

**Effective models of inflation from a nonlocal framework**Alexey S. Koshelev,<sup>1,2,3,\*</sup> K. Sravan Kumar,<sup>1,†</sup> and Paulo Vargas Moniz<sup>1,‡</sup><sup>1</sup>*Departamento de Física, Centro de Matemática e Aplicações (CMA-UBI),  
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The dilaton is a possible inflaton candidate following recent CMB data allowing a nonminimal coupling to the Ricci curvature scalar in the early Universe. In this paper, we introduce an approach that has seldom been used in the literature, namely dilaton inflation with non-local features. More concretely, employing non-local features expressed in [J. High Energy Phys. 04 (2007) 029], we study quadratic variations around a de Sitter geometry of an effective action with a nonlocal dilaton. The nonlocality refers to an infinite derivative kinetic term involving the operator  $\mathcal{F}(\square)$ . Algebraic roots of the characteristic equation  $\mathcal{F}(z) = 0$  play a crucial role in determining the properties of the theory. We subsequently study the cases when  $\mathcal{F}(\square)$  has one real root and one complex root, from which we retrieve two concrete effective models of inflation. In the first case we retrieve a class of single field inflations with universal prediction of  $n_s \sim 0.967$  with any value of the tensor to scalar ratio  $r < 0.1$  intrinsically controlled by the root of the characteristic equation. The second case involves a new class of two field conformally invariant models with a peculiar quadratic cross-product of scalar fields. In this latter case, we obtain Starobinsky-like inflation through a spontaneously broken conformal invariance. Furthermore, an uplifted minimum of the potential, which accounts for the vacuum energy after inflation is produced naturally through this mechanism intrinsically within our approach.

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Primordial inflation is a compelling paradigm for describing the early Universe. This is manifest through convincing observational data [1]. The end of inflation is characterized by primordial perturbations which are eventually responsible for the structure formation in the Universe. Their characteristics, namely, spectral tilts and the ratio of tensor to scalar power spectra  $r$ , have been recently measured:  $r$  has a well-established upper limit.<sup>1</sup>  $r < 0.1$  at 95% confidence level from Planck 2015 [1,3], whereas the scalar tilt is most precisely measured as  $n_s = 0.968 \pm 0.006$  at 95% confidence level. The CMB power spectra are so far found to be very much adiabatic, scale invariant and Gaussian [1,4], supporting thereby  $f(R)$  or single field inflation models. Among a broad class of models, the Starobinsky model based on the  $R + R^2$  gravity modification and the Higgs inflation [5–7] occupy a privileged position, with practically equal predictions in the  $(n_s, r)$  plane  $n_s = 1 - 2/N$ ,  $r = 12/N^2$ , where  $N$  is the number of  $e$ -foldings before the end of inflation. For the expected value  $N \approx 60$  the above predictions match very well the current observational values and constraints.

However, the physical nature of the inflaton and the corresponding mechanism driving the early universe accelerated expansion are still an open issue [8,9].

It can be added that according to the present observations, the Hubble parameter during inflation can be as large as  $10^{15}$  GeV, suggesting the scale of inflation to be of the order of  $M_I > 10^{15}$  GeV. These energy scales are acceptable in supergravity (SUGRA) and string theory, hence argued to play a crucial role [10]. Therefore, during the last years there have been many attempts to embed the inflationary picture into low energy effective theories derived from such fundamental approaches [11–15]. Furthermore, the observational data provided a special stimulus to studies of inflation in SUGRA and string theory. More precisely, flat potentials of the following form

$$V \sim (1 - e^{-\sqrt{2/3}B\phi})^{2n}. \quad (1)$$

became successful candidates for the description of inflation and appeared in various scenarios [8,9,11]. The parameter  $B$  in the above potential can lead to any value of  $r < 0.1$  with a universal value for  $n_s$  as it is in the  $R^2$  model, namely

$$n_s = 1 - \frac{2}{N}, \quad r = \frac{12B}{N^2}. \quad (2)$$

Such predictions are so far shown to occur in the low energy effective models of string theory/SUGRA and

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<sup>1</sup> $r < 0.07$  considering the recent results of BICEP2/Keck Array [2].

modified gravity [16–22]. In addition, several other models inspired from string theory were also successful in confronting Planck data [23–27].

Following the current observational status of inflation [8,9], a nonminimally coupled scalar was established as a suitable candidate. In this regard, it is possible consider a closed string dilaton in string theory as a candidate of interest. Embracing string theory as a key player in cosmological inflation, we take an inspiration from string field theory (SFT) [28,29] where nonlocality can naturally emerge in the action. Previous attempts considering inflation with non-locality features was done with  $p$ -adic strings [30,31]. Moreover, configurations of nonlocality lead to effective field theories with one or more scalar degrees of freedom [32].

In the context of the previous paragraphs, we will argue that interesting inflationary scenarios can be produced with nonlocal features of the dilaton. More precisely, the nonlocal nature of the dilaton is characterized by the function  $\mathcal{F}(\square) = \sum_{n=0}^{\infty} f_n \square^n$ , where  $\square$  is the d'Alembertian.

Depending on the number of roots of the characteristic equation  $\mathcal{F}(z) = 0$ , following the studies of [32–34], we can write effective actions that are equivalent up to the quadratic perturbations. More specifically, if  $\mathcal{F}(\square)$  has only one real root at  $z_1$ , the corresponding effective action contains just one propagating scalar where the kinetic term contains the parameter  $\mathcal{F}'(z_1)$  and any higher derivatives can be neglected assuming the field slow-rolls on a sufficiently flat potential. As a consequence, we can write a successful (albeit trivial) single field inflation with controlled slow-roll dynamics through the parameter  $\mathcal{F}'(z_1)$ , which leads to the prediction of  $r$  in (2). Far much more interesting if  $\mathcal{F}(\square)$  has a complex root the corresponding effective action contains instead two real scalar fields, which we will show to bear conformal invariance. In this case, the two scalar fields share an opposite sign of kinetic terms. From a spontaneous breaking of conformal symmetry, we gauge fix one of the scalar field and obtain a Starobinsky-like inflation, accompanied with a nontrivial uplifting of the inflaton potential towards a nonzero minimum.

This paper is organized as follows. In Sec. II, we start with a quadratic action with dilatonic perturbations around de Sitter (dS). We prescribe subsequently a method to write an effective action bearing nonlocality. In Sec. II B we study in detail two particular effective actions which leads to interesting inflationary scenarios. In Sec. III we summarize and discuss our inflationary scenarios. We refer to the Appendix A for additional notes on SFT and tachyon condensation (TC). Appendix B suggest a framework concerning nonlocal dilaton within string theory.

Through out the paper, we set the metric signature  $(-, +, +, +)$ , small Greek letters are the fully covariant indexes and the units  $\hbar = 1, c = 1, M_p^2 = \frac{1}{8\pi G}$ .

## II. EFFECTIVE FIELDS FROM A NONLOCAL FRAMEWORK

The attractor models of inflation leading to the predictions in (2) essentially have a Starobinsky like potential<sup>2</sup> with a parameter  $B$ . The realization of a Starobinsky like potential can be achieved via employing a nonminimally coupled scalar or conformal models [19,35]. The parameter  $B$  here<sup>3</sup> mainly defines the coefficient of the inflaton kinetic term. In string (field) theory the kinetic term of a scalar naturally comes with analytic infinite derivative function (non-locality). Assuming there exists a nonlocal dilaton<sup>4</sup> in a 4D effective version of string theory,<sup>5</sup> we can realize the parameter  $B$  by analyzing the linearization of the theory around a dS in a local limit.<sup>6</sup>

The second order action (scalar part) of an effective theory of nonlocal dilaton around dS should look like

$$\delta^{(2)}S = \frac{1}{2} \int d^4x \sqrt{-g} \varphi \mathcal{F}(\square) \varphi, \quad (3)$$

where

$$\mathcal{F}(\square) = M_p^2(2\square + 3R_0) + \tilde{\mathcal{F}}(\square) \quad (4)$$

where  $\varphi$  is the perturbation of the dilatonic field ( $\phi$ ),  $R_0$  is the scalar curvature around dS,  $M_p$  is the reduced Planck mass and  $\tilde{\mathcal{F}}(\square) = \sum_{n=1}^{\infty} f_n \square^n$ . Although during slow-roll inflation infinite derivatives are not so relevant, it is pertinent to identify the coefficients  $f_n$ .

To generate inflation we must have an appropriate potential in our setup. At present, the state of the art of the knowledge in string (field) theory lacks established methods to do so. In the course of this paper, we will employ potentials phenomenologically which we assume do not violate general principles of string theoretical construction (cf. the Appendices A, B for more discussions on this issue).

Considering therefore (3) for a general operator function  $\mathcal{F}(\square)$  we cannot convey inflationary physics straightforwardly. In general,  $\mathcal{F}(\square)$  being considered as an algebraic function may have many roots. That is, equation

$$\mathcal{F}(z) = 0 \quad (5)$$

can have more than one solution. We name it a characteristic equation. Because of that, the propagator for the field  $\varphi$  will have more than one pole. As such, it is equivalent to

<sup>2</sup>corresponding to canonical scalar field.

<sup>3</sup>i.e., so far shown to be obtained in SUGRA/string theory settings [16–22].

<sup>4</sup>That is naturally coupled to Ricci scalar.

<sup>5</sup>In Appendix. B we suggest a mechanism for obtaining non-local dilaton based on string (field) theory inspired set up. We defer further development of our Appendix in a subsequent study [36].

<sup>6</sup>It is natural to expect infinite derivatives are unimportant at the inflationary energy scales.

multiple degrees of freedom. Let us therefore write a local realization of (3). Originally, this was done in [32] and then formalized in [33,34,37]. We use the Weierstrass factorization [32] which prescribes that any entire function (we recall that SFT ensures that operators  $\mathcal{F}(\square)$  are analytic functions and in all existing computations they appear to be entire functions) can be written as

$$\mathcal{F}(z) = e^{\gamma(z)} \prod_j (z - z_j)^{m_j}, \quad (6)$$

where  $z_j$  are roots of the characteristic equation and  $m_j$  are their respective multiplicities. We further assume hereafter that all  $m_j = 1$  for simplicity.  $\gamma(z)$  is an entire function and as such its exponent has no roots on the whole complex plane. It was shown in [32] that for a quadratic Lagrangian of the type (3), a local equivalent quadratic Lagrangian can be constructed as

$$\delta^2 S_{\text{local}} = \frac{1}{2} \int d^4x \sqrt{-g} \sum_j \mathcal{F}'(z_j) \varphi_j(\square - z_j) \varphi_j, \quad (7)$$

where prime means derivative with respect to the argument  $z$  with the further evaluation at the point  $z_j$ . It is said to be equivalent because of the fact that solution for  $\varphi$ , which can be obtained from equations of motion following from (3), is connected to solutions for  $\varphi_j$  simply as

$$\varphi = \sum \varphi_j. \quad (8)$$

Roots  $z_j$  become therefore the most crucial elements and several comments are in order here:

- (i) Note that roots  $z_j$  can be complex in general. One real root  $z_1$  is the simplest situation (cf. Sec. II A). In this case, we have just a Lagrangian for a massive scalar. It is acceptable if  $\mathcal{F}'(z_1) > 0$  in order to evade a ghost in the spectrum.
- (ii) More than one real root apparently seems not to be a promising scenario. Since the function  $\mathcal{F}(z)$  is analytic (and therefore continuous), neighboring real roots will be accompanied with  $\mathcal{F}'(z_j)$  of opposite signs. In other words, one root is normal and the next to it is a ghost. We study an effective model corresponding to this case in Sec. II B.

### A. Effective model of single field inflation

If  $\mathcal{F}(z)$  has one real root, then (7) contains a single scalar degrees of freedom

$$\delta^2 S_{\text{local}} = \frac{1}{2} \int d^4x \sqrt{-g} \mathcal{F}'(z_1) \varphi(\square - z_1) \varphi \quad (9)$$

The effective action which is perturbatively equivalent up to quadratic order to (9) around dS background, looks like (taking  $M_p = 1$ )

$$S_1 = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \tilde{\Phi}^2 R - \frac{A}{2} \partial \tilde{\Phi}^2 - V(\tilde{\Phi}) \right], \quad (10)$$

where  $\tilde{\Phi}$  is an effective dilatonic field and the respective correspondence is

$$\begin{aligned} \mathcal{F}'(z_1) &= 6 + A \\ \mathcal{F}'(z_1) z_1 &= 3R_0 - V''(\tilde{\Phi}_0). \end{aligned} \quad (11)$$

Here  $R_0$  is scalar curvature of the dS vacuum solution for a constant  $\tilde{\Phi}$ . Assuming the generalized structure of from the proposed action (B2), the potential  $V(\tilde{\Phi})$  can be taken to be arbitrary. If we consider a potential  $V_J(\tilde{\Phi}) = V_0(-\tilde{\Phi}^2 + \tilde{\Phi}^4)^2$  which looks in the Einstein frame as

$$V_E = \tilde{V}_0 \left( 1 - e^{-\sqrt{\frac{2}{3|\mathcal{F}'(z_1)/6|}} \tilde{\phi}} \right)^2, \quad (12)$$

where  $\tilde{\phi}$  is canonically normalized field by defining  $\tilde{\Phi} = e^{-\sqrt{\frac{2}{3|\mathcal{F}'(z_1)/6|}} \tilde{\phi}}$ . The inflationary predictions corresponding to the potential in (12) are well known [16,19,38,39] and in particular we retrieve

$$n_s = 1 - \frac{2}{N}, \quad r = \frac{2\mathcal{F}'(z_1)}{N^2}, \quad (13)$$

where we consider  $N = 60$  number of  $e$ -foldings. We therefore conclude that provided the non-local operator  $\mathcal{F}(\square)$  contains one real root, it gives a successful inflation with a universal prediction of  $n_s = 0.967$  and the tensor to scalar ratio  $r < 0.1$ . The value of  $r$  can be varied to any value by varying the nonlocal parameter  $\mathcal{F}'(z_1)$ .

### B. Effective model of conformal inflation

If  $\mathcal{F}(z)$  has a complex root then we should write (7) for a scalar field and also for its complex conjugate. So considering such a pair of complex conjugate roots, we have

$$\begin{aligned} \delta^2 S_{\text{local}} &= \frac{1}{2} \int d^4x \sqrt{-g} [\mathcal{F}'(z_1) \varphi_1(\square - z_1) \varphi_1 \\ &\quad + \mathcal{F}'(\bar{z}_1) \bar{\varphi}_1(\square - \bar{z}_1) \bar{\varphi}_1], \end{aligned} \quad (14)$$

where a bar over represents the complex conjugates. To maintain the connection with (3) we should consider complex conjugate solutions to equations of motion, such that  $\varphi = \varphi_1 + \bar{\varphi}_1$  is real. The important feature is that the quadratic form of fields is already diagonal. Introducing  $\varphi_1 = \chi + i\sigma$ ,  $z_1 = \alpha + i\beta$ ,  $\mathcal{F}'(z_1) = c + is$  we can rewrite (14) in terms of real components as

$$\begin{aligned} \delta^2 S_{\text{local}} &= \int d^4x \sqrt{-g} [\chi(c\square - c\alpha + s\beta)\chi \\ &\quad - \sigma(c\square - c\alpha + s\beta)\sigma - 2\chi(s\square - s\alpha - c\beta)\sigma]. \end{aligned} \quad (15)$$

The above expression is inevitably non-diagonal and features a cross-product of real fields  $\sim \chi\sigma$ . In this

formulation, note that the two fields  $\chi$ ,  $\sigma$  share an opposite sign concerning their kinetic terms [40].

Let us now show that the following effective action with two fields with conformal invariance, can be perturbatively equivalent up to quadratic order to (15) around dS background

$$S_2 = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} [\tilde{\alpha}\tilde{\Phi}_1^2 - \tilde{\alpha}\tilde{\Phi}_2^2 - 2\tilde{\beta}\tilde{\Phi}_1\tilde{\Phi}_2] f\left(\frac{\tilde{\Phi}_2}{\tilde{\Phi}_1}\right) R + \frac{A}{2} [\tilde{\alpha}\partial\tilde{\Phi}_1^2 - \tilde{\alpha}\partial\tilde{\Phi}_2^2 - 2\tilde{\beta}\partial_\mu\tilde{\Phi}_1\partial^\mu\tilde{\Phi}_2] f\left(\frac{\tilde{\Phi}_2}{\tilde{\Phi}_1}\right) - V(\tilde{\Phi}_1, \tilde{\Phi}_2) \right], \quad (16)$$

where  $\tilde{\Phi}_1, \tilde{\Phi}_2$  are effective dilatonic fields.

We can write the quadratic Lagrangian for the spin-0 part, which contains 2 components  $\tilde{\chi} = \delta\tilde{\Phi}_1$  and  $\tilde{\sigma} = \delta\tilde{\Phi}_2$  (i.e., again the spin-0 metric perturbation is excluded by equations of motion), as

$$\delta^2 S_2 = \frac{1}{2} \int d^4x \sqrt{-g} [\tilde{\chi}\Delta_{\tilde{\chi}}\tilde{\chi} + \tilde{\sigma}\Delta_{\tilde{\sigma}}\tilde{\sigma} + \tilde{\chi}\Delta_{\tilde{\chi}\tilde{\sigma}}\tilde{\sigma}], \quad (17)$$

where

$$\begin{aligned} \Delta_{\tilde{\chi}} &= \frac{M_P^2}{2} \left( \frac{(\partial_{\tilde{\Phi}_1} I_0)^2}{I_0} (3\Box + R_0) + \frac{\partial^2 I_0}{\partial \tilde{\Phi}_1^2} R_0 \right) \\ &\quad - A\tilde{\alpha}f_0\Box - \frac{\partial^2 V_0}{\partial \tilde{\Phi}_1^2}, \\ \Delta_{\tilde{\sigma}} &= \frac{M_P^2}{2} \left( \frac{(\partial_{\tilde{\Phi}_2} I_0)^2}{I_0} (3\Box + R_0) + \frac{\partial^2 I_0}{\partial \tilde{\Phi}_2^2} R_0 \right) \\ &\quad + A\tilde{\alpha}f_0\Box - \frac{\partial^2 V_0}{\partial \tilde{\Phi}_2^2}, \\ \Delta_{\tilde{\chi}\tilde{\sigma}} &= \frac{M_P^2}{2} \left( \frac{\partial_{\tilde{\Phi}_1} I_0 \partial_{\tilde{\Phi}_2} I_0}{I_0} (3\Box + R_0) + \frac{\partial^2 I_0}{\partial \tilde{\Phi}_1 \partial \tilde{\Phi}_2} R_0 \right) \\ &\quad - A\tilde{\beta}f_0\Box - \frac{\partial^2 V_0}{\partial \tilde{\Phi}_1 \partial \tilde{\Phi}_2}, \end{aligned}$$

where  $R_0$  is the scalar curvature of dS vacuum for constant dilatonic fields  $\tilde{\Phi}_1 = \tilde{\Phi}_{1,0}$ ,  $\tilde{\Phi}_2 = \tilde{\Phi}_{2,0}$ . Here we define  $I(\tilde{\Phi}_1, \tilde{\Phi}_2) = [\tilde{\alpha}\tilde{\Phi}_1^2 - \tilde{\alpha}\tilde{\Phi}_2^2 - 2\tilde{\beta}\tilde{\Phi}_1\tilde{\Phi}_2] f(\tilde{\Phi}_2/\tilde{\Phi}_1)$  and  $I_0 \equiv I(\tilde{\Phi}_{1,0}, \tilde{\Phi}_{2,0})$ ,  $\partial_{\tilde{\Phi}_i} I_0 \equiv \partial I(\tilde{\Phi}_1, \tilde{\Phi}_2)/\partial \tilde{\Phi}_i$  are the quantities evaluated at the values of fields at dS vacuum and so on for analogous terms.

We make use of (15), in the case of two complex conjugate roots with the Lagrangian written in real fields. Hence, we juxtapose (15) and (17). The motivation for doing this is to establish a more fundamental correspondence<sup>7</sup> for

<sup>7</sup>This is, however much more involved than in Appendix II A with a single field.

the effective model (16). Essentially, the most important is to establish  $\Delta_{\tilde{\chi}} = -\Delta_{\tilde{\sigma}}$ . In this manner, we can neglect the second derivatives of the potential  $V$ . However, we must satisfy a number of constraints, namely, all parameters and vacuum fields values must be real and  $I_0$  strictly positive. And we want to have  $\tilde{\beta} \neq 0$ , which we will satisfy in the following. The greatly simplifying point is that we must require such an adjustment of coefficients of  $\Delta$ -s only in a single point ( $\tilde{\Phi}_1 = \tilde{\Phi}_{1,0}$ ,  $\tilde{\Phi}_2 = \tilde{\Phi}_{2,0}$ ). On top of this we emphasize again that we aim at retrieving a nearly dS phase, not an exact one. These requirements are generically satisfied altogether with the presence of a function  $f(\frac{\tilde{\Phi}_2}{\tilde{\Phi}_1})$  (apart from special situations which we discuss shortly). It is important that being a function of the ratio of fields it cannot spoil a possible conformal invariance.

Our argument and the construction is to establish an effective setting which can emulate (15). We claim that we have such an effective framework as long we can match quadratic actions for scalar modes around a dS background. We can thus establish a correspondence between (15) and (17) by means of the following:

- (i) During inflationary expansion we can assume that the scalar fields vary slowly and the kinetic terms can be neglected. We are thus mainly interested in whether  $\Delta_{\tilde{\chi}} = -\Delta_{\tilde{\sigma}}$  for the terms proportional to  $R_0$ . To have this we should require

$$\frac{(\partial_{\tilde{\Phi}_1} I_0)^2}{I_0} + \frac{\partial^2 I_0}{\partial \tilde{\Phi}_1^2} + \frac{(\partial_{\tilde{\Phi}_2} I_0)^2}{I_0} + \frac{\partial^2 I_0}{\partial \tilde{\Phi}_2^2} \approx 0 \quad (18)$$

- (ii) We can check that even in the very simple case of  $\tilde{\beta} = 0$ , a nonconstant function  $f$  is required to satisfy the above relation. A simple choice like

$$f = 1 + f_1 \tilde{\Phi}_2 / \tilde{\Phi}_1, \quad (19)$$

with just one free parameter  $f_1$  is sufficient. Otherwise, for  $f = \text{const}$  a condition  $\tilde{\Phi}_{1,0} = \pm i \tilde{\Phi}_{2,0}$  arises from (18). Therefore to build such an effective model the function  $f(\frac{\tilde{\Phi}_2}{\tilde{\Phi}_1})$  is very useful and important. The cross-product of fields may arise for  $\tilde{\beta} = 0$  but a quite involved nonpolynomial function  $f$  is required.

- (iii) For a non-trivial  $\tilde{\beta}$  the same function  $f$  as above in (19) is enough to arrange the condition (18). Moreover  $\tilde{\beta} \neq 0$  generates a cross-product of fields.
- (iv) In complete analogy we can consider the coefficients of the kinetic terms. We have to require a non-constant function  $f$ . We note that having opposite coefficients in front of d'Alembertian operators for different fields essentially means that one of these fields is a ghost.

Recalling expressions (7) and (15), we see that the presence of a cross-product is a special feature related to a complex root of the function  $\mathcal{F}(z)$  [which defines the nonlocal operator  $\mathcal{F}(\square)$ ]. This means that the parameter  $\beta$  found in (15) is essentially nonzero (notice that there is no direct simple relation between  $\tilde{\beta}$  and  $\beta$ ). In the limiting case of  $\beta \rightarrow 0$ , we should see the cross-product disappearing and this corresponds to  $\tilde{\beta} \rightarrow 0$  in the effective model (16). Another way to recognize the effective model (16) without a cross-product of fields is to consider directly (7) with two specially tuned real roots. This means that these roots are related as  $z_2 = -z_1$  and moreover  $\mathcal{F}'(z_2) = -\mathcal{F}'(z_1)$ .

To resolve the issue of a ghost in the spectrum requires an extra symmetry in order to gauge the ghost away. The most natural candidate is the conformal symmetry used in the building of similar models in [18,19,35]. The conformal invariance is restored in (16) if we assume  $A = 6$ . Our model without a cross-product resembles the conformal models studied in [41,42]. We stress that the cross-product appeared for the first time in the cosmological models and we have here provided an imperative explanation through the nonlocal dilaton.

Assuming  $f(\frac{\tilde{\Phi}_1}{\tilde{\alpha}}) \approx \text{constant}$  during inflation, then (16) can be written as

$$S_2 = \int d^4x \sqrt{-g} \left[ (\tilde{\alpha} \tilde{\Phi}_1^2 - \tilde{\alpha} \tilde{\Phi}_2^2 - 2\tilde{\beta} \tilde{\Phi}_1 \tilde{\Phi}_2) \frac{R}{12} + \frac{\tilde{\alpha}}{2} \partial \tilde{\Phi}_1^2 - \frac{\tilde{\alpha}}{2} \partial \tilde{\Phi}_2^2 - \tilde{\beta} \partial_\mu \tilde{\Phi}_1 \partial^\mu \tilde{\Phi}_2 - V_J(\tilde{\Phi}_1, \tilde{\Phi}_2) \right], \quad (20)$$

where we have set  $M_P = 1$  for simplicity and use the subscript  $J$  for the Jordan frame as before. Since the field  $\tilde{\Phi}_1$  has a wrong sign kinetic term (assuming  $\tilde{\alpha} > 0$ ), we can eliminate it by the choice of conformal gauge  $\tilde{\Phi}_1 = \sqrt{6}$

which spontaneously breaks the conformal invariance. To obtain a consistent inflation within this model we consider the following potential

$$V_J(\tilde{\Phi}_1, \tilde{\Phi}_2) = \frac{\lambda}{4} (\gamma_1 \tilde{\Phi}_2^2 + \gamma_2 \tilde{\Phi}_1 \tilde{\Phi}_2 + \gamma_3 \tilde{\Phi}_1^2) (\tilde{\Phi}_2 - \tilde{\Phi}_1)^2, \quad (21)$$

where  $\gamma_1, \gamma_2, \gamma_3$  are arbitrary constant parameters. The potential (21) is motivated from [18], which we generalize here to our conformal model with a term containing the cross-product of fields. The importance of this generalization will be explained in what follows. Note that if  $\tilde{\beta} = \gamma_2 = \gamma_3 = 0$ , the model reduces to the conformal model without a cross-product of fields studied in [18].

Rescaling the fields as  $\tilde{\Phi}_1 \rightarrow \frac{\tilde{\Phi}_1}{\sqrt{\tilde{\alpha}}}$  and  $\tilde{\Phi}_2 \rightarrow \frac{\tilde{\Phi}_2}{\sqrt{\tilde{\alpha}}}$  in action (20) and using the gauge  $\tilde{\Phi}_1 = \sqrt{6}$  we yield

$$S_2 = \int d^4x \sqrt{-g} \left[ \frac{R}{2} \left( 1 - \frac{\tilde{\Phi}_2^2}{6} - \frac{2\tilde{\beta}}{\sqrt{6}\tilde{\alpha}} \tilde{\Phi}_2 \right) - \frac{1}{2} \partial_\mu \tilde{\Phi}_2 \partial^\mu \tilde{\Phi}_2 - \frac{\lambda}{4\tilde{\alpha}^2} (\gamma_1 \tilde{\Phi}_2^2 + \gamma_2 \tilde{\Phi}_1 \tilde{\Phi}_2 + \gamma_3 \tilde{\Phi}_1^2) (\tilde{\Phi}_2 - \sqrt{6})^2 \right]. \quad (22)$$

Performing the conformal transformation  $g_{\mu\nu} \rightarrow [1 + \frac{\tilde{\beta}^2}{\tilde{\alpha}^2} - \frac{1}{6} (\tilde{\Phi}_2 + \frac{\tilde{\beta}}{\tilde{\alpha}} \sqrt{6})^2]^{-1} g_{\mu\nu}$  and shifting the field  $\tilde{\Phi}_2 \rightarrow \tilde{\Phi}_2 + \frac{\tilde{\beta}}{\tilde{\alpha}} \sqrt{6}$ , we arrive to the Einstein frame action

$$S_{2E} = \int d^4x \sqrt{-g_E} \left[ \frac{R_E}{2} - \frac{\omega}{2(\omega - \frac{\tilde{\Phi}_2^2}{6})^2} \partial_\mu \tilde{\Phi}_2 \partial^\mu \tilde{\Phi}_2 - V_E(\tilde{\Phi}_2) \right], \quad (23)$$

where  $\omega = 1 + \frac{\tilde{\beta}^2}{\tilde{\alpha}^2}$  and

$$V_E(\tilde{\Phi}_2) = \frac{9\lambda [\gamma_1 \tilde{\Phi}_2^2 + (\gamma_2 - 2\gamma_1 \frac{\tilde{\beta}}{\tilde{\alpha}}) \sqrt{6} \tilde{\Phi}_2 + 6(\gamma_1 \frac{\tilde{\beta}^2}{\tilde{\alpha}^2} - \gamma_2 \frac{\tilde{\beta}}{\tilde{\alpha}} + \gamma_3)] (\tilde{\Phi}_2 - \sqrt{6} \frac{\tilde{\beta}}{\tilde{\alpha}} - \sqrt{6})^2}{(\tilde{\beta}^2 - \tilde{\Phi}_2^2)^2}. \quad (24)$$

If  $\gamma_i$  are chosen such that  $\gamma_2 = 2\gamma_1 \frac{\tilde{\beta}}{\tilde{\alpha}}$  and  $\gamma_1 \frac{\tilde{\beta}^2}{\tilde{\alpha}^2} - \gamma_2 \frac{\tilde{\beta}}{\tilde{\alpha}} + \gamma_3 \gtrsim 0$ , we can obtain inflation with an uplifting of the potential at the minimum.

Being more concrete, let us consider a simple case with  $\gamma_1 = 1$ ,  $\gamma_2 = 2\frac{\tilde{\beta}}{\tilde{\alpha}}$  and  $\gamma_3 = 2\frac{\tilde{\beta}^2}{\tilde{\alpha}^2}$ , for which (24) reduces to the following form in terms of canonically normalized field  $\tilde{\Phi}_2 = \sqrt{6\omega} \tanh(\frac{\tilde{\phi}}{\sqrt{6}})$

$$V_E(\tilde{\phi}) = \mu^2 \left[ \sinh^2 \left( \frac{\tilde{\phi}}{\sqrt{6}} \right) + \frac{\tilde{\beta}^2}{(\tilde{\alpha}^2 + \tilde{\beta}^2)} \cosh^2 \left( \frac{\tilde{\phi}}{\sqrt{6}} \right) \right] \left[ \cosh \left( \frac{\tilde{\phi}}{\sqrt{6}} \right) - \frac{1}{1 + \frac{\tilde{\beta}}{\tilde{\alpha}}} \sqrt{1 + \frac{\tilde{\beta}^2}{\tilde{\alpha}^2}} \sinh \left( \frac{\tilde{\phi}}{\sqrt{6}} \right) \right]^2 \quad (25)$$

where  $\mu^2 = \frac{9\lambda(\tilde{\alpha} + \tilde{\beta})^2}{\tilde{\alpha}^2(\tilde{\alpha}^2 + \tilde{\beta}^2)}$ . In the limit  $\frac{\tilde{\beta}}{\tilde{\alpha}} \ll 1$ , the first term in (25) dominates during inflation while the second term is negligible. The potential (25) is always positive and in particular has a nonzero value at the minimum at  $\tilde{\phi} \approx 0$ . In general the shape of the potential is similar to the Starobinsky-like models in no-scale SUGRA [16].

Setting  $\tilde{\alpha} = 1$ , in the limit  $\tilde{\beta} \ll 1$ , we can approximate the potential in (25) as

$$V_E(\tilde{\phi}) \approx \frac{\mu^2}{4}(1 - e^{-\sqrt{\tilde{\beta}}\tilde{\phi}})^2 + \frac{\mu^2\tilde{\beta}^2}{4}(1 + e^{-\sqrt{\tilde{\beta}}\tilde{\phi}})^2, \quad (26)$$

where the first term dominates when  $\tilde{\phi} \gg 1$  and leads to a Starobinsky like inflation i.e.,  $n_s \sim 0.967$ ,  $r \sim 0.0033$  for  $N = 60$  and the second term gives a nonzero vacuum energy at the minimum of the potential<sup>8</sup> near  $\tilde{\phi} = 0$ . Here  $\mu \approx 2 \times 10^{-5}$  (in Planck units as we have set  $M_P = 1$ ) which can be determined from the observed amplitude of scalar perturbations  $A_s = 2.2 \times 10^{-9}$  at the horizon exit [1]. In particular  $\tilde{\beta} \sim 10^{-55}$  gives a vacuum energy that reproduces the present day cosmological constant  $\Lambda \sim 10^{-120}$ . Therefore, we conclude that a nonlocally induced cross-product of the fields  $\tilde{\Phi}_1$  and  $\tilde{\Phi}_2$  in (20) naturally uplifts the inflaton potential at the minimum and possibly explain the present day dark energy (assuming it is  $\Lambda$ CDM).

### III. CONCLUSIONS AND DISCUSSION

The dilaton is a possible inflaton candidate in the view of latest CMB data endorsing a nonminimal coupling to the Ricci curvature scalar. In this paper we investigated effective models of inflation emerged from an assumed second order action of non-local dilaton around dS. Here the nonlocality enters in the dilaton kinetic term with an analytic infinite derivative function  $\mathcal{F}(\square)$ . Using the non-local features explained in [32].

We analyze the cases corresponding to the roots  $z_j$  of the characteristic equation  $\mathcal{F}(z) = 0$ . The presence of the cross-product is a special feature related to a complex root of the function  $f(z)$  (which defines the non-local operator  $f(\square)$ ). Moreover, the derivatives  $\mathcal{F}'(z_j)$  play an important role. This is seen from action (7), which describes the evolution of scalar perturbations around a dS vacuum within a non-local context, non-locality being a guide in this process. Its importance is obvious as inflation is a dS like expansion and all the observable quantities related to scalars can be obtained from exploring the action for linear perturbations. A very important restriction is that no ghosts must be in the spectrum. This selects two configurations of roots.

First, there is a situation with one real root  $z_1$  accompanied with a correct sign of  $\mathcal{F}'(z_1)$ . In this case there is one scalar perturbative degree of freedom. Such a configuration can be obtained from the effective model description (10). It is important that coefficients in front of the Einstein-Hilbert term and the kinetic term of a scalar field are independent. We therefore conclude that provided the nonlocal operator  $\mathcal{F}(\square)$  contains one real root, it gives a successful inflation with a universal prediction of

<sup>8</sup>A potential of similar kind can be found in the  $\alpha$ -attractor models where the inflaton potential was uplifted due to the effect of a SUSY breaking mechanism [39].

$n_s = 0.967$  and tensor to scalar ratio as in (2) which can be adjusted to any value  $r < 0.1$  by means of the parameter  $\mathcal{F}'(z_1)$ . A future more accurate detection of parameter  $r$  from CMB [43] would indicate the values of  $z_1$  and  $\mathcal{F}'(z_1)$ .

Second, there was a case with two roots. They can be complex conjugate and then we should look at (15) which is written in manifestly real components. In this scenario, we inevitably get a quadratic cross-product of fields. Moreover, one field looks like a ghost. However, kinetic and mass terms have exactly opposite signs. This suggests that a conformal symmetry may help exorcizing the ghost. Indeed, building an effective model (16) we have taken the conformal symmetry into account and have shown that we indeed can make use of it to remove the unwanted degrees of freedom. The cross-product of fields naturally leads to an uplifting of the potential in the reheating point. In principle one can get a similar two-field model starting with two real roots which are related as  $z_1 = -z_2$  and  $\mathcal{F}'(z_1) = -\mathcal{F}'(z_2)$ . This latter case has no cross-product of fields and falls into the considerations of [41,42]. The novel feature here is that the conformally invariant models with a quadratic cross-product of scalar fields appear for the first time in a cosmological setup and can be naturally explained using the nonlocality of a dilaton.

More generic configurations with more than two fields may have no reasonably simple effective model counterpart. This is because more than one ghost would appear. In this case quite a peculiar structure may be required in order to arrange such a configuration that it will be possible to gauge away all the ghosts. However, although understanding a potential power of multifield models [42], we surely leave this as an open question. We also have skipped a case of multiple roots. It can be considered analogously but requires a more complicated formula mirroring (7).

We note that models of inflation obtained in this paper can be distinguished upon a deeper study of bi-spectrum and/or the reheating phase. This is because in such computations full nonlocal operators will come into play and these structures are unique for the presently studied class of models.

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### APPENDIX A: A REVIEW OF SFT AND TACHYON CONDENSATION

In generic words SFT is an off-shell description of interacting strings [28,29,44–48]. It describes a string by

means of a string field  $\Psi$ . This object is a shorthand for encoding all the string excitations in one instance. The corresponding action for open string field<sup>9</sup> can be written as

$$S = \frac{1}{g_o} \left( \frac{1}{2} \int \Psi \star Q \Psi + \frac{1}{3} \int \Psi \star \Psi \star \Psi \right) \quad (\text{A1})$$

where  $\star$  and  $\int$  are Witten product and integral for string fields respectively.  $Q$  is the Becchi-Rouet-Stora-Tyutin (BRST) charge. The first term clearly corresponds to the motion of free strings while the second term represents the interaction. The second term is the three-string vertex responsible for the nonperturbative physics.  $g_o$  is the open string coupling constant, it is dimensionless.

It has been understood [51–55] that the tachyon of open strings is responsible for the decay of unstable  $D$ -branes or  $D$ -brane-anti- $D$ -brane pairs. The corresponding process is the condensation of the tachyon (TC) to a non-perturbative minimum<sup>10</sup> Upon the TC the unstable brane (or pair) decays. It is the cornerstone of Sen's conjecture regarding TC that the depth of the tachyon potential minimum is exactly the tension of an unstable brane to which the string is attached to. The decay of a brane represents a configuration in which open strings must not exist, because the brane, to which they were attached, has decayed [58,59]. This being said, let us assume Sen's conjecture, which prescribes the disappearance of open string excitations. The latter phenomenon of open strings extinction can be formalized as follows in the field-theoretical language. Given a field  $\varphi$  the following quadratic Lagrangians are nondynamical

$$L = -m^2 \varphi^2 \quad \text{or} \quad L = \varphi e^{\gamma(\square)} \varphi. \quad (\text{A2})$$

The left Lagrangian is clearly a mass term without any dynamics. In the right Lagrangian,  $\square$  is the space-time d'Alembertian and  $\gamma$  is an entire function. Although it may look like  $\square$  produces dynamics as it is a differential operator, as long as we require that the function in the exponent is an entire function, the whole exponent has no eigenvalues as an operator. This means that the inverse of such an exponent gives no poles in the propagator and effectively we have no dynamics at all.

We further notice that the right Lagrangian in (A2) is an essentially nonlocal Lagrangian. It is obviously nondynamical on the quadratic level and as long as the field  $\varphi$  is alone. However, novel and unusual effects can be generated upon coupling to other fields or in the nonlinear physics [30,32,56,57,60].

<sup>9</sup>An action for a closed SFT can be written only in a nonpolynomial form, even for the bosonic strings [49,50].

<sup>10</sup>The TC process itself does not require a dynamical departure from a Minkowski background. This is supported by explicit papers [56,57] and related studies.

The essence of SFT is that as long as a string interaction is involved then the nonlocality of the above type emerges. Technically, we can understand this as follows. Strings are extended objects by construction. When a field-theoretic model describes strings, this property of an extended object is encoded in the nonlocality of interactions. SFT straightforwardly creates vertex terms of the form

$$\sim (e^{\alpha' \square} \varphi_1) (e^{\alpha' \square} \varphi_2) (e^{\alpha' \square} \varphi_3) \quad (\text{A3})$$

Here  $\alpha'$  is the string length squared (which may be different from the inverse of the Planck mass squared).

Note that upon lengthy computations [28,29], the quadratic Lagrangian of the open string tachyon  $\mathcal{T}$  near the vacuum is nondynamical of the form

$$L_{\mathcal{T}} = -\frac{T}{2} v(\square, \mathcal{T}). \quad (\text{A4})$$

For zero momenta, i.e., when  $\square = 0$  the resulting  $v(0, \mathcal{T})$  is exactly the tachyon potential. The dependence on  $\square$  is analytic and being linearized near the vacuum value of field  $\mathcal{T} = \mathcal{T}_0 + \tau$  it produces

$$L_{\tau} = -\frac{T}{2} \frac{v''(\mathcal{T} = \mathcal{T}_0)}{2} \tau e^{\gamma(\square)} \tau, \quad (\text{A5})$$

with some entire function  $\gamma(\square)$ . The coupling  $T$  is nothing but the tension of the unstable  $D$ -brane given as

$$T = \frac{1}{2\pi^2 g_o^2 (\alpha')^{\frac{p+1}{2}}}, \quad (\text{A6})$$

where  $\alpha'$  is the string length squared,  $g_o$  is the open string coupling constant and  $p$  comes from the dimensionality of the  $Dp$ -brane. Thus, as expected for a 3-brane,  $T$  has a dimension  $[\text{length}]^{-4}$  and the tachyon field  $\tau$  is dimensionless.

## APPENDIX B: A SFT INSPIRED FRAMEWORK FOR NONLOCAL DILATON

Let us start with the well-known action of a low energy open-closed SFT coupling, obtained in the framework of the linear dilaton conformal field theory [32,61] (see for instance [62]).

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} (\Phi^2 R + 4\partial_\mu \Phi \partial^\mu \Phi) - \frac{T}{2} \Phi [v(\square, \mathcal{T}) + 1] \right], \quad (\text{B1})$$

where we have redefined the dilaton field as  $\Phi = e^{-\phi}$ . In the above system, tachyon is assumed to be near the potential minimum and its dynamics can be neglected

(see Appendix A). A careful but quick analysis immediately shows that the above action does not support a dS background. We can easily see that the Minkowski background is the only option here that corresponds to an exact compensation of the tension of the initial  $D$ -brane by the tachyon energy at the bottom of the potential and the dilaton is a constant.

To produce inflation, in a nearly dS background, we include additional elements in (B1). Such terms may be invoked and expected from several arguments

- (i) Open-closed string interactions in general contain higher vertexes beyond the action above. These contributions generate new vertexes involving graviton, dilaton, and open string tachyon.
- (ii) The so called “marginal deformation” [63] excitation in the closed strings. This operator is also of a weight zero but in fact is nondynamical at a low-level considerations. However, its exclusion by equations of motion will generate additional terms to an effective action as well.
- (iii) Once a general (not linear) conformal field theory of the dilaton is considered the above analysis would not work. New interactions will be generated since the BRST algebra of the primary fields will get modified.

We therefore propose a broader [than (B1)] action that includes new possible interactions of tachyon of open string and the dilaton of closed string:

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} (\Phi^2 R + 4\partial_\mu \Phi \partial^\mu \Phi) - \frac{T}{2} \sum_{n=-\infty}^{\infty} \Phi^{n+1} v_n(\square, \mathcal{T}) \right]. \quad (\text{B2})$$

where  $R$  is Ricci scalar,  $T$  is the tension of the D-brane. Here, the term for  $v_0$  is the one appearing in (B1), i.e.,  $v_0 = v(\square, \mathcal{T}) + 1$ . The other terms  $v_n(\square, \mathcal{T})$  for  $n \neq 1$  would correspond and convey the higher order couplings of the tachyon potential to the dilaton, which in general depends on infinite number of d'Alembertian operators ( $\square$ ) based on the concepts of SFT (cf., Appendix A).

Action (B2) is different from (B1) by new terms involving coupling of dilaton and tachyon. Let us stress that we need to establish whether inflation is possible in this framework, keeping the dilaton constant in the vacuum then we will search for constant curvature solutions. This makes it irrelevant to consider higher curvature terms. Before proceeding, the appearance of an explicit dilaton potential does not contradict the “dilaton theorem” claim, as this was developed in a pure closed string framework. Moreover, results of [64] indicate that the open-closed SFT coupling will waive the “dilaton theorem” statement. Overall as such, action (B2) is a viable attempt to account open-closed strings couplings during the TC process. Explicit

computation of all such extra terms into the action within pure SFT considerations is beyond the scope of our present analysis.

To support action (B2) as a proposed framework to extract inflationary cosmology, we have to show a constant curvature (in particular dS) background solution is viable, when the dilaton field takes a constant value and the open string tachyon condenses to its minimum. Hence, varying (B2) with respect to the metric  $g_{\mu\nu}$ ,  $\mathcal{T}$  and  $\Phi$  we can show that the following configuration is a solution

$$\Phi = \Phi_0 = 1, \quad \mathcal{T} = \mathcal{T}_0, \quad g_{\mu\nu} \text{ is dS with} \\ R = R_0 = 2 \frac{T}{M_p^2} \sum_n v_{n,0}, \quad (\text{B3})$$

together with the following relations fulfilled

$$\sum_n v'_{n,0} = \sum_n v_{n,0} (3 - n) = 0, \quad (\text{B4})$$

where prime ' is the derivative with respect to an argument and the subscript 0 means that the function is evaluated at  $\mathcal{T} = \mathcal{T}_0$ . We note that  $\Phi_0$  can be any value and is irrelevant as long as it is finite, so we took  $\Phi_0 = 1$  for simplicity. We will discuss the question of how generic such configurations (B3), satisfying (B4), may arise in SFT in a separate forthcoming study [36]. Therefore, our proposed action (B2) can support dS solutions (B3).

## 1. Quadratic variations around de Sitter background

The quadratic variation of our background action (B2) can be written as two parts in the following way

$$\delta^{(2)} S = \delta^{(2)} S_{M_p^2} + \delta^{(2)} S_{\text{int}} \quad (\text{B5})$$

The perturbative modes are  $\varphi = \delta\Phi$ , trace of the metric perturbations  $h$  (we define  $\delta g_{\mu\nu} = h_{\mu\nu}$ ,  $h = h^\mu_\mu$ ) and  $\tau = \delta\mathcal{T}$ . Furthermore, different spins do not mix in the quadratic action, i.e., tensor modes do not mix with scalar modes. So, the first part of the quadratic varied action reads

$$\delta^{(2)} S_{M_p^2} = \int d^4x \sqrt{-g} \frac{M_p^2}{2} \varphi (2\square + 3R_0) \varphi. \quad (\text{B6})$$

where we substituted  $h$  from its equation of motion.

The second part, after a Taylor expansion of the tachyon potential  $v(\square, \mathcal{T})$  around  $\mathcal{T} = \mathcal{T}_0$ , reads

$$\delta^{(2)} S_{\text{int}} = -\frac{T}{2} \int d^4x \sqrt{-g} \sum_n \left[ (n+1) n \varphi^2 v_{n,0} + n v'_{n,0} \varphi f(\square) \tau + \frac{v''_{n,0}}{2} \tau e^{\gamma(\square)} \tau \right], \quad (\text{B7})$$

where we have used (A5). Accounting the fact that the open string tachyon on its own is not dynamical, the function  $\gamma(\square)$  in the exponent must be an entire function but the



operator  $f(\square)$  may have eigenvalues. Excluding  $\tau$  by its equation of motion is dictated by  $\tau = -\frac{\sum_n (nv'_{n,0})}{\sum_n v''_{n,0}} \times f(\square)e^{-\gamma(\square)}\varphi$ . Substituting this back into (B7) yields

$$\delta^{(2)}S_{\text{int}} = \frac{1}{2} \int d^4x \sqrt{-g} \varphi \bar{\mathcal{F}}(\square) \varphi, \quad (\text{B8})$$

where

$$\bar{\mathcal{F}}(\square) = -T \left[ \sum_n ((n+1)nv_{n,0}) - \frac{(\sum_n nv'_{n,0})^2}{2\sum_n v''_{n,0}} f(\square)^2 e^{-\gamma(\square)} \right]. \quad (\text{B9})$$

The second order action (B5) with (B6) and (B8) represents a nonlocal scalar (dilaton) degree of freedom around dS background. It is clear from the above expression that higher curvature corrections are not relevant for us. Indeed, suppose there is a term in the action like  $\sqrt{-g}\Phi^2 R^2$ , such a term would produce contributions to  $h^2$  and  $\varphi h$  but as long as our background has constant scalar curvature and constant dilaton field the final effect of such an additional term would be just renormalization of constants in action (B6). We see that both the spin-0

excitation of the metric and the dilaton field are combined into one joint scalar mode. Again, we can show by explicit computation that including other interactions, like for instance  $\sqrt{-g}\Phi^2 R^2 w(\square, \tau)$ , will result in the same net result when all but one scalar fields can be excluded by equations of motion which finally results in a single (nonlocal) scalar excitation.<sup>11</sup> We further mention that the open string sector contains only the tachyon, since higher mass fields have been integrated out, in the course of the brane decay consideration (cf. Appendix A).

Altogether, this Appendix provides a possible mechanism to motivate non-local dilaton from string theory. The further theoretical development of this part is deferred for future investigations.

<sup>11</sup>We here note that additional contributions to scalar and tensor modes can be generated by means of adding the curvature squared corrections, like  $R_{\mu\nu}^2$  or  $C^2$  where  $C$  is the Weyl tensor. Moreover, following the recent studies performed in [65,66] one has to pay special attention in order to maintain unitarity upon inclusion of terms which modify the Lagrangian for tensor modes beyond the Einstein's gravity. A standard minimal structure like  $C^2$  in the action will generate a massive spin-2 ghost (see [67] for the first comprehensive study of this question). We therefore leave the full consideration as an open question.

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