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Stationary electromagnetic fields of slowly rotating relativistic magnetized star in the braneworld

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The exterior electromagnetic fields of slowly rotating relativistic magnetized star in the braneworld are studied in detail. We have also obtained exact analytical solutions of the Maxwell equations for the magnetic and the electric fields inside the slowly rotating relativistic magnetized star in the braneworld. The dependence of the electromagnetic energy losses of the rotating magnetized star from the brane tension is also calculated and has been combined with the astrophysical data on pulsar period slowdown in order to get constraints on the brane parameter. We have found the upper limit for the brane parameter as $|Q^*| \lesssim 3 \times 10^{11}\ \mathrm{cm}^2$.

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I. INTRODUCTION

The extensive study of electromagnetic fields and radiation from the compact massive objects formed at the final stage of the stellar evolution is one of the most important tasks in the relativistic astrophysics for several reasons. First of all, the current observational status of these objects is based on the observations in the whole electromagnetic spectra of physical and astrophysical processes in the vicinity of the compact objects caused by the existence of the strong electromagnetic fields.

Definitely electromagnetic fields play an outstanding role in observation and detection of majority astrophysical objects in the late stage of their evolution especially compact relativistic stars, e.g., as pulsars and SGR (soft gamma ray repeaters) which can posses huge surface magnetic fields $B_0 \sim 10^{12}$ G for the standard neutron stars and $B_0 \sim 10^{15}$ G in the exceptional cases for magnetars [1,2]. The magnetic field of relativistic compact stars is one of the main quantities determining their observability, for example as radio pulsars through magneto-dipolar radiation. The magnetic field at the stellar surface determines energy losses from the star and therefore may be related with such observable parameters as period of pulsar and its spin-down through time derivation.

The second reason is that we may test various alternative theories of gravitation in the strong field regime through the study of astrophysical processes around compact objects for which general relativity effects are especially strong. Considering different space-time geometry around the relativistic stars one may investigate the effect of the different parameters on evolution and behavior of stellar interior and exterior magnetic fields. Then these models can be checked through comparison of theoretical results obtained with the precise observational data on the parameters of the compact objects as spin down of the pulsars.

The third reason may be seen in influence of stellar magnetic and electric field on different physical phenomena around the relativistic star, such as gravitational lensing, motion of test particles, and electromagnetic radiation from the accretion discs [3–8].

In the Newtonian approach the exact analytical solutions for the vacuum electromagnetic fields of rotating magnetized sphere are given in the classical paper of Deutsch [9] and interior solution for the electromagnetic fields of the constant magnetic density star are studied by many authors, see, for example, [10,11]. In the general relativistic approach the study of the magnetic field structure outside magnetized compact gravitational objects has been initiated in the pioneering work of Ginzburg and Ozernoy [12] and has been further extended by number of authors [13–23].

The astronomical observations performed in the last decades have confirmed the existence of new forms of matter which dominate in our Universe in the present epoch. For example, the detailed analysis of type Ia Supernovae (SNeIa) explosions in far galaxies, 10^{-5} order fluctuations in the cosmic microwave background radiation, and matter power spectrum inferred from large galaxy surveys in the Universe provide the strongest evidences for existence of new forms of matter in the observational cosmology. In particular, the known form of matter that is baryons contribute only for ∼4% of the total matter-energy, while the exotic cold dark matter (DM) interacting only gravitationally represents the bulk of the matter content (∼23%) and the so-called dark energy (DE) acting as antigravity represents (∼73%). Being the best fit to a wide range of the astronomical data, the standard cosmological model has some difficulties in the interpretation related to the unknown nature of DM and DE and due to this reason has motivated the numerous approaches for alternative theories of gravity (for details, see, e.g., [24]).

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Here we are interested in study the behavior of the electromagnetic fields of the slowly rotating relativistic magnetized star in one of the alternate theories of gravity in so-called braneworld model proposed by Randall and Sundrum [25] where the matter is confined to a three dimensional space so-called brane, embedded in a larger space so-called bulk in which only gravitation interaction can propagate. The static and spherically symmetric exterior vacuum solution of the braneworld models has been first obtained in the astrophysical scale in [26], which exactly coincides with the Reissner-Nordstrom solution with the only difference that the square of the electric charge has to be replaced by so-called brane parameter Q^*.

Braneworld corrections to the charged rotating black holes and to the perturbations in the electromagnetic potential around black holes are studied, for example, in [27,28]. In the paper [29], the stellar magnetic field configurations of the spherical symmetric relativistic stars in dependence on the brane tension have been studied where the numerical solutions for the exterior magnetic field of the relativistic star in braneworld are presented. Later this research has been extended in [30,31] for derivation of the magnetospheric and exterior vacuum solutions in the space-time metric of the slowly rotating relativistic stars in the braneworld and where approximate analytical results for the near-zone electromagnetic fields of the star in the braneworld have been presented. The stellar magnetic field configuration in the external background spacetime of relativistic magnetized stars in the Hořava-Lifshitz gravity and in modified $f(R)$ gravity has been studied in [32]. The Kerr-Newman black hole formed as result of the gravitational collapse of rotating and magnetized neutron stars has been recently studied in [33] while the energy extraction from the boosted and rotating black hole immersed in the uniform magnetic field in [34].

The paper is organized in the following way. Section II is devoted to the vacuum electromagnetic fields of a rotating relativistic magnetized star in the braneworld. In particular in the subsection II A we briefly explain structure of the slowly rotating star in the braneworld. Then in the subsections II B and II C we present exact analytical solutions for the magnetic and the electric fields exterior to the slowly rotating neutron star in the braneworld, by solving vacuum Maxwell equations analytically. We show that both magnetic and electric fields will be essentially modified by braneworld effects. In the next Sec. III, we analyze the exact analytical solutions for the vacuum electromagnetic fields obtained in previous section, and discuss the braneworld effects on the magnetic and the electric fields of the slowly rotating misaligned magnetized neutron star. In subsection III B we have checked effects of the brane tension on energy losses from the slowly rotating neutron star in the braneworld and have got astrophysical constraints on the value of the brane tension making comparison with the observational data. Finally, in Sec. IV we summarize our results and give future outlook related to the present work.

In the present work the functions and quantities denoted with upper index "*" are related to the ones belonging to the braneworld. We use a space-like signature $(-,+,+,+)$, a system of units in which $G = c = 1$ and we restore them when we need to compare the results with the observational data. Greek indices are taken to run from 0 to 3, Latin indices from 1 to 3.

II. THE VACUUM ELECTROMAGNETIC FIELDS OF A ROTATING RELATIVISTIC MAGNETIZED STAR IN THE BRANEWORLD

One of the most difficult mathematical problems is to obtain an analytical solution of the Einstein-Maxwell equations, which are coupled, but in the realistic astrophysical approximation when the electromagnetic field does not affect the space-time around the relativistic star one can get exact analytical solutions of the electromagnetic field equations (see, for the details, e.g. [22]). Throughout the paper we assume that the electromagnetic field and its energy are really small to change the space-time geometry, study the electromagnetic field in the fixed spacetime geometry, and examine the effects of the background gravitational field on the electromagnetic field of the slowly rotating relativistic star in the braneworld.

A. Spacetime of the slowly rotating star in the braneworld

The equations of the gravitational field are given by following form

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = k^2 T^{\rm eff}_{\mu\nu}, \tag{1}$$

where $R_{\mu\nu}$ and R are Ricci tensor and Ricci scalar, $g_{\mu\nu}$ is metric tensor of the spacetime and $k^2 = 8\pi$ is the Einstein constant, $T^{\rm eff}_{\mu\nu}$ is the effective energy-momentum tensor that can be considered as

$$T^{\rm eff}_{\mu\nu} = T_{\mu\nu} + T^*_{\mu\nu} + T^{\rm EM}_{\mu\nu}, \tag{2}$$

where $T_{\mu\nu}$ is a perfect fluid energy-momentum tensor

$$T_{\mu\nu} = \rho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu), \tag{3}$$

and $T^*_{\mu\nu}$ is the energy-momentum tensor arises from extra dimension (braneworld) that can be expressed in the following form

$$\begin{aligned} T^*_{\mu\nu} = & \frac{1}{2\lambda^*} \rho^2 u_\mu u_\nu + \frac{1}{2\lambda^*} \rho(\rho + 2p)(g_{\mu\nu} + u_\mu u_\nu) \\ & + \frac{1}{2\lambda^* k^4} \Big[\mathcal{U} u_\mu u_\nu + \mathcal{P} r_\mu r_\nu \\ & + (\mathcal{U} - \mathcal{P}) \left(g_{\mu\nu} + \frac{1}{3} u_\mu u_\nu \right) \Big], \end{aligned} \tag{4}$$

where u^μ is the four-velocity of the medium, r^μ is a unit radial vector, $\rho(r)$ and $p(r)$ are matter energy density and pressure, respectively. The nonlocal bulk effects are carried by the nonlocal energy density $\mathcal{U}(r)$ and nonlocal pressure $\mathcal{P}(r)$. The quantity λ^* is the brane tension parameter and the standard general relativity is recovered in the case when $\lambda^* \to \infty$.

Finally, the energy-momentum tensor for the electromagnetic field has a form after minimizing Yang-Mills action

$$T^{\rm EM}_{\mu\nu} = \frac{1}{4\pi}\left(F_{\mu\alpha}F^{\alpha}_{\nu} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right), \tag{5}$$

which is very small ($T^{\rm EM}_{\mu\nu} \ll T_{\mu\nu} + T^*_{\mu\nu}$) in comparison to the other two terms of the total energy-momentum tensor that is given in Eqs. (3)–(4) because (i) the equations are integrated from the inner radius where the electric currents are negligible and (ii) as we have mentioned above, the electromagnetic field does not contribute on the space-time geometry of the relativistic star. A_α is the vector potential of electromagnetic field, $F_{\alpha\beta} = \partial_\beta A_\alpha - \partial_\alpha A_\beta$ is antisymmetric tensor of the electromagnetic field.

In the coordinates (t, r, θ, ϕ) one can write the general form of the space-time metric of the slowly rotating relativistic star in the braneworld

$$ds^2 = e^{2\Lambda^*(r)}dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2 - e^{2\Phi^*(r)}dt^2 - 2\omega^*(r)r^2\sin^2\theta dt d\phi, \tag{6}$$

where $\omega^*(r)$ is the angular velocity of the dragging of inertial frames.

In general, $\Phi^*(r)$ and $\Lambda^*(r)$ are unknown functions which are satisfied by the following field equations inside the star, see [29,35]

$$k^2\rho^* = \frac{1}{r^2} - e^{-2\Lambda^*}\left(\frac{1}{r^2} - \frac{2}{r}\frac{d\Lambda^*}{dr}\right), \tag{7}$$

$$k^2\left(p^* + \frac{4}{k^4}\mathcal{P}\right) = -\frac{1}{r^2} + e^{-2\Lambda^*}\left(\frac{1}{r^2} + \frac{2}{r}\frac{d\Phi^*}{dr}\right), \tag{8}$$

$$0 = \frac{dp}{dr} + \frac{d\Phi^*}{dr}(\rho + p), \tag{9}$$

$$-\frac{k^2}{4}(\rho + p)\frac{d\rho}{dr} = \frac{1}{2}\frac{d\mathcal{U}}{dr} + \frac{d\Phi^*}{dr}(2\mathcal{U} + \mathcal{P}) + \frac{d\mathcal{P}}{dr} + \frac{3\mathcal{P}}{r}, \tag{10}$$

where $\rho^*(r)$ and $p^*(r)$ are the effective total energy density and the effective pressure defined as

$$\rho^* = \rho + \frac{1}{2\lambda^*}\left(\rho^2 + \frac{6}{k^4}\mathcal{U}\right), \tag{11}$$

$$p^* = p + \frac{1}{2\lambda^*}\left(\rho^2 + 2p\rho + \frac{4}{k^4}\mathcal{U}\right). \tag{12}$$

The external vacuum solution of the field equations outside the relativistic star (where $\rho = p = 0$) can be written as the well-known Reissner-Nordström-type and the exact form is given by [26]

$$e^{2\Phi^*} = e^{-2\Lambda^*} = N^{*2} = 1 - \frac{2M}{r} + \frac{Q^*}{r^2}, \qquad r \geq R, \tag{13}$$

where M is the total mass and R is the radius of the star and one can easily see that in the Reissner-Nordström-type solution the squared of electric charge is replaced by a brane parameter Q^*, so called a "tidal charge" or "Weyl charge" which arises from the projection on to the brane of the gravitational field in the bulk and it is negatively defined $Q^* = -3MR\rho_{r=R}/\lambda^* < 0$, (See [35]).

In the vacuum region the Lense-Thirring angular velocity can be written in term of the brane parameter in the following way

$$\omega^*(r) = \omega(r)\left(1 - \frac{Q^*}{2Mr}\right), \qquad \omega(r) = \frac{2J}{r^3}, \tag{14}$$

where the quantity $J = I\Omega$ is the total angular momentum of the star, with the moment of inertia I and the angular velocity Ω of the relativistic star, respectively.

We plan to study electromagnetic properties of the slowly rotating relativistic magnetized stars in the braneworld. In order to study the electromagnetic field of the relativistic star, one should consider the general relativistic form of the Maxwell equations [in particular in the spacetime geometry given by Eq. (6)]. The first pair of the general relativistic form of Maxwell equations is given by

$$\partial_\mu(\sqrt{-g}F^{\mu\nu}) = 4\pi\sqrt{-g}J^\nu, \tag{15}$$

which are the main equations along with the second pair of the Maxwell equations for the dual partner of the electromagnetic field tensor

$$\partial_\mu(\sqrt{-g}{}^*F^{\mu\nu}) = 0, \tag{16}$$

the so-called Bianchi identity for the electromagnetic field tensor, definition of the dual tensor of the electromagnetic field is given by ${}^*F^{\mu\nu} = (1/2\sqrt{-g})\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$, where $\varepsilon_{\alpha\beta\mu\nu}$ is the Levi-Civita symbol in four dimensional space and in general the four vector J_α is the electric current density which is the source of the electromagnetic field.

In Appendix A, the explicit form of the Maxwell equations are given in terms of the electric and the magnetic fields and related detail calculations are also shown.

B. The stationary magnetic field of the slowly rotating star in the braneworld

Before doing the calculation, let us assume that the magnetic moment of the magnetized star does not vary in time as a result of the high electrical conductivity of the medium inside the star $\sigma \to \infty$, see [36], and in case of the slowly rotation, one can ignore the deformation due to rotation that means the star has spherical shape and during the calculations we will consider only linear approximation for angular velocities as $\mathcal{O}(\omega)$ and $\mathcal{O}(\Omega)$.

We now look for the stationary solutions of the Maxwell equations for the components of the magnetic field in the following separable form, see [21,36]

$$B^{\hat{r}}(r,\theta,\phi,t) = F^*(r)[\cos\chi\cos\theta + \sin\chi\sin\theta\cos\lambda], \quad (17)$$

$$B^{\hat{\theta}}(r,\theta,\phi,t) = G^*(r)[\cos\chi\sin\theta - \sin\chi\cos\theta\cos\lambda], \quad (18)$$

$$B^{\hat{\phi}}(r,\theta,\phi,t) = H^*(r)\sin\chi\sin\lambda, \quad \lambda = \phi - \Omega t, \quad (19)$$

where unknown functions $F^*(r)$, $G^*(r)$, and $H^*(r)$ account for the general relativistic and braneworld extra dimension corrections to the magnetic field of the star and χ is the inclination angle of the magnetic field relative to the rotation axis of the star. Substituting the solutions in Eqs. (17)–(19) into the Maxwell equations (A2), (A7)–(A9) that is given in the Appendix A, and ignoring the higher order terms in the rotational parameters $\mathcal{O}(\omega)$, $\mathcal{O}(\Omega)$ and after making simple algebraic calculation one can easily obtain the following set of the differential equations for the unknown radial functions [21]

$$\frac{d}{dr}(r^2F^*) + \frac{2r}{N^*}G^* = 0, \quad (20)$$

$$\frac{d}{dr}(rN^*H^*) + F^* = 0, \quad (21)$$

$$H^* = G^*. \quad (22)$$

These equations can be expressed as a single, second order differential equation for the unknown function $F^*(r)$

$$\frac{d}{dr}\left[\left(1 - \frac{2M}{r} + \frac{Q^*}{r^2}\right)\frac{d}{dr}(r^2F^*)\right] - 2F^* = 0, \quad (23)$$

which is analytically solvable. The exact analytical form of the profile functions $F^*(r)$, $G^*(r)$, and $H^*(r)$ are given by the following form

$$F^*(r) = -\frac{3\mu}{4(M^2-Q^*)^{3/2}}\left[\frac{2\sqrt{M^2-Q^*}}{r}\left(1+\frac{M}{r}\right) + \left(1-\frac{Q^*}{r^2}\right)\ln\left(\frac{r-M-\sqrt{M^2-Q^*}}{r-M+\sqrt{M^2-Q^*}}\right)\right], \quad (24)$$

$$G^*(r) = H^*(r) = \frac{3\mu}{4(M^2-Q^*)^{3/2}}\left[\frac{2\sqrt{M^2-Q^*}}{rN^*}\left(1-\frac{M}{r}\right) + N^*\ln\left(\frac{r-M-\sqrt{M^2-Q^*}}{r-M+\sqrt{M^2-Q^*}}\right)\right], \quad (25)$$

where μ is the magnetic dipole moment.

It is always useful to get the right limiting values for the results obtained in the alternative theories of gravity in the general relativistic case by doing the following operation $Q^* \to 0$ (all detailed calculations are shown in the Appendix C). In the weak field approach one can easily obtain the following results for the profile functions (24)–(25)

$$F^*(r) = \frac{2\mu}{r^3}\left[1 + \frac{3M}{2r} + \frac{2M^2}{r^2}\left(1+\frac{\varrho^2}{5}\right) + \mathcal{O}\left(\frac{M^3}{r^3}\right)\right],$$

$$G^*(r) = \frac{\mu}{r^3}\left[1 + \frac{2M}{r} + \frac{3M^2}{r^2}\left(1+\frac{7\varrho^2}{30}\right) + \mathcal{O}\left(\frac{M^3}{r^3}\right)\right],$$

here we have introduced new parameter ϱ:

$$\varrho = \left(1 - \frac{Q^*}{M^2}\right)^{1/2}, \quad (26)$$

which is responsible for the effect of the braneworld tension.

In the Newtonian approximation the functions $F^*(r)$ and $G^*(r)$ take the following right limits:

$$\lim_{M/r\to 0} F^*(r) = \frac{2\mu}{r^3}, \qquad \lim_{M/r\to 0} G^*(r) = \frac{\mu}{r^3}. \quad (27)$$

Finally, the exterior solution for the stationary magnetic field of a misaligned magnetized star can be written in terms of the brane parameter Q^* (and ϱ) as

$$B^{\hat{r}}(r,\theta,\phi,t) = -\frac{3\mu}{4\varrho^3M^3}\left[\frac{2\varrho M}{r}\left(1+\frac{M}{r}\right) + \left(1-\frac{Q^*}{r^2}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right] \times (\cos\chi\cos\theta + \sin\chi\sin\theta\cos\lambda), \quad (28)$$

$$B^{\hat{\theta}}(r,\theta,\phi,t) = \frac{3\mu}{4\varrho^3M^3}\left[\frac{2\varrho M}{rN^*}\left(1-\frac{M}{r}\right) + N^*\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right] \times (\cos\chi\sin\theta - \sin\chi\cos\theta\cos\lambda), \quad (29)$$

$$B^{\hat{\phi}}(r,\theta,\phi,t) = \frac{3\mu}{4\varrho^3M^3}\left[\frac{2\varrho M}{rN^*}\left(1-\frac{M}{r}\right) + N^*\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right]\sin\chi\sin\lambda. \quad (30)$$

It can be easily found the interior solution for the magnetic field when the stellar matter consists of stiff matter with the equation of state $p = \rho$, see [21]

$$B^{\hat{r}} = \frac{C^{*}\mu}{R^{3}}(\cos\chi\cos\theta + \sin\chi\sin\theta\cos\lambda), \tag{31}$$

$$B^{\hat{\theta}} = -\frac{C^{*}\mu}{R^{3}e^{\Lambda^{*}(r)}}(\cos\chi\sin\theta - \sin\chi\cos\theta\cos\lambda), \tag{32}$$

$$B^{\hat{\phi}} = -\frac{C^{*}\mu}{R^{3}e^{\Lambda^{*}(r)}}\sin\chi\sin\lambda, \tag{33}$$

where C^{*} is an arbitrary constant that can be found after imposing the continuity of the radial magnetic field $B^{\hat{r}}$ across the star surface, or $[B^{\hat{r}}] = B^{\hat{r}}_{\rm ext}|_{r=R} - B^{\hat{r}}_{\rm int}|_{r=R} = 0$ as

$$\begin{aligned} C^{*} &= \frac{F^{*}(R)R^{3}}{\mu} \\ &= -\frac{3R^{3}}{4\varrho^{3}M^{3}}\left[\frac{2\varrho M}{R}\left(1+\frac{M}{R}\right) \right. \\ &\quad \left. + \left(1-\frac{Q^{*}}{R^{2}}\right)\ln\left(\frac{R-M(1+\varrho)}{R-M(1-\varrho)}\right)\right]. \end{aligned} \tag{34}$$

In a flat space this constant is satisfied through the following condition

$$\lim_{M/R\to 0, Q^{*}/R^{2}\to 0} C^{*} = 2.$$

C. The stationary electric field of the slowly rotating star in the braneworld

In this subsection we will find an exact analytical solutions of the Maxwell equations for the electric field of the slowly rotating magnetized star in the braneworld. By knowing the expressions for the components of the magnetic field one can immediately write the Maxwell equations for the electric field of the misaligned rotating relativistic star, see, e.g., [21].

One can look for the components of the electric field in the following separable form [21]

$$\begin{aligned} E^{\hat{r}}(r,\theta,\phi,t) &= [f_1^{*}(r) + f_3^{*}(r)]\cos\chi(3\cos^{2}\theta - 1) \\ &\quad + 3[g_1^{*}(r) + g_3^{*}(r)]\sin\chi\sin\theta\cos\theta\cos\lambda, \end{aligned} \tag{35}$$

$$\begin{aligned} E^{\hat{\theta}}(r,\theta,\phi,t) &= [f_2^{*}(r) + f_4^{*}(r)]\cos\chi\sin\theta\cos\theta \\ &\quad + [g_2^{*}(r) + g_4^{*}(r)]\sin\chi\cos\lambda \\ &\quad - [g_5^{*}(r) + g_6^{*}(r)](\cos^{2}\theta - \sin^{2}\theta)\sin\chi\cos\lambda, \end{aligned} \tag{36}$$

$$\begin{aligned} E^{\hat{\phi}}(r,\theta,\phi,t) &= [g_5^{*}(r) + g_6^{*}(r)]\sin\chi\cos\theta\sin\lambda \\ &\quad - [g_2^{*}(r) + g_4^{*}(r)]\sin\chi\cos\theta\sin\lambda. \end{aligned} \tag{37}$$

Substituting Eqs. (35)–(37) into the Maxwell equations (A3)–(A4) and (A6) in Appendix A, one can obtain the following set of systems of the linear differential equations for the unknown functions $f_1^{*}(r) - f_4^{*}(r)$ and $g_1^{*}(r) - g_6^{*}(r)$

$$N^{*}\frac{d}{dr}(r^{2}f_1^{*}) + rf_2^{*} = 0, \tag{38}$$

$$\frac{d}{dr}(rN^{*}f_2^{*}) + 6f_1^{*} = 0, \tag{39}$$

$$N^{*}\frac{d}{dr}(r^{2}f_3^{*}) + rf_4^{*} = 0, \tag{40}$$

$$\begin{aligned} \frac{d}{dr}(rNf_4^{*}) + 6f_3^{*} &= \frac{9\mu\omega^{*}r}{4\varrho^{3}M^{3}}\left[\frac{2\varrho M}{r}\left(1+\frac{M}{r}\right) \right. \\ &\quad \left. + \left(1-\frac{Q^{*}}{r^{2}}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right], \end{aligned} \tag{41}$$

$$N^{*}\frac{d}{dr}(r^{2}g_1^{*}) + 2rg_5^{*} = 0, \tag{42}$$

$$\frac{d}{dr}(rN^{*}g_5^{*}) + 3g_1^{*} = 0, \tag{43}$$

$$N^{*}\frac{d}{dr}(r^{2}g_3^{*}) + 2rg_6^{*} = 0, \tag{44}$$

$$\begin{aligned} \frac{d}{dr}(rN^{*}g_6^{*}) + 3g_1^{*} &= \frac{9\mu\omega^{*}r}{8\varrho^{3}M^{3}}\left[\frac{2\varrho M}{r}\left(1+\frac{M}{r}\right) \right. \\ &\quad \left. + \left(1-\frac{Q^{*}}{r^{2}}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right]. \end{aligned} \tag{45}$$

From these equations one can easily find interrelations between the unknown functions as [21]

$$g_1^{*} = f_1^{*}, \qquad g_3^{*} = f_3^{*}, \qquad g_5^{*} = \frac{1}{2}f_2^{*}, \qquad g_6^{*} = \frac{1}{2}f_4^{*},$$

and from Eq. (A3) one can directly obtain the expressions for the functions $g_2^{*}(r)$ and $g_4^{*}(r)$ as

$$\begin{aligned} g_2^{*}(r) &= \frac{3\mu\Omega r}{8\varrho^{3}M^{3}N^{*}}\left[\frac{2\varrho M}{r}\left(1+\frac{M}{r}\right) \right. \\ &\quad \left. + \left(1-\frac{Q^{*}}{r^{2}}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right], \end{aligned} \tag{46}$$

$$g_4^*(r) = -\frac{3\mu\omega^* r}{8\varrho^3 M^3 N^*}\left[\frac{2\varrho M}{r}\left(1+\frac{M}{r}\right) + \left(1-\frac{Q^*}{r^2}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right]. \quad (47)$$

After making algebraic calculations one can get the second order differential equation for the function $f_1^*(r)$:

$$\frac{d}{dr}\left[\left(1-\frac{2M}{r}+\frac{Q^*}{r^2}\right)\frac{d}{dr}(r^2 f_1^*)\right] - 6f_1^* = 0, \quad (48)$$

which has the following analytical solution

$$f_1^*(r) = \frac{\mu\Omega C^* C_1^*}{6c\varrho^5 R^2}\left[\varrho\left(\frac{2M^2}{3r^2}+\frac{2M}{r}+\frac{4Q^*}{3r^2}-4\right) + \left(3-\frac{2r}{M}-\frac{Q^*}{r^2}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right]. \quad (49)$$

One can immediately find a form of the function $f_2^*(r)$ as

$$f_2^*(r) = -\frac{\mu\Omega C^* C_1^*}{c\varrho^5 R^2}N^*\left[\left(1-\frac{r}{M}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right) - 2\varrho - \frac{2\varrho^3 M^2}{3r^2 N^{*2}}\right], \quad (50)$$

where a constant of integration C_1^* can be found from the boundary conditions.

It is necessary to solve one more equation for the functions $f_3^*(r)$ which comparing to the previous equation (48) has an additional term:

$$\frac{d}{dr}\left[\left(1-\frac{2M}{r}+\frac{Q^*}{r^2}\right)\frac{d}{dr}(r^2 f_3^*)\right] - 6f_3^* + \frac{9\mu J}{2\varrho^3 M^3 r^2}\left(1-\frac{Q^*}{2Mr}\right)\left[\frac{2\varrho M}{r}\left(1+\frac{M}{r}\right) + \left(1-\frac{Q^*}{r^2}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right] = 0. \quad (51)$$

Analytical solution of this equation has the following form

$$f_3^*(r) = \frac{15\mu J}{8c\varrho^5 M^5}\left\{C_3^*\left[\varrho\left(\frac{2M^2}{3r^2}+\frac{2M}{r}+\frac{4Q^*}{3r^2}-4\right) + \left(3-\frac{2r}{M}-\frac{Q^*}{r^2}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right] + f_3^{*p}(r)\right\}, \quad (52)$$

where $f_3^{*p}(r)$ is the particular solution of the Eq. (51) and its detailed form has shown in Appendix B. The function $f_4^*(r)$ can be easily found as

$$f_4^*(r) = -\frac{45\mu J}{4c\varrho^5 M^5}\left\{C_3^* N^*\left[\left(1-\frac{r}{M}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right) - 2\varrho - \frac{2\varrho^3 M^2}{3r^2 N^{*2}}\right] + \frac{N^*}{r}\frac{d}{dr}f_3^{*p}(r)\right\}, \quad (53)$$

where a constant of integration C_3^* can be found from the boundary conditions.

Finally collecting all equations, one can find the expression for the electric field produced by the magnetic dipole moment of the misaligned relativistic star in the braneworld

$$E^{\hat{r}}(r,\theta,\phi,t) = \left\{\frac{\mu\Omega C^* C_1^*}{6c\varrho^5 R^2}\left[\varrho\left(\frac{2M^2}{3r^2}+\frac{2M}{r}+\frac{4Q^*}{3r^2}-4\right) + \left(3-\frac{2r}{M}-\frac{Q^*}{r^2}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right] + \frac{15\mu\omega r^3}{16c\varrho^5 M^5}\left(C_3^*\left[\varrho\left(\frac{2M^2}{3r^2}+\frac{2M}{r}+\frac{4Q^*}{3r^2}-4\right) + \left(3-\frac{2r}{M}-\frac{Q^*}{r^2}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right] + f_3^{*p}(r)\right)\right\} \times [\cos\chi(3\cos^2\theta - 1) + 3\sin\chi\sin\theta\cos\theta\cos\lambda], \quad (54)$$

$$E^{\hat{\theta}}(r,\theta,\phi,t) = -\left\{\frac{\mu\Omega C^* C_1^*}{2c\varrho^5 R^2}N^*\left[\left(1-\frac{r}{M}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right) - 2\varrho - \frac{2\varrho^3 M^2}{3r^2 N^{*2}}\right] + \frac{45\mu\omega r^3}{16c\varrho^5 M^5}\left(C_3^* N^*\left[\left(1-\frac{r}{M}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right) - 2\varrho - \frac{2\varrho^3 M^2}{3r^2 N^{*2}}\right] + \frac{N^*}{r}\frac{d}{dr}f_3^{*p}(r)\right)\right\} \times [\cos\chi\sin 2\theta - \sin\chi\cos 2\theta\cos\lambda] + \frac{3\mu\bar{\omega} r}{8c\varrho^3 M^3 N^*}\left[\frac{2\varrho M}{r}\left(1+\frac{M}{r}\right) + \left(1-\frac{Q^*}{r^2}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right]\sin\chi\cos\lambda, \quad (55)$$

$$E^{\hat{\phi}}(r,\theta,\phi,t) = -\left\{\frac{\mu\Omega C^* C_1^*}{2c\varrho^5 R^2} N^* \left[\left(1-\frac{r}{M}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right) - 2\varrho - \frac{2\varrho^3 M^2}{3r^2 N^{*2}}\right]\right.$$
$$\left. + \frac{45\mu\omega r^3}{16c\varrho^5 M^5}\left(C_3^* N^*\left[\left(1-\frac{r}{M}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right) - 2\varrho - \frac{2\varrho^3 M^2}{3r^2 N^{*2}}\right] + \frac{N^*}{r}\frac{d}{dr} f_3^{*p}(r)\right)\right\} \sin\chi\cos\theta\sin\lambda$$
$$+ \frac{3\mu\bar{\omega} r}{8c\varrho^3 M^3 N^*}\left[\frac{2\varrho M}{r}\left(1+\frac{M}{r}\right) + \left(1-\frac{Q^*}{r^2}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right]\sin\chi\cos\theta\sin\lambda. \quad (56)$$

Under the assumption of the infinite perfect electric conductivity $\sigma \to \infty$ of the stellar medium in the region $R_{\rm IN} < r < R$, and by using the Ohm law the interior solution of the electric field has the form [21]

$$E^{\hat{r}} = \frac{\bar{\omega} r \sin\theta}{c e^{\Phi^*(r)}} B^{\hat{\theta}}, \quad (57)$$

$$E^{\hat{\theta}} = -\frac{\bar{\omega} r \sin\theta}{c e^{\Phi^*(r)}} B^{\hat{r}}, \quad (58)$$

$$E^{\hat{\phi}} = 0, \quad (59)$$

where $\bar{\omega} = \Omega - \omega^*$ is the angular velocity of the fluid as measured from the local free-falling frame.

The integration constants C_1^* and C_3^* can now be found from the following boundary conditions $[E^{\hat{\theta}}] = E^{\hat{\theta}}_{\rm ext}|_{r=R} - E^{\hat{\theta}}_{\rm int}|_{r=R} = 0$ and $[E^{\hat{\phi}}] = E^{\hat{\phi}}_{\rm ext}|_{r=R} - E^{\hat{\phi}}_{\rm int}|_{r=R} = 0$, that after imposing the continuity of the tangential electric field across the star surface [21]

$$C_1^* = \frac{\varrho^5}{N_R^{*2}}\left[\left(1-\frac{R}{M}\right)\ln\left(\frac{R-M(1+\varrho)}{R-M(1-\varrho)}\right) - 2\varrho - \frac{2\varrho^3 M^2}{3R^2 N_R^{*2}}\right]^{-1}, \quad (60)$$

$$C_3^* = -C_1^*\left[\frac{8M^5}{45R^5} C^* + \frac{N_R^{2*} R}{\varrho^5}\frac{d}{dr} f_3^{*p}(r)|_{r=R}\right]. \quad (61)$$

III. RESULTS AND DISCUSSIONS

A. Normalization of the physical quantities

Now we will analyze the analytical expressions for both magnetic and electric fields of rotating relativistic magnetized star in the braneworld. The expressions for the magnetic field (28)–(30) can be rewritten in terms of new normalized dimensionless radial variable $\eta = r/R$ and compactness of the star $\epsilon = 2M/R$ as

$$B^{\hat{r}} = -\frac{3B_0}{\varrho^3\epsilon^3}\left[\frac{\varrho\epsilon}{\eta}\left(1+\frac{\epsilon}{2\eta}\right) + \left(1-\frac{\epsilon^2(1-\varrho^2)}{4\eta^2}\right)\ln\left(\frac{2\eta-\epsilon(1+\varrho)}{2\eta-\epsilon(1-\varrho)}\right)\right][\cos\chi\cos\theta + \sin\chi\sin\theta\cos\lambda], \quad (62)$$

$$B^{\hat{\theta}} = \frac{3B_0}{\varrho^3\epsilon^3}\left[\frac{\varrho\epsilon}{\eta N_\eta^*}\left(1-\frac{\epsilon}{2\eta}\right) + N_\eta^*\ln\left(\frac{2\eta-\epsilon(1+\varrho)}{2\eta-\epsilon(1-\varrho)}\right)\right][\cos\chi\sin\theta - \sin\chi\cos\theta\cos\lambda], \quad (63)$$

$$B^{\hat{\phi}} = \frac{3B_0}{\varrho^3\epsilon^3}\left[\frac{\varrho\epsilon}{\eta N_\eta^*}\left(1-\frac{\epsilon}{2\eta}\right) + N_\eta^*\ln\left(\frac{2\eta-\epsilon(1+\varrho)}{2\eta-\epsilon(1-\varrho)}\right)\right]\sin\chi\sin\lambda. \quad (64)$$

Similarly the components of the electric field given by Eqs. (54)–(56) take the form

$$E^{\hat{r}} = \frac{E_0}{6\varrho^5}\left\{\kappa\left[\varrho\left(\frac{\epsilon}{\eta} + \frac{\epsilon^2(3-2\varrho^2)}{6\eta^2} - 4\right) + \left(3 - \frac{4\eta}{\epsilon} - \frac{\epsilon^2(1-\varrho^2)}{4\eta^2}\right)\ln\left(\frac{2\eta-\epsilon(1+\varrho)}{2\eta-\epsilon(1-\varrho)}\right)\right] + \frac{72}{\epsilon^4} f_3^{*p}(\eta)\right\}$$
$$\times[\cos\chi(3\cos^2\theta - 1) + 3\sin\chi\sin\theta\cos\theta\cos\lambda], \quad (65)$$

$$E^{\hat{\theta}} = -\frac{E_0}{2\varrho^5}\left\{\kappa N_\eta^*\left[\left(1-\frac{2\eta}{\epsilon}\right)\ln\left(\frac{2\eta-\epsilon(1+\varrho)}{2\eta-\epsilon(1-\varrho)}\right) - 2\varrho - \frac{\varrho^3\epsilon^2}{6\eta^2 N_\eta^{*2}}\right] + \frac{72}{\epsilon^4}\frac{N_\eta^*}{\eta}\frac{d}{d\eta} f_3^{*p}(\eta)\right\}$$
$$\times[\cos\chi\sin 2\theta - \sin\chi\cos 2\theta\cos\lambda]$$
$$+ \frac{3E_0\eta}{\varrho^3\epsilon^3 N_\eta^*}\left(1-\frac{2\epsilon}{5\eta^3}\right)\left[\frac{\varrho\epsilon}{\eta}\left(1+\frac{\epsilon}{2\eta}\right) + \left(1-\frac{\epsilon^2(1-\varrho^2)}{4\eta^2}\right)\ln\left(\frac{2\eta-\epsilon(1+\varrho)}{2\eta-\epsilon(1-\varrho)}\right)\right]\sin\chi\cos\lambda, \quad (66)$$

$$E^{\hat{\phi}} = -\frac{E_0}{2\varrho^5}\left\{\kappa N^*_\eta\left[\left(1-\frac{2\eta}{\epsilon}\right)\ln\left(\frac{2\eta-\epsilon(1+\varrho)}{2\eta-\epsilon(1-\varrho)}\right)-2\varrho-\frac{\varrho^3\epsilon^2}{6\eta^2N^{*2}_\eta}\right]+\frac{72}{\epsilon^4}\frac{N^*_\eta}{\eta}\frac{d}{d\eta}f^{*p}_3(\eta)\right\}\sin\chi\cos\theta\sin\lambda$$
$$+\frac{3E_0\eta}{\varrho^3\epsilon^3N^*_\eta}\left(1-\frac{2\epsilon}{5\eta^3}\right)\left[\frac{\varrho\epsilon}{\eta}\left(1+\frac{\epsilon}{2\eta}\right)+\left(1-\frac{\epsilon^2(1-\varrho^2)}{4\eta^2}\right)\ln\left(\frac{2\eta-\epsilon(1+\varrho)}{2\eta-\epsilon(1-\varrho)}\right)\right]\sin\chi\cos\theta\sin\lambda, \quad (67)$$

where $B_0 = 2\mu/R^3$ and $E_0 = B_0(\Omega R/c)$ are the Newtonian values of the magnetic and the electric field at the polar cap of the star.

We introduce the new constant $\kappa = C^*C^*_1 + 72C^*_3/\epsilon^4$ and the variable

$$N^{*2}_\eta = 1-\frac{\epsilon}{\eta}+\frac{\epsilon^2(1-\varrho^2)}{4\eta^2}.$$

Assuming that an angle between the magnetic and rotation axes is zero $\chi = 0$, the components of the magnetic fields take the following form

$$B^{\hat{r}} = -\frac{3B_0\cos\theta}{\varrho^3\epsilon^3}\left[\frac{\varrho\epsilon}{\eta}\left(1+\frac{\epsilon}{2\eta}\right)+\left(1-\frac{\epsilon^2(1-\varrho^2)}{4\eta^2}\right)\ln\left(\frac{2\eta-\epsilon(1+\varrho)}{2\eta-\epsilon(1-\varrho)}\right)\right], \quad (68)$$

$$B^{\hat{\theta}} = \frac{3B_0\sin\theta}{\varrho^3\epsilon^3}\left[\frac{\varrho\epsilon}{\eta N^*_\eta}\left(1-\frac{\epsilon}{2\eta}\right)+N^*_\eta\ln\left(\frac{2\eta-\epsilon(1+\varrho)}{2\eta-\epsilon(1-\varrho)}\right)\right], \quad (69)$$

$$B^{\hat{\phi}} = 0. \quad (70)$$

And in the case of the condition $\omega \ll \Omega$, one can rewrite the electric field as

$$E^{\hat{r}} = E_0\frac{C^*C^*_1}{6\varrho^5}\left[\varrho\left(\frac{\epsilon}{\eta}+\frac{\epsilon^2(3-2\varrho^2)}{6\eta^2}-4\right)+\left(3-\frac{4\eta}{\epsilon}-\frac{\epsilon^2(1-\varrho^2)}{4\eta^2}\right)\ln\left(\frac{2\eta-\epsilon(1+\varrho)}{2\eta-\epsilon(1-\varrho)}\right)\right](3\cos^2\theta-1), \quad (71)$$

$$E^{\hat{\theta}} = E_0\frac{C^*C^*_1}{\varrho^5}N^*_\eta\left[2\varrho+\frac{\varrho^3\epsilon^2}{6\eta^2N^{*2}_\eta}-\left(1-\frac{2\eta}{\epsilon}\right)\ln\left(\frac{2\eta-\epsilon(1+\varrho)}{2\eta-\epsilon(1-\varrho)}\right)\right]\sin\theta\cos\theta, \quad (72)$$

$$E^{\hat{\phi}} = 0. \quad (73)$$

Figures 1 and 2 draw the radial ($\eta = r/R$) dependence of the normalized radial and tangential magnetic field at the magnetic pole and in the equatorial plane of the magnetized star at the different sets of brane parameter. In both cases one can see that effects of the brane larger near the surface of the star as compared to a distance far from the star.

B. The dipolar electromagnetic radiation from the relativistic star in the braneworld

In this subsection, we will focus on the electromagnetic dipole radiation from the radio pulsar which is the

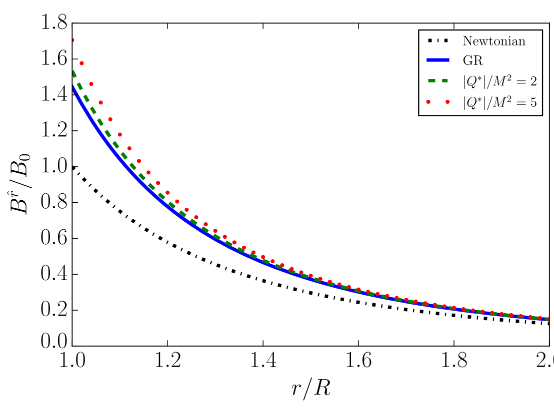

FIG. 1. The radial $\eta = r/R$ dependence of the normalized radial component of the magnetic field for several values of the brane parameter Q^*/M^2 when $\epsilon = 0.4$, $\chi = 0$ and $\theta = 0$.

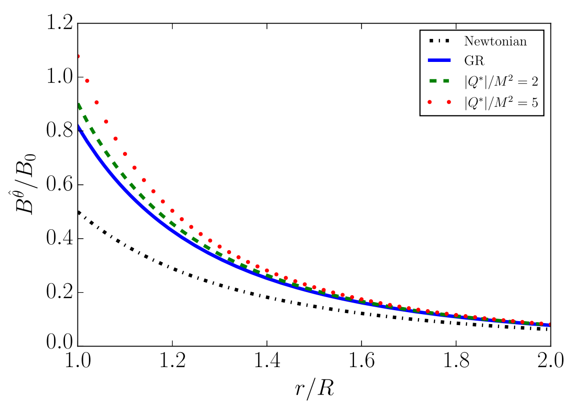

FIG. 2. The radial $\eta = r/R$ dependence of the normalized tangential component of the magnetic field for several values of the brane parameter Q^*/M^2 when $\epsilon = 0.4$, $\chi = 0$ and $\theta = \pi/2$.

observational evidence for the rotating magnetized (neutron) star. The luminosity of the magnetized star in the case of a purely electromagnetic dipolar radiation in the braneworld can be calculated as [22]

$$L^*_{\rm em} = \frac{\Omega^{*4}_{\rm R} R^6}{6c^3} B^{*2}_{\rm R} \sin^2\chi, \qquad (74)$$

where $\Omega^*_{\rm R} = \Omega/N^*_{\rm R}$ is angular velocity in observer frame and $B^*_{\rm R}$ is the value of the magnetic field strength at the surface of the star:

$$B^*_{\rm R} = f^*(\epsilon, Q^*) B_0, \qquad (75)$$

with

$$f^*(\epsilon,Q^*) = -\frac{3}{\varrho^3\epsilon^3}\left[\varrho\epsilon\left(1+\frac{\epsilon}{2}\right) + \left(1-\frac{\epsilon^2(1-\varrho^2)}{4}\right)\ln\left(\frac{2-\epsilon(1+\varrho)}{2-\epsilon(1-\varrho)}\right)\right], \qquad (76)$$

where subscript R indicates that the value of the quantity at $r = R$. From Eq. (74), one can easily see that the luminosity of the rotating magnetized neutron star in the braneworld is increased due to the amplification of the magnetic field and by the gravitational redshift of the effective rotational angular velocity $\Omega^*_{\rm R}$.

The Newtonian value of the luminosity in the case of pure dipole electromagnetic radiation has the following form [37]

$$L_{0\rm em} = \frac{\Omega^4 R^6}{6c^3} B_0^2 \sin^2\chi. \qquad (77)$$

In the presence of brane parameter the rate of the energy loss from the radio pulsar through dipolar electromagnetic radiation is [22]

$$\frac{L^*_{\rm em}}{L_{0\rm em}} = \left(\frac{\Omega^*_R}{\Omega}\right)^4 \left(\frac{B^*_R}{B_0}\right)^2 = \left(\frac{f^*}{N^{*2}_R}\right)^2. \qquad (78)$$

The dependence of the rate of the energy loss from the compactness of the magnetized neutron star in the braneworld for several values of the brane parameter is given in Fig. 3. The plots show the increase the rate of the energy loss with the increase of the compactness of the star.

In Fig. 4 it is shown the dependence of the rate of the energy loss of the magnetized neutron star in the braneworld from the module of the dimensionless brane parameter Q^*/M^2 for several values of the compactness of the star.

Assuming that the rotational energy of the star is converted into electromagnetic radiation one can relate the electromagnetic energy loss $L_{\rm em}$ with the loss of rotational kinetic energy as [22]

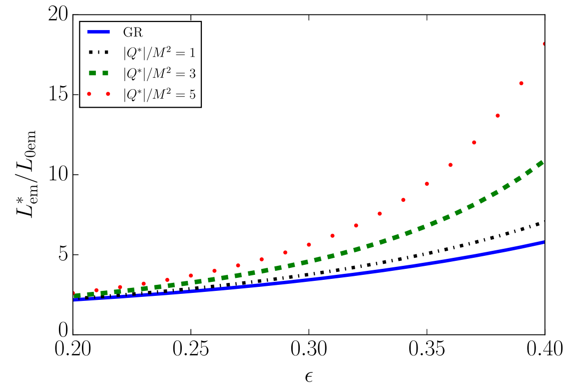

FIG. 3. The dependence of the energy losses $L^*_{\rm em}/L_{0\rm em}$ from the compactness ϵ of the star for several value of the brane parameter Q^*/M^2.

$$\dot{E}^*_{\rm rot} = \frac{d}{dt}\left(\frac{1}{2}\int d^3r\sqrt{\gamma^*}e^{-\Phi^*(r)}\rho^*(r)(u^i)^2\right) = -L^*_{\rm em}, \qquad (79)$$

where factor γ^* is defined as

$$\gamma^* = [-g_{00} + g_{ij}u^i u^j]^{-1/2} \sim e^{-\Phi^*},$$

with the three velocity of conducting medium inside the star $u^i = dx^i/dt$, see more details in [22,38].

Finding and measuring the moment of inertia of the neutron stars is one of the difficult problems in astrophysics. However under the assumption that the pressure of the matter inside the star is very small compared to its density $p \ll \rho$ one can find [22]

$$I^* = \int d^3r\sqrt{\gamma^*}e^{-\Phi^*(r)}\rho^*(r)r^2\sin^2\theta, \qquad (80)$$

in the Newtonian limit moment of inertia can be written as $I_0 = (2/5)MR^2$.

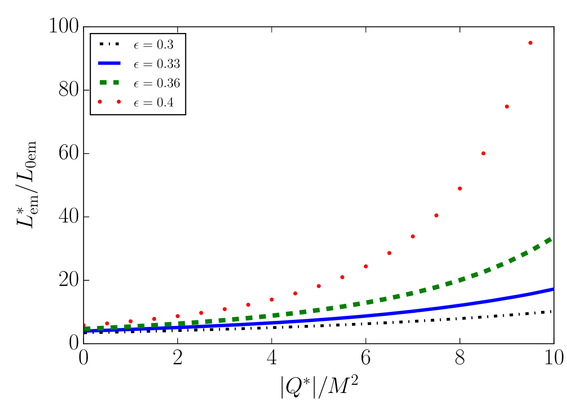

FIG. 4. The dependence of the energy losses $L^*_{\rm em}/L_{0\rm em}$ from the brane parameter Q^*/M^2 for several value of the compactness ϵ of the star.

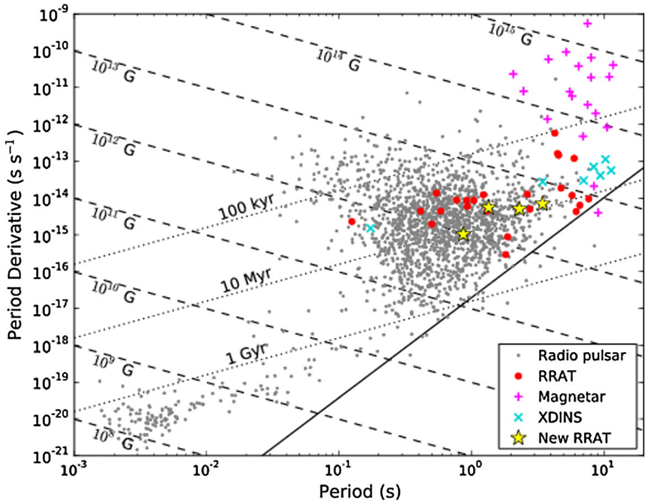

FIG. 5. $P - \dot{P}$ diagram for the observable pulsars and magnetars from the paper [45].

The magnetic field of the pulsar can also be written in terms of the most measurable quantities in the pulsar observation as period P and its time derivative $\dot{P} = dP/dt$ [22]:

$$P^* \dot{P}^* = \frac{2\pi^2}{3c^3} \frac{1}{N_R^4} \frac{B_R^* R^6}{I^*} = \left(\frac{f^{*2}}{N_R^{*4}} \frac{I_0}{I^*} \right) P_0 \dot{P}_0, \tag{81}$$

$$P_0 \dot{P}_0 = \frac{2\pi^2}{3c^3} \frac{B_0 R^6}{I_0}. \tag{82}$$

Finally one can get constraints for the brane parameter Q^* comparing the theoretical results with the observational data for the known rotating magnetized stars observed as pulsars and magnetars. In order to get the upper limit for the brane parameter Q^*, one can consider $P - \dot{P}$ diagram for the typical pulsars [39–44]. From observation data [45] which is shown in the Fig. 5, one can see that average value

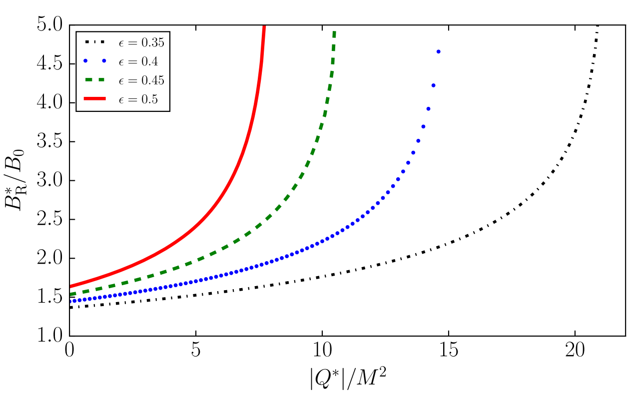

FIG. 6. The dependence of the magnetic field at the surface of the NS from the brane parameter for the different values of the compactness of the star.

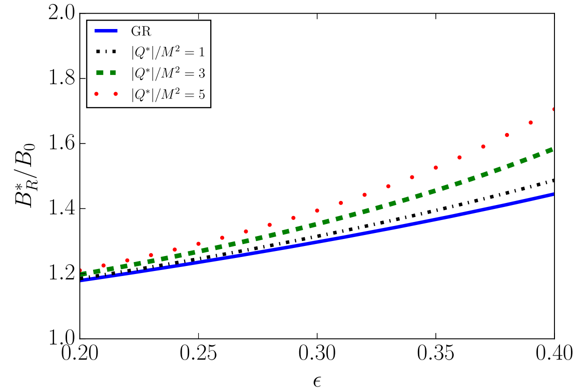

FIG. 7. The dependence of the magnetic field at the surface of the NS from the compactness of the star for the different values of the brane parameter.

of the magnetic field strength for the typical radio pulsar is about $B_{\rm av} \simeq 10^{12}$ G, and its period derivative is about $\simeq 10^{-15}$ s s^{-1}. Using these values and the magnetodipolar formula one can find the upper limit for the value of the brane parameter as $|Q^*| \lesssim 3 \times 10^{11}$ cm^2 $(|Q^*|/M^2 \simeq 8)$ which is in good agreement with the constraint obtained from the observations of the inner part of the accretion disks [28]. This statement is in agreement with the Figs. 6 and 7 on the dependence of the magnetic field at the surface of the NS from the brane parameter for the different values of the compactness of the star.

IV. SUMMARY

In the present work, we have studied modifications of the electromagnetic fields of a rotating magnetized neutron stars in the braneworld and their astrophysical applications. We have formulated the Maxwell equations for the slowly rotating magnetized compact star with nonzero brane tension and dipolar magnetic field configuration. As the analytical solutions are always more valuable for further applications we have derived exact analytical solution for the dipolar magnetic field in terms of the brane tension parameter Q^* and mass M which gives an opportunity to check the effects of the brane in the plasma magnetosphere of the rotating neutron stars, especially, when one calculates the Goldreich-Julian density which is very important quantity in the pulsar astrophysics.

Then we have solved the Maxwell equations for the electric field of the rotating magnetized star in the braneworld. The electric field strongly depends on the brane tension and becomes stronger with the increase of the brane tension.

As an important application of the obtained results we have calculated energy losses of slowly rotating magnetized neutron star in the braneworld through magnetodipolar radiation and found that the rotating star with nonzero brane tension will lose more energy compared to the typical rotating neutron star in general relativity. The obtained dependence has been combined with the astrophysical data on pulsar period slowdown in order to get constraints on the brane parameter. We have found the upper limit for the brane parameter as $|Q^*| \lesssim 3\times 10^{11}$ cm^2.

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APPENDIX A: MAXWELL EQUATIONS

The four-velocity in the ZAMO (zero angular momentum observer) frame can be written in the following form

$$u^\mu = \frac{1}{N^*}(1,0,0,\omega^*), \qquad u_\mu = N^*(-1,0,0,0). \tag{A1}$$

According to [21] the explicit form of the first pair of the general relativistic Maxwell equations in the spacetime (6) for the stationary electromagnetic fields in the orthonormal reference frame can be expressed as

$$N^*\sin\theta(r^2B^{\hat r})_{,r} + r(\sin\theta B^{\hat\theta})_{,\theta} + rB^{\hat\phi}_{,\phi} = 0, \tag{A2}$$

$$N^*[E^{\hat\theta}_{,\phi} - (\sin\theta E^{\hat\phi})_{,\theta}] - \omega^* r\sin\theta B^{\hat r}_{,\phi} = 0, \tag{A3}$$

$$N^*[\sin\theta(rN^*E^{\hat\phi})_{,r} - E^{\hat r}_{,\phi}] - \omega^* r\sin\theta B^{\hat\theta}_{,\phi} = 0, \tag{A4}$$

$$N^*E^{\hat r}_{,\theta} - N^*(rN^*E^{\hat\theta})_{,r} + N^*\sin\theta(\omega^* r^2 B^{\hat r})_{,r} + \omega^* r(\sin\theta B^{\hat\theta})_{,\theta} = 0, \tag{A5}$$

and the second pair is given by [21]

$$N^*\sin\theta(r^2E^{\hat r})_{,r} + r(\sin\theta E^{\hat\theta})_{,\theta} + rE^{\hat\phi}_{,\phi} = 4\pi r^2\sin\theta J^{\hat t}, \tag{A6}$$

$$N^*(\sin\theta B^{\hat\phi})_{,\theta} - N^*B^{\hat\theta}_{,\phi} - \omega^* r\sin\theta E^{\hat r}_{,\phi} = 4\pi N^* r\sin\theta J^{\hat r}, \tag{A7}$$

$$N^*B^{\hat r}_{,\phi} - N^*\sin\theta(rN^*B^{\hat\phi})_{,r} - \omega^* r\sin\theta E^{\hat\theta}_{,\phi} = 4\pi N^* r\sin\theta J^{\hat\theta}, \tag{A8}$$

$$N^*(rN^*B^{\hat\theta})_{,r} - N^*B^{\hat r}_{,\theta} + N^*\sin\theta(\omega^* r^2 E^{\hat r})_{,r} + \omega^* r(\sin\theta E^{\hat\theta})_{,\theta} = 4\pi N^* rJ^{\hat\phi} + 4\pi\omega^* r^2\sin\theta J^{\hat t}, \tag{A9}$$

where the four-current $J^{\hat\mu}$ is a sum of convection and conduction currents. According to Ohm's law [46,47], its components have the following form in the ZAMO frame [21]:

$$J^{\hat t} = \rho_e + \sigma\frac{\bar\omega r\sin\theta}{e^{\Phi^*}}E^{\hat\phi}, \tag{A10}$$

$$J^{\hat r} = \sigma\left(E^{\hat r} - \frac{\bar\omega r\sin\theta}{e^{\Phi^*}}B^{\hat\theta}\right), \tag{A11}$$

$$J^{\hat\theta} = \sigma\left(E^{\hat\theta} + \frac{\bar\omega r\sin\theta}{e^{\Phi^*}}B^{\hat r}\right), \tag{A12}$$

$$J^{\hat\phi} = \sigma E^{\hat\phi} + \frac{\bar\omega r\sin\theta}{e^{\Phi^*}}\rho_e. \tag{A13}$$

Here σ is the electrical conductivity and ρ_e is the proper charge density of the medium that is located in the region $R_{\rm IN} < r < R$ of the neutron star. Outside the star $\sigma = \rho_e = 0$.

APPENDIX B: PARTICULAR SOLUTION FOR THE FUNCTION $f_3^*(r)$

The differential equation for the unknown function $f_3^*(r)$ is given as

$$\frac{d}{dr}\left[\left(1-\frac{2M}{r}+\frac{Q^*}{r^2}\right)\frac{d}{dr}(r^2f_3^*)\right] - 6f_3^* + \frac{9\mu J}{2\varrho^3M^3r^2}\left(1-\frac{Q^*}{2Mr}\right)\left[\frac{2\varrho M}{r}\left(1+\frac{M}{r}\right)+\left(1-\frac{Q^*}{r^2}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right] = 0. \tag{B1}$$

The solution of this equation is

$$f_3^*(r) = \frac{15\mu J}{8c\varrho^5M^5}\left\{C_3^*\left[\varrho\left(\frac{2M^2}{3r^2}+\frac{2M}{r}+\frac{4Q^*}{3r^2}-4\right)+\left(3-\frac{2r}{M}-\frac{Q^*}{r^2}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right]+f_3^{*p}(r)\right\}. \tag{B2}$$

The explicit form of the particular solution of this equation is given by

$$
\begin{aligned}
f_3^{*p}(r) = {} & \frac{3\varrho^2 M}{10r^3}\left[2\varrho M^2 - Q^* \ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right] + \frac{1}{30\varrho M r Q^*}(18M^4 + 122M^2 Q^* + Q^{*2}) \\
& + \frac{3Q^*}{10\varrho^2 M r}\left[Li_2\left(\frac{r}{M(1+\varrho)}\right) - Li_2\left(\frac{r}{M(1-\varrho)}\right) + \ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\ln r\right] - \frac{6(M^2+Q^*)}{5\varrho M r}\ln r \\
& + \frac{Q^*}{40 M r \varrho^2 (1+\varrho)^2}[\varrho(3\varrho(8\varrho-1)-122)-91]\log\left(\frac{r}{M(1+\varrho)}\right) \\
& + \frac{Q^*}{40 M r \varrho^2 (1-\varrho)^2}[\varrho(3\varrho(8\varrho+1)-122)-91]\ln\left(\frac{r}{M(1-\varrho)}\right) \\
& + \frac{M}{10 r \varrho^2 (1+\varrho)}[6+\varrho(1-\varrho)(3-\varrho)-1+\varrho)\ln r]\ln[r-M(1+\varrho)] \\
& - \frac{M}{10 r \varrho^2 (1-\varrho)}[6-\varrho(1+\varrho)(3+\varrho)-1-\varrho)\ln r]\ln[r-M(1-\varrho)]. \qquad \text{(B3)}
\end{aligned}
$$

APPENDIX C: GENERAL RELATIVISTIC LIMIT VALUES FOR THE PROFILE FUNCTIONS

In general relativistic limit the functions $F^*(r)$ and $G^*(r)$ related to the magnetic field take the form (see, e.g., [21])

$$F(r) = \lim_{Q^*\to 0} F^*(r) = \lim_{\lambda^*\to\infty} F^*(r) = -\frac{3\mu}{4M^3}\left[\frac{2M}{r}\left(1+\frac{M}{r}\right) + \ln N^2\right], \qquad \text{(C1)}$$

$$G(r) = \lim_{Q^*\to 0} G^*(r) = \lim_{\lambda^*\to\infty} G^*(r) = \frac{3\mu N}{4rM^2}\left[1+\frac{1}{N^2}+\frac{r}{M}\ln N^2\right], \qquad \text{(C2)}$$

and other functions $f_i^*(r)$ related to the electric field take the form (see [21])

$$f_1(r) = \lim_{Q^*\to 0} f_1^*(r) = \lim_{\lambda^*\to\infty} f_1^*(r) = \frac{\mu\Omega C C_1}{6cR^2}\left[\frac{2M^2}{3r^2}+\frac{2M}{r}-4+\left(3-\frac{2r}{M}\right)\ln N^2\right], \qquad \text{(C3)}$$

$$f_2(r) = \lim_{Q^*\to 0} f_2^*(r) = \lim_{\lambda^*\to\infty} f_2^*(r) = -\frac{\mu\Omega C C_1}{cR^2}N\left[\left(1-\frac{r}{M}\right)\ln N^2 - 2 - \frac{2M^2}{3r^2N^2}\right], \qquad \text{(C4)}$$

$$f_3(r) = \lim_{Q^*\to 0} f_3^*(r) = \lim_{\lambda^*\to\infty} f_3^*(r) = \frac{15\mu\omega r^3}{8cM^5}\left\{\frac{2M^2}{5r^2}\ln N^2 + \frac{4M^3}{5r^3} + C_3\left[\frac{2M^2}{3r^2}+\frac{2M}{r}-4+\left(3-\frac{2r}{M}\right)\ln N^2\right]\right\}, \qquad \text{(C5)}$$

$$f_4(r) = \lim_{Q^*\to 0} f_4^*(r) = \lim_{\lambda^*\to\infty} f_4^*(r) = -\frac{45\mu\omega r^3}{4cM^5}\left\{\frac{4M^4}{15r^4N^2} + C_3 N\left[\left(1-\frac{r}{M}\right)\ln N^2 - 2 - \frac{2M^2}{3r^2N^2}\right]\right\}, \qquad \text{(C6)}$$

$$g_2(r) = \lim_{Q^*\to 0} g_2^*(r) = \lim_{\lambda^*\to\infty} g_2^*(r) = \frac{3\mu\Omega r}{8M^3 N}\left[\frac{2M}{r}\left(1+\frac{M}{r}\right)+\ln N^2\right], \qquad \text{(C7)}$$

$$g_4(r) = \lim_{Q^*\to 0} g_4^*(r) = \lim_{\lambda^*\to\infty} g_4^*(r) = -\frac{3\mu\omega r}{8M^3 N}\left[\frac{2M}{r}\left(1+\frac{M}{r}\right)+\ln N^2\right], \qquad \text{(C8)}$$

where the constants are

$$C = -\frac{3R^3}{4M^3}\left[\frac{2M}{R}\left(1+\frac{M}{R}\right)+\ln N_R^2\right], \qquad \text{(C9)}$$

$$C_1 = \frac{1}{N_R^2}\left[\left(1-\frac{R}{M}\right)\ln N_R^2 - 2 - \frac{2M^2}{3R^2N_R^2}\right]^{-1}, \quad \text{(C10)}$$

$$C_3 = \frac{2M^2}{15R^2}\left(\ln N_R^2 + \frac{2M}{R}\right)C_1. \quad \text{(C11)}$$

with $N = 1\text{–}2M/r$ and $N_R = 1\text{–}2M/R$.

APPENDIX D: THE NORMALIZED MAGNETIC FIELDS

The components of the normalized magnetic field are

$$B^{\hat{r}} = B_0\frac{f^*(\eta,\varrho)}{f^*(1,1)}\frac{1}{\eta^3}[\cos\chi\cos\theta + \sin\chi\sin\theta\cos\phi], \quad \text{(D1)}$$

$$B^{\hat{\theta}} = B_0\frac{g^*(\eta,\varrho)}{f^*(1,1)}\frac{1}{\eta^3}[\cos\chi\sin\theta - \sin\chi\cos\theta\cos\phi], \quad \text{(D2)}$$

$$B^{\hat{\phi}} = B_0\frac{g^*(\eta,\varrho)}{f^*(1,1)}\frac{1}{\eta^3}\sin\chi\sin\phi, \quad \text{(D3)}$$

where

$$f^*(\eta,\varrho) = -3\left(\frac{\eta}{\varrho\epsilon}\right)^3\left[\frac{\varrho\epsilon}{\eta}\left(1+\frac{\epsilon}{2\eta}\right) + \left(1-\frac{\epsilon^2(1-\varrho^2)}{4\eta^2}\right)\ln\left(\frac{2\eta-\epsilon(1+\varrho)}{2\eta-\epsilon(1-\varrho)}\right)\right], \quad \text{(D4)}$$

and

$$g^*(\eta,\varrho) = \left[\frac{3}{2N_\eta^*} - N_\eta^* f^*(\eta,\varrho)\right]\left(1-\frac{\epsilon^2(1-\varrho^2)}{4\eta^2}\right)^{-1}. \quad \text{(D5)}$$

APPENDIX E: THE NORMALIZED ELECTRIC FIELDS

Assuming that $\omega = 0$ one can rewrite the expressions for the electric field in much simple way

$$E^{\hat{r}} = E_0\frac{1}{\eta^3}\frac{p^*(\eta,\varrho)}{p^*(1,1)}[\cos\chi(3\cos^2\theta - 1) + 3\sin\chi\sin\theta\cos\theta\cos\lambda], \quad \text{(E1)}$$

$$E^{\hat{\theta}} = -E_0\frac{N_\eta^*}{\eta}\frac{d}{d\eta}\left(\frac{1}{\eta}\frac{p^*(\eta,\varrho)}{p^*(1,1)}\right) \times [\cos\chi\sin 2\theta - \sin\chi\cos 2\theta\cos\lambda] - E_0\frac{1}{\eta^2N_\eta^*}\frac{f^*(\eta,\varrho)}{f^*(1,1)}\sin\chi\cos\lambda, \quad \text{(E2)}$$

$$E^{\hat{\phi}} = -E_0\frac{N_\eta^*}{\eta}\frac{d}{d\eta}\left(\frac{1}{\eta}\frac{p^*(\eta,\varrho)}{p^*(1,1)}\right)\sin\chi\cos\theta\sin\lambda + E_0\frac{1}{\eta^2N_\eta^*}\frac{f^*(\eta,\varrho)}{f^*(1,1)}\sin\chi\cos\theta\sin\lambda, \quad \text{(E3)}$$

where

$$p^*(\eta,\varrho) = \frac{C^*C_1^*}{6\varrho^5}\left[\varrho\left(\frac{\epsilon^2(3-2\varrho^2)}{6\eta^2} + \frac{\epsilon}{\eta} - 4\right) + \left(3 - \frac{4\eta}{\epsilon} - \frac{\epsilon^2(1-\varrho^2)}{4\eta^2}\right)\ln\left(\frac{2\eta-\epsilon(1+\varrho)}{2\eta-\epsilon(1-\varrho)}\right)\right] \quad \text{(E4)}$$

and

$$N_1^{*2} = 1 - \epsilon + \frac{\epsilon^2(1-\varrho^2)}{4}.$$

We can easily find relations for the surface current density and the surface charge density for the star in the braneworld:

$$I^{*\hat{\theta}} = I^{\hat{\theta}}\frac{[B^{\hat{\phi}}]}{\{B^{\hat{\phi}}\}}, \qquad I^{*\hat{\phi}} = I^{\hat{\phi}}\frac{[B^{\hat{\theta}}]}{\{B^{\hat{\theta}}\}}, \quad \text{(E5)}$$

and

$$\sigma_s^* = \sigma_s\frac{[E^{\hat{r}}]}{\{E^{\hat{r}}\}}, \quad \text{(E6)}$$

where we have used the following shortage notation for the fields at the surface of the star $\{A\} = [A]|_{Q^*=0}$ in the Schwarzschild spacetime.

$$I^{*\hat{\phi}} = I^{\hat{\phi}}\frac{N_1}{2\varrho^2N_1^*}\left[1-\frac{\epsilon}{2}+\varrho^2\left(1+\frac{\epsilon}{2}\right) - \frac{1-\varrho^2}{\varrho\epsilon}N_1^{*2}\ln\left(\frac{2-\epsilon(1+\varrho)}{2-\epsilon(1-\varrho)}\right)\right] \quad \text{(E7)}$$

and

$$\sigma_s^* = \left\{1 - \frac{C}{6}\left[\frac{\epsilon^2}{6} + \epsilon - 4 + \left(3-\frac{4}{\epsilon}\right)\ln N_1^2\right]\right\}^{-1} \times \frac{C_1^*}{C_1}\left\{1 - \frac{bC^*}{6\varrho^5}\left[\varrho\left(\epsilon + \frac{\epsilon^2(3-2\varrho^2)}{6} - 4\right) + \left(3 - \frac{4}{\epsilon} - \frac{\epsilon^2(1-\varrho^2)}{4}\right)\ln\left(\frac{2-\epsilon(1+\varrho)}{2-\epsilon(1-\varrho)}\right)\right]\right\}\sigma_s, \quad \text{(E8)}$$

where $b = 1 + 2\cot^2\theta$.

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