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False vacuum decay catalyzed by black holes

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False vacuum states are metastable in quantum field theories, and true vacuum bubbles can be nucleated due to the quantum tunneling effect. It was recently suggested that an evaporating black hole (BH) can be a catalyst of bubble nucleations and dramatically shortens the lifetime of the false vacuum. In particular, in the context of the Standard Model valid up to a certain energy scale, even a single evaporating BH may spoil the successful cosmology by inducing the decay of our electroweak vacuum. In this paper, we reinterpret catalyzed vacuum decay by BHs, using an effective action for a thin-wall bubble around a BH to clarify the meaning of bounce solutions. We calculate bounce solutions in the limit of a flat spacetime and in the limit of negligible backreaction to the metric, where it is much easier to understand the physical meaning, and compare these results with the full calculations done in the literature. As a result, we give a physical interpretation of the enhancement factor: it is nothing but the probability of producing states with a finite energy. This makes it clear that all the other states such as plasma should also be generated through the same mechanism, and calls for finite density corrections to the tunneling rate, which tend to stabilize the false vacuum. We also clarify that the dominant process is always consistent with the periodicity indicated by the BH Hawking temperature after summing over all possible remnant BH masses or bubble energies, although the periodicity of each bounce solution as a function of a remnant BH can be completely different from the inverse temperature of the system, as mentioned in the previous literature.

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I. INTRODUCTION AND SUMMARY

The discovery of the Higgs boson has established the Standard Model (SM) [1,2]. For the current center value of the SM parameters, especially the measured top and Higgs masses [3–7], the Higgs potential develops a lower energy state than the electroweak vacuum at around the intermediate scale well below the Planck [8–20]. This fact implies that the quantum tunneling might lead to a disastrous decay of our vacuum [21,22]. Fortunately, it is known that, for the current favored value of SM parameters, its lifetime far exceeds the present age of the Universe, and thus our electroweak vacuum is believed to be metastable in the context of SM valid up to a very high energy scale.

Though this argument guarantees the safety of our vacuum in the present Universe, it does not mean that our metastable vacuum can survive throughout the history of the Universe. Thus, this scenario could in principle contradict various cosmological phenomena which can drive the vacuum decay. Many studies have been performed from this viewpoint. For instance, in the early Universe, it is believed that the Universe was filled with thermal plasma composed of SM particles. Since the Higgs interacts with SM particles including the Higgs itself, the thermal fluctuations might activate the decay of our vacuum [8,11,12,23], while these relativistic particles tend to stabilize the Higgs at the same time. It has been shown that this effect does not spoil our Universe for the best fit values of SM parameters [14–16]. If we further go back through the history of the Universe, we may encounter the phase of inflation and the subsequent (p)reheating. Since light fields with masses smaller than the Hubble parameter acquire fluctuations proportional to the Hawking temperature $H/2\pi$ during inflation [24–29], the Higgs might overcome the potential barrier via the Hawking-Moss instanton [30], which may be interpreted as the thermal hopping owing to the Hawking temperature. This observation puts a severe bound on the Hubble parameter of inflation: $H < \mathcal{O}(0.01)\Lambda_{\rm inst}$ with $\Lambda_{\rm inst}$ being the instability scale [14,31–44]. Although one might think that this constraint can be ameliorated by introducing a small coupling between the inflaton and the Higgs field, recent studies reveal that the (p)reheating stage after inflation could drive the catastrophic decay because the very interaction activates Higgs fluctuations due to the oscillating inflaton [45–50].

Recently, it was pointed out that a black hole (BH) can be a nucleation site just like a boiling stone in a superheated liquid system, and the vacuum transition rate can be dramatically enhanced (or the potential barrier becomes effectively smaller) around the BH [51–53] (see Refs. [54–59] for earlier work). The result of their calculation is independent of the periodicity of the Wick-rotated time coordinate, so that they insist that the result can be applied to an arbitrary low temperature system. In particular, the enhancement gets more significant for a smaller

BH, and in an extreme case, the Higgs can classically overcome the potential barrier, such as the thermal hopping and the Hawking-Moss transition. Applying this result to the Higgs field, they concluded that even a single small BH that evaporates within the current age of the Universe leads to the disaster of our vacuum [60].[1] And thus, there should not be such a small BH in our observable Universe. Although such a small BH may not be formed in the usual scenario of cosmology, their conclusion puts a stringent constraint on some cases, such as the formation of primordial BHs in the early Universe [63].[2]

In this paper, we reinterpret the earlier results derived via a Wick-rotated Euclidean field theory in Refs. [51–53] by invoking an effective action for a thin-wall bubble that can also describe the vacuum transition in scalar field theories [55,56,73]. We start with the bubble nucleation at a finite temperature in the flat spacetime, and recall that the final bubble nucleation rate can be factorized into the probability of producing states with a finite energy times a tunneling rate of a finite energy. This is also true if we have a BH. By extending the same procedure, we reformulate the bubble nucleation rate around a BH in the case where the back-reaction of the bubble on the spacetime can be neglected.[3]

We compare the bubble nucleation rate computed in this way with the full gravitational one in the limit of negligible backreaction. As a result, we clarify the meaning of the enhancement factor, that is, a probability of producing states with a finite energy E, which is a Boltzmann factor $e^{-E/T_{\rm BH}}$. It is hard to imagine that a BH only activates bubbles since quantum field theory has many other degrees of freedom to be excited. Hence, we expect all the states with a finite energy E should also be generated by the same mechanism. This argument clarifies the need of finite-density corrections to the bubble nucleation rate regardless of its origin, namely whether or not the Universe is filled by the plasma of the BH Hawking temperature, though the size of corrections depends on it.[4]

We also confirm that the periodicity of each bounce solution as a function of E is not necessarily related to the temperature of the system. However, after summing over all the possible transitions as a function of E, we find that the dominant process is always consistent with the periodicity indicated by the temperature of the system. This observation also holds if we have a BH. Although one still cannot determine the question raised in Ref. [63], whether or not the thermal plasma fills the whole Universe, by only looking at the periodicity of bounce solutions, our procedure indicates that the heart of the problem is free from a BH. The problem is whether or not a finite-volume heat reservoir can emit bubbles whose size is much larger than the size of the reservoir. We leave this issue as an open question.

The following is the summary of our results of this paper:

(a) In a flat spacetime with a finite-temperature plasma, we have shown $-\mathrm{d}B/\mathrm{d}E_* = 2\Delta\tau$ and $\mathrm{d}^2B/\mathrm{d}E_*^2 < 0$, where $-B$ is the exponent of the quantum tunneling rate, E_* is the energy of the bubble, and $2\Delta\tau$ is the periodicity of the bubble solution. Since the exponent of the Boltzmann factor satisfies $\mathrm{d}(-E_*/T_*)/\mathrm{d}E_* = -1/T_*$, the inequality $\mathrm{d}^2B/\mathrm{d}E_*^2 < 0$ implies that the dominant process is given either by $E_* = 0$ or $E_* = E_{\rm sp}$, where $E_{\rm sp}$ is the sphaleron energy.

(b) In the Schwarzschild–de Sitter spacetime, we eventually find $-\mathrm{d}B_{\rm bubble}/\mathrm{d}\Delta M = 2\Delta\tau$ and $-\mathrm{d}B_{\rm boundary}/\mathrm{d}\Delta M = -1/T_{\rm BH,-}$, where $T_{\rm BH,-}$ is the Hawking temperature associated with the remnant BH and $-B_{\rm boundary}$ and $-B_{\rm bubble}$ are the exponents of the quantum tunneling rate coming from the boundary of BH and the other contributions, respectively. The difference of the BH mass before and after the transition, ΔM, is equal to the bubble energy by the conservation of energy. We also show that $B_{\rm bubble}(\Delta M)$ coincides with $B(E_* = \Delta M)$ in the limit where the bubble radius is much larger than the BH radius once we identify the temperature as the Hawking temperature. In particular, $\mathrm{d}^2B_{\rm bubble}/\mathrm{d}\Delta M^2 < 0$ in that limit.

(c) In the fixed-background Schwarzschild–de Sitter spacetime with finite-temperature effects, we again obtain $-\mathrm{d}B/\mathrm{d}E_* = 2\Delta\tau$. The behavior of the second derivative is similar to the above full calculation. In the case that the effect of the change of the metric by the bubble is negligible, the nucleation rate coincides with the one derived by the above full calculation only if we identify the temperature of the system as the Hawking temperature of the BH. This observation clarifies that the enhancement factor is nothing but the probability of generating states with a finite energy, which is the Boltzmann factor with a BH Hawking temperature.

This paper is organized as follows. In Sec. II, we first review the calculation of the tunneling rate for a thin-wall bubble in a scalar field theory. We show that the transition rate is dominated either by a vacuum transition without an excited energy or by a sphaleron transition in this system. Next, we take into account gravity and consider the vacuum transition in the Schwarzschild–de Sitter spacetime in Sec. III. In particular, we calculate the bubble energy dependence of transition rate and show that its behavior is similar to the one in a finite-temperature system in a flat spacetime. We also use the effective action for the thin-wall bubble and show that the same nucleation rate in the

[1] See also Refs. [61,62] for related work.

[2] The bubble nucleation process is also important in the context of the multiverse, where bubbles continuously nucleate and observers may live in the baby universes [64–71]. The enhancement effect of the nucleation rate is applied in Ref. [72] to generate baby universes around BHs.

[3] This situation is practically important for realistic applications, for instance, to study the metastable Higgs vacuum [52].

[4] See also Sec. IV.

literature can be derived by the thermal activation of the BH Hawking temperature in a certain limit. Section IV is devoted to the conclusion and discussion, where we briefly explain the physics behind our result and discuss the possibility that the cost of such thermal plasma may significantly reduce the bubble nucleation rate.

II. TRANSITION WITHOUT GRAVITY

In this section, we review the calculation of the transition rate from a false vacuum to a true vacuum in quantum field theory without gravity, i.e., in the limit of $G \to 0$, where G $[\equiv 1/(8\pi M_{\rm Pl}^2)]$ is the Newton constant and $M_{\rm Pl}$ $(= 2.4 \times 10^{18}$ GeV) is the Planck scale. We take gravity into account in Sec. III.

A. Tunneling from a false vacuum

The action is given by

$$S[\phi] = \int \mathrm{d}^4x \left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \right], \qquad (2.1)$$

where $V(\phi)$ is a potential for the scalar field, which has a false vacuum at $\phi = \phi_{\rm FV}$ and the true vacuum at $\phi = \phi_{\rm TV}$.

The lifetime of the false vacuum can be calculated from the path integral as follows:

$$e^{-\Gamma t_0} = \frac{|\langle \phi_{\rm bubble}, t = t_0 | \phi_{\rm FV}, t = 0\rangle|^2}{|\langle \phi_{\rm FV}, t = t_0 | \phi_{\rm FV}, t = 0\rangle|^2} \qquad (2.2)$$

$$= \left| \int_{\phi(t=0)=\phi_{\rm FV}}^{\phi(t=t_0)=\phi_{\rm bubble}} \mathcal{D}\phi e^{iS[\phi]} \right|^2 \Big/ \left| \int_{\phi(t=0)=\phi_{\rm FV}}^{\phi(t=t_0)=\phi_{\rm FV}} \mathcal{D}\phi e^{iS[\phi]} \right|^2 \qquad (2.3)$$

$$= \int_{\rm bounce} \mathcal{D}\phi e^{iS[\phi]} \Big/ \int_{\phi(t\in[-t_0,t_0])=\phi_{\rm FV}} \mathcal{D}\phi e^{iS[\phi]} \qquad (2.4)$$

$$= \int_{\rm bounce} \mathcal{D}\phi e^{iS[\phi] - iS_{M,0}}, \qquad (2.5)$$

$$S_{\rm M,0} \equiv S[\phi(x) = \phi_{\rm FV}], \qquad (2.6)$$

with $t_0 \to \infty$, where the path integral $\int_{\rm bounce}$ is performed under the boundary conditions of $\phi(t = \pm t_0) = \phi_{\rm FV}$ and $\phi(t = 0) = \phi_{\rm bounce}$. The subscript "bubble" in ϕ means that it is a bubble configuration with a certain radius as we specify below. The denominator in the first line comes from the normalization of the initial and final states and gives the factor $e^{-iS_{\rm M,0}}$ in the last line, where the subscript "M" indicates this action is defined in Minkowski spacetime.

Now we take the imaginary time $\tau = it$ and rewrite Eq. (2.5) in terms of the Euclidean path integral. In the saddle point approximation in Euclidean theory, the path integral is approximated by $S_E[\phi_{\rm bounce}]$, which is calculated from a bounce solution of the classical Euclidean equation of motion. The action is minimized by a solution where ϕ bounces only once. In addition to the single bounce solution, there is an infinite number of solutions where ϕ bounces many times, which may be summed in the dilute gas approximation. Then we obtain

$$e^{-\Gamma t_0} \propto e^{-|K| t_0 \exp - (S_E[\phi_{\rm bounce}] - S_E[\phi = \phi_{\rm FV}])}, \qquad (2.7)$$

where $|K|$ is a prefactor that is not important for our discussion. Thus, we obtain

$$\Gamma \propto e^{-B}, \qquad (2.8)$$

$$B = S_{\rm bounce} - S_{\rm E,0}, \qquad (2.9)$$

where the Euclidean action is given by

$$S_E[\phi] = \int \mathrm{d}^4x \left[\frac{1}{2}\partial_\mu\phi\partial_\mu\phi + V(\phi) \right], \qquad (2.10)$$

and the normalization factor is given by

$$S_{\rm E,0} = S_E[\phi = \phi_{\rm FV}]. \qquad (2.11)$$

Let us emphasize that the action $S_{\rm bounce}$ is calculated from the bounce solution under the boundary condition of $\phi = \phi_{\rm FV}$ at $\tau = \pm\tau_0 (\to \pm\infty)$, and the tunneling process corresponds to a transition from $\phi = \phi_{\rm FV}$ to $\phi = \phi_{\rm bounce}$, i.e., a transition from the metastable ground state $\phi = \phi_{\rm FV}$ to the state at the other side of the potential barrier $\phi = \phi_{\rm bounce}$. Although we mention here the contribution from the perturbation $\delta\phi(\tau)$, it does not usually contribute to the exponential factor.[5]

The bounce solution $\phi_{\rm bounce}$ obeys the Euclidean equation of motion that is given by the variational principle of $S_E[\phi]$ in terms of ϕ:

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\tau^2} + \Delta\phi + U' = 0, \qquad (2.12)$$

$$U(\phi) = -V(\phi), \qquad (2.13)$$

where $\Delta = \partial_i^2$ is the Laplacian. In quantum field theory, the degrees of freedom is infinity because of the spacial dependence of the field. In many cases, however, we can use some symmetries to reduce the degrees of freedom to unity.

[5]If the system couples with light degrees of freedom, the prefactors could be significant [74–78]. In other words, we have to be careful what the "tree-level" action is in computing the bounce. Throughout this paper, we do not consider this issue further, and simply assume that we somehow know the tree-level action that is appropriate to compute the bounce.

The Euclidean action has an *O(4)* symmetry in quantum field theory, so let us first focus on *O(4)* symmetric solutions. In the thin-wall approximation, the scalar field configuration is approximated by the following *O(4)* symmetric configuration:

$$\phi(x) = \phi_{\rm thin}(\eta;\eta_*) \equiv \begin{cases} \phi_{\rm TV} & \text{for } \eta \ll \eta_*, \\ \phi_{\rm FV} & \text{for } \eta \gg \eta_*, \end{cases} \tag{2.14}$$

where η $(\equiv \sqrt{\tau^2 + x^2 + y^2 + z^2})$ is the radial coordinate of the Euclidean spacetime and η_* is the radius of the bubble. Note that this is an instanton solution where the time variable τ runs from $-\infty$ to ∞ and the configuration is nontrivial in a small interval $|\tau| \lesssim \eta_*$.

The number of degrees of freedom is reduced to be unity by the *O(4)* symmetry, so that we can consider a one-dimensional system with the variable η_*. Plugging the *O(4)* symmetric thin wall configuration back into the action, we can express the action as a function of the bubble radius η_*,

$$S_E = -\frac{1}{2}\pi^2\eta_*^4\epsilon + 2\pi^2\eta_*^3\sigma + S_{\rm E,0}, \tag{2.15}$$

where the first term comes from the contribution inside the bubble, while the second term comes from the surface of the bubble. We define the energy density difference as $\epsilon \equiv V(\phi_{\rm FV}) - V(\phi_{\rm TV})$ and the surface energy density of bubble σ as

$$\sigma \equiv \int_{\phi_{\rm TV}}^{\phi_{\rm FV}} {\rm d}\phi \sqrt{2[V(\phi) - V(\phi_{\rm FV})]}. \tag{2.16}$$

The variable η_* obeys a constraint that originates from the Euclidean equation of motion. The same equation can be derived from the variational principle of $S_E[\phi(\eta;\eta_*)]$ in terms of the variable η_*. Then we find the following results:

$$\eta_* = \eta_0 \equiv \frac{3\sigma}{\epsilon}, \tag{2.17}$$

$$S_E = \frac{27\pi^2}{2}\frac{\sigma^4}{\epsilon^3} + S_{\rm E,0}(\phi_{\rm FV}). \tag{2.18}$$

We find

$$B = B_0 \equiv \frac{27\pi^2}{2}\frac{\sigma^4}{\epsilon^3}. \tag{2.19}$$

It is instructive to consider the same theory with an *O(3)* spherical symmetric assumption, which can be generalized to calculate transition rates in a finite temperature. In the thin-wall approximation, the worldsheet metric is written as (see, e.g., Ref. [73])

$${\rm d}s_3^2 = -\left[1 - \left(\frac{{\rm d}r_*}{{\rm d}t}\right)^2\right]{\rm d}t^2 + r_*^2{\rm d}\Omega. \tag{2.20}$$

The action for the domain wall is given by the worldsheet area in addition to the difference of potential energy inside and outside bubble, so that we obtain

$$S = \int {\rm d}t\left[-4\pi r_*^2\sigma\gamma^{-1} + \frac{4}{3}\pi r_*^3\epsilon\right] + S_{M,0}, \tag{2.21}$$

where $\gamma = [1 - ({\rm d}r_*/{\rm d}t)^2]^{-1/2}$ may be regarded as the gamma factor of the domain wall. The first term is the surface term, which contains the kinetic energy of bubble, and the second term comes from the contribution inside the bubble. The conserved energy can be derived from

$$E = \frac{\partial L}{\partial \dot{r}_*}\dot{r}_* - L, \tag{2.22}$$

where the Lagrangian L can be read from the above action. This is rewritten as

$$4\pi r_*^2\sigma\gamma - \frac{4}{3}\pi r_*^3\epsilon = E_* \tag{2.23}$$

$$= 0. \tag{2.24}$$

In the second equality, we use the fact that the initial energy is zero. Taking the imaginary time $\tau = it$, this can be rewritten as

$$\left(\frac{{\rm d}r_*}{{\rm d}\tau}\right)^2 = \left(\frac{3\sigma}{\epsilon}\right)^2\frac{1}{r_*^2} - 1. \tag{2.25}$$

The bounce solution is given by

$$r_* = [(3\sigma/\epsilon)^2 - \tau^2]^{1/2}, \tag{2.26}$$

so that we obtain the value of the Euclidean bounce action as

$$S_E = \frac{27\pi^2}{2}\frac{\sigma^4}{\epsilon^3} + S_{\rm E,0}. \tag{2.27}$$

Noting that $r_*^2 + \tau^2 = (3\sigma/\epsilon)^2 = \eta_*^2$, these results are consistent with the above results using the *O(4)* symmetric assumption.

B. Tunneling with a finite energy

Now we can consider a transition from an excited state around the false vacuum by using the *O(3)* approximation and the thin-wall approximation. As one can see from Eq. (2.23) and Fig. 1, a state with a finite energy E_* allows an *O(3)* symmetric bubble with ${\rm d}r/{\rm d}t = 0$, whose radius is r_{*1} or $r_{*,2}$. The amplitude of the transition from a bubble

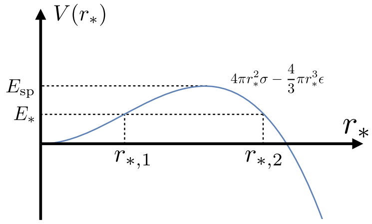

FIG. 1. Example of bubble potential as a function of its radius r_*.

with $r_* = r_{*,1}$ at $\tau = 0$ to another one with $r_* = r_{*2}$ at $\tau = \tau_0$, which is going to expand, is obtained from

$$\langle r_{*,2}, \tau = \tau_0 | r_{*,1}, \tau = 0 \rangle = \int_{r(\tau=0)=r_{*,1}}^{r(\tau=\tau_0)=r_{*,2}} \mathcal{D}[\phi] e^{-S[\phi]}. \tag{2.28}$$

The transition rate is then given by

$$e^{-\Gamma t_0} \propto |\langle r_{*,2}, \tau = \tau_0 | r_{*,1} \tau = 0 \rangle|^2 \tag{2.29}$$

$$= \left| \int_{r(\tau=0)=r_{*,1}}^{r(\tau=\tau_0)=r_{*,2}} \mathcal{D}[\phi] e^{-S[\phi]} \right|^2 \tag{2.30}$$

$$= \int_{\mathrm{bounce}} \mathcal{D}[\phi] e^{-S[\phi]}, \tag{2.31}$$

where the last path integral is performed under the boundary conditions such that $r = r_{*,1}$ at $\tau = -\tau_0$, $r = r_{*,2}$ at $\tau = 0$, and $r = r_{*,1}$ at $\tau = \tau_0$. By using the saddle-point approximation, the path integral is replaced by the dominant contribution $e^{-S[\phi_{\mathrm{bounce}}]}$ where ϕ_{bounce} is the bounce solution obeying the Euclidean equation of motion with the boundary condition of $r_* \in [r_{*,1}, r_{*2}]$. The amplitude is normalized by $|\langle r_{*,1}, \tau = \tau_0 | r_{*,1}, \tau = 0 \rangle|^2$. As a result, the transition rate can be expressed as

$$\Gamma \propto e^{-B(E_*)}, \quad B(E_*) = S_{\mathrm{bounce}}(E_*) - S_{\mathrm{E},0}(E_*), \tag{2.32}$$

where $S_{\mathrm{E},0}(E_*)$ comes from the normalization. Note again that $S_{\mathrm{bounce}}(E_*)$ is obtained from the Euclidean equation of motion from $r_{*,1}$ to $r_{*,2}$, while $S_{\mathrm{E},0}(E_*)$ is an *O(3)* symmetric bubble with a fixed radius $r_{*,1}$.

The bounce with a finite energy E_* satisfies

$$4\pi r_*^2 \sigma\gamma - \frac{4}{3}\pi r_*^3 \epsilon = E_*, \tag{2.33}$$

where E_* is the initial energy. There are two solutions with a vanishing wall velocity $\dot{r} = 0$, $r_{*,1}$ and $r_{*,2}$, which are obtained from $E_* = 4\pi\sigma r_*^2 - 4\pi r_*^3 \epsilon/3$ as can be seen from Fig. 1. Note here that the initial energy should be small enough to have these two solutions. The critical energy, above which we do not have solutions for Eq. (2.23), is obtained from the condition $\mathrm{d}r_*/\mathrm{d}\tau = \mathrm{d}^2 r_*/\mathrm{d}\tau^2 = 0$. The critical solution is

$$r_{*,\mathrm{sp}} = \frac{2\sigma}{\epsilon}, \tag{2.34}$$

$$E_{\mathrm{sp}} = \frac{16\pi\sigma^3}{3\epsilon^2}. \tag{2.35}$$

Here, the subscript "sp" indicates that this is nothing but the sphaleron, as we will see in the next Sec. II C. When we regard r_* as a position variable of a particle in a one-dimensional system, the constraint Eq. (2.33) can be rewritten as the following conservation law of "energy":

$$\frac{1}{2}\left(\frac{\mathrm{d}r_*}{\mathrm{d}\tau}\right)^2 + U(r_*) = 0, \tag{2.36}$$

$$2U(r_*) = 1 - \left[\frac{\epsilon}{3\sigma} r_* + \frac{E_*}{4\pi r_*^2 \sigma}\right]^{-2}. \tag{2.37}$$

Plugging the solutions into Eq. (2.21), we get the bubble nucleation rate for $E_* < E_{\mathrm{sp}}$,

$$B(E_*) = \int \mathrm{d}r_* \sqrt{(4\pi r_*^2 \sigma)^2 - \left(\frac{4}{3}\pi r_*^3 \epsilon + E_*\right)^2} \tag{2.38}$$

$$= \int \mathrm{d}\tau 4\pi r_*^2 \sigma\gamma \left(\frac{\mathrm{d}r_*}{\mathrm{d}\tau}\right)^2, \tag{2.39}$$

where we use $S_{\mathrm{E},0}(E_*) = E_* \int \mathrm{d}\tau + S_{\mathrm{E},0}$. Once we regard the factor $4\pi r_*^2 \sigma\gamma$ as the effective mass of the bubble, the result is similar to that in the one-dimensional quantum mechanical system.

Note again that the above transition means that a bubble with a radius $r_{*,1}$, which is not $r_* = 0$, tunnels into the one with a radius $r_{*,2}$. Hence, we need to specify the way to excite the initial state to the bubble with the radius $r_* = r_{*,1}$. If such bubbles are continuously produced and collapse in the initial state with a finite probability, the vacuum decay rate may be expressed as the probability of creating bubbles with $r_* = r_{*,1}$ times the probability of tunneling from $r_* = r_{*,1}$ to $r_* = r_{*,2}$, namely $e^{-B(E)}$. Here, note that we first assume the field configuration as Eq. (2.14) and reduce the number of degrees of freedom to unity. Since there are infinite degrees of freedom for the scalar field, it is generally difficult to give the energy so that all the energy is converted to such a macroscopic configuration, that is, the initial bubble with a radius $r_{*,1}$. The thermal state is an example that we have such an excited initial condition naturally. In this case, all degrees of freedom have a typical energy of order T_* with the Boltzmann weight, and hence the probability of creating the initial bubble is nothing but $\exp(-E_*(r_* = r_{*,1})/T)$ and is nonzero.

C. Tunneling with a thermal energy

Now we can consider a transition in the scalar field theory in a thermal background with a temperature of T_*. The transition rate can be calculated by the integral of the Boltzmann factor times quantum tunneling rate (see, e.g., Ref. [79]):

$$\Gamma = \Gamma_q + \Gamma_c, \tag{2.40}$$

$$\Gamma_q \sim \int_0^{E_{\rm sp}} dE e^{-E/T_*} e^{-B(E)}, \tag{2.41}$$

$$\Gamma_c \sim \int_{E_{\rm sp}}^{\infty} dE e^{-E/T_*}, \tag{2.42}$$

where Γ_c is the classical transition rate. Note that, for $E_* > E_{\rm sp}$, the bubble nucleation rate is unity, $B(E) = 1$, where $E_{\rm sp}$ is the sphaleron energy as explained below.

Let us evaluate the transition rate approximately. As discussed in the case of one-dimensional quantum mechanics, the question traces back to the behavior of d^2B/dE^2. If $d^2B/dE^2 > 0$ for $0 \leq E \leq E_{\rm sp}$, one may evaluate the integral via the steepest descent method by expanding the exponent as follows:

$$\frac{E}{T_*} + B(E) = \frac{E_{\rm cr}}{T_*} + B(E_{\rm cr}) + \left[\frac{1}{T_*} + B'(E_{\rm cr})\right](E_* - E_{\rm cr}) + \frac{B''(E_{\rm cr})}{2}(E - E_{\rm cr})^2 + \ldots. \tag{2.43}$$

If one finds the solution of

$$1/T_* = -B'(E_{\rm cr}) \tag{2.44}$$

$$= 2\int_{r_*(\tau=-\Delta\tau)=r_{*,1}}^{r_*(\tau=0)=r_{*,2}} dr_* \left(\left|\frac{dr_*}{d\tau}\right|\right)^{-1} \tag{2.45}$$

$$= 2\Delta\tau \quad \text{for } 0 \leq E \leq E_{\rm sp}, \tag{2.46}$$

where $2\Delta\tau$ is the time periodicity of the bounce solution, the integral may be approximated by the Gaussian. In this case the first integral is dominated by the energy $E_{\rm cr}$,

$$\Gamma_q \sim e^{-E_{\rm cr}/T_*} e^{-(S_{\rm bounce}(E_{\rm cr}) - S_{\rm E,0}(E_{\rm cr}))} \tag{2.47}$$

$$= e^{-S_{\rm bounce}(E_{\rm cr})}, \tag{2.48}$$

where we use $S_{\rm E,0}(E_{\rm cr}) = E_{\rm cr}/T_*$. This equation indicates that the transition rate can be decomposed into two parts as Eq. (2.47): the Boltzmann factor and the quantum tunneling rate. On the one hand, the quantum tunneling rate tells one that the bubble with $r_* = r_{*,1}$, which is not equal to $r_* = 0$, tunnels to $r_* = r_{*,2}$. On the other hand, the Boltzmann factor represents the probability that the bubble with energy $E_{\rm cr}$ reaches the point of $r_* = r_{*,1}$. Therefore, in total, Eq. (2.48) gives the transition rate where the bubble goes from $r_* = 0$ to $r_* = r_{*,1}$ via thermal excitation and then to $r_* = r_{*,2}$ via quantum tunneling. If there is no solution to Eq. (2.46), the integral is dominated by the boundary between Eq. (2.41) and Eq. (2.42), which ends up $e^{-E_{\rm sp}/T_*}$.

However, at least for the thermal transition in quantum field theory under the thin-wall approximation with the *O(3)* symmetry, the second derivative of the bounce action with respect to its energy is always $d^2B/dE^2 \leq 0$. Let us first confirm this property. Figure 2 shows the bounce B and its first derivative, $-B'(E)$, as a function of the energy E. One can see that $-B'(E)$ is an increasing function with respect to E, and thus $dB^2/dE^2 \leq 0$. Therefore, the saddle point is not a minimum of the exponent in Eq. (2.41).

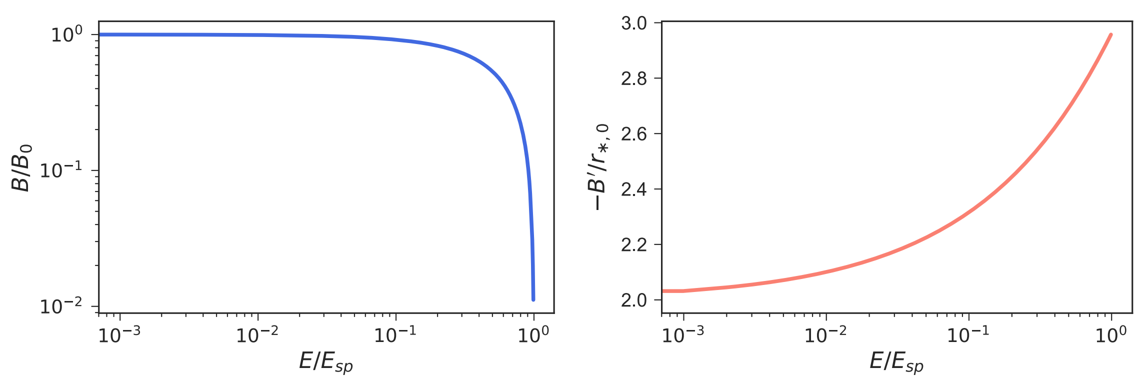

FIG. 2. Plot of $B(E)$ (blue line) and $-dB(E)/dE$ (pink line) as a function of E for a thin-wall bubble in quantum field theory. Here, we have normalized quantities by those at the zero-temperature: $B_0 = 27\pi^2\sigma^4/2\epsilon^3$ and $r_{*,0} = 3\sigma/\epsilon$. The results with these combinations are independent of σ and ϵ.

Rather, the integral is dominated by its edges, $E = 0$ or $E_{\rm sp}$. As a result, the transition rate can be expressed as

$$\Gamma \sim \Gamma_q + \Gamma_c, \tag{2.49}$$

$$\Gamma_q \sim e^{-B(0)}, \tag{2.50}$$

$$\Gamma_c \sim e^{-E_{\rm sp}/T_*}. \tag{2.51}$$

Recalling the sphaleron energy $E_{\rm sp} = 16\pi\sigma^3/3\epsilon^2$ and the bounce action $B(0) = 27\pi^2\sigma^4/2\epsilon^3$, we can go further. It is clear that the sphaleron transition dominates the decay process when the temperature T_* is large enough to satisfy $E_{\rm sp}/T_* < B(0)$. The threshold temperature is given by

$$T_{*,\rm th} = \frac{E_{\rm sp}}{B(0)} = \frac{32\epsilon}{81\pi\sigma} \tag{2.52}$$

$$= \frac{32}{27\pi}\eta_*^{-1} \ll \eta_*^{-1}. \tag{2.53}$$

Thus, we conclude that the transition rate is summarized as

$$\Gamma \sim e^{-B(0)} \quad \text{for } T_* \le T_{*,\rm th}, \tag{2.54}$$

$$\Gamma \sim e^{-E_{\rm sp}/T_*} \quad \text{for } T_{*,\rm th} \le T_*. \tag{2.55}$$

Before closing this section, we would like to explain the relation between the above results and the well-known method of putting the theory on $S^1_{T_*^{-1}} \times \mathbb{R}^3$ (i.e., periodic Wick-rotated time $\tau \sim \tau + T_*^{-1}$) and evaluating the imaginary part of the free energy $F = \ln Z$ [23]. In this case, the geometry forces all the configurations to be periodic, including the bounce. The quantum one, Eq. (2.54), corresponds to the dominant periodic instanton. Interestingly, though there exist other branches of periodic instantons with a finite energy E, the vacuum one, $E = 0$, dominates for the thin-wall approximation under the *O(3)* symmetry. Note here that, since $T_{*,\rm th}^{-1}$ is much larger than the radius of the vacuum bubble η_* as can be seen from Eq. (2.53), the vacuum bubble may be embedded in $S^1_{T_*^{-1}} \times \mathbb{R}^3$ for $T_* \le T_{*,\rm th}$. The classical one, Eq. (2.55), is obtained by the dimensional reduction of $S^1_{T_*^{-1}}$, which is a good approximation if the energy scale of the bubble is much smaller than the temperature. It is clear that the static solution can be embedded in the periodic spacetime.

III. TRANSITION IN THE SCHWARZSCHILD–DE SITTER SPACETIME

Gravity changes the spacetime in accordance with finite-energy objects. In the case of our interest, not only the BH but the bubble could be the origin of such distortions. Roughly speaking, in the context of the vacuum decay, the change of the spacetime affects the cost to nucleate the bubble. One can guess that gravity should modify the vacuum decay rate. Therefore, in this section, we switch on gravity and investigate its effect on the vacuum decay. The action is given by [80]

$$S = S_{\rm bubble} + S_G, \tag{3.1}$$

$$S_{\rm bubble} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\partial\phi)^2 - V(\phi)\right], \tag{3.2}$$

$$S_G = \frac{1}{2}\int_Y R\sqrt{-g}d^4x + S_{\rm boundary}, \tag{3.3}$$

where g is the determinant of the metric and R is the Ricci scalar. We take the Planck unit, $M_{\rm Pl}$ ($\simeq 2.4 \times 10^{18}$ GeV) $= 1$ (i.e., the Newton constant G is taken to be $8\pi G = 1$), unless otherwise stated. The curvature scalar R contains terms with second derivatives, which can be removed by integration by parts and write the action only by first derivatives so that we can use the path integral approach in the gravitational theory. As a result, the boundary term $S_{\rm boundary}$ arises, which is the integral of the trace of the second fundamental form of the boundary K [81],

$$S_{\rm boundary} = \int_{\partial Y} K\sqrt{-h}d^3x = \frac{\partial}{\partial n}\int_{\partial Y}\sqrt{-h}d^3x = \frac{\partial}{\partial n}V_{\rm boundary}, \tag{3.4}$$

where h is the determinant of the three-dimensional metric on the boundary surface, $V_{\rm boundary}$ is the volume of the boundary, and n is the unit normal. Note that the region of boundary depends on the metric, and hence the boundary term $S_{\rm boundary}$ is determined only after we specify the metric.

In particular, we consider a bubble nucleation in (anti-)de Sitter spacetime with a BH, which is described by the Schwarzschild-de Sitter metric,

$$ds^2 = -f_{\rm SdS}(r)dt^2 + \frac{dr^2}{f_{\rm SdS}(r)} + r^2 d\Omega, \tag{3.5}$$

$$f_{\rm SdS}(r) = 1 - \frac{M_{\rm BH}}{4\pi r} - \frac{\Lambda r^2}{3}, \tag{3.6}$$

where $M_{\rm BH}$ is a BH mass and Λ is a vacuum energy. The areas of the boundary, $\partial V_{\rm boundary}/\partial n$, at the surface of the BH ($\equiv A_{\rm BH}$) and at the cosmological horizon ($\equiv A_{\rm dS}$) are given by $A_{\rm BH} \simeq M_{\rm BH}^2/4\pi$ and $A_{\rm dS} \simeq 4\pi/H^2$ in this metric, respectively, where H is the inverse of the apparent horizon length related to the vacuum energy as $H^2 = \Lambda/3$. This metric respects only an *O(3)* symmetry. We use the *O(3)* symmetric assumption to find bounce solutions in the following analysis.

A. Bubble nucleation via tunneling

Here, we illustrate how to evaluate the bounce action in the presence of a BH with gravitational backreaction following Refs. [51–53,60]. Since we employ a thin-wall approximation, we may use the Euclidean metric defined separately in the outer and inner regions of the bubble,

$$\mathrm{d}s^2 = f_\pm(r)\mathrm{d}\tau_\pm^2 + \frac{\mathrm{d}r^2}{f_\pm(r)} + r^2\mathrm{d}\Omega, \tag{3.7}$$

$$f_\pm(r) = 1 - \frac{M_\pm}{4\pi r} - \frac{\Lambda_\pm r^2}{3}, \tag{3.8}$$

where M_+ is the initial BH mass and M_- is the remnant BH mass after the bubble nucleation. The zeros of $f_\pm$ define the horizon of each patch. We have at most two zeros for each $f_\pm$: the BH horizon $R_{\mathrm{BH},\pm}$ and the de Sitter horizon $R_{\mathrm{dS},\pm}$ for $\Lambda_\pm > 0$, respectively. The natural periodicities to eliminate the conical deficits at each horizon are the following: $T^{-1}_{\mathrm{BH},\pm} = 4\pi R_{\mathrm{BH},\pm}/(1-\Lambda_\pm R^2_{\mathrm{BH},\pm})$ at $r = R_{\mathrm{BH},\pm}$ and $T^{-1}_{\mathrm{dS},\pm} = 4\pi R_{\mathrm{dS},\pm}/(\Lambda_\pm R^2_{\mathrm{dS},\pm} - 1)$ at $r = R_{\mathrm{dS},\pm}$.

We now specify the setup of our interest. In the following, we focus on the case where we initially have a BH with M_+. Also, we assume that the vacuum energy of the scalar field changes from Λ_+ to Λ_- due to this transition. On the other hand, the remnant BH mass M_- is taken to be an arbitrary parameter. We do not impose that the Hawking temperature of the initial black hole $T_{\mathrm{BH},+} = 4\pi R_{\mathrm{BH},+}/(1-\Lambda_+ R^2_{\mathrm{BH},+})$ should coincide with that inferred from the periodicity of the bubble solution. This mismatch will cause a conical deficit at the BH horizon, and thus we have to cope with it appropriately, as done in Ref. [51].

Before evaluating the bounce action, we briefly illustrate how to obtain the wall trajectory of the nucleated bubble with fixed M_+, $\Lambda_\pm$, and T_{ini}. The wall trajectory $r_*(\lambda)$ is parametrized by the proper time of a comoving observer of the wall,

$$f_\pm \dot{\tau}_\pm^2 + \frac{\dot{r}_*^2}{f_\pm} = 1, \tag{3.9}$$

where the dot denotes the derivative with respect to the proper time λ. The Israel junction condition yields

$$f_+\gamma_+ - f_-\gamma_- = -\frac{1}{2}\sigma r_*, \tag{3.10}$$

where σ is the tension of the bubble and

$$\gamma_\pm \equiv \dot{\tau}_\pm = \frac{1}{\sqrt{f_\pm(r_*) + \frac{1}{f_\pm(r_*)}\left(\frac{\mathrm{d}r_*}{\mathrm{d}\tau_\pm}\right)^2}}. \tag{3.11}$$

We can explicitly rewrite it as

$$f_\pm\gamma_\pm = \left(\frac{\Delta\Lambda}{3\sigma} \mp \frac{\sigma}{4}\right) r + \frac{\Delta M}{4\pi\sigma r^2}. \tag{3.12}$$

The junction condition implies that the wall velocity has to satisfy the following conservation law of "energy":

$$\frac{1}{2}\left(\frac{\mathrm{d}\tilde{r}_*}{\mathrm{d}\tilde{\lambda}}\right)^2 + U(\tilde{r}) = 0, \tag{3.13}$$

$$2U(\tilde{r}) = \left(\tilde{r}_* + \frac{k_2}{\tilde{r}_*^2}\right)^2 + \frac{k_1}{\tilde{r}_*} - 1, \tag{3.14}$$

where $\tilde{r}_* = \alpha r_*/\gamma$ and $\tilde{\lambda} = \alpha\lambda/\gamma$, and

$$k_1 = \frac{\alpha M_-}{4\pi\gamma} + \frac{(1-\alpha)\alpha\Delta M}{2\pi\sigma\gamma^2}, \qquad k_2 = \frac{\alpha^2\Delta M}{4\pi\sigma\gamma^2},$$
$$\gamma_{\mathrm{GMW}} = \frac{\sigma l^2}{1+\sigma^2 l^2/4}, \qquad \alpha^2 = 1 + \frac{\Lambda_-\gamma^2}{3}, \qquad l^2 = \frac{3}{\Delta\Lambda}, \tag{3.15}$$

where $\Delta M \equiv M_+ - M_-$ and $\Delta\Lambda \equiv \Lambda_+ - \Lambda_-$ $(\equiv\epsilon)$.[6] Once we fix the all the parameters $M_\pm$ and $\Lambda_\pm$, we can obtain the wall trajectory as a function of the proper time, $r_*(\lambda)$, in principle. In our setup, we have fixed M_+ and $\Lambda_\pm$ and hence we have a family of solutions as a function of the remnant BH mass, $r_*(\lambda; M_-)$.

Now we are in a position to evaluate the Euclidean action by the solution to Eq. (3.13). The gravitational Euclidean action is given by

$$S_G = S_{\mathrm{boundary}} - \frac{1}{2}\left[\int_{Y_+}\sqrt{g}R + \int_{Y_-}\sqrt{g}R\right] + \left[\int_{\partial Y_+}\sqrt{h}K + \int_{\partial Y_-}\sqrt{h}K\right], \tag{3.16}$$

where Y_- and Y_+ represent the regions inside and outside of the bubble, respectively, and $\partial Y_\pm$ represents the boundary induced by the bubble. Note again that S_{boundary} accounts for the boundaries at the horizons. The Einstein equation and the Israel junction condition imply that the action can be rewritten as

$$S_G = S_{\mathrm{boundary}} - \frac{1}{2}\left[\int_{Y_+}\sqrt{g}4\Lambda_+ + \int_{Y_-}\sqrt{g}4\Lambda_-\right] - \frac{3}{2}\left[\int_{\partial Y}\sqrt{h}\sigma\right]. \tag{3.17}$$

[6]Note that the γ_{GMW} here is different from the gamma factor of the domain wall, γ, defined below.

In the thin-wall approximation, the bubble Euclidean action is given by

$$S_{\rm bubble} = \int d^4x\sqrt{g}\left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right] \tag{3.18}$$

$$= \int d\lambda\left[4\pi r_*^2\sigma + \frac{4\pi}{3}r_*^3(\Lambda_-\gamma_- - \Lambda_+\gamma_+) - \frac{4\pi}{3}R_{\rm BH,-}^3\Lambda_-\gamma_- + \frac{4\pi}{3}R_{\rm dS,+}^3\Lambda_+\gamma_+\right]. \tag{3.19}$$

If there is no de Sitter horizon, we have to drop the last term.

The bubble nucleation rate around the BH is calculated from

$$B = S_{\rm E} - S_{\rm E,0}, \tag{3.20}$$

where $S_{\rm E}$ is the total Euclidean action ($= S_G + S_{\rm bubble}$) and $S_{\rm E,0}$ is the action without the bubble. If the gravitational action changes, the boundary term may also change. Assuming that the bubble is not as large as the de Sitter horizon, one can see that the boundary from the BH horizon only contributes to the difference. The explicit form of B is given by

$$B(M_-) = B_{\rm bubble}(M_-) + B_{\rm boundary}(M_-), \tag{3.21}$$

$$B_{\rm bubble}(M_-) = 4\pi\int r_*(f_+ d\tau_+ - f_- d\tau_-) + \int\frac{4\pi}{3}r_*^3(\Lambda_+ d\tau_+ - \Lambda_- d\tau_-) - \frac{1}{2}\int(M_+ d\tau_+ - M_- d\tau_-), \tag{3.22}$$

$$B_{\rm boundary}(M_-) = 8\pi^2(R_{\rm BH,+}^2 - R_{\rm BH,-}^2), \tag{3.23}$$

where we use the Israel junction $\sigma r_*/2 = -(f_+\dot\tau_+ - f_-\dot\tau_-)$. The third term in $B_{\rm bubble}$ is related to the contribution of the conical deficit. The term $B_{\rm boundary}$ accounts for the change of the BH entropy which comes from the area of the boundary at the horizon. We regard this result as a function of M_- for later convenience. In summary, the bubble nucleation rate from the initial state (a BH of M_+, vacuum energy of Λ_+) to the final state (a BH of M_-, vacuum energy Λ_-) is given by

$$\Gamma(M_-) \sim e^{-B(M_-)}. \tag{3.24}$$

We emphasize that the initial mass of the BH M_+, the initial and final energy densities $\Lambda_\pm$, and the surface energy density of the bubble σ are determined by the initial conditions and the potential of the scalar field, while the remnant BH mass M_- after the bubble nucleation is not yet determined in the above calculation. In the sense of the path integral approach, we should sum over all the nucleation rates $\Gamma(M_-) \sim e^{-B(M_-)}$ in terms of the variable M_-. Thus, we need to find a minimal value of the action in terms of M_-,

$$\Gamma \sim \int dM_-\Gamma(M_-) \sim \int dM_- e^{-B_{\rm boundary}-B_{\rm bubble}} \sim e^{-B_{\rm min}}. \tag{3.25}$$

As has been pointed out in Ref. [51], it is possible that the remnant BH mass M_- is larger than the initial BH mass M_+ though it usually gives subdominant contributions. Here, we comment on the lower bound on the integral Eq. (3.25). If M_+ is sufficiently large, there is a lower bound of M_-, below which the "potential" $U(\tilde r)$ is always larger than zero and there is no solution to Eq. (3.13). At the critical point, $U(\tilde r) = U'(\tilde r) = 0$ and the solution is static. See also the discussion below.

Let us take the variation of the bounce action $B = B_{\rm boundary} + B_{\rm bubble}$ with respect to M_- so as to approximate the integral of Eq. (3.25). The boundary term gives

$$\frac{d}{dM_-}B_{\rm boundary} = -\frac{4\pi R_{\rm BH,-}}{1 - \Lambda_- R_{\rm BH,-}^2} = -T_{\rm BH,-}^{-1}. \tag{3.26}$$

By numerically solving the equation of motion and calculating the transition rate (See Figs. 3 and 4), we also find that the variation of the other terms satisfies

$$\frac{d}{dM_-}B_{\rm bubble} = \int d\tau_- = 2\Delta\tau_-. \tag{3.27}$$

Combining these results, we obtain

$$\frac{d}{dM_-}B = 2\Delta\tau_- - T_{\rm BH,-}^{-1}. \tag{3.28}$$

The above result Eq. (3.28) is similar to the one obtained in the previous sections [see Eq. (2.46)]. In particular, if we require $dB(M_-)/dM_- = 0$, we find the relation $2\Delta\tau_- = T_{\rm BH,-}^{-1}$, which may imply that the transition is due to the thermal effect with Hawking temperature. However, the transition rate is not minimized at the saddle point unless $d^2B(M_-)/dM_-^2 > 0$ as we discussed in the previous section. In fact, we numerically check that this condition is not always satisfied. We show an example in Fig. 3, where we assume $R_{\rm BH,+} = 0.1r_{*,0}$, $\Lambda_+ = 0$, and $\Lambda_- = -0.03/r_{*,0}^2$.[7] These parameters lead to $l = 10r_{*,0}$ and $\sigma l/M_{\rm Pl}^2 = r_{*,0}/l = 0.1$. Note that the $r_{*,0}$ dependence can be trivially factorized in our results such that

[7]Note that $r_{*,0} \equiv 3\sigma/\Delta\epsilon$ ($\epsilon = \Delta\Lambda$).

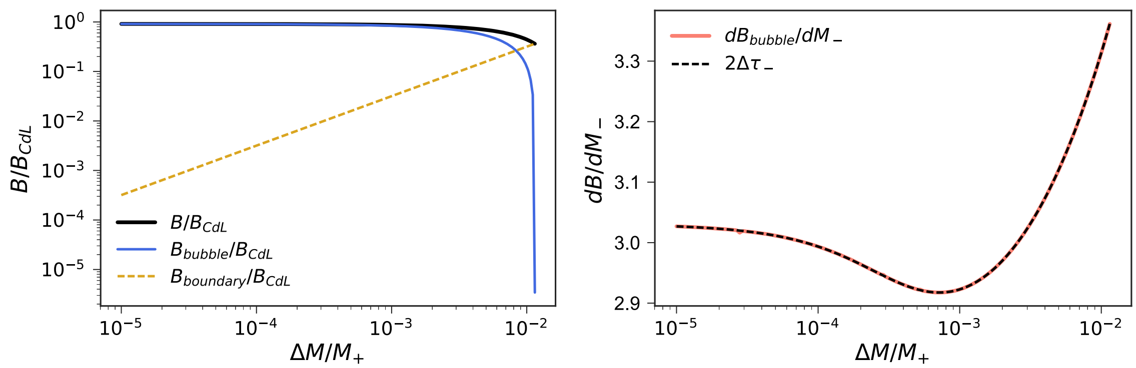

FIG. 3. *Left*: $B_{\rm bubble}$ (blue line) as a function of $\Delta M(\equiv M_+ - M_-)$, and *Right*: $\mathrm{d}B_{\rm bubble}/\mathrm{d}M_-$ (pink line) as a function of ΔM, for a thin-wall bubble in the Schwarzschild–de Sitter spacetime. In the left panel, we also plot B (black thick line) and $B_{\rm boundary}$ (yellow dashed line); in the right panel, $2\Delta\tau_-$ (black dashed line) is shown. We take $R_{\rm BH,+} = 0.1 r_{*,0}$, $\Lambda_+ = 0$, and $\Lambda_- = -0.03/r_{*,0}^2$. In this case, one can show that $l = 10 r_{*,0}$ and $\sigma l/M_{\rm Pl}^2 = r_{*,0}/l = 0.1 \ll 1$. We take $r_{*,0} = 1/M_{\rm Pl}$ to plot the results, but the $r_{*,0}$ dependence can be trivially factorized such that B, $B_{\rm CdL} \propto r_{*,0}^2$, $\Delta M, M_+ \propto r_{*,0}$, and $\mathrm{d}B/\mathrm{d}M_- \propto r_{*,0}$. Note that $\mathrm{d}B_{\rm bubble}/\mathrm{d}M_- = -\mathrm{d}B_{\rm bubble}/\mathrm{d}\Delta M$.

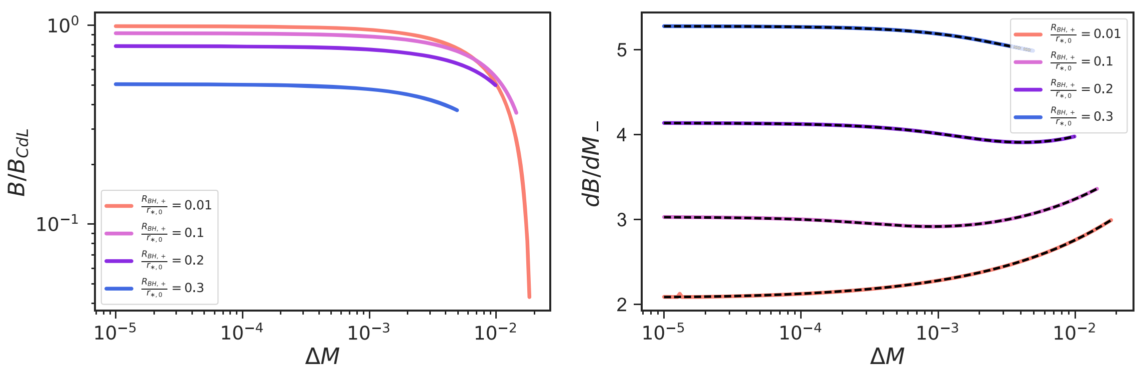

FIG. 4. *Left*: B as a function of $\Delta M(\equiv M_+ - M_-)$, and *Right*: $\mathrm{d}B_{\rm bubble}/\mathrm{d}M_-$ as a function of ΔM, for a thin-wall bubble in the Schwarzschild–de Sitter spacetime. We take the parameters to be the same as in Fig. 3 except that we take $R_{\rm BH,+}/r_{*,0} = 0.01$ (red line), 0.1 (pink line), 0.2 (violet line), and 0.3 (blue line).

$B, B_{\rm CdL} \propto r_{*,0}^2$, $\Delta M, M_+ \propto r_{*,0}$, and $\mathrm{d}B/\mathrm{d}M_- \propto r_{*,0}$ though we take $r_{*,0} = 1/M_{\rm Pl}$ as a reference value to plot them. In the left panel of Fig. 3, $B(\Delta M)$ monotonically decreases as ΔM increases and it is minimized at the maximal value of ΔM, which corresponds to the static solution. Note again that, if the seed BH mass, M_+, is sufficiently heavy, there exists a maximal value of ΔM, above which we do not have solutions to Eq. (3.13) and at which the solution becomes static. In the right panel, $\mathrm{d}B_{\rm bubble}(\Delta M)/\mathrm{d}M_-$ increases as ΔM increases for moderately large ΔM, which implies that there is no saddle point at least in that range of ΔM.

We can see that the result coincides with Fig. 2 when we take a limit of $r_{*,0} \gg R_{\rm BH,+}$ (and $\Delta M \ll M_+$ as we discuss in the next subsection), where the effect of curved spacetime on the bubble configuration is negligible. Figure 4 shows the result of B and its derivative with respect to M_- for the cases of $R_{\rm BH,+}/r_{*,0} = 0.01$ (red line), 0.1 (pink line), 0.2 (violet line), and 0.3 (blue line). We can see that the red line in the figure, where we take $R_{\rm BH,+}/r_{*,0} = 0.01 \ll 1$, coincides with that in Fig. 2. Note however that the result coincides with the one in flat spacetime only when we identify T_* in flat spacetime as the Hawking temperature $T_{\rm BH,-}$ of the BH according to the correspondence between Eqs. (2.46) and (3.28). As we see below, this temperature is much larger than $T_{*,\rm th}$ defined by Eq. (2.53), so that the transition rate is dominated by the static solution. This is also true for larger values of $R_{\rm BH,+}/r_{*,0}$, which then implies that the static solution dominates the transition rate even if $\mathrm{d}^2 B_{\rm bubble}/\mathrm{d}\Delta M_+^2 < 0$ for moderately small ΔM and large $R_{\rm BH,+}/r_{*,0}$. These results show that the effect of the boundary term is relevant and is crucial even if the curved spacetime does not affect the bubble configuration.

Since the transition rate is smaller for smaller M_-, it is minimized at the critical point for a large initial BH mass. In fact, this property has been pointed out in Ref. [51] and so the static solution is studied in detail in the literature. To clarify the situation, we shall take the limit of $M_- = M_{-,\rm min} + \varepsilon$ with $\varepsilon \to +0$ and see what happens. $r_{*,1}$ and $r_{*,2}$ coincide with each other at $r_* = r_{*,\rm sp}$, where

$$\tilde{r}_{*,\rm sp} \equiv \sqrt{\frac{1}{9}(1 + A^{-1/3} + A^{1/3})} \geq \frac{1}{3}, \qquad (3.29)$$

$$A \equiv 2^{-1/3}\left(2 + (27k_2)^2 + 27|k_2|\sqrt{4 + (27k_2)^2}\right), \qquad (3.30)$$

and the amplitude of the solution decreases to zero for $\varepsilon \to +0$. However, the second derivative $U''(\tilde{r})$ is nonzero in that limit. Since the potential can be approximated to be a quadratic potential for a sufficiently small ε, the periodicity is determined solely by the second derivative. It is given by

$$-\frac{\partial^2 U}{\partial \tilde{r}^2} = 9\Bigg[1 - A^{1/3} + \frac{1}{(27k_2)^2} A^{1/3}(-B + (B - (27k_2)^2)A^{1/3}/2)\Bigg], \qquad (3.31)$$

$$B \equiv 27|k_2|\sqrt{4 + (27k_2)^2}. \qquad (3.32)$$

Note that $-(1/2)\partial^2 U/\partial\tilde{r}^2 = 3\,(9)$ for $k_2 = 0\,(\pm\infty)$. This is plotted in Fig. 5, where the red dashed line represents the asymptotic value for $k_2 \to \pm\infty$. If $\Lambda_+ = 0$, there is an upper bound on k_2 indicated by the dotted line. We find that the periodicity of $\tilde{\lambda}$ is in the range of $(2\pi/3, 2\pi/\sqrt{3})$ and is of order unity. Thus, the periodicity of $\tau_\pm$ for $M_- = M_{-,\rm min} + \varepsilon$ with $1 \gg \epsilon > 0$ is related to that of λ as follows:

$$\lim_{\varepsilon \to +0} 2\Delta\tau_\pm(M_{-,\rm min} + \varepsilon) = \int \gamma_\pm \mathrm{d}\lambda = \frac{1}{\sqrt{f_\pm(r_{*,\rm sp})}} \frac{\gamma_{\rm GMW}}{\alpha} 2\Delta\tilde{\lambda}. \qquad (3.33)$$

Noting that $\gamma/\alpha \simeq r_{*,0}$ for $\sigma^2 l^2 \ll 1$ and $\Lambda_+ = 0$, we find that the resulting periodicity is of the same order as the radius of the Coleman solution. Therefore, the periodicity of the bubble is much larger than the radius of the BH, indicating that $\mathrm{d}B/\mathrm{d}M_- = 2\Delta\tau_- - T^{-1}_{\rm BH,-} \simeq 2\Delta\tau_- > 0$ for $M_- = M_{-,\rm min} + \varepsilon$ with $1 \gg \varepsilon > 0$. This observation justifies that the transition rate tends to be minimized by the smallest value of M_-, namely the static solution. For a sufficiently small $M_{\rm BH}$, the minimum mass of the remnant BH becomes zero.[8] In this special case, $\mathrm{d}B/\mathrm{d}M_- = \mathrm{d}B_{\rm bubble}/\mathrm{d}M_- = 2\Delta\tau_-$, which indicates that the transition rate is minimized at $M_- = 0$ if possible.

[8]Such a seed mass is referred to as a critical mass in Refs. [51,53].

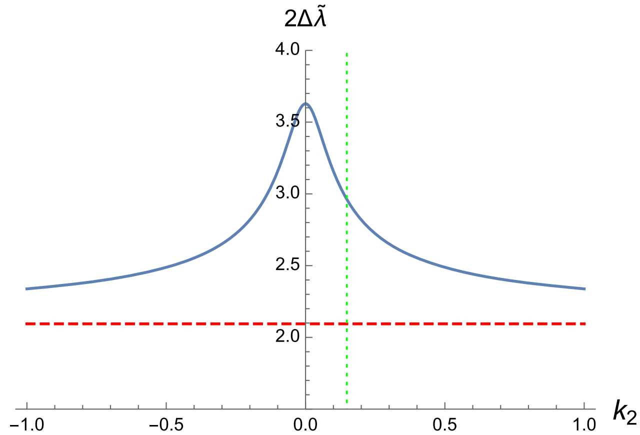

FIG. 5. Relation between $2\Delta\tilde{\lambda}$ and k_2 for static solutions. The red dashed line represents the asymptotic value of $2\Delta\tilde{\lambda}$ for $k_2 \to \pm\infty$. The green dashed line represents an upper bound of k_2 for the case of $\Lambda_+ = 0$.

Let us interpret the obtained result. First, note that we can identify $\Delta M \equiv M_+ - M_-$ as the energy of the bubble because of the energy conservation. Then we can find some similarities between the above results and the ones in the previous section. First, the derivative of B with respect to ΔM (or $-M_-$) gives the periodicity of the instanton solution and the inverse of the Hawking temperature of remnant BH $T^{-1}_{\rm BH-}$. This corresponds to Eq. (2.46) once we identify the temperature as the Hawking temperature. Second, the second derivative tends to be negative (at least for a moderately large ΔM and/or $r_{*,0} \gg R_{\rm BH,+}$). Depending on the seed BH mass, the transition rate is dominated by $M_- = 0$ or a static solution, which are the boundary of the allowed value of M_-. This is analogous to the thermal transition for a high enough temperature $T_* > T_{*,\rm th}$ discussed in Sec. II C. (In fact, the Hawking temperature $T_{\rm BH,-}$ is much larger than the threshold temperature $T_{*,\rm th}$.) There, the periodicity of the bubble for $E < E_{\rm sp}$ never coincides with T_*^{-1}. This is not a problem because those solutions with $E < E_{\rm sp}$ do not dominate the path integral in this case; rather, the static solution $E = E_{\rm sp}$ is realized and this bounce can be embedded into the Euclidean spacetime with the periodicity of T_*^{-1}. We will further clarify these correspondences in the subsequent subsections.

B. Bubble nucleation without backreaction to metric

In this section, we neglect the change of the metric by the bubble in this subsection and regard the metric as a background [73]. Comparing the result computed in this way with the full gravitational one, the meaning of the enhancement factor in Refs. [51–53] is clarified. In the thin-wall approximation, the worldsheet metric in the Schwarzschild–de Sitter spacetime background is written as

$$\mathrm{d}s_3^2 = -\left[f(r_*) - \frac{1}{f(r_*)}\left(\frac{\mathrm{d}r_*}{\mathrm{d}t}\right)^2\right]\mathrm{d}t^2 + r_*^2\mathrm{d}\Omega^2. \quad (3.34)$$

The Euclidean effective action for the domain wall is given by the worldsheet area in addition to the difference of potential energy inside and outside bubble, and thus we obtain

$$S_E = \int \mathrm{d}\tau\left[4\pi r_*^2\sigma\gamma^{-1} - \frac{4\pi}{3}(r_*^3 - R_{\mathrm{BH}}^3)\epsilon\right], \quad (3.35)$$

where $\epsilon = \Lambda_+ - \Lambda_-$ ($\equiv \Delta\Lambda$) and $R_{\mathrm{BH}} = M_{\mathrm{BH}}/4\pi$ is the BH horizon radius. Recall that the Euclidean gamma factor γ is given by

$$\gamma = \frac{1}{\sqrt{f(r_*) + \frac{1}{f(r_*)}\left(\frac{\mathrm{d}r_*}{\mathrm{d}\tau}\right)^2}}. \quad (3.36)$$

The conserved energy is

$$E = \frac{\partial L}{\partial \dot{r}_*}\dot{r}_* - L, \quad (3.37)$$

where the Lagrangian L can be read from the above action. It can be expressed as

$$f(r_*)4\pi r_*^2\sigma\gamma - \frac{4\pi}{3}(r_*^3 - R_{\mathrm{BH}}^3)\epsilon = E_*, \quad (3.38)$$

where E_* is the total energy in the initial state. We should take $E_* = 0$ for the vacuum transition without the thermal effect, in which case the bubble nucleates from $r_* = R_{\mathrm{BH}}$, i.e., from the surface of the BH, to a certain radius. The conservation law can be rewritten as

$$\frac{1}{f^2(r_*)}\left(\frac{\mathrm{d}r_*}{\mathrm{d}\tau}\right)^2 = f(r_*)\left[\frac{\epsilon}{3\sigma}r_* + \frac{E_*}{4\pi r_*^2\sigma}\right]^{-2} - 1, \quad (3.39)$$

where we redefine $E_* - 4\pi R_{\mathrm{BH}}^3\epsilon/3$ as E_*.

It is convenient to introduce the proper time λ of the bubble trajectory,

$$\mathrm{d}\lambda = \gamma^{-1}\mathrm{d}\tau, \quad (3.40)$$

which gives

$$f(r_*)\left(\frac{\mathrm{d}\tau}{\mathrm{d}\lambda}\right)^2 = 1 - \frac{1}{f(r_*)}\left(\frac{\mathrm{d}r_*}{\mathrm{d}\lambda}\right)^2. \quad (3.41)$$

Then Eq. (3.39) can be rewritten as

$$\left(\frac{\mathrm{d}r_*}{\mathrm{d}\lambda}\right)^2 = f(r_*) - \left(\frac{\epsilon}{3\sigma}r_* + \frac{E_*}{4\pi\sigma r_*^2}\right)^2, \quad (3.42)$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}\lambda} = \frac{1}{f(r_*)}\left(\frac{\epsilon}{3\sigma}r_* + \frac{E_*}{4\pi\sigma r_*^2}\right). \quad (3.43)$$

Recalling that

$$\left[1 - \frac{k_1}{\tilde{r}_*} - \left(\tilde{r}_* + \frac{k_2}{\tilde{r}_*^2}\right)^2\right]\left[\frac{\tilde{r}_*}{\alpha} + \frac{k_2}{\tilde{r}_*^2}\right]^{-2} = f_-(r_*)\left[\frac{k_2}{\tilde{r}_*^2} + \frac{\tilde{r}_*}{\alpha}\right]^{-2} - 1, \quad (3.44)$$

we find that this equation is consistent with the result in the previous subsection in the limit of $f_-(r) \approx f(r)$, $\gamma_{\mathrm{GMW}} \approx 3\sigma/\epsilon = r_{*,0}$, and $\Delta M = E_*$. Here, extra terms in γ_{GMW} other than $3\sigma/\epsilon$ in Refs. [51–53,60] come from the backreaction to the bubble, as we see below. Thus our result is consistent with their result in the limit of the negligible backreaction.

Here, we would like to explain that the exact form of γ_{GMW} can be understood as the backreaction to the bubble. In the next-to-leading order approximation, we have to take into account the self-gravitational energy of the bubble in the Newtonian limit. It may be calculated by

$$E_{\mathrm{self}} = \frac{1}{2}\int r_*^2\mathrm{d}\Omega'^2\int r_*^2\mathrm{d}\Omega^2\frac{G\sigma\sigma}{|\mathbf{r} - \mathbf{r}'|} \quad (3.45)$$

$$= \pi r_*^3\sigma^2, \quad (3.46)$$

where $G = 1/(8\pi)$ is the Newton constant. When we include this contribution to the bubble action, it may be absorbed by the redefinition of ϵ,

$$\epsilon \to \epsilon\left(1 - \frac{3\sigma^2}{4\epsilon}\right). \quad (3.47)$$

This is because the ϵ dependence comes only through $\gamma_{\mathrm{GMW}} \simeq 3\sigma/\epsilon$ in the previous leading order results. Then the next-to-leading order contribution is

$$\gamma_{\mathrm{GMW}} \simeq \frac{3\sigma}{\epsilon}\left(1 - \frac{3\sigma^2}{4\epsilon}\right), \quad (3.48)$$

which is consistent with the expansion of the exact γ_{GMW} under $\sigma^2/\epsilon \ll 1$.

Now we are ready to discuss the bubble nucleation with a finite energy under this metric, and see its relation to the result in the previous section. First we need to derive the bubble nucleation rate with an initial energy of E_*, where the bubble solution obeys Eq. (3.38). It is given by

$$B(E_*) = S_{\rm bounce}(E_*) - S_{\rm E,0}(E_*) \tag{3.49}$$

$$= \int {\rm d}r_* \sqrt{\frac{1}{f(r_*)}(4\pi r_*^2\sigma)^2 - \frac{1}{f^2(r_*)}\left(\frac{4}{3}\pi r_*^3\epsilon + E_*\right)^2} \tag{3.50}$$

$$= \int {\rm d}\tau 4\pi r_*^2 \sigma\gamma \frac{1}{f(r_*)}\left(\frac{{\rm d}r_*}{{\rm d}\tau}\right)^2, \tag{3.51}$$

with $S_{\rm E,0}(E_*) = E_*\Delta\tau$. To get the vacuum decay rate, we have to multiply the probability of producing a bubble with $r_{*,1}$. Let us consider the situation where bubbles with E_* are produced with a rate of $e^{-P(E_*)}$. Then the bubble nucleation rate for $E_* < E_{\rm sp}$ is given by

$$\Gamma_q(E_*;[P]) \sim e^{-P(E_*)}e^{-B(E_*)}. \tag{3.52}$$

Up to here, the probability function $P(E)$ is generic. In the following, we would like to specify its form so that it reproduces the previous result in a certain limit. Then, we discuss its consequence. First, we show that $B(E_*)$ coincides with $B_{\rm bubble}$ [Eq. (3.22)] in the limit of $f_- \approx f$, $\gamma_{\rm GMW} \approx 3\sigma/\epsilon = r_{*,0}$ and $\Delta M = E_*$. This is obvious because we have already seen that the master Eq. (3.43) coincides with that in the previous section [Eqs. (3.13) and (3.14)]. For clarity, we also plotted $B(E_*)$ and $-B'(E_*)$ in Figs. 6 and 7 for parameters which are close to the case shown in Figs. 3 and 4, respectively. One can see that $B(E_*)$ and $-B'(E_*)$ coincide with those in Figs. 3 and 4, since, for the parameters in these figures, the backreaction is quite small because of $\sigma l = r_{*,0}/l = 0.1 \ll 1$ and $M_+ \gg E_{\rm sp} \geq \Delta M$. Next, let us see how $B_{\rm boundary}$ behaves in this limit. Since we have $M_{\rm BH} \approx M_+ \approx M_- \gg \Delta M$ and $1 \gg r_{*,0}/l \gtrsim M_-/4\pi l$, the boundary term can be expressed as

$$B_{\rm boundary} = \frac{1}{2}\left[M_+^2 - M_-^2\left(1 + \frac{(M_-/4\pi)^2}{l^2} + \ldots\right)^2\right]$$
$$\approx \frac{\Delta M}{1/M_{\rm BH}} = \frac{E_*}{T_{\rm BH}}. \tag{3.53}$$

In the last equality, we use $T_{\rm BH} = 1/M_{\rm BH}$ and identify $E_* = \Delta M$. Now it is clear that the result in the previous section is reproduced for the probability function of

$$P(E_*) = \frac{E_*}{T_{\rm BH}}, \tag{3.54}$$

which is nothing but the canonical ensemble with a temperature T_* that coincides with the Hawking temperature of the background BH, $T_* = T_{\rm BH}$.

Before discussing the physical meaning of $P(E_*)$, let us evaluate the vacuum decay rate. It is given by

$$\Gamma = \int {\rm d}E_* \Gamma_q(E_*;[P]) \sim \int {\rm d}E_* e^{-P(E_*)}e^{-B(E_*)}. \tag{3.55}$$

Again, let us first try to find the saddle point. A first derivative of the exponent yields

$$\frac{{\rm d}P}{{\rm d}E_*} = T_{\rm BH}^{-1}, \tag{3.56}$$

$$-\frac{{\rm d}B}{{\rm d}E_*} = \int {\rm d}\tau \frac{(4/3)\pi r_*^3\epsilon + E_*}{f(r_*)4\pi r_*^2\sigma\gamma} = \int {\rm d}\tau = 2\Delta\tau. \tag{3.57}$$

If the second derivative of the bounce with respect to E_* is always positive, one may approximate the integral by the saddle point satisfying $2\Delta\tau = T_{\rm BH}^{-1}$. However, the second derivative can be negative in some cases (see Figs. 6 and 7). Actually, the sign is always negative in the quantum field theory in the flat spacetime as we have discussed in Sec. II C. If $B''(E)$ is negative for $0 \leq E_* \leq E_{\rm sp}$, the transition rate may not be dominated by the saddle point

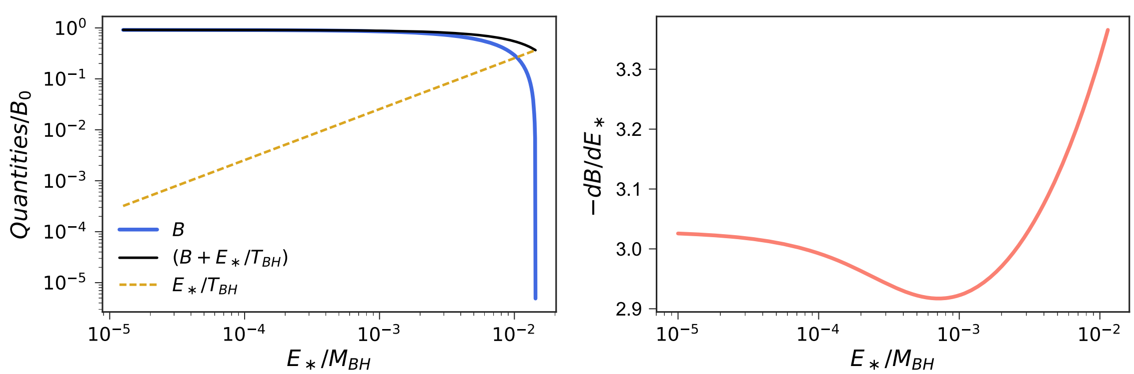

FIG. 6. Plot of $B(E_*)$ (left panel) and $-{\rm d}B(E_*)/{\rm d}E_*$ (right panel) as a function of E_* for a thin-wall bubble in the Schwarzschild spacetime without the backreaction of the bubble to the metric. In the left panel, we also plot $B + E_*/T_{\rm BH}$ (black thick line) and $E_*/T_{\rm BH}$ (yellow dashed line). We take $R_{\rm BH,+} = 0.1r_{*,0}$ and $l = 10r_{*,0}$. We also take $r_{*,0} = 1/M_{\rm Pl}$ though the $r_{*,0}$ dependence can be trivially factorized as ${\rm d}B/{\rm d}E_* \propto r_{*,0}$. One can see that the result almost coincides with Fig. 3.

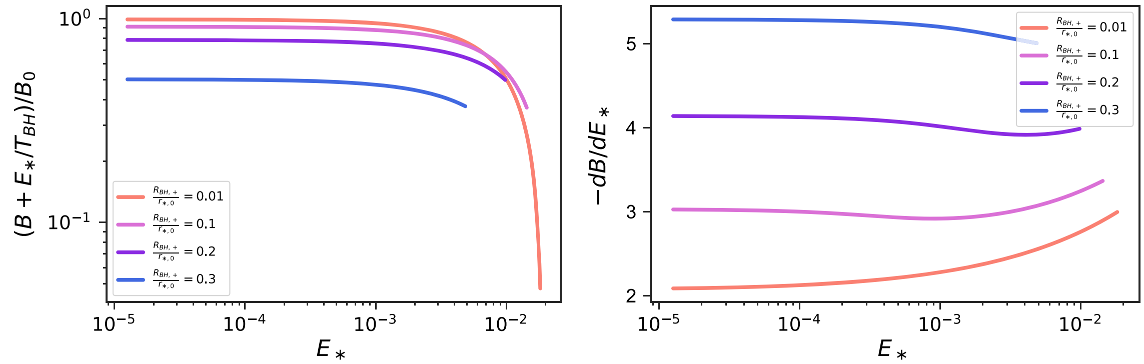

FIG. 7. Plot same as Fig. 6 but with $R_{\mathrm{BH},+}/r_{*,0} = 0.01$ (red line), 0.1 (pink line), 0.2 (violet line), and 0.3 (blue line). One can see that the result almost coincides with Fig. 4.

but by either boundary of E_*. One of the boundaries of E_* is, of course, $E_* = 0$, which is just the quantum tunneling process from a false vacuum in the presence of a BH with M_{BH} [54]. The other boundary E_{sp} comes from the condition that $B(E_{\mathrm{sp}}) = 0$, which corresponds to the sphaleron process at high temperature and is determined by the static solution. For the sample parameters shown in Figs. 6 and 7, one can see that $E_* = E_{\mathrm{sp}}$ dominates for $T_* = T_{\mathrm{BH},-}$. Here, we assume $E_* \ll M_+$ so that we can neglect the change of metric due to the bubble nucleation. Hence, we require $E_{\mathrm{sp}} \ll M_+$. Note again that the dominant process is always consistent with the periodicity indicated by the BH Hawking temperature T_{BH}, although a family of bounce solutions as a function of E_* is not.

Now we would like to discuss the consequence of $P(E_*)$. Comparing the bubble nucleation rate computed in this section with the full gravitational one, we have seen that the enhancement factor can be regarded as a probability of generating bubbles with a finite energy E. However, there are many other degrees of freedom in quantum field theory, and hence it is hard to imagine that a BH only excites bubbles. Therefore, all the states with an energy E other than bubbles should also be generated by the same mechanism. This observation makes it clear that one needs to take into account finite-density corrections to the bubble nucleation rate induced by the presence of plasma. Up to here, the conclusion is independent of whether or not the plasma fills the whole Universe. However, the size of corrections does depend. If $P(E_*)$ originates from Hawking radiation emitted from the BH horizon, the flux decreases as we move away from the BH. If it originates from the thermal plasma of the Hawking temperature filling the Universe, the thermal plasma is present even far away from the BH. We expect that the finite-density corrections in the former case are milder than those in the latter. See Sec. IV for further discussion.

C. Bubble nucleation via sphaleron

At a sufficiently high temperature, the classical transition rate Γ_c dominates over the transition rate [30], which corresponds to a static solution of Eq. (3.38). Then, the transition rate is given by e^{-E_{sp}/T_*}, where E_{sp} is the bubble energy for the static solution. This is nothing but the sphaleron transition. In this subsection, we study this sphaleron transition in more detail.

1. de Sitter spacetime

First, let us focus on the Hawking-Moss transition in the de Sitter spacetime, where the metric in the static patch can be written as

$$\mathrm{d}s^2 = -f_{\mathrm{dS}}(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f_{\mathrm{dS}}(r)} + r^2\mathrm{d}\Omega, \tag{3.58}$$

$$f_{\mathrm{dS}}(r) = 1 - \frac{\Lambda r^2}{3}. \tag{3.59}$$

This coordinate has an apparent horizon with a radius of $H^{-1} = \sqrt{3/\Lambda}$.

Suppose that the transition occurs in the whole region inside the horizon, in which case we should take into account the boundary term of the gravitational action. Since the scalar field is static inside the bubble and the whole region is contained inside the bubble, the solution is static and the transition rate is dominated by the boundary term,

$$B = 8\pi^2\left(\frac{3}{V(\phi)} - \frac{3}{V_{\mathrm{FV}}}\right), \tag{3.60}$$

where $V(\phi)$ is the local maximal of the scalar potential. In gravity theory, the static solution results in $H = 0$ (i.e., $E = 0$) by the Hamiltonian constraint, which is the consequence of the least action principle for Laps function and some other components in the metric [80]. Therefore, the tunneling rate is determined only by the change of the boundary term ΔS_T for the static solution as Eq. (3.60) [82], analogous to the thermodynamic transition with a conserved energy, like a *microcanonical* picture.

A similar result can be derived when we neglect the effect of the change of metric and regard the metric as a background. This case is analogous to the thermodynamic transition with unchanged temperature, like a *canonical* picture, as we will see. First, note that we use the metric of Eq. (3.59), which has an apparent boundary at $r_H = H^{-1}$. It can be rewritten as

$$\mathrm{d}s^2 = -\mathrm{d}T^2 + \mathrm{d}S^2 + \mathrm{d}X^2 + \mathrm{d}Y^2 + \mathrm{d}Z^2, \tag{3.61}$$

with

$$-T^2 + S^2 + X^2 + Y^2 + Z^2 = H^{-2}, \tag{3.62}$$

$$T = \sqrt{H^{-2} - r^2}\sinh[Ht], \tag{3.63}$$

$$S = \sqrt{H^{-2} - r^2}\cosh[Ht], \tag{3.64}$$

$$X = r\sin\theta\cos\phi, \tag{3.65}$$

$$Y = r\sin\theta\sin\phi, \tag{3.66}$$

$$Z = r\cos\theta. \tag{3.67}$$

The metric has a singularity at $T = S = 0$ unless the time variable t is periodic in the imaginary part with a period of $T_{\mathrm{dS}}^{-1} = 2\pi/H$. Therefore, for an equilibrium state in de Sitter space, the propagator of a scalar field has a periodicity in $i(t - t')$ with a period of $T_{\mathrm{dS}}^{-1} = 2\pi/H$, which implies that the scalar field is mimicked as in a thermal system with a temperature of $T_{\mathrm{dS}} = H/2\pi$.[9] Thus, the scalar field has thermal fluctuation, and in fact Hawking and Moss have pointed out that the thermal transition of the Universe inside the horizon can occur due to the thermal effect of Hawking radiation in de Sitter spacetime. When we consider a nucleation of a bubble with the size of Hubble volume, it is static and its energy is given by

$$E_{\mathrm{sp}} = \frac{4\pi}{3}H^{-3}\Delta V(\phi). \tag{3.68}$$

The static bubble is nucleated by the following sphaleron rate:

$$\Gamma_{\mathrm{sp}} \propto e^{-E_{\mathrm{sp}}/T_{\mathrm{dS}}}. \tag{3.69}$$

This transition process is just the Hawking-Moss transition and the exponential factor is equal to the result [Eq. (3.60)] in the limit of $\Delta V \ll V$ $([V(\phi) - V_{\mathrm{FV}}]/V^2(\phi) \simeq 1/V(\phi) - 1/V_{\mathrm{FV}})$. This equivalence also implies that the above calculation of Eq. (3.60), where we include the boundary term as proposed in Refs. [51–53], corresponds to the transition due to the thermal effect with the Hawking temperature.

Next, let us consider the same theory with a metric which can describe the outer region of the boundary [29]. This result can be reproduced by calculating a distribution function of ϕ for a mode larger than the Hubble horizon scale. The metric is written by

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a_0^2 e^{2Ht}\mathrm{d}x^2 = (H\eta)^{-2}(\mathrm{d}\eta^2 - \mathrm{d}x^2), \tag{3.70}$$

where η is the conformal time defined by $a_0 e^{Ht} = (-H\eta)^{-1}$ for $-\infty < \eta < 0$. Let us first assume that the scalar field is massless, in which case the equation of motion is given by

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \frac{k^2}{a^2(t)}\phi_k = 0 \tag{3.71}$$

in the Fourier space. The solution is given by

$$\phi_k = \sqrt{\frac{\pi}{4}}H(-\eta)^{3/2}H_{3/2}^{(1)}(-k\eta) = \frac{H}{\sqrt{2k}}\left(\eta - \frac{i}{k}\right)e^{-ik\eta}, \tag{3.72}$$

where $H_{3/2}^{(1)}$ is the Hankel function of the first class. Here, we implicitly assume that the vacuum state annihilated by a_k's corresponds to the usual adiabatic vacuum in the limit of $\eta \to -\infty$. Such a vacuum is chosen in the context of inflation and corresponds to the one naturally defined in the metric Eq. (3.59). This is because well inside the horizon the scalar field is just a free theory one, which is equivalent to the one without metric (i.e., Minkowski metric). When we focus on a mode with the wavelength of the order of the Hubble length in a de Sitter vacuum, the particle distribution function $\rho(\phi, t)$ is calculated as

$$\rho(\phi, t) = N^{-1}\exp\left(-\frac{8\pi^2}{3H^4}\Delta V(\phi)\right), \tag{3.73}$$

where N is a normalization factor [29]. Therefore, the probability for a transition to ϕ_2 is given by

$$\Gamma \propto \exp[-8\pi^2/(3H^4)(V(\phi_2) - V_{\mathrm{FV}})]. \tag{3.74}$$

Since $T_{\mathrm{dS}} = H/(2\pi)$, this result is consistent with the above ones.

Here, we summarize these methods and results. When we consider the metric with a boundary at $r = H^{-1}$ and regard it as a background, the Hawking-Moss transition occurs due to the thermal effect [see Eq. (3.68)]. When we consider the same metric and take into account the Einstein equation as well as the equation of motion for the scalar field, the transition is a static solution and its rate is given by the surface term (or entropy) [see Eq. (3.60)]. When we consider the metric without a boundary and use the

[9] Unless it has a conformal coupling $\xi = 1/6$.

Bunch-Davis (BD) vacuum, the transition can be calculated from the method used by Starobinsky and Yokoyama [see Eq. (3.73)]. These calculations are equivalent because the initial state is the same in each.

We may obtain some insights into the origin of the thermal plasma of the Hawking temperature in the above calculation by noting that there are ambiguities in the meaning of a vacuum state in curved spacetime [81,83–85]. There is no apparent boundary in the metric Eq. (3.70), so it is natural to consider a nonsingular vacuum. It is actually the case for the BD vacuum $|0_{\rm BD}\rangle$ [85]. Then the transition rate is calculated from

$$e^{-\Gamma t_0} = \frac{|\langle \phi_{\rm bubble}, \tau = \tau_0 | \phi_{\rm FV}, \tau = 0, {\rm BD}\rangle|^2}{|\langle \phi_{\rm FV}, \tau = \tau_0, {\rm BD} | \phi_{\rm FV}, \tau = 0, {\rm BD}\rangle|^2} \tag{3.75}$$

$$= \int \mathcal{D}\phi e^{iS[\phi] - iS[\phi_{\rm FV}]}, \tag{3.76}$$

as explained above. Then, after the coordinate transformation to the metric Eq. (3.59), the imaginary time should be periodic so as to avoid the singularity at the apparent horizon. Since the periodicity of imaginary time effectively leads to a thermal effect on the scalar field, the BD vacuum state is seen as a thermally excited state by the observer using the latter metric. Therefore, for the latter observer, the transition rate from the same state is schematically written as

$$e^{-\Gamma t_0} = \frac{|\sum_{E,i} \langle \phi_{\rm bubble}, \tau = \tau_0 | E, i\rangle \langle E, i | \phi_{\rm FV}, \tau = 0, {\rm BD}\rangle|^2}{|\langle \phi_{\rm FV}, \tau = \tau_0, {\rm BD} | \phi_{\rm FV}, \tau = 0, {\rm BD}\rangle|^2}, \tag{3.77}$$

where we insert a complete set $\sum_{E,i} |E, i\rangle\langle E, i| = 1$. Since the BD vacuum is a thermally excited state with the temperature of $T_{\rm dS} = H/(2\pi)$ from the observer with the metric with the apparent boundary, we use $\langle E, i | \phi_{\rm FV}, \tau = 0, {\rm BD}\rangle \sim e^{-E/T_{\rm dS}}$ and rewrite it as

$$\Gamma \sim \int_0^{V_{\rm top}} {\rm d}E e^{-E/T_{\rm dS}} e^{-B(E)} + \int_{V_{\rm top}}^{\infty} {\rm d}E e^{-E/T_{\rm dS}}. \tag{3.78}$$

Note that we also have to include the boundary term in $B(E)$ because there is the apparent singularity. Of course, these results represent the nucleation rate from the same state $|\phi_{\rm FV}, \tau = 0, {\rm BD}\rangle$, so they should be equal to each other.

This example implies that the effective thermal effect is implicitly included when we calculate transition rates in a Euclidean background metric. This should also be true in the Schwarzschild–de Sitter spacetime. In fact, it has been discussed that a no-boundary wave function of BH in the Euclidean geometry coincides with the Hartle-Hawking vacuum state [86,87]. This fact may indicate that the bubble nucleation rate is enhanced by the thermal effect of the Hawking radiation.

2. Schwarzschild–de Sitter spacetime

Now, we move to the sphaleron transition in the Schwarzschild–de Sitter spacetime.

Let us first take the viewpoint of Sec. III A. There may be no solution of Eq. (3.13) for small values of M_- when M_+ is larger than a certain threshold value [51–53,60]. In particular, this is the case for $k_1 \gg 1$ and/or $k_2 \gg 1$. In this case, the transition rate is dominated by a static solution and the sum of the matter and the gravitational action vanishes due to the Hamiltonian constraint except for the boundary term. Thus, the transition rate is just given by the boundary term in ΔS_G as $2\pi \Delta A = (M_+^2 - M_-^2)/2$. Note that since the total energy is conserved in this calculation, the bubble energy is equal to the change of the BH mass: $\Delta E_{\rm bubble} = \Delta M$. Then the bounce action can be interpreted as a change of the entropy associated with the Hawking temperature ${\rm d}S = {\rm d}U/T_{{\rm BH},-}$ because

$$B_{\rm boundary} = \int {\rm d}M_- \frac{\rm d}{{\rm d}M_-} B_{\rm boundary} = -\int \frac{{\rm d}M_-}{T_{{\rm BH},-}} = \int {\rm d}S, \tag{3.79}$$

where we use Eq. (3.26). One can see that this viewpoint is analogous to the thermodynamic transition where the energy is conserved during that process, like a microcanonical picture.

Then, let us discuss the viewpoint of Sec. III B. There may be no solution satisfying the equation of motion Eq. (3.39) for the energy larger than a certain threshold value. This implies that the transition occurs classically with the probability function of Eq. (3.54) for such a large initial energy. Thus, we obtain

$$\Gamma_{\rm BH} \sim \int_0^{E_{\rm sp}} {\rm d}E_* e^{-P(E_*) - B(E_*)} + \int_{E_{\rm sp}} {\rm d}E_* e^{-P(E_*)}, \tag{3.80}$$

where the first term is just Eq. (3.52) and the second term is the classical contribution. The sphaleron energy $E_{\rm sp}$ is given by the bubble energy for the static solution. It is clear that we have the classical transition process from this viewpoint. When we use Eq. (3.54) and consider a system with a sufficiently high temperature, the classical transition rate Γ_c dominates over the transition rate in Eq. (2.40), which corresponds to a static solution of Eq. (3.38). In this case, the transition rate is given by $e^{-E_{\rm sp}/T_*}$, where $E_{\rm sp}$ is the bubble energy for the static solution. This viewpoint is analogous to the thermodynamic transition where the temperature is unchanged instead of the energy, like a canonical picture.

Note that these two pictures give the same result in a certain limit, as in the case of the equivalence between the microcanonical and canonical ensembles. In the viewpoint of Sec. III A, the energy is conserved $E_{\rm sp} = \Delta M$. Thus the transition rate of the sphaleron process in Sec. III B is interpreted as to $e^{-\Delta M/T_*}$. This can be farther rewritten as $e^{-M_{\rm BH}\Delta M} \simeq e^{-(M_+^2 - M_-^2)/2} = e^{-2\pi\Delta A}$, where we use $M_+ \simeq M_-$, which should be satisfied to match the result in Sec. III B regarding the metric as a background. Thus, both results coincide if the backreaction of the bubble energy to the metric can be neglected. This result also implies that the transition occurs via the thermal fluctuation of the Hawking temperature.

IV. CONCLUSIONS AND DISCUSSION

In this paper, we reconsidered the bubble nucleation around a BH by using an effective theory of a thin-wall bubble in the Schwarzschild–de Sitter spacetime. We calculated the bubble nucleation rate in Schwarzschild–de Sitter spacetime using three different methods. The first one is proposed in Refs. [51–53] and is the full calculation taking into account the backreaction to the metric. The other two are calculations in certain limits: a flat spacetime limit and a fixed-background limit. The bubble nucleation rate in these latter two methods can be decomposed into two factors: a quantum tunneling rate from a finite energy E and a probability of producing states with an energy E. The latter factor is just the Boltzmann factor in a finite-temperature system. Comparing these results with that of the full calculation used in the literature, we clarified the physical meaning of the enhancement factor due to the existence of BH. Namely, the enhancement factor can be interpreted as a probability of producing states with an energy ΔM, where ΔM is the difference of BH mass before and after the transition. This makes it clear that all the other states, such as plasma, are also generated through the same mechanism, and calls for finite-density corrections to the tunneling rate, which tend to stabilize the false vacuum. We showed also that the probability is just equal to the Boltzmann factor for the energy $E = \Delta M$. This means that the results of the latter two calculations coincide with that of the former only when we consider an activation due to the finite temperature and the temperature should be identified with the Hawking temperature associated with the BH horizon. This implies that, in the former calculation, the finite-temperature effect is implicitly taken into account. The enhancement of the bubble nucleation rate should be related to the Hawking radiation.

We showed also that the periodicity of the bounce solution is not necessarily related to the temperature of the system, but the consistency of those limits indicates that the bounce solutions around a BH coincide with the thermally activated tunneling associated with the BH Hawking temperature. Although the periodicity of *bounce solutions* as a function of its energy can be different from the one indicated by the conical singularity of BH horizon, we find that the dominant process can always be embedded in the Euclidean *spacetime* with a periodicity of the Hawking temperature.

It may be instructive to interpret two methods in the curved spacetime in terms of statistical mechanics. The former, where we fully include the effect of gravity, may correspond to the microcanonical ensemble. The energy is fixed and should be conserved before and after the transition. This is satisfied in the above calculation because the energy of the bubble is equal to the mass difference of BH ΔM. This interpretation is also supported by the fact that the transition is determined by the change of the BH entropy above a certain threshold. The other method, where we neglect the backreaction to the metric and consider a thermal activation, may correspond to the canonical ensemble, where temperature is fixed but the energy of the bubble is not necessarily conserved before and after the transition. In this case, the free energy should be minimized for the dominant transition process. In any case, the result should be the same in a limit where the backreaction can be neglected because these interpretations are irrelevant for the physical results.

According to our results and the above discussion, we conclude that the bubble nucleation around a BH is associated with a thermal effect of the Hawking temperature, and the enhancement factor is nothing but a probability of generating bubbles with a finite energy. The point here is that all the states other than bubbles should also be generated since it is hard to imagine a mechanism which only activates bubbles, though we have many other degrees of freedom in quantum field theory. There are two possible interpretations of the probability function. The enhancement may be due to a thermal plasma which fills a whole space outside of the BH as shown in the left panel of Fig. 8. On the other hand, it may be possible that the bubble nucleation occurs at the BH horizon with a nonzero kinetic energy, and it expands to a critical bubble (see the right panel in Fig. 8). If the former interpretation is correct, the obtained bounce solutions may not correspond to the realistic case, where a BH resides in an almost empty space, but rather to the case where a BH is surrounded by the thermal plasma of the BH Hawking temperature. This has been pointed out in Refs. [79,88,89] in de Sitter background. Moreover, the existence of such thermal plasma may reduce the probability of bubble nucleations. Its effect typically makes the transition difficult because the scalar field prefers a symmetric point in the field space due to the thermal mass. As a result, the bubble nucleation rate may not be so drastically enhanced even around a small BH. This viewpoint and question are also discussed in Ref. [63] (see also Refs. [61,62]). On the other hand, if the latter interpretation is correct, the Universe is not necessarily filled with the thermal plasma and the bounce solutions can be applied to the realistic case. Even in this case, we need to take into account the finite-density

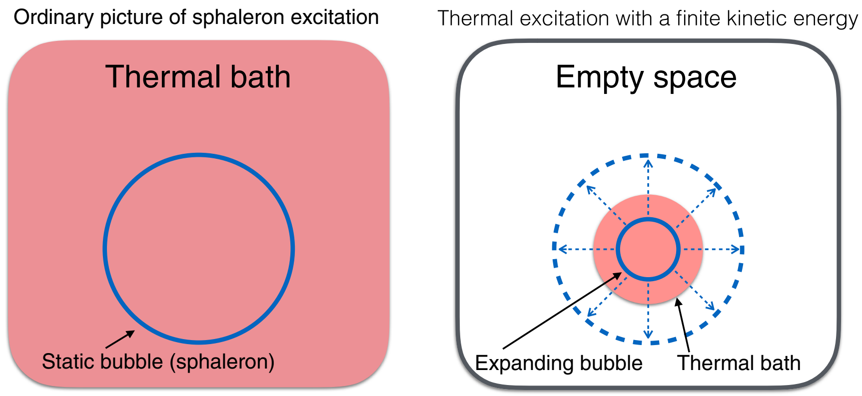

FIG. 8. Schematic figures that describe the excitation of a sphaleron bubble in the thermal plasma with an infinite volume (left panel) and in the thermal plasma with a finite volume whose size is smaller than the sphaleron solution (right panel). The process that the bubble nucleation at the BH radius by the Hawking radiation corresponds to is described in the right panel.

corrections to the effective potential because there exists a flux of Hawking radiation emitted from the BH as done in Ref. [90] (see, e.g., Refs. [91–95] for related work). Notice that, however, this interpretation leaves an open question whether or not a bubble can be excited with a nonzero kinetic energy by a finite-volume thermal bath (or at the BH horizon) whose size is much smaller than the critical bubble, and expands to a critical bubble. Nevertheless, we postpone the conclusion in this paper; rather we explain how to include the finite-density corrections of Hawking radiation. Detailed studies on this aspect will be presented elsewhere [96].

Thermal effects originate from the ambiguity of the vacuum state at the false vacuum. When we perform the Wick rotation and calculate the Euclidean action, we use a specific metric where the components of the metric do not depend on the (imaginary) time variable. However, it is well known that the vacuum state has an observer dependence [81,83–85]. For example, a vacuum state defined by a freely falling observer around a BH is a thermally excited state for a static observer. This ambiguity of the vacuum state may add an implicit assumption on the initial state when we calculate a Euclidean action in general relativity. Regarding the observer dependence, we have also obtained a consistent result in the case of bubble nucleations in a de Sitter universe which supports this viewpoint. Our consideration clarifies that we have to take particular care of the initial vacuum state in the Wick-rotated Euclidean spacetime to calculate the bubble nucleation rate around a BH. This is also supported by the fact that a no-boundary wave function of BH in the Euclidean geometry coincides with the Hartle-Hawking vacuum state [86,87].

Strictly speaking, a BH surrounded by an infinite thermal plasma of the BH Hawking temperature in a de Sitter or flat spacetime is thermodynamically unstable. This is because a larger/smaller BH gets fat/light by accreting/emitting particles from/to the thermal plasma, and thus the system is unstable under the perturbation. Although we naively expect that this effect could be neglected if one restricts the validity of the calculation to the case where the lifetime of our vacuum is much shorter than the evaporation time scale of the BH, a more rigorous way to test the procedure in Refs. [51–53] may be to study the anti–de Sitter Schwarzschild metric as done recently in Ref. [97].

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[1] G. Aad *et al.* (ATLAS Collaboration), Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. B **716**, 1 (2012).

[2] S. Chatrchyan *et al.* (CMS Collaboration), Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, Phys. Lett. B **716**, 30 (2012).

[3] P. P. Giardino, K. Kannike, I. Masina, M. Raidal, and A. Strumia, The universal Higgs fit, J. High Energy Phys. 05 (2014) 046.

[4] G. Aad *et al.* (ATLAS Collaboration), Measurements of Higgs boson production and couplings in diboson final states with the ATLAS detector at the LHC, Phys. Lett. B **726**, 88 (2013); Corrigendum, Phys. Lett. B **734**, 406(E) (2014).

[5] S. Chatrchyan *et al.* (CMS Collaboration), Measurement of the properties of a Higgs boson in the four-lepton final state, Phys. Rev. D **89**, 092007 (2014).

[6] G. Aad *et al.* (ATLAS and CMS Collaborations), Combined Measurement of the Higgs Boson Mass in pp Collisions at $\sqrt{s} = 7$ and 8 TeV with the ATLAS and CMS Experiments, Phys. Rev. Lett. **114**, 191803 (2015).

[7] V. Khachatryan *et al.* (CMS Collaboration), Measurement of the top quark mass using proton-proton data at $\sqrt{(s)} = 7$ and 8 TeV, Phys. Rev. D **93**, 072004 (2016).

[8] M. Sher, Electroweak Higgs potentials and vacuum stability, Phys. Rep. **179**, 273 (1989).

[9] G. W. Anderson, New cosmological constraints on the Higgs boson and top quark masses, Phys. Lett. B **243**, 265 (1990).

[10] P. B. Arnold, Can the electroweak vacuum be unstable?, Phys. Rev. D **40**, 613 (1989).

[11] P. B. Arnold and S. Vokos, Instability of hot electroweak theory: Bounds on m(H) and M(t), Phys. Rev. D **44**, 3620 (1991).

[12] J. R. Espinosa and M. Quiros, Improved metastability bounds on the standard model Higgs mass, Phys. Lett. B **353**, 257 (1995).

[13] G. Isidori, G. Ridolfi, and A. Strumia, On the metastability of the standard model vacuum, Nucl. Phys. **B609**, 387 (2001).

[14] J. R. Espinosa, G. F. Giudice, and A. Riotto, Cosmological implications of the Higgs mass measurement, J. Cosmol. Astropart. Phys. 05 (2008) 002.

[15] J. Ellis, J. R. Espinosa, G. F. Giudice, A. Hoecker, and A. Riotto, The probable fate of the standard model, Phys. Lett. B **679**, 369 (2009).

[16] J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, A. Riotto, and A. Strumia, Higgs mass implications on the stability of the electroweak vacuum, Phys. Lett. B **709**, 222 (2012).

[17] F. Bezrukov, M. Yu. Kalmykov, B. A. Kniehl, and M. Shaposhnikov, Higgs boson mass and new physics, J. High Energy Phys. 10 (2012) 140.

[18] G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, and A. Strumia, Higgs mass and vacuum stability in the Standard Model at NNLO, J. High Energy Phys. 08 (2012) 098.

[19] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio, and A. Strumia, Investigating the near-criticality of the Higgs boson, J. High Energy Phys. 12 (2013) 089.

[20] A. V. Bednyakov, B. A. Kniehl, A. F. Pikelner, and O. L. Veretin, Stability of the Electroweak Vacuum: Gauge Independence and Advanced Precision, Phys. Rev. Lett. **115**, 201802 (2015).

[21] S. R. Coleman, Fate of the false vacuum: Semiclassical theory, Phys. Rev. D **15**, 2929 (1977); Erratum, Phys. Rev. D **16**, 1248(E) (1977).

[22] S. R. Coleman and F. De Luccia, Gravitational effects on and of vacuum decay, Phys. Rev. D **21**, 3305 (1980).

[23] A. D. Linde, Decay of the false vacuum at finite temperature, Nucl. Phys. **B216**, 421 (1983); Erratum, Nucl. Phys. **B223**, 544(E) (1983).

[24] A. H. Guth and S. Y. Pi, Fluctuations in the New Inflationary Universe, Phys. Rev. Lett. **49**, 1110 (1982).

[25] A. A. Starobinsky, Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations, Phys. Lett. **117B**, 175 (1982).

[26] S. W. Hawking, The development of irregularities in a single bubble inflationary universe, Phys. Lett. **115B**, 295 (1982).

[27] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Spontaneous creation of almost scale-free density perturbations in an inflationary universe, Phys. Rev. D **28**, 679 (1983).

[28] S. W. Hawking and I. G. Moss, Fluctuations in the inflationary universe, Nucl. Phys. **B224**, 180 (1983).

[29] A. A. Starobinsky and J. Yokoyama, Equilibrium state of a selfinteracting scalar field in the De Sitter background, Phys. Rev. D **50**, 6357 (1994).

[30] S. W. Hawking and I. G. Moss, Supercooled phase transitions in the very early universe, Phys. Lett. **110B**, 35 (1982).

[31] O. Lebedev and A. Westphal, Metastable electroweak vacuum: Implications for inflation, Phys. Lett. B **719**, 415 (2013).

[32] K. Enqvist, T. Meriniemi, and S. Nurmi, Generation of the Higgs condensate and its decay after inflation, J. Cosmol. Astropart. Phys. 10 (2013) 057.

[33] A. Kobakhidze and A. Spencer-Smith, Electroweak vacuum (in)stability in an inflationary universe, Phys. Lett. B **722**, 130 (2013).

[34] M. Herranen, T. Markkanen, S. Nurmi, and A. Rajantie, Spacetime Curvature and the Higgs Stability During Inflation, Phys. Rev. Lett. **113**, 211102 (2014).

[35] A. Hook, J. Kearney, B. Shakya, and K. M. Zurek, Probable or Improbable Universe? Correlating electroweak vacuum instability with the scale of inflation, J. High Energy Phys. 01 (2015) 061.

[36] A. Kobakhidze and A. Spencer-Smith, The Higgs vacuum is unstable, arXiv:1404.4709.

[37] K. Kamada, Inflationary cosmology and the standard model Higgs with a small Hubble induced mass, Phys. Lett. B **742**, 126 (2015).

[38] K. Enqvist, T. Meriniemi, and S. Nurmi, Higgs dynamics during inflation, J. Cosmol. Astropart. Phys. 07 (2014) 025.

[39] M. Fairbairn and R. Hogan, Electroweak Vacuum Stability in Light of BICEP2, Phys. Rev. Lett. **112**, 201801 (2014).

[40] J. R. Espinosa, G. F. Giudice, E. Morgante, A. Riotto, L. Senatore, A. Strumia, and N. Tetradis, The cosmological Higgstory of the vacuum instability, J. High Energy Phys. 09 (2015) 174.

[41] J. Kearney, H. Yoo, and K. M. Zurek, Is a Higgs vacuum instability fatal for high-scale inflation?, Phys. Rev. D **91**, 123537 (2015).

[42] K. Kohri and H. Matsui, Electroweak vacuum instability and renormalized Higgs Field vacuum fluctuations in the inflationary universe, J. Cosmol. Astropart. Phys. 08 (2017) 011.

[43] W. E. East, J. Kearney, B. Shakya, H. Yoo, and K. M. Zurek, Spacetime dynamics of a Higgs vacuum instability during inflation, Phys. Rev. D **95**, 023526 (2017).

[44] A. Joti, A. Katsis, D. Loupas, A. Salvio, A. Strumia, N. Tetradis, and A. Urbano, (Higgs) vacuum decay during inflation, J. High Energy Phys. 07 (2017) 058.
[45] M. Herranen, T. Markkanen, S. Nurmi, and A. Rajantie, Spacetime Curvature and Higgs Stability After Inflation, Phys. Rev. Lett. **115**, 241301 (2015).
[46] Y. Ema, K. Mukaida, and K. Nakayama, Fate of electroweak vacuum during preheating, J. Cosmol. Astropart. Phys. 10 (2016) 043.
[47] K. Kohri and H. Matsui, Higgs vacuum metastability in primordial inflation, preheating, and reheating, Phys. Rev. D **94**, 103509 (2016).
[48] K. Enqvist, M. Karciauskas, O. Lebedev, S. Rusak, and M. Zatta, Postinflationary vacuum instability and Higgs-inflaton couplings, J. Cosmol. Astropart. Phys. 11 (2016) 025.
[49] M. Postma and J. van de Vis, Electroweak stability and non-minimal coupling, J. Cosmol. Astropart. Phys. 05 (2017) 004.
[50] Y. Ema, M. Karciauskas, O. Lebedev, and M. Zatta, Early Universe Higgs dynamics in the presence of the Higgs-inflaton and non-minimal Higgs-gravity couplings, J. Cosmol. Astropart. Phys. 06 (2017) 054.
[51] R. Gregory, I. G. Moss, and B. Withers, Black holes as bubble nucleation sites, J. High Energy Phys. 03 (2014) 081.
[52] P. Burda, R. Gregory, and I. Moss, Gravity and the Stability of the Higgs Vacuum, Phys. Rev. Lett. **115**, 071303 (2015).
[53] P. Burda, R. Gregory, and I. Moss, Vacuum metastability with black holes, J. High Energy Phys. 08 (2015) 114.
[54] W. A. Hiscock, Can black holes nucleate vacuum phase transitions?, Phys. Rev. D **35**, 1161 (1987).
[55] V. A. Berezin, V. A. Kuzmin, and I. I. Tkachev, O(3) invariant tunneling in General Relativity, Phys. Lett. B **207**, 397 (1988).
[56] P. B. Arnold, Gravity and false vacuum decay rates: O(3) solutions, Nucl. Phys. **B346**, 160 (1990).
[57] V. A. Berezin, V. A. Kuzmin, and I. I. Tkachev, Black holes initiate false vacuum decay, Phys. Rev. D **43**, R3112 (1991).
[58] A. Gomberoff, M. Henneaux, C. Teitelboim, and F. Wilczek, Thermal decay of the cosmological constant into black holes, Phys. Rev. D **69**, 083520 (2004).
[59] J. Garriga and A. Megevand, Decay of de Sitter vacua by thermal activation, Int. J. Theor. Phys. **43**, 883 (2004).
[60] P. Burda, R. Gregory, and I. Moss, The fate of the Higgs vacuum, J. High Energy Phys. 06 (2016) 025.
[61] N. Tetradis, Black holes and Higgs stability, J. Cosmol. Astropart. Phys. 09 (2016) 036.
[62] D. Canko, I. Gialamas, G. Jelic-Cizmek, A. Riotto, and N. Tetradis, On the catalysis of the electroweak vacuum decay by black holes at high temperature, arXiv:1706.01364.
[63] D. Gorbunov, D. Levkov, and A. Panin, Fatal youth of the Universe: Black hole threat for the electroweak vacuum during preheating, arXiv:1704.05399.
[64] K. Sato, M. Sasaki, H. Kodama, and K.-i. Maeda, Creation of wormholes by first order phase transition of a vacuum in the early universe, Prog. Theor. Phys. **65**, 1443 (1981).
[65] K.-i. Maeda, K. Sato, M. Sasaki, and H. Kodama, Creation of de Sitter–Schwarzschild Wormholes by a cosmological first order phase transition, Phys. Lett. **108B**, 98 (1982).
[66] K. Sato, H. Kodama, M. Sasaki, and K.-i. Maeda, Multi-production of universes by first order phase transition of a vacuum, Phys. Lett. **108B**, 103 (1982).
[67] V. A. Berezin, V. A. Kuzmin, and I. I. Tkachev, Thin wall vacuum domains evolution, Phys. Lett. **120B**, 91 (1983).
[68] S. K. Blau, E. I. Guendelman, and A. H. Guth, Dynamics of false vacuum bubbles, Phys. Rev. D **35**, 1747 (1987).
[69] J. Garriga and A. Vilenkin, Recycling universe, Phys. Rev. D **57**, 2230 (1998).
[70] J. Garriga, A. Vilenkin, and J. Zhang, Black holes and the multiverse, J. Cosmol. Astropart. Phys. 02 (2016) 064.
[71] H. Deng, J. Garriga, and A. Vilenkin, Primordial black hole and wormhole formation by domain walls, J. Cosmol. Astropart. Phys. 04 (2017) 050.
[72] N. Oshita and J. Yokoyama, Creation of an inflationary universe out of a black hole, arXiv:1601.03929.
[73] R. Basu, A. H. Guth, and A. Vilenkin, Quantum creation of topological defects during inflation, Phys. Rev. D **44**, 340 (1991).
[74] A. O. Caldeira and A. J. Leggett, Quantum tunneling in a dissipative system, Ann. Phys. (N.Y.) **149**, 374 (1983).
[75] L.-D. Chang and S. Chakravarty, Quantum decay in a dissipative system, Phys. Rev. B **29**, 130 (1984).
[76] E. J. Weinberg, Vacuum decay in theories with symmetry breaking by radiative corrections, Phys. Rev. D **47**, 4614 (1993).
[77] S. Matsumoto, K. I. Nagao, M. Nakamura, and M. Senami, Fate of the false vacuum revisited, arXiv:1009.1927.
[78] B. Garbrecht and P. Millington, Self-consistent solitons for vacuum decay in radiatively generated potentials, Phys. Rev. D **92**, 125022 (2015).
[79] A. R. Brown and E. J. Weinberg, Thermal derivation of the Coleman–De Luccia tunneling prescription, Phys. Rev. D **76**, 064003 (2007).
[80] S. W. Hawking and G. T. Horowitz, The Gravitational Hamiltonian, action, entropy and surface terms, Classical Quantum Gravity **13**, 1487 (1996).
[81] G. W. Gibbons and S. W. Hawking, Action integrals and partition functions in quantum gravity, Phys. Rev. D **15**, 2752 (1977).
[82] N. Oshita and J. Yokoyama, Entropic interpretation of the Hawking-Moss bounce, Prog. Theor. Exp. Phys. (**2016**), 051E02 2016.
[83] W. G. Unruh, Notes on black hole evaporation, Phys. Rev. D **14**, 870 (1976).
[84] G. W. Gibbons and S. W. Hawking, Cosmological event horizons, thermodynamics, and particle creation, Phys. Rev. D **15**, 2738 (1977).
[85] T. S. Bunch and P. C. W. Davies, Quantum field theory in de Sitter space: Renormalization by point splitting, Proc. R. Soc. A **360**, 117 (1978).
[86] J. B. Hartle and S. W. Hawking, Path integral derivation of black hole radiance, Phys. Rev. D **13**, 2188 (1976).
[87] A. O. Barvinsky, V. P. Frolov, and A. I. Zelnikov, Wavefunction of a black hole and the dynamical origin of entropy, Phys. Rev. D **51**, 1741 (1995).
[88] S. H. Henry Tye, D. Wohns, and Y. Zhang, Coleman–de Luccia tunneling and the Gibbons-Hawking temperature, Int. J. Mod. Phys. A **25**, 1019 (2010).

[89] A. Masoumi and E. J. Weinberg, Bounces with $O(3) \times O(2)$ symmetry, Phys. Rev. D **86**, 104029 (2012).
[90] C. Cheung and S. Leichenauer, Limits on new physics from black holes, Phys. Rev. D **89**, 104035 (2014).
[91] P. Candelas, Vacuum polarization in Schwarzschild spacetime, Phys. Rev. D **21**, 2185 (1980).
[92] I. G. Moss, Black-hole bubbles, Phys. Rev. D **32**, 1333 (1985).
[93] J. H. MacGibbon, B. J. Carr, and D. N. Page, Do evaporating black holes form photospheres?, Phys. Rev. D **78**, 064043 (2008).
[94] S. B. Giddings, Hawking radiation, the Stefan-Boltzmann law, and unitarization, Phys. Lett. B **754**, 39 (2016).
[95] S. Emelyanov, Near-horizon physics of an evaporating black hole: One-loop effects in the $\lambda\Phi^4$-theory, arXiv: 1608.05318.
[96] N. Oshita, K. Mukaida, M. Yamada, and J. Yokoyama (unpublished).
[97] P. Chen, G. Domnech, M. Sasaki, and D.-h. Yeom, Thermal activation of thin-shells in anti–de Sitter black hole spacetime, J. High Energy Phys. 07 (2017) 134.