

Low-energy effects in a higher-derivative gravity model with real and complex massive poles

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(Received 9 November 2016; published 8 November 2017)

The most simple superrenormalizable model of quantum gravity is based on the general local covariant six-derivative action. In addition to graviton such a theory has massive scalar and tensor modes. It was shown recently that in the case when the massive poles emerge in complex conjugate pairs, the theory has also unitary S -matrix and hence can be seen as a candidate to be a consistent quantum gravity theory. In the present work we construct the modified Newton potential and explore the gravitational light bending in a general six-derivative theory, including the most interesting case of complex massive poles. In the case of the light deflection the results are obtained within classical and semiclassical approaches.

DOI: [10.1103/PhysRevD.96.104004](https://doi.org/10.1103/PhysRevD.96.104004)

I. INTRODUCTION

Quantum gravity is an important part of a modern quantum field theory (QFT) and of the gravitational physics. Since there are relatively small chances to observe quantum corrections to the action of gravity, one of the main targets of quantum gravity is to establish the classical action capable of providing a consistent quantum theory. Thinking in this direction we immediately realize the relevant role played by higher-derivative terms, as well as the difficult problem they represent. Even at the semiclassical level one has to include fourth derivative terms into the gravitational action in order to provide renormalizability [1] (see also [2,3] for an introduction and [4] for a recent review), and the same is also true for the quantum theory of the gravitational field itself [5]. On the other hand the fourth derivative terms lead to ghost (or tachyonic ghost) degrees of freedom in the physical spectrum of the theory, which implies that there will be instabilities in the classical solutions. The existence of a physically real ghost particle is a theoretical disaster: such a particle has negative kinetic energy, therefore it will accelerate emitting plenty of gravitons. As a result the absolute value of its negative energy rapidly goes to infinity and an infinitely powerful gravitational explosion will occur. Since nothing of this sort was observed so far, the problem should have some theoretical resolution.

During many years the discussions about the problem of ghosts were based on the following approaches:

- (i) Treating all higher derivatives, together with the corresponding quantum contributions, as small corrections [6], in the same way as it is done in QED to avoid the run-away solutions [7]. Within this approach one has to ignore a great difference which exists between gravity and QED, since the latter is renormalizable without higher derivative terms. As a consequence, in QED the higher derivative terms are not running, and one can always assume that they are just part of a more general action which is presumably nonlocal and free of ghosts. A very simple example of an artificial ghost appearance was recently discussed in the context of effective approach to quantum field theory [8]. In the case of gravity the assumption of smallness of the higher-derivative terms is much more *ad hoc* and is certainly suitable only at the energy scales much below the Planck scale. A natural question is why we need a theory of quantum gravity which works only at low energies, and this question remains without answer.

Another alternative is to assume that the higher derivative ghosts exist only as virtual excitations, but for some unknown reason they are not generated as physical particles at the sub-Planck energies. The creation of a Planck-mass ghost from vacuum requires a concentration of gravitons with Planck density, and in some gravity models this may be impossible [9]. A strong support for this hypothesis comes from the low-energy stability of the classical cosmological solutions in higher derivative gravity models [10–12].

- (ii) In string theory the space-time metric is regarded as an effective composite field and one can redefine it

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in such a way that the ghost degrees of freedom disappear [13]. Formally this solves the problem.¹ However, there are two difficulties in this approach. *First*, the procedure is ambiguous. For instance one can remove or not R^2 , R^3 , and other similar terms, or one can just modify their coefficients. All of these terms do not contribute to ghosts, but at the same time they do affect classical gravitational solutions [14], giving rise to a great uncertainty in the predictions of the theory. *Second*, the removal of all the terms which produce ghosts, such as $R_{\mu\nu}\square^k R^{\mu\nu}$ for $k \geq 0$ and $R\square^k R$ for $k \geq 1$ must be performed with absolute precision. Any infinitesimal deviation from zero in any of these coefficients means that the ghost comes back and that its mass is huge, even compared to the Planck mass. Then the effect of such a ghost (e.g., instability of Minkowski space) will be even stronger and not weaker, as one can imagine.

Furthermore, at lower energies our experience shows that the appropriate description of quantum effects is within the QFT, not string theory. However, the loop corrections within QFT typically break down an absolutely precise fine-tuning which is required to avoid ghosts (see, e.g., the discussion in [15]). Therefore, string theory is helpful in solving ghost problems only if we assume that all the low-energy quantum physics, in all its details, is a consequence of a string theory, that provides the requested cancellations. Such an assumption looks very strong and certainly difficult to believe in without further arguments.

- (iii) In the framework of four derivative quantum gravity one can assume that the dressed gravitational propagator, with quantum corrections, makes the ghost unstable. Then the theory could possibly have a unitary S -matrix. This idea was nicely introduced in [16–18], but the final conclusion was that the information which can be obtained via the perturbative QFT approaches is not sufficient to decide whether this mechanism is working or not [19].
- (iv) Another possibility is to start from a nonlocal theory with infinitely many derivatives of the metric. One can consider nonlocal form factors such as $R_{\mu\nu}\Phi(\square)R^{\mu\nu}$ and $R\Psi(\square)R$ so that no other poles will exist in the propagator besides the massless one corresponding to the graviton. This procedure can be applied either in string theory [20] as an alternative to the metric reparametrization of [13], or in quantum gravity [21] (see also [22–24] for recent developments and further references). The main disadvantage of this approach is that the functions

$\Phi(\square)$ and $\Psi(\square)$ must be chosen with absolute precision. As a result the ghost-free conditions can not survive any kind of low-energy quantum corrections [15]. After the specially tuned form of the form factors gets modified, there is an infinite amount of ghostlike states, all of them corresponding to complex poles.

- (v) The last possibility is to consider local gravitational theories with more than four derivatives. These theories have remarkable quantum properties. Typically they are superrenormalizable [25] and also, in case of massive complex poles, can be unitary in the Lee-Wick sense [26]. Therefore, these theories are capable to solve the conflict between UV renormalizability and unitarity in quantum gravity. Regardless of remaining problems, these models are unitary without any sort of fine-tuning and hence they represent simpler alternatives to the nonlocal models.

Of course, at the present level the higher derivative theories with complex massive poles cannot be seen as a complete solution of the quantum gravity problem, but they look as strong candidates. Therefore, it makes sense to explore their IR properties at the classical level and identify observables which might be useful for experimental detection of higher derivatives. The model of our interest is the simplest theory which admits complex poles, with the action of the form

$$\mathcal{S} = \int d^4x \sqrt{-g} \left\{ \frac{2}{\kappa^2} R + \frac{\alpha}{2} R^2 + \frac{\beta}{2} R_{\mu\nu}^2 + \frac{A}{2} R\square R + \frac{B}{2} R_{\mu\nu}\square R^{\mu\nu} + \mathcal{L}_M \right\}, \quad (1)$$

where \mathcal{L}_M is the matter Lagrangian, $\kappa^2/2 = 16\pi G = M_P^{-2}$, G is Newton's constant, and M_P is the reduced Planck mass. α , β , A , and B are free parameters, the first two being dimensionless, and A and B carry dimension of $(\text{mass})^{-2}$. The values of these parameters should be determined by experimental data.

Let us note that the structure of the poles in the dressed propagator of gravitons was considered in some recent publications, for example in [27–29], where physical effects of complex poles were discussed. In particular, one of the results of [27] is that the perturbative unitarity can be restored by the resummation. In general, the approach of the present work differs from the one in these references since we regard the higher-derivative model as a fundamental rather than effective. In the IR sector considered here, however, the difference between the approaches is supposed to be irrelevant, as heavy degrees of freedom should decouple in the long-distance limit. In our present work this is not the case nonetheless, because we are partially dealing with the propagation of massive degrees of freedom up to the cosmic scales, or at

¹It is worth noting that the same can be achieved in the semiclassical theory of gravity.

least up to the scale of laboratory. Indeed, since there are no direct experiments on quantum gravity, very different approaches to the problem should be seen as legitimate in this area.

The model (1) is the particular case of the superrenormalizable quantum gravity theory formulated in [25]. One can generalize it by adding $\mathcal{O}(R^3)$ -terms to the action, but these terms should be irrelevant for our purposes, since we are interested in the effects related to the linear gravitational perturbations.

In the present paper we will explore in detail the two most obvious low-energy observables which can be used to falsify the presence of fourth- and six-derivative terms in the theory (1). The first part of the work is about the modified Newtonian potential. If compared to the previous works on the subject (see, e.g., [30–32]), we include here the cases of complex and multiple real poles, that provides a better perspective and understanding for the modified potential in general polynomial higher derivative models.

The second part of the paper is devoted to the bending of light in the theories with higher derivatives. This issue is attracting a great deal of attention, especially in relation to quantum gravity and quantum field theory effects. Indeed, quantum effects can be partially taken into account in the low-energy domain by the use of semiclassical methods. In the case of gravity let us mention, e.g., the influence of the one-loop vacuum polarization in the propagation of photons on a curved background. This issue was explored in the papers [33,34] using two different approaches. In the former work the effect is described by the differential cross section. It gives the correct leading term for the gravitational bending angle of an unpolarized beam plus a semiclassical correction, which depends on the energy of the photons. On the other hand, in [34] the semiclassical correction is introduced in the interaction potential between an external gravitational field and a photon. As a result the deflection angle depends on the photon's polarization, but it is nondispersive. According to [34], this version of the semiclassical consideration is the correct one, since it assumes that for macroscopic systems the photon is better described by a compact wave packet with a definite path in the gravitational field. In Sec. V one can find the discussion of this issue in the context of higher-derivative gravity. In particular, we elaborate on the explanation concerning the limits of applicability of the semiclassical approach similar to the one of [33], and explain why the method based on the cross sections usually can not be used to describe the bending of light at astronomical scales.

The bending of light in the theory (1) is briefly discussed in the parallel work [35] which is devoted to the possibility of a specific seesaw mechanism in higher derivative quantum gravity. The much more detailed treatment of this issue here complements the discussion of the parallel work.

The paper is organized as follows. In Sec. II a generalization of a theorem by Teyssandier [36] for the six-order gravity is formulated. The theorem, which is proved in Appendix A, presents the general solution for the linearized sixth-order gravity as a linear combination of five auxiliary fields. In Sec. III we study the modified Newtonian potential of the theory. The poles of the propagator can be either real (simple or degenerate) or complex. In particular, we show that the potential is regular at the origin, extending the result of [31].² Section IV is devoted to the study of the classical gravitational deflection of light rays, for each of the possible types of poles. The quantum mechanical formulation of the scattering process and the restricted applicability of such an approach to macroscopic systems is discussed in Sec. V. In Sec. VI we draw our conclusions.

Our notations are as follows. The units correspond to $\hbar = c = 1$. The signature is $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, and the Riemann and Ricci tensors are

$$R^\rho{}_{\lambda\mu\nu} = \partial_\mu \Gamma^\rho_{\lambda\nu} - \partial_\nu \Gamma^\rho_{\lambda\mu} + \Gamma^\sigma_{\lambda\nu} \Gamma^\rho_{\sigma\mu} - \Gamma^\sigma_{\lambda\mu} \Gamma^\rho_{\sigma\nu} \quad (2)$$

and $R_{\mu\nu} = R^\rho{}_{\mu\rho\nu}$. This choice of notations is intended to facilitate the comparison of our calculations with the previous work [37] on the four-derivative gravity.

II. FIELD GENERATED BY A POINTLIKE MASS IN REST

It is clear that the right choice of fields parametrization and of a suitable gauge condition may lead to an essential simplification of the field equations. This is especially important for the higher-order gravity models, which have rather complicated dynamical equations. In 1989 Teyssandier [36] introduced a useful form of the third-order coordinate condition for the linearized fourth-order gravity described by the action

$$\mathcal{S}_4 = \int d^4x \sqrt{-g} \left\{ \frac{2}{\kappa^2} R + \frac{\alpha}{2} R^2 + \frac{\beta}{2} R_{\mu\nu}^2 + \mathcal{L}_M \right\}, \quad (3)$$

In the Teyssandier gauge the general solution of the linearized field equations are written as a linear combination of three decoupled fields [36]. In terms of these auxiliary fields the weak gravitational field generated by a static source can be promptly computed, as well as the classical potential of the theory, which is proportional to the (00)-component of the metric perturbation.

Our goal is to obtain similar representation in the framework of the sixth-order gravity model (1). The variational principle applied to the action $\mathcal{S}[g_{\mu\nu}]$ leads to the field equations of the sixth-order gravity:

²A more general treatment of this issue is given in the parallel work [32].

$$\begin{aligned}
 & \frac{2}{\kappa^2} \left(R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} \right) + \frac{\alpha}{2} \left[2RR_{\mu\nu} + 2\nabla_\mu \nabla_\nu R - 2g_{\mu\nu} \square R - \frac{R^2}{2} g_{\mu\nu} \right] + \frac{\beta}{2} \left[-\frac{1}{2} g_{\mu\nu} R^2_{\alpha\beta} + \nabla_\mu \nabla_\nu R + 2R_{\mu\sigma\rho\nu} R^{\sigma\rho} - \frac{1}{2} g_{\mu\nu} \square R - \square R_{\mu\nu} \right] \\
 & + \frac{A}{2} \left[R_{\mu\nu} \square R + R \square R_{\mu\nu} + 2\square \nabla_\mu \nabla_\nu R - 2g_{\mu\nu} \square^2 R - (\nabla_\mu R)(\nabla_\nu R) + \frac{1}{2} g_{\mu\nu} (\nabla R)^2 \right] \\
 & + \frac{B}{2} \left[\square \nabla_\mu \nabla_\nu R + 2\square (R_{\mu\sigma\rho\nu} R^{\sigma\rho}) - \square^2 \left(R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} R \right) - 4(\nabla_\sigma R_{\nu\rho})(\nabla^\sigma R_{\mu}{}^\rho) \right. \\
 & + 2(\nabla_\sigma R_{\rho(\nu})(\nabla_{\mu)} R^{\sigma\rho}) + R_{\nu\sigma} \nabla_\rho \nabla_\mu R^{\rho\sigma} + R_{\mu\sigma} \nabla_\rho \nabla_\nu R^{\rho\sigma} + \frac{1}{2} g_{\mu\nu} (\nabla_\lambda R_{\rho\sigma})^2 \\
 & \left. - (\nabla^\sigma R + 2R^{\rho\sigma} \nabla_\rho) \nabla_{(\mu} R_{\nu)\sigma} - (\nabla_\mu R_{\rho\sigma})(\nabla_\nu R^{\rho\sigma}) - 2R_{\sigma(\mu} \square R_{\nu)}{}^\sigma \right] = -\frac{1}{2} T_{\mu\nu}, \tag{4}
 \end{aligned}$$

where the parenthesis in the indices denote symmetrization, e.g.,

$$\nabla_{(\mu} R_{\nu)\sigma} \equiv \frac{1}{2} (\nabla_\mu R_{\nu\sigma} + \nabla_\nu R_{\mu\sigma}).$$

In the weak field regime the metric can be considered as a fluctuation around the flat space,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \tag{5}$$

with $|\kappa h_{\mu\nu}| \ll 1$. The Ricci tensor $R_{\mu\nu}$ and the scalar curvature R up to the first order in κ are

$$R_{\mu\nu}^{(1)} = \frac{\kappa}{2} [\square h_{\mu\nu} - \eta^{\lambda\rho} (\gamma_{\lambda\mu,\nu\rho} + \gamma_{\lambda\nu,\mu\rho})], \tag{6}$$

$$R^{(1)} = \kappa \left(\frac{1}{2} \square h - \eta^{\lambda\rho} \eta^{\mu\nu} \gamma_{\lambda\mu,\nu\rho} \right). \tag{7}$$

In the last expressions we used the notations

$$\gamma_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h, \quad h = \eta^{\mu\nu} h_{\mu\nu}. \tag{8}$$

Since the equations of motion are already expanded to the order κ^2 , the d'Alembertian is calculated using the flat metric, $\square = \eta^{\mu\nu} \partial_\mu \partial_\nu$.

Using the expressions (6)–(8), the linearized equations of motion (4) are

$$\begin{aligned}
 & \left(\frac{2}{\kappa^2} - \frac{\beta}{2} \square - \frac{B}{2} \square^2 \right) \left(R_{\mu\nu}^{(1)} - \frac{1}{2} \eta_{\mu\nu} R^{(1)} \right) \\
 & - \left(\alpha + \frac{\beta}{2} + A \square + \frac{B}{2} \square \right) (\eta_{\mu\nu} \square R^{(1)} - \partial_\mu \partial_\nu R^{(1)}) = -\frac{1}{2} T_{\mu\nu}. \tag{9}
 \end{aligned}$$

The trace of Eq. (9) has the form

$$\begin{aligned}
 & \left(\alpha + \frac{\beta}{2} + A \square + \frac{B}{2} \square \right) \square R^{(1)} \\
 & = -\frac{1}{3} \left(\frac{2}{\kappa^2} - \frac{\beta}{2} \square - \frac{B}{2} \square^2 \right) R^{(1)} + \frac{1}{6} T. \tag{10}
 \end{aligned}$$

Replacing (10) into (9) yields

$$\begin{aligned}
 & \left(\frac{2}{\kappa^2} - \frac{\beta}{2} \square - \frac{B}{2} \square^2 \right) \left(R_{\mu\nu}^{(1)} - \frac{1}{6} \eta_{\mu\nu} R^{(1)} \right) \\
 & + \left(\alpha + \frac{\beta}{2} + A \square + \frac{B}{2} \square \right) \partial_\mu \partial_\nu R^{(1)} = \frac{1}{6} T \eta_{\mu\nu} - \frac{1}{2} T_{\mu\nu}. \tag{11}
 \end{aligned}$$

Inserting the expression (6) for the first order Ricci tensor into the preceding equation we obtain

$$\begin{aligned}
 & \left[\frac{\kappa^2}{4} (\beta + B \square) \square - 1 \right] \left(\square h_{\mu\nu} - \frac{1}{3\kappa} R^{(1)} \eta_{\mu\nu} \right) + \Gamma_{(\mu,\nu)} \\
 & = 2 \left(T_{\mu\nu} - \frac{1}{3} T \eta_{\mu\nu} \right), \tag{12}
 \end{aligned}$$

where we defined the quantities

$$\begin{aligned}
 \Gamma_\mu & = \left(1 - \frac{\kappa^2 \beta}{4} \square - \frac{\kappa^2 B}{4} \square^2 \right) \gamma_{\mu\rho}{}^{\rho} \\
 & - \frac{\kappa}{2} \left(\alpha + \frac{\beta}{2} + A \square + \frac{B}{2} \square \right) R_{,\mu}^{(1)}. \tag{13}
 \end{aligned}$$

Hence, implementing the gauge condition $\Gamma_\mu = 0$ makes the problem of solving the linearized field equations (9) for $h_{\mu\nu}$ to be equivalent to the system consisting of the gauge condition and of Eq. (12). The convenience of this gauge is to allow the solution to be expressed in terms of auxiliary fields. Let us formulate this statement as a Theorem, with the proof postponed to Appendix A.

Theorem. The general solution of the system constituted by (12) and the gauge condition $\Gamma_\mu = 0$ can be presented in the form

$$h_{\mu\nu} = h_{\mu\nu}^{(E)} + \Psi_{\mu\nu} + \bar{\Psi}_{\mu\nu} - \eta_{\mu\nu}\Phi - \eta_{\mu\nu}\bar{\Phi}, \quad (14)$$

where the auxiliary fields $h_{\mu\nu}^{(E)}$, $\Psi_{\mu\nu}$, $\bar{\Psi}_{\mu\nu}$, Φ and $\bar{\Phi}$ satisfy the second order equations

$$\square h_{\mu\nu}^{(E)} = \frac{\kappa}{2} \left(\frac{1}{2} T \eta_{\mu\nu} - T_{\mu\nu} \right), \quad (15)$$

$$\gamma_{\mu\nu}^{(E),\nu} = 0, \quad \gamma_{\mu\nu}^{(E)} \equiv h_{\mu\nu}^{(E)} - \frac{1}{2} \eta_{\mu\nu} h^{(E)}, \quad (16)$$

$$(m_{2+}^2 + \square) \Psi_{\mu\nu} = \frac{\kappa}{2} \left(T_{\mu\nu} - \frac{1}{3} T \eta_{\mu\nu} \right), \quad (17)$$

$$(m_{2-}^2 + \square) \bar{\Psi}_{\mu\nu} = m_{2+}^2 \Psi_{\mu\nu}, \quad (18)$$

$$(\Psi_{\mu\nu} + \bar{\Psi}_{\mu\nu})^{,\mu\nu} = \square (\Psi + \bar{\Psi}), \quad (19)$$

$$(m_{0+}^2 + \square) \Phi = \frac{\kappa}{12} T, \quad (20)$$

$$(m_{0-}^2 + \square) \bar{\Phi} = m_{0+}^2 \Phi. \quad (21)$$

Here and in what follows we use the condensed notations

$$\Psi = \eta^{\mu\nu} \Psi_{\mu\nu}, \quad \sigma_1 = 3\alpha + \beta, \quad \sigma_2 = 3A + B \quad (22)$$

and

$$m_{2\pm}^2 = \frac{-\frac{\beta|B|}{B} \pm \sqrt{\beta^2 + \frac{16}{\kappa^2} B}}{-2|B|},$$

$$m_{0\pm}^2 = \frac{\frac{\sigma_1|\sigma_2|}{\sigma_2} \pm \sqrt{\sigma_1^2 - \frac{8}{\kappa^2} \sigma_2}}{2|\sigma_2|}. \quad (23)$$

According to the Theorem 1 formulated above, it is possible to split the field $h_{\mu\nu}$ into a linear combination of the five fields: a massless tensor representing the solution of linearized Einstein's equations in de Donder gauge, two massive tensor fields $\Psi_{\mu\nu}$ and $\bar{\Psi}_{\mu\nu}$ and two scalars Φ and $\bar{\Phi}$. Let us stress that in the present case the massive fields with the same spin are not dynamically independent. For this reason, as it will be shown in the next section, the theory under discussion has a finite modified Newtonian potential, regardless of the (complex or real) nature of the quantities $m_{2\pm}$ and $m_{0\pm}$.

Using the previous theorem it is straightforward to calculate the field generated by a pointlike mass in rest at $\mathbf{r} = 0$. The corresponding energy-momentum tensor is $T_{\mu\nu}(\mathbf{r}) = M \eta_{\mu 0} \eta_{\nu 0} \delta^{(3)}(\mathbf{r})$. The solution for $h_{\mu\nu}^{(E)}$ is the same as in Einstein's gravity in the de Donder gauge:

$$h_{\mu\nu}^{(E)}(\mathbf{r}) = \frac{M\kappa}{16\pi r} (\eta_{\mu\nu} - 2\eta_{\mu 0} \eta_{\nu 0}). \quad (24)$$

The solutions for the massive tensor fields read

$$\Psi_{\mu\nu}(\mathbf{r}) = \frac{M\kappa}{8\pi} \left(\eta_{\mu 0} \eta_{\nu 0} - \frac{1}{3} \eta_{\mu\nu} \right) \frac{e^{-m_{2+}r}}{r} \quad (25)$$

and

$$\bar{\Psi}_{\mu\nu}(\mathbf{r}) = \frac{M\kappa}{8\pi} \left(\eta_{\mu 0} \eta_{\nu 0} - \frac{1}{3} \eta_{\mu\nu} \right) \times \frac{m_{2+}^2}{m_{2+}^2 - m_{2-}^2} \left(\frac{e^{-m_{2-}r}}{r} - \frac{e^{-m_{2+}r}}{r} \right). \quad (26)$$

It is easy to verify that these solutions satisfy the subsidiary gauge condition (19).

For the scalar modes we have

$$\Phi(\mathbf{r}) = \frac{M\kappa}{48\pi} \frac{e^{-m_{0+}r}}{r}, \quad (27)$$

$$\bar{\Phi}(\mathbf{r}) = \frac{M\kappa}{48\pi} \frac{m_{0+}^2}{m_{0+}^2 - m_{0-}^2} \left(\frac{e^{-m_{0-}r}}{r} - \frac{e^{-m_{0+}r}}{r} \right). \quad (28)$$

By inserting the last five expressions into Eq. (14) one finds the nonzero components of the metric, h_{00} and $h_{11} = h_{22} = h_{33}$, in the form

$$h_{00}(\mathbf{r}) = \frac{M\kappa}{16\pi} \left(-\frac{1}{r} + \frac{4}{3} F_2 - \frac{1}{3} F_0 \right), \quad (29)$$

$$h_{11}(\mathbf{r}) = \frac{M\kappa}{16\pi} \left(-\frac{1}{r} + \frac{2}{3} F_2 + \frac{1}{3} F_0 \right), \quad (30)$$

where ($k = 0, 2$ labels the spin of the particle)

$$F_k = \frac{m_{k+}^2}{m_{k+}^2 - m_{k-}^2} \frac{e^{-m_{k-}r}}{r} + \frac{m_{k-}^2}{m_{k-}^2 - m_{k+}^2} \frac{e^{-m_{k+}r}}{r}. \quad (31)$$

Equations (29) and (30) represent the weak field generated by a point mass in the general sixth-order gravity. In the previous work [31] the (00)-component of the metric perturbation has been computed in the more general case containing terms \square^n of arbitrary order in the action, but only for real and nondegenerate massive poles of the propagator. The expressions (29) and (30) apply to all types of poles.

III. MODIFIED NEWTONIAN POTENTIAL IN THE SIXTH-ORDER GRAVITY

The modified Newtonian potential (we shall use simply "potential" in what follows) of the sixth-order gravity can

be directly read off from the solution (29) for the field generated by a pointlike mass in rest,

$$V(r) = \frac{\kappa}{2} h_{00}(r) = MG \left(-\frac{1}{r} + \frac{4}{3} F_2 - \frac{1}{3} F_0 \right), \quad (32)$$

with the functions $F_{0,2}$ defined in Eq. (31).

In this section we analyse the possible types of ‘‘masses’’ allowed by the sixth-order gravity and their influence on the potential. The calculations require only h_{00} ; the other components of the metric will prove relevant in the further sections dedicated to the gravitational light deflection. The relevant quantities to be analyzed are F_k , hence the results will also be useful later on.

The following observation is in order. Complex massive poles are not allowed in the fourth-order gravity, since they would imply nonphysical complex values for the potential. However, in the sixth-order gravity the massive modes of the same spin form dynamically dependent pairs. As we shall see in short, this makes complex poles admissible and leads to a real potential with oscillatory modes.

A. Real poles

In what follows we explore three different possibilities for real poles, namely, pairs of different poles, including the special situation in which one of the poles is much heavier than the other, and the case of multiple (degenerate) poles.

1. Real simple poles

Real simple poles occur in the propagator of the massive tensor field provided that

$$\beta < 0, \quad B < 0, \quad \beta^2 + \frac{16B}{\kappa^2} > 0, \quad (33)$$

which enables one to redefine $m_{2\pm}^2$ as

$$m_{2\pm,\text{real}}^2 = \frac{\beta \pm \sqrt{\beta^2 + \frac{16}{\kappa^2} B}}{2B}. \quad (34)$$

These masses satisfy the condition $m_{2-} > m_{2+}$, where the lightest one corresponds to the well-known ghost mode and the other is a healthy particle [25].

With respect to the scalar field, the conditions $m_{0\pm}^2 > 0$ and $m_{0+} \neq m_{0-}$ yield

$$\begin{aligned} \sigma_1 &= 3\alpha + \beta > 0, & \sigma_2 &= 3A + B > 0, \\ (3\alpha + \beta)^2 - \frac{8(3A + B)}{\kappa^2} &> 0. \end{aligned} \quad (35)$$

Under these conditions one can redefine the scalar masses as

$$m_{0\pm,\text{real}}^2 = \frac{\sigma_1 \pm \sqrt{\sigma_1^2 - \frac{8\sigma_2}{\kappa^2}}}{2\sigma_2}. \quad (36)$$

Note that if (33) holds, then α and A must be positive. For the scalar field $m_{0+} > m_{0-}$, but now the largest mass corresponds to the ghost mode [25,31]. The reason for the qualitative difference between the scalar and tensor cases is that in the latter there exists the graviton, which is a healthy massless particle.

The expression for the potential is

$$\begin{aligned} V_{\text{real}}(r) &= -\frac{MG}{r} \\ &+ \frac{4MG}{3} \left(\frac{m_{2+}^2}{m_{2+}^2 - m_{2-}^2} \frac{e^{-m_{2-}r}}{r} + \frac{m_{2-}^2}{m_{2-}^2 - m_{2+}^2} \frac{e^{-m_{2+}r}}{r} \right) \\ &- \frac{MG}{3} \left(\frac{m_{0+}^2}{m_{0+}^2 - m_{0-}^2} \frac{e^{-m_{0-}r}}{r} + \frac{m_{0-}^2}{m_{0-}^2 - m_{0+}^2} \frac{e^{-m_{0+}r}}{r} \right), \end{aligned} \quad (37)$$

which is just a particular case of the result obtained in Ref. [31] by means of a different technique. This potential is regular at the origin. Our following calculations will show that this feature is also present if the massive poles are degenerate or complex.

2. Real degenerate poles

The condition for having degenerate poles in the propagator of the tensor or scalar fields is, respectively,

$$B = -\frac{\beta^2 \kappa^2}{16} \quad \text{and} \quad \sigma_2 = \frac{\sigma_1^2 \kappa^2}{8}. \quad (38)$$

These formulas correspond to transforming the last inequalities in Eqs. (33) and (35) into equalities. Thus, the masses m_k are defined by $m_2 = \sqrt{\beta/(2B)}$ and $m_0 = \sqrt{\sigma_1/(2\sigma_2)}$.

It proves useful to consider this situation starting from the assumption that the difference between the two real masses is small,

$$\begin{aligned} m_{2-} &= m_{2+} + \epsilon_2 = m_2 + \epsilon_2 \\ m_{0+} &= m_{0-} + \epsilon_0 = m_0 + \epsilon_0, \end{aligned} \quad (39)$$

with $0 < \epsilon_k/m_k \ll 1$. Then the quantity F_k reads

$$\begin{aligned} F_k &= \left(-\frac{m_k}{2\epsilon_k} + \frac{1}{4} - \frac{\epsilon_k}{8m_k} \right) \frac{e^{-(m_k+\epsilon_k)r}}{r} \\ &+ \left(\frac{m_k}{2\epsilon_k} + \frac{3}{4} + \frac{\epsilon_k}{8m_k} \right) \frac{e^{-m_k r}}{r} + \mathcal{O}\left(\frac{\epsilon_k^2}{m_k^2}\right). \end{aligned} \quad (40)$$

The limit $\epsilon_k \rightarrow 0$ is smooth, and we arrive at the expression for F_k for real degenerate poles,

$$F_k \rightarrow \left(\frac{1}{r} + \frac{m_k}{2} \right) e^{-m_k r}. \quad (41)$$

The potential for two pairs of degenerate real poles assumes the form

$$V_{\text{degen}}(r) = MG \left[-\frac{1}{r} + \frac{4}{3} \left(\frac{1}{r} + \frac{m_2}{2} \right) e^{-m_2 r} - \frac{1}{3} \left(\frac{1}{r} + \frac{m_0}{2} \right) e^{-m_0 r} \right], \quad (42)$$

which is indeed finite at the origin,

$$V_{\text{degen}}(0) = -\frac{MG}{3} \left(2m_2 - \frac{m_0}{2} \right). \quad (43)$$

The result (42) is in agreement with [30], where it was considered the particular case $\beta = B = 0$.

3. Real poles with strong hierarchy

Another possibility allowed by the sixth-order gravity is to have one of the masses of the auxiliary fields some (or many) orders of magnitude smaller than the other:

$$m_{2-} \gg m_{2+} \quad \text{and/or} \quad m_{0+} \gg m_{0-}. \quad (44)$$

This situation leads to potentially observable effects of higher derivatives at low energies, e.g., through modifications of inverse-square force law which could be detected in laboratory experiments.³ The possibility of such a strong hierarchy is discussed in detail in the parallel paper [35], which is mainly devoted to this issue in general higher-derivative gravities. Hence we will give here just a brief comment. The conditions (44) can be achieved, respectively, provided that $16|B| \ll \kappa^2 \beta^2$ and/or that $8\sigma_2 \ll \kappa^2 \sigma_1^2$. It is easy to see that if both conditions hold, in the leading order in m_{2+}/m_{2-} (and m_{0-}/m_{0+}) the potential reduces to the approximate form

$$V_4(r) = MG \left(-\frac{1}{r} + \frac{4}{3} \frac{e^{-m_{2+}r}}{r} - \frac{1}{3} \frac{e^{-m_{0-}r}}{r} \right). \quad (45)$$

As it should be expected, this expression coincides with that obtained in Ref. [5] within the fourth-order gravity, i.e., the theory defined by the action (3). Qualitatively, this means that at longer distances the heaviest masses have no effect.

Let us remember that the only possibility of reducing the lightest masses in (44) is to increase the coefficients α and β of the fourth-derivative terms. In other words, tuning the sixth-order coefficients do not reduce the lightest masses.

³Another consequence of this possibility is related to the alleged protection against Ostrogradsky-type instabilities [11,12], which would be less efficient.

Further results on the viability of a gravitational seesaw-like mechanism can be found in [35].

B. Complex poles

Complex poles in the propagator of the spin-2 field can occur provided that

$$\beta^2 + \frac{16B}{\kappa^2} < 0, \quad \left(\beta^2 + \frac{16B}{\kappa^2} \right)^{1/2} = ic_2, \quad (46)$$

with $c_2 > 0$ for definiteness. The first condition requires $B < 0$, while the four-derivative parameter β can be either positive or negative—different from the real poles case, Eq. (33).

The positions of the poles are defined by

$$m_{2\pm}^2 = \frac{\beta \pm ic_2}{2B}. \quad (47)$$

The square root of these quantities yield the “masses”

$$m_{2+} = a_2 - ib_2 \quad \text{and} \quad m_{2-} = a_2 + ib_2, \quad (48)$$

where $a_2, b_2 > 0$ are defined through

$$a_2^2 = \frac{-\beta + \sqrt{\beta^2 + c_2^2}}{4|B|} = \frac{-\beta + \sqrt{\frac{16|B|}{\kappa^2}}}{4|B|},$$

$$b_2^2 = \frac{\beta + \sqrt{\beta^2 + c_2^2}}{4|B|} = \frac{\beta + \sqrt{\frac{16|B|}{\kappa^2}}}{4|B|}. \quad (49)$$

One can always assume that m_{2+} and m_{2-} have positive real parts. A short comment is in order here. If choosing $m_{2\pm}$ with negative real part in Eq. (48), then the decreasing real exponentials would turn to be increasing, introducing into the potential growing oscillating modes at large distances. To avoid these growing modes one would have to choose growing exponentials as solution of the system (17)–(19). In this case the negative real part of the “masses” would combine with the increasing exponentials yielding decreasing oscillatory modes, resulting precisely in Eq. (50) below. Hence, the generality is not lost due to our choice of signs.

Finally, replacing (48) into the expression for F_2 leads us to

$$F_2 = \left[\cos(b_2 r) - \frac{\beta}{c_2} \sin(b_2 r) \right] \frac{e^{-a_2 r}}{r}, \quad (50)$$

which is a real quantity.

The condition for complex “masses” in the scalar field reads

$$\sigma_1^2 - \frac{8\sigma_2}{\kappa^2} < 0 \Rightarrow \sigma_2 > 0, \quad \sigma_1 \in \mathbb{R}. \quad (51)$$

Similar to the spin-2 case, we define

$$ic_0 = \sqrt{\sigma_1^2 - \frac{8\sigma_2}{\kappa^2}}, \quad m_{0\pm} = a_0 \pm ib_0, \quad (52)$$

where $(a_0, b_0 > 0)$

$$a_0^2 = \frac{\sigma_1 + \sqrt{\frac{8\sigma_2}{\kappa^2}}}{4\sigma_2}, \quad b_0^2 = \frac{-\sigma_1 + \sqrt{\frac{8\sigma_2}{\kappa^2}}}{4\sigma_2}. \quad (53)$$

The contribution of the scalar field to the potential is

$$F_0 = \left[\cos(b_0 r) + \frac{\sigma_1}{c_0} \sin(b_0 r) \right] \frac{e^{-a_0 r}}{r}. \quad (54)$$

Taking together the contributions (50) and (54) we arrive at the potential in the case of complex poles,

$$V_C(r) = -\frac{MG}{r} + \frac{4MG}{3} \left[\cos(b_2 r) - \frac{\beta}{c_2} \sin(b_2 r) \right] \frac{e^{-a_2 r}}{r} - \frac{MG}{3} \left[\cos(b_0 r) + \frac{\sigma_1}{c_0} \sin(b_0 r) \right] \frac{e^{-a_0 r}}{r}. \quad (55)$$

It is straightforward to verify that this potential is finite at $r = 0$. Indeed, this feature can be extended to the theory of arbitrary order in the derivatives, including the case of multiple complex poles [32].

The main distinguished feature of the complex poles case is the presence of oscillating terms. Depending on which quantity is greater in the pair (a_k, b_k) , the oscillatory terms can be more or less relevant in the potential. For example, in the case of the spin-2 field, $\beta < 0$ implies $a_2 > b_2$. Since the characteristic length of the Yukawa potential is $2\pi/a_2$ and the period of the oscillating terms is $2\pi/b_2$, the oscillations can be smooth, yielding an appreciable contribution only at distances larger than the Yukawa length $2\pi/a_2$. There, the potential associated to this field has an oscillating sign, but with a small absolute value due to the suppression caused by a_2 . Hence, at these distances the potential is dominated by the Newtonian term owed to the graviton.

On the other hand, if $\beta > 0$ it follows that $a_2 < b_2$. Then the space period of oscillations is typically smaller than the range of the Yukawa factor. This situation implies a significant change in the behavior of the potential at small distances, with the contribution of the spin-2 field changing its sign. The same argument applies *mutatis mutandis* to the scalar field. In case $a_k = 0$ the ‘‘masses’’ are purely imaginary quantities which correspond to tachyonic modes. In this case F_k loses its damping term yielding a

non-Newtonian behaviour in the infinity. It is clear that this case can be ruled out.

It is noteworthy that when we allowed massive complex poles, the constraints on β and σ_1 were relaxed. As we have just mentioned, important changes in the ultraviolet behavior of the potential occur if, contrary to the real mass case, it is chosen $\beta \geq 0$ and/or $\sigma_1 \leq 0$. The case of $\beta = 0$ and/or $\sigma_1 = 0$ makes the real and imaginary parts a_k and b_k to assume the same value, hence only the cosine functions remain in the expression for the potential.⁴

Likewise the case of real poles, one might suppose a sort of natural seesaw mechanism which could reduce a_k and b_k and bring the phenomenology of those modes to the low-energy scale. Indeed, this can only happen for unnatural values of the massive parameters at the action. Namely, in order to have $a_2 \approx b_2 \ll M_P$ one has to impose $|B| \gg M_P^2$. A more detailed and general discussion on this subject can be found in Ref. [35].

Before closing this section, let us return to the Theorem 1. In Sec. II it was mentioned that the auxiliary fields of the same spin have coupled dynamics. At the same time, the equations for spin-2 and spin-0 components are factorized. Due to this fact the cancelation of the Newtonian singularity occurs independently of the (complex or real) nature and the multiplicity of the massive poles. In brief, such a cancelation takes place if there is at least one massive state in each of the sectors [32].

IV. LIGHT BENDING: CLASSICAL APPROACH

Up to this point all the discussions were related to the (00)-component of the metric (29). Here and in the following section we shall use this and also other components to study the weak-field regime of the gravitational deflection of light within the sixth-order gravity. This issue has already been analyzed in the framework of the fourth-derivative theory (see [37] and references therein). In these works the phenomenon of light bending was used to derive a lower-bound on the mass of the tensor mode of the metric. The sixth-order model which we deal with here has a richer variety of possible scenarios. The main purpose of our present study is to systematically explore all of them for different types of poles in the gravitational propagator.

The gravitational light bending problem has been explored by using both classical or semiclassical approaches. In several works it was explained that these two methods may lead to different results (see, e.g., [33,34,37–40]). In the present section we analyze the phenomenon from a classical point of view, that is, by treating both gravity and light as classical fields. In the next section we describe the semiclassical approach and discuss its applicability, so as to explain the mentioned difference.

⁴When the first version of the present work was under preparation, we learned that the potential for the particular case $\beta = \alpha = 0$ and $a_2 = a_0$ was derived in Ref. [23].

In order to arrive at a better understanding of the qualitative features of the gravitational deflection of a light ray passing close to a massive body we shall use the so-called $\epsilon - \mu$ -form of Maxwell equations in curved space-time [41,42]. This formalism can be applied to static, spherically symmetric gravitational fields, since under such circumstances it is always possible to find a coordinate system where the metric has the isotropic form

$$g_{00} = g_{00}(r), \quad g_{0i} = 0, \quad g_{ij} = -\delta_{ij}f(r), \quad (56)$$

for some function $f(r)$, where $r = |\mathbf{r}|$. Using this metric it is not difficult to show that the inhomogeneous Maxwell equations

$$F^{\mu\nu}{}_{;\mu} = J^\nu \quad (57)$$

can be cast into the form

$$\nabla \cdot (\epsilon \mathbf{E}) = \rho, \quad \frac{\partial}{\partial t}(\epsilon \mathbf{E}) - \nabla \times \left(\frac{\mathbf{B}}{\mu} \right) = \mathbf{j}, \quad (58)$$

where

$$\epsilon = \mu = \sqrt{\frac{f(r)}{g_{00}(r)}}. \quad (59)$$

These equations have the form of the usual (flat-space) Gauss's and Ampère's laws in a medium with refractive index

$$n(r) = \sqrt{\epsilon \mu} = \sqrt{\frac{f(r)}{g_{00}(r)}}. \quad (60)$$

Thus, in the geometric optics limit, i.e., if the wavelength of light is much smaller than the curvature scale, the influence of gravity on light can be taken into account through (58), which can be naturally interpreted as if gravity endows the flat space-time with an effective refractive index. For example, the deflection of a light ray passing close to a massive body can then be evaluated using the Snell-Descartes law. Following the calculations of [42], the deflection angle θ for a light ray passing in the vicinity of a massive body with the impact parameter ρ is given, to the first order in G , by the expression

$$\theta = - \int_{-\infty}^{+\infty} \frac{\rho}{rn(r)} \frac{dn(r)}{dr} dx, \quad (61)$$

where $r = \sqrt{x^2 + \rho^2}$

and the trajectory of the photon is parametrized by x . A small observation concerning the limits of integration in (61) is on order. According to the scheme introduced in [42]

the integration is performed starting at the position x of the light source (a distant star, for example), up to the position of the observer, respectively to the massive scattering object. Since we consider deflection caused by the Sun, it is natural to suppose that both the light source and the Earth correspond to the space infinities. However, for a precise calculation in more exotic scenarios (e.g., those with complex poles), the upper limit related to Earth's position may need to be redefined.

Since the field generated by a pointlike mass in rest found in Sec. II is already in the isotropic form, it is straightforward to evaluate the effective refractive index associated to the sixth-order gravity. From Eqs. (29) and (30) it follows that, to the first order,

$$\begin{aligned} n(r) &= \sqrt{\frac{1 - \kappa h_{11}(r)}{1 + \kappa h_{00}(r)}} \\ &= 1 - MG \left(-\frac{1}{r} + \frac{4}{3} F_2 - \frac{1}{3} F_0 \right) - MG \left(-\frac{1}{r} + \frac{2}{3} F_2 + \frac{1}{3} F_0 \right) \\ &= n_{\text{GR}}(r) - 2MG F_2, \end{aligned} \quad (62)$$

where

$$n_{\text{GR}}(r) \equiv 1 + \frac{2MG}{r} \quad (63)$$

is the effective refractive index of general relativity.

The immediate conclusion which follows from the expression (62) is that light bending in this theory does not depend directly on the scalar excitations $m_{0\pm}$, and hence on the sectors R^2 and $R\Box R$. This result is rather expected, since both sectors can be regarded as the result of a conformal transformation on the weak-field metric. Since the curved-space Maxwell equations are conformally invariant, these terms have no direct effect on the light deflection, in the leading approximation. Let us note that the semiclassical derivation of the same statement for $R + R^2$ can be found in [43] and will be extended to the theory with $R\Box R$ term in the next section.

On the other hand, the scalar modes may have an indirect influence on the bending of light, through the redefinition of Newton's constant G and the related calibration of mass of astronomical bodies. This effect is typical in the literature on the massless Brans-Dicke theory [44].⁵ The situation for the massive Brans-Dicke theory can be very different, as explained, e.g., in the Refs. [46,47].

Indeed, it is even easier to understand the difference between massless and massive cases for the model of $R + \alpha R^2$ -gravity, than for the classically equivalent Brans-Dicke theory. According to our previous considerations,

⁵An important consideration concerning the effective Newton constant in metric-scalar models, including cosmological aspects of the problem, was given in [45].

the modified Newtonian potential in this case has the form

$$V(r) = -\frac{GM}{r} \left(1 + \frac{1}{3} e^{-m_0 r} \right), \quad (64)$$

where the mass of the scalar mode m_0 can be very small only for a huge value of the parameter α . In the case when the enormous value of α can overwhelmingly compensate the “natural” value of m_0 (which is of the Planck order of magnitude), the scalar mass becomes incredibly small and the exponential in Eq. (64) can be considered as constant unity at the astronomical scale. This is exactly what we observe for the massless limit of the Brans-Dicke model. In such an exotic situation one cannot measure a real value of the product GM in laboratory experiments or in the Solar System observations, and will observe $(4/3)GM$ instead. At the same time the bending of light will be measuring the real value GM , so some discrepancy is unavoidable between the two sets of observational and experimental data.

In general, we will not bother with the redefinition of the product GM , since we are not interested in such huge values of β . We will come back to this discussion only at one point, when comparing the effect of the deflection of light to the modified Newtonian potential.

For the sake of completeness we show explicitly how the scalar contributions appear as a conformal transformation. Starting from the auxiliary fields representation in Eq. (14), if $\alpha = A = 0$ the general solution of the field equations reads

$$h_{\mu\nu}^{(\alpha=A=0)} = h_{\mu\nu}^{(E)} + \Psi_{\mu\nu} + \bar{\Psi}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} (\Psi + \bar{\Psi}), \quad (65)$$

which yields the metric

$$g_{\mu\nu}^{(\alpha=A=0)} = \eta_{\mu\nu} + \kappa \left[h_{\mu\nu}^{(E)} + \Psi_{\mu\nu} + \bar{\Psi}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} (\Psi + \bar{\Psi}) \right]. \quad (66)$$

Thus, the metric associated to the full sixth-order gravity can be expressed in the conformal form

$$g_{\mu\nu} = \left[1 - \kappa(\Phi + \bar{\Phi}) + \frac{\kappa}{2}(\Psi + \bar{\Psi}) \right] g_{\mu\nu}^{(\alpha=A=0)}, \quad (67)$$

keeping, as usual, terms up to first order in the metric fluctuation.

In what follows we consider systematically the results for the light deflection according to the nature of the massive tensor excitations. Namely, we analyze the effective refractive index for the different versions of F_2 , as described in the previous section.

A. Deflection with real simple poles

In the case of real simple poles the effective refractive index is given by the general formula

$$n(r) = n_{\text{GR}}(r) + 2MG \left(\frac{m_{2+}^2}{m_{2-}^2 - m_{2+}^2} \frac{e^{-m_{2-}r}}{r} - \frac{m_{2-}^2}{m_{2-}^2 - m_{2+}^2} \frac{e^{-m_{2+}r}}{r} \right). \quad (68)$$

Since $m_{2-} > m_{2+}$, the m_{2-} -term yields an attractive force and produces an increase of $n(r)$, while the m_{2+} -term gives a negative contribution to the refractive index, which is responsible for the well-known repulsive force caused by the ghost mode [31,37]. This repelling force is stronger than that of the healthy massive mode, since

$$\left| \frac{m_{2-}^2}{m_{2-}^2 - m_{2+}^2} \frac{e^{-m_{2+}r}}{r} \right| > \left| \frac{m_{2+}^2}{m_{2+}^2 - m_{2-}^2} \frac{e^{-m_{2-}r}}{r} \right|. \quad (69)$$

As a consequence $n(r) < n_{\text{GR}}(r)$, implying that light deflects less in the sixth-order gravity than in general relativity.

It is easy to show that for a fixed value of β there is also the relation

$$n(r) > n_4(\beta, r) = n_{\text{GR}}(r) - \frac{2MG}{r} \exp\left(-\frac{4r}{|\beta|\kappa^2}\right), \quad (70)$$

where the r.h.s. is the effective refractive index of the fourth-order gravity with the same β , i.e., with $A = B = 0$. In order to prove inequality (70), we note that $\partial m_{2+}^2 / \partial B < 0$, therefore the smallest value for m_{2+}^2 can be achieved by taking the limit $B \rightarrow 0$ (remember $B < 0$), hence

$$\lim_{B \rightarrow 0} m_{2+}^2 = -\frac{4}{\beta\kappa^2} = m_{2(4)}^2, \quad (71)$$

which is precisely the square of the mass of the fourth-order gravity’s ghost [5]. Since the Yukawa potential is stronger for a smaller mass, if $m_{2+}^2 = m_{2(4)}^2$ the repulsive term achieves its maximum strength, while the attractive massive term tends to zero. We conclude that $n(r) > n_4(r)$ for the same value of β in six- and fourth-derivative models. In particular, $n(r) > 1$, which means that the balance of the three forces never results in a net outward deflection.

The previous discussion can be summarized by the following chain of inequalities, where the last two hold with the same value of β :

$$n_{\text{GR}}(r) > n(r) > n_4(r) > 1. \quad (72)$$

Those inequalities become true equalities, respectively, in the following limits:

$$\begin{aligned}
 & i) \quad m_{2\pm} \rightarrow \infty; \\
 & ii) \quad m_{2+}/m_{2-} \rightarrow 0; \\
 & iii) \quad m_{2\pm} \rightarrow 0.
 \end{aligned} \tag{73}$$

The possibility (ii) corresponds to the fourth-derivative gravity theory, with two disproportional masses as explained in Sec. III A 1, and with $B \rightarrow 0$ as we have discussed above.

For the sake of completeness we write the result for the deflection angle of a light ray with impact parameter ρ , given by Eq. (61) with the effective refractive index (68),

$$\theta = \theta_{\text{GR}} + 2MG\rho(I_- - I_+), \tag{74}$$

$$I_{\pm} = \frac{m_{2\mp}^2}{|m_{2\pm}^2 - m_{2\mp}^2|} \int_{-\infty}^{+\infty} \left(\frac{1}{r} + m_{2\pm} \right) \frac{e^{-rm_{2\pm}}}{r^2} dx,$$

where $r = \sqrt{x^2 + \rho^2}$. (75)

Here $\theta_{\text{GR}} \equiv 4GM/\rho$ is the bending angle predicted by general relativity. In a higher derivative theory the ghost term I_+ enters with a “wrong” sign, tending to reduce the deflection angle.

The magnitude of deflection depends on the three length scales, defined by the inverse masses of the tensor modes and by the impact parameter. There is a region delimited by $r_1 = 1/m_{2-}$ and $r_2 = 1/m_{2+}$ where the dominant contribution to the deflection is owed to the ghost mode and the graviton. If there is a strong hierarchy $m_{2-} \gg m_{2+}$ between the masses, then the massive healthy tensor mode is irrelevant along the trajectory of the light ray, and the deflection angle is approximately that of the fourth-order gravity [37],

$$\theta \approx \theta_{\text{GR}} - 2MG\rho \int_{-\infty}^{+\infty} \left(\frac{1}{r} + m_2 \right) \frac{e^{-rm_2}}{r^2} dx. \tag{76}$$

Here the definition of r is the same as in (75). One can observe that outside of the sphere of the radius $1/m_{2+}$, the dominant contribution to the light deflection comes from the graviton sector and the effect of the massive modes is suppressed.

B. Deflection with real degenerate poles

If the masses of the tensor excitations are approximately the same, one can use the quantity F_2 given by Eq. (40), or Eq. (41) in the limit $m_{2-} = m_{2+} = m_2$. The latter yields the effective refractive index

$$n_{\text{degen}}(r) = n_{\text{GR}}(r) - 2MG \left(\frac{1}{r} + \frac{m_2}{2} \right) e^{-m_2 r}. \tag{77}$$

As far as the mentioned limit is smooth, it is possible to restrict our consideration to the limit of equal masses. Without the hierarchy between the masses, the relation (69) and its implications do not hold. Then for a sufficiently small r it is possible to have $n_{\text{degen}} < 0$. In this case the repulsive force is strong enough to cause a net outward deflection at this region. Hence, the chain of inequalities of Eq. (72) simplifies to $n_{\text{GR}} > n_{\text{degen}}$, formally without a lower bound.

In this scenario, the expression for the deflection angle θ reads

$$\theta_{\text{degen}} = \theta_{\text{GR}} - 2MG\rho \int_{-\infty}^{+\infty} \left(\frac{m_2^2 r}{2} + m_2 + \frac{1}{r} \right) \frac{e^{-rm_2}}{r^2} dx, \tag{78}$$

with r the same as in (75), which can be recognized as the deflection angle in the fourth-derivative gravity *with the same mass m_2* according to Eq. (76), minus an extra correction owed to the healthy massive (degenerate) excitation. Indeed, the effective refractive index (77) can be cast into the form

$$n_{\text{degen}}(m_2, r) = n_4(m_2, r) - MGm_2 e^{-m_2 r}, \tag{79}$$

where $n_4(m_2, r)$ corresponds to fourth-order gravity with the mass m_2 .

It is important to stress that the strong repulsive force occurs only at distances smaller than $1/m_2$. For the Planck-order mass $m_2 \propto M_P$ this distance is of the order of 10^{-43} cm, so the repulsive effect does not affect the light deflected by astronomical bodies including our Sun.

C. Deflection with complex poles

The expression for the effective refractive index of the sixth-order gravity in the presence of complex massive poles follows from Eqs. (50) and (62),

$$n_{\text{C}}(r) = n_{\text{GR}}(r) - 2MG \left[\cos(b_2 r) - \frac{\beta}{c_2} \sin(b_2 r) \right] \frac{e^{-a_2 r}}{r}. \tag{80}$$

Accordingly, the deflection of a light ray with impact parameter ρ is given by

$$\begin{aligned}
 \theta = \theta_{\text{GR}} - 2MG\rho \int_{-\infty}^{+\infty} dx \left\{ \left[b_2 - \frac{\beta}{c_2} \left(a_2 + \frac{1}{r} \right) \right] \sin(b_2 r) \right. \\
 \left. + \left(a_2 + \frac{1}{r} + \frac{\beta}{c_2} b_2 \right) \cos(b_2 r) \right\} \frac{e^{-a_2 r}}{r^2},
 \end{aligned} \tag{81}$$

where we use the standard parametrization of (75).

Since the expressions presented above for the deflection angles carry the assumption that these angles are small, to all practical purposes the impact parameter coincides with the closest approach distance [42]. Thus one can define the trajectory scale by ρ^{-1} . In the complex poles cases there are also three length scales: the one of the Yukawa part, the typical length period of the oscillation and the impact parameter. The analysis is complicated due to the presence of the oscillating terms, hence in what follows we describe only two simple but illustrating examples.

1. The case of $a_2 \gg b_2$

From the definitions of Sec. III. 2, it follows that $a_2 > b_2$ if and only if $\beta < 0$. Besides, if c_2 is sufficiently small, such that $c_2^2/\beta^2 \ll 1$, it is possible to have the real part of the “mass” much larger than the imaginary part. In such a scenario, the massive quantities a_2 and b_2 may be approximated by

$$a_2^2 \approx \frac{4}{\kappa^2|\beta|} \left(2 - \frac{3c_2^2}{2\beta^2}\right), \quad b_2^2 \approx \frac{2}{\kappa^2|\beta|} \frac{c_2^2}{\beta^2}. \quad (82)$$

Furthermore, the condition $a_2 \gg b_2$ means that the Yukawa potential has a very short range if compared to the large space period of the oscillatory terms.

It remains possible for the Yukawa and oscillation scales to be either small or large with respect to the impact parameter. Assuming that $\rho^{-1} \ll a_2$, the correction due to the higher-derivatives is always tiny against the general relativity’s term, hence $\theta \approx \theta_{\text{GR}}$. The only interesting situation is therefore $b_2 \ll \rho^{-1}$, with ρ^{-1} comparable to a_2 . Accordingly we may write $\cos(b_2 r) \approx 1$ and $\sin(b_2 r) \approx b_2 r$, which reduces the deflection angle to

$$\begin{aligned} \theta &\approx \theta_{\text{GR}} - 2MG\rho \int_{-\infty}^{+\infty} dx \left(-\frac{\beta a_2 b_2}{c_2} r + a_2 + \frac{1}{r} \right) \frac{e^{-a_2 r}}{r^2} \\ &\approx \theta_{\text{GR}} - 2MG\rho \int_{-\infty}^{+\infty} dx \left(\frac{a_2^2}{2} r + a_2 + \frac{1}{r} \right) \frac{e^{-a_2 r}}{r^2}. \end{aligned} \quad (83)$$

It is easy to see that this is roughly the same expression (78) for the real degenerate poles. This result should be expected, since the condition $b_2 \ll \rho^{-1} \sim a_2$ means that the imaginary part of $m_{2\pm}$ is tiny with respect to all other scales of the system. Hence, to the leading order both scenarios turn out to be the same, confirming the correctness of our calculations. Differences start to emerge only when second- and first-order corrections in $b_2 r$ and b_2/a_2 , respectively, are taken into account.

2. The case of $b_2 \gg a_2$

This condition only holds provided that $\beta > 0$ and $c_2^2/\beta^2 \ll 1$. The quantities a_2 and b_2 now read, to the leading order,

$$a_2^2 \approx \frac{2}{\kappa^2|\beta|} \frac{c_2^2}{\beta^2}, \quad b_2^2 \approx \frac{4}{\kappa^2|\beta|} \left(2 - \frac{3c_2^2}{2\beta^2}\right), \quad (84)$$

and therefore $b_2^2/a_2^2 \approx 4\beta^2/c_2^2$. As a consequence

$$b_2 \frac{|\beta|}{c_2} \gg b_2 > \frac{b_2}{2} \approx a_2 \frac{|\beta|}{c_2} \gg a_2. \quad (85)$$

Let us remember that the condition $a_2 \ll b_2$ means that the range of the Yukawa term is much larger than the space period of the trigonometric functions in the expression for the effective refractive index. Then many oscillations typically occur before the exponential factor makes the whole expression negligible. This regime, therefore, has a much stronger dependence on the impact parameter if compared to the analysis which was described before.

In the regime $\rho^{-1} \gg b_2$ one may approximate the argument of the trigonometric functions by the leading constant value ρb_2 , for $r \approx \rho$, where the amplitude of the correction term is maximum. It is clear that the change of impact parameter by even a small fraction of its original value can produce a large variation of the correction from the higher-derivative terms, including altering the sign of this correction.

This strong dependence on ρ is only suppressed for $\rho^{-1} < a_2$, due to the exponential damping. In view of Eq. (85), the expression for the deflection angle simplifies to

$$\theta \approx \theta_{\text{GR}} - MG\rho \frac{\beta b_2}{c_2} \int_{-\infty}^{+\infty} dx \cos(b_2 r) \frac{e^{-a_2 r}}{r^2}. \quad (86)$$

D. Final comments on classical deflection

Some general comments are in order. The two previous simple examples show that the corrections due to the higher-derivative terms can manifest strong dependence on the impact parameter in the case of complex poles. The origin of this effect is the oscillatory behaviour of the effective refractive index. In the realistic situations, however, the only feasible scenarios are those where the real part is large enough to damp the oscillations far beyond the current experimental bounds. For instance, the most precise measurements of deflection of light rays close to the Sun, carried out by modeling solar occultations of radio sources, have confirmed general relativity’s prediction within the uncertainty of a few parts in 100,000 [48]. In the visible spectrum, the astrometry of stars during solar eclipses yield the verification of the deflection angle to the precision of 1% [49].

The deflection of light rays close to the solar limb in the four-derivative gravity (76) corresponds to the Yukawa potential with mass $m_2 > 10^{-23}$ GeV [37]. Such a figure, nevertheless, is far too small if one takes into account laboratory tests of the inverse-square force law. Torsion-balance experiments currently yield a much stricter bound

on the order of $m_2 > 10^{-12}$ GeV for one additional Yukawa potential [50,51]. These bounds may be viewed as first estimates to a lower-limit on the real component a_2 , if we assume that it is large enough to damp the oscillations up to this length scale. However, no bound on the imaginary part can be established from this preliminary analysis. Precise modeling of experimental data, especially those from torsion-balances, are required in order to detect a possible oscillatory behavior of the gravitational potential. A stimulating discussion on the perspective of detecting oscillations in the gravitational potential was set about in the recent work [52].

V. LIGHT BENDING: SEMICLASSICAL APPROACH

Let us now consider the photon as a quantum particle which interacts with the classical external gravitational field. The main virtue of the semiclassical calculation using Feynman diagrams is to consider the background metric not as a completely sterile medium, but as an external field whose massive modes are excited depending on the energy of the interacting particle. In the case of the purely massless gravitational excitation both classical and the semiclassical approaches are equivalent, but in the presence of a massive parameter m the semiclassical scattering starts to depend on the ratio between m and the energy of the photon [37,39]. As we have already mentioned in the Introduction, the question of whether and when the semiclassical approach can be used has been discussed in the literature [34] and the general conclusion is that its pertinence is restricted to scattering processes with very small impact parameter. Yet, this approach looks interesting for it clarifies some general features of the scattering, and therefore we include it here. In what follows we present the results of the calculations for the cross section formulas, and then discuss their applicability to the bending of light in the Solar System.

At the tree level the only diagram contributing to the scattering of a photon by a classical external gravitational field is the one in Fig. 1, producing the vertex function

$$V_{\mu\nu}(p, p') = \frac{\kappa}{2} h_{\text{ext}}^{\lambda\rho}(\mathbf{k}) F_{\mu\nu\lambda\rho}(p, p'), \quad (87)$$

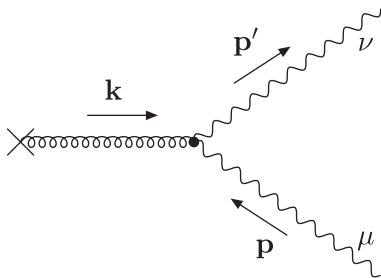


FIG. 1. Photon scattering by an external gravitational field. Here $|\mathbf{p}| = |\mathbf{p}'|$.

where p and p' are the four-momenta of the initial and final states of the photon, while $h_{\text{ext}}^{\lambda\rho}(\mathbf{k})$ is the linearized gravitational field in the momentum-space representation. The function in (87) has the form

$$\begin{aligned} F_{\mu\nu\lambda\rho}(p, p') = & -\eta_{\mu\nu}\eta_{\lambda\rho}P \cdot P' + \eta_{\lambda\rho}P'_\mu P_\nu \\ & + 2(\eta_{\mu\nu}P_\lambda P'_\rho - \eta_{\nu\rho}P_\lambda P'_\mu \\ & - \eta_{\mu\lambda}P_\nu P'_\rho + \eta_{\mu\lambda}\eta_{\nu\rho}P \cdot P'). \end{aligned} \quad (88)$$

Since, according to the Theorem 1, the gravitational field $h_{\text{ext}}^{\lambda\rho}$ can be written as the sum of five auxiliary fields, and owed to the linearity of the Fourier transform, the vertex function assumes the form

$$\begin{aligned} V_{\mu\nu} = & \mathcal{M}_{\mu\nu}^{(E)} + \mathcal{M}_{\mu\nu}^{(\Psi)} + \mathcal{M}_{\mu\nu}^{(\bar{\Psi})} + \mathcal{M}_{\mu\nu}^{(\Phi)} + \mathcal{M}_{\mu\nu}^{(\bar{\Phi})}, \\ \mathcal{M}_{\mu\nu}^{(\dots)} = & \frac{\kappa}{2} h_{\text{ext}}^{(\dots)\lambda\rho} F_{\mu\nu\lambda\rho}, \end{aligned} \quad (89)$$

where the last equation is valid for all five auxiliary fields.

We point out that for a photon the dispersion relation is $p^2 = E^2 - \mathbf{p}^2 = 0 = p'^2$. Furthermore, we can assume the field to be weak and hence neglect the possible energy exchange between the photon and gravitational field. Therefore it follows that $|\mathbf{p}| = |\mathbf{p}'|$. Bearing this in mind, it is easy to verify that $\eta^{\lambda\rho} F_{\mu\nu\lambda\rho} = 0$. Hence $\mathcal{M}_{\mu\nu}^{(\Phi)} = \mathcal{M}_{\mu\nu}^{(\bar{\Phi})} = 0$ and the Feynman amplitudes related to the scalar modes of the gravitational field are null.

From the perspective of Feynman diagrams, the contribution of the scalar mode of the metric vanishes because it interacts with the photon through the trace of the energy-momentum tensor. This trace is null for the electromagnetic field, and as a consequence *none* of the scalar components contribute to the scattering of light. This confirms the result which we obtained in Sec. IV within the classical framework. One of the manifestations of this is that the R^2 - and $R\Box R$ -terms do not affect light deflection, except in the recalibration of the product GM in the special case of very light scalar mode(s).

The Feynman amplitude for the scattering of photons with initial polarization vector $\epsilon_r^\mu(\mathbf{p})$ and final polarization $\epsilon_{r'}^\nu(\mathbf{p}')$ is given by

$$\mathcal{M}_{rr'} = V_{\mu\nu}(p, p') \epsilon_r^\mu(\mathbf{p}) \epsilon_{r'}^\nu(\mathbf{p}'). \quad (90)$$

Taking into account Eq. (88) and the completeness relation for the polarization vectors,

$$\begin{aligned} \sum_{r=1}^2 \epsilon_r^\mu(\mathbf{p}) \epsilon_r^\nu(\mathbf{p}) = & -\eta^{\mu\nu} - \frac{p^\mu p^\nu}{(p \cdot n)^2} + \frac{p^\mu n^\nu + p^\nu n^\mu}{p \cdot n}, \\ & (n^\mu n_\mu = 1). \end{aligned} \quad (91)$$

The sum over all the polarizations yields the unpolarized cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{2(4\pi)^2} \sum_{r,r'} |\mathcal{M}_{rr'}|^2 = \frac{1}{2(4\pi)^2} V_{\mu\nu} V^{\mu\nu}. \quad (92)$$

Furthermore, it is cursory to show that

$$\eta^{\lambda 0} \eta^{\rho 0} F_{\lambda\rho\mu\nu} \eta_{\alpha 0} \eta_{\beta 0} F^{\alpha\beta\mu\nu} = 2E^4 (1 - \cos\theta)^2, \quad (93)$$

where $E = E'$ is the energy of the photon and θ is the deflection angle between \mathbf{p} and \mathbf{p}' .

Using Eqs. (89), (92), and (93) it follows that

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{\kappa^4 M^2 E^4 (1 + \cos\theta)^2}{(16\pi)^2} \\ &\times \left[\frac{1}{\mathbf{k}^2} - \frac{1}{m_{2-}^2 - m_{2+}^2} \left(\frac{m_{2-}^2}{\mathbf{k}^2 + m_{2+}^2} - \frac{m_{2+}^2}{\mathbf{k}^2 + m_{2-}^2} \right) \right]^2. \end{aligned} \quad (94)$$

In the formula (94) one can recognize the standard gravitational version of the Rutherford formula, plus the correction coming from the massive modes.

In what follows we assume that the bending angle is small and calculations are performed in the leading order in θ . Then $\mathbf{k}^2 \approx 2\mathbf{p}^2(1 - \cos\theta) \approx E^2\theta^2$ and the previous expression reduces to

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= 16G^2 M^2 \left(\frac{1}{\theta^2} - \frac{m_{2-}^2}{m_{2-}^2 - m_{2+}^2} \frac{E^2}{E^2\theta^2 + m_{2+}^2} \right. \\ &\quad \left. + \frac{m_{2+}^2}{m_{2-}^2 - m_{2+}^2} \frac{E^2}{E^2\theta^2 + m_{2-}^2} \right)^2, \end{aligned} \quad (95)$$

where we omitted $\mathcal{O}(\theta^{-3})$ and other relatively small terms.

It is clear that the propagation of photons in this model is dispersive, i.e., depends on the energy of the photon. The same general feature was established in Ref. [37] for the photons in the fourth-order theory. However, in the six-derivative case there are several possible scenarios, depending on the type of the quantities $m_{2\pm}$. In what follows we treat each case separately.

A. Scattering with real simple poles

Let us start by recalling that in the simpler fourth-order gravity, for a given θ , the cross section is smaller than in general relativity,

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega} \right)_4 &= 16G^2 M^2 \left(\frac{1}{\theta^2} - \frac{E^2}{E^2\theta^2 + m_2^2} \right)^2 \\ &< \frac{16G^2 M^2}{\theta^4} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{GR}}. \end{aligned} \quad (96)$$

This happens because the $R_{\mu\nu}^2$ -sector yields a repulsive dispersive interaction such that more energetic photons are less scattered.

In the sixth-order gravity, in addition to the attractive nondispersive force coming from the R -sector and the repulsive dispersive force due to the $R_{\mu\nu}^2$ -sector, there is another attractive, dispersive, force due to the term $R_{\mu\nu} \square R^{\mu\nu}$. This makes the ‘‘tug of war’’ between those forces more complicated than in the fourth-order gravity; yet, the qualitative conclusions are the same. One can summarize the results as follows:

- (i) Light is less scattered than in general relativity. The hierarchy $m_{2-} > m_{2+}$ implies that

$$\frac{m_{2-}^2 E^2}{E^2\theta^2 + m_{2+}^2} > \frac{m_{2+}^2 E^2}{E^2\theta^2 + m_{2-}^2}, \quad (97)$$

and hence [see Eq. (95)]

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{GR}} > \frac{d\sigma}{d\Omega} > \left(\frac{d\sigma}{d\Omega} \right)_4 > 0, \quad (98)$$

where $\left(\frac{d\sigma}{d\Omega} \right)_4$ is the cross section for the fourth-order gravity with the same β (see discussion in the Sec. IV A). The second inequality tends to equality in the case of strong hierarchy $m_{2-} \gg m_{2+}$, while the last inequality in (98) tends to equality in the limit $E \rightarrow \infty$, when no deflection occurs.

- (ii) More energetic photons undergo less deflection. This happens because they interact strongly with the dispersive terms and, as one can see in (97), among the dispersive forces the repelling one is always bigger. Physically, the reason is that the coupling constant is the same for all intermediate tensor bosons, thus the one with larger mass makes smaller effect.

The dependence on E cannot be observed in the classical approach. But it is interesting to note that besides the dispersive behavior, the general qualitative conclusions of the classical approach are verified at the quantum level. In order to see this, one can compare, for instance, the chain of inequalities in Eqs. (72) and (98).

B. Scattering with real degenerate poles

The cross section for the case of real degenerate poles can be explored using the general expression for the cross section (95). We start from the case of a weak hierarchy

$$m_{2-} = m_{2+} + \epsilon = m_2 + \epsilon, \quad \text{with} \quad \frac{\epsilon}{m_2} \ll 1,$$

and then take the limit $\epsilon \rightarrow 0$, which smoothly yields

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{degen}} = 16G^2M^2 \left[\frac{1}{\theta^2} - \frac{E^4\theta^2 + 2m_2^2E^2}{(E^2\theta^2 + m_2^2)^2} \right]^2. \quad (99)$$

It is straightforward to verify that this cross section is bounded by zero (for $m_2/E \rightarrow 0$) and by the general relativity cross section as $E/m_2 \rightarrow 0$. Therefore the qualitative conclusions of the case with real simple poles apply here too; namely, light deflects less than in general relativity, and more energetic photons are less scattered.

C. Scattering with complex poles

The unpolarized cross section for the situation where the poles of the propagator are complex can be evaluated by inserting the quantities (48) and (49) into the general formula (95). This procedure yields

$$\left(\frac{d\sigma}{d\Omega}\right)_C = 16G^2M^2 \left(\frac{1}{\theta^2} - f \right)^2, \quad (100)$$

where

$$f = \frac{E^4\theta^2 + 2E^2(a_2^2 - b_2^2)}{(E^2\theta^2 + a_2^2 - b_2^2)^2 + 4a_2^2b_2^2}. \quad (101)$$

Differently from the case of real poles, for certain angles and combinations of B , β , and E it is possible to have $(\frac{d\sigma}{d\Omega})_C \geq (\frac{d\sigma}{d\Omega})_{\text{GR}}$. A useful example is as follows:

$$\theta^2 = \frac{\beta \pm \sqrt{\beta^2 + \frac{8B}{\kappa^2}}}{2|B|E^2} \Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_C(\theta) = 4 \left(\frac{d\sigma}{d\Omega}\right)_{\text{GR}}(\theta). \quad (102)$$

It is good to remember that Eq. (102) only holds if $\beta > 0$, otherwise $\theta^2 < 0$.

It is natural to ask whether it is possible to have $(\frac{d\sigma}{d\Omega})_C > (\frac{d\sigma}{d\Omega})_{\text{GR}}$ with $\beta < 0$. In order to answer this question, we must note that the quantity f which appears on the cross section (100) is always positive if $a_2 > b_2$, but has indefinite sign if $a_2 < b_2$. In view of this fact, we analyze each possibility separately, as well as the special case $a_2 = b_2$.

1. The case of $a_2 > b_2$

It is straightforward to verify that f in Eq. (101) is not only positive, but is also a strictly increasing function on E , if $a_2 > b_2$ (or, equivalently, $\beta < 0$). In fact, the sign of $\partial f/\partial E$ is determined by its numerator,

$$\text{sgn}\left(\frac{\partial f}{\partial E}\right) = \text{sgn}[4a_2^2b_2^2(a_2^2 - b_2^2) + 4E^2\theta^2a_2^2b_2^2 + (a_2^2 - b_2^2)^3 + E^2\theta^2(a_2^2 - b_2^2)^2]. \quad (103)$$

Hence, if $\beta < 0$, the function f grows with the increase of E . Besides, $f \rightarrow 1/\theta^2$ when $E \rightarrow \infty$, which means that sending photons with higher energy can, at most, cancel the Einstein's term $1/\theta^2$ in the cross section expression. We conclude that if $\beta < 0$ then light would always scatter less than in general relativity, and even less for high-energy photons. This is qualitatively the same behavior as in the case of real poles.

In the strong hierarchy regime $a_2 \gg b_2$, the cross section formula (100) boils down to

$$\frac{d\sigma}{d\Omega} \approx 16G^2M^2 \left[\frac{1}{\theta^2} - \frac{E^4\theta^2 + 2a_2^2E^2}{(E^2\theta^2 + a_2^2)^2} \right]^2. \quad (104)$$

As one ought to expect, this expression corresponds to the cross section for real degenerate poles (99). Indeed, in Sec. IV C 1 it was argued that both situations are equivalent if terms of order b_2/a_2 are not taken into account.

2. The case of $a_2 < b_2$

In the six-derivative theory one can set $\beta > 0$ and still have a stable massless tensor mode. Then $a_2 < b_2$, hence it is possible to have $f < 0$ and $\frac{\partial f}{\partial E} < 0$, according to the conditions

$$f < 0 \Leftrightarrow b_2^2 - a_2^2 > \frac{E^2\theta^2}{2}, \quad (105)$$

$$\frac{\partial f}{\partial E} < 0 \Leftrightarrow b_2^2 - a_2^2 > E^2\theta^2. \quad (106)$$

It is easy to see that the two following regimes may occur, in addition to the usual behavior of the previously described scenario. First, if $f < 0$ but $\frac{\partial f}{\partial E} > 0$, then the correction term f will sum up with the general relativity term $1/\theta^2$, making the cross section larger than the general relativity one. At the same time more energetic photons still have smaller cross section. For low energy photons, the cross section increases with the energy up to the point where $E^2\theta^2 = b_2^2 - a_2^2$. Below this value of energy the sign of derivative changes and the cross section starts to decrease.

The zero point of the derivative $\partial f/\partial E$ corresponds to the unique local minimum of $f(E)$. For lower energy photons both $f < 0$ and $\frac{\partial f}{\partial E} < 0$, hence the cross section is still greater than in general relativity, but it decreases to $(\frac{d\sigma}{d\Omega})_{\text{GR}}$ as $E \rightarrow 0$. At this region more energetic photons undergo more scattering. Therefore, f is bounded between $-\frac{E^4\theta^2}{4a_2^2b_2^2}$ and θ^{-2} , and the cross section satisfies the conditions

$$0 \leq \frac{d\sigma}{d\Omega} \leq 16MG \left[\frac{1}{\theta^2} + \frac{E^4\theta^2}{4a_2^2b_2^2} \right]^2. \quad (107)$$

One can note that if the massive parameters of the action are of the order of the Planck mass, then the upper bound on the cross section is going to be very close to the cross section of general relativity. Hence, this scenario is not ruled out in principle. It is interesting to notice that this is the only scenario where the upper-bound on the cross section is not trivial.

3. The case $a_2 \approx b_2$

The condition $a_2 \approx b_2 = \mu$ is fulfilled provided that $\kappa^2 \beta^2 \ll 16|B|$. Under such an assumption the massive parameter reads $\mu \approx (\kappa^2 |B|)^{-1/4}$, and the cross section becomes

$$\frac{d\sigma}{d\Omega} \approx 16G^2 M^2 \left[\frac{1}{\theta^2} - \frac{E^4 \theta^2}{4\mu^4 + E^4 \theta^4} \right]^2 \leq \left(\frac{d\sigma}{d\Omega} \right)_{GR}. \quad (108)$$

As expected, $\frac{d\sigma}{d\Omega} \rightarrow \left(\frac{d\sigma}{d\Omega} \right)_{GR}$ when $\mu/E \rightarrow \infty$. Hence, as in the scenario with real poles or with $a_2 > b_2$, the cross section decreases with the energy of the photon.

D. On the applicability of semiclassical approach

Let us now comment on the applicability of the diagrammatic approach for the gravitational light bending, which has been described in this section. It is well known that this method is equivalent to the classical one for evaluation of the modified Newtonian potential in both general relativity and higher derivative gravity. This approach also works pretty well in general relativity for the description of the bending of light. At the same time, we know that for the fourth-derivative gravity the results of the classical and semiclassical methods diverge [37], and we have just seen that the situation is the same in the six-derivative gravity case. Therefore, it is necessary to explain the discrepancy between the two methods and understand which of them is correct and which is not.

The semiclassical approach implies the evaluation of the scattering amplitude, representing the interaction of a photon with a massive matter source. It is usually assumed that this massive particle is heavy and remains static, since it represents a heavy body such as a star or a galaxy, while the photon plays the role of a test particle. At the tree level this corresponds to a Feynman diagram as displayed in Fig. 1. In the case of general relativity, the cross section for the exchange of one graviton is simply a reduced case of Eq. (94),

$$\left(\frac{d\sigma}{d\Omega} \right)_{GR} = \frac{\kappa^4 M^2 E^4}{(8\pi)^2 \mathbf{k}^4}, \quad (109)$$

which in the small-angle approximation boils down to

$$\left(\frac{d\sigma}{d\Omega} \right)_{GR} = \frac{16G^2 M^2}{\theta^4}. \quad (110)$$

This matches the small-angle classical cross section for general relativity [53], but it is not a trivial fact. It only happens because of the special form of the interaction, which has an infinite range or, in other words, does not have an intrinsic scale [34]. This interaction classically corresponds to the Newtonian potential, and its remarkable feature is that the classical, the Born-approximated and the exact quantum cross sections do coincide [54].

In the very simple terms we can understand the validity of the semiclassical approximation in this case as follows. The underlying assumption in the quantum formulation is that the initial and final states of the photon are described by a wave which has no space localization. Therefore, the massless intermediate particle provide a non-scale description, such that the absence of localization of the free photon in the quantum formalism does not manifest as a trouble in the calculations. However, if the same scheme is applied to the theory with massive intermediate particles, the result may be incorrect, especially if the range of the force is not much larger than the impact parameter of the given scattering process. In other words, in the case of a massive intermediate particle one cannot regard the initial and final states of the scattered particle as a free wave without space localization, unless the impact parameter is sufficiently small.

Let us consider the issue in more detail. In order to apply quantum cross sections for evaluating the deflection of a photon passing close to an astronomical body, one has to compare quantum and classical cross sections [33,37–39,55,56],

$$\frac{b}{\sin \theta} \left| \frac{db}{d\theta} \right| = \left(\frac{d\sigma}{d\Omega} \right)_{\text{quantum}}. \quad (111)$$

Solving the differential equation we arrive at the answer for θ as a function of the impact parameter b . As we have already noted above, the semiclassical methodology based on (111) cannot be safely used in most of the cases, since it gives the correct result only in the case of tree-level general relativity [34] (see also more recent discussion in [40]).

The reason for the failure of using (111) is related to the fundamental difference of the terms on both sides of this equation. In the classical scattering theory there is a direct relation between the impact parameter and the scattered angle. At the same time the quantum cross section has an intrinsic probabilistic meaning, for it is related to the amplitude of the scattered wave function. It assumes that the incoming particle can be well represented by a plane wave, and that the scattered particle is going to be detected far away from the interaction zone. Such assumptions should not be taken for granted in all cases.

Consider as an example the sixth-order gravity with real poles. Following the extremely “mild” assumption which was already used in Sec. IV, let us assume that the masses $m_{2\pm}$ are such that the Yukawa potentials have ranges on the

submillimeter scale, in agreement to the lower bounds from laboratory experiments [50,51]. Then, classically, a light ray with impact parameter of one solar radius R_\odot would undergo roughly the same deflection as in general relativity. On the other hand, the quantum cross section depends on the energy of the photon, and can become arbitrarily small provided that $E \gg m_{2\pm}$. The last means that no appreciable deflection should occur if the wavelength of the photon is short enough, e.g., at the submicrometer scale.

The contradiction occurs because in the present case it is not correct to use the quantum cross section as for large impact parameters we are not in the quantum regime. The light emitted by a distant star and deflected by the Sun with the massive intermediate particle cannot be represented by a probabilistic plane wave interacting with the corresponding Yukawa potential. Instead, it ought to be described by a compact wave packet arriving with a definite impact parameter $b \sim R_\odot$ and hence it is passing far away from the center of the potential. The quantum cross section should be used only when the impact parameter is comparable to the size of the wave packet. The typical scale involved in the problem of our interest is the one defined by the massive tensor modes. It is clear that this condition cannot be achieved at the macroscopic astronomical scales.

The correct way to use the tree-level scattering amplitudes for evaluating the gravitational bending of light by astronomical bodies is via its Fourier transform, which provides the classical interaction potential. This quantity may be used within the classical scattering theory to compute the bending angle. The methodology is equivalent to the classical analysis presented in Sec. IV and agrees with the common lore that the tree-level computations, in general, should agree with the classical physics results.

VI. CONCLUSIONS

The six-derivative model represents the simplest version of the large class of quantum gravity theories which are local (i.e., polynomial in derivatives), superrenormalizable and that enable one to have only complex conjugate pairs of massive poles in the propagator. According to the recent paper [26] this kind of theories have unitary S -matrix and therefore resolve an old-standing conflict between renormalizability and unitarity in quantum gravity. Another class of theories which possess similar properties are nonlocal, (or non-polynomial in derivatives) and have no massive poles at the tree level [20,21,57,58]. However, in these theories an infinite number of ghostlike states with complex poles emerge when any kind of quantum loop corrections are taken into account [15]. For this reason the theory with higher derivatives and complex massive poles is quite general in quantum gravity, and therefore it deserves serious investigation not only in the UV, but also in the IR limit.

In the present work we made the first step in exploring the low-energy manifestations of complex higher-derivative

states. For the sake of completeness we also considered the cases of real massive poles, both simple and multiple. It turned out that the effect of complex poles on the modified Newtonian potential and on the gravitational bending of light is partially similar to the one of the massive real ghost mode in the four-derivative gravity theory. At the same time, there are some new and remarkable features, such as the oscillatory behavior of the potential $V(r)$, which takes place in the case of complex poles.

We have shown that there is a difference between the classical and quantum cross sections for the gravitational scattering of the photon. In the former case one has to treat photon as a particle moving in the determined classical background of a weak gravitational field. Contrary to this, within the semiclassical approach the tree-level cross section is used to evaluate the same scattering. We have confirmed, for the six-derivative models, the previous conclusions of [34], that the semiclassical approach used, e.g., in [33] cannot be applied for higher-derivative models, except in the case of extremely small impact parameters.

Still in the quantum domain, it was shown that the cross sections vanish for photons with energies $E \gg |m_{2\pm}|$. This feature was also noticed in the case of fourth-order gravity [37] and it can be qualitatively explained recalling the uncertainty principle. In fact, this is the energy necessary to localize a particle with uncertainty smaller than $1/|m_{2\pm}|$. In such case the Coulomb-shielding property of the Yukawa short-range potentials is broken, and the photon is able to probe the inner parts of the potential, where it tends to behave like $1/r$. As the contribution of the Yukawa-type potentials approaches that of Rutherford scattering, they cancel out the authentic Rutherford term owed to the (massless) graviton. From the diagrammatic perspective, this can be understood as the backreaction of the photon on the background and its capability of exciting the massive modes. The effect becomes significant for high-energy photons with small impact parameters, with frequencies comparable to the mass of the tensor excitations.

From the phenomenological side, our investigation has shown that the gravitational light bending in the Solar System cannot predict new dispersive phenomena such as in lensing or arriving time delays, nor give tight constraints to the massive modes. It is more likely to detect the influence of the higher-derivative terms in laboratory experiments using torsion-balance or in the cosmological observations. The analysis of these possibilities would be quite interesting and should represent an interesting subject for future work.

ACKNOWLEDGMENTS

B. L. G. and I.Sh. are very grateful to A. Starobinsky for clarifying discussions and criticism. A. A. acknowledges CNPq and FAPERJ for financial support. B. L. G. is thankful to CNPq for supporting his Ph.D. project. I. Sh. is grateful to CNPq, FAPEMIG and ICTP for partial

support of his work. B. L. G. wishes to thank the Department of Physics at Universidade Federal de Juiz de Fora for the kind hospitality during his visit, when the initial version of this work was completed.

APPENDIX: PROOF OF THE THEOREM 1

Let us prove Theorem 1 which enables one to write the general of the field equations in terms of auxiliary fields.

It is easy to show that the gauge condition $\Gamma_\mu = 0$ can be achieved by means of coordinate transformation $x^\mu \rightarrow x'^\mu = x^\mu + \kappa \xi^\mu(x)$. The transformation of the linearized perturbations are

$$h'_{\mu\nu} = h_{\mu\nu} - (\xi_{\mu,\nu} + \xi_{\nu,\mu}), \quad (\text{A1})$$

$$\gamma'_{\mu\nu} = \gamma_{\mu\nu} - (\xi_{\mu,\nu} + \xi_{\nu,\mu}) + \eta_{\mu\nu} \xi_{\lambda,\lambda}. \quad (\text{A2})$$

Since for the scalar curvature $R' = R$, it is easy to derive

$$\Gamma_\mu \rightarrow \Gamma'_\mu = \Gamma_\mu - \left(1 - \frac{\kappa^2 \beta}{4} \square - \frac{\kappa^2 B}{4} \square^2\right) \square \xi_\mu. \quad (\text{A3})$$

The next step consists in the following proposition:

Proposition 1 The general solution of the system

$$\left(1 - \frac{\kappa^2 \beta}{4} \square - \frac{\kappa^2 B}{4} \square^2\right) \left(-\square h_{\mu\nu} + \frac{R}{3\kappa} \eta_{\mu\nu}\right) = \frac{\kappa}{2} \left(T_{\mu\nu} - \frac{T}{3} \eta_{\mu\nu}\right), \quad (\text{A4})$$

$$\begin{aligned} \Gamma_\mu &= \left(1 - \frac{\kappa^2 \beta}{4} \square - \frac{\kappa^2 B}{4} \square^2\right) \gamma_{\mu\rho,\rho} \\ &\quad - \frac{\kappa}{2} \left(\alpha + \frac{\beta}{2} + A \square + \frac{B}{2} \square\right) R_{,\mu} = 0 \end{aligned} \quad (\text{A5})$$

has the form

$$\begin{aligned} h_{\mu\nu} &= h_{\mu\nu}^{(E)} + (m_{2+}^2 + m_{2-}^2 + \square) \psi_{\mu\nu} \\ &\quad - \eta_{\mu\nu} (m_{0+}^2 + m_{0-}^2 + \square) \phi, \end{aligned} \quad (\text{A6})$$

where the fields $h_{\mu\nu}^{(E)}$, $\psi_{\mu\nu}$ and ϕ satisfy the equations

$$\square h_{\mu\nu}^{(E)} = \frac{\kappa}{2} \left(\frac{1}{2} T \eta_{\mu\nu} - T_{\mu\nu}\right), \quad (\text{A7})$$

$$\gamma_{\mu\nu}^{(E),\nu} = 0, \quad \text{where } \gamma_{\mu\nu}^{(E)} = h_{\mu\nu}^{(E)} - \frac{1}{2} \eta_{\mu\nu} h^{(E)}, \quad (\text{A8})$$

$$(m_{2+}^2 + \square)(m_{2-}^2 + \square) \psi_{\mu\nu} = \frac{\kappa}{2} \left(T_{\mu\nu} - \frac{1}{3} T \eta_{\mu\nu}\right), \quad (\text{A9})$$

$$(m_{2+}^2 + m_{2-}^2 + \square)(\psi_{\mu\nu}{}^{,\mu\nu} - \square \psi) = 0, \quad (\text{A10})$$

$$(m_{0+}^2 + \square)(m_{0-}^2 + \square) \phi = \frac{\kappa T}{12}. \quad (\text{A11})$$

Here we used notations (22) and (23).

Proof.—The first parenthesis in Eq. (A4) can be factorized as

$$-\frac{\kappa^2 B}{4} (m_{2+}^2 + \square)(m_{2-}^2 + \square), \quad (\text{A12})$$

provided that

$$m_{2+}^2 + m_{2-}^2 = \frac{\beta}{B} \quad \text{and} \quad m_{2+}^2 m_{2-}^2 = -\frac{4}{\kappa^2 B}, \quad (\text{A13})$$

that corresponds to the definition (23). Defining

$$\psi_{\mu\nu} = -\frac{\kappa^2 B}{4} \left(-\square h_{\mu\nu} + \frac{1}{3\kappa} R \eta_{\mu\nu}\right), \quad (\text{A14})$$

Eq. (A4) results in

$$(m_{2+}^2 + \square)(m_{2-}^2 + \square) \psi_{\mu\nu} = \frac{\kappa}{2} \left(T_{\mu\nu} - \frac{1}{3} T \eta_{\mu\nu}\right), \quad (\text{A15})$$

which is precisely (9). In terms of the field $\psi_{\mu\nu}$, Eq. (A4) can be rewritten as

$$\square^2 \psi_{\mu\nu} + \frac{\beta}{B} \square \psi_{\mu\nu} - \square h_{\mu\nu} + \frac{R}{3\kappa} \eta_{\mu\nu} = \frac{\kappa}{2} \left(T_{\mu\nu} - \frac{1}{3} T \eta_{\mu\nu}\right). \quad (\text{A16})$$

This equation can be cast in a more useful form by means of the following expressions:

(i) Trace of (A4),

$$\left[1 - \frac{\kappa^2}{4} (\beta + B \square)\right] \left(\square h - \frac{4}{3\kappa} R\right) = \frac{\kappa}{6} T. \quad (\text{A17})$$

(ii) Divergence of Γ_μ in (5)

$$\begin{aligned} 0 &= \left[1 - \frac{\kappa^2}{4} (\beta \square + B \square^2)\right] \gamma_{\mu\rho}{}^{,\mu\rho} \\ &\quad - \frac{\kappa}{2} \left(\alpha + \frac{\beta}{2} + A \square + \frac{B}{2} \square\right) \square R. \end{aligned} \quad (\text{A18})$$

(iii) Summing up the last two equations and using (7) yields

$$\frac{R}{3\kappa} = \frac{\kappa}{12} T - \frac{\kappa}{2} \left(\alpha + \frac{\beta}{3} + A \square + \frac{B}{3} \square\right) \square R. \quad (\text{A19})$$

Then, inserting (A19) into (A16) gives

$$\square h_{\mu\nu}^{(E)} = -\frac{\kappa}{2} \left(T_{\mu\nu} - \frac{T}{2} \eta_{\mu\nu} \right) \quad (\text{A20})$$

where we defined the new field

$$h_{\mu\nu}^{(E)} = -\square \psi_{\mu\nu} - \frac{\beta}{B} \psi_{\mu\nu} + h_{\mu\nu} + \frac{\kappa}{2} \left(\alpha + \frac{\beta}{3} + A\square + \frac{B}{3}\square \right) R \eta_{\mu\nu}. \quad (\text{A21})$$

One can rewrite (A21) in an alternative useful form

$$h_{\mu\nu} = h_{\mu\nu}^{(E)} + (m_{2+}^2 + m_{2-}^2 + \square) \psi_{\mu\nu} - \frac{\kappa}{2} \left(A + \frac{B}{3} \right) \left(\frac{3\alpha + \beta}{3A + B} + \square \right) R \eta_{\mu\nu}. \quad (\text{A22})$$

The only field which remains to be defined is the scalar ϕ . Equation (A19) can be rewritten in the factorized form

$$\frac{\kappa}{2} \left(A + \frac{B}{3} \right) (m_{0+}^2 + \square)(m_{0-}^2 + \square) R = \frac{\kappa}{12} T, \quad (\text{A23})$$

where the quantities m_{0+}^2 and m_{0-}^2 satisfy

$$m_{0+}^2 + m_{0-}^2 = \frac{3\alpha + \beta}{3A + B}, \quad (\text{A24})$$

$$m_{0+}^2 m_{0-}^2 = \frac{2}{\kappa^2 (3A + B)}. \quad (\text{A25})$$

It is straightforward to verify that the solution of this system is the second relation in (23). Hence one can define the scalar field

$$\phi = \frac{\kappa}{2} \left(A + \frac{B}{3} \right) R, \quad (\text{A26})$$

while its equation of motion follows from (A23),

$$(m_{0+}^2 + \square)(m_{0-}^2 + \square)\phi = \frac{\kappa}{12} T. \quad (\text{A27})$$

The general solution (A22) of the system (12) can be presented in the form

$$h_{\mu\nu} = h_{\mu\nu}^{(E)} + (m_{2+}^2 + m_{2-}^2 + \square) \psi_{\mu\nu} - \eta_{\mu\nu} (m_{0+}^2 + m_{0-}^2 + \square) \phi. \quad (\text{A28})$$

Up to this point we have shown that the general solution of (A4) is written as a combination of three independent fields which satisfy the equations of motion (A15), (A20), and (A27). In order to complete the proof one has to show that the tensor fields $h_{\mu\nu}^{(E)}$ and $\psi_{\mu\nu}$ satisfy the gauge conditions.

In terms of $\gamma_{\mu\nu}^{(E)} = h_{\mu\nu}^{(E)} - \frac{1}{2} h^{(E)} \eta_{\mu\nu}$, Eq. (A20) can be written as

$$\square \gamma_{\mu\nu}^{(E)} = -\frac{\kappa}{2} T_{\mu\nu}. \quad (\text{A29})$$

One can note that the gauge condition $\Gamma_\mu = 0$ is equivalent to $\Omega_{\mu\nu}{}^{,\nu} = 0$, where

$$\Omega_{\mu\nu} = \left[1 - \frac{\kappa^2}{4} (\beta\square + B\square^2) \right] \gamma_{\mu\nu} - \frac{\kappa}{2} \left(\alpha + \frac{\beta}{2} + A\square + \frac{B}{2}\square \right) R \eta_{\mu\nu}. \quad (\text{A30})$$

According to Eq. (A28) it follows

$$\gamma_{\mu\nu} = \gamma_{\mu\nu}^{(E)} + \left(\frac{\beta}{B} + \square \right) \left(\psi_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \psi \right) + \left(\frac{\sigma_1}{\sigma_2} + \square \right) \phi \eta_{\mu\nu}. \quad (\text{A31})$$

By combining Eqs. (A15), (A26), (A29), and (A31), it is easy to show that

$$\Omega_{\mu\nu} = \gamma_{\mu\nu}^{(E)}. \quad (\text{A32})$$

Therefore, the gauge condition (5) implies

$$\gamma_{\mu\nu}^{(E),\nu} = 0. \quad (\text{A33})$$

Together with the equation of motion (A20), this means $h_{\mu\nu}^{(E)}$ is the solution of linearized general relativity in de Donder gauge.

The gauge condition for the field $\psi_{\mu\nu}$ can be obtained by remembering that [see Eq. (7)]

$$\gamma_{\mu\nu}{}^{,\mu\nu} = \frac{1}{2} \square h - \frac{1}{\kappa} R. \quad (\text{A34})$$

Taking into account (A28), (A31), and (A33) in the previous expression it can be shown that

$$(m_{2+}^2 + m_{2-}^2 + \square) (\psi_{\mu\nu}{}^{,\mu\nu} - \square \psi) = 0, \quad (\text{A35})$$

completing the proof.

The Theorem 1 can then be regarded as a corollary of the previous proposition which follows from the change of variables

$$\bar{\Psi}_{\mu\nu} = m_{2+}^2 \psi_{\mu\nu}, \quad \Psi_{\mu\nu} = (m_{2-}^2 + \square) \psi_{\mu\nu}, \quad (\text{A36})$$

$$\bar{\Phi} = m_{0+}^2 \phi, \quad \Phi = (m_{0-}^2 + \square) \phi \quad (\text{A37})$$

in Eqs. (A6)–(A11).

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