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Supersymmetry breaking in spatially modulated vacua

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We study spontaneous supersymmetry breaking in spatially modulated stable or metastable vacua in supersymmetric field theories. Such spatial modulation can be realized in a higher derivative chiral model for which vacuum energies are positive, negative, or zero, depending on the model parameters. There appears a Nambu-Goldstone boson associated with the spontaneous breaking of the translational and $U(1)$ symmetries without the quadratic kinetic term and with a quartic derivative term in the modulated direction, and a gapless Higgs mode. We show that there appears a Goldstino associated with the supersymmetry breaking at a metastable vacuum, where energy is positive, while it becomes a fermionic ghost in the negative energy vacuum, and zero norm state and disappears from the physical sector in the zero energy vacuum.

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I. INTRODUCTION

Finding vacua where supersymmetry (SUSY) is spontaneously broken is an important problem in supersymmetric field theories, since it is obviously broken if it exists in nature. The famous examples of spontaneous SUSY breaking include the O'Raifeartaigh model for chiral superfields [1] and supersymmetric gauge theories [2], where the positive energy vacuum is characterized by a constant vacuum expectation value of scalar fields. Remarkably, it is desirable that these spontaneous SUSY breakings are caused by the dynamics of models [3]. However, the severe constraint by the Witten index [4] makes it hard to construct a phenomenologically viable model where dynamical SUSY breaking is possible. A large number of efforts has been devoted to construct a model for the dynamical SUSY breaking. The constraint of the Witten index can be circumvented if one employs a local minima, not the global minimum, for the SUSY breaking vacua. Even though the local minima are metastable false vacua decaying into the global vacuum in a finite time, they are nevertheless useful candidates of phenomenologically possible vacua if the lifetime of the vacua is longer than that of our Universe. This is the idea of the SUSY breaking in the metastable vacua [5,6]. It is worthwhile to emphasize that almost all of the SUSY breaking vacua discussed in the literature respect the translational symmetry in the relativistic field theories, for which the order parameter of vacua is constant.

On the other hand, space-time symmetry breakings have been discussed in a vast literature. Nonlinear realizations for spontaneously broken space-time symmetry were first formulated in Ref. [7] as the so-called inverse Higgs mechanism, and corresponding Nambu-Goldstone (NG) modes were discussed in Ref. [8]. Phenomenology of the spontaneous Lorentz symmetry breakings have been intensively studied in the past [9–13]. The ghost condensation [14] also gives an example. The presence of a brane or soliton also breaks translational symmetry perpendicular to the brane as well as a Lorentz symmetry tilting the brane. In this case, the NG modes associated with the broken symmetries appear as massless fields in the world-volume theory [15–17] (and references in [18]). Spontaneous breakings of the (super-)Poincaré symmetry have also been discussed in the context of Bogomol'nyi-Prasad-Sommerfield (BPS) [19–21] as well as non-BPS branes [22]. In addition to spontaneous breakings, there are also studies on the explicit Lorentz violations from the viewpoints of quantum gravity [23,24], massive gravity [25,26], and particle physics [27–29]. The explicit Lorentz symmetry violations in SUSY theories [30–32], including formal aspects of Lorentz violating SUSY breaking [33], have been also discussed.

Among other things, it is becoming more important to consider the possibility of spatially inhomogeneous ground states in condensed matter physics [34,35] and QCD [36–38]. For such a kind of ground states, the order parameter is characterized by a spatially varying function, and several translational symmetries are spontaneously broken there. We have recently proposed that such modulation can occur in relativistic field theories [39], and we have found that the NG boson appears as a consequence of spontaneous symmetry breaking of translational and $U(1)$ symmetries. Despite the physical importance of the spatially modulated vacua, there have been no studies on such vacua in

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supersymmetric contexts. It would therefore be plausible to admit SUSY breaking in spatially inhomogeneous vacua where parts of space-time symmetries in theories are also broken.

In this paper, we study spontaneous SUSY breaking in modulated vacua where the translational symmetry is broken. This possibility may open up phenomenologically viable model buildings based on a new kind of SUSY breaking. Our model contains a SUSY breaking modulated vacuum in addition to the SUSY preserving vacuum. The modulated vacuum that we find is metastable with positive vacuum energy, stable and degenerated with the SUSY preserving vacuum that has zero vacuum energy, or unstable with negative vacuum energy, depending on the model parameters. In addition to the NG boson associated with spontaneously broken translational symmetry [39], there appears a massless fermion, a Goldstino, in any case, as a consequence of the SUSY breaking. In the case of the positive energy vacuum, the Goldstino propagates with the correct sign of the kinetic term along both the modulation and the transverse directions. For the zero energy vacuum, the Goldstino has zero norm and disappears from the spectrum, even in the presence of the SUSY breaking. Although the negative energy vacuum is stable in the bosonic sector [39], it is unstable in the fermionic sector: the Goldstino becomes a fermionic ghost in the orthogonal direction, thereby leading to the instability. It can have a kinetic term with the wrong sign even along the modulated direction when the vacuum energy is negative or zero. One of the interesting features of our model is that SUSY is broken even though auxiliary field F does not have a vacuum expectation value (VEV), unlike usual SUSY breakings.

To find such a kind of modulated vacua, we introduce supersymmetric higher derivative chiral models. From a viewpoint of low-energy effective theories, supersymmetric field theories generically receive higher derivative corrections. Here "the higher derivative" means that terms contain more than two space-time derivatives. There are a variety of higher derivative supersymmetric chiral models. We concentrate on models where only the single space-time derivative acts on fields, such as $\partial^m\varphi$. In this paper, we never consider terms that contain more than two derivatives on one field such as $\partial^2\varphi$ that cannot be removed by partial integrations in the action. Terms with this kind of interactions suffer from a potential instability of systems [40]. This instability results in the existence of ghosts, and it is known as the Ostrogradski instability [41].[1] It is convenient to employ the off-shell superfield formalism to construct supersymmetric theories. One often encounters the so-called auxiliary field problem implying that the equation of motion for the auxiliary field F ceases to be algebraic [43]. Then it is not so easy to write down the on-shell Lagrangians. This has been seen in various supersymmetric higher derivative models, such as a supersymmetric Wess-Zumino-Witten (WZW) term [44] and supersymmetric Skyrme models [45]. The supersymmetric higher derivative models free from the auxiliary field problem have been discussed in various contexts. For example, higher derivative corrections to the ordinary quadratic kinetic terms appear in low-energy effective theories of supersymmetric models [46,47]. Other examples include the supersymmetric generalization of the Wess-Zumino-Novikov-Witten (WZNW) term [48], the world-volume action of supersymmetric branes [49], higher derivative chiral models coupled with supergravities [50], supersymmetric Skyrme-like models [51,52], and an inflation model driven by supersymmetric higher derivative terms of inflatons [53]. Two of the present authors have studied BPS states in supersymmetric higher derivative theories [54] and higher derivative corrections to manifestly supersymmetric nonlinear realization of the NG multiplet [55]. In particular, all four possible derivative terms free from the auxiliary field problem and ghosts have been classified in Ref. [56], and they have been generalized to an arbitrary number of derivatives in Refs. [54,55], which we use in this paper.

The organization of this paper is as follows. In Sec. II, we introduce the supersymmetric chiral model with higher derivative terms that is free from the auxiliary field problem in the bosonic sector. In Sec. III, we focus on a specific model where spatially modulated ground states are allowed. Supersymmetry is spontaneously broken in the modulated vacua. We show that the modulated vacuum is classified according to the vacuum energy. In Sec. IV, we discuss the NG modes in the modulated vacua. We demonstrate that the quadratic kinetic terms of bosonic NG modes associated with the spontaneous breaking of bosonic symmetries in the modulated vacua vanish in general. On the other hand, a Higgs mode, perpendicular to the NG mode, appears as a massless boson. For the spontaneous breaking of SUSY, the corresponding NG mode, i.e., the Goldstino, becomes a ghost when the vacuum energy is negative while it becomes a zero norm state when the vacuum has zero energy. In Sec. V, we introduce a superpotential in our model. Section VI is devoted to the conclusion and discussions. The component expansions of the higher derivative parts of the chiral superfield are found in the Appendix.

II. SUPERSYMMETRIC HIGHER DERIVATIVE MODEL

In this section we introduce the supersymmetric higher derivative model that is free from the auxiliary field problem in the bosonic sector. The Lagrangian of the model is given by

[1] There is a way to remove a ghost by gauging [42], but we do not consider such a possibility.

$$\mathcal{L} = \int d^4\theta K(\Phi^i, \Phi^{\dagger\bar{j}}) + \frac{1}{16}\int d^4\theta \Lambda_{ik\bar{j}\bar{l}}(\Phi, \Phi^\dagger) D^\alpha \Phi^i D_\alpha \Phi^k \bar{D}_{\dot{\alpha}} \Phi^{\dagger\bar{j}} \bar{D}^{\dot{\alpha}} \Phi^{\dagger\bar{l}} + \left(\int d^2\theta W(\Phi^i) + \text{H.c.}\right). \quad (1)$$

Here $\Phi^i = \varphi^i(y) + \sqrt{2}\theta^\alpha \psi^i_\alpha(y) + \theta^2 F^i(y) (i = 1, \ldots, N)$ are the four-dimensional $\mathcal{N} = 1$ chiral superfields in the chiral base $y^m = x^m + i\theta\sigma^m\bar{\theta} (m = 0, 1, 2, 3)$ whose component fields are complex scalars φ^i, Weyl fermions ψ^i, and auxiliary fields F^i. K is a Kähler potential, W is a superpotential, and $\Lambda_{ik\bar{j}\bar{l}}$ is a (2,2) Kähler tensor whose (anti)holomorphic indices are symmetrized. We basically follow the conventions and notations of Wess and Bagger [57]. The flat metric is given by $\eta_{mn} = \text{diag}(-1, 1, 1, 1)$. The first and the third terms in (1) are the ordinary kinetic and potential terms of supersymmetric chiral models while the second term provides higher derivative terms. A specific property of the second term is that the purely bosonic components included in there saturate the Grassmann coordinates,

$$\frac{1}{16} D^\alpha \Phi^i D_\alpha \Phi^k \bar{D}_{\dot{\alpha}} \bar{\Phi}^{\dagger\bar{j}} \bar{D}^{\dot{\alpha}} \Phi^{\dagger\bar{l}} = \theta^2\bar{\theta}^2 [(\partial_m \varphi^i \partial^m \varphi^k)(\partial_n \bar{\varphi}^{\bar{j}} \partial^n \bar{\varphi}^{\bar{l}}) - 2\partial_m \varphi^i F^k \partial^m \bar{\varphi}^{\bar{j}} \bar{F}^{\bar{l}} + F^i \bar{F}^{\bar{j}} F^k \bar{F}^{\bar{l}}] + I_\text{f}. \quad (2)$$

Here I_f represents terms that include fermions. Therefore only the lowest components in the Kähler tensor $\Lambda_{ik\bar{j}\bar{l}}(\Phi, \Phi^\dagger)$ contribute to the purely bosonic parts of the Lagrangian. Then the component Lagrangian is given by

$$\mathcal{L} = \frac{\partial^2 K}{\partial\varphi^i \partial\bar{\varphi}^{\bar{j}}}(-\partial_m \varphi^i \partial^m \bar{\varphi}^{\bar{j}} + F^i \bar{F}^{\bar{j}}) + \frac{\partial W}{\partial \varphi^i} F^i + \frac{\partial \bar{W}}{\partial \bar{\varphi}^{\bar{j}}} \bar{F}^{\bar{j}} + \Lambda_{ik\bar{j}\bar{l}}(\varphi, \bar{\varphi})\{(\partial_m \varphi^i \partial^m \varphi^k)(\partial_n \bar{\varphi}^{\bar{j}} \partial^n \bar{\varphi}^{\bar{l}}) - 2\partial_m \varphi^i \partial^m \bar{\varphi}^{\bar{j}} F^k \bar{F}^{\bar{l}} + F^i \bar{F}^{\bar{j}} F^k \bar{F}^{\bar{l}}\} + \mathcal{L}_\text{fermions}, \quad (3)$$

where $\mathcal{L}_\text{fermions}$ are terms that include fermionic fields. Note that $\Lambda_{ik\bar{j}\bar{l}}(\Phi, \Phi^\dagger)$ generically contains space-time derivatives of the chiral superfields and there is an arbitrary order of derivative terms in the Lagrangian (3). The equation of motion for $\bar{F}$ is

$$\frac{\partial^2 K}{\partial\varphi^i \partial\bar{\varphi}^{\bar{j}}} F^i - 2\Lambda_{ik\bar{j}\bar{l}} \partial_m \varphi^i F^k \partial^m \bar{\varphi}^{\bar{l}} + 2\Lambda_{ik\bar{j}\bar{l}} F^i F^k \bar{F}^{\bar{l}} + \frac{\partial \bar{W}}{\partial \bar{\varphi}^{\bar{j}}} = 0. \quad (4)$$

As we have advertised, this equation does not contain any space-time derivatives on F. Then, in principle, Eq. (4) is algebraically solvable. However, it is not so straightforward to solve the equation for general N since it is a simultaneous equation of cubic order. Only a few solutions have been known. For example, for $N = 1$ single chiral superfield models, one can solve the cubic order equation (4) by Cardano's method [53]. Consequently, there are multiple distinct on-shell branches associated with the independent solutions to the auxiliary fields. To see this explicitly, let us begin with a single superfield model without superpotential. The equation for $\bar{F}$ becomes

$$K_{\varphi\bar{\varphi}} F - 2\Lambda \partial_m \varphi \partial^m \bar{\varphi} F + 2\Lambda F^2 \bar{F} = 0, \quad (5)$$

where $K_{\varphi\bar{\varphi}}$ is the Kähler metric. There are two kinds of solutions to this equation. One is the trivial solution $F = 0$, and the bosonic part of the on-shell Lagrangian for this solution is

$$\mathcal{L} = -K_{\varphi\bar{\varphi}} \partial_m \varphi \partial^m \bar{\varphi} + \Lambda(\partial_m \varphi \partial^m \varphi)(\partial_n \bar{\varphi} \partial^n \bar{\varphi}). \quad (6)$$

We call the theory for the solution $F = 0$ canonical branch. The Lagrangian (6) represents the ordinary quadratic kinetic term of the complex scalar field φ with the higher derivative interactions governed by the tensor $\Lambda(\varphi, \bar{\varphi})$. It is evident that the higher derivative corrections are introduced as a perturbation to the quadratic kinetic term.

On the other hand, there is another nontrivial solution to the auxiliary field equation (5),

$$F\bar{F} = -\frac{K_{\varphi\bar{\varphi}}}{2\Lambda} + \partial_m \varphi \partial^m \bar{\varphi}. \quad (7)$$

The bosonic part of the on-shell Lagrangian for solution (8) is

$$\mathcal{L} = (|\partial_m \varphi \partial^m \varphi|^2 - (\partial_m \varphi \partial^m \bar{\varphi})^2)\Lambda - \frac{(K_{\varphi\bar{\varphi}})^2}{4\Lambda}. \quad (8)$$

In this branch, the quadratic canonical kinetic term disappears and the last term is interpreted as a potential term. We call this a noncanonical branch. Compare to the canonical branch, the higher derivative terms are not introduced perturbatively. We cannot take the limit $\Lambda \to 0$ in this branch.

Even though the higher derivative interactions appear in a different way in Lagrangians (6) and (8), SUSY is manifestly realized in each branch. A specific feature, for example, BPS states in the single chiral superfield models, is discussed in Refs. [51,54]. The higher derivative corrections to the NG supermultiplets in supersymmetric vacua were discussed in Ref. [55]. For multiple or matrix-valued fields models, it is not so straightforward to solve the equation for the auxiliary fields, but only one example can be found in Ref. [52] in which the authors solved the equation corresponding to Eq. (4) for the $SU(2)$-valued auxiliary field and found a supersymmetric extension of the Skyrme model.

In the next section we focus on a single chiral superfield model and discuss SUSY breaking in spatially modulated vacua.

III. SPATIALLY MODULATED VACUUM IN SUPERSYMMETRIC HIGHER DERIVATIVE MODEL

In this section, we investigate spatially modulated vacua in the supersymmetric field theories with higher derivative terms. For simplicity, we consider single superfield models without superpotential where the Kähler metric is a constant $K_{\varphi\bar{\varphi}} = k > 0$ and we focus on the canonical branch. For a generic Λ, the energy density is given by

$$\begin{aligned}\mathcal{E} = {} & k(|\dot{\varphi}|^2 + |\partial_i\varphi|^2) + \Lambda\{3|\dot{\varphi}|^4 - \dot{\varphi}^2(\partial_i\bar{\varphi})^2 \\ & - \dot{\bar{\varphi}}^2(\partial_i\varphi)^2 - (\partial_i\varphi)^2(\partial_j\bar{\varphi})^2\} \\ & - \frac{\partial\Lambda}{\partial\dot{\varphi}}|\dot{\varphi}|^2\{(-\dot{\varphi}^2 + (\partial_i\varphi)^2)(-\dot{\bar{\varphi}}^2 + (\partial_i\bar{\varphi})^2)\} \\ & - \frac{\partial\Lambda}{\partial\dot{\bar{\varphi}}}|\dot{\varphi}|^2\{(-\dot{\varphi}^2 + (\partial_i\varphi)^2)(-\dot{\bar{\varphi}}^2 + (\partial_i\bar{\varphi})^2)\},\end{aligned} \tag{9}$$

where $i,\ j = 1,\ 2,\ 3$ and $\dot{\varphi} = \frac{\partial\varphi}{\partial x^0}$, $\dot{\bar{\varphi}} = \frac{\partial\bar{\varphi}}{\partial x^0}$. Note that, in general, the Hermitian Kähler tensor Λ is a function of φ, $\partial_m\varphi$ and their Hermitian conjugate. Vacua are defined such that the configurations minimize the energy density $\mathcal{E}$. We are interested in models where static, spatially modulated configurations are realized as vacua. Namely, we look for a Kähler tensor Λ for which a spatial derivative of the field φ develops constant nonzero VEVs. The simplest example is the one-dimensional spatial modulation. To determine Λ, which realizes a modulated vacuum, we assume the configuration $\dot{\varphi} = \partial_2\varphi = \partial_3\varphi = 0$ and nonzero $\partial_1\varphi$. Then the energy density becomes

$$\mathcal{E} = k|\partial_1\varphi|^2 - \Lambda|\partial_1\varphi|^4. \tag{10}$$

Configurations $\partial_1\varphi = \text{const} \neq 0$ that minimize (10) are spatially modulated vacua along the x^1 direction. Since $\partial_1\varphi$ appears as the absolute value in (10), we further assume that Λ is a function of $|\partial_1\varphi|$ only. This results in the situation where the shift symmetry $\varphi \to \varphi + c$ is preserved. Here c is a constant. Then the energy density (10), which is a function of $X \equiv |\partial_1\varphi| \geq 0$, is interpreted as a potential for X,

$$\mathcal{E} = kX^2 - \Lambda(X)X^4, \qquad k > 0, \qquad X \geq 0. \tag{11}$$

One easily finds that for the simplest choice $\Lambda = \lambda = \text{const}$, there are no minima other than $X = 0$. The next simplest choice of Λ is

$$\Lambda = \lambda - \alpha|\partial_1\varphi|^2, \tag{12}$$

where α is a real constant. This corresponds to the choice

$$\Lambda = \lambda - \alpha\partial_m\Phi\partial^m\Phi^\dagger. \tag{13}$$

Then the energy density for a one-dimensional modulation $\partial_1\varphi \neq 0$ becomes

$$\mathcal{E} = \alpha X^6 - \lambda X^4 + kX^2. \tag{14}$$

As we will see below, for $\lambda > 0$, $\alpha > 0$ there are local (global) minima at $X \neq 0$. Note that for this choice of Λ, the bosonic part of Lagrangian (6) becomes the one that we studied in Ref. [39] which allows for a spatially modulated vacuum. In the following, we make a brief summary of the modulated vacuum found in Ref. [39]. We also note that although the theory manifestly realizes SUSY, the energy functional (9) is not positive (semi)definite. Therefore, vacua of the theory need not have zero energy in general even in supersymmetric theories. Indeed, the spatially modulated vacuum allows the negative energy as we will see below.

Since the energy density is a function of $|\partial_1\varphi|^2$, it is convenient to define $x \equiv |\partial_1\varphi|^2$ and treat $\mathcal{E}$ as a function of x,

$$\mathcal{E}(x) \equiv \alpha x^3 - \lambda x^2 + kx, \qquad x \geq 0. \tag{15}$$

All minima of the function $\mathcal{E}(x)$ that satisfy the equation of motion are vacua of the model. At first, one finds the minimum $x = 0$ in which the scalar field has a constant or vanishing VEV. In addition to this trivial vacuum, the function $\mathcal{E}(x) (x \geq 0)$ can have another minimum at $x \neq 0$ in which the space-time derivative of φ has nonzero VEVs. This is indeed the case when the parameters k, λ, α satisfy the condition $\lambda^2 - 3\alpha k > 0$. The potential $\mathcal{E}(x)$ has a minimum at

$$x_+ = \frac{\lambda + \sqrt{\lambda^2 - 3\alpha k}}{3\alpha}, \tag{16}$$

which is obviously nonzero. At the vacuum $|\partial_1\varphi|^2 = x_+$, we found the following spatially modulated configuration:

$$\varphi(x^1) = \varphi_0 e^{ipx^1}, \qquad \varphi_0, p \in \mathbb{R}, \tag{17}$$

where the constants p, φ_0 satisfy $p^2\varphi_0^2 = x_+$. The period of the modulation is given by $2\pi/p$. In the previous paper [39], we have found that the configuration (17) satisfies the equation of motion, and it is a completely consistent vacuum of the theory. The modulated vacuum (17) spontaneously breaks the translational symmetry along the x^1 direction and the rotational symmetries in the (x^1, x^2), (x^1, x^3) planes, as well as the $U(1)$ symmetry $\varphi \to e^{i\theta}\varphi$.

We have shown that there remain symmetries of the $(2+1)$-dimensional Poincaré group $ISO(2,1)$ and simultaneous operations of the translation P^1 along the x^1 direction and the $U(1)$ transformation $[P^1 \times U(1)]_{\rm sim}$. We have also pointed out that only the breaking pattern $P^1 \times U(1) \to [P^1 \times U(1)]_{\rm sim}$ gives rise to an NG boson.

To clarify the SUSY breaking in the modulated vacuum (17), we recall the SUSY variation of the fermion ψ. In the canonical branch, this is given by

$$\delta\psi_\alpha = i\sqrt{2}(\sigma^m)_{\alpha\dot{\alpha}}\bar{\xi}^{\dot{\alpha}}\partial_m\varphi + \sqrt{2}\xi_\alpha F(\varphi,\bar{\varphi}) = i\sqrt{2}\sigma^1\bar{\xi}\partial_1\varphi. \tag{18}$$

Here, ξ, $\bar{\xi}$ are parameters of the SUSY transformation. It is clear that SUSY is preserved in the vacuum $x = |\partial_1\varphi|^2 = 0$. On the other hand, in the modulated vacuum (17), one finds that the variation (18) does not vanish and SUSY is spontaneously broken there. A particular emphasis is placed on the fact that nonzero values of the auxiliary field F are not an order parameter of the SUSY breaking anymore. This is a reflection of the fact that the energy density (9) of this model is not positive (semi)definite. To illustrate this issue, we examine the sign of the vacuum energy $\mathcal{E}$. The vacuum energy at $x = x_+$ is calculated as

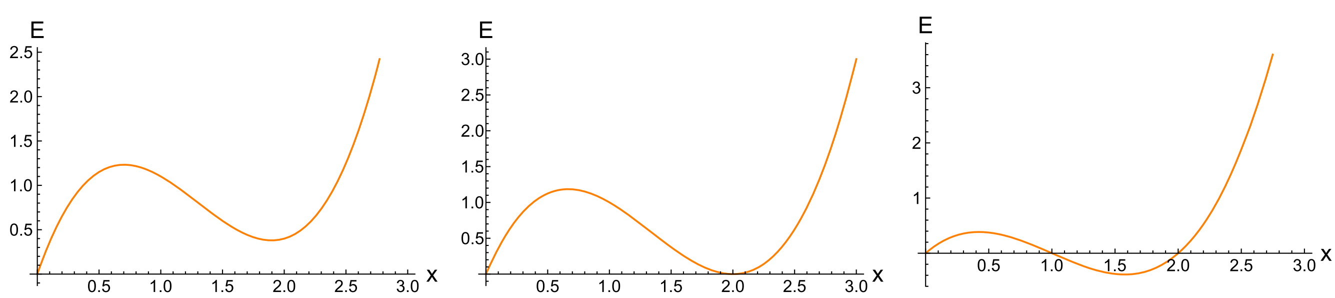
$$\mathcal{E}(x_+) = -\frac{1}{27\alpha^2}(\lambda + \sqrt{\lambda^2 - 3\alpha k}) \times \{-6\alpha k + \lambda(\lambda + \sqrt{\lambda^2 - 3\alpha k})\}. \tag{19}$$

It is evident that the quantity (19) is not always positive semidefinite. We have found that the sign of the vacuum energy is classified according to the discriminant condition of the function $\alpha x^2 - \lambda x + k$. Depending on the parameters k, λ, α, we have three distinct types of vacua. In the following, we assume that all the parameters satisfy the condition $\lambda^2 - 3\alpha k > 0$, which guarantees that the potential has a local minimum given in Eq. (16).

(i) *Positive energy vacuum*: When the parameters satisfy the discriminant condition $\lambda^2 - 4\alpha k < 0$, then the function $\mathcal{E}(x) = \alpha x^3 - \lambda x^2 + kx$ is positive definite. If this is the case, the potential function $\mathcal{E}(x)$ looks like Fig. 1(a). We find that the local vacuum energy at $x = x_+$ is positive $\mathcal{E}(x_+) > 0$ and the SUSY breaking vacuum at $x = x_+$ is metastable. It seems that the metastable vacuum decays to the global supersymmetric vacuum at $x = 0$ within a finite time. However, we can make the lifetime of the metastable vacuum longer by choosing parameters of the potential appropriately. If the lifetime is longer than that of the Universe, this kind of metastable vacuum becomes a possible candidate of a phenomenologically viable grand state. Indeed, the dynamical SUSY breaking in a metastable vacua was discussed in the framework of supersymmetric effective theories [5].

(ii) *Zero energy vacuum*: When the parameters k, λ, α satisfy the condition $\lambda^2 - 4\alpha k = 0$, then a schematic picture of the function $\mathcal{E}(x)$ is given by Fig. 1(b). In addition to the SUSY vacuum $x = 0$, we have a local vacuum $x = x_+$ in which $\mathcal{E}(x_+) = 0$. They are actually degenerated global vacua. Interestingly, although $\mathcal{E}(x_+) = 0$, this does not imply that the vacuum preserves SUSY. In fact, we have seen that SUSY is broken by the condition in Eq. (18). This results in the fact that the Goldstino in this vacuum becomes nondynamical and does not propagate in the directions transverse to the modulation as we will see later.

(iii) *Negative energy vacuum*: Finally, we consider the condition $\lambda^2 - 4\alpha k > 0$. When this is the case, the function $\mathcal{E}(x)$ looks like Fig. 1(c). Now the supersymmetric vacuum $x = 0$ becomes metastable and the SUSY breaking vacuum at $x = x_+$ is energetically favored. Therefore the global vacuum is located at $x = x_+$ in which $\mathcal{E}(x_+) < 0$. In this vacuum, SUSY is again broken by the condition



(a) $\mathcal{E}(x_+) > 0$ $(\alpha = 1, \lambda = 3.9, k = 4)$ (b) $\mathcal{E}(x_+) = 0$ $(\alpha = 1, \lambda = 4, k = 4)$ (c) $\mathcal{E}(x_+) < 0$ $(\alpha = 1, \lambda = 3, k = 2)$

FIG. 1. Profiles of the energy function $\mathcal{E}(x)$. The vertical and the horizontal axes stand for the energy $\mathcal{E}(x)$ and x. The local vacua for (a) positive, (b) zero, and (c) negative vacuum energies with examples of the parameters are shown.

(18). We will discuss the Goldstone mode associated with the SUSY breaking in the negative vacuum state in the next section.

IV. NAMBU-GOLDSTONE MODES IN SUPERSYMMETRY BREAKING MODULATED VACUUM

In this section, we study NG modes in the SUSY-breaking spatially modulated vacuum (17). There are two kinds of NG modes. One is the bosonic mode that appears due to the spontaneously broken symmetry $P^1 \times U(1) \to [P^1 \times U(1)]_{\rm sim}$ in the modulated vacuum. We note that the translation P^1 and the rotations in the (x^1, x^2) and (x^1, x^3) planes are not independent of each other [58]. This is a particular example of the inverse Higgs effect [7]. Therefore, there is only one bosonic NG mode. The other is the fermionic NG mode (Goldstino) associated with the SUSY breaking. In the following, we discuss bosonic and fermionic NG modes separately.

A. Bosonic sector

We first summarize the bosonic NG mode in the modulated vacuum in the model characterized by (13), which is identical to the one studied in Ref. [39]. We shift the field from the modulated vacuum (17) and introduce the fluctuation $\tilde{\varphi}$ as a dynamical field,

$$\varphi \to \langle\varphi\rangle + \tilde{\varphi}, \tag{20}$$

where $\langle\varphi\rangle = \varphi_0 e^{ipx^1}$ is the modulating VEV. The quadratic terms of the dynamical scalar field $\tilde{\varphi}$ are extracted from Lagrangian (6). The result is

$$\mathcal{L}_{\mathrm{quad}\tilde{\varphi}} = -\frac{1}{2}\vec{\varphi}^\dagger \mathbf{M}\vec{\varphi}. \tag{21}$$

Here we have defined the following quantities:

$$\vec{\varphi} = \begin{pmatrix} \partial_{\hat{m}}\tilde{\varphi} \\ \partial_{\hat{m}}\tilde{\varphi}^\dagger \\ \partial_1\tilde{\varphi} \\ \partial_1\tilde{\varphi}^\dagger \end{pmatrix}, \qquad \mathbf{M} = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}. \tag{22}$$

We have separated the terms to the $SO(2,1)$ invariant transverse sector $(\hat{m} = 0, 2, 3)$ and the modulation sector. In the 4×4 Hermitian matrix $\mathbf{M}$, each block element is given by

$$M_1 = \begin{pmatrix} k + \alpha x_+^2 & 2(\lambda - \alpha x_+)x_+ e^{-2ipx^1} \\ 2(\lambda - \alpha x_+)x_+ e^{2ipx^1} & k + \alpha x_+^2 \end{pmatrix},$$
$$M_2 = \begin{pmatrix} 9\alpha x_+^2 - 4\lambda x_+ + k & 2(\lambda - 3\alpha x_+)x_+ e^{-2ipx^1} \\ 2(\lambda - 3\alpha x_+)x_+ e^{2ipx^1} & 9\alpha x_+^2 - 4\lambda x_+ + k \end{pmatrix}. \tag{23}$$

The eigenvalues of M_1 and M_2 determine the coefficients of the quadratic kinetic terms in the $SO(2,1)$ invariant transverse and the modulation directions, respectively. In our previous paper [39], we have found that M_1 and M_2 have zero and positive eigenvalues, respectively. The quadratic kinetic term for the zero eigenvalue modes vanish. We pointed out that the mode associated with the zero eigenvalue of M_2 in the modulation direction corresponds to the NG mode that appears due to the spontaneous breaking of $P^1 \times U(1)$. We have also shown that cubic derivative terms for the bosonic NG mode are absent and a quartic derivative term of the NG mode appears in the Lagrangian. On the other hand, the positive eigenvalue mode in the M_2 sector is the Higgs mode, which has a quadratic kinetic term. This is apparently a gapless mode. This is a generalization of the ordinary NG theorem where the NG and the Higgs modes appear as zero and positive eigenvalue modes for the quadratic curvature of the potential energy. The difference from the ordinary NG theorem is that we have VEVs for the derivative of fields but not fields themselves. The zero eigenvalue of M_1 in the $SO(2,1)$ invariant sector corresponds to a flat direction of the potential.

B. Fermionic sector

We next investigate fermions in the modulated vacuum. The situation is quite different from the bosonic sector. To see this, let us consider the $\mathcal{N} = 1$ SUSY algebra,

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2(\sigma^m)_{\alpha\dot{\alpha}}P_m, \tag{24}$$

where Q_α, $\bar{Q}_{\dot{\alpha}}$ are supercharges and P^m is the generator of translation. Then, the energy for a state $|\Psi\rangle$ is given by

$$E_\Psi = \langle\Psi|P^0|\Psi\rangle = \frac{1}{4}\sum_{\alpha,\dot{\alpha}=1,2}(\|Q_\alpha|\Psi\rangle\|^2 + \|\bar{Q}_{\dot{\alpha}}|\Psi\rangle\|^2). \tag{25}$$

From the expression (25), one finds that when the energy for a state $|\Psi\rangle$ is negative $E_\Psi < 0$, then there are negative norm states (ghosts) in the system. In particular, for a vacuum $|0\rangle$, since SUSY is spontaneously broken there, the states $Q_\alpha|0\rangle, \bar{Q}_{\dot{\alpha}}|0\rangle \neq 0$ are identified with the Goldstino in the zero-momentum associated with the SUSY breaking. We therefore expect that there are ghost Goldstino in the negative energy modulated vacuum.

To see this explicitly, we evaluate the coefficient of the kinetic term of ψ in the chiral multiplet which is a unique candidate of the Goldstino. As one finds in Eq. (A2) in the Appendix, the fermion field in the Lagrangian appears with the auxiliary field accompanied by the space-time derivative. Eventually, the equation of motion for the auxiliary field becomes nonalgebraic when the fermion field is included. To write down the quadratic kinetic term of

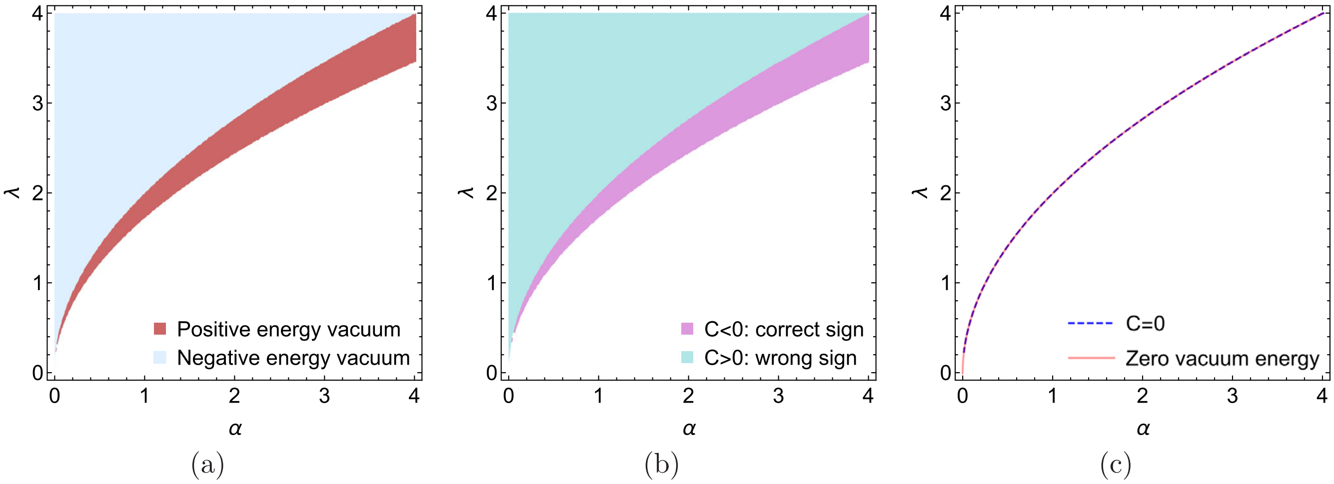


FIG. 2. (a) The parameter regions (α, λ) for the modulated vacuum with positive and negative energies. (b) The parameter region where the coefficient C of the fermionic kinetic term has wrong sign $C > 0$. (c) The parameter region (curves) of (α, λ) for the zero energy vacuum and the vanishing fermion kinetic term $C = 0$. Here all the examples are shown with $k = 1$ fixed.

the fermion in the canonical branch, we solve the auxiliary field equation in the perturbation of ψ. Since the fermion emerges as a bilinear in the solution of F, we have $F = 0 + \mathcal{O}(\psi^2)$ in the canonical branch. Using this fact, the quadratic terms of the fermion ψ in the Lagrangian are found to be

$$\begin{aligned}\mathcal{L}_{\mathrm{quad}\psi} = {}& i\{-k + (\lambda - \alpha x_+)x_+\}\bar{\psi}\bar{\sigma}^{\hat{m}}\partial_{\hat{m}}\psi \\ &+ \frac{i}{2}\{-k + 3(\lambda - \alpha x_+)x_+ + 2\alpha x_+^2 e^{2ipx^1}\}\bar{\psi}\bar{\sigma}^1\partial_1\psi \\ &+ \frac{i}{2}\{-k + 3(\lambda - \alpha x_+)x_+ + 2\alpha x_+^2 e^{-2ipx^1}\}\psi\sigma^1\partial_1\bar{\psi} \\ &+ px_+\{\alpha x_+ - (\lambda - \alpha x_+)px_+\varphi_0\}\psi\sigma^1\bar{\psi}. \end{aligned} \qquad (26)$$

Here we have again separated the terms to the $SO(2,1)$ invariant transverse and the modulation sectors. The coefficient of the $SO(2,1)$ Lorentz invariant fermion kinetic term can be calculated as

$$\begin{aligned} C \equiv {}& -k + x_+(\lambda - \alpha x_+) = -k + \frac{1}{3\alpha}(\lambda + \sqrt{\lambda^2 - 3\alpha k}) \\ &\times \left(\lambda - \frac{1}{3}(\lambda + \sqrt{\lambda^2 - 3\alpha k})\right). \end{aligned} \qquad (27)$$

Whether the fermion becomes a ghost or not can be read off from the sign of the coefficient C. When $C > 0$ ($C < 0$), this is the wrong (correct) sign of the fermionic kinetic term, and then ψ is (not) a fermionic ghost. The sign of C is determined by the parameters k, λ, α, which are related to the sign of the vacuum energy we have classified before. The parameter regions of the positive and negative vacuum energies are found in Fig. 2(a). The regions where the coefficient of the fermion kinetic term C has the correct $C < 0$ and the wrong $C > 0$ signs are shown in Fig. 2(b). One finds that the regions of the positive energy and the correct sign $C < 0$ and those of the negative energy and the wrong sign $C > 0$ completely coincide. With this result at hand, we find that the Goldstino propagates in the transverse direction in the correct way; i.e., it never becomes a ghost in the metastable modulated vacuum. On the other hand, the Goldstino becomes a ghost in the negative energy vacuum. This is consistent with the SUSY algebra in Eq. (25). The norm of the Goldstino is positive (negative) for positive (negative) vacuum energies.

Since the sign of C changes continuously, one notices that at the boundary of two regions, the kinetic term vanishes. Indeed, the parameter curves for the zero vacuum energy and $C = 0$ coincide as in Fig. 2(c). We therefore expect that the Goldstino becomes nondynamical in the zero energy vacuum. This is a conceivable result from the observation of the SUSY algebra in Eq. (25). The fact that SUSY is broken in the zero vacuum energy results in the relation

$$0 = \sum_{\alpha,\dot{\alpha}=1,2}(\|Q_\alpha|0\rangle\|^2 + \|\bar{Q}_{\dot{\alpha}}|0\rangle\|^2), \qquad (28)$$

for the state $Q_\alpha|0\rangle \neq 0$. Namely, the Goldstino becomes a zero norm state, and it disappears from the physical sector. This is quite different from the ordinary SUSY breaking.

Things get more involved when we look at the kinetic term in the modulation direction. To clarify the sign of the coefficient of the kinetic term in the modulation direction, we perform the partial integration in the third term in Eq. (26). We then find

$$\begin{aligned}\mathcal{L}_{\mathrm{quad}\psi} = {}& iC\bar{\psi}\bar{\sigma}^{\hat{m}}\partial_{\hat{m}}\psi + iC_{\mathrm{mod}}\bar{\psi}\bar{\sigma}^1\partial_1\psi \\ &+ px_+^2\{\alpha - (\lambda - \alpha x_+)p\varphi_0 - 2pe^{-2ipx^1}\}\psi\sigma^1\bar{\psi}. \end{aligned} \qquad (29)$$

Here we have defined

$$C_{\rm mod} \equiv C - 2\alpha x_+^2 (1 - \cos(2px^1)). \qquad (30)$$

Because of the modulated vacuum, the coefficient $C_{\rm mod}$ oscillates in the x^1 direction. However, since the inequality $C_{\rm mod} \leq C$ always holds, the coefficient $C_{\rm mod}$ takes negative values in the parameter region for $C < 0$. We therefore conclude that the Goldstino in the positive energy vacuum propagates in the correct way even in the modulation direction. Then the modulated vacuum with positive energy we found is a completely consistent (meta)stable vacuum even in the fermionic sector.

On the other hand, because the minimum value of $C_{\rm mod}$,

$$\begin{aligned} \min C_{\rm mod} &= -k + (\lambda - \alpha x_+) x_+ - 4\alpha x_+^2 \\ &= -2\alpha x_+^2 - \lambda x_+ < 0, \end{aligned} \qquad (31)$$

is negative even in the region for $C \geq 0$, the modulation direction can have the correct sign of the fermionic kinetic term even in the negative or zero energy vacua. This also indicates the fact that the Goldstino has a nonzero kinetic term along the modulated direction even in the zero energy vacuum. Presumably, this is because the modulated vacuum (17) breaks the translational symmetry along x^1. We can perform the Lorentz boost of the zero-momentum Goldstino $Q_\alpha|0\rangle$, whether it is a ghost or not, to obtain the one that has a nonzero momentum $P^{\hat{m}}$. The resulting Goldstino has nonzero kinetic term $\bar{\psi}\bar{\sigma}^{\hat{m}}\partial_{\hat{m}}\psi$ in the $SO(2,1)$ Lorentz invariant sector. Since the sign of the norm does not change under the Lorentz transformation, there is a one-to-one correspondence between the sign of C and the norm of $Q_\alpha|0\rangle$ in the Lorentz invariant sector. However, this discussion does not hold in the modulated direction. We are not able to perform the translational transformation along the x^1 direction to obtain ψ that has a nonzero kinetic term $\bar{\psi}\bar{\sigma}^1\partial_1\psi$. Therefore the sign of $C_{\rm mod}$ does not help in judging the existence of ghosts in the modulated direction. In summary, we cannot say anything about ghosts in the modulated direction.

V. ANALYSIS WITH SUPERPOTENTIAL

In this section we introduce an example of the higher derivative chiral model where a superpotential W exists. We demonstrate that superpotentials generically change the "potential" of the derivative terms and a variety of modulated vacua is possible. The equation of motion for the auxiliary field in the single superfield model with general Λ becomes

$$K_{\varphi\bar{\varphi}}F + (-2F\partial_m\varphi\partial^m\bar{\varphi} + 2F^2\bar{F})\Lambda + \frac{\partial\bar{W}}{\partial\bar{\varphi}} = 0, \qquad (32)$$

where we have introduced only the bosonic fields. After eliminating $\bar{F}$ in the above equation, we have the equation only for F,

$$2\Lambda\frac{\partial W}{\partial\varphi}F^3 + \frac{\partial\bar{W}}{\partial\bar{\varphi}}(K_{\varphi\bar{\varphi}} - 2\Lambda\partial_m\varphi\partial^m\bar{\varphi})F + \left(\frac{\partial\bar{W}}{\partial\bar{\varphi}}\right)^2 = 0. \qquad (33)$$

The solutions are given by the Cardano's formula,

$$\begin{aligned} F^{(a)} = \omega^a &\sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} \\ &+ \omega^{3-a}\sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}. \end{aligned} \qquad (34)$$

Here $\omega^3 = 1$ and $a = 0, 1, 2$. We have defined the following quantities:

$$p = \frac{1}{2\Lambda}\left(\frac{\partial W}{\partial\varphi}\right)^{-1}\left(\frac{\partial\bar{W}}{\partial\bar{\varphi}}\right)(K_{\varphi\bar{\varphi}} - 2\Lambda\partial_m\varphi\partial^m\bar{\varphi}), \qquad (35)$$

$$q = \frac{1}{2\Lambda}\left(\frac{\partial W}{\partial\varphi}\right)^{-1}\left(\frac{\partial\bar{W}}{\partial\bar{\varphi}}\right)^2. \qquad (36)$$

The purely bosonic terms of the on-shell Lagrangian is calculated as

$$\begin{aligned} \mathcal{L}^{(a)} &= -K_{\varphi\bar{\varphi}}\partial_m\varphi\partial^m\bar{\varphi} + \Lambda(\partial_m\varphi\partial^m\varphi)(\partial_n\bar{\varphi}\partial^n\bar{\varphi}) \\ &\quad + F^{(a)}\bar{F}^{(a)}(-K_{\varphi\bar{\varphi}} + 2\Lambda\partial_m\varphi\partial^m\bar{\varphi}) - 3(F^{(a)}\bar{F}^{(a)})^2\Lambda. \end{aligned} \qquad (37)$$

Here $F^{(a)}$, $\bar{F}^{(a)}$ are one of the solutions for $a = 0, 1, 2$ in Eq. (34). Apparently there are three distinct branches corresponding to $a = 0, 1, 2$.

For simplicity, we choose the $a = 0$ branch and employ the ansatz for static, one-dimensional spatial configurations along the x^1 direction, $\varphi = \varphi(x^1)$. Then the energy functional becomes

$$\begin{aligned} \mathcal{E} &= K_{\varphi\bar{\varphi}}|\partial_1\varphi|^2 - \Lambda(\partial_1\varphi)^2(\partial_1\bar{\varphi})^2 \\ &\quad - |F^{(0)}|^2(K_{\varphi\bar{\varphi}} - 2\Lambda|\partial_1\varphi|^2) - 3|F^{(0)}|^4\Lambda. \end{aligned} \qquad (38)$$

Again, we consider the model characterized by the tensor (13) with the following simplest superpotential:

$$W = \beta\Phi. \qquad (39)$$

Here β is a real constant. The energy functional becomes a function of $x = |\partial_1\varphi|^2$,

$$\begin{aligned} \mathcal{E} &= kx - (\lambda - \alpha x)x^2 - |F(x)|^2(k - 2(\lambda - \alpha x)x) \\ &\quad - 3(\lambda - \alpha x)|F(x)|^4. \end{aligned} \qquad (40)$$

The auxiliary field in the $a = 0$ branch is

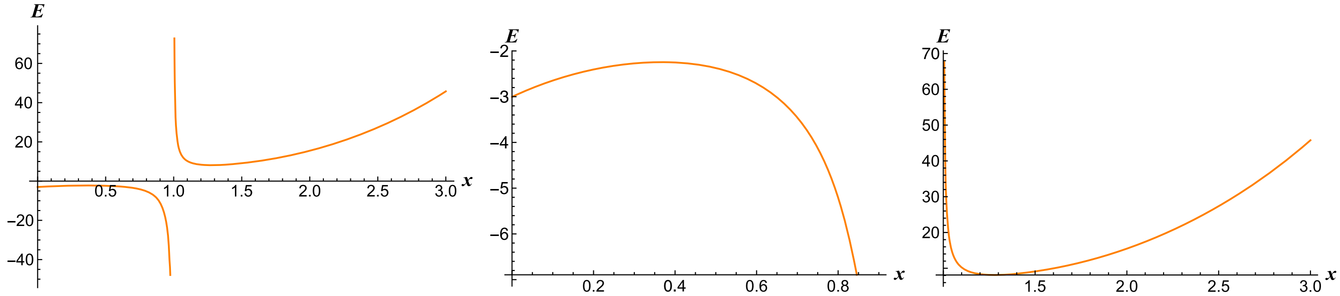


FIG. 3. Energy plot for $\Lambda = \lambda - \alpha|\partial_m \varphi|^2$, $W = \beta\Phi$ with $k = 1$, $\lambda = 1$, $\alpha = 1$, $\beta = 1$. Left: The global structure of the energy functional $\mathcal{E}$. Middle: Enlarged view of $\mathcal{E}$ around the origin $0 \le x \le 0.9$. Right: Enlarged view $\mathcal{E}$ in the region $x \ge 1$. The vertical and the horizontal axes represent the energy $\mathcal{E}(x)$ and $x = |\partial_1 \varphi|^2$.

$$F(x) = \left[-\frac{\beta}{4}(\lambda - \alpha x)^{-1} + \sqrt{\frac{\beta^2}{16}(\lambda - \alpha x)^{-2} + \frac{1}{6^3}(\lambda - \alpha x)^{-3}(k - 2(\lambda - \alpha x)x)^3}\right]^{\frac{1}{3}} + \left[-\frac{\beta}{4}(\lambda - \alpha x)^{-1} - \sqrt{\frac{\beta^2}{16}(\lambda - \alpha x)^{-2} + \frac{1}{6^3}(\lambda - \alpha x)^{-3}(k - 2(\lambda - \alpha x)x)^3}\right]^{\frac{1}{3}}. \quad (41)$$

A schematic picture of $\mathcal{E}(x)$ is found in Fig. 3. One finds that for the region in $0 < x < 1$, the potential is not bounded from below and the system becomes unstable (see the middle figure in Fig. 3). The origin is a metastable supersymmetric vacuum (although the vacuum energy in the example in Fig. 3 is negative and we expect that a ghost appears there). However, in the region $x > 1$, there is a global vacuum around $x = |\partial_1 \varphi|^2 \sim 1.280$ where SUSY is spontaneously broken (see the right figure in Fig. 3). The vacuum at $x \sim 1.280$ is clearly stable against decay, and it has positive energy ($\mathcal{E} \sim 8.126$). This is an acceptable, stable supersymmetry breaking vacuum. Even though the system becomes unstable in the small x region, this is an example where superpotential drastically changes the stability of modulated vacua.

VI. SUMMARY AND DISCUSSIONS

In this paper, we have studied the spatially modulated vacua in a supersymmetric theory with higher derivative terms. We have focused on the model where the famous Ostrogradsky instability is absent. Even though the scalar fields in the chiral multiplet appear with higher derivatives, the model exhibits no propagating auxiliary fields. The higher derivative part of the theory is defined by the Kähler tensor Λ. There are distinct on-shell branches corresponding to the different solutions to the equation of motion for the auxiliary field. We first consider the canonical branch in the model where the Kähler tensor Λ is given in Eq. (13) and no superpotential. The energy functional of this model is determined by the derivative terms of the scalar fields. We have found that the potential for the derivative terms allows a local vacuum where SUSY is spontaneously broken. In the SUSY breaking vacuum, we have shown that the translational symmetry along one direction and the rotational symmetries in the $(x^1, x^2), (x^1, x^3)$ planes are broken. However, the simultaneous transformation of P^1 and $U(1)$ is preserved in the modulated vacuum. This modulated vacuum is completely consistent with the equation of motion. The vacuum energy depends on the parameters of the Kähler metric and tensor. There are vacua where the vacuum energy is positive, zero, and negative.

We have demonstrated that the quadratic canonical kinetic term for the bosonic NG mode associated with the breaking of $P^1 \times U(1)$ vanishes while the Higgs boson that is orthogonal to the NG mode remains nonzero with the correct sign. This is a generalization of the NG theorem in higher derivative theories. On the other hand, the nature of the NG fermion (Goldstino) in the modulated vacua is quite different from the bosonic modes. We have found the SUSY breaking vacua where the vacuum energies take positive, negative, and zero values. For the positive vacuum energy, the modulated vacuum is metastable against decaying to the global supersymmetric vacuum. However, sufficiently large possibilities of allowed parameters k, α, Λ for the metastable vacuum indicate that one can make the decay rate be so small compared with the lifetime of the Universe [5]. The Goldstino in this vacuum is well behaved; namely, it has the correct sign of the kinetic term both in the $SO(2,1)$ Lorentz invariant and in the modulated sectors. We have also shown that when SUSY is spontaneously broken in the vacuum where the vacuum energy is zero, then the kinetic term of the Goldstino vanishes and it becomes nondynamical. This is consistent with the SUSY algebra in which the norm of the zero-momentum Goldstino states becomes zero. This is quite different from the ordinary supersymmetric theories where the zero energy vacuum corresponds to the supersymmetric vacuum. For the negative vacuum energy, the modulated vacuum is the global minimum and it is the true vacuum. The SUSY algebra together with the negative vacuum energy implies that the Goldstino has a negative norm; i.e., it becomes a fermionic ghost. We have explicitly shown that there appears the wrong sign for the kinetic term of ψ in the negative energy vacuum. Although goldstinos accompanied by the negative vacuum energy are problematic in a physical theory [59], there are several ways to remove undesirable ghost states from the physical sector [42,60,61]. We therefore conclude that the spatially modulated state with positive vacuum energy is the physically acceptable supersymmetry breaking vacuum in our model.

We have also studied a model with a superpotential. Although the on-shell Lagrangian is complicated due to the solution to the equation of motion for the auxiliary field, we have been able to explicitly draw the potential energy for the derivative terms. As an example, a simple model where the linear superpotential is introduced to the prototypical model is analyzed. We have found that at large $x = |\partial_1\varphi|^2$, there is a modulated vacuum that is stable against decaying. We expect there are no ghost Goldstino in this vacuum. However, in the vicinity of the origin, the energy is not bounded from below and the system suffers from the serious instability and ghosts. In particular, in the supersymmetric vacuum in the origin, we expect a ghost Goldstino. Alternative choices of K, Λ, and W would help us to find a modulated vacuum that is the global minimum and has positive energy.

We have explicitly shown that the spontaneous SUSY breaking on a spatially inhomogeneous vacuum actually occurs in a simple SUSY model where no propagating auxiliary fields and no Ostrogradsky's ghost [41] exist. We stress that the spontaneous SUSY breaking in the spatially modulated vacua—that attract the greater attention recently [36–38]—together with the ubiquity of the Lorentz violation [28], opens up robust possibilities of model buildings for particle physics and cosmology [26,62].

Before closing the paper, we give several discussions. In this paper we have discussed a new mechanism for spontaneous supersymmetry breaking based on the modulated vacua studied in [39]. There are several interesting issues on the modulated vacua in supersymmetric theories. In this paper we have studied spatially modulated vacua along only one direction. However, it is possible to find higher dimensional modulation [63]. It is also interesting to find a temporal modulation [64]. It is conceivable that modulated vacua including the one presented in this paper are ubiquitous in supersymmetric higher derivative theories. We expect that these kinds of modulated vacua are utilized for phenomenological model buildings. Embedding to supergravity [50,56] is one of the future directions.

Most notably, it is always true that the ordinary quadratic kinetic term of the bosonic NG modes disappears in the modulated vacua and there are derivative interactions of quartic type [39]. Although these quartic derivative interactions do not show any problematic behavior in the classical mechanics, they may cause some (yet unknown) problems in the quantum regime. To our knowledge, there are no systematic analysis on consistent quantum theories for such a vanishing quadratic kinetic term model. It would therefore be interesting to study a quantum mechanical model where no quadratic kinetic term of dynamical variables exists. We will come back to these issues in future research.

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APPENDIX: COMPONENT EXPANSION OF THE HIGHER DERIVATIVE TERMS

The component expansion including fermions of the $\mathcal{N} = 1$ chiral superfield in the central basis is

$$\begin{aligned}\Phi &= \varphi + i(\theta\sigma^m\bar{\theta})\partial_m\varphi + \frac{1}{4}\theta^2\bar{\theta}^2\Box\varphi + \sqrt{2}\theta^\alpha\psi_\alpha - \frac{i}{\sqrt{2}}\theta^2\partial_m\psi^\alpha(\sigma^m)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} + \theta^2 F,\\ \Phi^\dagger &= \bar{\varphi} - i(\theta\sigma^m\bar{\theta})\partial_m\bar{\varphi} + \frac{1}{4}\theta^2\bar{\theta}^2\Box\bar{\varphi} + \sqrt{2}\bar{\theta}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} + \frac{i}{\sqrt{2}}\bar{\theta}^2\theta^\alpha(\sigma^m)_{\alpha\dot{\alpha}}\partial_m\bar{\psi}^{\dot{\alpha}} + \bar{\theta}^2\bar{F}.\end{aligned} \tag{A1}$$

The component expansion of the higher derivative term is [56]

$$\begin{aligned}
\frac{1}{16}(D\Phi)^2(\bar{D}\Phi^\dagger)^2 = \theta^2\bar{\theta}^2\Big[&(\partial_m\varphi)^2(\partial_n\bar{\varphi})^2 - 2\bar{F}F\partial_m\varphi\partial^m\bar{\varphi} + \bar{F}^2F^2 \\
&-\frac{i}{2}(\psi\sigma^m\bar{\sigma}^n\sigma^p\partial_p\bar{\psi})\partial_m\varphi\partial_n\bar{\varphi} + \frac{i}{2}(\partial_p\psi\sigma^p\bar{\sigma}^m\sigma^n\bar{\psi})\partial_m\varphi\partial_n\bar{\varphi} \\
&+ i\psi\sigma^m\partial^n\bar{\psi}\partial_m\varphi\partial_n\bar{\varphi} - i\partial^m\sigma^n\bar{\psi}\partial_m\varphi\partial_n\bar{\varphi} + \frac{i}{2}\psi\sigma^m\bar{\psi}(\partial_m\bar{\varphi}\Box\varphi - \partial_m\varphi\Box\bar{\varphi}) \\
&+ \frac{1}{2}(F\Box\varphi - \partial_m F\partial^m\varphi)\bar{\psi}^2 + \frac{1}{2}(\bar{F}\Box\bar{\varphi} - \partial_m\bar{F}\partial^m\bar{\varphi})\psi^2 \\
&+ \frac{1}{2}F\partial_m\varphi(\bar{\psi}\bar{\sigma}^m\sigma^n\partial_n\bar{\psi} - \partial_n\bar{\psi}\bar{\sigma}^m\sigma^n\bar{\psi}) + \frac{1}{2}\bar{F}\partial_m\bar{\varphi}(\partial_n\sigma^n\bar{\sigma}^m\psi - \psi\sigma^n\bar{\sigma}^m\partial_n\psi) \\
&+ \frac{3}{2}i\bar{F}F(\partial_m\psi\sigma^m\bar{\psi} - \psi\sigma^m\partial_m\bar{\psi}) + \frac{i}{2}\psi\sigma^m\bar{\psi}(F\partial_m\bar{F} - \bar{F}\partial_m F)\Big] \\
&+ \sqrt{2}i\bar{\theta}^2(\partial_m\varphi)^2(\theta\sigma^n\bar{\psi})\partial_n\bar{\varphi} - \sqrt{2}i\theta^2(\partial_m\bar{\varphi})^2(\psi\sigma^n\bar{\theta})\partial_n\varphi \\
&+ \sqrt{2}\theta^2 F\partial_m\bar{\varphi}(i\bar{F}(\psi\sigma^m\bar{\theta}) + (\bar{\theta}\bar{\sigma}^m\sigma^n\bar{\psi})\partial_m\varphi) \\
&+ \sqrt{2}\bar{\theta}^2\bar{F}\partial_m\varphi(-iF(\theta\sigma^m\bar{\psi}) + (\psi\sigma^m\bar{\sigma}^n\theta)\partial_n\bar{\varphi}) \\
&- \frac{1}{2}\bar{\theta}^2(\partial_m\varphi)^2\bar{\psi}\bar{\psi} - \frac{1}{2}\theta^2(\partial_m\bar{\varphi})^2\psi\psi + 2(\psi\sigma^m\bar{\theta})(\theta\sigma^n\bar{\psi})\partial_m\varphi\partial_n\bar{\varphi} \\
&+ 2\bar{F}F(\theta\psi)(\bar{\theta}\bar{\psi}) + i(\theta\sigma^m\bar{\theta})(F\partial_m\varphi\bar{\psi}\bar{\psi} - \bar{F}\partial_m\bar{\varphi}\psi\psi) + \frac{1}{2}\theta^2F^2\bar{\psi}\bar{\psi} + \frac{1}{2}\bar{\theta}^2\bar{F}^2\psi\psi \\
&+ \sqrt{2}\bar{F}F(\bar{F}(\theta\psi) + F(\bar{\theta}\bar{\psi})) + i(\psi\sigma^m\bar{\psi})(F\partial_m\bar{\varphi} - \bar{F}\partial_m\varphi). \qquad \text{(A2)}
\end{aligned}$$

The component expansion of the Λ function for the model $\Lambda = \lambda - \alpha\partial_m\Phi\partial^m\Phi^\dagger$ is calculated using the following expression:

$$\begin{aligned}
\partial_m\Phi\partial^m\Phi^\dagger = &\partial_m\varphi\partial^m\bar{\varphi} + \sqrt{2}(\theta\partial_m\psi)\partial^m\bar{\varphi} + \sqrt{2}(\bar{\theta}\partial_m\bar{\psi})\partial^m\varphi + \theta^2\partial_m\bar{\varphi}\partial^m F + \bar{\theta}^2\partial_m\varphi\partial^m\bar{F} \\
&+ \theta^\alpha\bar{\theta}^{\dot{\alpha}}[i(\sigma^p)_{\alpha\dot{\alpha}}(\partial_m\bar{\varphi}\partial_p\partial^m\varphi - \partial_p\partial_m\bar{\varphi}\partial^m\varphi) - 2\partial_m\bar{\psi}_{\dot{\alpha}}\partial^m\psi_\alpha] \\
&+ \theta^2\bar{\theta}^{\dot{\alpha}}\left[\frac{i}{\sqrt{2}}(\sigma^p)_{\alpha\dot{\alpha}}(\partial_m\bar{\varphi}\partial_p\partial^m\psi^\alpha - \partial_p\partial_m\bar{\varphi}\partial^m\psi^\alpha) - \sqrt{2}\partial_m F\partial^m\bar{\psi}_{\dot{\alpha}}\right] \\
&+ \bar{\theta}^2\theta^\alpha\left[-\frac{i}{\sqrt{2}}(\sigma^p)_{\alpha\dot{\alpha}}(\partial_m\bar{\psi}^{\dot{\alpha}}\partial_p\partial^m\varphi - \partial_p\partial_m\bar{\psi}^{\dot{\alpha}}\partial^m\varphi) + \sqrt{2}\partial_m\bar{F}\partial^m\psi_\alpha\right] \\
&+ \theta^2\bar{\theta}^2\Big[\partial_m F\partial^m\bar{F} + \frac{1}{4}\partial_m\bar{\varphi}\Box\partial^m\varphi + \frac{1}{4}\Box\partial_m\bar{\varphi}\partial^m\varphi - \frac{1}{2}\partial_m\partial_p\bar{\varphi}\partial^m\partial^p\varphi \\
&+ \frac{i}{2}\partial_m\partial_p\bar{\psi}\bar{\sigma}^p\partial^m\psi - \frac{i}{2}\partial_m\bar{\psi}\bar{\sigma}^p\partial_p\partial^m\psi\Big]. \qquad \text{(A3)}
\end{aligned}$$

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