Nonuniversal anomaly-free U(1) model with three Higgs doublets and one singlet scalar field

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The flavor problem, neutrino physics, and the fermion mass hierarchy are important motivations to extend the Standard Model to the TeV scale. A new family nonuniversal extension is presented with three Higgs doublets, one Higgs singlet, and one scalar dark matter candidate. Exotic fermions are included in order to cancel chiral anomalies and to allow family nonuniversal $\mathrm{U}(1)_X$ charges. By implementing an additional $\mathbb{Z}_2$ symmetry, the Yukawa coupling terms are suited in such a way that the fermion mass hierarchy is obtained without fine-tuning. The neutrino sector includes Majorana fermions to implement the inverse seesaw mechanism. The effective mass matrix for Standard Model neutrinos is fitted to current neutrino oscillation data to check the consistency of the model with experimental evidence, obtaining that the normal-ordering scheme is preferred over the inverse ones, and the values of the neutrino Yukawa coupling constants are shown. Finally, the $h \to \tau\mu$ lepton-flavor-violation process is addressed with the rotation matrices of the *CP*-even scalars, left- and right-handed charged leptons, yielding definite regions where the model is consistent with CMS reports of $\mathrm{BR}(h \to \tau\mu)$.

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I. INTRODUCTION

Although the Standard Model (SM) [1–3] has been successful in explaining the experimental low-energy observations in particle physics, there is some theoretical and observational evidence that suggests an underlying beyond-the-Standard Model (BSM) extension. Two of these pieces of evidence are the fermion mass hierarchy problem and the neutrino oscillation. In the hierarchy problem [4], the mass of the fermions and their mixing requires arbitrary fine-tuning of the Yukawa coupling constants. Some approaches in the framework of BSM extensions involves schemes to explain this puzzle in the framework of zero-texture structures of the Yukawa matrices [5,6]. Moreover, in these schemes, the neutrino oscillation problem could be addressed, obtaining satisfactory models of flavor physics.

The confirmation of neutrino oscillations and the massive nature of neutrinos as well as their mixing angles have been widely confirmed by precision measurements done by a huge number of experiments [7–28]. References [29,30] show the most recent fit from the experimental data. The massive nature of neutrinos motivates different scenarios BSM in which the origin of the smallness of their masses could be understood. The preferred mechanism to obtain small masses is the *seesaw mechanism* (SSM) [31–35], which introduces new Majorana fermions with their corresponding mass terms in the Lagrangian in such a way that the enormous scale of their masses (10^{14} GeV) suppresses the electroweak ones, yielding small neutrino masses of the active neutrinos at the eV scale. However, the large scale of the Majorana neutrinos is unreachable by current or future high-energy experiments. There exists another mechanism, the *inverse SSM* [36–39], which introduces additional neutrinos that reduce the Majorana mass scale to the experimentally accessible energies. Also, a Majorana neutrino could induce matter-antimatter asymmetries through the *leptogenesis mechanism* [40].

On the other hand, there are different models BSM with extended scalar sectors. The interest in these type of extensions has grown since the detection of the Higgs boson at the ATLAS [41] and CMS [42] experiments at the LHC, the best known being the two-Higgs-doublet model (2HDM), which introduces two charged $H^{\pm}$, one *CP*-odd A, and two *CP*-even h and H scalar bosons by proposing the existence of a second Higgs doublet [43]. Such models arise naturally in supersymmetric extensions. Also, 2HDM yield scenarios in which the large hierarchy between the t and b quarks can be understood by proposing a vacuum hierarchy between the two doublets [44]. Other models extend to the next-to-minimal 2HDM (N2HDM) by adding to the minimal 2HDM a scalar SM singlet, which could yield the spontaneous symmetry breakdown (SSB) of additional U(1) gauge symmetries. Another scenario proposes an additional scalar field as a candidate for dark matter (DM) [45–51], which does not receive a vacuum expectation value (VEV).

Regarding the Abelian extensions of the SM [52], different issues can be addressed such as neutrino physics [53–55], flavor physics [56–58], and DM phenomenology

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[59–64]. The direct consequence of adding an Abelian gauge symmetry is the appearance of an additional neutral gauge boson Z'_μ, which may modify some electroweak observables [65–67] through the mixing with the ordinary Z_μ boson after two SSBs, the first one triggered by some scalar singlet and the second one being the electroweak SSB. There are other extensions which are nonuniversal, i.e., they distinguish among families, with different phenomenological consequences, such as quark mass hierarchy to dark matter detection [68–72].

The main goal of this article is to obtain predictable mass structures and parameters from the neutrino oscillation data by introducing a nonuniversal anomaly free $\mathrm{U}(1)'$ extension with three Higgs doublets and one Higgs singlet. The three doublets will generate the electroweak symmetry breaking, while the singlet induces the $\mathrm{U}(1)'$ breaking spontaneously. The fermion and gauge sectors are also extended by new extra quarks and leptons (including Majorana fermions) with a $\mathbb{Z}_2$ symmetry and the Z' gauge boson with nonuniversal interactions. A scalar singlet without a vacuum expectation value is also included. Section II presents the model, its particle content, and the Yukawa Lagrangian. The bosonic sector of the model is presented in Secs. III and IV. In Sec. V, the mass expressions for all fermions as well as their mixing angles are obtained. Then, Sec. VI presents a procedure to test the consistency of the model with current neutrino oscillation data by fitting the Yukawa coupling constants of the neutrino Yukawa Lagrangian, and Sec. VII explores how adequate the model is in studying lepton flavor violation (LFV) in Higgs decays. Finally, a discussion about the main results and some conclusions are outlined in Sec. VIII.

II. SOME REMARKS OF THE MODEL

The model proposes the existence of a nonuniversal gauge group $\mathrm{U}(1)_X$ of which the gauge boson and coupling constant are Z'_μ and g_X, respectively. This additional gauge symmetry introduces new triangle chiral anomaly equations

$$\begin{aligned}
[\mathrm{SU}(3)_C]^2\mathrm{U}(1)_X \to A_C &= \sum_Q X_{Q_L} - \sum_Q X_{Q_R} \\
[\mathrm{SU}(2)_L]^2\mathrm{U}(1)_X \to A_L &= \sum_\ell X_{\ell_L} + 3\sum_Q X_{Q_L} \\
[\mathrm{U}(1)_Y]^2\mathrm{U}(1)_X \to A_{Y^2} &= \sum_{\ell,Q}[Y^2_{\ell_L}X_{\ell_L} + 3Y^2_{Q_L}X_{Q_L}] - \sum_{\ell,Q}[Y^2_{\ell_R}X_{L_R} + 3Y^2_{Q_R}X_{Q_R}] \\
\mathrm{U}(1)_Y[\mathrm{U}(1)_X]^2 \to A_Y &= \sum_{\ell,Q}[Y_{\ell_L}X^2_{\ell_L} + 3Y_{Q_L}X^2_{Q_L}] - \sum_{\ell,Q}[Y_{\ell_R}X^2_{\ell_R} + 3Y_{Q_R}X^2_{Q_R}] \\
[\mathrm{U}(1)_X]^3 \to A_X &= \sum_{\ell,Q}[X^3_{\ell_L} + 3X^3_{Q_L}] - \sum_{\ell,Q}[X^3_{\ell_R} + 3X^3_{Q_R}] \\
[\mathrm{Grav}]^2\mathrm{U}(1)_X \to A_\mathrm{G} &= \sum_{\ell,Q}[X_{\ell_L} + 3X_{Q_L}] - \sum_{\ell,Q}[X_{\ell_R} + 3X_{Q_R}],
\end{aligned} \tag{1}$$

which can be solved by assigning nontrivial X-quantum numbers to the fermions of the SM [68,72]. Nonuniversal solutions emerge naturally if new quarks and leptons are added with the condition that they must acquire masses at a larger scale than the electroweak ones. All the new particles are assumed to be singlets under the gauge $\mathrm{SU}(2)_L$ group. So, they acquire masses by the VEV of a new Higgs singlet (1HS) field, χ, which has $\mathrm{U}(1)_X$ charge ($X = -1/3$) in such a way that it spontaneously breaks the new gauge symmetry. Another scalar singlet σ identical to χ but without a VEV is introduced [69–71].

On the other hand, the fermion mass hierarchy can be understood with a three Higgs doublet model (3HDM), with VEVs $v_1 > v_2 > v_3$ and the general constraint $v^2 = v_1^2 + v_2^2 + v_3^2$, with $v = 246$ GeV the electroweak breaking scale. The first VEV, v_1, can be associated with the mass of the top quark t at 10^2 GeV. Second, the tau lepton τ and the bottom quark b might acquire mass through v_2 at 1 GeV. Third, the muon μ and the strange quark s may get mass by v_3 at the 10^2 MeV scale. Furthermore, the charm quark c and the first family (u, d, e) could acquire mass through seesaw mechanisms or radiative corrections in order to get smaller masses without unpleasant fine-tuning procedures. Thus, the combination among the 3HDM, the 1HS, and the requirement of specific $\mathbb{Z}_2$ transformations leads us to predictable mass structures of the fermions.

The chosen particle spectrum is presented in Table I, in which three new quarks ($\mathcal{T}$, $\mathcal{J}^{1,2}$), two charged leptons ($\mathcal{E}^{1,2}$), and three right-handed neutrinos $\nu_R^{e,\mu,\tau}$ are introduced such that the model is free from chiral anomalies. By replacing the X charges shown in Table I in Eqs. (1), the complete set of chiral anomalies gets canceled identically,

TABLE I. Nonuniversal X quantum number and $\mathbb{Z}_2$ parity for SM and non-SM fermions.

Bosons	$X^{\pm}$	Quarks	$X^{\pm}$	Leptons	$X^{\pm}$
Scalar doublets			SM fermionic doublets		
$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{h_1+v_1+i\eta_1}{\sqrt{2}} \end{pmatrix}$	$+2/3^+$	$q_L^1 = \binom{u^1}{d^1}_L$	$+1/3^+$	$\ell_L^e = \binom{\nu^e}{e^e}_L$	0^+
$\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{h_2+v_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$	$+1/3^-$	$q_L^2 = \binom{u^2}{d^2}_L$	0^-	$\ell_L^\mu = \binom{\nu^\mu}{e^\mu}_L$	0^+
$\Phi_3 = \begin{pmatrix} \phi_3^+ \\ \frac{h_3+v_3+i\eta_3}{\sqrt{2}} \end{pmatrix}$	$+1/3^+$	$q_L^3 = \binom{u^3}{d^3}_L$	0^+	$\ell_L^\tau = \binom{\nu^\tau}{e^\tau}_L$	-1^+
Scalar singlets			SM fermionic singlets		
		$u_R^{1,3}$	$+2/3^+$	e_R^e	$-4/3^+$
$\chi = \frac{\xi_\chi+v_\chi+i\zeta_\chi}{\sqrt{2}}$	$-1/3^+$	u_R^2	$+2/3^-$	e_R^μ	$-1/3^+$
σ	$-1/3^-$	$d_R^{1,2,3}$	$-1/3^-$	e_R^τ	$-4/3^-$
Gauge bosons		Non-SM quarks		Non-SM leptons	
$W_\mu^\pm W_\mu^3$	$0^+ 0^+$	$\mathcal{T}_L \mathcal{T}_R$	$+1/3^- +2/3^-$	$\nu_R^{e,\mu,\tau} \mathcal{N}_R^{e,\mu,\tau}$	$+1/3^+ 0^+$
B_μ	0^+	$\mathcal{J}_L^{1,2}$	0^+	$\mathcal{E}_L^1, \mathcal{E}_R^2$	-1^+
Ξ_μ	0^+	$\mathcal{J}_R^{1,2}$	$-1/3^+$	$\mathcal{E}_R^1, \mathcal{E}_L^2$	$-2/3^+$

$$
\begin{aligned}
A_C &= 1 - 1, \\
A_L &= -2 + 3(2/3), \\
A_{Y^2} &= [-26/3 + 3(2/3)] - [-56/3 + 3(4)], \\
A_Y &= [-44/9 + 3(2/9)] - [-92/9 + 3(2)], \\
A_X &= [-89/27 + 3(1/9)] - [-161/27 + 3(1)], \\
A_G &= [-11/3 + 3(3)] - [-11/3 + 3(3)]. \qquad (2)
\end{aligned}
$$

The $\mathbb{Z}_2$ parities are also shown as superscripts in the X charges. Note that, despite the scalar doublets Φ_2 and Φ_3 having the same X charge, they have opposite $\mathbb{Z}_2$ parity such that their couplings to fermions are complementary.

The addition of right-handed neutrinos ν_R allows Yukawa couplings with ℓ_L through Φ_3 so as active neutrinos get massive. However, the experiments suggest that their masses are smaller than their charged lepton partners by many orders of magnitude. This huge difference could be explained by the well-known inverse SSM, which is implemented here by introducing three Majorana fermions, $\mathcal{N}_R$, which couple to ν_R via the scalar singlet χ. The existence of the corresponding Majorana mass term induces the inverse SSM, yielding to three light and three quasi-degenerated heavy neutrinos at the TeV scale. One important consequence of the smallness of v_3 is that the mass of the Majorana neutrinos can be as low as the MeV scale.

The Yukawa Lagrangians of the model for the uplike, downlike, neutral, and charged fermions are, respectively,

$$
\begin{aligned}
-\mathcal{L}_U = {}& h_{3u}^{11}\overline{q_L^1}\tilde{\Phi}_3 u_R^1 + h_{2u}^{12}\overline{q_L^1}\tilde{\Phi}_2 u_R^2 + h_{3u}^{13}\overline{q_L^1}\tilde{\Phi}_3 u_R^3 \\
&+ h_{1u}^{22}\overline{q_L^2}\tilde{\Phi}_1 u_R^2 + h_{1u}^{31}\overline{q_L^3}\tilde{\Phi}_1 u_R^1 + h_{1u}^{33}\overline{q_L^3}\tilde{\Phi}_1 u_R^3 \\
&+ h_{2\mathcal{T}}^1\overline{q_L^1}\tilde{\Phi}_2\mathcal{T}_R + h_{1\mathcal{T}}^2\overline{q_L^2}\tilde{\Phi}_1\mathcal{T}_R + g_{\sigma u}^1\overline{\mathcal{T}_L}\sigma u_R^1 \\
&+ g_{\chi u}^2\overline{\mathcal{T}_L}\chi u_R^2 + g_{\sigma u}^3\overline{\mathcal{T}_L}\sigma u_R^3 + g_{\chi\mathcal{T}}\overline{\mathcal{T}_L}\chi\mathcal{T}_R + \text{H.c.}, \qquad (3)
\end{aligned}
$$

$$
\begin{aligned}
-\mathcal{L}_D = {}& h_{1\mathcal{J}}^{11}\overline{q_L^1}\Phi_1\mathcal{J}_R^1 + h_{2\mathcal{J}}^{21}\overline{q_L^2}\Phi_2\mathcal{J}_R^1 + h_{3\mathcal{J}}^{31}\overline{q_L^3}\Phi_3\mathcal{J}_R^1 \\
&+ h_{1\mathcal{J}}^{12}\overline{q_L^1}\Phi_1\mathcal{J}_R^2 + h_{2\mathcal{J}}^{22}\overline{q_L^2}\Phi_2\mathcal{J}_R^2 + h_{3\mathcal{J}}^{32}\overline{q_L^3}\Phi_3\mathcal{J}_R^2 \\
&+ h_{3d}^{21}\overline{q_L^2}\Phi_3 d_R^1 + h_{3d}^{22}\overline{q_L^2}\Phi_3 d_R^2 + h_{3d}^{23}\overline{q_L^2}\Phi_3 d_R^3 \\
&+ h_{2d}^{31}\overline{q_L^3}\Phi_2 d_R^1 + h_{2d}^{32}\overline{q_L^3}\Phi_2 d_R^2 + h_{2d}^{33}\overline{q_L^3}\Phi_2 d_R^3 \\
&+ g_{\sigma d}^{11}\overline{\mathcal{J}_L^1}\sigma^* d_R^1 + g_{\sigma d}^{11}\overline{\mathcal{J}_L^1}\sigma^* d_R^2 + g_{\sigma d}^{13}\overline{\mathcal{J}_L^1}\sigma^* d_R^3 \\
&+ g_{\sigma d}^{21}\overline{\mathcal{J}_L^2}\sigma^* d_R^1 + g_{\sigma d}^{22}\overline{\mathcal{J}_L^2}\sigma^* d_R^2 + g_{\sigma d}^{23}\overline{\mathcal{J}_L^2}\sigma^* d_R^3 \\
&+ g_{\chi\mathcal{J}}^1\overline{\mathcal{J}_L^1}\chi^*\mathcal{J}_R^1 + g_{\chi\mathcal{J}}^2\overline{\mathcal{J}_L^2}\chi^*\mathcal{J}_R^2 + \text{H.c.}, \qquad (4)
\end{aligned}
$$

$$
\begin{aligned}
-\mathcal{L}_N = {}& h_{3\nu}^{ee}\overline{\ell_L^e}\tilde{\Phi}_3\nu_R^e + h_{3\nu}^{e\mu}\overline{\ell_L^e}\tilde{\Phi}_3\nu_R^\mu + h_{3\nu}^{e\tau}\overline{\ell_L^e}\tilde{\Phi}_3\nu_R^\tau \\
&+ h_{3\nu}^{\mu e}\overline{\ell_L^\mu}\tilde{\Phi}_3\nu_R^e + h_{3\nu}^{\mu\mu}\overline{\ell_L^\mu}\tilde{\Phi}_3\nu_R^\mu + h_{3\nu}^{\mu\tau}\overline{\ell_L^\mu}\tilde{\Phi}_3\nu_R^\tau \\
&+ g_{\chi\mathcal{N}}^{ij}\overline{\nu_R^{iC}}\chi^*\mathcal{N}_R^j + \frac{1}{2}\overline{\mathcal{N}_R^{iC}}M_{\mathcal{N}}^{ij}\mathcal{N}_R^j + \text{H.c.}, \qquad (5)
\end{aligned}
$$

$$
\begin{aligned}
-\mathcal{L}_E = {}& h_{3e}^{e\mu}\overline{\ell_L^e}\Phi_3 e_R^\mu + h_{3e}^{\mu\mu}\overline{\ell_L^\mu}\Phi_3 e_R^\mu + h_{3e}^{\tau e}\overline{\ell_L^\tau}\Phi_3 e_R^e \\
&+ h_{2e}^{\tau\tau}\overline{\ell_L^\tau}\Phi_2 e_R^\tau + h_{1\mathcal{E}}^{e1}\overline{\ell_L^e}\Phi_1\mathcal{E}_R^1 + h_{1\mathcal{E}}^{\mu 1}\overline{\ell_L^\mu}\Phi_1\mathcal{E}_R^1 \\
&+ g_{\chi e}^{1e}\overline{\mathcal{E}_L^1}\chi^* e_R^e + g_{\chi e}^{2\mu}\overline{\mathcal{E}_L^2}\chi e_R^\mu + g_{\chi\mathcal{E}}^1\overline{\mathcal{E}_L^1}\chi\mathcal{E}_R^1 \\
&+ g_{\chi\mathcal{E}}^2\overline{\mathcal{E}_L^2}\chi^*\mathcal{E}_R^2 + \text{H.c.}, \qquad (6)
\end{aligned}
$$

where $\tilde{\Phi} = i\sigma_2\Phi^*$ are the scalar doublet conjugates and the Majorana mass components are denoted as $M_{\mathcal{N}}^{ij}$. The next section presents the acquisition of masses in the fermion sectors.

III. GAUGE BOSONS AND MASSES

The gauge bosons of the model comprise the vector sector of the SM plus the additional Ξ_μ gauge boson of the Abelian extension $\mathrm{U}(1)_X$. The gauge Lagrangian is

$$\mathcal{L}_{\rm Gauge} = -\frac{1}{4}{\rm Tr}(\mathbf{W}^{\mu\nu}\mathbf{W}_{\mu\nu}) - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}\Xi^{\mu\nu}\Xi_{\mu\nu}, \quad (7)$$

where $\Xi_{\mu\nu}$ is the strength-field tensor of the Ξ_μ gauge boson

$$\Xi_{\mu\nu} = \partial_\mu\Xi_\nu - \partial_\nu\Xi_\mu. \quad (8)$$

The gauge boson masses, on the other hand, come from the kinetic part of the Higgs Lagrangian

$$\mathcal{L}^{\rm Kin}_{\rm Higgs} = \frac{1}{2}\sum_{1,2,3}(D^\mu\Phi_i)^\dagger(D_\mu\Phi_i) + \frac{1}{2}(D^\mu\chi)^*(D_\mu\chi), \quad (9)$$

where the covariant derivatives are

$$D_\mu\Phi_i = \partial_\mu\Phi_i - ig\mathbf{W}_\mu\Phi_i - ig'YB_\mu\Phi_i - ig_X X_i\Xi_\mu\Phi_i, \quad (10a)$$

$$D_\mu\chi = \partial_\mu\chi + \frac{ig_X}{3}\Xi_\mu\chi. \quad (10b)$$

By evaluating the Higgs fields at their VEVs, the gauge boson masses appear. The mass of the $W^\pm_\mu$ is

$$m_W^2 = \frac{g^2}{4}(v_1^2 + v_2^2 + v_3^2) = \frac{g^2v^2}{4}, \quad (11)$$

where v is the complete electroweak VEV. Regarding the neutral gauge bosons, the mass matrix in the basis $\mathbf{W}^0_\mu = (B_\mu, W^3_\mu, \Xi_\mu)$ is

$$M^2_{W^0} \approx \frac{1}{4}\begin{pmatrix} g'^2v^2 & -gg'v^2 & \frac{4}{3}g'g_Xv^2 \\ -gg'v^2 & g^2v^2 & -\frac{4}{3}gg_Xv^2 \\ \frac{4}{3}g'g_Xv^2 & -\frac{4}{3}gg_Xv^2 & \frac{4}{9}g_X^2v_\chi^2 \end{pmatrix}. \quad (12)$$

Its determinant is null as is hoped because of the existence of a massless gauge boson, the photon A_μ. In addition, there are two massive gauge bosons, the electroweak Z_μ at GeV scale and the new Z'_μ at TeV,

$$m_Z^2 \approx \frac{g^2 + g'^2}{4}v^2 = \frac{g^2v^2}{4c_W^2}, \qquad m_{Z'}^2 \approx \frac{g_X^2v_\chi^2}{9}. \quad (13)$$

The mass eigenstates $\mathbf{Z}_\mu = (A_\mu, Z_\mu, Z'_\mu)$ are obtained as $\mathbf{Z}_\mu = R_{W^0}\mathbf{W}^0_\mu$ through the mixing matrix R_{W^0}. In the Cabbibo–Kobayashi–Maskawa (CKM) parametrization, its angles are

$$\tan\theta^{W^0}_{12} = \frac{g'}{g}, \quad \tan\theta^{W^0}_{13} = 0, \quad \tan\theta^{W^0}_{23} \approx \frac{3gv^2}{g_Xc_Wv_\chi^2}, \quad (14)$$

and the first angle turns out to be the well-known Weinberg angle $\tan\theta^{W^0}_{12} = t_W$.

IV. HIGGS POTENTIAL AND SCALAR MASSES

The scalar potential of the model is established according to the $U(1)_X$ charges and $\mathbb{Z}_2$ parities shown in Table I. So, the most general potential invariant under the $G_{\rm SM}\otimes U(1)_X\otimes\mathbb{Z}_2$ symmetry is

$$\begin{aligned} V_{\rm H} ={}& \mu_1^2\Phi_1^\dagger\Phi_1 + \mu_2^2\Phi_2^\dagger\Phi_2 + \mu_3^2\Phi_3^\dagger\Phi_3 + \mu_\chi^2\chi^*\chi + \lambda_{\chi\chi}(\chi^*\chi)^2 - \frac{f_2}{\sqrt{2}}(\Phi_1^\dagger\Phi_2\chi + \text{H.c.}) - \frac{f_3}{\sqrt{2}}(\Phi_1^\dagger\Phi_3\chi + \text{H.c.}) \\ &+ \lambda_{11}(\Phi_1^\dagger\Phi_1)^2 + \lambda_{12}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - \lambda'_{12}(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \lambda_{22}(\Phi_2^\dagger\Phi_2)^2 + \lambda_{23}(\Phi_2^\dagger\Phi_2)(\Phi_3^\dagger\Phi_3) - \lambda'_{23}(\Phi_2^\dagger\Phi_3)(\Phi_3^\dagger\Phi_2) \\ &+ \lambda_{33}(\Phi_3^\dagger\Phi_3)^2 + \lambda_{13}(\Phi_1^\dagger\Phi_3)(\Phi_3^\dagger\Phi_1) - \lambda'_{13}(\Phi_1^\dagger\Phi_3)(\Phi_3^\dagger\Phi_1) + \lambda_{1\chi}(\Phi_1^\dagger\Phi_1)(\chi^*\chi) + \lambda_{2\chi}(\Phi_2^\dagger\Phi_2)(\chi^*\chi) + \lambda_{3\chi}(\Phi_3^\dagger\Phi_3)(\chi^*\chi). \end{aligned} \quad (15)$$

A. Minimization of the potential

The previous potential is minimized by differentiating it with respect to each one of the VEVs and isolating the quadratic constants μ_α where $\alpha, \beta = 1, 2, 3, \chi$. Thus, the following constants are obtained,

$$-\mu_1^2 = \sum_{\alpha=1}^{\chi}\Lambda_{1\alpha}v_\alpha^2 - \frac{v_\chi v_2 f_2 + v_\chi v_3 f_3}{2v_1}, \quad (16a)$$

$$-\mu_2^2 = \sum_{\alpha=1}^{\chi}\Lambda_{2\alpha}v_\alpha^2 - \frac{f_2v_1v_\chi}{2v_2}, \quad (16b)$$

$$-\mu_3^2 = \sum_{\alpha=1}^{\chi}\Lambda_{3\alpha}v_\alpha^2 - \frac{f_3v_1v_\chi}{2v_3}, \quad (16c)$$

$$-\mu_\chi^2 = \sum_{\alpha=1}^{\chi}\Lambda_{\chi\alpha}v_\alpha^2 - \frac{v_1v_2f_2 + v_1v_3f_3}{2v_\chi}, \quad (16d)$$

where the constants $\Lambda_{\alpha\beta} = \Lambda_{\beta\alpha}$ are (i, j = 1, 2, 3)

$$\begin{cases} \Lambda_{\alpha\alpha} = \lambda_{\alpha\alpha}, \\ \Lambda_{ij} = (\lambda_{ij} - \lambda'_{ij})/2 \\ \Lambda_{i\chi} = \lambda_{i\chi}/2. \end{cases} \quad (17)$$

B. Charged scalar boson masses

The mass matrix of the charged bosons is obtained by calculating the Hessian matrix with respect to the charged components of the Higgs doublets. In the basis $\phi^{\pm} = (\phi_1^{\pm}, \phi_2^{\pm}, \phi_3^{\pm})$, it turns out to be

$$M_{\mathrm{C}}^2 \approx \frac{1}{4}\begin{pmatrix} \frac{v_i f_i v_\chi}{v_1} & -f_2 v_\chi & -f_3 v_\chi \\ -f_2 v_\chi & \frac{f_2 v_\chi v_1}{v_2} & 0 \\ -f_3 v_\chi & 0 & \frac{f_3 v_\chi v_1}{v_3} \end{pmatrix}, \tag{18}$$

where $v_i f_i = v_2 f_2 + v_3 f_3$. Its determinant is null as it is hoped because of the existence of $G_W^{\pm}$, the Goldstone bosons of $W_\mu^{\pm}$. Additionally, there exist two physical charged bosons, $H_1^{\pm}$ and $H_2^{\pm}$, which acquire mass at the TeV scale.

The masses of the physical charged bosons are

$$m_{H_{1,2}^{\pm}}^2 \approx \frac{f_2(v_1^2+v_2^2)v_\chi}{8v_1v_2} + \frac{f_3(v_1^2+v_3^2)v_\chi}{8v_1v_3} \pm \sqrt{\frac{f_2^2(v_1^2+v_2^2)^2 v_\chi^2}{64v_1^2v_2^2} - \frac{f_2f_3(v_1^4-v_2^2v_3^2)v_\chi^2}{32v_1^2v_2v_3} + \frac{f_3^2(v_1^2+v_3^2)^2v_\chi^2}{64v_1^2v_3^2}}. \tag{19}$$

The mixing matrix R_{C} diagonalizes the mass matrix M_{C}^2 obtaining the mass eigenstates $\mathbf{H}^{\pm} = R_{\mathrm{C}}\phi^{\pm}$, which are expressed in the basis $\mathbf{H}^{\pm} = (G_W^{\pm}, H_1^{\pm}, H_2^{\pm})$. Its corresponding mixing angles in the CKM parametrization are

$$\tan^2\theta_{12}^C = \frac{v_2^2}{v_1^2}, \quad \tan^2\theta_{13}^C = \frac{v_3^2}{v_1^2+v_2^2}, \quad \tan^2\theta_{23}^C \approx 0. \tag{20}$$

C. *CP*-odd boson masses

The mass matrix of the *CP*-odd (pseudoscalar) bosons is obtained by calculating the Hessian matrix with respect to the *CP*-odd components of the Higgs doublets. In the basis $\boldsymbol{\eta} = (\eta_1, \eta_2, \eta_3, \zeta_\chi)$, it turns out to be

$$M_{\mathrm{odd}}^2 = \frac{1}{4}\begin{pmatrix} \frac{v_i f_i v_\chi}{v_1} & -f_2 v_\chi & -f_3 v_\chi & -f_i v_i \\ -f_2 v_\chi & \frac{f_2 v_\chi v_1}{v_2} & 0 & f_2 v_1 \\ -f_3 v_\chi & 0 & \frac{f_3 v_\chi v_1}{v_3} & f_3 v_1 \\ -f_i v_i & f_2 v_1 & f_3 v_1 & \frac{v_i f_i v_1}{v_\chi} \end{pmatrix}, \tag{21}$$

where $v_i f_i = v_2 f_2 + v_3 f_3$. Its determinant is null as is hoped because of the existence of G_Z and $G_{Z'}$, the Goldstone bosons of Z_μ and Z'_μ, respectively. Additionally, there exist two physical pseudoscalar bosons, A_1 and A_2, which acquire mass at the TeV scale.

The masses of the physical pseudoscalar bosons are

$$m_{A_{1,2}}^2 \approx \frac{f_2(v_1^2+v_2^2)v_\chi}{8v_1v_2} + \frac{f_3(v_1^2+v_3^2)v_\chi}{8v_1v_3} \pm \sqrt{\frac{f_2^2(v_1^2+v_2^2)^2 v_\chi^2}{64v_1^2v_2^2} - \frac{f_2f_3(v_1^4-v_2^2v_3^2)v_\chi^2}{32v_1^2v_2v_3} + \frac{f_3^2(v_1^2+v_3^2)^2v_\chi^2}{64v_1^2v_3^2}}, \tag{22}$$

which are equal to the charged bosons masses at $\mathcal{O}(v^2)$. The mixing matrix R_{odd} diagonalizes the mass matrix M_{odd}^2, obtaining the mass eigenstates $\mathbf{A} = R_{\mathrm{odd}}\boldsymbol{\eta}$, which are expressed in the basis $\mathbf{A} = (G_Z, A_1, A_2, G_{Z'})$. Moreover, the diagonalization in this case is a little more complicated because there are four bosons instead of three in comparison with the charged scalar boson sector. So, it was implemented an extended-CKM parametrization, which includes mixings with a fourth component. Thereby, the corresponding mixing angles are

$$\tan^2\theta_{12}^A = \frac{v_2^2}{v_1^2 c_{14}^2}, \qquad \tan^2\theta_{13}^A = \frac{v_3^2}{v_1^2 c_{14}^2 + v_2^2}, \tag{23a}$$

$$\tan^2\theta_{23}^A \approx 0, \qquad \tan^2\theta_{14}^A = \frac{v_1}{v_\chi}, \tag{23b}$$

where $c_{14}^2 = \cos^2\theta_{14}^{\mathrm{odd}}$.

D. *CP*-even boson masses

The mass matrix of the *CP*-even (true scalar) bosons is obtained by calculating the Hessian matrix with respect to the CP-even components of the Higgs doublets. In the basis $\mathbf{h} = (h_1, h_2, h_3, \xi_\chi)$, the CP-even mass matrix is

$$M^2_{\text{even}} = \begin{pmatrix} \mathcal{M}_{hh} & \mathcal{M}_{h\xi} \\ \mathcal{M}^{\text{T}}_{h\xi} & \mathcal{M}_{\xi\xi} \end{pmatrix}, \tag{24}$$

where the blocks are defined as

$$\mathcal{M}_{hh} = \begin{pmatrix} \Lambda_{11} v_1^2 & \Lambda_{12} v_1 v_2 & \Lambda_{13} v_1 v_3 \\ \Lambda_{12} v_1 v_2 & \Lambda_{22} v_2^2 & \Lambda_{23} v_2 v_3 \\ \Lambda_{13} v_1 v_3 & \Lambda_{23} v_2 v_3 & \Lambda_{33} v_3^2 \end{pmatrix} + M^2_{\text{C}} \tag{25a}$$

$$\mathcal{M}_{h\xi} = \begin{pmatrix} \Lambda_{1\chi} v_1 v_\chi - \frac{f_i v_i}{4} \\ \Lambda_{2\chi} v_2 v_\chi - \frac{f_2 v_1}{4} \\ \Lambda_{3\chi} v_3 v_\chi - \frac{f_3 v_1}{4} \end{pmatrix} \tag{25b}$$

$$\mathcal{M}_{\xi\xi} = \Lambda_{\chi\chi} v_\chi^2 + \frac{v_i f_i v_1}{4 v_\chi}. \tag{25c}$$

The mixing matrix R_{even}, which diagonalizes the mass matrix M^2_{even}, gives the mass eigenstates $\mathbf{H} = R_{\text{even}} \mathbf{h}$, which are expressed in the basis $\mathbf{H} = (h, H_1, H_2, \mathcal{H})$. Moreover, R_{even} splits into a seesaw rotation $R^{\text{SS}}_{\text{even}}$ and a block-diagonal rotation R^{B}_{even} such that $R_{\text{even}} = R^{\text{B}}_{\text{even}} R^{\text{SS}}_{\text{even}}$.

Since $|\mathcal{M}_{hh}| < |\mathcal{M}_{h\xi}| < |\mathcal{M}_{\xi\xi}|$, the seesaw procedure will be implemented by following Ref. [72], which block diagonalizes $\mathcal{M}_{hh}$ such that the h scalars get separated from the ξ ones. The following approximations are made on the blocks in order to avoid cumbersome expressions after rotating out the ξ scalars:

$$\mathcal{M}_{h\xi} \approx \begin{pmatrix} \Lambda_{1\chi} v_1 v_\chi \\ \Lambda_{2\chi} v_2 v_\chi \\ \Lambda_{3\chi} v_3 v_\chi \end{pmatrix}, \qquad \mathcal{M}_{\xi\xi} \approx \Lambda_{\chi\chi} v_\chi^2. \tag{26}$$

The seesaw rotation $R_{\text{even,SS}}$ and its angle Θ_{even} are

$$R^{\text{SS}}_{\text{even}} = \begin{pmatrix} 1 & -\Theta^\dagger_{\text{even}} \\ \Theta_{\text{even}} & 1 \end{pmatrix}, \tag{27}$$

$$\Theta^\dagger_{\text{even}} = \mathcal{M}^{-1}_{\xi\xi} \mathcal{M}_{h\xi} = \begin{pmatrix} \frac{\Lambda_{1\chi} v_1}{\Lambda_{\chi\chi} v_\chi} \\ \frac{\Lambda_{2\chi} v_2}{\Lambda_{\chi\chi} v_\chi} \\ \frac{\Lambda_{3\chi} v_3}{\Lambda_{\chi\chi} v_\chi} \end{pmatrix}. \tag{28}$$

The block diagonalization acts as $(R^{\text{SS}}_{\text{even}})^{\text{T}}$

$$R^{\text{SS}}_{\text{even}} \mathcal{M}_{hh} (R^{\text{SS}}_{\text{even}})^{\text{T}} = \begin{pmatrix} M^2_{hh} & 0 \\ 0 & M^2_{\xi\xi} \end{pmatrix}, \tag{29}$$

where the new blocks are

$$M^2_{hh} \approx \mathcal{M}_{hh} - \mathcal{M}_{h\xi} \mathcal{M}^{-1}_{\xi\xi} \mathcal{M}^{\text{T}}_{h\xi}, \qquad M^2_{\xi\xi} \approx \mathcal{M}_{\xi\xi}. \tag{30}$$

The resulting matrix M_{hh} has the same algebraic structure of $\mathcal{M}_{hh}$ with new definitions of the constants Λ_{ij}'s, where $i, j = 1, 2, 3$. The matrix turns out to be

$$M^2_{hh} \approx \mathcal{M}_{hh} - \mathcal{M}_{h\xi} \mathcal{M}^{-1}_{\xi\xi} \mathcal{M}^{\text{T}}_{h\xi} \approx \begin{pmatrix} \tilde{\Lambda}_{11} v_1^2 & \tilde{\Lambda}_{12} v_1 v_2 & \tilde{\Lambda}_{13} v_1 v_3 \\ \tilde{\Lambda}_{12} v_1 v_2 & \tilde{\Lambda}_{22} v_2^2 & \tilde{\Lambda}_{23} v_2 v_3 \\ \tilde{\Lambda}_{13} v_1 v_3 & \tilde{\Lambda}_{23} v_2 v_3 & \tilde{\Lambda}_{33} v_3^2 \end{pmatrix} + M^2_{\text{C}}, \tag{31}$$

where the tilde constants are

$$\tilde{\Lambda}_{11} = \Lambda_{11} - \frac{\Lambda^2_{1\chi}}{\Lambda_{\chi\chi}}, \qquad \tilde{\Lambda}_{12} = \Lambda_{12} - \frac{\Lambda_{1\chi}\Lambda_{2\chi}}{\Lambda_{\chi\chi}},$$
$$\tilde{\Lambda}_{22} = \Lambda_{22} - \frac{\Lambda^2_{2\chi}}{\Lambda_{\chi\chi}}, \qquad \tilde{\Lambda}_{23} = \Lambda_{23} - \frac{\Lambda_{2\chi}\Lambda_{3\chi}}{\Lambda_{\chi\chi}},$$
$$\tilde{\Lambda}_{33} = \Lambda_{33} - \frac{\Lambda^2_{3\chi}}{\Lambda_{\chi\chi}}, \qquad \tilde{\Lambda}_{13} = \Lambda_{13} - \frac{\Lambda_{1\chi}\Lambda_{3\chi}}{\Lambda_{\chi\chi}}. \tag{32}$$

By neglecting the electroweak VEVs in the matrix M^2_{hh}, one obtains $M^2_{hh} \approx M^2_{\text{C}}$. Thus, M^2_{hh} should have the two mass eigenvalues $m^2_{H_{1,2}} \approx m^2_{H^\pm_{1,2}}$ at the TeV scale and a third one m^2_h at hundreds of GeV, which would be zero if the electroweak vacuum v were neglected. However, the nonvanishing determinant of M^2_{hh} shows the existence of the smallest eigenvalue, which can be obtained by dividing the determinant of M^2_{hh} by the product of the two largest eigenvalues

$$m^2_h \approx \frac{\text{Det}[M^2_{hh}]}{m^2_{H_1} m^2_{H_2}} \approx \Lambda_{hh} v^2 = \left(\sum_{i=1}^{i=3} \tilde{\Lambda}_{ij} v_i^2 v_j^2 \right) v^2, \tag{33}$$

where Λ_{hh} is the effective coupling constant of the 125 GeV Higgs boson.

The mixing matrix R^{hh}_{even}, which diagonalizes M^2_{hh}, can be approximated to R_{C} because of the method employed in the eigenvalue search. Thus, the corresponding mixing angles of R^{hh}_{even} are

$$\tan^2\theta^h_{12} \approx \frac{v_2^2}{v_1^2}, \quad \tan^2\theta^h_{13} \approx \frac{v_3^2}{v_1^2 + v_2^2}, \quad \tan^2\theta^h_{23} \approx 0. \tag{34}$$

Finally, the transformation R^{B}_{even}, which diagonalizes each one of the blocks after the seesaw procedure, turns out to be

$$R^{B}_{\text{even}} = \begin{pmatrix} R^{hh}_{\text{even}} & 0 \\ 0 & 1 \end{pmatrix}. \tag{35}$$

E. Summary of masses of the scalar sector

The scalar sector of the model includes the following:

(i) Three pairs of charged bosons: one pair corresponding to the W_μ's Goldstone bosons $G_W^\pm$ and two pairs of physical charged scalars with masses given by

$$m^2_{H^\pm_{1,2}} \approx \frac{f_2(v_1^2+v_2^2)v_\chi}{8v_1v_2} + \frac{f_3(v_1^2+v_3^2)v_\chi}{8v_1v_3} \pm \sqrt{\frac{f_2^2(v_1^2+v_2^2)^2v_\chi^2}{64v_1^2v_2^2} - \frac{f_2f_3(v_1^4-v_2^2v_3^2)v_\chi^2}{32v_1^2v_2v_3} + \frac{f_3^2(v_1^2+v_3^2)^2v_\chi^2}{64v_1^2v_3^2}}.$$

(ii) Four CP-odd bosons: two Goldstone bosons G_Z and $G_{Z'}$ corresponding to the gauge fields Z_μ and Z'_μ, respectively, and two physical CP-odd scalars with masses given by

$$m^2_{A_{1,2}} \approx \frac{f_2(v_1^2+v_2^2)v_\chi}{8v_1v_2} + \frac{f_3(v_1^2+v_3^2)v_\chi}{8v_1v_3} \pm \sqrt{\frac{f_2^2(v_1^2+v_2^2)^2v_\chi^2}{64v_1^2v_2^2} - \frac{f_2f_3(v_1^4-v_2^2v_3^2)v_\chi^2}{32v_1^2v_2v_3} + \frac{f_3^2(v_1^2+v_3^2)^2v_\chi^2}{64v_1^2v_3^2}}.$$

(iii) Four CP-even bosons: the SM-Higgs boson with mass given by $m_H^2 = \Lambda_{hh}v^2$, two new CP-even scalar with masses given by

$$m^2_{H_{1,2}} \approx \frac{f_2(v_1^2+v_2^2)v_\chi}{8v_1v_2} + \frac{f_3(v_1^2+v_3^2)v_\chi}{8v_1v_3} \pm \sqrt{\frac{f_2^2(v_1^2+v_2^2)^2v_\chi^2}{64v_1^2v_2^2} - \frac{f_2f_3(v_1^4-v_2^2v_3^2)v_\chi^2}{32v_1^2v_2v_3} + \frac{f_3^2(v_1^2+v_3^2)^2v_\chi^2}{64v_1^2v_3^2}},$$

and a CP-even boson with mass given by $\lambda_{\chi\chi}v_\chi^2$.

Finally, the scalar sector of the model in Ref. [72] can be recovered by neglecting v_3 since the previous model has two doublets and one singlet, in contrast to the three doublets and the singlet of this model.

V. FERMION MASSES

First of all, the fermions of each sector can be described employing two bases: the flavor basis $\mathbf{F}$ or the mass basis $\mathbf{f}$. In the flavor basis, after the Yukawa Lagrangian is evaluated at VEVs, the mass terms can be written as

$$-\mathcal{L}_F = \overline{\mathbf{F}_L}\mathbb{M}_F\mathbf{F}_R + \text{H.c.} \tag{36}$$

Since the mass matrix $\mathbb{M}_F$ is not Hermitian, it has to be diagonalized by the biunitary transformation

$$\mathbb{M}_F^{\text{diag}} = (\mathbb{V}_L^F)^\dagger\mathbb{M}_F\mathbb{V}_R^F, \tag{37}$$

and consequently the mass and flavor bases will be related via the mixing matrices $\mathbb{V}_L^F$ and $\mathbb{V}_R^F$ in the following way:

$$\mathbf{F}_L = \mathbb{V}_L^F\mathbf{f}_L, \qquad \mathbf{F}_R = \mathbb{V}_R^F\mathbf{f}_R. \tag{38}$$

In particular, the left-handed mixing matrix can be expressed as the product of two mixing matrices

$$\mathbb{V}_L^F = \mathbb{V}_{L,\text{SS}}^F\mathbb{V}_{L,\text{B}}^F. \tag{39}$$

The former matrix rotates out the exotic fermions through a seesaw procedure by taking into account the fact that $v_\chi \gg v_{1,2,3}$. For this, first, we split the whole symmetric mass matrices in blocks ($\mathbb{M}_F\mathbb{M}_F^\dagger$ for charged fermions and $\mathbb{M}_N$ for neutrinos) [73],

$$\mathbb{M}_F^{\text{sym}} = \begin{pmatrix} \mathcal{M}^f_{3\times3} & \mathcal{M}^{f\mathcal{F}}_{3\times n} \\ \mathcal{M}^{\mathcal{F}f}_{n\times3} & \mathcal{M}^{\mathcal{F}}_{n\times n}\end{pmatrix}, \tag{40}$$

where $\mathcal{M}^{\mathcal{F}f} = (\mathcal{M}^{f\mathcal{F}})^{\text{T}}$ and n is the number of exotic fermions for each sector (1 for up quarks, 2 for down quarks and charged leptons, and 6 for neutrinos). The seesaw rotation matrix is

$$\mathbb{V}^F_{L,\mathrm{SS}} = \begin{pmatrix} 1 & \Theta^{F\dagger}_L \\ -\Theta^F_L & 1 \end{pmatrix}, \tag{41}$$

where $\Theta^F_L = (\mathcal{M}^{\mathcal{F}})^{-1}\mathcal{M}^{\mathcal{F}f}$. The resulting block-diagonal mass matrix is

$$(\mathbb{V}^F_{L,\mathrm{SS}})^{\mathrm{T}}\mathbb{M}^{\mathrm{sym}}_F\mathbb{V}^F_{L,\mathrm{SS}} = \begin{pmatrix} m^{\mathrm{sym}}_{F,\mathrm{SM}} & 0_{3\times n} \\ 0_{n\times 3} & M^{\mathrm{sym}}_{F,\mathrm{exot}} \end{pmatrix}, \tag{42}$$

where $m^{\mathrm{sym}}_{F,\mathrm{SM}}$ is the SM mass matrix given by

$$m^{\mathrm{sym}}_{F,\mathrm{SM}} \approx \mathcal{M}^f - \mathcal{M}^{f\mathcal{F}}(\mathcal{M}^{\mathcal{F}})^{-1}\mathcal{M}^{\mathcal{F}f} \tag{43}$$

and $M^{\mathrm{sym}}_{F,\mathrm{exot}} \approx \mathcal{M}^{\mathcal{F}}$ is the exotic mass matrix. The latter matrix in Eq. (39), $\mathbb{V}^F_{L,\mathrm{B}}$, describes the diagonalization of $m^{\mathrm{sym}}_{F,\mathrm{SM}}$ and $M^{\mathrm{sym}}_{F,\mathrm{exot}}$. It has the structure

$$\mathbb{V}^F_{\mathrm{B}} = \begin{pmatrix} V^F_{\mathrm{SM}} & 0_{3\times n} \\ 0_{n\times 3} & V^F_{\mathrm{exot}} \end{pmatrix}, \tag{44}$$

where V^F_{SM} is parametrized by

$$V^F_{\mathrm{SM}} = R_{13}(\theta^F_{13}, \delta^F_{13})R_{23}(\theta^F_{23}, \delta^F_{23})R_{12}(\theta^F_{12}, \delta^F_{12}) \tag{45}$$

and the matrices R_{ij} are

$$R_{12}(\theta^F_{12}) = \begin{pmatrix} c^F_{12} & s^F_{12} & 0 \\ -s^{F*}_{12} & c^F_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{46a}$$

$$R_{13}(\theta^F_{13}) = \begin{pmatrix} c^F_{13} & 0 & s^F_{13} \\ 0 & 1 & 0 \\ -s^{F*}_{13} & 0 & c^F_{13} \end{pmatrix}, \tag{46b}$$

$$R_{23}(\theta^F_{23}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c^F_{23} & s^F_{23} \\ 0 & -s^{F*}_{23} & c^F_{23} \end{pmatrix}, \tag{46c}$$

where $c^F_{ij} = \cos\theta^F_{ij}$ and $s^F_{ij} = \sin\theta^F_{ij}\exp(i\delta^F_{ij})$. The angles θ^F_{ij} are specified by their tangents $t^F_{ij} = \tan\theta^F_{ij}$, which could be calculated exactly or approximately using the vacuum hierarchy of the three Higgs doublet outlined in Sec. II. On the other hand, the Dirac phases δ^F_{ij} can be chosen in such a way that they correspond to the experimental measurements. Regarding neutrinos, the Majorana phases have to be included [see Eq. (89)].

The mass matrices and their mass eigenvalues and mixing angles (involving SM and exotic fermions) can be obtained by using the vacuum hierarchy of the Higgs doublets as shown in the next subsections.

A. Uplike quarks

The uplike quark sector is described in the bases **U** and **u**, where the former is the flavor basis, while the latter is the mass basis

$$\mathbf{U} = (u^1, u^2, u^3, \mathcal{T}),$$
$$\mathbf{u} = (u, c, t, T). \tag{47}$$

The mass term in the flavor basis turns out to be

$$-\mathcal{L}_U = \overline{\mathbf{U}_L}\mathbb{M}_U\mathbf{U}_R + \mathrm{H.c.}, \tag{48}$$

where $\mathbb{M}_U$ is

$$\mathbb{M}_U = \frac{1}{\sqrt{2}}\begin{pmatrix} h^{11}_{3u}v_3 & h^{12}_{2u}v_2 & h^{13}_{3u}v_3 & h^{1}_{2\mathcal{T}}v_2 \\ 0 & h^{22}_{1u}v_1 & 0 & h^{2}_{1\mathcal{T}}v_1 \\ h^{31}_{1u}v_1 & 0 & h^{33}_{1u}v_1 & 0 \\ 0 & g^2_{\chi u}v_\chi & 0 & g_{\chi\mathcal{T}}v_\chi \end{pmatrix}. \tag{49}$$

Since the determinant of $\mathbb{M}_U$ is nonvanishing, the four uplike quarks acquire masses. The mass eigenvalues can be calculated by applying different seesaw schemes: the first one rotates out the exotic $\mathcal{T}$ quark, while the second consists of taking advantage of the large hierarchy between the t quark and the lightest ones in the VEVs $v_1 > v_2 > v_3$. Consequently, the four mass eigenvalues are

$$m^2_u = \frac{(h^{11}_{3u}h^{33}_{1u} - h^{13}_{3u}h^{31}_{1u})^2}{(h^{33}_{1u})^2 + (h^{31}_{1u})^2}\frac{v^2_3}{2},$$
$$m^2_c = \frac{(h^{22}_{1u}g_{\chi\mathcal{T}} - h^{2}_{1\mathcal{T}}g^2_{\chi u})^2}{(g_{\chi\mathcal{T}})^2 + (g^2_{\chi u})^2}\frac{v^2_1}{2},$$
$$m^2_t = [(h^{33}_{1u})^2 + (h^{31}_{1u})^2]\frac{v^2_1}{2},$$
$$m^2_{\mathcal{T}} = [(g_{\chi\mathcal{T}})^2 + (g^2_{\chi u})^2]\frac{v^2_\chi}{2}, \tag{50}$$

and the corresponding left-handed rotation matrix can be expressed by

$$\mathbb{V}^U_L = \mathbb{V}^U_{L,\mathrm{SS}}\mathbb{V}^U_{L,\mathrm{B}}, \tag{51}$$

where the seesaw angle is

$$\Theta^{U\dagger}_L = \begin{pmatrix} \frac{h^{1}_{2\mathcal{T}}g_{\chi\mathcal{T}} + h^{12}_{2u}g^2_{\chi u}}{(g_{\chi\mathcal{T}})^2 + (g^2_{\chi u})^2}\frac{v_2}{v_\chi} \\ \frac{h^{2}_{1\mathcal{T}}g_{\chi\mathcal{T}} + h^{22}_{1u}g^2_{\chi u}}{(g_{\chi\mathcal{T}})^2 + (g^2_{\chi u})^2}\frac{v_1}{v_\chi} \\ 0 \end{pmatrix}, \tag{52}$$

while $\mathbb{V}^U_{L,\mathrm{B}}$ diagonalizes only the SM up quarks. Its angles are given by

$$
\begin{aligned}
t_{12}^{U} &= \frac{h_{2u}^{12} g_{\chi\mathcal{T}} - h_{2\mathcal{T}}^{1} g_{\chi u}^{2}}{h_{1u}^{22} g_{\chi\mathcal{T}} - h_{1\mathcal{T}}^{2} g_{\chi u}^{2}} \frac{v_2}{v_1},\\
t_{13}^{U} &= \frac{h_{3u}^{13} h_{1u}^{33} + h_{3u}^{11} h_{1u}^{31}}{(h_{1u}^{33})^2 + (h_{1u}^{31})^2} \frac{v_3}{v_1},\\
t_{23}^{U} &= 0.
\end{aligned}
\tag{53}
$$

The heavy quarks T and t acquire masses at tree-level through v_χ and v_1, respectively. The c quark acquires mass also through v_1; however, this exhibits two suppression mechanisms: by the seesaw with the exotic quark T and the difference of the Yukawa coupling constants. Finally, the u quark acquires mass through v_3 with the same suppression mechanisms of the c quark but with t instead of T.

B. Downlike quarks

The downlike quarks are described in the bases $\mathbf{D}$ and $\mathbf{d}$, where the former is the flavor basis, while the latter is the mass basis

$$
\begin{aligned}
\mathbf{D} &= (d^1, d^2, d^3, \mathcal{J}^1, \mathcal{J}^2),\\
\mathbf{d} &= (d, s, b, J^1, J^2).
\end{aligned}
\tag{54}
$$

The mass term in the flavor basis is

$$
-\mathcal{L}_D = \overline{\mathbf{D}_L} \mathbb{M}_D \mathbf{D}_R + \text{H.c.}, \tag{55}
$$

where $\mathbb{M}_D$ turns out to be

$$
\mathbb{M}_D = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & 0 & 0 & h_{1\mathcal{J}}^{11} v_1 & h_{1\mathcal{J}}^{12} v_1\\
h_{3d}^{21} v_3 & h_{3d}^{22} v_3 & h_{3d}^{23} v_3 & h_{2\mathcal{J}}^{21} v_2 & h_{2\mathcal{J}}^{22} v_2\\
h_{2d}^{31} v_2 & h_{2d}^{32} v_2 & h_{2d}^{33} v_2 & h_{3\mathcal{J}}^{31} v_3 & h_{3\mathcal{J}}^{32} v_3\\
0 & 0 & 0 & g_{\chi\mathcal{J}}^{1} v_\chi & 0\\
0 & 0 & 0 & 0 & g_{\chi\mathcal{J}}^{2} v_\chi
\end{pmatrix}. \tag{56}
$$

Unlike the previous cases, the determinant of $\mathbb{M}_D$ vanishes. Actually, the rank of the mass matrix is not 5 but 4. Consequently, the lightest quark d remains massless. However, this quark can generate a small mass through radiative corrections according to Fig. 1. The contribution of this diagram is

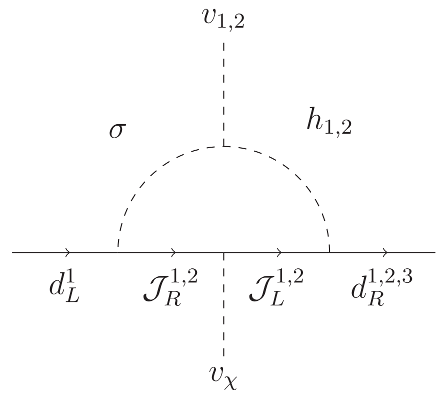


FIG. 1. One-loop correction to the d^1-quark propagator.

$$
\Sigma_d^{1k} = \sum_{i=1,2} \frac{f_\sigma g_{\sigma d}^{i1\,*} h_{k\mathcal{J}}^{ki} v_k}{(4\pi)^2 m_{Ji}} C_0\left(\frac{m_\sigma}{m_{Ji}}, \frac{m_{hk}}{m_{Ji}}\right), \tag{57}
$$

where $k = 1, 2, 3$; f_σ is the trilinear coupling constant involving σ and doublets $\Phi_{1,2,3}$; and the function $C_0(x, y)$ is given by [74]

$$
C_0(x, y) = \frac{1}{(1 - x^2)(1 - y^2)(x^2 - y^2)} \times \left\{ x^2 y^2 \ln\left(\frac{x^2}{y^2}\right) - x^2 \ln x^2 + y^2 \ln y^2 \right\}. \tag{58}
$$

Thus, up to one-loop correction, the mass matrix is

$$
\mathbb{M}_D = \frac{1}{\sqrt{2}} \begin{pmatrix}
\Sigma_d^{11} & \Sigma_d^{12} & \Sigma_d^{13} & h_{1\mathcal{J}}^{11} v_1 & h_{1\mathcal{J}}^{12} v_1\\
h_{3d}^{21} v_3 & h_{3d}^{22} v_3 & h_{3d}^{23} v_3 & h_{2\mathcal{J}}^{21} v_2 & h_{2\mathcal{J}}^{22} v_2\\
h_{2d}^{31} v_2 & h_{2d}^{32} v_2 & h_{2d}^{33} v_2 & h_{3\mathcal{J}}^{31} v_3 & h_{3\mathcal{J}}^{32} v_3\\
0 & 0 & 0 & g_{\chi\mathcal{J}}^{1} v_\chi & 0\\
0 & 0 & 0 & 0 & g_{\chi\mathcal{J}}^{2} v_\chi
\end{pmatrix}, \tag{59}
$$

the determinant of which does not vanish. Its diagonalization is straightforward by considering the hierarchy $\Sigma_{dj} \ll v_3 \ll v_2 \ll v_\chi$.

The masses of the d and s quarks are given by

$$
\begin{aligned}
m_d^2 &= \frac{[(\Sigma_d^{11} h_{3d}^{22} - \Sigma_d^{12} h_{3d}^{21}) h_{2d}^{33} + (\Sigma_d^{13} h_{3d}^{21} - \Sigma_d^{11} h_{3d}^{23}) h_{2d}^{32} + (\Sigma_d^{12} h_{3d}^{23} - \Sigma_d^{13} h_{3d}^{22}) h_{2d}^{31}]^2}{[(h_{3d}^{21})^2 + (h_{3d}^{22})^2](h_{2d}^{33})^2 + [(h_{3d}^{23})^2 + (h_{3d}^{21})^2](h_{2d}^{32})^2 + [(h_{3d}^{22})^2 + (h_{3d}^{23})^2](h_{2d}^{31})^2},\\
m_s^2 &= \frac{[(h_{3d}^{21})^2 + (h_{3d}^{22})^2](h_{2d}^{33})^2 + [(h_{3d}^{23})^2 + (h_{3d}^{21})^2](h_{2d}^{32})^2 + [(h_{3d}^{22})^2 + (h_{3d}^{23})^2](h_{2d}^{31})^2}{(h_{2d}^{33})^2 + (h_{2d}^{32})^2 + (h_{2d}^{31})^2} \frac{v_3^2}{2},
\end{aligned}
\tag{60}
$$

while the masses of b, J^1 and J^2 are

$$m_b^2 = [(h_{2d}^{33})^2 + (h_{2d}^{32})^2 + (h_{2d}^{31})^2]\frac{v_2^2}{2},$$
$$m_{J1}^2 = (g_{\chi\mathcal{J}}^1)^2\frac{v_\chi^2}{2}, m_{J2}^2 = (g_{\chi\mathcal{J}}^2)^2\frac{v_\chi^2}{2}. \qquad (61)$$

The corresponding left-handed rotation matrix is

$$\mathbb{V}_L^D = \mathbb{V}_{L,\mathrm{SS}}^D \mathbb{V}_{L,\mathrm{B}}^D, \qquad (62)$$

where the seesaw angle that rotates out species J^i is

$$\Theta_L^{D\dagger} = \begin{pmatrix} \frac{h_{1\mathcal{J}}^{11}}{g_{\chi\mathcal{J}}^1}\frac{v_1}{v_\chi} & \frac{h_{1\mathcal{J}}^{12}}{g_{\chi\mathcal{J}}^1}\frac{v_2}{v_\chi} \\ \frac{h_{2\mathcal{J}}^{11}}{g_{\chi\mathcal{J}}^1}\frac{v_2}{v_\chi} & \frac{h_{2\mathcal{J}}^{12}}{g_{\chi\mathcal{J}}^2}\frac{v_2}{v_\chi} \\ \frac{h_{3\mathcal{J}}^{11}}{g_{\chi\mathcal{J}}^1}\frac{v_3}{v_\chi} & \frac{h_{3\mathcal{J}}^{12}}{g_{\chi\mathcal{J}}^2}\frac{v_3}{v_\chi} \end{pmatrix} \qquad (63)$$

and the SM angles of $\mathbb{V}_{L,\mathrm{B}}^D$ are given by

$$t_{12} = \frac{\Sigma_d^{11}h_{3d}^{21} + \Sigma_d^{12}h_{3d}^{22} + \Sigma_d^{13}h_{3d}^{23}}{(h_{3d}^{21})^2 + (h_{3d}^{22})^2 + (h_{3d}^{23})^2 v_3},$$
$$t_{13} = \frac{\Sigma_d^{11}h_{2d}^{31} + \Sigma_d^{12}h_{2d}^{32} + \Sigma_d^{13}h_{2d}^{33}}{(h_{2d}^{31})^2 + (h_{2d}^{32})^2 + (h_{2d}^{33})^2 v_2},$$
$$t_{23} = \frac{h_{3d}^{21}h_{2d}^{31} + h_{3d}^{22}h_{2d}^{32} + h_{3d}^{23}h_{2d}^{33}}{(h_{2d}^{31})^2 + (h_{2d}^{32})^2 + (h_{2d}^{33})^2}\frac{v_3}{v_2}. \qquad (64)$$

The heaviest quarks J^1 and J^2 acquire masses at the TeV scale due to v_χ, while the b quark obtains mass through v_2 at the GeV scale. The strange quark acquires mass proportional to v_3 at hundreds of MeV with the suppression due to the b quark. The lightest d quark does not acquire mass at tree level but at one loop, where the radiative correction works as a suppression mechanism.

As an alternative scenario, if σ acquires a VEV v_σ smaller than v_3, the entries of the fourth and fifth rows of the matrix in Eq. (56) are not null. In this case, the mass matrix is

$$\mathbb{M}_D = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & h_{1\mathcal{J}}^{11}v_1 & h_{1\mathcal{J}}^{12}v_1 \\ h_{3d}^{21}v_3 & h_{3d}^{22}v_3 & h_{3d}^{23}v_3 & h_{2\mathcal{J}}^{21}v_2 & h_{2\mathcal{J}}^{22}v_2 \\ h_{2d}^{31}v_2 & h_{2d}^{32}v_2 & h_{2d}^{33}v_2 & h_{3\mathcal{J}}^{31}v_3 & h_{3\mathcal{J}}^{32}v_3 \\ g_{\sigma d}^{11}v_\sigma & g_{\sigma d}^{12}v_\sigma & g_{\sigma d}^{13}v_\sigma & g_{\chi\mathcal{J}}^1 v_\chi & 0 \\ g_{\sigma d}^{21}v_\sigma & g_{\sigma d}^{22}v_\sigma & g_{\sigma d}^{23}v_\sigma & 0 & g_{\chi\mathcal{J}}^2 v_\chi \end{pmatrix}, \qquad (65)$$

the determinant of which is nonvanishing and consequently the d quark is massive, with mass given by

$$m_d^2 = \frac{(g_{\sigma d}^{11})^2(h_{1\mathcal{J}}^{12})^2}{(g_{\chi\mathcal{J}}^1 v_\chi)^2}\frac{v_\sigma^2 v_1^2}{v_\chi^2}, \qquad (66)$$

where for simplicity h_{3d}^{21}, h_{3d}^{23}, and h_{2d}^{31} have been set to zero in order to simplify the expression.

C. Neutral leptons

The neutrinos involve both Dirac and Majorana masses in their Yukawa Lagrangian. The flavor and mass bases are, respectively,

$$\mathbf{N}_L = (\nu_L^{e,\mu,\tau}, \nu_R^{e,\mu,\tau C}, \mathcal{N}_R^{e,\mu,\tau C}),$$
$$\mathbf{n}_L = (\nu_L^{1,2,3}, N_L^{1,2,3}, \tilde{N}_L^{1,2,3}). \qquad (67)$$

The mass term expressed in the flavor basis is

$$-\mathcal{L}_N = \frac{1}{2}\overline{\mathbf{N}_L^C}\mathbb{M}_N\mathbf{N}_L, \qquad (68)$$

where the mass matrix has the block structure

$$\mathbb{M}_N = \begin{pmatrix} 0 & \mathcal{M}_\nu^{\mathrm{T}} & 0 \\ \mathcal{M}_\nu & 0 & \mathcal{M}_\mathcal{N}^{\mathrm{T}} \\ 0 & \mathcal{M}_\mathcal{N} & M_\mathcal{N} \end{pmatrix}, \qquad (69)$$

with $\mathcal{M}_\mathcal{N} = \mathrm{diag}(h_\mathcal{N}^1, h_\mathcal{N}^2, h_\mathcal{N}^3)\frac{v_\chi}{\sqrt{2}}$ the Dirac mass in the $(\nu_R^C, \mathcal{N}_R)$ basis, and

$$\mathcal{M}_\nu = \frac{v_3}{\sqrt{2}}\begin{pmatrix} h_{3\nu}^{ee} & h_{3\nu}^{e\mu} & h_{3\nu}^{e\tau} \\ h_{3\nu}^{\mu e} & h_{3\nu}^{\mu\mu} & h_{3\nu}^{\mu\tau} \\ 0 & 0 & 0 \end{pmatrix} \qquad (70)$$

is a Dirac mass matrix for (ν_L, ν_R). $M_\mathcal{N} = \mu_\mathcal{N}\mathbb{I}_{3\times3}$ is the Majorana mass of $\mathcal{N}_R$.

By employing the inverse SSM, taking into account the hierarchy $v_\chi \gg v_3 \gg |M_\mathcal{N}|$, we find that

$$(\mathbb{V}_{L,\mathrm{SS}}^N)^\dagger\mathbb{M}_N\mathbb{V}_{L,\mathrm{SS}}^N = \begin{pmatrix} m_\nu & 0 & 0 \\ 0 & m_N & 0 \\ 0 & 0 & m_{\tilde{N}} \end{pmatrix}, \qquad (71)$$

where the resultant 3×3 blocks are [38,39]

$$m_\nu = \mathcal{M}_\nu^{\mathrm{T}}(\mathcal{M}_\mathcal{N})^{-1}M_\mathcal{N}(\mathcal{M}_\mathcal{N}^{\mathrm{T}})^{-1}\mathcal{M}_\nu,$$
$$M_N \approx \mathcal{M}_\mathcal{N} - M_\mathcal{N}, \qquad M_{\tilde{N}} \approx \mathcal{M}_\mathcal{N} + M_\mathcal{N}. \qquad (72)$$

The most important details of these matrices are discussed in Sec. VI.

D. Charged leptons

The charged leptons are described in the bases $\mathbf{E}$ and $\mathbf{e}$, where the former is the flavor basis, while the latter is the mass basis

$$\mathbf{E} = (e^e, e^\mu, e^\tau, \mathcal{E}^1, \mathcal{E}^2),$$
$$\mathbf{e} = (e, \mu, \tau, E^1, E^2). \tag{73}$$

The mass term obtained from the Yukawa Lagrangian is

$$-\mathcal{L}_E = \overline{\mathbf{E}_L}\mathbb{M}_E\mathbf{E}_R + \text{H.c.}, \tag{74}$$

where $\mathbb{M}_E$ turns out to be

$$\mathbb{M}_E = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & h_{3e}^{e\mu}v_3 & 0 & h_{1\mathcal{E}}^{e1}v_1 & 0 \\ 0 & h_{3e}^{\mu\mu}v_3 & 0 & h_{1\mathcal{E}}^{\mu1}v_1 & 0 \\ h_{3e}^{\tau e}v_3 & 0 & h_{2e}^{\tau\tau}v_2 & 0 & 0 \\ g_{\chi e}^{1e}v_\chi & 0 & 0 & g_{\chi\mathcal{E}}^{1}v_\chi & 0 \\ 0 & g_{\chi e}^{2\mu}v_\chi & 0 & 0 & g_{\chi\mathcal{E}}^{2}v_\chi \end{pmatrix}. \tag{75}$$

The determinant of $\mathbb{M}_E$ is nonvanishing, ensuring that the five charged leptons acquire masses. Although its eigenvalues and the mixing matrix $\mathbb{V}_L^E$ have large analytical solutions, we can obtain predictable expressions by implementing the vacuum hierarchy of the Higgs doublets. The resulting eigenvalues are

$$m_e^2 = \frac{(h_{3e}^{e\mu}h_{1\mathcal{E}}^{\mu1} - h_{3e}^{\mu\mu}h_{1\mathcal{E}}^{e1})^2}{(h_{1\mathcal{E}}^{e1})^2 + (h_{1\mathcal{E}}^{\mu1})^2}\frac{v_3^2}{2},$$
$$m_\mu^2 = \frac{(h_{3e}^{e\mu}h_{1\mathcal{E}}^{e1} + h_{3e}^{\mu\mu}h_{1\mathcal{E}}^{\mu1})^2}{(h_{1\mathcal{E}}^{e1})^2 + (h_{1\mathcal{E}}^{\mu1})^2}\frac{v_3^2}{2} + \frac{(h_{3e}^{\tau e})^2 v_3^2}{2},$$
$$m_\tau^2 = \frac{(h_{2e}^{\tau\tau})^2 v_2^2}{2},$$
$$m_{E1}^2 = [(g_{\chi\mathcal{E}}^1)^2 + (g_{\chi e}^{1e})^2]\frac{v_\chi^2}{2},$$
$$m_{E2}^2 = [(g_{\chi\mathcal{E}}^2)^2 + (g_{\chi e}^{2\mu})^2]\frac{v_\chi^2}{2}. \tag{76}$$

The exotic charged leptons E^1 and E^2 have acquired masses at the TeV scale, while the heaviest SM lepton τ has acquired mass at the GeV scale, proportional to v_2. On the other hand, the charged leptons μ and e have acquired mass through v_3, which constitutes the smallest VEV. Both of them participate in a sort of seesaw with the matrix entries proportional to v_1, which helps us to suppress their masses. Moreover, the e mass is further suppressed because of the difference between the Yukawa coupling constants in the first equation of (76), which can be assumed to be at the same order of magnitude.

1. Left-handed rotation

The unitary transformation that diagonalizes the matrix $\mathbb{M}_L^E = \mathbb{M}_E\mathbb{M}_E^\dagger$ can be split as follows:

$$\mathbb{V}_L^E = \mathbb{V}_{L,\mathrm{SS}}^E\mathbb{V}_{L,\mathrm{B}}^E. \tag{77}$$

The seesaw procedure is done by $\mathbb{V}_{L,\mathrm{SS}}^E$ because $\mathbb{M}_L^E$ has the suited hierarchy in its sub-blocks. The corresponding seesaw angle turns out to be

$$\Theta_L^{E\dagger} = \begin{pmatrix} \frac{h_{1\mathcal{E}}^{e1}g_{\chi\mathcal{E}}^{1}v_1v_\chi}{2m_{E^1}^2} & \frac{h_{3e}^{e\mu}g_{\chi e}^{2\mu}v_3v_\chi}{2m_{E^2}^2} \\ \frac{h_{1\mathcal{E}}^{\mu1}g_{\chi\mathcal{E}}^{1}v_1v_\chi}{2m_{E^1}^2} & \frac{h_{3e}^{\mu\mu}g_{\chi e}^{2\mu}v_3v_\chi}{2m_{E^2}^2} \\ \frac{(h_{2e}^{\tau e}g_{\chi e}^{1e} + h_{2e}^{\tau\tau}g_{\chi e}^{1\tau})v_3v_\chi}{2m_{E^1}^2} & 0 \end{pmatrix}. \tag{78}$$

Then, the transformation $\mathbb{V}_{L,\mathrm{B}}^E$ only diagonalizes SM leptons with angles given by

$$t_{L,12}^E \approx \frac{h_{1E}^{e1}}{h_{1E}^{\mu1}},$$
$$t_{L,23}^E \approx -\frac{2g_{\chi E}^{1\,3}h_{2e}^{\tau e}h_{2e}^{\tau\tau\,2}}{g_{\chi e}^{e1\,3}h_{1E}^{e1\,2}h_{1E}^{\mu1}}\frac{v_2{}^2v_3}{v_1{}^3},$$
$$t_{L,13}^E \approx \frac{g_{\chi E}^{1}h_{2e}^{\tau e}}{g_{\chi e}^{e1}h_{1E}^{e1}}\frac{v_3}{v_1}. \tag{79}$$

2. Right-handed rotation

On the other hand, the right-handed matrix $\mathbb{M}_R^E = \mathbb{M}_E^\dagger\mathbb{M}_E$ cannot be diagonalized by means of the seesaw procedure because of the presence of v_χ^2 terms in the top-left 3×3 block. Therefore, finite angles are required to rotate out any v_χ^2 in contrast with the diagonalization procedure applied on $\mathbb{M}_L^E$. These angles can be approximately obtained by neglecting any electroweak vacuum in $\mathbb{M}_R^E$,

$$\mathbb{M}_R^E \approx \frac{v_\chi^2}{2}\begin{pmatrix} g_{\chi e}^{e1\,2} & 0 & g_{\chi e}^{e1}g_{\chi e}^{\tau1} & g_{\chi\mathcal{E}}^{1}g_{\chi e}^{e1} & 0 \\ 0 & g_{\chi e}^{\mu2\,2} & 0 & 0 & g_{\chi\mathcal{E}}^{2}g_{\chi e}^{\mu2} \\ g_{\chi e}^{e1}g_{\chi e}^{\tau1} & 0 & g_{\chi e}^{\tau1\,2} & g_{\chi\mathcal{E}}^{1}g_{\chi e}^{\tau1} & 0 \\ g_{\chi\mathcal{E}}^{1}g_{\chi e}^{e1} & 0 & g_{\chi\mathcal{E}}^{1}g_{\chi e}^{\tau1} & g_{\chi\mathcal{E}}^{1}{}^{2} & 0 \\ 0 & g_{\chi\mathcal{E}}^{2}g_{\chi e}^{\mu2} & 0 & 0 & g_{\chi\mathcal{E}}^{2}{}^{2} \end{pmatrix}, \tag{80}$$

and diagonalizing it in such a way that the masses of the exotic species E^1 and E^2 result in the bottom-right block. This rotation may be expressed by the parametrization

$$\mathbb{V}^E_{R,v_\chi} = R_{25}(\theta^E_{R,25})R_{34}(\theta^E_{R,34})R_{14}(\theta^E_{R,14}), \tag{81}$$

and the corresponding angles are given by

$$t^E_{R,25} \approx \frac{g^{\mu 2}_{\chi e}}{g^2_{\chi E}},$$
$$t^E_{R,34} \approx \frac{g^{\tau 1}_{\chi e}}{g^1_{\chi E}},$$
$$t^E_{R,14} \approx \frac{g^{e1}_{\chi e}}{\sqrt{(g^1_{\chi \mathcal{E}})^2 + (g^{\tau 1}_{\chi e})^2}}. \tag{82}$$

Consequently, the diagonalization is done by the transformation

$$\mathbb{V}^E_R = \mathbb{V}^E_{R,v_\chi}\mathbb{V}^E_{R,\mathrm{B}}. \tag{83}$$

After rotating out v_χ from the top-left 3×3 block, it is viable to implement a similar rotation to $\mathbb{V}^E_{L,\mathrm{SM}}$ with the following angles:

$$t^E_{R,12} \approx -\frac{g^{e1}_{\chi e}(h^{e1\,2}_{1E} + h^{\mu 1\,2}_{1E})}{g^1_{\chi E}(h^{e1}_{1E}h^{e\mu}_{3e} + h^{\mu 1}_{1E}h^{\mu\mu}_{3e})}\frac{v_1}{v_3},$$
$$t^E_{R,23} \approx \frac{g^1_{\chi E}h^{\tau e}_{2e}(h^{e1}_{1E}h^{e\mu}_{3e} + h^{\mu 1}_{1E}h^{\mu\mu}_{3e})}{g^{e1}_{\chi e}h^{\tau\tau}_{2e}(h^{e1\,2}_{1E} + h^{\mu 1\,2}_{1E})}, \qquad \frac{{v_3}^2}{v_1 v_2}$$
$$t^E_{R,13} \approx \frac{{g^1_{\chi E}}^2 h^{\tau e}_{2e}h^{\tau\tau}_{2e}}{{g^{e1}_{\chi e}}^2(h^{e1\,2}_{1E} + h^{\mu 1\,2}_{1E})}\frac{v_2 v_3}{{v_1}^2}. \tag{84}$$

Summarizing, the fermion mass hierarchy is induced by the generation of a hierarchy of the vacuum of the Higgs doublets together with the mass matrices obtained from the Yukawa Lagrangian, the terms of which are constrained by the nonuniversal $\mathrm{U}(1)_X$ gauge and $\mathbb{Z}_2$ discrete symmetries. The fermion masses are outlined in Table IV.

TABLE II. Three-flavor oscillation parameters fitting at 1σ reported by Refs. [29,30]. $\ell = 1$ for NO and 2 for IO.

	NO	IO
$\sin^2\theta_{12}$	$0.308^{+0.013}_{-0.012}$	$0.308^{+0.013}_{-0.012}$
$\sin^2\theta_{23}$	$0.440^{+0.023}_{-0.019}$	$0.584^{+0.018}_{-0.022}$
$\sin^2\theta_{13}$	$0.02163^{+0.00074}_{-0.00074}$	$0.02175^{+0.00075}_{-0.00074}$
δ_{CP}	289^{+38}_{-51}	269^{+39}_{-45}
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.49^{+0.19}_{-0.17}$	$7.49^{+0.19}_{-0.17}$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.526^{+0.039}_{-0.037}$	$-2.518^{+0.038}_{-0.037}$

VI. NEUTRINO PARAMETERS

The consistency of this model with the current neutrino oscillation data shown in Table II is tested by exploring the parameter space of the neutral sector of the Yukawa Lagrangian. For simplicity, we choose a basis for ν_R where $\mathcal{M}_\mathcal{N}$ is diagonal, and $M_\mathcal{N}$ is proportional to the identity

$$\mathcal{M}_\mathcal{N} = \mathrm{diag}(\,h^1_\mathcal{N}, h^2_\mathcal{N}, h^3_\mathcal{N}\,)\frac{v_\chi}{\sqrt{2}} \tag{85}$$

$$M_\mathcal{N} = \mu_\mathcal{N}\mathbb{I}_{3\times 3}, \tag{86}$$

where $\mu_\mathcal{N}$ fixes the Majorana mass such that the light neutrinos acquire masses at the eV scale. On the other hand, the coupling constants $h^1_\mathcal{N}$, $h^2_\mathcal{N}$, and $h^3_\mathcal{N}$ determine the masses of the heaviest neutrinos.

By replacing the Dirac mass matrix from (70) in the light mass eigenvalues in (72), the explicit expression of the SM neutrino mass matrix is obtained:

$$m_\nu = \frac{\mu_\mathcal{N} v_3^2}{(h^1_\mathcal{N})^2 v_\chi^2}\begin{pmatrix} (h^{ee}_{3\nu})^2 + (h^{\mu e}_{3\nu})^2\rho^2 & h^{ee}_{3\nu}h^{e\mu}_{3\nu} + h^{\mu e}_{3\nu}h^{\mu\mu}_{3\nu}\rho^2 & h^{ee}_{3\nu}h^{e\tau}_{3\nu} + h^{\mu e}_{3\nu}h^{\mu\tau}_{3\nu}\rho^2 \\ h^{ee}_{3\nu}h^{e\mu}_{3\nu} + h^{\mu e}_{3\nu}h^{\mu\mu}_{3\nu}\rho^2 & (h^{e\mu}_{3\nu})^2 + (h^{\nu\mu}_{3\mu})^2\rho^2 & h^{e\mu}_{3\nu}h^{e\tau}_{3\nu} + h^{\mu\mu}_{3\nu}h^{\mu\tau}_{3\nu}\rho^2 \\ h^{ee}_{3\nu}h^{e\tau}_{3\nu} + h^{\mu e}_{3\nu}h^{\mu\tau}_{3\nu}\rho^2 & h^{e\mu}_{3\nu}h^{e\tau}_{3\nu} + h^{\mu\mu}_{3\nu}h^{\mu\tau}_{3\nu}\rho^2 & (h^{e\tau}_{3\nu})^2 + (h^{\mu\tau}_{3\nu})^2\rho^2 \end{pmatrix}. \tag{87}$$

The ratio $\rho = h^1_\mathcal{N}/h^2_\mathcal{N}$ describes the heavy neutrino hierarchy. Since the matrix m_ν has a null determinant, at least one neutrino is massless. The above matrix is diagonalized by

$$(V^N_{L,\mathrm{SM}})^\dagger m_\nu V^N_{L,\mathrm{SM}} = m_\nu^{\mathrm{diag}},$$

which together with $V^E_{L,\mathrm{SM}}$ constitutes the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [75,76]

$$U_\ell = (V^E_{L,\mathrm{SM}})^\dagger V^N_{L,\mathrm{SM}}. \tag{88}$$

The parametrization for the PMNS matrix follows the convention shown in Eq. (45) given by

$$U_\ell = D(1, \delta^N_2, \delta^N_3)R_{23}(\theta^N_{23})R_{13}(\theta^N_{13}, \delta^N_{13})R_{12}(\theta^N_{12}), \tag{89}$$

where $D(1, \delta^N_2, \delta^N_3)$ is the Majorana phase matrix

$$D(1, \delta^N_2, \delta^N_3) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\delta^N_2} & 0 \\ 0 & 0 & e^{i\delta^N_3} \end{pmatrix}.$$

The angles can be obtained following the convention presented by the Particle Data Group [77],

$$s_{13}^2 = |U_{e3}|^2,$$
$$s_{23}^2 = \frac{|U_{\mu 3}|^2}{1-|U_{e3}|^2},$$
$$s_{12}^2 = \frac{|U_{e2}|^2}{1-|U_{e3}|^2}. \tag{90}$$

The resulting angles obtained from the experimental data are shown in Table II, which have been fitted in Refs. [29,30], in which the convention (90) was employed.

A. Numerical exploration of m_ν consistent with current data

Since the components of the neutrino mass matrix m_ν are quadratic forms of the Yukawa couplings, it is useful to do some coordinate transformation to simplify them. The Yukawa couplings are expressed in their "Cartesian" fashion, but their "polar" form can be written as

$$\begin{cases} h_{3\nu}^{ee} = h_\nu^e \cos\theta_\nu^e \\ \rho h_{3\nu}^{\mu e} = h_\nu^e \sin\theta_\nu^e, \end{cases}$$
$$\begin{cases} h_{3\nu}^{e\mu} = h_\nu^\mu \cos\theta_\nu^\mu \\ \rho h_{3\nu}^{\mu\mu} = h_\nu^\mu \sin\theta_\nu^\mu, \end{cases}$$
$$\begin{cases} h_{3\nu}^{e\tau} = h_\nu^\tau \cos\theta_\nu^\tau \\ \rho h_{3\nu}^{\mu\tau} = h_\nu^\tau \sin\theta_\nu^\tau. \end{cases} \tag{91}$$

The Dirac mass matrix becomes

$$\mathcal{M}_\nu = \frac{v_3}{\sqrt{2}\rho}\begin{pmatrix} \rho h_\nu^e c_\nu^e & \rho h_\nu^\mu c_\nu^\mu & \rho h_\nu^\tau c_\nu^\tau \\ h_\nu^e s_e & h_\nu^\mu s_\mu & h_\nu^\tau s_\tau \\ 0 & 0 & 0 \end{pmatrix}, \tag{92}$$

and consequently the neutrino mass matrix is

$$m_\nu = \frac{\mu_\mathcal{N} v_3^2}{(h_\mathcal{N}^1)^2 v_\chi^2}\begin{pmatrix} (h_\nu^e)^2 & h_\nu^e h_\nu^\mu c_\nu^{e\mu} & h_\nu^e h_\nu^\tau c_\nu^{e\tau} \\ h_\nu^e h_\nu^\mu c_\nu^{e\mu} & (h_\nu^\mu)^2 & h_\nu^\mu h_\nu^\tau c_\nu^{\mu\tau} \\ h_\nu^e h_\nu^\tau c_\nu^{e\tau} & h_\nu^\mu h_\nu^\tau c_\nu^{\mu\tau} & (h_\nu^\tau)^2 \end{pmatrix}, \tag{93}$$

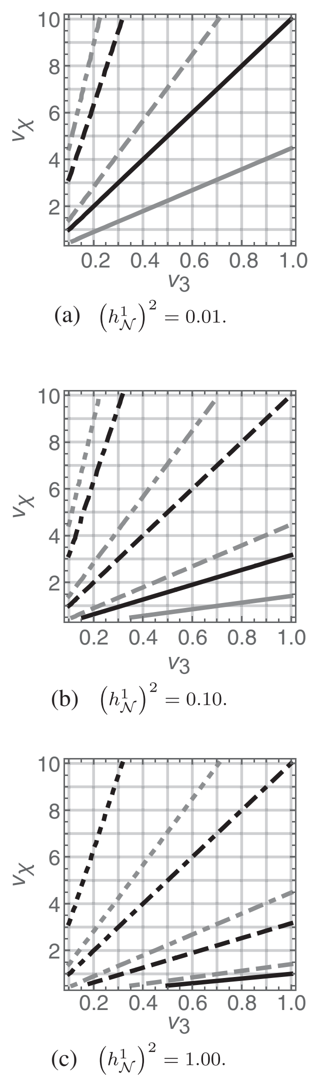
where $c_\nu^{\alpha\beta} = \cos(\theta_\nu^\alpha - \theta_\nu^\beta)$. It is also possible to obtain the mass matrix by defining the vectors in the neutrino Yukawa coupling space,

$$\mathbf{h}_\nu^e = (h_\nu^e c_\nu^e, h_\nu^e s_\nu^e),$$
$$\mathbf{h}_\nu^\mu = (h_\nu^\mu c_\nu^\mu, h_\nu^\mu s_\nu^\mu),$$
$$\mathbf{h}_\nu^\tau = (h_\nu^\tau c_\nu^\tau, h_\nu^\tau s_\nu^\tau), \tag{94}$$

in such a way that the mass matrix is obtained by dot multiplying these vectors,

$$m_\nu = \frac{\mu_\mathcal{N} v_3^2}{(h_\mathcal{N}^1)^2 v_\chi^2}\begin{pmatrix} |\mathbf{h}_\nu^e|^2 & \mathbf{h}_\nu^e\cdot\mathbf{h}_\nu^\mu & \mathbf{h}_\nu^e\cdot\mathbf{h}_\nu^\tau \\ \mathbf{h}_\nu^e\cdot\mathbf{h}_\nu^\mu & |\mathbf{h}_\nu^\mu|^2 & \mathbf{h}_\nu^\mu\cdot\mathbf{h}_\nu^\tau \\ \mathbf{h}_\nu^e\cdot\mathbf{h}_\nu^\tau & \mathbf{h}_\nu^\mu\cdot\mathbf{h}_\nu^\tau & |\mathbf{h}_\nu^\tau|^2 \end{pmatrix}. \tag{95}$$

The new matrix can be diagonalized yielding the corresponding eigenvalues and eigenvectors and also the mixing matrix and its angles using the definitions in (90). Moreover, to make this model consistent with neutrino oscillation data [29,30], the Yukawa parameters (h_ν^e, θ_ν^e), $(h_\nu^\mu, \theta_\nu^\mu)$, $(h_\nu^\tau, \theta_\nu^\tau)$, and θ_{12}^E should be fitted. Such a procedure is done with the Monte Carlo method by generating one billion trials in the parameter space and accepting points



(a) $(h_\mathcal{N}^1)^2 = 0.01$.

(b) $(h_\mathcal{N}^1)^2 = 0.10$.

(c) $(h_\mathcal{N}^1)^2 = 1.00$.

FIG. 2. Contour plots of v_χ (TeV) vs v_3 (GeV) from Eq. (98) for different values of $(h_\mathcal{N}^1)^2$ and $\mu_\mathcal{N}$. From below to above are the corresponding contour plots for the following values of $\mu_\mathcal{N}$: 10 keV (gray line), 50 keV (black line), 100 keV (gray dashed line), 500 keV (black dashed line), 1 MeV (gray dotted-dashed line), 5 MeV (black dotted-dashed line), 10 MeV (gray dotted line), and 50 MeV (black dotted line).

that match the mass matrix to the experimental data. It is worth mentioning that the other two rotation parameters θ^E_{13} and θ^E_{23} from Eq. (79) are approximated to m_τ/m_t.

On the other hand, the appropriate mass scale and mass ordering can be obtained by adjusting the outer factor of the mass matrix and the ratio ρ. For both normal-ordering (NO) and inverted-ordering (IO) schemes, the Yukawa coupling constants can be set to

$$(h^1_{\mathcal{N}})^2 = 0.02,\quad \rho^2 = 0.5, \tag{96}$$

while the mass scale is set to

$$v_3 = 0.5\ \text{GeV},\quad v_\chi = 5\ \text{TeV},\quad \mu_{\mathcal{N}} = 0.1\ \text{MeV}. \tag{97}$$

The above values fix the outer factor of the mass matrix (87) at 50 meV, which yields to the correct squared-mass differences. Nevertheless, there exist other possible values for the parameters $\mu_{\mathcal{N}}$, $h_{N1\chi}$, v_χ, and v_3 that give the factor of 50 meV. The only condition required to get the correct mass scale is

$$\frac{\mu_{\mathcal{N}} v_3^2}{(h^1_{\mathcal{N}})^2 {v_\chi}^2} = 50\ \text{meV}. \tag{98}$$

By taking the above constraint and isolating $\mu_{\mathcal{N}}$, it is possible to obtain other solutions. These solutions are shown in Fig. 2, in which some contour plots on the v_χ vs v_3 plane for different values of $\mu_{\mathcal{N}}$ from 10 keV to 50 MeV are shown. One should note that, due to the smallness of v_3, the Majorana mass scale $\mu_{\mathcal{N}}$ does not need to be small, contrary to other models [38,39]. Specifically, values at the MeV scale are consistent with the observable data.

Table III shows the values of the parameters found with the Monte Carlo procedure consistent with the reported values in Refs. [29,30] for NO and IO. From the fact that the mass matrix is isotropic in the parameter space, θ^e_ν is set to zero, and any other solution with $\theta^e_\nu \neq 0$ is obtained by doing three-dimensional rotations in the neutrino parameter space. The other angles are determined by θ^μ_ν and $\theta^\tau_\nu - \theta^\mu_\nu$.

The NO scheme has several disconnected regions (there are 12 shown in Table III) distributed in 3000 solutions consistent with neutrino data that demonstrate the high consistency of the model with this scheme. Such values can be replaced in (93) in order to obtain the correct squared-mass differences and angles. However, the IO scheme has only four regions distributed in less than a thousand solutions. It is due to the more restrictive constraints implied in this scheme. The masslessness of ν^3_L, the values of the mixing angles, and the quasidegenerated masses of ν^1_L and ν^2_L restrict the parameter regions enormously, as is shown in Table III.

The stringent constraint on θ^E_{12} in the IO scheme is noteworthy. The NO one allows it to vary from 0° to 45°, while the IO one allows it to vary only from 0° to 3°. The narrowing is observable in the h^μ_ν, h^τ_ν, θ^μ_ν, and $\theta^\tau_\nu - \theta^\mu_\nu$ widths in the last four rows of Table III. On the contrary, there does not exist such stringent constraints in the NO scheme.

VII. $h \to \tau\tau$ AND $h \to \tau\mu$

The Higgs lepton flavor violation (HLFV) processes comprise some of the new hints in searching for new

TABLE III. Parameter domains that reproduce neutrino data for NO and IO from Refs. [29,30]. $\theta^e_\nu = 0$.

θ^E_{12}	h^e_ν	h^μ_ν	h^τ_ν	θ^μ_ν	$\theta^\tau_\nu - \theta^\mu_\nu$
			Normal ordering		
0°	0.270 ± 0.007	0.738 ± 0.040	0.747 ± 0.040	$\pm(39.49 \pm 2.99)$	$\pm(38.79 \pm 0.78)$
	0.271 ± 0.007	0.741 ± 0.041	0.745 ± 0.041	$\pm(140.24 \pm 2.93)$	$\mp(38.69 \pm 0.84)$
	0.274 ± 0.007	0.737 ± 0.043	0.745 ± 0.043	$\pm(40.39 \pm 2.80)$	$\mp(141.25 \pm 0.74)$
	0.275 ± 0.008	0.754 ± 0.040	0.729 ± 0.040	$\pm(78.17 \pm 2.61)$	$\mp(218.52 \pm 0.65)$
15°	0.294 ± 0.008	0.737 ± 0.045	0.738 ± 0.043	$\pm(66.73 \pm 1.02)$	$\pm(33.32 \pm 0.81)$
	0.362 ± 0.014	0.722 ± 0.033	0.725 ± 0.041	$\pm(51.41 \pm 2.81)$	$\mp(43.99 \pm 0.51)$
	0.358 ± 0.015	0.720 ± 0.036	0.727 ± 0.041	$\pm(50.75 \pm 3.29)$	$\mp(224.51 \pm 0.74)$
30°	0.400 ± 0.008	0.689 ± 0.035	0.734 ± 0.035	$\pm(46.38 \pm 1.91)$	$\pm(27.53 \pm 1.12)$
	0.471 ± 0.019	0.625 ± 0.021	0.751 ± 0.029	$\pm(42.39 \pm 1.94)$	$\pm(129.04 \pm 0.73)$
	0.402 ± 0.010	0.694 ± 0.043	0.729 ± 0.045	$\pm(46.16 \pm 2.21)$	$\mp(152.30 \pm 1.40)$
45°	0.495 ± 0.003	0.548 ± 0.004	0.796 ± 0.005	$\pm(42.61 \pm 0.82)$	$\pm(19.10 \pm 0.75)$
	0.498 ± 0.002	0.547 ± 0.007	0.791 ± 0.003	$\pm(41.96 \pm 0.78)$	$\pm(160.04 \pm 0.58)$
			Inverted ordering		
0°	0.984 ± 0.006	0.725 ± 0.031	0.700 ± 0.032	$\pm(81.88 \pm 0.84)$	$\mp(163.17 \pm 0.56)$
1°	0.982 ± 0.006	0.732 ± 0.030	0.695 ± 0.031	$\pm(81.57 \pm 0.74)$	$\pm(161.91 \pm 0.55)$
2°	0.980 ± 0.006	0.747 ± 0.022	0.681 ± 0.022	$\pm(81.54 \pm 0.51)$	$\mp(160.77 \pm 0.50)$
3°	0.978 ± 0.006	0.759 ± 0.014	0.671 ± 0.013	$\pm(81.51 \pm 0.32)$	$\mp(159.56 \pm 0.46)$

physics BSM. From them, the process $h \to \tau\mu$ suggests new physics since CMS 8 TeV had reported the branching ratio [78]

$$\mathrm{BR}(h \to \tau\mu) = (0.84^{+0.39}_{-0.37})\%. \tag{99}$$

In comparison, ATLAS reported [79]

$$\mathrm{BR}(h \to \tau\mu) = (0.53 \pm 0.51)\%, \tag{100}$$

consistent with CMS. Thus, the process $h \to \tau\mu$ may be giving the first evidence on nonoscillatory LFV, together with the well-known neutrino oscillations. Now, according to the present model, the piece of the Lagrangian that predicts this process is

$$-\mathcal{L}_{E,\mathrm{SM},h} = \frac{h^{e\mu}_{3e}}{\sqrt{2}}\overline{e^{e}_{L}}h_3 e^{\mu}_{R} + \frac{h^{\mu\mu}_{3e}}{\sqrt{2}}\overline{e^{\mu}_{L}}h_3 e^{\mu}_{R} + \frac{h^{\tau e}_{2e}}{\sqrt{2}}\overline{e^{\tau}_{L}}h_2 e^{e}_{R} + \frac{h^{\tau\tau}_{2e}}{\sqrt{2}}\overline{e^{\tau}_{L}}h_2 e^{\tau}_{R} + \mathrm{H.c.} \tag{101}$$

involving the interaction between charged leptons and Higgs doublets. Then, the replacement of the flavor states by the corresponding mass eigenstates with the rotation matrices

$$\mathbf{h} = R^{\mathrm{T}}_{\mathrm{even}}\mathbf{H}, \tag{102a}$$

$$\mathbf{E}_L = \mathbb{V}^{E}_{L}\mathbf{e}_L, \tag{102b}$$

$$\mathbf{E}_R = \mathbb{V}^{E}_{R}\mathbf{e}_R \tag{102c}$$

is required in order to get the suited couplings. The rotation matrices can be expressed, at leading order, as

$$R^{\mathrm{T}}_{\mathrm{even}} = \begin{pmatrix} 1 & -s^h_{12} & -s^h_{13} \\ s^h_{12} & 1 & 0 \\ s^h_{13} & 0 & 1 \end{pmatrix} \tag{103a}$$

$$\mathbb{V}^{E}_{L,\mathrm{SM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & s^E_{L,13} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & s^E_{L,23} \\ \frac{s^E_{L,23}-s^E_{L,13}}{\sqrt{2}} & -\frac{s^E_{L,23}+s^E_{L,13}}{\sqrt{2}} & 1 \end{pmatrix}, \tag{103b}$$

$$\mathbb{V}^{E}_{R} = \begin{pmatrix} c^E_{R,12} & 1 & s^E_{R,13} \\ -1 & c^E_{R,12} & s^E_{R,23} \\ s^E_{R,23} & -s^E_{R,13} & 1 \end{pmatrix}. \tag{103c}$$

After rotating the flavor basis into the mass eigenbasis, the Yukawa charged lepton Lagrangian can be expressed as

$$-\mathcal{L}_{E,\mathrm{SM},h} = y_{ij}\overline{e^{i}_{L}}e^{j}_{R}h + \mathrm{H.c.}, \tag{104}$$

where y_{ii} (y_{ij}) are the conserving (violating) flavor lepton number coupling constants.

A. Conserving lepton number process $h \to \tau\tau$

The process $h \to \tau\tau$ is prediced by the piece

$$-\mathcal{L}_{h\tau\tau} = y_{\tau\tau}\bar{\tau}\tau h \tag{105}$$

with

$$y_{\tau\tau} = \frac{m_\mu s^h_{13} s^E_{R,13}}{\sqrt{2}\rho_3 v} + \frac{m_\tau s^h_{12}}{\rho_2 v}. \tag{106}$$

Thus, s^h_{12} controls how much h decays into $\tau\tau$. Additionally, the ratio

$$\frac{\sigma(h \to \tau\tau)}{\sigma(h \to \tau\tau)_{\mathrm{SM}}} = \begin{cases} 0.90 \pm 0.28 & \text{CMS} \\ 1.43 - 0.37 + 0.43 & \text{ATLAS} \end{cases} \tag{107}$$

comprises an important hint of BSM physics in flavor violation. In the present model, it turns out to be

$$\frac{\sigma(h \to \tau\tau)}{\sigma(h \to \tau\tau)_{\mathrm{SM}}} \approx \left(\frac{s^h_{12}}{\rho_2}\right)^2, \tag{108}$$

where $\rho_2 \approx 0.05$ in order to obtain the masses of b and τ. The dependences on $s^E_{L,123}$, $s^E_{R,13}$, $s^E_{R,23}$, and s^h_{13} are strongly suppressed by factors m^2_μ/m^2_τ. Therefore, the CMS and ATLAS limits yield the regions

$$s^h_{12} = \begin{cases} (2.67 \pm 0.85) \times 10^{-2} & \text{CMS} \\ (3.86 \pm 0.06) \times 10^{-2} & \text{ATLAS}, \end{cases} \tag{109}$$

constraining the available domains in the parameter space, which is presented in the next paragraph in light of the HLFV $h \to \tau\mu$ decay.

B. HLFV $h \to \tau\mu$

The process $h \to \tau\mu$ is predicted by the piece

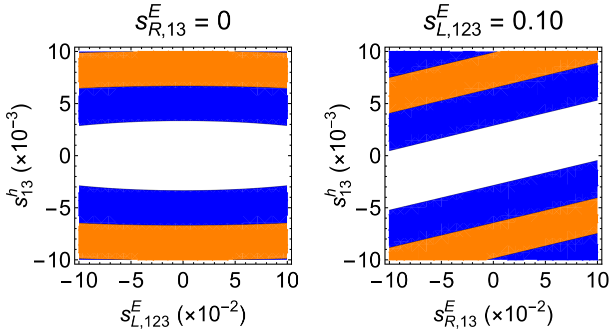
$$-\mathcal{L}_{h\tau\mu} = y_{\mu\tau}\overline{\mu_L}\tau_R h + y_{\tau\mu}\overline{\tau_L}\mu_R h + \mathrm{H.c.}, \tag{110}$$

where the LFV couplings are

$$y_{\mu\tau} = \frac{m_\mu s^h_{13} s^E_{R,23}}{\rho_3 v} - \frac{m_\tau s^h_{12} s^E_{L,123}}{\sqrt{2}\rho_2 v}, \tag{111a}$$

$$y_{\tau\mu} = \frac{m_\mu s^h_{13}}{\sqrt{2}\rho_3 v} - \frac{m_\tau s^h_{12} s^E_{R,13}}{\rho_2 v}, \tag{111b}$$

and $s^E_{L,123} = s^E_{L,13} + s^E_{L,23}$. The branching ratio (BR) from the Lagrangian is given by



(a) (b)

FIG. 3. Contour plots of $\bar{y}_{\mu\tau}$ in the planes s^h_{13} vs $s^E_{L,123}$ and s^h_{13} vs $s^E_{R,13}$ with $s^E_{R,23} = 0.01$ and $s^h_{12} = 0.05$. The orange (blue) regions shows 68% (95%) C.L.

$$\mathrm{BR}(h \to \tau\mu) = \frac{m_h}{8\pi\Gamma_h}\bar{y}_{\mu\tau} \approx 1200\bar{y}_{\mu\tau}, \tag{112}$$

with the new parameter $\bar{y}_{\mu\tau}$ written as

$$\bar{y}_{\mu\tau} = \sqrt{y^2_{\mu\tau} + y^2_{\tau\mu}}. \tag{113}$$

According to Ref. [80], the parameter $\bar{y}_{\mu\tau}$ lies in the region

$$0.002(0.001) < \bar{y}_{\mu\tau} < 0.003(0.004) \tag{114}$$

at 68% (95%) C.L. in such a way the CMS result might be explained.

By fixing the value of $\bar{y}_{\mu\tau}$ at 68% and 95% C.L., some parameter spaces can be drafted to observe how much the model is consistent with the CMS report. First of all, $\bar{y}_{\mu\tau}$ does not depend on $s^R_{R,23}$ due to the $m_\mu/\rho_3 v$ factor in the coefficient $y_{\mu\tau}$. Second, according to Fig. 3(a), the dependence on $s^E_{L,123}$ is slighter compared with s^h_{13} and $s^E_{R,13}$ because of the constraint on $s^h_{12} \approx 0.05$. On the other hand, there exists a direct proportionality between s^h_{13} and $s^E_{R,13}$ shown in Fig. 3(b) in order to satisfy the CMS result. Moreover, how s^h_{13} determines the actual value of $\bar{y}_{\mu\tau}$ in comparison with $s^E_{L,123}$ and s^h_{13} is observed. Summarizing, s^h_{13} should be larger than 5 in order to get the suited $\mathrm{BR}(h \to \tau\mu)$ in accordance with the CMS report.

VIII. DISCUSSION AND CONCLUSIONS

Some issues not explained by the SM, such as the flavor problem, neutrino masses, and mixing can be addressed by employing the addition of Abelian symmetries and the extension of the particle spectrum. The model shown here exhibits nonuniversal $U(1)_X$ quantum numbers with $\mathbb{Z}_2$ parities, which require extended scalar and fermion sectors in order to cancel chiral anomalies and to avoid massless charged fermions. The model implements three scalar doublets and one scalar singlet with an additional scalar field without a VEV. The fermion sector includes three exotic quarks $\mathcal{T}$ and $\mathcal{J}^{1,2}$, five exotic leptons $\mathcal{E}^{1,2}$ and $\nu^{e,\mu,\tau}_R$, and three Majorana fermions $\mathcal{N}^{e,\mu,\tau}_R$, which allow different mass mechanisms in such a way that the fermion mass hierarchy is obtained naturally, as it was shown in Sec. V in Eqs. (76), (50), (60), and (61). A summary about the fermion mass acquisition is presented in Table IV.

TABLE IV. Summary of fermion masses showing their VEVs as well as the suppression mechanism if it is involved. The orders of magnitude of v_χ, v_1, v_2, v_3, and $\mu_\mathcal{N}$ are units of TeV, hundreds of GeV, units of GeV, hundreds of MeV, and units of MeV, respectively.

	Leptons		Quarks	
Family		Mass		Mass
1	ν^1_L	$\frac{\mu_\mathcal{N} v_3^2}{(h^1_\mathcal{N})^2 v_\chi^2} h^2_{\nu 1}$	u	$\frac{h_u^2 - h_u'^2}{h_t}\frac{v_3}{\sqrt{2}}$
2	ν^2_L	$\frac{\mu_\mathcal{N} v_3^2}{(h^1_\mathcal{N})^2 v_\chi^2} h^2_{\nu 2}$	c	$\frac{h_c^2 - h_c'^2}{h_T}\frac{v_1}{\sqrt{2}}$
3	ν^3_L	$\frac{\mu_\mathcal{N} v_3^2}{(h^1_\mathcal{N})^2 v_\chi^2} h^2_{\nu 3}$	t	$\frac{h_t v_1}{\sqrt{2}}$
Exotic	N^i_L	$\frac{h^i_\mathcal{N} v_\chi}{\sqrt{2}} \mp \mu_\mathcal{N}$	T	$\frac{h_T v_\chi}{\sqrt{2}}$
1	e	$\frac{h_\ell^2 - h_\ell'^2}{h_{v1}}\frac{v_3}{\sqrt{2}}$	d	$\frac{\Sigma_d h_d^2}{h_s^2 + h_s'^2}$
2	μ	$\frac{h_\ell^2 + h_\ell'^2}{h_{v1}}\frac{v_3}{\sqrt{2}}$	s	$\frac{h_s^2 + h_s'^2}{h_b}\frac{v_3}{\sqrt{2}}$
3	τ	$\frac{h_\tau v_2}{\sqrt{2}}$	b	$\frac{h_b v_2}{\sqrt{2}}$
Exotic	E^1	$\frac{h_{E1} v_\chi}{\sqrt{2}}$	J^1	$\frac{h_{J1} v_\chi}{\sqrt{2}}$
Exotic	E^2	$\frac{h_{E2} v_\chi}{\sqrt{2}}$	J^2	$\frac{h_{J2} v_\chi}{\sqrt{2}}$

A vacuum hierarchy among the three Higgs doublets is obtained from the electroweak VEV $v = 246$ GeV together with the third-generation fermion masses, especially the t-quark mass at $m_t = 173.21$ GeV. First, according to the SM, the electroweak bosons acquire mass through the VEVs of the three doublets, such that the effective electroweak VEV turns out to be

$$v^2 = v_1^2 + v_2^2 + v_3^2 = (246\ \mathrm{GeV})^2. \tag{115}$$

Second, the t-quark mass in the model is given by

$$m_t = \sqrt{(h^{33}_{1u})^2 + (h^{31}_{1u})^2}\frac{v_1}{\sqrt{2}} = 173.21 \pm 0.71\ \mathrm{GeV}. \tag{116}$$

Additionally, if the Yukawa coupling constants are assumed at order 1, we obtain that v_1 is close to the value of the electroweak VEV since $\sqrt{2}m_t \approx v_1$. Therefore, $v_1 \approx 245.9$ GeV is the dominant contribution to the electroweak VEV, leaving a small gap to be filled by v_2 and v_3. Third, the v_2 and v_3 vacua are determined by the b-quark and τ-lepton masses together with the muon and neutrino masses, respectively. Regarding v_2, it is observed that

$v_2 \approx \sqrt{2}m_b \approx 6$ GeV, and since the τ/b mass ratio is of order

$$\frac{m_\tau}{m_b} = \sqrt{\frac{(h_{2e}^{\tau\tau})^2}{(h_{2d}^{33})^2 + (h_{2d}^{32})^2 + (h_{2d}^{31})^2}} \approx \frac{1}{2}, \qquad (117)$$

the assumption to assign such a numerical value to v_2 is adequate. In addition, the fact that $v_3 \approx \sqrt{2}m_\mu \approx 0.2$ GeV with the neutrino mass scale factor obtained from the inverse seesaw mechanism

$$\frac{\mu_{\mathcal{N}} v_3^2}{v_\chi^2} = 50 \text{ meV} \qquad (118)$$

suggests that v_3 should be about 200 MeV. Consequently, the vacuum hierarchy $v_1 \ll v_2 \ll v_3$ is consistent with the current phenomenological observations.

On the other hand, the masses of the u quark, c quark, and the electron appears as subtractions between Yukawa couplings, which gives an additional suppression of their masses. This feature is not accidental but comes from the form of the seesaw formula (43),

$$m_{F,\text{SM}}^{\text{sym}} \approx \mathcal{M}^f - \mathcal{M}^{f\mathcal{F}}(\mathcal{M}^{\mathcal{F}})^{-1}\mathcal{M}^{\mathcal{F}f},$$

and the vacuum hierarchy. For instance, the c mass comes from the following sub-block in the uplike quark mass matrix:

$$\mathbb{M}_{c\mathcal{T}} \propto \begin{pmatrix} h_{1u}^{22}v_1 & | & h_{1\mathcal{T}}^{2}v_1 \\ — & — & — \\ g_{\chi u}^{2}v_\chi & | & g_{\chi\mathcal{T}}v_\chi \end{pmatrix}.$$

By taking into account the fact that $v_\chi \ll v_1$, the eigenvalues

$$m_c^2 = \frac{(h_{1u}^{22}g_{\chi\mathcal{T}} - h_{1\mathcal{T}}^{2}g_{\chi u}^{2})^2}{(g_{\chi\mathcal{T}})^2 + (g_{\chi u}^{2})^2}\frac{v_1^2}{2},$$
$$m_{\mathcal{T}}^2 = [(g_{\chi\mathcal{T}})^2 + (g_{\chi u}^{2})^2]\frac{v_\chi^2}{2} + \frac{(h_{1u}^{22}g_{\chi u}^{2} + h_{1\mathcal{T}}^{2}g_{\chi\mathcal{T}})^2}{(g_{\chi\mathcal{T}})^2 + (g_{\chi u}^{2})^2}\frac{v_1^2}{2}$$

are obtained, such that the exotic $\mathcal{T}$ quark suppress the c-quark mass. The same scenario appears in the u-quark mass, where the corresponding sub-block is

$$\mathbb{M}_{ut} \propto \begin{pmatrix} h_{3u}^{11}v_3 & | & h_{3u}^{13}v_3 \\ — & — & — \\ h_{1u}^{31}v_1 & | & h_{1u}^{33}v_1 \end{pmatrix},$$

the eigenvalues of which turn out to be (with the assumption $v_3/v_1 \ll 1$)

$$m_u^2 = \frac{(h_{3u}^{11}h_{1u}^{33} - h_{3u}^{13}h_{1u}^{31})^2}{(h_{1u}^{33})^2 + (h_{1u}^{31})^2}\frac{v_3^2}{2},$$
$$m_t^2 = [(h_{1u}^{33})^2 + (h_{1u}^{31})^2]\frac{v_1^2}{2} + \frac{(h_{3u}^{11}h_{1u}^{33} + h_{3u}^{13}h_{1u}^{31})^2}{(h_{1u}^{33})^2 + (h_{1u}^{31})^2}\frac{v_3^2}{2},$$

and it is observed again how a heavier mass, in this case the t-quark mass, suppresses the mass of the light u quark. Finally, the e and μ leptons shows a similar behavior, but the suppression is between v_3 in the second column with the v_1 in the fourth column of the charged lepton mass matrix

$$\mathbb{M}_{e\mu} \propto \begin{pmatrix} h_{3e}^{e\mu}v_3 & | & h_{1\mathcal{E}}^{e1}v_1 \\ — & — & — \\ h_{3e}^{\mu\mu}v_3 & | & h_{1\mathcal{E}}^{\mu1}v_1 \end{pmatrix}, \qquad (119)$$

such that the masses of the lightest charged leptons are

$$m_e^2 = \frac{(h_{3e}^{e\mu}h_{1\mathcal{E}}^{\mu1} - h_{3e}^{\mu\mu}h_{1\mathcal{E}}^{e1})^2}{(h_{1\mathcal{E}}^{e1})^2 + (h_{1\mathcal{E}}^{\mu1})^2}\frac{v_3^2}{2},$$
$$m_\mu^2 = \frac{(h_{3e}^{e\mu}h_{1\mathcal{E}}^{e1} + h_{3e}^{\mu\mu}h_{1\mathcal{E}}^{\mu1})^2}{(h_{1\mathcal{E}}^{e1})^2 + (h_{1\mathcal{E}}^{\mu1})^2}\frac{v_3^2}{2}. \qquad (120)$$

These suppression mechanisms are induced by the vacuum hierarchy together with the zero texture matrices obtained from the nonuniversal $\text{U}(1)_X$ interaction and the $\mathbb{Z}_2$ parity. Consequently, the subtraction of Yukawa coupling constants is a natural consequence of the diagonalization of the mass matrices by taking into account the vacuum hierarchy outlined in the previous paragraphs.

Furthermore, the model is not only consistent with the fermion mass hierarchy of the SM, but also it is consistent with some phenomenological reports in the lepton sector. Regarding the consistency of the model with current neutrino oscillation data, this scheme is consistent with both mass orderings, NO and IO, but the former is preferred because of the large abundance of solutions in comparison with the latter. The PMNS angles and the squared-mass differences are satisfied without needing fine-tunings due to the fact that the parameters h_ν^e, h_ν^μ, and h_ν^τ vary from 0 to 1, and similarly the angles θ_ν^μ and θ_ν^τ span $\pm 180°$. Moreover, the suited neutrino mass scale is fitted by the Majorana mass $\mu_{\mathcal{N}}$, the Yukawa coupling $h_{\mathcal{N}}^1$, and the vacua v_3 and v_χ, so the model has a large set of solutions in order to be consistent with neutrino oscillation data. Additionally, the model is adequate for understanding the CMS report about the $h \to \tau\mu$ branching ratio. The mixing angles of the CP-even scalars, the left- and right-handed charged leptons, together with the vacuum hierarchy yield definite regions of consistency in the mixing angle space, where the most important relation is between s_{13}^h and $s_{R,13}^E$.

Regarding the exotic neutral sector N and $\tilde{N}$, Eqs. (85) and (96) allow us to set the pseudo-Dirac neutrinos masses from 100 to 700 GeV by setting $\mu_{\mathcal{N}}$ from 4 keV to 0.1 GeV such that N and $\tilde{N}$ can be observed at current energy scales at particle colliders. Moreover, since $h^3_{\mathcal{N}}$ in Eq. (85) does not matter in setting the correct squared-mass differences of light neutrinos, N^3 and $\tilde{N}^3$ masses are not constrained.

The present model shows how the introduction of new nonuniversal quantum numbers and an extended scalar sector present a fertile scenario in which some issues—which the SM cannot explain, such as the fermion mass hierarchy; HLFV processes; or, last but not least, the evidence of the massive nature of neutrinos in their oscillations—can be understood with the introduction of the least number of new particles and symmetries.

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[1] S. L. Glashow, Partial-symmetries of weak interactions, Nucl. Phys. **22**, 579 (1961).
[2] A. Salam, *Elementary Particle Physics (Nobel Symp. No. 8)*, edited by N. Svartholm (Almquist and Wiksells, Stockholm, 1968), p. 367.
[3] S. Weinberg, A Model of Leptons, Phys. Rev. Lett. **19**, 1264 (1967).
[4] H. Georgi, The flavor problem, Phys. Lett. B **169**, 231 (1986).
[5] H. Fritzsch, Weak-interaction mixing in the six-quark theory, Phys. Lett. B **73**, 317 (1978).
[6] M. Fukugita, M. Tanimoto, and T. Yanagida, Phenomenological lepton mass matrix, Prog. Theor. Phys. **89**, 263 (1993).
[7] R. Davis, Jr., D. S. Harmer, and K. C. Hoffman, Search for Neutrinos from the Sun, Phys. Rev. Lett. **20**, 1205 (1968).
[8] J. N. Abdurashitov *et al.*, Measurement of the solar neutrino capture rate with gallium metal. III. Results for the 2002–2007 data-taking period, Phys. Rev. C **80**, 015807 (2009).
[9] F. Kaether, W. Hampel, G. Heusser, J. Kiko, and T. Kirsten, Reanalysis of the *GALLEX* solar neutrino flux and source experiments, Phys. Lett. B **685**, 47 (2010).
[10] B. T. Cleveland, T. Daily, R. Davis, Jr., J. R. Distel, K. Lande, C. Lee, P. S. Wildenhain, and J. Ullman, Measurement of the solar electron neutrino flux with the *H*omestake chlorine detector, Astrophys. J. **496**, 505 (1998).
[11] B. Aharmim *et al.*, Combined analysis of all three phases of solar neutrino data from the Sudbury Neutrino Observatory, Phys. Rev. C **88**, 025501 (2013).
[12] G. Bellini *et al.*, Measurement of the solar B 8 neutrino rate with a liquid scintillator target and 3 MeV energy threshold in the Borexino detector, Phys. Rev. D **82**, 033006 (2010).
[13] G. Bellini *et al.* (Borexino Collaboration), Neutrinos from the primary proton-proton fusion process in the Sun, Nature (London) **512**, 383 (2014).
[14] J. Hosaka *et al.*, Solar neutrino measurements in Super-Kamiokande-I, Phys. Rev. D **73**, 112001 (2006).
[15] J. Hosaka *et al.*, Three flavor neutrino oscillation analysis of atmospheric neutrinos in Super-Kamiokande, Phys. Rev. D **74**, 032002 (2006).
[16] J. Cravens *et al.*, Solar neutrino measurements in Super-Kamiokande-II, Phys. Rev. D **78**, 032002 (2008).
[17] M. Aartsen *et al.*, Determining neutrino oscillation parameters from atmospheric muon neutrino disappearance with three years of IceCube DeepCore data, Phys. Rev. D **91**, 072004 (2015).
[18] A. Gando *et al.*, Constraints on θ_{13} from a three-flavor oscillation analysis of reactor antineutrinos at KamLAND, Phys. Rev. D **83**, 052002 (2011).
[19] M. Apollonio *et al.*, Limits on neutrino oscillations from the CHOOZ experiment, Phys. Lett. B **466**, 415 (1999).
[20] A. Piepke *et al.* (Palo Verde Collaboration), Final results from the Palo Verde neutrino oscillation experiment, Prog. Part. Nucl. Phys. **48**, 113 (2002).
[21] F. An *et al.*, New Measurement of Antineutrino Oscillation with the Full Detector Configuration at Daya Bay, Phys. Rev. Lett. **115**, 111802 (2015).
[22] S.-B. Kim *et al.* (RENO Collaboration), Measurement of neutrino mixing angle θ_{13} and mass difference from reactor antineutrino disappearance in the RENO experiment, Nucl. Phys. **B908**, 94 (2016).
[23] J. Kopp, P. A. Machado, M. Maltoni, and T. Schwetz, Sterile neutrino oscillations: The global picture, J. High Energy Phys. 05 (2013) 050.
[24] P. Adamson *et al.*, Measurement of Neutrino and Antineutrino Oscillations using Beam and Atmospheric Data in MINOS, Phys. Rev. Lett. **110**, 251801 (2013).
[25] P. Adamson *et al.*, Electron Neutrino and Antineutrino Appearance in the Full MINOS Data Sample, Phys. Rev. Lett. **110**, 171801 (2013).
[26] K. Abe *et al.*, Observation of Electron Neutrino Appearance in a Muon Neutrino Beam, Phys. Rev. Lett. **112**, 061802 (2014).
[27] L. Kolupaeva, Current results of the NOνA experiment, Eur. Phys. J. Web Conf. **125**, 01002 (2016).
[28] P. Adamson *et al.*, Constraints on Oscillation Parameters from ν_e Appearance and ν_μ Disappearance in NOνA, Phys. Rev. Lett. **118**, 231801 (2017).
[29] I. Esteban, M. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, and T. Schwetz, Updated fit to three neutrino mixing:

Exploring the accelerator-reactor complementarity, J. High Energy Phys. 01 (2017) 087.
[30] Nufit 3.0, www.nu-fit.org, 2016.
[31] P. Minkowski, $\mu \to e\gamma$ at a rate of one out of 109 muon decays?, Phys. Lett. B **67**, 421 (1977).
[32] M. Gell-Mann and P. Ramond, and R. Slansky, *Supergravity*, edited by F. Nieuwenhuizen and D. Friedman (North-Holland, Amsterdam, 1979), p. 315.
[33] T. Yanagida, Horizontal symmetry and masses of neutrinos, Prog. Theor. Phys. **64**, 1103 (1980).
[34] R. N. Mohapatra and G. Senjanović, Neutrino Mass and Spontaneous Parity Nonconservation, Phys. Rev. Lett. **44**, 912 (1980).
[35] J. Schechter and J. W. Valle, Neutrino masses in $SU(2) \otimes \mathrm{U}(1)$ theories, Phys. Rev. D **22**, 2227 (1980).
[36] R. N. Mohapatra, Mechanism for Understanding Small Neutrino Mass in Superstring Theories, Phys. Rev. Lett. **56**, 561 (1986).
[37] R. N. Mohapatra and J. W. Valle, Neutrino mass and baryon-number nonconservation in superstring models, Phys. Rev. D **34**, 1642 (1986).
[38] E. Cataño, R. Martinez, and F. Ochoa, Neutrino masses in a 331 model with right-handed neutrinos without doubly charged higgs bosons via inverse and double seesaw mechanisms, Phys. Rev. D **86**, 073015 (2012).
[39] A. G. Dias, C. d. S. Pires, P. R. da Silva, and A. Sampieri, Simple realization of the inverse seesaw mechanism, Phys. Rev. D **86**, 035007 (2012).
[40] M. Fukugita and T. Yanagida, Barygenesis without grand unification, Phys. Lett. B **174**, 45 (1986).
[41] G. Aad *et al.*, Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. B **716**, 1 (2012).
[42] S. Chatrchyan *et al.*, Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, Phys. Lett. B **716**, 30 (2012).
[43] J. F. Gunion, H. E. Haber, G. Kane, and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, New York, 1990), Vol. 2.
[44] G. C. Branco, P. Ferreira, L. Lavoura, M. Rebelo, M. Sher, and J. P. Silva, Theory and phenomenology of two-Higgs-doublet models, Phys. Rep. **516**, 1 (2012).
[45] X. G. He, T. Li, X. Q. Li, J. Tandean, and H. C. Tsai, The simplest dark-matter model, CDMS II results, and Higgs detection at LHC, Phys. Lett. B **688**, 332 (2010).
[46] B. Grzadkowski and P. Osland, Tempered two-Higgs-doublet model, Phys. Rev. D **82**, 125026 (2010).
[47] S. Profumo, Fundamental physics from the sky: Cosmic rays, gamma rays and the hunt for dark matter, J. Phys. Conf. Ser. **485**, 012007 (2014).
[48] X. G. He and J. Tandean, New LUX and PandaX-II results illuminating the simplest Higgs-portal dark matter models, J. High Energy Phys. 12 (2016) 74.
[49] H. Han and S. Zheng, Higgs-portal scalar dark matter: Scattering cross section and observable limits, Nucl. Phys. **B914**, 248 (2017).
[50] S. von Buddenbrock, N. Chakrabarty, A. S. Cornell, D. Kar, M. Kumar, T. Mandal, B. Mellado, B. Mukhopadhyaya, R. G. Reed, and X. Ruan, Phenomenological signatures of additional scalar bosons at the LHC, Eur. Phys. J. C **76**, 580 (2016).
[51] M. Carena, H. E. Haber, I. Low, N. R. Shah, and C. E. Wagner, Alignment limit of the NMSSM Higgs sector, Phys. Rev. D **93**, 035013 (2016).
[52] P. Langacker, The physics of heavy Z' gauge bosons, Rev. Mod. Phys. **81**, 1199 (2009).
[53] E. Ma, Neutrino masses in an extended gauge model with E_6 particle content, Phys. Lett. B **380**, 286 (1996).
[54] V. Barger, P. Langacker, and H.-S. Lee, Primordial nucleosynthesis constraints on Z' properties, Phys. Rev. D **67**, 075009 (2003).
[55] S. King, S. Moretti, and R. Nevzorov, Theory and phenomenology of an exceptional supersymmetric standard model, Phys. Rev. D **73**, 035009 (2006).
[56] P. Langacker and M. Plümacher, Flavor changing effects in theories with a heavy Z' boson with family nonuniversal couplings, Phys. Rev. D **62**, 013006 (2000).
[57] K. Leroux and D. London, Flavour-changing neutral currents and leptophobic Z' gauge bosons, Phys. Lett. B **526**, 97 (2002).
[58] S. Baek, J. H. Jeon, and C. Kim, $B_s^0 - \bar{B}_s^0$ mixing in leptophobic Z' model, Phys. Lett. B **641**, 183 (2006).
[59] T. Hur, H.-S. Lee, and S. Nasri, Supersymmetric $\mathrm{U}(1)'$ model with multiple dark matters, Phys. Rev. D **77**, 015008 (2008).
[60] G. Bélanger, A. Pukhov, and G. Servant, Dirac neutrino dark matter, J. Cosmol. Astropart. Phys. 01 (2008) 009.
[61] S. Profumo and F. S. Queiroz, Constraining the z' mass in 331 models using direct dark matter detection, Eur. Phys. J. C **74**, 2960 (2014).
[62] A. Alves, A. Berlin, S. Profumo, and F. S. Queiroz, Dirac-fermionic dark matter in $\mathrm{U}(1)_X$ models, J. High Energy Phys. 10 (2015) 76.
[63] A. Alves, G. Arcadi, P. Dong, L. Duarte, F. S. Queiroz, and J. W. Valle, R-parity as a residual gauge symmetry: Probing a theory of cosmological dark matter, Phys. Lett. B **772**, 825 (2017).
[64] G. Arcadi, M. Dutra, P. Ghosh, M. Lindner, Y. Mambrini, M. Pierre, S. Profumo, and F. S. Queiroz, The Waning of the WIMP? A Review of Models, Searches, and Constraints, arXiv:1703.07364.
[65] A. Leike, The phenomenology of extra neutral gauge bosons, Phys. Rep. **317**, 143 (1999).
[66] E. Accomando, C. Corianò, L. D. Rose, J. Fiaschi, C. Marzo, and S. Moretti, Z', Higgses and heavy neutrinos in $\mathrm{U}(1)_X$ models: From the LHC to the GUT scale, J. High Energy Phys. 07 (2016) 086.
[67] L. D. Rose, S. Khalil, and S. Moretti, Explanation of the 17 MeV Atomki Anomaly in a $\mathrm{U}(1)'$-Extended 2-Higgs Doublet Model, arXiv:1704.03436.
[68] R. Martinez, J. Nisperuza, F. Ochoa, and J. Rubio, Some phenomenological aspects of a new $\mathrm{U}(1)'$ model, Phys. Rev. D **89**, 056008 (2014).
[69] R. Martinez, J. Nisperuza, F. Ochoa, and J. Rubio, Scalar dark matter with CERN-LEP data and Z' search at the LHC in an $\mathrm{U}(1)'$ model, Phys. Rev. D **90**, 095004 (2014).
[70] R. Martinez, J. Nisperuza, F. Ochoa, J. Rubio, and C. Sierra, Scalar coupling limits and diphoton Higgs decay from LHC

in an $\mathrm{U}(1)'$ model with scalar dark matter, Phys. Rev. D **92**, 035016 (2015).

[71] R. Martinez and F. Ochoa, Spin-independent interferences and spin-dependent interactions with scalar dark matter, J. High Energy Phys. 05 (2016) 113.

[72] S. F. Mantilla, R. Martinez, and F. Ochoa, Neutrino and cp-even Higgs boson masses in a nonuniversal $u(1)'$ extension, Phys. Rev. D **95**, 095037 (2017).

[73] W. Grimus and L. Lavoura, The seesaw mechanism at arbitrary order: disentangling the small scale from the large scale, J. High Energy Phys. 11 (2001) 042.

[74] A. C. Hernandez, R. Martinez, and F. Ochoa, Radiative seesaw-type mechanism of quark masses in $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$, Phys. Rev. D **87**, 075009 (2013).

[75] B. Pontecorvo, Inverse beta processes and nonconservation of lepton charge, Zh. Eksp. Teor. Fiz. **34** (1958).

[76] Z. Maki, M. Nakagawa, and S. Sakata, Remarks on the unified model of elementary particles, Prog. Theor. Phys. **28**, 870 (1962).

[77] C. Patrignani *et al.* (Particle Data Group Collaboration), Review of particle physics, Chin. Phys. C **40**, 100001 (2016).

[78] V. Khachatryan *et al.* (CMS Collaboration), Search for lepton-flavour-violating decays of the Higgs boson, Phys. Lett. B **749**, 337 (2016).

[79] G. e. Aad, Search for lepton-flavour-violating decays of the Higgs and z bosons with the atlas detector, Eur. Phys. J. C **77**, 70 (2017).

[80] J. Herrero-García, N. Rius, and A. Santamaria, Higgs lepton flavour violation: UV completions and connection to neutrino masses, J. High Energy Phys. 11 (2016) 84.