$ to get this solution with Eq. (3.4).

[5]Working with this replacement is equivalent to working in units of $U_{KK} = M_{KK} = 1$ in this model, as in Ref. [44].

where $a = \frac{1}{216\pi^3}$ and $K(z) = 1 + z^2$. To see the dependence of λ, it would be convenient to employ the rescaling used in Ref. [44], which is

$$\begin{aligned}&(x^0, x^M) \to (x^0, \lambda^{-1/2} x^M),\\&(\mathcal{A}_0, \Phi_0) \to (\mathcal{A}_0, \Phi_0),\\&(\mathcal{A}_M, \Phi_M) \to (\lambda^{1/2}\mathcal{A}_M, \lambda^{1/2}\Phi_M),\end{aligned} \tag{3.14}$$

where M, N run over $1, 2, 3, z$ and i, $j = 1, 2, 3$. Using Eq. (3.14) in the large-λ limit, Eq. (3.13) becomes

$$\begin{aligned}\mathcal{L}_{\mathrm{YM}}^{\mathrm{D8}/\overline{\mathrm{D8}}} &= -aN_c b^{3/2}\mathrm{Tr}\left[\frac{\lambda}{2}\boldsymbol{\mathcal{F}}_{MN}^2 - bz^2\left(\frac{5}{12} - \frac{1}{4b}\right)\boldsymbol{\mathcal{F}}_{ij}^2 + \frac{bz^2}{2}\left(1 + \frac{1}{b}\right)\boldsymbol{\mathcal{F}}_{iz}^2 - \boldsymbol{\mathcal{F}}_{0M}^2\right]\\&\equiv aN_c b^{3/2}\mathcal{L}_{\mathrm{YM}}^L + aN_c b^{3/2}\lambda\mathcal{L}_0^H + aN_c b^{3/2}\mathcal{L}_1^H + \mathcal{O}(\lambda^{-1}),\end{aligned} \tag{3.15}$$

where $\mathcal{L}_{\mathrm{YM}}^L$ represents the Lagrangian for the light hadrons (which was derived in Ref. [50]), and the explicit form of $\mathcal{L}_{\mathrm{YM}}^L$ can be found in Eqs. (A1)–(A2) in the Appendix. Substituting Eq. (3.15) for Eq. (3.9), we obtain

$$\begin{aligned}\mathcal{L}_0^H &= -(D_M\Phi_N^\dagger - D_N\Phi_M^\dagger)(D_M\Phi_N - D_N\Phi_M) + 2\Phi_M^\dagger\mathcal{F}^{MN}\Phi_N,\\\mathcal{L}_1^H &= 2(D_0\Phi_M^\dagger - D_M\Phi_0^\dagger)(D_0\Phi_M - D_M\Phi_0) - 2\Phi_0^\dagger\mathcal{F}^{0M}\Phi_M - 2\Phi_0^\dagger\mathcal{F}^{0M}\Phi_M + \tilde{\mathcal{L}}_1^H,\end{aligned} \tag{3.16}$$

where $D_M\Phi_N = \partial_M\Phi_N + \mathcal{A}_{[M}\Phi_{N]}$ and

$$\begin{aligned}\tilde{\mathcal{L}}_1^H &= bz^2\left(\frac{5}{6} - \frac{1}{2b}\right)(D_i\Phi_j - D_j\Phi_i)^\dagger(D_i\Phi_j - D_j\Phi_i) - bz^2\left(1 + \frac{1}{b}\right)(D_i\Phi_z - D_z\Phi_i)^\dagger(D_i\Phi_z - D_z\Phi_i)\\&\quad - bz^2\left(\frac{5}{3} - \frac{1}{b}\right)\Phi_i^\dagger\mathcal{F}^{ij}\Phi_j + bz^2\left(1 + \frac{1}{b}\right) \times (\Phi_z^\dagger\mathcal{F}^{zi}\Phi_i + \mathrm{c.c.}).\end{aligned} \tag{3.17}$$

The action S_Ψ in Eq. (3.10) is

$$\begin{aligned}S_\Psi &= -\frac{T_8(2\pi\alpha')^2}{4}\int d^9\xi\sqrt{-\det G}e^{-\Phi} \times \mathrm{Tr}\{-2D_a\varphi^i D_a\varphi^i + [\varphi^i, \varphi^j]^2\}\\&= \tilde{T}_8\int d^4x dz\sqrt{-\det G}e^{-\Phi} \times \mathrm{Tr}\left\{\frac{1}{2}D_a\Psi D_a\Psi - \frac{1}{4}[\Psi, \Psi]^2\right\},\end{aligned} \tag{3.18}$$

with $D_a\Psi = \partial_a\Psi + i[\mathcal{A}_a, \Psi]$. According to Ref. [73], one can define the moduli by the extrema of the potential contribution or $[\Psi, [\Psi, \Psi]] = 0$ in Eq. (3.18). So the moduli solution of Ψ for N_f light branes separated from one heavy brane can be defined with a finite VEV v as

$$\Psi = \begin{pmatrix} -\frac{v}{N_f}\mathbf{1}_{N_f} & 0 \\ 0 & v \end{pmatrix}. \tag{3.19}$$

With the solution (3.19), we have

$$S_\Psi = -\tilde{T}U_{KK}^{-1}v^2\frac{2(N_f + 1)^2}{N_f^2} \times \int d^4x dz H_0^{3/2}U^2(g^{zz}\Phi_z^\dagger\Phi_z + g^{\mu\nu}\Phi_\mu^\dagger\Phi_\nu). \tag{3.20}$$

Again, we introduce the dimensionless variable Φ_a by imposing $z \to zU_{KK}$, $x^\mu \to x^\mu/M_{KK}$, $\boldsymbol{\mathcal{A}}_z \to \boldsymbol{\mathcal{A}}_z/U_{KK}$, and $\boldsymbol{\mathcal{A}}_\mu \to \boldsymbol{\mathcal{A}}_\mu M_{KK}$, which means that we require an additional replacement $v \to \frac{M_{KK}^{1/2}}{U_{KK}^{1/2}}v$. Then, using the λ rescaling as in Eq. (3.14), we finally obtain

$$S_\Psi = -aN_c b^{3/2}\int d^4x dz 2m_H^2\Phi_M^\dagger\Phi_M + \mathcal{O}(\lambda^{-1}), \tag{3.21}$$

where $m_H = \frac{1}{\sqrt{6}}\frac{N_f+1}{N_f}vb^{1/4}$.

For a Dp-brane, there is a CS term in the total action whose standard form is

$$\begin{aligned}S_{\mathrm{CS}}^{D_\mathrm{p}} &= \mu_p\int_{D_p}\sum_q C_{q+1}\wedge\mathrm{Tr}e^{2\pi\alpha'\mathcal{F}}\\&= \mu_p\int_{D_p}\sum_n C_{p-2n+1}\wedge\frac{1}{n!}(2\pi\alpha')^n\mathrm{Tr}\mathcal{F}^n.\end{aligned} \tag{3.22}$$

In our D0-D4 background, the nonvanishing terms for the probe D8-/$\overline{\mathrm{D8}}$-branes are

$$\begin{aligned}S_{\mathrm{CS}}^{\mathrm{D8}/\overline{\mathrm{D8}}} &= \frac{1}{3!}(2\pi\alpha')^3\mu_8\int_{\mathrm{D8}/\overline{\mathrm{D8}}}C_3\wedge\mathrm{Tr}[\mathcal{F}^3]\\&\quad + 2\pi\alpha'\mu_8\int_{\mathrm{D8}/\overline{\mathrm{D8}}}C_7\wedge\mathrm{Tr}[\mathcal{F}].\end{aligned} \tag{3.23}$$

The first term in Eq. (3.23) can be integrated out by using $dC_3 = f_4$ [given in Eq. (2.2)], which yields a CS 5-form,

$$S_{\mathrm{CS}}^{\mathrm{D8}/\overline{\mathrm{D8}}} = \frac{N_c}{24\pi^2}\int_{\mathbb{R}^{4+1}}\boldsymbol{\mathcal{A}}\boldsymbol{\mathcal{F}}^2 - \frac{1}{2}\boldsymbol{\mathcal{A}}^3\boldsymbol{\mathcal{F}} + \frac{1}{10}\boldsymbol{\mathcal{A}}^5, \tag{3.24}$$

and this term is invariant under the λ rescaling (3.14). However, explicit calculations show that the second term in Eq. (3.23) becomes $\mathcal{O}(\lambda^{-1})$ in the large-λ limit. So on the light-flavor branes, only Eq. (3.24) in the CS term survives in the strong-coupling limit. Inserting Eqs. (3.8) and (3.9) into Eq. (3.24) with the dimensionless variables, Eq. (3.24) becomes

$$\mathcal{L}_{\rm CS}^{\rm D8/\overline{D8}} = \mathcal{L}_{\rm CS}^{L}(\mathcal{A}) + \mathcal{L}_{\rm CS}^{H}, \tag{3.25}$$

where $\mathcal{L}_{\rm CS}^{L}(\mathcal{A})$ represents the CS term for the light hadrons given in Eqs. (A1)–(A2) (which was studied in Refs. [44,50]), and

$$\begin{aligned}\mathcal{L}_{\rm CS}^{H} = &-\frac{iN_c}{24\pi^2}(d\Phi^\dagger \mathcal{A} d\Phi + d\Phi^\dagger d\mathcal{A}\Phi + \Phi^\dagger d\mathcal{A} d\Phi)\\ &-\frac{iN_c}{16\pi^2}(d\Phi^\dagger \mathcal{A}^2\Phi + \Phi^\dagger \mathcal{A}^2 d\Phi + \Phi^\dagger \mathcal{A} d\mathcal{A}\Phi\\ &+\Phi^\dagger d\mathcal{A}\mathcal{A}\Phi) - \frac{5iN_c}{48\pi^2}\Phi^\dagger\mathcal{A}^3\Phi + \mathcal{O}(\Phi^4,\mathcal{A}).\end{aligned} \tag{3.26}$$

Therefore, the action for the light-heavy interaction can be collected from Eqs. (3.16), (3.17), (3.21), and (3.26) on the light-flavor branes.

IV. THE ZERO MODES

In the limit $\lambda\to\infty$ followed by $m_H\to\infty$, the heavy meson in the bulk can be treated as the instanton configuration on the flavor branes, which can be effectively treated as a spinor. It forms a four-dimensional flavored zero mode which can be interpreted as a bound of either the heavy flavor or anti-heavy flavor in the spacetime of $\{x^\mu\}$. However, in the Skyrme model, the Wess-Zumino-Witten term is time-odd, which carries opposite signs for heavy particles and antiparticles. While this is difficult for antiparticles using holography, it is remarkable.

A. Equations of motion

Let us consider the solution of the heavy meson field Φ_M. Notice that Φ_M is independent of Φ_0, so the equations of motion from the action (3.16), (3.17), (3.21), and (3.26) read

$$D_M D_M \Phi_N - D_N D_M \Phi_M + 2\mathcal{F}_{NM}\Phi_M + \mathcal{O}(\lambda^{-1}) = 0, \tag{4.1}$$

and the equation of motion for Φ_0 is

$$\begin{aligned}&D_M(D_0\Phi_M - D_M\Phi_0) - \mathcal{F}^{0M}\Phi_M\\ &-\frac{1}{64\pi^2 ab^{3/2}}\epsilon_{MNPQ}\mathcal{K}_{MNPQ} + \mathcal{O}(\lambda^{-1}) = 0,\end{aligned} \tag{4.2}$$

where the 4-form $\mathcal{K}_{MNPQ}$ is given as

$$\begin{aligned}\mathcal{K}_{MNPQ} = &\,\partial_M\mathcal{A}_N\partial_P\Phi_Q + \mathcal{A}_M\mathcal{A}_N\partial_P\Phi_Q + \partial_M\mathcal{A}_N\mathcal{A}_P\Phi_Q\\ &+\frac{5}{6}\mathcal{A}_M\mathcal{A}_N\mathcal{A}_P\Phi_Q.\end{aligned} \tag{4.3}$$

In the heavy-quark limit, we follow Ref. [61] to redefine $\Phi_M = \phi_M e^{-im_H x^0}$ for particles, with the replacement $m_H\to -m_H$ for antiparticles.

B. The double limit

It is very difficult to calculate all of the contributions from the heavy meson field Φ_M. Hence, we consider the limit $\lambda\to\infty$ followed by $m_H\to\infty$, which is called the "double limit". So the leading contributions come from the light effective action presented in Ref. (2.5), which is of the order of λm_H^0, while the next-to-leading contributions come from the heavy-light interaction Lagrangian $\mathcal{L}_1^H$ in Eq. (3.16) and $\mathcal{L}_{\rm CS}^H$ in Eq. (3.26), which is of the order of $\lambda^0 m_H$. The double limit is valid if we assume that the heavy meson field Φ_M is very massive, which means that the separation of the heavy and light branes is very large, as shown in Fig. 1. So the straight string takes a value at $z = z_H$ which satisfies

$$\begin{aligned}m_H &= \frac{1}{\pi l_s^2}\lim_{z_H\to\infty}\int_0^{z_H} dz\sqrt{-g_{00}g_{zz}}\\ &\simeq \frac{1}{\pi l_s^2}U_{KK}^{1/3}z_H^{2/3} + \mathcal{O}(z_H^0).\end{aligned} \tag{4.4}$$

It would be convenient to rewrite Eq. (4.4) with the dimensionless variables by the replacements $m_H\to m_H M_{KK}$ and $z_H\to z_H U_{KK}$. Then, using Eq. (2.13), we have

$$\frac{m_H}{\lambda} = \frac{2b}{9\pi}z_H^{2/3}. \tag{4.5}$$

According to the above discussion, the derivative of Φ_M can be replaced by $D_0\Phi_M \to (D_0 \pm im_H)\Phi_M$, with "−" for particles and "+" for antiparticles. Then we collect the order $\lambda^0 m_H$ from our heavy-light action, which is

$$\begin{aligned}\mathcal{L}_{m_H} &= \mathcal{L}_{1,m} + \mathcal{L}_{{\rm CS},m},\\ \mathcal{L}_{1,m} &= ab^{3/2}N_c[4im_H\phi_M^\dagger D_0\phi_M\\ &\quad - 2im_H(\phi_0^\dagger D_M\phi_M - {\rm c.c.})],\\ \mathcal{L}_{{\rm CS},m} &= \frac{m_H N_c}{16\pi^2}\epsilon_{MNPQ}\phi_M^\dagger\mathcal{F}_{NP}\phi_Q\\ &= \frac{m_H N_c}{8\pi^2}\phi_M^\dagger\mathcal{F}_{MN}\phi_N.\end{aligned} \tag{4.6}$$

The equation of motion (4.2) suggests a considerable simplification ($D_M\Phi_M = 0$), which implies that Φ_M is a covariantly transverse mode.

C. Vector to spinor

In the $N_f = 2$ case of the D0-D4/D8 system, the small instanton is described by a flat-space four-dimensional instanton solution of $SU(2)$ Yang-mills theory [50] in the large-λ limit, which is

$$\mathcal{A}_M^{cl} = -\bar{\sigma}_{MN}\frac{x^N}{x^2+\rho^2},$$
$$\mathcal{A}_0^{cl} = -\frac{i}{8\pi^2 ab^{3/2}x^2}\left[1-\frac{\rho^4}{(x^2+\rho^2)^2}\right], \tag{4.7}$$

where $x^2 = (x^M - X^M)^2$ and X^M is a constant. Notice that in Eq. (4.7) $\mathcal{A}_0^{cl}$ is Abelian, while $\mathcal{A}_M^{cl}$ is non-Abelian. It carries a field strength

$$\mathcal{F}_{MN} = \frac{2\bar{\sigma}_{MN}\rho^2}{(x^2+\rho^2)^2}. \tag{4.8}$$

By defining $f_{MN} = \partial_{[M}\phi_{N]} + \mathcal{A}_{[M}\phi_{N]}$, $\mathcal{L}_0^H$ in Eq. (3.16) can be rewritten as follows:

$$\begin{aligned}\mathcal{L}_0^H &= -f_{MN}^\dagger f_{MN} + 2\phi_M^\dagger \mathcal{F}_{MN}\phi_N\\ &= -f_{MN}^\dagger f_{MN} + 2\epsilon_{MNPQ}\phi_M^\dagger D_N D_P \phi_Q\\ &= -f_{MN}^\dagger f_{MN} + f_{MN}^\dagger \star f_{MN}\\ &= -\frac{1}{2}(f_{MN} - \star f_{MN})^\dagger (f_{MN} - \star f_{MN}),\end{aligned} \tag{4.9}$$

where $\star$ represents the Hodge dual. Therefore, the equations of motion (4.1) can be replaced by

$$f_{MN} - \star f_{MN} = 0,$$
$$D_M\phi_M = 0, \tag{4.10}$$

which is equivalent to

$$\sigma_M D_M \psi = 0, \quad \text{with} \quad \psi = \bar{\sigma}_M \phi_M. \tag{4.11}$$

So, ϕ_M can be solved from Eq. (4.10) as

$$\phi_M = \bar{\sigma}_M \xi \frac{\rho}{(x^2+\rho^2)^{3/2}} \equiv \bar{\sigma}_M f(x)\xi, \tag{4.12}$$

which is in agreement with Ref. [61]. As ξ is a two-component spinor, Eq. (4.11) is remarkable since it shows that a heavy vector meson holographically binds to an instantonic configuration in the bulk, and thus a vector zero mode is equivalently described by a spinor.

V. QUANTIZATION

The classical moduli of the bound instanton zero mode should be quantized by slowly rotating and translating the bound state, since it breaks rotational and translational symmetry. The instantonic and standard quantization of the leading λN_c contribution can be found in Ref. [50], while the quantization of the subleading $\lambda^0 m_H$ contribution involving zero modes in the D0-D4/D8 system is new. We will employ the quantization applied on D4/D8 as in Ref. [61].

A. Collectivization

As in Refs. [5,9], we assume that the zero modes slowly rotate, translate, and deform through

$$\Phi_M \to V[a_I(t)]\Phi_M[X^0(t), Z(t), \rho(t), \chi(t)],$$
$$\Phi_0 \to 0 + \delta\phi_0, \tag{5.1}$$

where X^0 and Z are the centers in the x^i and z directions, respectively. a_I is the $SU(2)$ gauge rotation. They are represented by $X^\alpha = (X^i, Z, \rho)$, with

$$-iV^\dagger\partial_0 V = \Phi = -\partial_t X^M \mathcal{A}_M + \chi^i \Phi_i,$$
$$\chi^i = -i\mathrm{Tr}(\tau^i a_I^{-1}\partial_t a_I). \tag{5.2}$$

Here, the a^I's carry the quantum numbers of isospin and angular momentum, and the τ^i's are Pauli matrices. Since Eq. (4.2) has to be satisfied, $\delta\phi_0$ is fixed at the next-to-leading order,

$$-D_M^2\delta\phi_0 + D_M\bar{\sigma}_M\left[\partial_t X^i \frac{\partial(f\chi)}{\partial X^i} + \partial_t\chi\right]$$
$$+ i(\partial_t X^\alpha \partial_\alpha \Phi_M - D_M\Phi)\bar{\sigma}_M\chi + \delta S_{\mathrm{CS}} = 0. \tag{5.3}$$

For a general quantization of the ensuing moduli, we can solve Eq. (5.3) and then insert the solution back into the action.

B. Leading order of the heavy mass term

The heavy mass terms in the double limit are given in Eq. (4.6). Imposing Eqs. (5.1) and (5.2) on Eq. (4.6), the contributions to order $\lambda^0 m_H$ in $\mathcal{L}_{1,m}$ come from three terms, which are

$$\mathcal{L}_{1,m} = ab^{3/2}N_c(16im_H\xi^\dagger\partial_t\xi f^2 + 16im_H\xi^\dagger\xi\mathcal{A}_0 f^2$$
$$- 16m_H f^2\xi^\dagger\sigma_\mu\Phi\bar{\sigma}_\mu\xi), \tag{5.4}$$

where $\mathcal{A}_0$ is the rescaled $U(1)$ gauge field. With the gauge field strength (4.8), the CS term in Eq. (4.6) can be written as

$$\mathcal{L}_{\mathrm{CS},m} = \frac{3m_H N_c}{\pi^2}\frac{f^2\rho^2}{(x^2+\rho^2)^2}\xi^\dagger\xi. \tag{5.5}$$

Notice that the third term in Eq. (5.4) vanishes owing to the identity $\sigma_\mu\tau^i\bar{\sigma}_\mu = 0$.

There is a Coulomb-like backreaction according to the coupling $\xi^\dagger\xi\mathcal{A}_0$ in Eq. (5.4). To clarify this, let us introduce a Coulomb-like potential defined as $\varphi = -i\mathcal{A}_0$. We collect all of the $U(1)$ couplings from Eq. (4.6) up to $\mathcal{O}(\lambda^0 m_H)$ as

$$\mathcal{L}_{U(1)} = -ab^{3/2}N_c\left[\frac{1}{2}(\nabla\varphi)^2 + \varphi(\rho_0 - 16m_H f^2\xi^\dagger\xi)\right], \tag{5.6}$$

where ρ_0 is the $U(1)$ "charge," which is given as

$$\rho_0 = \frac{1}{64\pi^2 ab^{3/2}}\epsilon_{MNPQ}\mathcal{F}_{MN}\mathcal{F}_{PQ}. \tag{5.7}$$

Solving the equation of motion from Eq. (5.6) for φ, one obtains its on-shell action as

$$\mathcal{L}_{U(1)} = \mathcal{L}_{U(1)}[\rho_0] + 16ab^{3/2}N_c m_H f^2\xi^\dagger\xi(-i\mathcal{A}_0^{cl}) - \frac{ab^{3/2}N_c}{24\pi^2\rho^2}(16m_H\xi^\dagger\xi)^2. \tag{5.8}$$

The last term is the Coulomb-like self-interaction, which is repulsive and tantamount to fermion number repulsion in holography.

C. Moduli effective action

All of the contributions up to $\mathcal{O}(\lambda^0 m_H)$ in the effective moduli action can be collected from Eqs. (5.4), (5.6), and (5.8). Let us summarize them as follows:

$$\mathcal{L} = \mathcal{L}^L[a_I, X^\alpha] + 16im_H ab^{3/2}N_c\xi^\dagger\partial_t\xi\int d^4x f^2 - 16m_H ab^{3/2}N_c\xi^\dagger\xi \times\int d^4x\left[i\mathcal{A}_0^{cl}f^2 - \frac{3}{16\pi^2 ab^{3/2}}\frac{f^2\rho^2}{(x^2+\rho^2)^2}\right] - \frac{ab^{3/2}N_c}{24\pi^2\rho^2}(16m_H\xi^\dagger\xi)^2. \tag{5.9}$$

Here $\mathcal{L}^L$ refers to the effective action on the moduli space from the contribution of the light hadrons, which is identical to the derivation in Ref. [50]. Equation (5.9), explicitly shows the new contribution due to the bound heavy meson. At the leading order, the coupling of the light collective degrees of freedom should be a general reflection of heavy quark symmetry. However, in Eq. (5.9) there is no such coupling on the order of $\mathcal{O}(\lambda^0 m_H)$ to the heavy spinor degree of freedom ξ. Notice that the coupling to the instanton size ρ does not upset this symmetry. In order to calculate Eq. (5.9), we follow the steps in Ref. [12], i.e., we use the normalization $\int d^4x f^2 = 1$, insert the explicit form of $\mathcal{A}_0^{cl}$, and rescale $\xi \to \xi/\sqrt{16ab^{3/2}N_c m_H}$. Finally, we obtain

$$\mathcal{L} = \mathcal{L}^L[a_I, X^\alpha] + i\xi^\dagger\partial_t\xi + \frac{3}{32\pi^2 ab^{3/2}\rho^2}\xi^\dagger\xi - \frac{(\xi^\dagger\xi)^2}{24\pi^2 ab^{3/2}\rho^2 N_c}. \tag{5.10}$$

This shows the zero mode of the vector to the instanton transmutation of a massive spinor with a repulsively Coulomb-like self-interaction in the presence of the D0 charge. A negative mass term also means that the energy of the heavy meson decreases, so the preceding arguments are also suitable for an anti-heavy meson in the presence of an instanton with a positive mass term, leading to Eq. (5.10). The energy of this meson increases in the presence of the instanton to order λ^0. It originates from the Chern-Simons term in the holographical action which is the analogue of the effects due to the Wess-Zumino-Witten term in the Skyrme model.

D. Heavy-light spectrum

The steps to quantize the Lagrangian (5.9) are the same as those presented in Ref. [50] for $\mathcal{L}^L[a_I, X^\alpha]$. We use $\mathcal{H}^L[a_I, X^\alpha]$ to represent the Hamiltonian associated to $\mathcal{L}^L[a_I, X^\alpha]$; then, the Hamiltonian for Eq. (5.10) takes the form

$$\mathcal{H} = \mathcal{H}^L[a_I, X^\alpha] - \frac{3}{32\pi^2 ab^{3/2}\rho^2}\xi^\dagger\xi + \frac{(\xi^\dagger\xi)^2}{24\pi^2 ab^{3/2}\rho^2 N_c}. \tag{5.11}$$

The quantization rule for the spinor ξ should be chosen as

$$\xi_i\xi_j^\dagger + \xi_j^\dagger\xi_i = \delta_{ij}. \tag{5.12}$$

So the rotation of the spinor ξ is equivalent to a spatial rotation of the heavy vector meson field ϕ_M since $U^{-1}\bar{\sigma}_M U = \Lambda_{MN}\bar{\sigma}_M$, where U and Λ represent the rotation of a spinor and a vector, respectively, e.g., $\xi \to U\xi$, $\phi_M \to \Lambda_{MN}\phi_N$. The parity of ξ is positive, which is opposite to ϕ_M.

The spectrum of Eq. (5.11) is the same as that in Ref. [50]. Since Eq. (5.11) contains only two terms proportional to ρ^{-2}, by comparing Eq. (5.11) with the $\mathcal{H}^L[a_I, X^\alpha]$ presented in Ref. [50] the heavy-light spectrum can be obtained by modifying Q as

$$Q = \frac{N_c}{40ab^{3/2}\pi^2} \to \frac{N_c}{40ab^{3/2}\pi^2}\left[1 - \frac{15}{4N_c}\xi^\dagger\xi + \frac{5}{3N_c^2}(\xi^\dagger\xi)^2\right]. \tag{5.13}$$

Let us use $\mathbf{J}$ and $\mathbf{I}$ to represent the spin and isospin; they are related by

$$\vec{\mathbf{J}} = -\vec{\mathbf{I}} + \vec{\mathbf{S}} = -\vec{\mathbf{I}} + \xi^\dagger\frac{\vec{\tau}}{2}\xi. \tag{5.14}$$

We notice that we have $\mathbf{J}+\mathbf{I}=0$ in the absence of the heavy-light meson, as expected from the spin-flavor hedgehog character. The quantum states for a single bound state, i.e., $N_Q \equiv \xi^\dagger \xi = 1$ and IJ^π assignments are labeled by

$$|N_Q, J_M, l_m, n_z, n_\rho > \quad \text{with} \quad IJ^\pi = \frac{l}{2}\left(\frac{l}{2} \pm \frac{1}{2}\right)^\pi. \tag{5.15}$$

Here n_z, $n_\rho = 0, 1, 2\ldots$ represent the number of quanta associated to the collective motion and the radial breathing of the instanton core, respectively. Following Refs. [44,50], the spectrum of the bound heavy-light state in the D0-D4/D8 system is

$$M_{N_Q} = M_0 + N_Q m_H + M_{KK}\sqrt{\frac{3-b}{3}}(n_\rho + n_z + 1) + M_{KK}\left[\frac{(l+1)^2(3-b)}{12} + \frac{3-b}{15}N_c^2 \times \left(1 - \frac{15}{4N_c}N_Q + \frac{5}{3N_c^2}N_Q^2\right)\right]^{1/2}, \tag{5.16}$$

where M_{KK} is the Kaluza-Klein mass and $M_0 = \frac{\lambda N_c b^{3/2}}{27\pi} M_{KK}$.

1. Single heavy-baryon spectrum

The lowest heavy states with one heavy quark are characterized by $N_Q = 1$, $l = \text{even}$, $N_c = 3$ and n_z, $n_\rho = 0$, 1. So the mass spectrum is given as

$$M_{\text{single}} = M_0 + m_H + M_{KK}(n_\rho + n_z + 1)\sqrt{\frac{3-b}{3}} + M_{KK}\left[\frac{(l+1)^2(3-b)}{12} - \frac{7}{180}(3-b)\right]^{1/2}. \tag{5.17}$$

Let us consider the states with $n_z = n_\rho = 0$ and identify the state with $l = 0$ and the assignments $IJ^\pi = 0\frac{1}{2}^+$ as the heavy-light isosinglet Λ_Q. Then, we identify the state with $l = 2$ and the assignments $IJ^\pi = 1\frac{1}{2}^+, 1\frac{3}{2}^+$ as the heavy-light isotriplets Σ_Q, Σ_Q^*, respectively. Subtracting the nucleon mass M_N (which is identified as the state with $l = 0$ of the light-baryon spectrum) from Eq. (5.17), we have

$$\begin{aligned} M_{\Lambda_Q} - M_N - m_H &\simeq -0.76\sqrt{3-b}M_{KK}, \\ M_{\Sigma_Q} - M_N - m_H &\simeq -0.12\sqrt{3-b}M_{KK}, \\ M_{\Sigma_Q^*} - M_N - m_H &\simeq -0.12\sqrt{3-b}M_{KK}. \end{aligned} \tag{5.18}$$

Thus we see the explicit dependence on the D0 charge in the baryon spectrum in this model. Next, we can study the excited heavy baryons with Eq. (5.17). Let us consider the low-lying breathing modes R ($n_\rho = 1$) with the even assignments $IJ^\pi = 0\frac{1}{2}^+, 1\frac{1}{2}^+, 1\frac{3}{2}^+$, and the odd-parity excited states O ($n_z = 1$) with the even assignments $IJ^\pi = 0\frac{1}{2}^-, 1\frac{1}{2}^-, 1\frac{3}{2}^-$. Using Eq. (5.17), we have ($E = O, R$)

$$\begin{aligned} M_{\Lambda_{EQ'}}(b) &= +0.23M_{\Lambda_Q}(b) + 0.77M_N(b) - 0.23m_H + m'_H, \\ M_{\Sigma_{EQ'}}(b) &= -0.59M_{\Lambda_Q}(b) + 1.59M_N(b) + 0.59m_H + m'_H, \end{aligned} \tag{5.19}$$

where the holographically model-independent relations from Ref. [61],

$$\begin{aligned} M_{\Lambda_{Q'}} &= M_{\Lambda_Q} + m_{H'} - m_H, \\ M_{\Sigma_{Q'}} &= 0.84M_N + m_{H'} + 0.16(M_{\Lambda_Q} - m_H), \end{aligned} \tag{5.20}$$

have been imposed.

2. Double-heavy baryons

Since heavy baryons also contain anti-heavy quarks, let us return to the preceding arguments using the reduction $\Phi_M = \phi_M e^{+im_H x^0}$, in order to consider an anti-heavy-light meson. Most of the calculations are similar except for pertinent minus signs in the effective Lagrangian. In the form of a zero mode, if we bind one heavy-light and one anti-heavy-light meson, the effective Lagrangian now reads

$$\mathcal{L} = \mathcal{L}^L[a_I, X^\alpha] + i\xi_Q^\dagger \partial_t \xi_Q + \frac{3}{32\pi^2 a b^{3/2}\rho^2}\xi_Q^\dagger \xi_Q - i\xi_{\bar{Q}}^\dagger \partial_t \xi_{\bar{Q}} - \frac{3}{32\pi^2 a b^{3/2}\rho^2}\xi_{\bar{Q}}^\dagger \xi_{\bar{Q}} + \frac{(\xi_Q^\dagger \xi_Q - \xi_{\bar{Q}}^\dagger \xi_{\bar{Q}})^2}{24\pi^2 a b^{3/2}\rho^2 N_c}. \tag{5.21}$$

The contributions of the mass from a heavy-light and anti-heavy-light mesons are opposite, as we have indicated. So the mass spectrum for baryons with N_Q heavy quarks and $N_{\bar{Q}}$ anti-heavy quarks can be calculated as

$$M_{Q\bar{Q}} = M_0 + (N_Q + N_{\bar{Q}})m_H + M_{KK}\sqrt{\frac{3-b}{3}}(n_\rho + n_z + 1) + M_{KK}\left\{\frac{(l+1)^2(3-b)}{12} + \frac{3-b}{15}N_c^2 \times \left[1 - \frac{15(N_Q - N_{\bar{Q}})}{4N_c} + \frac{5(N_Q - N_{\bar{Q}})^2}{3N_c^2}\right]\right\}^{1/2}. \tag{5.22}$$

The lowest state ($N_Q = N_{\bar{Q}} = 1, n_\rho = n_z = 0, l = 1$) with the assignments $IJ^\pi = \frac{1}{2}\frac{1}{2}^-, \frac{1}{2}\frac{3}{2}^-$ can be identified as pentaquark baryonic states, and the masses are given as

$$M_{Q\bar{Q}}(b) - M_N(b) - 2m_H = 0, \tag{5.23}$$

which obviously does not depend on the D0 charge.

For the excited pentaquark states, we identify the lowest state as O with odd parity, assignments $IJ^\pi = \frac{1}{2}\frac{1}{2}^+, \frac{1}{2}\frac{3}{2}^+$, and quantum numbers $N_Q = N_{\bar{Q}} = 1, n_\rho = 0, n_z = 1, l = 1$.

The state with quantum numbers $N_Q = N_{\bar{Q}} = 1, n_\rho = 1, n_z = 0, l = 1$ and the same assignments identifies breathing or Roper R pentaquarks as the ground state. So the mass relations for these states are given as ($E = O$, R)

$$M_{EQ\bar{Q}}(b) - M_N(b) - 2m_H \simeq 0.58\sqrt{3-b}M_{KK}. \tag{5.24}$$

On the other hand, the delta-type pentaquarks can be identified as the states with quantum numbers $N_Q = N_{\bar{Q}} = 1, n_\rho = n_z = 0, l = 3$. Altogether, we have one $IJ^\pi = \frac{3}{2}\frac{1}{2}^-$, two $IJ^\pi = \frac{3}{2}\frac{3}{2}^-$, and one $IJ^\pi = \frac{3}{2}\frac{5}{2}^-$ state, so the masses with heavy flavors are given as

$$M_{\Delta Q\bar{Q}}(b) - M_N(b) - 2m_H \simeq 0.42\sqrt{3-b}M_{KK}. \tag{5.25}$$

VI. SUMMARY

Using the Witten-Sakai-Sugimoto model in the D0-D4 background [48,49] and the mechanism proposed in Refs. [59–61], we have extended the analysis in Refs. [50,51] to involve the heavy flavors using a top-down holographic approach to the single- and double-heavy baryon spectra. The heavy-light interaction was introduced into this model by considering a pair of heavy-flavor branes which are separated from the light-flavor branes. The heavy baryon emerges from the zero mode of the reduced vector meson field to order λm_H^0. The binding of the heavy and anti-heavy mesons is equivalent to the instanton configurations of the gauge field on the flavor branes at leading order in λ, even in the presence of the Chern-Simons term. The smeared D0 charge was turned on in the D4-soliton background, so our calculation contains the excited states with nonzero $\mathrm{Tr}[\mathcal{F}\wedge\mathcal{F}]$ or a nonzero θ angle in the dual field theory. The θ dependence is through a parameter b (or $\tilde{\kappa}$) which is monotonically increasing with θ.

Following the quantization in Ref. [61], the bound state moduli gives a rich spectrum. It contains the coupled rotational, vibrational, and translational modes. There are also some newly excited states in the spectrum which have yet to be observed. The charmed pentaquark can be naturally identified as a pair of degenerate heavy isodoublets with $IJ^\pi = \frac{1}{2}\frac{1}{2}^-, \frac{1}{2}\frac{3}{2}^-$ in the spectra when it is extended to the double-heavy baryon case. Our calculation also shows the D0 charge moduli in some new pentaquarks with hidden charm and bottom, and five delta-like pentaquarks with hidden charm in the spectra. Notice that our discussion returns to that in Ref. [61] if $b = 1$, i.e., there is no D0 charge. Particularly for the $b > 3$ case, we notice that the spectrum becomes complex which indicates that baryons cannot be stable, and this is in agreement with the previous study [50,51] of the holographic baryons in this model.

As in most applications of gauge/gravity duality, our analysis was done in the large-N_c and large 't Hooft coupling λ limits, but now with large m_H. Since the baryon spectrum demonstrates the behavior of light baryons shown in Refs. [50,51], we expect that this model will also capture the qualitative $\tilde{\kappa}$ (or θ angle) behavior, at least for small $\tilde{\kappa}$ (or θ angle) in a QCD-like theory when the heavy-light interaction is involved. Although we have compared our results with the real-world nuclei or quark states by setting $N_c = 3$ (as in Refs. [50,51]), there is still a long way to go before we can obtain a realistic baryon spectrum.

ACKNOWLEDGMENTS

I would like to thank Wenhe Cai for her helpful discussion. S. W. L. would like to acknowledge support by the Thousand Young Talents Program and Fudan University.

APPENDIX: THE QUATIZATION OF THE LIGHT FLAVORS

In this appendix, we collect the essential steps to quantize the light-baryon Lagrangian $\mathcal{L}^L$ presented in this manuscript. The details can be systematically reviewed in Refs. [44,50]. With the dimensionless variables, the explicit formula of $\mathcal{L}^L[a_I, X^\alpha]$ is given as

$$\mathcal{L}^L = \mathcal{L}^L_{\mathrm{YM}} + \mathcal{L}^L_{CS}, \tag{A1}$$

where

$$\begin{aligned}\mathcal{L}^L_{\mathrm{YM}} &= -a\lambda N_c b^{1/2}\int d^4x dz H_0^{1/2}(U)\\ &\quad\times\mathrm{Tr}\left[\frac{1}{2}\frac{U_{KK}}{U}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{U^3}{U_{KK}^3}b\mathcal{F}_{\mu z}\mathcal{F}^{\mu z}\right],\\ \mathcal{L}^L_{CS} &= \frac{N_c}{24\pi^2}\mathrm{Tr}\left[\mathcal{A}\wedge\mathcal{F}\wedge\mathcal{F} - \frac{1}{2}\mathcal{A}^3\wedge\mathcal{F} + \frac{1}{10}\mathcal{A}^5\right].\end{aligned} \tag{A2}$$

For the two-flavor case, the $U(2)$ gauge field $\mathcal{A}$ can be decomposed into a $SU(2)$ part A and a $U(1)$ part $\hat{A}$ as

$$\mathcal{A} = A + \frac{1}{2}\hat{A}, \tag{A3}$$

whose gauge field strength is

$$\mathcal{F} = F + \frac{1}{2}\hat{F}. \tag{A4}$$

In the large-λ limit, imposing the λ rescale (3.14), the equation of motion from Eq. (A2) can be obtained as

$$\begin{aligned}&D_M F_{MN} + \mathcal{O}(\lambda^{-1})\\ &\quad= 0, D_M F_{0M} + \frac{1}{64\pi^2 ab^{3/2}}\epsilon_{MNPQ}\hat{F}_{MN}F_{PQ} + \mathcal{O}(\lambda^{-1}) = 0,\\ &\partial_M\hat{F}_{0M} + \mathcal{O}(\lambda^{-1})\\ &\quad= 0, \partial_M\hat{F}_{0M} + \frac{1}{64\pi^2 ab^{3/2}}\epsilon_{MNPQ}\mathrm{Tr}[F_{MN}F_{PQ}]\\ &\qquad+ \mathcal{O}(\lambda^{-1}) = 0,\end{aligned} \tag{A5}$$

and the solution is given in Eq. (4.7).

In order to obtain the spectrum, we require the moduli of the solution to be time dependent, i.e.,

$$X^\alpha, \quad a^I \to X^\alpha(t), \quad a^I(t). \tag{A6}$$

Here $a^I(t)$ refers to the $SU(2)$ orientation. So the $SU(2)$ gauge transformation also becomes time dependent,

$$\begin{aligned}&\mathcal{A}_M \to V(\mathcal{A}_M^{cl} - i\partial_M)V^{-1},\\&\mathcal{F}_{MN} \to V\mathcal{F}_{MN}^{cl}V^{-1},\\&F_{0M} \to V(\dot{X}^\alpha\partial_\alpha\mathcal{A}_M^{cl} - D_M^{cl}\Phi)V^{-1},\end{aligned} \tag{A7}$$

where $\Phi = -iV^\dagger\partial_0 V, V^\dagger = V^{-1}$.

The motion of the collective coordinates could be characterized by the effective Lagrangian in the moduli space. Up to $\mathcal{O}(\lambda^{-1})$, it is

$$\begin{aligned}L &= \frac{1}{2}m_X g_{\alpha\beta}\dot{X}^\alpha\dot{X}^\beta - U(X^\alpha) + \mathcal{O}(\lambda^{-1})\\&= \frac{1}{2}m_X\dot{\vec{X}}^2 + \frac{1}{2}m_Z\dot{Z}^2 + \frac{1}{2}m_y\dot{y}_I\dot{y}_I - U(X^\alpha),\end{aligned} \tag{A8}$$

where a dot represents a derivative with respect to t, $g_{\alpha\beta}$ is the metric of the moduli space parametrized by X^α which satisfies $ds^2 = g_{\alpha\beta}dX^\alpha dX^\beta = d\vec{X}^2 + dZ^2 + 2dy_I dy_I$, and $\sum_{I=1}^4 y_I y_I = \rho^2$. $U(X^\alpha)$ is the effective potential associated to the on-shell Lagrangian with the instanton solution (4.7), i.e.,

$$\int d^3x dz \mathcal{L}^L[a_I, X^\alpha]_{\text{onshell}} = -U(X^\alpha). \tag{A9}$$

The baryon spectrum can be obtained by quantizing Eq. (A8) (soliton) at rest. The quantization procedure simply replaces the momenta in the Lagrangian with the corresponding differential operators, which can act on the wave function of baryon states. So the quantized Hamiltonian associated to Eq. (A8) is

$$\begin{aligned}&H = H_X + H_Z + H_y,\\&H_X = \frac{1}{2m_X}P_X^2 + M_0 = -\frac{1}{2m_X}\sum_{i=1}^3\frac{\partial^2}{\partial X_i^2} + M_0,\\&H_Z = \frac{1}{2m_Z}P_Z^2 + \frac{1}{2}m_Z\omega_Z^2 Z^2 = -\frac{1}{2m_Z}\frac{\partial^2}{\partial Z^2} + \frac{1}{2}m_Z\omega_Z^2 Z^2,\\&H_y = \frac{1}{2m_y}P_y^2 + \frac{1}{2}m_y\omega_y^2\rho^2 + \frac{Q}{\rho^2}\\&\quad = -\frac{1}{2m_y}\sum_{I=1}^4\frac{\partial^2}{\partial y_I^2} + \frac{1}{2}m_y\omega_y^2\rho^2 + \frac{Q}{\rho^2}.\end{aligned} \tag{A10}$$

In units of $U_{KK} = M_{KK} = 1$, or equivalently with the replacements $z \to zU_{KK}$, $x^\mu \to x^\mu/M_{KK}$, $\boldsymbol{\mathcal{A}}_z \to \boldsymbol{\mathcal{A}}_z/U_{KK}$, and $\boldsymbol{\mathcal{A}}_\mu \to \boldsymbol{\mathcal{A}}_\mu M_{KK}$, we have the following dimensionless values:

$$\begin{aligned}&M_0 = 8\pi^2\lambda ab^{3/2}N_c, \qquad \omega_Z = \frac{1}{3}(3-b),\\&\omega_y = \frac{1}{12}(3-b), \qquad Q = \frac{N_c}{40\pi^2 ab^{3/2}}.\end{aligned} \tag{A11}$$

The eigenstates of H_Z are nothing but harmonic-oscillator states. The eigenfunctions of H_y are represented by $T^l(a_I)R_{l,n_\rho}(\rho)$, where $T^l(a_I)$ are the spherical harmonic functions on S^3. They are in the representations of $(\frac{l}{2},\frac{l}{2})$ under the transformation of $SO(4) = SU(2)\times SU(2)/Z_2$. The former $SU(2)$ corresponds to the isometric rotation, while the latter is the space rotation in $\{x^i\}$. The states with $I = J = \frac{l}{2}$ are described by this quantization, so the nucleon state is realized as the lowest state with $l = 1, n_\rho = n_z = 0$ of the Hamiltonian (A10).

[1] E. V. Shuryak, Hadrons containing a heavy quark and QCD sum rules, Nucl. Phys. **B198**, 83 (1982); N. Isgur and M. B. Wise, Spectroscopy with Heavy-Quark Symmetry, Phys. Rev. Lett. **66**, 1130 (1991); A. V. Manohar and M. B. Wise, *Heavy Quark Physics* (Cambridge University Press, Cambridge, England, 2000).

[2] B. Aubert *et al.* (*BABAR* Collaboration), Observation of a narrow meson decaying to $D_s^+\pi^0$ at a mass of 2.32 GeV$/c^2$, Phys. Rev. Lett. **90**, 242001 (2003).

[3] D. Besson *et al.* (CLEO Collaboration), Observation of a narrow resonance of mass 2.46 GeV$/c^2$ decaying to $D_s^{*+}\pi^0$ and confirmation of the $D_{sJ}^*(2317)$ state, Phys. Rev. D **68**, 032002 (2003).

[4] M. A. Nowak, M. Rho, and I. Zahed, Chiral effective action with heavy quark symmetry, Phys. Rev. D **48**, 4370 (1993).

[5] M. A. Nowak, M. Rho, and I. Zahed, Chiral doubling of heavy-light hadrons: *BABAR* 2317 MeV and CLEO 2463 MeV discoveries, Acta Phys. Pol. B **35**, 2377 (2004).

[6] W. A. Bardeen and C. T. Hill, Chiral dynamics and heavy quark symmetry in a toy field theoretic model, Phys. Rev. D **49**, 409 (1994).

[7] W. A. Bardeen, E. J. Eichten, and C. T. Hill, Chiral multiplets of heavy-light mesons, Phys. Rev. D **68**, 054024 (2003).

[8] D0 Collaboration, Evidence for a $B_s^0\pi^\pm$ state, Phys. Rev. Lett. **117**, 022003 (2016).

[9] LHCb Collaboration, Observation of $J/\psi\varphi$ structures consistent with exotic states from amplitude analysis of $B^+ \to J/\psi\varphi K^+$ decays, Phys. Rev. Lett. **118**, 022003 (2017).
[10] LHCb Collaboration, Amplitude analysis of $B^+ \to J/\psi\varphi K^+$ decays, Phys. Rev. D **95**, 012002 (2017).
[11] LHCb Collaboration, Observation of $J/\psi p$ resonances consistent with pentaquark states in $\Lambda_b^0 \to J/\psi K^- p$ decays, Phys. Rev. Lett. **115**, 072001 (2015).
[12] LHCb Collaboration, Model-independent evidence for $J/\psi p$ contributions to $\Lambda_b^0 \to J/\psi p K^-$ decays, Phys. Rev. Lett. **117**, 082002 (2016).
[13] LHCb Collaboration, Evidence for exotic hadron contributions to $\Lambda_b^0 \to J/\psi p \pi^-$ decays, Phys. Rev. Lett. **117**, 082003 (2016).
[14] LHCb Collaboration, Observation of five new narrow Ω_c^0 states decaying to $\Xi_c^+ K^-$, Phys. Rev. Lett. **118**, 182001 (2017).
[15] D. Kharzeev, R. D. Pisarski, and M. H. G. Tytgat, Possibility of spontaneous parity violation in hot QCD, Phys. Rev. Lett. **81**, 512 (1998).
[16] D. Kharzeev, R. D. Pisarski, and M. H. G. Tytgat, Parity odd bubbles in hot QCD, arXiv:hep-ph/9808366.
[17] D. E. Kharzeev, R. D. Pisarski, and M. H. G. Tytgat, Aspects of parity, *CP*, and time reversal violation in hot QCD, in *Strong and Electroweak Matter 2000: Proceedings of the SEWM2000 Meeting, Marseille, France, 13–17 June, 2000*, edited by C. P. Korthals Altes (World Scientific, Singapore, 2001), p. 32.
[18] K. Buckley, T. Fugleberg, and A. Zhitnitsky, Can induced Θ vacua be created in heavy-ion collisions?, Phys. Rev. Lett. **84**, 4814 (2000).
[19] D. Kharzeev, Parity violation in hot QCD: Why it can happen, and how to look for it, Phys. Lett. B **633**, 260 (2006).
[20] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, The effects of topological charge change in heavy ion collisions: Event by event *P* and *CP* violation, Nucl. Phys. **A803**, 227 (2008).
[21] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, The chiral magnetic effect, Phys. Rev. D **78**, 074033 (2008).
[22] D. E. Kharzeev, The chiral magnetic effect and anomaly-induced transport, Prog. Part. Nucl. Phys. **75**, 133 (2014).
[23] L. Del Debbio, G. Mario Manca, H. Panagopoulos, A. Skouroupathis, and E. Vicari, θ dependence of the spectrum of SU(N) gauge theories, J. High Energy Phys. 06 (2006) 005.
[24] M. D'Elia and F. Negro, θ dependence of the deconfinement temperature in Yang-Mills theories, Phys. Rev. Lett. **109**, 072001 (2012).
[25] M. D'Elia and F. Negro, Phase diagram of Yang-Mills theories in the presence of a θ term, Phys. Rev. D **88**, 034503 (2013).
[26] E. Witten, Theta dependence in the large N limit of four-dimensional gauge theories, Phys. Rev. Lett. **81**, 2862 (1998).
[27] E. Vicari and H. Panagopoulos, θ dependence of SU(N) gauge theories in the presence of a topological term, Phys. Rep. **470**, 93 (2009).
[28] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri, and Y. Oz, Large N field theories, string theory and gravity, Phys. Rep. **323**, 183 (2000).
[29] J. M. Maldacena, The large N limit of superconformal field theories and supergravity, Int. J. Theor. Phys. **38**, 1113 (1999).
[30] E. Witten, Anti–de Sitter space, thermal phase transition, and confinement in gauge theories, Adv. Theor. Math. Phys. **2**, 505 (1998).
[31] T. Sakai and S. Sugimoto, Low energy hadron physics in holographic QCD, Prog. Theor. Phys. **113**, 843 (2005).
[32] T. Sakai and S. Sugimoto, More on a holographic dual of QCD, Prog. Theor. Phys. **114**, 1083 (2005).
[33] E. Witten, Baryons and branes in anti–de Sitter space, J. High Energy Phys. 07 (1998) 006.
[34] D. J. Gross and H. Ooguri, Aspects of large N gauge theory dynamics as seen by string theory, Phys. Rev. D **58**, 106002 (1998).
[35] O. Aharony, J. Sonnenschein, and S. Yankielowicz, A holographic model of deconfinement and chiral symmetry restoration, Ann. Phys. (Amsterdam) **322**, 1420 (2007).
[36] O. Bergman, G. Lifschytz, and M. Lippert, Holographic nuclear physics, J. High Energy Phys. 11 (2007) 056.
[37] A. Rebhan, The Witten-Sakai-Sugimoto model: A brief review and some recent results, EPJ Web Conf. **95**, 02005 (2015).
[38] S.-w. Li, A. Schmitt, and Q. Wang, From holography towards real-world nuclear matter, Phys. Rev. D **92**, 026006 (2015).
[39] N. R. Constable and R. C. Myers, Spin-two glueballs, positive energy theorems and the AdS/CFT correspondence, J. High Energy Phys. 10 (1999) 037.
[40] R. C. Brower, S. D. Mathur, and C.-I. Tan, Glueball spectrum for QCD from AdS supergravity duality, Nucl. Phys. **B587**, 249 (2000).
[41] K. Hashimoto, C.-I. Tan, and S. Terashima, Glueball decay in holographic QCD, Phys. Rev. D **77**, 086001 (2008).
[42] F. Brünner, D. Parganlija, and A. Rebhan, Glueball decay rates in the Witten-Sakai-Sugimoto model, Phys. Rev. D **91**, 106002 (2015).
[43] S.-w. Li, Glueball-baryon interactions in holographic QCD, Phys. Lett. B **773**, 142 (2017).
[44] H. Hata, T. Sakai, S. Sugimoto, and S. Yamato, Baryons from instantons in holographic QCD, Prog. Theor. Phys. **117**, 1157 (2007).
[45] H. Liu and A. A. Tseytlin, D3-brane-D-instanton configuration and $N = 4$ super YM theory in constant self-dual background, Nucl. Phys. **B553**, 231 (1999).
[46] J. L. F. Barbon and A. Pasquinucci, Aspects of instanton dynamics in AdS/CFT duality, Phys. Lett. B **458**, 288 (1999).
[47] K. Suzuki, D0-D4 system and $(3 + 1)$-dimensional QCD, Phys. Rev. D **63**, 084011 (2001).
[48] C. Wu, Z. Xiao, and D. Zhou, Sakai-Sugimoto model in D0-D4 background, Phys. Rev. D **88**, 026016 (2013).
[49] S. Seki and S.-J. Sin, A new model of holographic QCD and chiral condensate in dense matter, J. High Energy Phys. 10 (2013) 223.
[50] W. Cai, C. Wu, and Z. Xiao, Baryons in the Sakai-Sugimoto model in the D0-D4 background, Phys. Rev. D **90**, 106001 (2014).
[51] S.-w. Li and T. Jia, Matrix model and holographic baryons in the D0-D4 background, Phys. Rev. D **92**, 046007 (2015).
[52] S.-w. Li and T. Jia, Three-body force for baryons from the D0-D4/D8 matrix model, Phys. Rev. D **93**, 065051 (2016).

[53] W. Cai, K.-l. Li, and S.-w. Li, Electromagnetic instability and Schwinger effect in holographic QCD with a topological charge, arXiv:1612.07087.

[54] F. Bigazzi, A. L. Cotrone, and R. Sisca, Notes on theta dependence in holographic Yang-Mills, J. High Energy Phys. 08 (2015) 090.

[55] L. Bartolini, F. Bigazzi, S. Bolognesi, A. L. Cotrone, and A. Manenti, Theta dependence in holographic QCD, J. High Energy Phys. 02 (2017) 029.

[56] W. Cai and S.-w. Li, Sound waves in the compactified D0-D4 brane system, Phys. Rev. D **94**, 066012 (2016).

[57] C. Wu, Y. Chen, and M. Huang, Chiral vortical effect from the compactified D4-branes with smeared D0-brane charge, J. High Energy Phys. 03 (2017) 082.

[58] S. Sugimoto and K. Takahashi, QED and string theory, J. High Energy Phys. 04 (2004) 051.

[59] Y. Liu and I. Zahed, Holographic heavy-light chiral effective action, Phys. Rev. D **95**, 056022 (2017).

[60] Y. Liu and I. Zahed, Heavy-light mesons in chiral AdS/QCD, Phys. Lett. B **769**, 314 (2017).

[61] Y. Liu and I. Zahed, Heavy baryons and their exotics from instantons in holographic QCD, Phys. Rev. D **95**, 116012 (2017).

[62] N. N. Scoccola, D. O. Riska, and M. Rho, On the pentaquark candidates $P_c^+(4380)$ and $P_c^+(4450)$ within the soliton picture of baryons, Phys. Rev. D **92**, 051501 (2015).

[63] E. Bergshoeff, A. Collinucci, A. Ploegh, S. Vandoren, and T. Van Riet, Non-extremal D-instantons and the AdS/CFT correspondence, J. High Energy Phys. 01 (2006) 061.

[64] K. Hashimoto, N. Iizuka, and P. Yi, A matrix model for baryons and nuclear forces, J. High Energy Phys. 10 (2010) 003.

[65] A. Paredes and P. Talavera, Multiflavour excited mesons from the fifth dimension, Nucl. Phys. **B713**, 438 (2005).

[66] J. Erdmenger, N. Evans, and J. Grosse, Heavy-light mesons from the AdS/CFT correspondence, J. High Energy Phys. 01 (2007) 098.

[67] J. Erdmenger, K. Ghoroku, and I. Kirsch, Holographic heavy-light mesons from non-Abelian DBI, J. High Energy Phys. 09 (2007) 111.

[68] C. P. Herzog, S. A. Stricker, and A. Vuorinen, Remarks on heavy-light mesons from AdS/CFT, J. High Energy Phys. 05 (2008) 070.

[69] Y. Bai and H.-C. Cheng, A holographic model of heavy-light mesons, J. High Energy Phys. 08 (2013) 074.

[70] K. Hashimoto, N. Ogawa, and Y. Yamaguchi, Holographic heavy quark symmetry, J. High Energy Phys. 06 (2015) 040.

[71] J. Sonnenschein and D. Weissman, A tetraquark or not a tetraquark? A holography inspired stringy hadron (HISH) perspective, Nucl. Phys. **B920**, 319 (2017).

[72] N. Itzhaki, I. R. Klebanov, P. Ouyang, and L. Rastelli, Is $\Theta^+(1540)$ a kaon-Skyrmion resonance?, Nucl. Phys. **B684**, 264 (2004).

[73] R. C. Myers, Dielectric-branes, J. High Energy Phys. 12 (1999) 022.