Comment on "Lattice gluon and ghost propagators and the strong coupling in pure $SU(3)$ Yang-Mills theory: Finite lattice spacing and volume effects"

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The authors of [Phys. Rev. D **94**, 014502 (2016)] reported about a careful analysis of the impact of lattice artifacts on the $SU(3)$ gauge-field propagators. In particular, they found that the low-momentum behavior of the renormalized propagators depends on the lattice bare coupling and interpreted this as the result of its being affected by discretization artifacts. We discuss here a different interpretation for these results.

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I. INTRODUCTION

The understanding of the IR dynamics of QCD has been very much boosted in the past years by the endeavors in obtaining a very detailed picture for the fundamental Green's functions of the theory in both lattice [1–8] and continuum QCD [9–30]. Namely, a consensus has been reached about both the fact that the gluon propagator takes a nonzero finite value at vanishing momentum (corresponding to a dynamical generation of an effective gluon mass [31–34]) and the fact that the ghost propagator essentially behaves, at asymptotically low momenta, as its tree-level expression dictates. These findings have recently contributed, for instance, to establish a striking connection between gauge and matter sectors in defining an interaction kernel for a symmetry-preserving truncation of Schwinger-Dyson equations (SDEs) able to reproduce the observable properties of hadrons [35]; as well as to the construction of a process-independent strong running coupling which agrees very well with the Bjorken sum-rule effective charge [36].

Very recently, the authors of [1] have performed a thorough study of the effect of lattice artifacts on pure Yang-Mills $SU(3)$ gluon and ghost propagators in Landau gauge, as a result of which they claimed that they both depend on the lattice spacing, a, in the infrared domain, while finite volume effects appear to be very mild when lattice volumes are larger than $(6.5\ \mathrm{fm})^4$, in physical units. Specifically, the authors concluded that the zero-momentum gluon propagator dropped roughly by a factor of 10% when the lattice spacing increases from 0.06 fm ($\beta = 6.3$) up to 0.18 fm ($\beta = 5.7$). This is attributed in [1] to a discretization artifact but, in our view, the latter cannot fully explain their findings.

Standard discretization artifacts, mainly owing their origin to the breaking of the $O(4)$ rotational symmetry and taking place at the length scale a, can hardly be felt by gluon modes with characteristic wavelengths of $1/p \gg a$, corresponding to deep infrared momenta. Furthermore, one should expect for them, controlled by powers of ap, not to be stronger at low infrared than at large UV momenta (otherwise, the very precise matching for the lattice estimates of the Taylor running coupling from many different simulations found in [37–39] would not have been possible). On the other hand, the Gribov ambiguity has been recently argued [40] to induce, seen through an alternative lattice implementation of Landau gauge, a different kind of discretization effect, affecting the gluon and ghost fields specially at low momenta. Other than by discretization artifacts, as will be discussed below, the findings of [1] can be also accounted by deviations in the lattice scale setting. In this note, we will preliminary check with the gluon propagator lattice data of ref. [41,42], exploited there for different purposes, whether similar lattice spacing effects are also present at low momenta and, if so, whether they can be removed, or smoothed, by assuming small lattice scale deviations. In ref. [40], the Taylor coupling has been seen to be enough affected by the gauge-fixing, albeit it is not clear at what extent it impacts on the gluon propagator, as the ghost propagator (also involved in the coupling definition) is known to be more sensitive to the Gribov ambiguity. Here, we will only focus on the gluon propagator, conjecture that it receives no important contribution from the gauge-fixing discretization artifact and make thus the low-momentum lattice spacing effects to be wiped out by a lattice scale resetting. Our conjecture can be only supported, *a posteriori*, by the practical success of this removal of lattice spacing effects, as well as by a further scrutiny of the data analyzed by the authors of [1].

II. LATTICE SCALE DEVIATIONS

Let us focus on the Landau-gauge gluon propagator, defined as

$$D_{\mu\nu}^{ab}(p) = \langle A_\mu^a(p) A_\nu^b(-p) \rangle = \delta^{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) D(p^2) \tag{2.1}$$

where A_μ^a is the gauge field in momentum space, latin (greek) indices correspond to color (Lorentz) degrees of freedom, $\langle\cdot\rangle$ expresses the integration over the gauge fields, which is replaced by the average over gauge field configurations in lattice QCD, and $D(p^2)$ is the so-called gluon propagator which, as explained in [1], is to be renormalized on the lattice by applying the MOM prescription,

$$D_R(p^2, \zeta^2)|_{p^2=\zeta^2} = Z_3^{-1}(\zeta^2) D(\zeta^2) = \frac{1}{\zeta^2}; \tag{2.2}$$

where ζ^2 is the renormalization point, fixed at 4 GeV in Ref. [1]. The details of the computation of the gluon propagator on the lattice can be found in the literature, for instance in some previous works of the authors of [1], as [43], or in previous works of some of us such as [8].

In a very recent lattice analysis of the three-gluon vertex and running coupling [41], we have also computed the gluon propagator for different lattice bare couplings. In particular, we obtained the results displayed in Fig. 1, for $\beta = 5.6$ and $\beta = 5.8$ from quenched simulations with the Wilson action in 48^4 lattices. Details of the lattice set-ups can be found in Table I. Both our physical volumes, namely $(7.1\ \text{fm})^4$ and $(11.3\ \text{fm})^4$, are larger than $(6.5\ \text{fm})^4$, above which the authors of [1] found a negligible impact from finite volume effects. The statistical errors have been estimated by applying the jackknife method. The propagators are displayed as a function of the lattice momenta $p_\mu = 2\pi/(Na)n_\mu$, with $n_\mu = 0, 1, \ldots N/4$, instead of the tree-level improved $\hat{p}_\mu = 2/a \sin(ap_\mu/2)$. We have applied the $H(4)$-extrapolation [45], which has been proven to be a very efficient prescription to cure the data from the hypercubic artifacts [45–47]. In addition, we have also employed such a kinematical cut that $ap \leq \pi/2$, thus lessening the impact of any remaining discretization artifacts. As a consequence of this, the largest accessible momentum for the simulation at $\beta = 5.6$ is not much above the momentum, $\zeta = 1.3$ GeV, which we take here for the renormalization point. Indeed, imposing the renormalization condition at $\zeta = 4$ GeV, for which $a\zeta \sim 1.5\pi$ at $\beta = 5.6$ and $\sim\pi$ at $\beta = 5.8$, might imply to incorporate sizable discretization artifacts and, as the propagators are thus required to take there the same value, $1/\zeta^2$, propagate these artifacts down to low IR momenta.

The latter is a possible source, partially at least, for the lattice spacing effect reported in [1]. However, our propagators displayed in the upper panel of Fig. 1, renormalized at $\zeta = 1.3$ GeV, show the same effect: the data obtained with a larger value of the lattice spacing (lower β) appear to deviate upwards when the momentum decreases. Alternatively, we argue this effect may also result from a systematic uncertainty in the lattice scale setting. Indeed, if one admits a small deviation in the lattice scale, $a^{(\delta)} = a(1+\delta)$, such that $a^{(\delta)}$ is a better estimate for the lattice spacing; any dimensionless lattice result, obtained at a physical momentum p with the scale a, would correspond to a new physical momentum $p/(1+\delta)$ with the scale $a^{(\delta)}$. Thus, one can write

$$(a^{(\delta)})^{-2} D^{(\delta)}(p^2/(1+\delta)^2) = a^{-2} D(p^2), \tag{2.3}$$

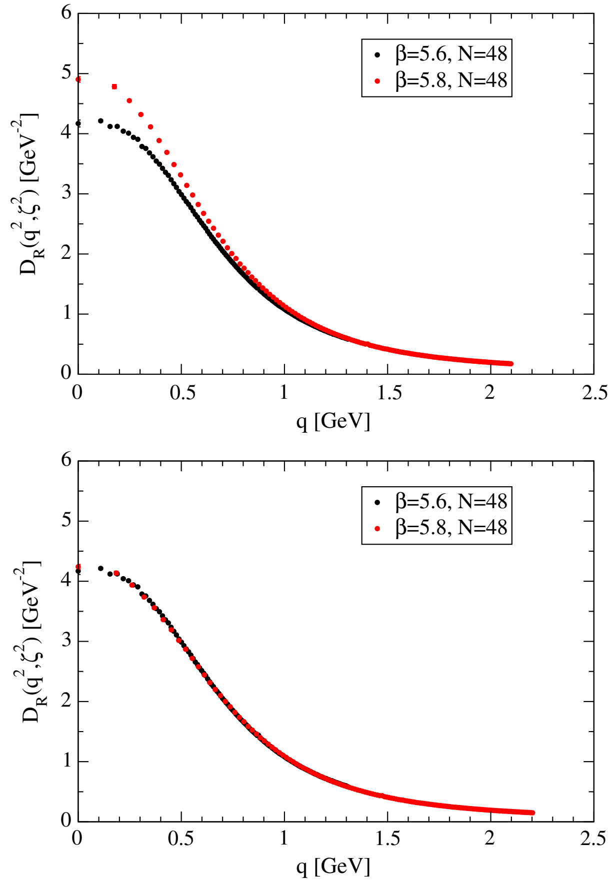

FIG. 1. Upper panel.- Lattice gluon propagator results for the set-ups given in Table I. Lower panel.- The same gluon propagator results after applying to the data at $\beta = 5.8$ the "*recalibration*" described in the text through Eqs. (2.4), (2.5), with $\delta = -0.05$ for the deviation parameter.

TABLE I. Lattice setups specifying the bare lattice coupling $\beta = 6/g_0^2$, the number of lattice sites in any of the directions, N, the lattice spacing, a, and the number of gauge-field configurations exploited. The lattice scales have been taken from [44], where statistical errors of 0.3-0.6% are obtained from the scale setting.

β	N	a [fm]	Configurations
5.6	48	0.236	1920
5.8	48	0.147	960

where D and $D^{(\delta)}$ stand for the bare gluon propagator resulting from setting the scale with, respectively, a and $a^{(\delta)}$. After replacing $p/(1+\delta) \to p$, the *recalibrated* gluon propagator can be then recast as

$$D^{(\delta)}(p^2) = (1+\delta)^2 D((1+\delta)^2 p^2), \tag{2.4}$$

and, after renormalization at $p^2 = \zeta^2$, one would get

$$D_R^{(\delta)}(p^2, \zeta^2) = \frac{D((1+\delta)^2 p^2)}{\zeta^2 D((1+\delta)^2 \zeta^2)}. \tag{2.5}$$

Therefore, the systematic deviation in the scale setting expressed by δ would result in a nontrivial transformation of the data that might well account for the low-momentum discrepancies shown by the upper panel of Fig. 1.

In order to check the validity of this conjecture, we just consider the results obtained at $\beta = 5.6$ as non-deviated and estimate the deviation parameter δ at $\beta = 5.8$ required to get rid of the low-momentum discrepancies and get the data from both simulations lying on top of each other. This can be strikingly seen in the lower panel of Fig. 1, to be left with which one needs to apply $\delta = -0.05$. Properly interpreted, the latter means that all the discrepancies can be explained if we accept a 5% of deviation in the ratio between the lattice spacings at $\beta = 5.8$ and at $\beta = 5.6$, with respect to the values quoted in Table I. These values have been obtained in [44] by using the Sommer parameter, r_0, and are compatible with those from [1] set by the string tension in [48]. In both cases, the scale setting procedures refer to the force between external static charges. The relative accuracy of r_0/a resulting from the thorough statistical analysis of [44] is of the order 0.3–0.6%, but a larger cut-off-dependent systematical uncertainty can be sensibly conceived. The 5% of deviation for the ratio of lattice spacings can very well result from a combination of deviations of around 2–3% in the lattice space setting for both simulations. The same might be enough to explain the effects at low-momentum reported in [1]. Other scale setting prescriptions as the more precise ones grounded on the Wilson flow [49–51] could presumably result on reduced systematic uncertainties. The comparison of the running of renormalized propagators can anyhow be of much help to check these uncertainties and refine the scale setting.

III. CONCLUSIONS

We suggest that the lattice spacing effects discussed by the authors of [1], taking place in the low-momentum domain of the quenched gluon propagators, can also result from a small systematic deviations in the lattice scale setting based on the definition of the force between external static charges. We have made the conjecture that neither gauge-fixing induced nor other possible discretization artifacts have a visible impact on the gluon propagator. Since the lattice scale resetting that we have here applied results in a nontrivial modification of the propagator's low-momentum behaviour, the good agreement of the latter obtained from lattice ensembles with different lattice spacings supports that conjecture. However, albeit discretization artifacts might have impact on the low-momentum gluon propagator, lattice scale deviations will undoubtedly affect it and should be properly considered. A further and detailed study of data for the ghost propagator and the Taylor coupling, along the line of this note, would be very welcome as it would help to pinpoint the extent of the gauge-fixing discretization artifacts in the low-momentum running of the gauge field propagators.

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