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UV completion of five-dimensional scalar QED and Lorentz symmetry

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We study a five-dimensional Horava-Lifshitz–like scalar QED with dynamical exponent $z = 2$. Consistency of the renormalization procedure requires the presence of four quartic and one sixfold scalar couplings besides the terms bilinear in the scalar fields. We compute one-loop radiative corrections to the parameters in the original Lagrangian, employing dimensional regularization in the spatial part of the Feynman integrals and prove the relevant Ward identities. By using renormalization group methods, we determine the behavior of the coupling constants with changes in the energy and discuss the emergence of Lorentz symmetry at low energies.

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I. INTRODUCTION

The use of Lagrangians exhibiting space-time anisotropy and equipped with high spatial derivative terms, Horava-Lifshitz (HL)-like models [1,2], has attracted considerable attention in the recent years. This is because they allow for an ultraviolet completion of otherwise nonrenormalizable models and, in particular, may lead to a consistent quantum gravity theory [1,3]. It should be noticed that originally high spatial derivatives were used in the description of Lifshitz points in statistical mechanics studies [2]. Further applications to statistical mechanics and condensed matter may be found in [4,5].

A considerable amount of work has been devoted to study different facets of HL models. These studies encompass quantum gravitational issues, such as black holes [6], renormalization features [7], and other aspects [8]. Besides that, many studies have also been dedicated to nongravitational models [9]. In particular, for scalar models, renormalization aspects have been treated in [10,11], gauge theories similar to QED were studied in [12,13], and Ward identities and anomalies were considered in [14–16].

The basic assumption behind these proposals is that, asymptotically, the equations of motion are invariant under the rescaling $x^i \to bx^i$, $t \to b^z t$, where z, the so-called dynamical critical exponent, is related with the ultraviolet behavior of the models. As space-time anisotropy breaks Lorentz symmetry, to physically validate HL models at the low-energy scale of the present Universe, it is necessary to prove that Lorentz invariance is at least approximately realized at small energies. Renormalization group arguments indicate that, to achieve this behavior, it is required that the effective coefficients of the high derivative terms in the Lagrangian should monotonically decrease as the energy decreases.

In the last two decades, models in more than four dimensions have aroused a great deal of interest (see [17] and references therein). The reason is that compactification of extra dimensions introduces new scales and new physics in the desert separating the electroweak unification scale (10^2 GeV) from the Planck scale (10^{19} GeV) of the quantization of gravitation, the hierarchy problem. However, usual quantum field models ($z = 1$) are, in general, nonrenormalizable in more than four dimensions. This work is dedicated to the study of $z = 2$ scalar quantum electrodynamics in five-dimensions, the highest dimension where this model is renormalizable. Actually, the model is super renormalizable or nonrenormalizable for dimensions lower or higher than five, respectively.

We would like to point out some earlier studies related to this subject. Reference [13] provided a study of Lorentz symmetry restoration and a discussion of anomalies in a four-dimensional HL-like spinor and scalar QED. That work was followed by [18], in which, also in four dimensions, the anomalous magnetic momentum was determined and a complete one-loop renormalization analysis was presented. In five-dimensions, we are aware of the work [12] on spinor HL-like QED showing that a great simplification occurs at very high energies where the usual spatial terms, i.e., linear terms in the spatial derivatives, may be neglected; in particular, the gauge coupling constant is not renormalized. This simplicity was also pursued in [13,18], the usual term being also absent. Differently, we consider here the dynamics of the more general renormalizable scalar model obeying gauge symmetry and charge conjugation. The presence of usual terms, quadratic in the spatial derivatives, make the complete calculation of the Green functions infeasible. In spite of this, it is still possible to obtain one-loop renormalization constants that allow for the determination of relevant renormalization group β functions. Using these results, we analyze the evolution of the parameters of the theory and determine a range for which Lorentz symmetry may be restored.

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One possible usefulness of this work is the following. five-dimensional scalar QED with $z=1$ is nonrenormalizable; in this situation, one may still use it as an effective theory for small energies up to some scale Λ. To fix Λ, we may consider the Lagrangian with $z=2$, which is renormalizable but breaks Lorentz symmetry. However, if we could find, for small energies, a range of values for which the Lorentz symmetry is approximately realized, we may take these energies as the ones where the effective theory with $z=1$ is approximately correct.

This work is organized as follows. In Sec. II we introduce the model, state the Feynman rules needed to compute the radiative corrections, present the degree of superficial divergence, and show the results for the one-loop vertex functions. Explicit calculations of the divergences and renormalization are provided in the Appendix, where we also verified the Ward identities obeyed by the vertex functions. In Sec. III, by using renormalization group methods, the relevant β functions are computed. Finally, in Sec. IV we present a summary and the conclusions of this work.

II. THE MODEL

In this work, we study a $z=2$ version of five-dimensional scalar QED described by the Lagrangian density

$$\mathcal{L}=\frac{1}{2}F_{0i}F_{0i}-\frac{a_1^2}{4}F_{ij}F_{ij}-\frac{a_2^2}{4}\partial_l F_{ij}\partial_l F_{ij}+(D_0\phi)^*D_0\phi - b_1^2(D_i\phi)^*D_i\phi - b_2^2(D_iD_j\phi)^*D_iD_j\phi - m^2\phi^*\phi - ieb_3^2F_{ij}(D_i\phi)^*D_j\phi-\frac{e^2}{2}b_4^2F_{ij}F_{ij}\phi^*\phi, \quad (1)$$

where $D_{0,i}=\partial_{0,i}-ieA_{0,i}$ is the gauge covariant derivative. The parameters a_2 and b_i with $i=2,3,4$, which control the high derivative terms, are taken to be dimensionless in momentum units. From that, and taking into account the dimension six of $\mathcal{L}$, we get that the dimensions of ϕ and A_i are equal to one, whereas the dimension of A_0 is two. The parameters a_1^2, b_1^2, and m have dimension two and e is dimensionless. The above expression is the most general gauge invariant Lagrangian containing, at most, two scalar fields. Integrating by parts, other possible terms, for example, $\partial_j F_{ij}\partial_l F_{il}$, may be reduced to the ones in (1).

We choose to work in a strict Coulomb gauge by adding to (1) the gauge fixing Lagrangian

$$\mathcal{L}_{\rm GF}=\frac{\eta}{2}(\partial_i A_i)^2 \quad (2)$$

and letting η tend to infinity. Notice that gauge invariance and charge conjugation ($\phi\leftrightarrow\phi^*$ and $A_\mu\to -A_\mu$) forbid the appearance of pure gauge monomials, without scalar field factors and containing more than two gauge fields. However, we will show shortly that terms with four and six scalar fields have to be included. Using the above Lagrangian, we obtain the propagators and interacting vertices:

(1) For the gauge field,

$$\langle TA_i(k)A_j(-k)\rangle = i\frac{\delta_{ij}-\frac{k_ik_j}{\vec{k}^2}}{k_0^2-a_1^2\vec{k}^2-a_2^2\vec{k}^4+i\epsilon};$$
$$\langle TA_0(k)A_0(-k)\rangle=\frac{i}{\vec{k}^2}; \quad (3)$$

and $\langle TA_0(k)A_i(-k)\rangle=0$.

(2) For the scalar field,

$$\langle T\phi(k)\phi^*(-k)\rangle=\frac{i}{k_0^2-b_1^2\vec{k}^2-b_2^2\vec{k}^4-m^2+i\epsilon}. \quad (4)$$

There are four three-linear vertices, which we label as V_{3X}, $X=A, B, C, D$. By taking the Fourier transforms of these interaction terms and taking the momenta always entering at the vertex, one finds their expressions in momenta space to be

$$V_{3A}(p,k,k')=eA_0(p)\phi(k)\phi^*(k')\times(p_0+2k_0), \quad (5)$$

$$V_{3B}(p,k,k')=-eb_1^2A_i(p)\phi(k)\phi^*(k')\times(p_i+2k_i), \quad (6)$$

$$V_{3C}(p,k,k')=-eb_2^2A_i(p)\phi(k)\phi^*(k')\times(p_j+2k_j)\{(p_i+k_i)(p_j+k_j)+k_ik_j\}, \quad (7)$$

$$V_{3D}(p,k,k')=-eb_3^2A_i(p)\phi(k)\phi^*(k')\times\{k_i(\vec{k}'^2+\vec{k}'\cdot\vec{k})-k_i'(\vec{k}^2+\vec{k}'\cdot\vec{k})\}, \quad (8)$$

where $k'=-k-p$. There are also five four-linear vertices,

$$V_{4A}(p,p',k,k')=e^2A_0(p)A_0(p')\phi(k)\phi^*(k'), \quad (9)$$

$$V_{4B}(p,p',k,k')=-e^2b_1^2A_i(p)A_i(p')\phi(k)\phi^*(k'), \quad (10)$$

$$V_{4C}(p,p',k,k')=-e^2b_2^2A_i(p)A_j(p')\phi(k)\phi^*(k')\times\{k_ik_j+k_i'k_j'-k_i'k_j-k_ik_j' -2\vec{k}\cdot\vec{k}'\delta_{ij}-\vec{p}\cdot\vec{p}'\delta_{ij}-p_i'(k_j+k_j') -\vec{p}'\cdot(\vec{k}+\vec{k}')\delta_{ij}\}, \quad (11)$$

$$V_{4D}(p,p',k,k')=-e^2b_3^2A_i(p)A_j(p')\phi(k)\phi^*(k')\times\{p_i'(k_j+k_j')-\delta_{ij}\vec{p}'\cdot(\vec{k}+\vec{k}')\} \quad (12)$$

$$V_{4E}(p,p',k,k') = -e^2 b_4^2 A_i(p)A_j(p')\phi(k)\phi^*(k') \times \{p'_i p_j - \delta_{ij}\vec{p'}\cdot\vec{p}\}, \quad (13)$$

where the momenta satisfy $k' = -k - p - p'$. There is also a vertex with five fields given by

$$V_5(p_1,p_2,p_3,k,k') = -2e^3 b_2^2 A_i(p_1)A_i(p_2)A_j(p_3)\phi(k) \times \phi^*(k')(k_j - k'_j), \quad (14)$$

where $p_1 + p_2 + p_3 + k + k' = 0$, and a vertex with six fields,

$$V_6(p_1,p_2,p_3,p_4,k,k') = e^4 A_i(p_1)A_i(p_2)A_j(p_3)A_j(p_4)\phi(k)\phi^*(k'), \quad (15)$$

with the momenta satisfying $\sum_{i=1}^4 p_i + k + k' = 0$.

By using these expressions, we may compute the degree of superficial divergence for a generic Feynman diagram γ

$$\delta(\gamma) = 6 - N_\phi - N_{A_i} - 2N_{A_0} - 2\nu_{3B} - 2\nu_{4B}, \quad (16)$$

where $N_{\mathcal{O}}$ denotes the number of external lines of the field $\mathcal{O}$ and $\nu_{\mathcal{O}}$ is the number of vertices of the type $V_{\mathcal{O}}$ in γ. Notice from (16) that graphs without external gauge field lines, but either with four or six external scalar lines, are quadratically and logarithmically divergent, respectively. Therefore, for consistency of the renormalization process, one should enlarge our model and add to (1) the terms given by

$$\mathcal{L}_\phi = \xi_1[\phi^*(D_iD_i\phi) + (D_iD_i\phi)^*\phi]\phi^*\phi + \xi_2[\phi^*(D_i\phi)\phi^*(D_i\phi) + (D_i\phi)^*\phi(D_i\phi)^*\phi] + \xi_3\phi^*(D_i\phi)(D_i\phi)^*\phi - \frac{\lambda}{4}(\phi^*\phi)^2 - \frac{g}{6}(\phi^*\phi)^3. \quad (17)$$

Notice that, except for the term with λ, all these vertices have ultraviolet dimension six and are therefore renormalizable; they do not modify the power counting given in (16). The vertex with the coupling λ has operator dimension four, it is super renormalizable, and modifies the power counting; a term $-2\nu_\lambda$ has to be added to the rhs of (16). To keep the ultraviolet divergences under control, the spatial part of the Feynman integrals will be regularized to $d = 4 - \epsilon$ dimensions. It is also convenient to introduce a parameter μ with momentum dimension two and make the following replacements:

$$e \to e\mu^{\epsilon/4}, \qquad \lambda \to \lambda\mu^{\epsilon/2}, \qquad g \to g\mu^{\epsilon}, \qquad \xi_n \to \xi_n\mu^{\epsilon/2}, \quad (18)$$

with $n = 1, 2, 3$. After the pole part of the integrals have been removed, we will let $\epsilon \to 0$. In the Appendix, we determined the counterterms needed to eliminate these would be divergences. Using those results, we obtain the gauge field two-point vertex functions

$$\Gamma_{00}^{(2)}(p) = \left(1 + \frac{\alpha}{8}\ln\mu\right)\vec{p}^2 + (\text{finite part}), \quad (19)$$

$$\Gamma_{0i}^{(2)}(p) = \Gamma_{i0}^{(2)} = \left(1 + \frac{\alpha}{8}\ln\mu\right)p_0 p_i + (\text{finite part}), \quad (20)$$

and

$$\Gamma_{ij}^{(2)}(p) = \left(1 + \frac{\alpha}{8}\ln\mu\right)\delta_{ij}p_0^2 - \left(a_1^2 - \frac{\alpha}{8}R\ln\mu\right)(\delta_{ij}\vec{p}^2 - p_ip_j) - \left(a_2^2 - \frac{\alpha}{8}S\ln\mu\right)(\delta_{ij}\vec{p}^2 - p_ip_j)\vec{p}^2 + (\text{finite part}), \quad (21)$$

where $\alpha = \frac{e^2}{16\pi^2 b_2}$, and R and S are defined in (A12).

It should be stressed that the two-point vertex function of the gauge field that we are considering is restricted to its transverse part because its longitudinal part is meaningless.

We have also

$$\Gamma^{(2)}(p) = \left(1 - \frac{\alpha}{2}\ln\mu\right)p_0^2 - \left(b_1^2 - \frac{Q_1}{2}\ln\mu\right)\vec{p}^2 - \left(b_2^2 - \frac{Q_2}{2}\ln\mu\right)\vec{p}^4 - \left(m^2 - \frac{Q_3}{2}\ln\mu\right) + (\text{finite part}), \quad (22)$$

for the renormalized two-point function of the scalar field, with Q_1, Q_2, and Q_3 given in (A18)–(A20),

$$\Gamma_0^{(3)}(p-p') = e\left(1 - \frac{\alpha}{2}\ln\mu\right)(p_0 + p'_0) + (\text{finite part}), \quad (23)$$

for the three-point vertex function $\langle A_0\phi^*\phi\rangle$, and

$$\Gamma_i^{(3)}(p,-p') = e\left(b_1^2 - \frac{Q_1}{2}\ln\mu\right)(p_i + p'_i) \times e\left(b_2^2 - \frac{Q_2}{2}\ln\mu\right)(p_i(\vec{p}^2 + \vec{p}\cdot\vec{p'}) + p'_i(\vec{p'}^2 + \vec{p}\cdot\vec{p'})) \times e\left(b_3^2 - \frac{K_3}{2}\ln\mu\right)(p_i(\vec{p'}^2 - \vec{p}\cdot\vec{p'}) + p'_i(\vec{p}^2 - \vec{p}\cdot\vec{p'})) + (\text{finite part}), \quad (24)$$

for the three-point vertex function $\langle A^i\phi^*\phi\rangle$, with K_3 defined in (A27). As argued in the Appendix, these functions satisfy the simplest Ward identities associated with current conservation.

In the next section, we employ these expressions to find some of the β functions of the model.

III. RENORMALIZATION GROUP AND EFFECTIVE COUPLINGS

We may now fix the renormalization group flows of the parameters of the model. The vertex functions $\Gamma^{(N_{A_0},N_{A_i},N_\phi)}(p)$ satisfy the 't Hooft–Weinberg renormalization group equation

$$\left[\mu\frac{\partial}{\partial\mu}+a_1\beta_{a_1}\frac{\partial}{\partial a_1}+\beta_{a_2}\frac{\partial}{\partial a_2}+b_1\beta_{b_1}\frac{\partial}{\partial b_1}+\beta_{b_2}\frac{\partial}{\partial b_2}+\beta_{b_3}\frac{\partial}{\partial b_3}+\beta_{b_4}\frac{\partial}{\partial b_4}+\beta_e\frac{\partial}{\partial e}+\lambda\beta_\lambda\frac{\partial}{\partial\lambda}+\beta_g\frac{\partial}{\partial g}+\sum_{n=1}^{3}\beta_{\xi_n}\frac{\partial}{\partial\xi_n}+m^2\delta\frac{\partial}{\partial m^2}-\gamma_\Gamma\right]\Gamma^{(N)}=0, \tag{25}$$

where $2\gamma_\Gamma = N_\phi\gamma_\phi + N_{A_0}\gamma_{A_0} + N_{A_i}\gamma_{A_i}$, and

$$\beta_{a_1}=\frac{\mu}{a_1}\frac{da_1}{d\mu},\quad \beta_{a_2}=\mu\frac{da_2}{d\mu},\quad \beta_{b_1}=\frac{\mu}{b_1}\frac{db_1}{d\mu},$$
$$\beta_{b_2}=\mu\frac{db_2}{d\mu},\quad \beta_{b_3}=\mu\frac{db_3}{d\mu},\quad \beta_{b_4}=\mu\frac{db_4}{d\mu},$$
$$\beta_e=\mu\frac{de}{d\mu},\quad \beta_\lambda=\frac{\mu}{\lambda}\frac{d\lambda}{d\mu},\quad \beta_g=\mu\frac{dg}{d\mu},$$
$$\beta_{\xi_n}=\mu\frac{d\xi_n}{d\mu},\quad \delta=\frac{\mu}{m^2}\frac{dm^2}{d\mu},\quad\text{and}\quad \gamma_\Gamma=\frac{\mu}{Z_\Gamma}\frac{dZ_\Gamma}{d\mu}. \tag{26}$$

To obtain the above functions, we proceed as follows. We substitute the vertex functions listed in the previous section in the renormalization group equation and equate to zero the coefficient of each power of the momentum and each power of the coupling constants. In the case of the pure gauge functions, for instance, we determine

$$\beta_{a_1}=\frac{\alpha}{16}\left[\frac{R+a_1^2}{a_1^2}\right]\quad\text{and}\quad\beta_{a_2}=\frac{\alpha}{16}\left[\frac{S+a_2^2}{a_2}\right], \tag{27}$$

and also

$$\gamma_A\equiv\gamma_{A_0}=\gamma_{A_i}=\frac{\alpha}{8}. \tag{28}$$

Furthermore, by inserting the scalar field two-point function into the renormalization group equation, we get

$$\delta=\frac{1}{2}\left[\frac{Q_3-\alpha m^2}{m^2}\right], \tag{29}$$

$$\gamma_\phi=-\frac{\alpha}{2}, \tag{30}$$

and

$$\beta_{b_1}=\frac{1}{4}\left[\frac{Q_1-\alpha b_1^2}{b_1^2}\right]\quad\text{and}\quad\beta_{b_2}=\frac{1}{4}\left[\frac{Q_2-\alpha b_2^2}{b_2^2}\right]. \tag{31}$$

Similarly, using the three-point vertex function, we get

$$\beta_e=\frac{e\alpha}{16},\qquad\beta_{b_3}=\frac{1}{4}\left[\frac{K_3-\alpha b_3^2}{b_3^2}\right]. \tag{32}$$

Even without calculating the radiative corrections for the vertices with more than three fields, the results obtained so far, together with some reasonable assumptions, allow us to examine relevant questions related to the possible emergence of Lorentz symmetry at low energies. For that purpose, we recall that, as a function of the momenta and the parameters of the model, $\Gamma^{(N)}$ has dimension $6-N_\phi-N_{A_i}-2N_{A_0}$ and therefore satisfies

$$\left[2p_0\frac{\partial}{\partial p_0}+p\frac{\partial}{\partial p}+2\mu\frac{\partial}{\partial\mu}+a_1\frac{\partial}{\partial a_1}+b_1\frac{\partial}{\partial b_1}+2\lambda\frac{\partial}{\partial\lambda}+4m^2\frac{\partial}{\partial m^2}-(6-N_\phi-N_{A_i}-2N_{A_0})\right]\Gamma^{(N)}=0, \tag{33}$$

where p_0 and p symbolically stand for the sets of timelike and spacelike parts of the momenta. From (33) and the renormalization group equation, we may now write

$$\left[-\frac{\partial}{\partial t}+\left(\beta_{a_1}-\frac{1}{2}\right)a_1\frac{\partial}{\partial a_1}+\left(\beta_{b_1}-\frac{1}{2}\right)b_1\frac{\partial}{\partial b_1}+(\beta_\lambda-1)\lambda\frac{\partial}{\partial\lambda}+\beta_{a_2}\frac{\partial}{\partial a_2}+\sum_{i=2}^{4}\left(\beta_{b_i}\frac{\partial}{\partial b_i}\right)+\sum_{n=1}^{3}\beta_{\xi_n}\frac{\partial}{\partial\xi_n}+\beta_e\frac{\partial}{\partial_e}+\beta_g\frac{\partial}{\partial_g}+(\delta-2)m^2\frac{\partial}{\partial m^2}+\frac{1}{2}(6-N_\phi-N_{A_i}-2N_{A_0})-\gamma_\Gamma\right]\Gamma^{(N)}(e^tp_0,e^{t/2}p,x)=0, \tag{34}$$

where x designates the set of parameters of the model, specified in (1) and (17). To solve this equation, we introduce running couplings. For the coefficients of the renormalizable (marginal) vertices, generically denoted by $\bar{a}(a,t)$, they obey

$$\frac{\partial \bar{a}}{\partial t} = \beta_{\bar{a}} \tag{35}$$

and the initial condition $\bar{a}(a, 0) = a$. On the other hand, the running couplings associated with the coefficients of the super-renormalizable (relevant) vertices $\bar{m}(m, t)$, $\bar{a}_1(a_1, t)$, $\bar{b}_1(b_1, t)$, and $\bar{\lambda}(\lambda, t)$ must satisfy

$$\frac{\partial \bar{m}^2}{\partial t} = (\delta - 2)\bar{m}^2, \qquad \frac{\partial \bar{a}_1}{\partial t} = \left(\beta_{\bar{a}_1} - \frac{1}{2}\right)\bar{a}_1,$$
$$\frac{\partial \bar{b}_1}{\partial t} = \left(\beta_{\bar{b}_1} - \frac{1}{2}\right)\bar{b}_1, \qquad \frac{\partial \bar{\lambda}}{\partial t} = (\beta_{\bar{\lambda}} - 1)\bar{\lambda}, \tag{36}$$

also subject to the condition that at $t = 0$ they are equal to the original parameters. Thus, for the couplings a_2, b_i with $i = 2, 3, 4$ and ξ_n with $n = 1, 2, 3$, Lorentz symmetry demands that the corresponding β functions be positive for small energies. This, however, will not be enough if $a_1 \neq b_1$. Thus, we set $a_1 = b_1 = c$ as a starting condition for these parameters in the original Lagrangian and require $\beta_{\bar{a}_1} = \beta_{\bar{b}_1}$ so that they remain equal as t varies.

We may now factorize c^2 out from the Lagrangian and redefine $c^{-1}\partial_0 \to \partial_0$, $c^{-1}A_0 \to A_0$, $c^{-2}m^2 \to m^2$, $c^{-2}\lambda \to \lambda$ and $c^{-2}g \to g$. We get a new Lagrangian with the usual terms of the $4 + 1$ scalar QED and with the high derivative terms divided by c^2. For the emergence of the Lorentz symmetry to take place, the coefficients of these terms should be small. Let a^2/c^2 be one of these coefficients; we shall have then

$$\frac{\partial}{\partial t}\left(\frac{\bar{a}^2}{\bar{c}^2}\right) = \frac{2\bar{a}}{\bar{c}^2}\left[\beta_{\bar{a}} - \bar{a}\left(\beta_{\bar{c}} - \frac{1}{2}\right)\right] > 0. \tag{37}$$

One simplification is to set the $\xi_n = 0$, assuming that, at least to one loop, they are not generated by the radiative corrections. The choice $a_1 = b_1 = c$ corresponds to the assumption that, in the absence of high derivatives terms, the speed of light is well defined. The imposition that $\beta_{\bar{a}_1} = \beta_{\bar{b}_1}$ implies that this velocity remains well defined, although it may change with the energy. However, the system of equations is still very complicated, so we restrict our analysis to the situation in which $\beta_{\bar{c}} = 0$. This condition allows one to fix b_3 and b_4 as functions of a_2 and b_2

$$b_3 = \sqrt{\frac{b_2}{3(a_2^2 + a_2 b_2 + b_2^2)}}[3b_2(3a_2^2 - a_2 b_2 - b_2^2) \pm \sqrt{3}\sqrt{-2a_2^6 - 4a_2^5 b_2 + 8a_2^4 b_2 + 7a_2^3 b_2^3 + 18a_2^2 b_2^4 + 18a_2 b_2^5 + 9b_2^6}]^{1/2} \tag{38}$$

and

$$b_4 = \sqrt{\frac{b_2}{6(a_2^2 + a_2 b_2 + b_2^2)}}[3b_2(a_2^2 - 7a_2 b_2 - 7b_2^2) \pm 4\sqrt{3}\sqrt{-2a_2^6 - 4a_2^5 b_2 + 8a_2^4 b_2 + 7a_2^3 b_2^3 + 18a_2^2 b_2^4 + 18a_2 b_2^5 + 9b_2^6}]^{1/2}, \tag{39}$$

with the use of the signs + or − in these expressions to be discussed shortly. By using (38), we can eliminate the dependence on b_3 and b_4 from β_{a_2}, β_{b_2}, and β_{b_3} so that they become

$$\beta_{a2} = \frac{e^2}{27648\pi^2 a_2 b_2}\left\{108a_2^2 + 27b_2^2 + \frac{42b_2(3b_2 P_1(a_2,b_2) \mp 2\sqrt{3}\sqrt{P_2(a_2,b_2)})}{P_3(a_2,b_2)} + \frac{(3b_2 P_1(a_2,b_2) \mp 2\sqrt{3}\sqrt{P_2(a_2,b_2)})^2}{(P_3(a_2,b_2))^2}\right\}, \tag{40}$$

$$\beta_{b_2} = \frac{e^2}{2304\pi^2}\left\{-36 + \frac{1}{a_2(a_2+b_2)^3}\left[9b_2^2(23a_2^2 + 37a_2 b_2 + 16b_2^2) - \frac{a_2(a_2+3b_2)(3b_2 P_1(a_2,b_2) \mp 2\sqrt{3}\sqrt{P_2(a_2,b_2)})^2}{(P_3(a_2,b_2))^2} - \frac{6b_2(7a_2^2 + 9a_2 b_2 + 4b_2^2)(3b_2 P_1(a_2,b_2) \mp 2\sqrt{3}\sqrt{P_2(a_2,b_2)})}{P_3(a_2,b_2)}\right]\right\}, \tag{41}$$

and

$$\beta_{b_3} = \frac{e^2 P_3(a_2,b_2)}{2304\pi^2 a_2 b_2 (a_2+b_2)^3(-3b_2P_1(a_2,b_2) \pm 2\sqrt{3}\sqrt{P_2(a_2,b_2)})} \times \Bigg\{ 9b_2(3a_2^4+9a_2^3b_2+25a_2^2b_2^2+41a_2b_2^3+20b_2^4) + \frac{3(21a_2^4+63a_2^3b_2+23a_2^2b_2^2-63a_2b_2^3-40b_2^4)(3b_2P_1(a_2,b_2)\mp 2\sqrt{3}\sqrt{P_2(a_2,b_2)})}{P_3(a_2,b_2)} + \frac{2b_2(8a_2^2+15a_2b_2+6b_2^2)(3b_2P_1(a_2,b_2)\mp 2\sqrt{3}\sqrt{P_2(a_2,b_2)})^2}{(P_3(a_2,b_2))^2}\Bigg\}, \tag{42}$$

where the polynomials $P_1(a_2,b_2)$, $P_2(a_2,b_2)$, and $P_3(a_2,b_2)$ were introduced to simplify the writing of the above expressions; they are given by

$$\begin{aligned} P_1(a_2,b_2) &= -3a_2^2+a_2b_2+b_2^2, \\ P_2(a_2,b_2) &= -2a_2^6-4a_2^5b_2+8a_2^4b_2^2+7a_2^3b_2^3+18a_2^2b_2^4 \\ &\quad +18a_2b_2^5+9b_2^6, \\ P_3(a_2,b_2) &= a_2^2+a_2b_2+b_2^2. \end{aligned} \tag{43}$$

Because of the complexity of these expressions, we will employ numerical methods to find regions where the parameters decrease by lowering the energy: first, we find zeros of (42) and then analyze the behavior of these functions as perturbed around the zeros. Then, we do the same for (40) and (41) and obtain the following:

(1) In the interval $0 \le \frac{b_2}{a_2} < 0.62429879$, we have $\beta_{a_2} > 0$.
(2) In the interval $\frac{b_2}{a_2} > 0.48792827$, we have $\beta_{b_2} > 0$.
(3) In the interval $\frac{b_2}{a_2} > 0.49508332$, we have $\beta_{b_3} > 0$.

Finally, concerning the behavior of β_{b_4} we notice that, as b_4 is a function of a_2 and b_2,

$$\beta_{b_4} = \beta_{a_2}\frac{\partial b_4}{\partial a_2} + \beta_{b_2}\frac{\partial b_4}{\partial b_2}, \tag{44}$$

and performing the same analysis described above, we find that β_{b_4} is positive for $\frac{b_2}{a_2} > 0.50848002$. Thus, by collecting all these results, we find that, in the interval

$$0.50848002 < \frac{b_2}{a_2} < 0.62429879, \tag{45}$$

all β functions are positive. Lorentz symmetry may emerge but this requires a fine tuning procedure, as described.

IV. SUMMARY AND CONCLUSIONS

In this work, we studied the $z=2$ scalar quantum electrodynamics in five spacetime dimensions. We regularized the Feynman amplitudes by promoting the spatial part of the Feynman integrals to $d = 4-\epsilon$ and the renormalization of the model was accomplished by removing the pole parts of the result (minimum subtraction procedure). We explicitly checked that these pole parts satisfy, as they should, the Ward identities characteristics of the model. By determining the relevant β functions, we analyzed possible scenarios for the evolution of various coupling constants. We verified that the emergence of the Lorentz symmetry may occur in the low-energy limit, but this requires a fine tuning procedure. Another possibility is to have an ultraviolet regime in which the usual, quadratic terms in the derivatives, become negligible. This is a great simplification, making it possible to completely determine the one-loop integrals. As a third scenario, there is the opposite situation in which the usual terms may be very large, which would be interesting for applications to the physics of the early Universe.

Finite temperature/density effects may be considered using standard methods. In particular, that extension does not significantly alter the ultraviolet structure we analyzed in this work. This is so because the conserved charge density still has the usual form

$$j_0(x) = i(\phi^* D_0\phi - \phi(D_0\phi)^*), \tag{46}$$

and therefore the chemical potential vertex $\mu_0 j_0$ is super-renormalizable. Its impact on the UV behavior manifests itself through the inclusion of a term $-2\nu_{\mu_0}$ in the power counting. A finite temperature T may also be considered by discretizing the temporal part of the momenta through the Matsubara replacement $p_0 \to (2i\pi n)T$. Of course, the resulting ultraviolet structure is the same as before the replacement.

Similarly, to make contact with the four-dimensional physics, we may compactify one spatial dimension in a circle. Imposing to the fields periodic boundary conditions in that fifth dimension, we get towers of Kaluza-Klein modes of increasing masses. As in the case of finite temperature, this construction does not alter the ultraviolet structure discussed in this work. The phenomenological aspects of this structure have not been treated here and will be the subject of future work.

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APPENDIX: ONE-LOOP CORRECTIONS AND RENORMALIZATION

In this Appendix, we will examine the ultraviolet structure of the model by analyzing the possible divergences as specified by (16). As mentioned before, our Feynman integrals are dimensionally regulated by promoting their spatial parts to $d = 4 - \epsilon$ dimensions. These integrals are not analytically feasible, and to extract their divergent parts, we Taylor expand their integrands in powers of the external momenta. For a Feynman amplitude I_Γ of a graph Γ we use

$$I_\Gamma(p) = \sum_{s=0}^{[\frac{\delta(\Gamma)}{2}]} \frac{p_0^s}{s!}\frac{\partial^s}{\partial p_0^s} \sum_{n=0}^{\delta(\Gamma)-2s} \frac{p_{i_1}\cdots p_{i_n}}{n!}\frac{\partial}{\partial p_{i_1}}\cdots\frac{\partial}{\partial p_{i_n}} I_\Gamma + \text{finite part}, \tag{A1}$$

where $\delta(\Gamma)$ is the degree of superficial divergence of Γ, $[x]$ is the greatest integer less than or equal to x, p_0^s symbolically stands for the product of s timelike components of an independent set of external momenta; p_i denotes the ith spacelike component and all derivatives are computed at zero external momenta.

By using (A1), for the coefficients of the Taylor expansion we obtain integrals of the type

$$J(x,y,z) = \int \frac{dk_0}{2\pi}\frac{d^dk}{(2\pi)^d}\frac{k_0^x|\vec{k}|^y}{[k_0^2 - b_1^2\vec{k}^2 - b_2^2\vec{k}^4 - m^2]^z} \tag{A2}$$

or

$$\int \frac{dk_0}{2\pi}\frac{d^dk}{(2\pi)^d}\frac{1}{(k_0^2 - b_1^2\vec{k}^2 - a_1^2(\vec{k}^2)^2 - m_1^2)^{z_1}} \times \frac{1}{(k_0^2 - b_2^2\vec{k}^2 - a_2^2(\vec{k}^2)^2 - m_2^2)^{z_2}}, \tag{A3}$$

if there are propagators with different denominators in the loop integral. In this last case, we use Feynman's trick

$$\frac{1}{A^{z_1}B^{z_2}} = \frac{\Gamma(z_1+z_2)}{\Gamma(z_1)\Gamma(z_2)}\int_0^1 dx\frac{x^{z_1-1}(1-x)^{z_2-1}}{[Ax + B(1-x)]^{z_1+z_2}} \tag{A4}$$

to obtain an integral similar to (A2). The divergent part of this integral may be calculated using standard methods (see appendix in [14] for details), yielding the result

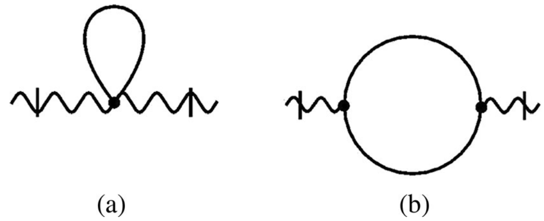

FIG. 1. Graphs contributing to the polarization tensor (the continuous and wavy lines represent the scalar and gauge field propagators): (a) tadpole graph and (b) fish graph.

$$J(x,y,z) = \frac{i^{1+x-2z}}{(4\pi)^{(d+2)/2}}\frac{[(-1)^x + 1]}{2}\frac{\Gamma(\frac{x+1}{2})}{\Gamma(\frac{d}{2})\Gamma(z)} \times \sum_{n=0}^{2}\frac{(-b_1^2)^n}{n!}\frac{\Gamma(\frac{d+y+2n}{4})\Gamma(\omega + \frac{(n-1)}{2})}{(b_2^2)^{(d+y+2n)/4}(m^2)^{\omega+\frac{(n-1)}{2}}}, \tag{A5}$$

where $w = (4z - 2x - y - d)/4$. We now analyze the possible divergences on the effective action, as indicated in (16). We have:

1. Pure gauge sector, i.e., graphs with $N_\phi = 0$. In what follows, $\Pi_{\mu\nu}$ will denote the correction to the kernel of the term with two gauge fields in the effective action, i.e., the term $A^\mu(p)\Pi_{\mu\nu}A^\nu(-p)$. Due to gauge invariance, the counterterms must depend on the potential A_μ only through the gauge field strength $F_{\mu\nu}$. Also, charge conjugation symmetry restricts the number of external lines to be even.

1a. For $N_{A_0} = 2$, the divergences are quadratic. Using the Feynman rules stated before, we found the following contributions coming from the graphs depicted in Fig 1,

$$\Pi_{00}(p) = e^2\left[-\int [dk]\frac{1}{\Omega_b[k^2]} + \frac{1}{2}\int [dk]\frac{(p_0 + 2k_0)^2}{\Omega_b[k^2]\Omega_b[(k+p)^2]}\right], \tag{A6}$$

where here and henceforth we employ the notation $\Omega_b(k) \equiv k_0^2 - b_1^2\vec{k}^2 - b_2^2\vec{k}^4 - m^2$ and $[dk] \equiv \mu^{\epsilon/2}dk_0d^dk/(2\pi)^{d+1}$. Due to the presence of quadratic and quartic terms in the denominators of the integrands, the above integrals do not produce simple analytic expressions. The pole part of the result may nevertheless be easily computed by expanding the integrands in power series and using (A5) as described before. Proceeding in this way, we found

$$\Pi_{00}(p) = \frac{i}{4}\alpha\mu^{\epsilon/2}\left[\frac{1}{\epsilon}\vec{p}^2 + (\text{finite part})\right], \tag{A7}$$

where $\alpha = \frac{e^2}{16\pi^2 b_2}$. For $N_{A_0} = 1$ and $N_{A_i} = 1$, the graphs have the same topology as before but different polynomials at the vertices. We have

$$\Pi_{0i}(p) = -\frac{e^2}{4}\left\{b_1^2\int[dk]\frac{(p_0+2k_0)(p_i+2k_i)}{\Omega_b[k^2]\Omega_b[(k+p)^2]} + b_2^2\int[dk]\frac{(p_0+2k_0)(p_l+2k_l)\{(p_i+k_i)(p_l+k_l)+k_ik_l\}}{\Omega_b[k^2]\Omega_b[(k+p)^2]}\right\}, \tag{A8}$$

yielding

$$\Pi_{0i}(p) = \Pi_{i0}(p) = \frac{i}{4}\alpha\mu^{\epsilon/2}\left[\frac{1}{\epsilon}p_0p_i + (\text{finite part})\right]. \tag{A9}$$

1b. Similarly, for $N_{A_i} = 2$, divergences arise only if $\nu_{3B} = 0$ and, in that case, the degree of superficial divergence is four. Explicit calculation gives

$$\begin{aligned}\Pi_{ij}(p) = {}& -e^2\delta_{ij}\left\{(b_1^2+b_2^2\vec{p}^2)\int[dk]\frac{1}{\Omega_b[k^2]} + \frac{2}{d(d+2)}b_2^2\int[dk]\frac{\vec{k}^2}{\Omega_b[k^2]}\right\}\\ &+\frac{e^2}{2}\int[dk]\left\{b_1^4\frac{(p_i+2k_i)(p_j+2k_j)}{\Omega_b[k^2]\Omega_b[(k+p)^2]}\right.\\ &+b_2^4\frac{(p_l+2k_l)(p_m+2k_m)\{(p_i+k_i)(p_l+k_l)+k_ik_l\}\{(p_j+k_j)(p_m+k_m)+k_jk_m\}}{\Omega_b[k^2]\Omega_b[(k+p)^2]}\\ &\left.+2b_1^2b_2^2\frac{(p_i+2k_i)(p_l+2k_l)\{(p_j+k_j)(p_l+k_l)+k_jk_l\}}{\Omega_b[k^2]\Omega_b[(k+p)^2]}\right\},\end{aligned} \tag{A10}$$

so that

$$\begin{aligned}\Pi_{ij}(p) = \frac{i\alpha\mu^{\epsilon/2}}{4}&\left[\frac{1}{\epsilon}(\delta_{ij}p_0^2 + R(\delta_{ij}\vec{p}^2 - p_ip_j)\right.\\ &\left.+S(\delta_{ij}\vec{p}^2-p_ip_j)\vec{p}^2) + (\text{finite part})\right],\end{aligned} \tag{A11}$$

where

$$R = \frac{2b_1^2}{b_2^2}(2b_2^2 - b_3^2 + b_4^2) \quad \text{and}$$
$$S = \frac{1}{12b_2^2}(3b_2^4 - 14b_2^2b_3^2 + b_3^4). \tag{A12}$$

Thus, the counterterms have the forms $C_1F_{i0}F_{i0}$, $C_2F_{ij}F_{ij}$, and $C_3\partial_lF_{ij}\partial_lF_{ij}$, where

$$C_1 = \frac{\alpha}{8}\frac{1}{\epsilon}, \qquad C_2 = \frac{\alpha}{4}\frac{b_1^2}{b_2^2}\frac{(2b_2^2-b_3^2+b_4^2)}{\epsilon},$$
$$C_3 = \frac{\alpha}{96b_2^2}\frac{(3b_2^4-14b_2^2b_3^2+b_3^4)}{\epsilon}. \tag{A13}$$

The above results also show that the wave renormalization functions for the fields A_0 and A_i are equal. Observe that for $b_1 = 0$ there is no contribution to the term $F_{ij}F_{ij}$, as expected, because of conformal invariance.

1c. $N_{A_i} = 4$. Here the relevant graphs are quadratically divergent, but because the counterterms necessarily depend on the potential only through the field strength, four momentum factors are needed to produce a nonzero result. In this case, the contribution is finite.

2. Matter/gauge field mixed sector. First, we have $N_A = 0$ and $N_\phi = 2$. In this case, we found one-loop corrections of the form $\phi^*\Delta\Gamma^{(2)}\phi$, coming from graphs with three different topologies, as shown in Fig 2. The tadpole graphs [Fig 2(a)] have one internal scalar line and the vertex is either the λ vertex or one of the vertices with the couplings ξ_n. They furnish

$$\begin{aligned}\Delta\Gamma_1^{(2)} = \lambda\int[dk]\frac{1}{\Omega_b[k^2]} &+ (4\xi_1 - \xi_3)\\ &\times\left\{\int[dk]\frac{\vec{k}^2}{\Omega_b[k^2]} + \vec{p}^2\int[dk]\frac{1}{\Omega_b[k^2]}\right\}.\end{aligned} \tag{A14}$$

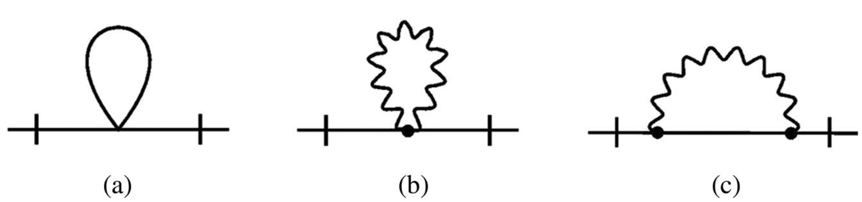

FIG. 2. Radiative corrections for the scalar matter field: (a) tadpole graph with an internal scalar line, (b) tadpole graph with internal gauge field line, and (c) graph with two vertices.

There is a also a tadpole graph with internal spatial gauge field propagator, as shown in Fig 2(b) (notice that, since the spatial part is dimensionally regularized, the would-be contribution of the tadpole graph with internal timelike gauge propagator vanishes),

$$\Delta\Gamma_2^{(2)} = -e^2(1-d)\int [dk]\frac{b_1^2 + b_2^2(k^2 + \frac{4}{d}\vec{p}^2)}{\Omega_a[k^2]}. \tag{A15}$$

There are, finally, the contributions from the graphs with two trilinear vertices (see Fig. 2(c))

$$\begin{aligned}\Delta\Gamma_3^{(2)} = e^2\int [dk]\Bigg\{&\frac{(k_0+2p_0)^2}{\vec{k}^2\Omega_b[(k+p)^2]} + \frac{4b_1^4 p_i p_j(\delta_{ij} - \frac{k_i k_j}{\vec{k}^2})}{\Omega_a[k^2]\Omega_b[(k+p)^2]}\\ &+\frac{b_2^4 p_i p_j(\vec{k}+2\vec{p})^4(\delta_{ij} - \frac{k_i k_j}{\vec{k}^2})}{\Omega_a[k^2]\Omega_b[(k+p)^2]}\\ &+\frac{4b_1^2 b_2^2 p_i p_j(\vec{k}+2\vec{p})^2(\delta_{ij} - \frac{k_i k_j}{\vec{k}^2})}{\Omega_a[k^2]\Omega_b[(k+p)^2]}\Bigg\}. \end{aligned}\tag{A16}$$

By performing the indicated integrals in the above expressions, we obtain the total correction $\Delta\Gamma^{(2)} = \Delta\Gamma_1^{(2)} + \Delta\Gamma_2^{(2)} + \Delta\Gamma_3^{(2)}$ to the two-point vertex function for the scalar field,

$$\begin{aligned}\Delta\Gamma^{(2)} = i\mu^{\epsilon/2}\Bigg[&\frac{1}{\epsilon}(-\alpha p_0^2 + Q_1\vec{p}^2 + Q_2\vec{p}^4 + Q_3)\\ &+ (\text{finite part})\Bigg],\end{aligned}\tag{A17}$$

where Q_i with $i = 1, 2, 3$ are

$$\begin{aligned}Q_1 = &\frac{3\alpha}{8a_2^3 b_2^2(a_2+b_2)^2}\{2a_1^2 a_2 b_2^2(11b_2^4 - 2b_2^2 b_3^2 - b_3^4)\\ &+ a_1^2 b_2^3(11b_2^4 - 2b_2^2 b_3^2 - b_3^4) + a_2^3 b_1^2(7b_2^4 + 6b_2^2 b_3^2 - b_3^4)\\ &+ 2a_2^2 b_2(6a_1^2 b_2^4 + b_1^2(3b_2^4 + 2b_2^2 b_3^2 - b_3^4))\}\\ &+ (4\xi_1 - \xi_3)\frac{b_1^2}{32\pi^2 b_2^3},\end{aligned}\tag{A18}$$

$$\begin{aligned}Q_2 = &\frac{\alpha}{4a_2(a_2+b_2)^3}\{b_2^4(23a_2^2 + 37a_2 b_2 + 16b_2^2)\\ &+ 2b_2^2(7a_2^2 + 9a_2 b_2 + 4b_2^2)b_3^2 - a_2(a_2 + 3b_2)b_3^4\},\end{aligned}\tag{A19}$$

and

$$\begin{aligned}Q_3 = &\frac{\alpha}{8a_2^5 b_2^2}\{12a_1^2 a_2^2 b_1^2 b_2^3 - 9a_1^4 b_2^3(b_2^2 + b_4^2) + a_2^5 b_1^4\\ &- 4a_2^5 b_2^2 m^2\} + \lambda\frac{b_1^2}{32\pi^2 b_2^3} + (4\xi_1 - \xi_3)\frac{(4b_2^2 m^2 - 3b_1^2)}{32\pi^2 b_2^5}.\end{aligned}\tag{A20}$$

It should be noted that, for $a_1 = b_1 = 0$, Q_1 vanishes so that corrections to the lowest order terms in the spatial derivatives do not occur. Observe that, if also $m = 0$, then Q_3 vanishes so that conformal invariance is preserved.

3. Three-point vertex function associated with the product $\phi(p)\phi^*(-p')A_\mu(p-p')$ (see graphs in Fig 3).

3a. The contributions for the correction for the vertex V_{3A} ($N_{A_0} = 1$ and $N_\phi = 2$) was found to be

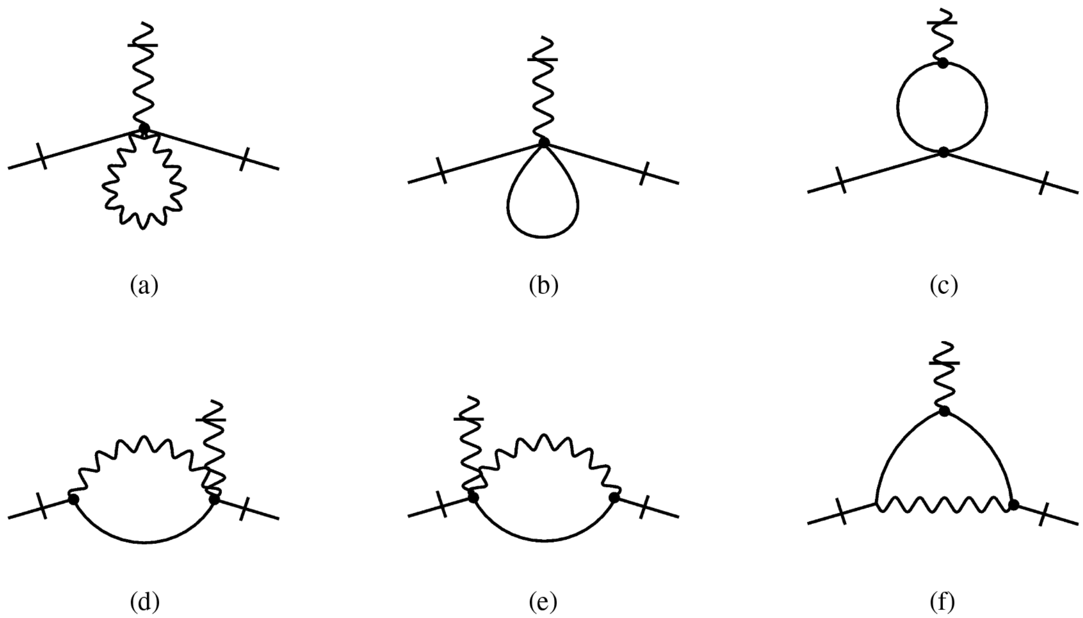

FIG. 3. General aspect of graphs contributing to the three point vertex function: (a) tadpole graph with an internal gauge field line, (b) tadpole graph with an internal scalar line, (c) graph with one sclar four vertex, (d) and (e) graphs with two vertices and (f) graph with three vertice.

$$\Delta\Gamma_1^{(3)0} = e^3 \int [dk]\left[\frac{p_0}{(\vec{k}+\vec{p})^2\Omega_b[k^2]} + \frac{p'_0}{(\vec{k}+\vec{p'})^2\Omega_b[k^2]}\right], \tag{A21}$$

coming from the graphs with two vertices and

$$\begin{aligned}\Delta\Gamma_2^{(3)0} = ie^3 \int [dk](p_0+p'_0+2k_0)\Bigg\{&\frac{(k_0+2p_0)(k_0+2p'_0)}{\vec{k}^2\Omega_b[(k+p)^2]\Omega_b[(k+p')^2]} + \frac{4b_1^4 p_i p'_j(\delta_{ij}-\frac{k_ik_j}{\vec{k}^2})}{\Omega_a[k^2]\Omega_b[(k+p)^2]\Omega_b[(k+p')^2]}\\ &+\frac{b_2^4 p_i p'_j(\vec{k}+2\vec{p})^2(\vec{k}+2\vec{p}')^2(\delta_{ij}-\frac{k_ik_j}{\vec{k}^2})}{\Omega_a[k^2]\Omega_b[(k+p)^2]\Omega_b[(k+p')^2]}\\ &+\frac{2b_1^2b_2^2 p_i p'_j(\vec{k}+2\vec{p}')^2(\delta_{ij}-\frac{k_ik_j}{\vec{k}^2})}{\Omega_a[k^2]\Omega_b[(k+p)^2]\Omega_b[(k+p')^2]} + \frac{2b_1^2b_2^2 p_i p'_j(\vec{k}+2\vec{p})^2(\delta_{ij}-\frac{k_ik_j}{\vec{k}^2})}{\Omega_a[k^2]\Omega_b[(k+p)^2]\Omega_b[(k+p')^2]}\Bigg\},\end{aligned} \tag{A22}$$

coming from graphs with three vertices. After performing the integrations, we obtain

$$\begin{aligned}\Delta\Gamma^{(3)0}(p,p') &\equiv \Delta\Gamma_1^{(3)0}(p,p') + \Delta\Gamma_2^{(3)0}(p,p')\\ &= i\frac{e^3\mu^{\epsilon/2}}{16\pi^2 b_2}\left[-\frac{1}{\epsilon}(p^0+p'^0) + (\text{finite part})\right].\end{aligned} \tag{A23}$$

3b. The divergent contribution to the three-point vertex function with spatial A_i is more cumbersome. It involves graphs (ten with three vertices, six with two vertices, and four tadpoles). The final result is

$$\begin{aligned}\Delta\Gamma^{(3)i}(p,p') = ie\mu^{\epsilon/2}\Bigg\{&\frac{1}{\epsilon}[K_1(p_i+p'_i)\\ &+K_2(p_i(\vec{p}^2+\vec{p}\cdot\vec{p'}) + p'_i(\vec{p'}^2+\vec{p}\cdot\vec{p'}))\\ &+K_3(p_i(\vec{p'}^2-\vec{p}\cdot\vec{p'})\\ &+p'_i(\vec{p}^2-\vec{p}\cdot\vec{p'}))] + (\text{finite part})\Bigg\},\end{aligned} \tag{A24}$$

where

$$\begin{aligned}K_1 = \frac{3\alpha}{8a_2^3b_2^2(a_2+b_2)^2}\{&2a_1^2a_2b_2^2(11b_2^4-2b_2^2b_3^2-b_3^4)\\ &+a_1^2b_2^3(11b_2^4-2b_2^2b_3^2-b_3^4)\\ &+a_2^3b_1^2(7b_2^4+6b_2^2b_3^2-b_3^4)\\ &+2a_2^2b_2(6a_1^2b_2^4+b_1^2(3b_2^4+2b_2^2b_3^2-b_3^4))\}\\ &+\frac{(4\xi_1-\xi_3)b_1^2}{32\pi^2b_2^3},\end{aligned} \tag{A25}$$

$$\begin{aligned}K_2 = \frac{\alpha}{4a_2(a_2+b_2)^3}\{&b_2^4(23a_2^2+37a_2b_2+16b_2^2)\\ &+2b_2^2(7a_2^2+9a_2b_2+4b_2^2)b_3^2 - a_2(a_2+3b_2)b_3^4\},\end{aligned} \tag{A26}$$

and

$$\begin{aligned}K_3 = \frac{\alpha}{12a_2(a_2+b_2)^3}\{&3a_2^4b_2^2+9a_2^3b_2^3+25a_2^2b_2^4\\ &+41a_2b_2^5+20b_2^6-3a_2^4b_3^2-9a_2^3b_2b_3^2\\ &+31a_2^2b_2^2b_3^2+81a_2b_2^3b_3^2+40b_2^4b_3^2\\ &+16a_2^2b_3^4+30a_2b_2b_3^4+12b_2^2b_3^4\}\\ &+\frac{(3b_3^2-7b_2^2)(4\xi_2-\xi_3)}{192\pi^2b_2^3}.\end{aligned} \tag{A27}$$

These results allow us to prove the simplest Ward identities of the model, namely,

$$p_\mu\Pi^{\mu\nu} = 0 \quad \text{and} \quad (p'_\mu - p_\mu)\Gamma^{(3)\mu} = e[\Gamma^{(2)}(p') - \Gamma^{(2)}(p)], \tag{A28}$$

where $\Gamma^{(2)}(p)$ is the two-point vertex function of the scalar fields and $\Gamma^{(3)\mu}$ denotes the three-point vertex function of the product of fields $A^\mu(p'-p)\phi^*(-p')\phi(p)$. The first identity may be verified straightforwardly using the previous results for the components of the polarization tensor. It shows that the radiative correction to the gauge field two-point function is transversal; in the tree approximation, that function also has a longitudinal part due to the gauge fixing. The second identity may also be verified using that, before renormalization,

$$\begin{aligned}\Gamma^{(2)} &= i[p_0^2 - b_1^2\vec{p}^2 - b_2^2\vec{p}^4 - m^2] + \Delta\Gamma^{(2)}\\ &= i\left[\left(1-\frac{\alpha}{\epsilon}\right)p_0^2 - \left(b_1^2-\frac{1}{\epsilon}Q_1\right)\vec{p}^2 - \left(b_2^2-\frac{1}{\epsilon}Q_2\right)\vec{p}^4\right.\\ &\quad\left.-\left(m^2-\frac{1}{\epsilon}Q_3\right) + (\text{finite part})\right]\end{aligned} \tag{A29}$$

and

$$\Gamma^{(3)0} = ie\left[\left(1-\frac{\alpha}{\epsilon}\right)(p_0+p'_0) + (\text{finite part})\right], \tag{A30}$$

$$\Gamma^{(3)i} = -ie\left[\left(b_1^2 - \frac{1}{\epsilon}K_1\right)(p_i + p_i') + \left(b_2^2 - \frac{1}{\epsilon}K_2\right)(p_i(\vec{p}^2 + \vec{p}\cdot\vec{p}') + p_i'(\vec{p}'^2 + \vec{p}\cdot\vec{p}')) + \left(b_3^2 - \frac{1}{\epsilon}K_3\right)(p_i(\vec{p}'^2 - \vec{p}\cdot\vec{p}') + p_i'(\vec{p}^2 - \vec{p}\cdot\vec{p}')) + (\text{finite part})\right], \quad \text{(A31)}$$

where the expressions for $Q_1 = K_1$, $Q_2 = K_2$, Q_3, and K_3 were given in (A18), (A19), (A20), and (A27), respectively.

[1] P. Horava, Phys. Rev. D **79**, 084008 (2009).
[2] E. M. Lifshitz, Zh. Eksp. Teor. Fiz. **11**, 255 (1941); **11**, 269 (1941).
[3] P. Horava, Classical Quantum Gravity **28**, 114012 (2011).
[4] E. Fradkin, *Field Theories of Condensed Matter Physics* (Cambridge University Press, New York, 2013).
[5] P. R. S. Gomes, P. F. Bienzobaz, and M. Gomes, Phys. Rev. D **88**, 025050 (2013).
[6] R.-G. Cai, L.-M. Cao, and N. Ohta, Phys. Rev. D **80**, 024003 (2009); R. A. Konoplya, Phys. Lett. B **679**, 499 (2009).
[7] A. O. Barvinsky, D. Blas, M. Herrero-Valea, S. M. Sibiryakov, and C. F. Steinwachs, Phys. Rev. D **93**, 064022 (2016).
[8] H. Lu, C. Mei, and C. N. Pope, Phys. Rev. Lett. **103**, 091301 (2009); H. Nastase, arXiv: 0904.3604; C. R. Muniz, V. B. Bezerra, and M. S. Cunha, Ann. Phys. (Amsterdam) **359**, 55 (2015).
[9] A. F. Ferrari, H. O. Girotti, M. Gomes, A. Yu. Petrov, and A. J. da Silva, Mod. Phys. Lett. A **28**, 1350052 (2013); I. J. Morales Ulion, E. R. Bezerra de Mello, and A. Yu. Petrov, Int. J. Mod. Phys. A **30**, 1550220 (2015); P. R. S. Gomes, Int. J. Mod. Phys. A **31**, 1630009 (2016).
[10] D. Anselmi, Ann. Phys. (Amsterdam) **324**, 874 (2009); **324**, 1058 (2009); P. R. S. Gomes and M. Gomes, Phys. Rev. D **85**, 085018 (2012).
[11] R. Iengo, J. Russo, and M. Serone, J. High Energy Phys. 11 (2009) 020; J. M. Romero, J. A. Santiago, O. Gonzalez-Gaxiola, and A. Zamora, Mod. Phys. Lett. A **25**, 3381 (2010).
[12] R. Iengo and M. Serone, Phys. Rev. D **81**, 125005 (2010).
[13] M. Gomes, T. Mariz, J. R. Nascimento, J. M. Queiruga, A. Yu. Petrov, and A. J. da Silva, Phys. Rev. D **92**, 065028 (2015).
[14] P. R. S. Gomes and M. Gomes, Phys. Rev. D **85**, 065010 (2012).
[15] I. Arav, S. Chapman, and Y. Oz, J. High Energy Phys. 02 (2015) 078.
[16] I. Adam, I. V. Melnikov, and S. Theisen, J. High Energy Phys. 09 (2009) 130.
[17] M. Shifman, Int. J. Mod. Phys. A **25**, 199 (2010).
[18] M. Gomes, T. Mariz, J. R. Nascimento, A. Yu. Petrov, and A. J. da Silva, Phys. Lett. B **764**, 277 (2017).