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Neutrino mass matrices with three or four vanishing cofactors and nondiagonal charged lepton sector

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We investigate the texture structures of lepton mass matrices with four (five) nonzero elements in the charged lepton mass matrix and three (four) vanishing cofactors in the neutrino mass matrix. Using weak basis transformations, all possible textures for three and four vanishing cofactors in M_ν are grouped into seven classes, and predictions for the unknown parameters—such as the Dirac CP-violating phase and the effective Majorana mass—for the phenomenologically allowed textures are obtained. We, also, illustrate how such texture structures can be realized using discrete Abelian flavor symmetries.

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I. INTRODUCTION

Solar, atmospheric, and reactor neutrino experiments in addition to the more recent neutrino production from acceleration-based beams have provided some novel results over the last two decades or so and, invariably, strengthened the flavor Standard Model (SM). However, some critical ingredients like leptonic CP violation, the neutrino mass hierarchy and neutrino masses are still missing. Furthermore, the nature of neutrinos (Majorana/Dirac) and absolute neutrino masses are still open issues. While the developments over the past two decades have brought out a coherent picture of neutrino mixing, the neutrino mass hierarchy, which is strongly correlated with the neutrino masses and the CP phase δ is still unknown. Specifically, the sign of $|\Delta m_{31}^2| = |m_3^2 - m_1^2|$ is still unconstrained and is the focal issue for several ongoing and forthcoming experiments. In addition, recent neutrino oscillation data hint towards a nonmaximal atmospheric mixing angle (θ_{23}) which implies two possibilities: $\theta_{23} < \frac{\pi}{4}$ or $\theta_{23} > \frac{\pi}{4}$ [1] which when combined with the θ_{13}-δ [2] and the $\pm\Delta m_{31}^2$-δ degeneracy [3] leads to an overall eightfold degeneracy [4]. In the SM, all fermion masses are Dirac masses which are generated via the Higgs mechanism. In order to have massive Dirac neutrinos, one has to necessarily enlarge the SM particle content by introducing right-handed neutrinos ν_R. The ν_L and ν_R form a Dirac spinor $\Psi_\nu = \nu_L + \nu_R$ where ν_R are the additional spin states for the neutrinos. However, the gauge singlets ν_R can have a Majorana mass term $\nu_R^T C^{-1} M_R \nu_R$ where M_R, in general, is not diagonal in the flavor basis where M_D is diagonal. Diagonalizing the full mass term leads to Majorana neutrinos and new mass eigenstates. Of course, one can attempt to forbid M_R by postulating an additional symmetry such as lepton number conservation.

The mass matrix for Majorana neutrinos is, in general, complex symmetric containing nine physical parameters which include the three mass eigenvalues (m_1, m_2, m_3), the three mixing angles (θ_{13}, θ_{12}, θ_{23}) and the three CP-violating phases (α, β, δ). The two mass-squared differences (Δm_{12}^2, Δm_{23}^2) and the three neutrino mixing angles (θ_{12}, θ_{23}, θ_{13}) have been measured in solar, atmospheric and reactor neutrino experiments. While the Dirac-type CP-violating phase δ will be probed in the forthcoming neutrino oscillation experiments the neutrinoless double beta decay searches will provide additional constraints on the neutrino mass scale. The last unknown mixing angle (θ_{13}) has been measured with a fairly good precision by a number of recent [5–9] experiments. It is, however, clear that the neutrino mass matrix which encodes the neutrino properties has several unknown neutrino parameters which will remain undetermined even in the near future. Thus, the phenomenological approaches aimed at reducing the number of independent parameters are bound to play a crucial role in further development. There are several classes of predictive models in the literature such as texture zeros [10–14], vanishing cofactors [15–17], hybrid textures [18] and equality between elements [19] which explain the presently available neutrino oscillation data.

Neutrino mass matrices with texture zeros and vanishing cofactors are particularly interesting due to their connections to flavor symmetries. Neutrino mass models with texture zeros and vanishing cofactors have been widely studied in the literature [10–17,20,21] for this reason. The lepton mass matrices with texture zeros and vanishing cofactors in both the charged lepton mass matrix M_l and the neutrino mass matrix M_ν have been systematically studied in Refs. [13,14,20]. Lepton mass matrices where the charged lepton mass matrix M_l has four (five) nonzero elements while the Majorana neutrino mass matrix M_ν has

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three (two) nonzero matrix elements have been studied recently in Ref. [22] and an inverted neutrino mass ordering with a nonmaximal Dirac-type CP-violating phase δ are predicted for all viable textures. The recent confirmation of a nonzero and not-so-small reactor mixing angle θ_{13} has emerged as an important discriminator of neutrino mass models and many models based on discrete symmetries have been discarded as these models require the breaking of these symmetries to accommodate the current neutrino data.

In the present work, we consider new textures of lepton mass matrices and systematically, investigate their predictions for the unknown parameters. We show that these new textures can be realized on the basis of discrete Abelian Z_n symmetries. Specifically, we investigate textures of lepton mass matrices with four (five) nonzero elements in M_l and three (four) vanishing cofactors in M_ν. The textures considered in the present work are as predictive as the textures with two texture zeros/two vanishing cofactors in the flavor basis. Moreover, vanishing cofactors in M_ν can be seen as zero entries in M_R and M_D within the framework of the type-I seesaw mechanism [23]:

$$M_\nu = -M_D M_R^{-1} M_D^T, \tag{1}$$

where M_D is the Dirac neutrino mass matrix and M_R is the right-handed Majorana neutrino mass matrix.

The texture structures related by weak basis transformations lead to the same predictions for neutrino parameters; hence, one cannot distinguish mass matrix structures related by weak basis transformations. The charged lepton mass matrix having four nonzero matrix elements with a nonzero determinant (as none of the charged lepton masses is zero), can have the following form:

$$M_l = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & 0 & \times \end{pmatrix}. \tag{2}$$

Other possible structures can be obtained by considering all possible reorderings of rows and columns of M_l. The Hermitian products $H_l = M_l M_l^\dagger$ corresponding to charged lepton mass matrices are given by

$$H_{l1} = \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}, \qquad H_{l2} = \begin{pmatrix} \times & 0 & \times \\ 0 & m_\mu^2 & 0 \\ \times & 0 & \times \end{pmatrix},$$
$$H_{l3} = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix} \tag{3}$$

where the diagonalization of H_l gives the value of V_l as $V_l^\dagger H_l V_l = \mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2)$.

The neutrino mass matrices with three vanishing cofactors have the following 20 distinct possible structures which have been classified into six classes:

Class-I

$$M_{\nu 1} = \begin{pmatrix} \times & \Delta & \Delta \\ \Delta & \times & \Delta \\ \Delta & \Delta & \times \end{pmatrix}, \tag{4}$$

Class-II

$$M_{\nu 2} = \begin{pmatrix} \Delta & \Delta & \times \\ \Delta & \times & \Delta \\ \times & \Delta & \times \end{pmatrix}, \qquad M_{\nu 3} = \begin{pmatrix} \times & \Delta & \Delta \\ \Delta & \Delta & \times \\ \Delta & \times & \times \end{pmatrix},$$
$$M_{\nu 4} = \begin{pmatrix} \Delta & \times & \Delta \\ \times & \times & \Delta \\ \Delta & \Delta & \times \end{pmatrix}, \qquad M_{\nu 5} = \begin{pmatrix} \times & \Delta & \Delta \\ \Delta & \times & \times \\ \Delta & \times & \Delta \end{pmatrix},$$
$$M_{\nu 6} = \begin{pmatrix} \times & \times & \Delta \\ \times & \Delta & \Delta \\ \Delta & \Delta & \times \end{pmatrix}, \qquad M_{\nu 7} = \begin{pmatrix} \times & \Delta & \times \\ \Delta & \times & \Delta \\ \times & \Delta & \Delta \end{pmatrix}, \tag{5}$$

Class-III

$$M_{\nu 8} = \begin{pmatrix} \times & \times & \Delta \\ \times & \Delta & \times \\ \Delta & \times & \Delta \end{pmatrix}, \qquad M_{\nu 9} = \begin{pmatrix} \times & \Delta & \times \\ \Delta & \Delta & \times \\ \times & \times & \Delta \end{pmatrix},$$
$$M_{\nu 10} = \begin{pmatrix} \Delta & \times & \times \\ \times & \times & \Delta \\ \times & \Delta & \Delta \end{pmatrix}, \qquad M_{\nu 11} = \begin{pmatrix} \Delta & \Delta & \times \\ \Delta & \times & \times \\ \times & \times & \Delta \end{pmatrix},$$
$$M_{\nu 12} = \begin{pmatrix} \Delta & \times & \times \\ \times & \Delta & \Delta \\ \times & \Delta & \times \end{pmatrix}, \qquad M_{\nu 13} = \begin{pmatrix} \Delta & \times & \Delta \\ \times & \Delta & \times \\ \Delta & \times & \times \end{pmatrix}, \tag{6}$$

Class-IV

$$M_{\nu 14} = \begin{pmatrix} \Delta & \Delta & \Delta \\ \Delta & \times & \times \\ \Delta & \times & \times \end{pmatrix}, \qquad M_{\nu 15} = \begin{pmatrix} \times & \times & \Delta \\ \times & \times & \Delta \\ \Delta & \Delta & \Delta \end{pmatrix},$$
$$M_{\nu 16} = \begin{pmatrix} \times & \Delta & \times \\ \Delta & \Delta & \Delta \\ \times & \Delta & \times \end{pmatrix}, \tag{7}$$

Class-V

$$M_{\nu 17} = \begin{pmatrix} \Delta & \times & \times \\ \times & \Delta & \times \\ \times & \times & \Delta \end{pmatrix}, \tag{8}$$

Class-VI

$$M_{\nu 18} = \begin{pmatrix} \times & \times & \times \\ \times & \Delta & \Delta \\ \times & \Delta & \Delta \end{pmatrix}, \qquad M_{\nu 19} = \begin{pmatrix} \Delta & \times & \Delta \\ \times & \times & \times \\ \Delta & \times & \Delta \end{pmatrix},$$
$$M_{\nu 20} = \begin{pmatrix} \Delta & \Delta & \times \\ \Delta & \Delta & \times \\ \times & \times & \times \end{pmatrix} \tag{9}$$

where Δ at the (i,j) position represents the vanishing cofactor corresponding to element (i,j) while $\times$ denotes a nonzero arbitrary entry. Neutrino mass matrices in each class are related by S_3 permutation symmetry as $M_\nu \to S^T M_\nu S$, where S denotes the permutation matrices corresponding to the S_3 group:

$$S_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad S_{123} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$
$$S_{132} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \qquad S_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
$$S_{13} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad S_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{10}$$

A. Analysis

The mass term for charged leptons and Majorana neutrinos can be written as

$$-\mathcal{L}_{\text{mass}} = \bar{l}_L M_l l_R - \frac{1}{2}\nu_L^T C^{-1} M_\nu \nu_L + \text{H.c.} \tag{11}$$

where C is the charge conjugation matrix. The charged lepton and the Majorana neutrino mass matrix can be diagonalized as

$$V_l^\dagger M_l M_l^\dagger V_l = (M_l^D)^2, \qquad V_\nu^T M_\nu V_\nu = M_\nu^D \tag{12}$$

where $M_l^D = \text{diag}(m_e, m_\mu, m_\tau)$, and $M_\nu^D = \text{diag}(m_1, m_2, m_3)$. V_l and V_ν are unitary matrices connecting mass eigenstates to the flavor eigenstates. The lepton mixing matrix also known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [24] is given by

$$U = V_l^\dagger V_\nu \tag{13}$$

which can be parametrized in terms of three mixing angles and three CP-violating phases in the standard parametrization as [25]

$$U = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -e^{i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i(\beta+\delta)} \end{pmatrix} \tag{14}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. α and β are the two Majorana CP-violating phases and δ is the Dirac-type CP-violating phase.

The CP violation in neutrino oscillation experiments can be reflected in terms of the Jarlskog rephasing invariant quantity J_{CP} [26] with

$$J_{CP} = \text{Im}\{U_{11}U_{22}U_{12}^*U_{21}^*\}$$
$$= \sin\theta_{12}\sin\theta_{23}\sin\theta_{13}\cos\theta_{12}\cos\theta_{23}\cos^2\theta_{13}\sin\delta. \tag{15}$$

The effective Majorana neutrino mass $|m_{ee}|$, which determines the rate of neutrinoless double beta decay is given by

$$|m_{ee}| = |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2|. \tag{16}$$

There are a large number of experiments such as CUORICINO [27], CUORE [28], MAJORANA [29], SuperNEMO [30], and EXO [31] which aim to achieve a sensitivity up to 0.01 eV for $|m_{ee}|$.

Recent cosmological observations provide more stringent constraints on the absolute neutrino mass scale. Planck satellite data [32] combined with WMAP, cosmic microwave background and baryon acoustic oscillation experiments limit the sum of neutrino masses $\sum_{i=1}^3 m_i \leq 0.23$ eV at the 95% confidence level (C.L.).

II. NUMERICAL ANALYSIS

In this section we present a detailed numerical analysis along with the main predictions for viable textures. The charged lepton mixing matrices corresponding to structures H_{l1}, H_{l2}, H_{l3} are given by

$$V_{l1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & e^{i\phi_l}\sin\theta \\ 0 & -e^{-i\phi_l}\sin\theta & \cos\theta \end{pmatrix},$$

$$V_{l2} = \begin{pmatrix} \cos\theta & 0 & e^{i\phi_l}\sin\theta \\ 0 & 1 & 0 \\ -e^{-i\phi_l}\sin\theta & 0 & \cos\theta \end{pmatrix},$$

$$V_{l3} = \begin{pmatrix} \cos\theta & e^{i\phi_l}\sin\theta & 0 \\ -e^{-i\phi_l}\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (17)$$

respectively, with $\cos\theta = \sqrt{\frac{m_y - m}{m_y - m_x}}$. The parameters m_x, m_y are defined as $m_x = m_\mu^2$, $m_y = m_\tau^2$ for the structure H_{l1}, $m_x = m_e^2$, $m_y = m_\tau^2$ for the structure H_{l2} and $m_x = m_e^2$, $m_y = m_\mu^2$ for the structure H_{l3}. Also, the parameter m is constrained as $m_x < m < m_y$. The charged lepton mass eigenvalues are $m_e = 0.510998928$ MeV, $m_\mu = 105.6583715$ MeV and $m_\tau = 1776.86$ MeV [33].

Neutrino mass matrices of Class-I and Class-II lead to one or more zeros in the lepton mixing matrix U and, hence, both classes are phenomenologically excluded. Neutrino mass matrices of Class-VI lead to two degenerate neutrino mass eigenvalues, which is inconsistent with the current experimental data and hence this class is, also, phenomenologically ruled out. Therefore, we focus on the other three nontrivial classes i.e., Classes-III, -IV and -V. The ten possible structures for M_ν from Classes-III, -IV, -V along with the three structures for H_l, lead to a total of $10 \times 3 = 30$ possible combinations of charged lepton and neutrino mass matrices. But all possible combinations for H_l and M_ν are not independent of each other as the transformations $M_\nu \to S M_\nu S^T$ and $H_l \to S H_l S^\dagger$ relate some of the textures with each other. Table I contains all possible independent texture structures of M_ν and H_l and their viabilities for normal mass ordering (NO) and inverted mass ordering (IO).

TABLE I. Possible independent structures of H_l, M_ν and their viability.

Class	Texture	H_l, M_ν	Viability NO	Viability IO
III	III-(A)	H_{l1}, $M_{\nu 8} \sim H_{l1}$, $M_{\nu 9}$	×	×
	III-(B)	H_{l2}, $M_{\nu 10} \sim H_{l2}$, $M_{\nu 11}$	×	×
	III-(C)	H_{l3}, $M_{\nu 13} \sim H_{l3}$, $M_{\nu 12}$	×	×
	III-(D)	H_{l1}, $M_{\nu 10} \sim H_{l1}$, $M_{\nu 12}$	×	×
	III-(E)	H_{l2}, $M_{\nu 8} \sim H_{l2}$, $M_{\nu 13}$	×	×
	III-(F)	H_{l3}, $M_{\nu 11} \sim H_{l3}$, $M_{\nu 9}$	×	×
	III-(G)	H_{l1}, $M_{\nu 11} \sim H_{l1}$, $M_{\nu 13}$	×	×
	III-(H)	H_{l3}, $M_{\nu 8} \sim H_{l3}$, $M_{\nu 10}$	√	×
	III-(I)	H_{l2}, $M_{\nu 9} \sim H_{l2}$, $M_{\nu 12}$	√	×
IV	IV-(A)	H_{l1}, $M_{\nu 14}$	×	×
	IV-(B)	H_{l2}, $M_{\nu 16}$	×	×
	IV-(C)	H_{l3}, $M_{\nu 15}$	×	×
	IV-(D)	H_{l3}, $M_{\nu 14} \sim H_{l3}$, $M_{\nu 16}$	√	√
	IV-(E)	H_{l2}, $M_{\nu 14} \sim H_{l2}$, $M_{\nu 15}$	√	√
	IV-(F)	H_{l1}, $M_{\nu 16} \sim H_{l1}$, $M_{\nu 15}$	√	√
V	V-(A)	H_{l1}, $M_{\nu 17}$	×	×
	V-(B)	H_{l2}, $M_{\nu 17}$	×	×
	V-(C)	H_{l3}, $M_{\nu 17}$	×	×

A. Class-III

1. Texture III-(H)

First, we analyze the texture structure III-(H) in which M_ν has vanishing cofactors corresponding to the (1,3), (2,2) and (3,3) elements and H_l has texture zeros at the (1,3) and (2,3) positions. The texture structures of neutrino and charged lepton mass matrices have the following form:

$$M_{\nu 8} = \begin{pmatrix} \times & \times & \Delta \\ \times & \Delta & \times \\ \Delta & \times & \Delta \end{pmatrix} \quad \text{and} \quad H_{l3} = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix}. \quad (18)$$

All the nonzero elements of M_ν are, in general, complex. The neutrino mass matrix can be made real as $M_\nu = P_\nu M_\nu^r P_\nu^T$, with the phase matrix $P_\nu = \mathrm{diag}(e^{i\psi_1}, e^{i\psi_2}, e^{i\psi_3})$. The matrix M_ν^r is diagonalized by the orthogonal matrix O_ν as

$$M_\nu^r = O_\nu M_\nu^D O_\nu^T \quad (19)$$

and the neutrino mixing matrix is $V_\nu = P_\nu O_\nu$. The PMNS mixing matrix is given by

$$U = V_l^\dagger V_\nu = V_l^\dagger P_\nu O_\nu. \quad (20)$$

We use the invariants $\mathrm{Tr}[M_\nu^r]$, $\mathrm{Tr}[M_\nu^{r^2}]$ and $\mathrm{Det}[M_\nu^r]$ to redefine mass matrix elements in terms of mass eigenvalues. The eigenvalues of the neutrino mass matrix for NO are m_1, $-m_2$ and m_3. The eigenvalues m_2 and m_3 can be calculated from the mass-squared differences Δm_{21}^2 and Δm_{31}^2 using the relation

$$m_2 = \sqrt{\Delta m_{21}^2 + m_1^2} \quad \text{and} \quad m_3 = \sqrt{\Delta m_{31}^2 + m_1^2}. \quad (21)$$

The orthogonal mixing matrix O_ν for NO is given by

$$O_{\nu 8|NO} = \begin{pmatrix} -\frac{\sqrt{m_2 m_3}\sqrt{(m_2-m_1)(m_1+m_3)}}{\sqrt{(m_1+m_2)(m_3-m_1)a}} & -\frac{\sqrt{m_1 m_3}\sqrt{(m_2-m_1)(m_3-m_2)}}{\sqrt{(m_1+m_2)(m_2+m_3)a}} & \frac{\sqrt{m_1 m_2}\sqrt{(m_1+m_3)(m_3-m_2)}}{\sqrt{(m_3-m_1)(m_2+m_3)a}} \\ \frac{\sqrt{m_1(m_3-m_2)}}{\sqrt{(m_1+m_2)(m_3-m_1)}} & -\frac{\sqrt{m_2(m_1+m_3)}}{\sqrt{(m_1+m_2)(m_2+m_3)}} & \frac{\sqrt{(m_1-m_2)m_3}}{\sqrt{(m_1-m_3)(m_2+m_3)}} \\ \frac{m_1\sqrt{m_1(m_3-m_2)}}{\sqrt{(m_1+m_2)(m_3-m_1)a}} & \frac{m_2\sqrt{m_2(m_1+m_3)}}{\sqrt{(m_1+m_2)(m_2+m_3)a}} & \frac{m_3\sqrt{(m_2-m_1)m_3}}{\sqrt{(m_3-m_1)(m_2+m_3)a}} \end{pmatrix} \quad (22)$$

where $a = m_1(m_2 - m_3) + m_2 m_3$. For IO, with the neutrino mass eigenvalues $(-m_1, m_2, m_3)$, the orthogonal matrix O_ν is given by

$$O_{\nu 8|IO} = \begin{pmatrix} -\frac{\sqrt{(m_2-m_1)(m_1-m_3)}\sqrt{m_2 m_3}}{\sqrt{b(m_1+m_2)(m_1+m_3)}} & \frac{\sqrt{m_1 m_3}\sqrt{(m_2-m_1)(m_2+m_3)}}{\sqrt{b(m_1+m_2)(m_2-m_3)}} & -\frac{\sqrt{m_1 m_2}\sqrt{(m_1-m_3)(m_2+m_3)}}{\sqrt{b(m_2-m_3)(m_1+m_3)}} \\ -\frac{\sqrt{m_1(m_2+m_3)}}{\sqrt{(m_1+m_2)(m_1+m_3)}} & \frac{\sqrt{m_2(m_1-m_3)}}{\sqrt{(m_1+m_2)(m_2-m_3)}} & \frac{\sqrt{(m_2-m_1)m_3}}{\sqrt{(m_2-m_3)(m_1+m_3)}} \\ \frac{m_1\sqrt{m_1(m_2+m_3)}}{\sqrt{b(m_1+m_2)(m_1+m_3)}} & \frac{m_2\sqrt{m_2(m_1-m_3)}}{\sqrt{b(m_1+m_2)(m_2-m_3)}} & \frac{m_3\sqrt{(m_2-m_1)m_3}}{\sqrt{b(m_2-m_3)(m_1+m_3)}} \end{pmatrix} \quad (23)$$

where $b = m_1(m_2 + m_3) - m_2 m_3$. The mass eigenvalues can be calculated by using the relations

$$m_2 = \sqrt{\Delta m_{23}^2 + m_3^2} \quad \text{and} \quad m_1 = \sqrt{\Delta m_{13}^2 + m_3^2}. \quad (24)$$

The charged lepton mixing matrix is given by

$$V_{l3} = \begin{pmatrix} \cos\theta_l & e^{i\phi_l}\sin\theta_l & 0 \\ -e^{-i\phi_l}\sin\theta_l & \cos\theta_l & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (25)$$

where $\cos\theta_l = \sqrt{\frac{m_\mu^2 - m}{m_\mu^2 - m_e^2}}$ and $m_e^2 < m < m_\mu^2$.

The lepton mixing matrix for texture structure III-(H) can be written as

$$\begin{aligned} U &= V_{l3}^\dagger V_{\nu 8} = V_{l3}^\dagger P_\nu O_{\nu 8|NO(IO)} \\ &= \begin{pmatrix} \cos\theta_l & -e^{i\phi_l}\sin\theta_l & 0 \\ e^{-i\phi_l}\sin\theta_l & \cos\theta_l & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &\quad\times \begin{pmatrix} e^{i\psi_1} & 0 & 0 \\ 0 & e^{i\psi_2} & 0 \\ 0 & 0 & e^{i\psi_3} \end{pmatrix} O_{\nu 8|NO(IO)} \\ &= P' \begin{pmatrix} \cos\theta_l & -e^{i\eta}\sin\theta_l & 0 \\ \sin\theta_l & e^{i\eta}\cos\theta_l & 0 \\ 0 & 0 & 1 \end{pmatrix} O_{\nu 8|NO(IO)} \end{aligned} \quad (26)$$

where $\eta = \psi_2 - \psi_1 + \phi_l$ and the phase matrix $P' = \mathrm{diag}(e^{i\psi_1}, e^{i(\psi_1 - \phi_l)}, e^{i\psi_3})$.

2. Texture III-(I)

For texture structure III-(I), M_ν has vanishing cofactors corresponding to the (1,2), (2,2), and (3,3) elements while H_l has zeros at the (1,2) and (2,3) elements. The neutrino and the charged lepton mass matrices for texture III-(I) have the following form:

$$M_{\nu 9} = \begin{pmatrix} \times & \Delta & \times \\ \Delta & \Delta & \times \\ \times & \times & \Delta \end{pmatrix} \quad \text{and} \quad H_{l2} = \begin{pmatrix} \times & 0 & \times \\ 0 & m_\mu^2 & 0 \\ \times & 0 & \times \end{pmatrix}. \quad (27)$$

The neutrino mass matrix $M_{\nu 9}$ is related to $M_{\nu 8}$ by permutation symmetry, $M_{\nu 9} = S_{23} M_{\nu 8} S_{23}^T$. Therefore, the neutrino mixing matrix for texture $M_{\nu 9}$ can be written in terms of $V_{\nu 8}$ as $V_{\nu 9} = S_{23} V_{\nu 8}$. Therefore, the PMNS mixing matrix for texture III-(I) is given by

$$\begin{aligned} U &= V_{l2}^\dagger V_{\nu 9} = V_{l2}^\dagger S_{23} V_{\nu 8} = V_{l2}^\dagger S_{23} P_\nu O_{\nu 8} \\ &= \begin{pmatrix} \cos\theta_l & 0 & -e^{i\phi_l}\sin\theta_l \\ 0 & 1 & 0 \\ e^{-i\phi_l}\sin\theta_l & 0 & \cos\theta_l \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\ &\quad\times \begin{pmatrix} e^{i\psi_1} & 0 & 0 \\ 0 & e^{i\psi_2} & 0 \\ 0 & 0 & e^{i\psi_3} \end{pmatrix} O_{\nu 8|NO(IO)} \\ &= P' \begin{pmatrix} \cos\theta_l & -e^{i\eta}\sin\theta_l & 0 \\ 0 & 0 & 1 \\ \sin\theta_l & e^{i\eta}\cos\theta_l & 0 \end{pmatrix} O_{\nu 8|NO(IO)}. \end{aligned} \quad (28)$$

For NO and IO, the orthogonal matrix $O_{\nu 8}$ is given in Eqs. (22) and (23), respectively.

The three lepton mixing angles in terms of the lepton mixing matrix elements are given by

$$\sin\theta_{13} = |U_{13}|, \qquad \sin\theta_{23} = \frac{|U_{23}|}{\sqrt{1-|U_{13}|^2}} \quad \text{and}$$
$$\sin\theta_{12} = \frac{|U_{12}|}{\sqrt{1-|U_{13}|^2}}. \tag{29}$$

The CP-violating phase δ can be calculated using Eq. (15):

$$\sin\delta = \frac{\mathrm{Im}(U_{11}U_{22}U_{12}^{\dagger}U_{21}^{\dagger})}{\sin\theta_{12}\sin\theta_{23}\sin\theta_{13}\cos\theta_{12}\cos\theta_{23}\cos^2\theta_{13}} \tag{30}$$

where the elements of U are given in Eq. (26) for texture III-(H) and Eq. (28) for texture III-(I).

Similarly, the three mixing angles can be calculated by substituting the elements of U from Eqs. (26) and (28) into Eq. (29) for textures III-(H) and III-(I), respectively. The effective Majorana mass $|m_{ee}|$ can be calculated using Eq. (16).

There exist the following relations between the neutrino oscillation parameters of texture structures III-(H) and III-(I):

$$\theta_{12}^{H} = \theta_{12}^{I}, \qquad \theta_{13}^{H} = \theta_{13}^{I}, \qquad \theta_{23}^{H} = \frac{\pi}{2} - \theta_{23}^{I}. \tag{31}$$

For the numerical analysis, we have generated random numbers of the order of $\sim 10^7$ for parameters Δm_{21}^2 and $\Delta m_{31}^2(\Delta m_{32}^2)$ for NO (IO) within their experimentally allowed 3σ ranges. $m_1(m_3)$ have been generated randomly between 0–0.33 eV. We vary the parameter η randomly within the range $(0-2\pi)$ and the parameter m has been varied randomly within the range $(m_e^2–m_\mu^2)$ for texture III-(H) and $(m_e^2–m_\tau^2)$ for texture III-(I). We use the experimental constraints on neutrino oscillation parameters as given in Table II. In this analysis, the upper bound on the sum of neutrino masses is set to be $\sum m_i \leq 1$ eV. It turns out that textures III-(H) and III-(I) are, phenomenologically, viable only for normal mass ordering. Figures 1 and 2 depict the predictions for textures III-(H) and III-(I), respectively. It can be seen from Fig. 1 that $\sin\delta$ lies in the range (−1–1) and the Jarlskog CP invariant parameter J_{CP} varies in the range (−0.036–0.036) for NO in III-(H) whereas for texture III-(I) the ranges for $\sin\delta$ and J_{CP} as shown in Fig. 2 are (−1–1) and ±(0.004–0.036), respectively. The correlation plots in the $(\theta_{23} - \sin\delta)$ plane in Figs. 1 and 2 show that δ is more favored to lie near $\delta \sim \pm\frac{\pi}{2}$. These results are consistent with the recent observations in the long-baseline neutrino oscillation experiments like T2K and NOvA [34] which show a preference for the CP-violating phase δ to lie around $\delta \sim -\frac{\pi}{2}$. The range for the smallest neutrino mass m_1 is (0.0087–0.015) eV for texture III-(H) and (0.0084–0.014) eV for texture III-(I). The sum of neutrino masses $\sum m_i$ lies in the ranges (0.071–0.086) eV and (0.07–0.085) eV for textures III-(H) and III-(I), respectively. The charged lepton correction θ_l for Class-III turns out to be very large and lies in the range (62°–69°) for both allowed textures. Parameter $|m_{ee}|$ is constrained to lie in the range (0.0025–0.0071) eV for texture III-(H) and (0.0024–0.007) eV for III-(I). All textures of Class-III with inverted mass ordering are ruled out at 3σ C.L.

B. Class-IV

The textures IV-(A), IV-(B) and IV-(C) are, phenomenologically, nonviable for the current 3σ ranges of neutrino oscillation parameters due to the presence of one zero element in the lepton mixing matrix U. We discuss the viable textures of Class-IV and their phenomenology below.

1. *Texture IV-(D)*

Here M_ν has vanishing cofactors corresponding to elements (1,1), (1,2), and (1,3) which is equivalent to scaling the neutrino mass matrix [36] and the charged lepton mass matrix H_l has zeros at the (1,3) and (2,3) positions. The mass matrices M_ν and H_l are given by

TABLE II. Current neutrino oscillation parameters from global fits [35] with $\Delta m_{3l}^2 \equiv \Delta m_{31}^2 > 0$ for NO and $\Delta m_{3l}^2 \equiv \Delta m_{32}^2 < 0$ for IO.

	Normal ordering		Inverted ordering	
Neutrino Parameter	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
θ_{12}°	$33.56_{-0.75}^{+0.77}$	31.38 → 35.99	$33.56_{-0.75}^{+0.77}$	31.38 → 35.99
θ_{23}°	$41.6_{-1.2}^{+1.5}$	38.4 → 52.8	$50.0_{-1.4}^{+1.1}$	38.8 → 53.1
θ_{13}°	$8.46_{-0.15}^{+0.15}$	7.99 → 8.90	$8.49_{-0.15}^{+0.15}$	8.03 → 8.93
δ_{CP}°	261_{-59}^{+51}	0.0 → 360	277_{-46}^{+40}	145 → 391
$\Delta m_{21}^2/10^{-5}$ eV^2	$7.50_{-0.17}^{+0.19}$	7.03 → 8.09	$7.50_{-0.17}^{+0.19}$	7.03 → 8.09
$\Delta m_{3l}^2/10^{-3}$ eV^2	$+2.524_{-0.040}^{+0.039}$	2.407 → 2.643	$-2.514_{-0.041}^{+0.038}$	−2.635 → −2.399

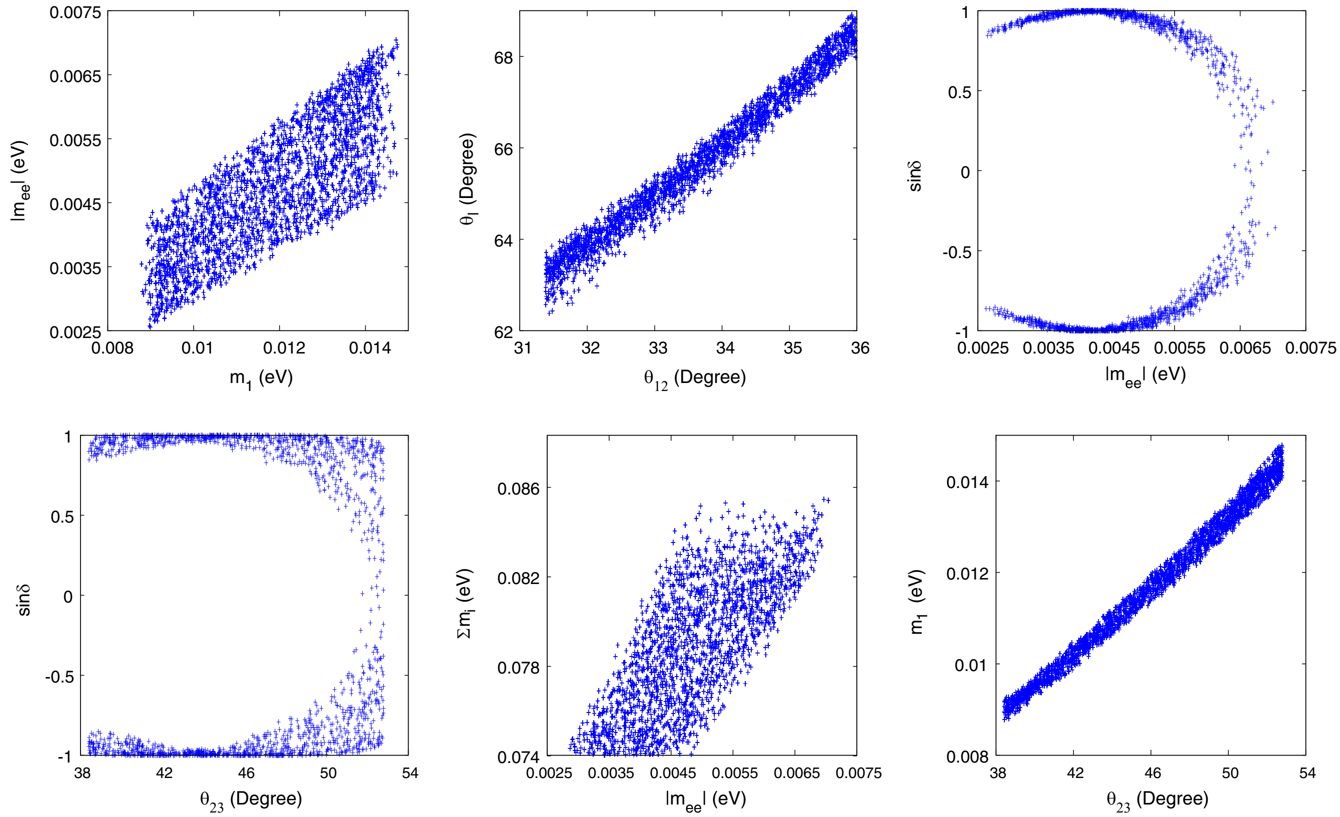

FIG. 1. Correlation plots between different parameters for NO in texture III-(H).

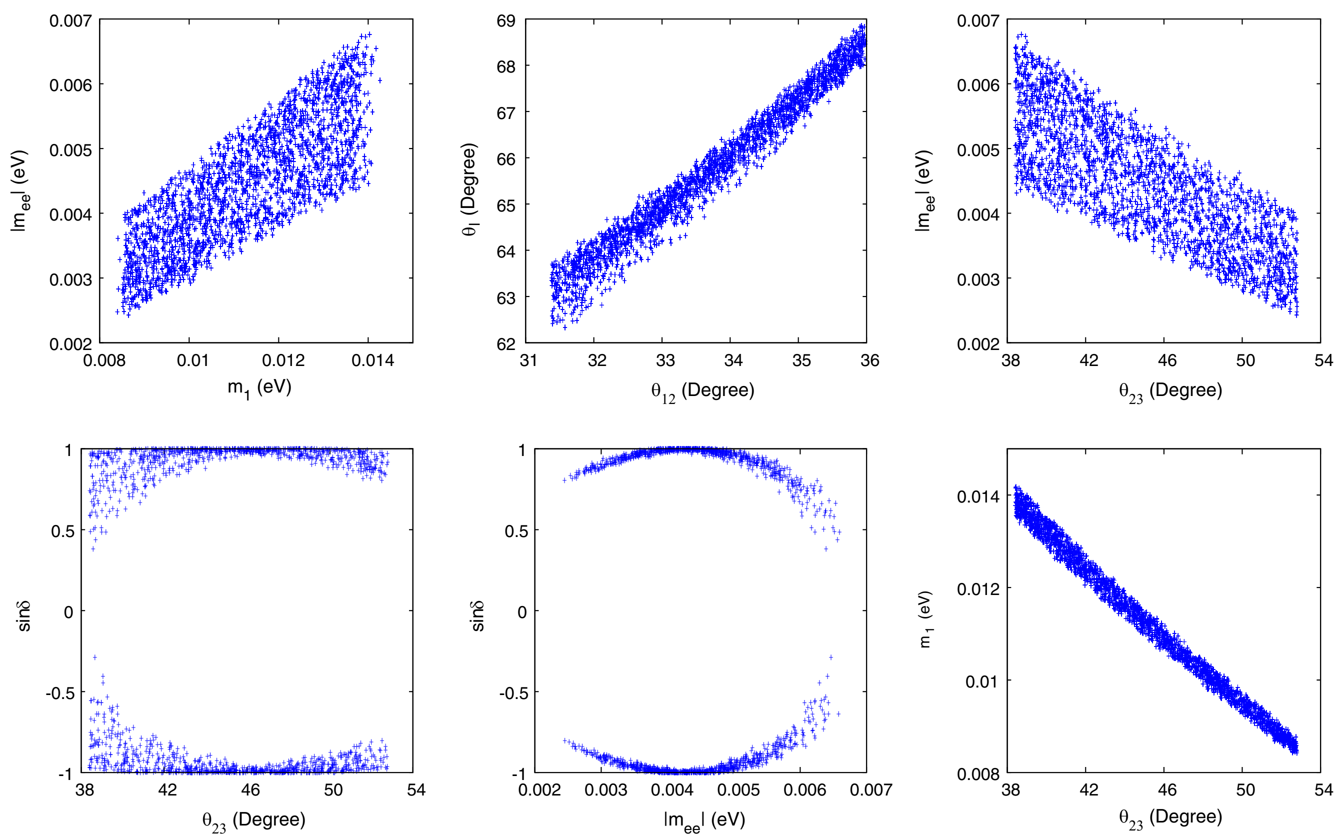

FIG. 2. Correlation plots between different parameters for NO in texture III-(I).

$$M_{\nu 14} = \begin{pmatrix} \Delta & \Delta & \Delta \\ \Delta & \times & \times \\ \Delta & \times & \times \end{pmatrix} \equiv \begin{pmatrix} Ae^{i\phi_a} & Be^{i\phi_b} & \frac{Be^{i\phi_b}}{c} \\ Be^{i\phi_b} & De^{i\phi_d} & \frac{De^{i\phi_d}}{c} \\ \frac{Be^{i\phi_b}}{c} & \frac{De^{i\phi_d}}{c} & \frac{De^{i\phi_d}}{c^2} \end{pmatrix}, \tag{32}$$

$$H_{l3} = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix}. \tag{33}$$

The neutrino mass matrix which, in general, is complex symmetric for this class cannot be diagonalized directly due to the presence of a nonremovable phase. Instead, we diagonalize the Hermitian product $M_\nu M_\nu^\dagger$ which gives the neutrino mixing matrix as

$$V_\nu^\dagger M_{\nu 14} M_{\nu 14}^\dagger V_\nu = \mathrm{diag}(m_1^2, m_2^2, m_3^2). \tag{34}$$

Therefore, we have the following Hermitian matrix:

$$M_{\nu 14} M_{\nu 14}^\dagger = \begin{pmatrix} a & be^{i\phi} & \frac{be^{i\phi}}{c} \\ be^{-i\phi} & d & \frac{d}{c} \\ \frac{be^{-i\phi}}{c} & \frac{d}{c} & \frac{d}{c^2} \end{pmatrix} = P_\nu^\dagger \begin{pmatrix} a & b & \frac{b}{c} \\ b & d & \frac{d}{c} \\ \frac{b}{c} & \frac{d}{c} & \frac{d}{c^2} \end{pmatrix} P_\nu \tag{35}$$

where

$$a = A^2 + B^2\left(1 + \frac{1}{c^2}\right),$$
$$be^{i\phi} = ABe^{-i(\phi_a+\phi_b)} + BDe^{i(\phi_b+\phi_d)}\left(1 + \frac{1}{c^2}\right),$$
$$d = B^2 + D^2\left(1 + \frac{1}{c^2}\right) \tag{36}$$

and $P_\nu = \mathrm{diag}(e^{i\phi}, 1, 1)$ is the phase matrix. The mixing matrix for $M_{\nu 14}$ is given by

$$V_{\nu 14} = P_\nu O_{\nu 14} \tag{37}$$

where $O_{\nu 14}$ is an orthogonal unitary matrix which diagonalizes the real symmetric mass matrix given in Eq. (35).

For NO, the neutrino mass eigenvalues are $m_1 = 0$, $m_2 = \sqrt{\Delta m_{21}^2}$, $m_3 = \sqrt{\Delta m_{31}^2}$ and the orthogonal matrix $O_{\nu 14}$ is given by

$$O_{\nu 14|NO} = \begin{pmatrix} 0 & \frac{ac^2-(c^2+1)d-\sqrt{x}}{\sqrt{4b^2c^4+4b^2c^2+(-ac^2+dc^2+d+\sqrt{x})^2}} & \frac{ac^2-(c^2+1)d+\sqrt{x}}{\sqrt{4b^2c^4+4b^2c^2+(ac^2-(c^2+1)d+\sqrt{x})^2}} \\ -\frac{1}{\sqrt{c^2+1}} & \frac{2bc^2}{\sqrt{4b^2c^4+4b^2c^2+(-ac^2+dc^2+d+\sqrt{x})^2}} & \frac{2bc^2}{\sqrt{4b^2c^4+4b^2c^2+(ac^2-(c^2+1)d+\sqrt{x})^2}} \\ \frac{c}{\sqrt{c^2+1}} & \frac{2bc}{\sqrt{4b^2c^4+4b^2c^2+(-ac^2+dc^2+d+\sqrt{x})^2}} & \frac{2bc}{\sqrt{4b^2c^4+4b^2c^2+(ac^2-(c^2+1)d+\sqrt{x})^2}} \end{pmatrix} \tag{38}$$

where

$$x = (c^2(d-a)+d)^2 + 4b^2(c^4+c^2), \quad c = \frac{\sqrt{d}}{\sqrt{-a-d+m_2^2+m_3^2}}, \quad \text{and} \quad b = \frac{\sqrt{d}\sqrt{(m_3^2-a)(a-m_2^2)}}{\sqrt{-a+m_2^2+m_3^2}}. \tag{39}$$

The free parameters a, d are constrained to lie in the range $m_2^2 < a < m_3^2$ and $d < m_3^2$. For IO, the mass eigenvalues are $m_1 = \sqrt{\Delta m_{23}^2 - \Delta m_{21}^2}$, $m_2 = \sqrt{\Delta m_{23}^2}$ and $m_3 = 0$ and the corresponding orthogonal matrix $O_{\nu 14}$ is given by

$$O_{\nu 14|IO} = \begin{pmatrix} \frac{ac^2-(c^2+1)d-\sqrt{x}}{\sqrt{4b^2c^4+4b^2c^2+(-ac^2+dc^2+d+\sqrt{x})^2}} & \frac{ac^2-(c^2+1)d+\sqrt{x}}{\sqrt{4b^2c^4+4b^2c^2+(ac^2-(c^2+1)d+\sqrt{x})^2}} & 0 \\ \frac{2bc^2}{\sqrt{4b^2c^4+4b^2c^2+(-ac^2+dc^2+d+\sqrt{x})^2}} & \frac{2bc^2}{\sqrt{4b^2c^4+4b^2c^2+(ac^2-(c^2+1)d+\sqrt{x})^2}} & -\frac{1}{\sqrt{c^2+1}} \\ \frac{2bc}{\sqrt{4b^2c^4+4b^2c^2+(-ac^2+dc^2+d+\sqrt{x})^2}} & \frac{2bc}{\sqrt{4b^2c^4+4b^2c^2+(ac^2-(c^2+1)d+\sqrt{x})^2}} & \frac{c}{\sqrt{c^2+1}} \end{pmatrix} \tag{40}$$

where

$$x = (c^2(d-a)+d)^2 + 4b^2(c^4+c^2),$$
$$c = \frac{\sqrt{d}}{\sqrt{-a-d+m_1^2+m_2^2}},$$
$$b = \frac{\sqrt{d}\sqrt{(m_2^2-a)(a-m_1^2)}}{\sqrt{-a+m_1^2+m_2^2}} \tag{41}$$

with $m_1^2 < a < m_2^2$ and $d < m_2^2$.

The PMNS mixing matrix for texture IV-(D) is given by

$$U = V_{l3}^\dagger V_{\nu 14} = V_{l3}^\dagger P_\nu O_{\nu 14|NO(IO)}$$
$$= \begin{pmatrix} \cos\theta_l & -e^{i\phi_l}\sin\theta_l & 0 \\ e^{-i\phi_l}\sin\theta_l & \cos\theta_l & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\phi} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times O_{\nu 14|NO(IO)}$$
$$= \begin{pmatrix} e^{i\phi} & 0 & 0 \\ 0 & e^{i(\phi-\phi_l)} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_l & -e^{i(\phi_l-\phi)}\sin\theta_l & 0 \\ \sin\theta_l & e^{i(\phi_l-\phi)}\cos\theta_l & 0 \\ 0 & 0 & 1 \end{pmatrix} \times O_{\nu 14|NO(IO)}$$
$$= P' \begin{pmatrix} \cos\theta_l & -e^{i\eta}\sin\theta_l & 0 \\ \sin\theta_l & e^{i\eta}\cos\theta_l & 0 \\ 0 & 0 & 1 \end{pmatrix} O_{\nu 14|NO(IO)} \tag{42}$$

where $\eta = \phi_l - \phi$ and $\cos\theta_l = \sqrt{\frac{m_\mu^2 - m}{m_\mu^2 - m_e^2}}$ for $m_e^2 < m < m_\mu^2$. The phase matrix is given by $P' = \mathrm{diag}(e^{i\psi}, e^{i(\phi-\phi_l)}, 1)$. For IO of texture IV-(D), the neutrino oscillation parameters are related as

$$\sin\theta_{13} = \frac{\sin\theta_l}{\sqrt{c^2+1}}, \tan\theta_{23} = \frac{\cos\theta_l}{c}. \tag{43}$$

2. Texture IV-(E)

Texture IV-(E) has three vanishing cofactors in M_ν corresponding to elements (1,1), (1,2), (1,3) and two zeros in H_l at the (1,2) and (2,3) positions. The mass matrices of neutrinos and charged leptons are given by

$$M_{\nu 14} = \begin{pmatrix} \Delta & \Delta & \Delta \\ \Delta & \times & \times \\ \Delta & \times & \times \end{pmatrix} \quad \text{and} \quad H_{l2} = \begin{pmatrix} \times & 0 & \times \\ 0 & m_\mu^2 & 0 \\ \times & 0 & \times \end{pmatrix}. \tag{44}$$

The charged lepton mixing matrix for H_{l2} is given by

$$V_{l2} = \begin{pmatrix} \cos\theta_l & 0 & e^{i\phi_l}\sin\theta_l \\ 0 & 1 & 0 \\ -e^{-i\phi_l}\sin\theta_l & 0 & \cos\theta_l \end{pmatrix} \tag{45}$$

where $\cos\theta_l = \sqrt{\frac{m_\tau^2 - m}{m_\tau^2 - m_e^2}}$ for $m_e^2 < m < m_\tau^2$.

The PMNS mixing matrix for this texture is given by

$$U = V_{l2}^\dagger V_{\nu 14} = V_{l2}^\dagger P_\nu O_{\nu 14|NO(IO)}$$
$$= P' \begin{pmatrix} \cos\theta_l & 0 & -e^{i\eta}\sin\theta_l \\ 0 & 1 & 0 \\ \sin\theta_l & 0 & e^{i\eta}\cos\theta_l \end{pmatrix} O_{\nu 14|NO(IO)} \tag{46}$$

where $\eta = \phi_l - \phi$ and $P' = \mathrm{diag}(e^{i\psi}, e^{i(\phi-\phi_l)}, 1)$. The orthogonal matrix $O_{\nu 14|NO(IO)}$ is given in Eqs. (38) and (40) for normal and inverted mass orderings. The neutrino oscillation parameters for IO of texture IV-(E) are related as follows:

$$\sin\theta_{13} = \sin\theta_l \frac{c}{\sqrt{c^2+1}}, \qquad \tan\theta_{23} = \frac{\sec\theta_l}{c}. \tag{47}$$

3. Texture IV-(F)

In this texture structure, the neutrino mass matrix has three vanishing cofactors corresponding to the (1,2), (2,2), and (2,3) positions and the charged lepton mass matrix has zero elements at the (1,2) and (1,3) positions. M_ν and H_l have the following structure:

$$M_{\nu 16} = \begin{pmatrix} \times & \Delta & \times \\ \Delta & \Delta & \Delta \\ \times & \Delta & \times \end{pmatrix} \quad \text{and} \quad H_{l1} = \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}. \tag{48}$$

The neutrino mass matrix $M_{\nu 16}$ is related to $M_{\nu 14}$ by S_3 permutation symmetry as $M_{\nu 16} = S_{12} M_{\nu 14} S_{12}^T$, where S_{12} is an element of the permutation group S_3. Therefore, the mixing matrix for structure $M_{\nu 16}$ is given by

$$V_{\nu 16} = S_{12} V_{\nu 14}. \tag{49}$$

The PMNS mixing matrix for texture IV-(F) is given by

$$U = V_{l1}^\dagger V_{\nu 16} = V_{l1}^\dagger S_{12} V_{\nu 14}$$
$$= P' \begin{pmatrix} 0 & 1 & 0 \\ \cos\theta_l & 0 & -e^{i\eta}\sin\theta_l \\ \sin\theta_l & 0 & e^{i\eta}\cos\theta_l \end{pmatrix} O_{\nu 14|NO(IO)} \tag{50}$$

where Eqs. (38) and (40) give $O_{\nu 14}$ for NO and IO, respectively. For IO of texture IV-(F), the neutrino oscillation parameters are related as

$$\sin\theta_{13} = \frac{1}{\sqrt{c^2+1}}, \qquad \tan\theta_{23} = \tan\theta_l. \tag{51}$$

The numerical results of Class-IV are presented in Figs. 3, 4 and 5 for both normal and inverted mass orderings. It can be seen from these figures that $\sin\delta$ spans the range (−1–1)

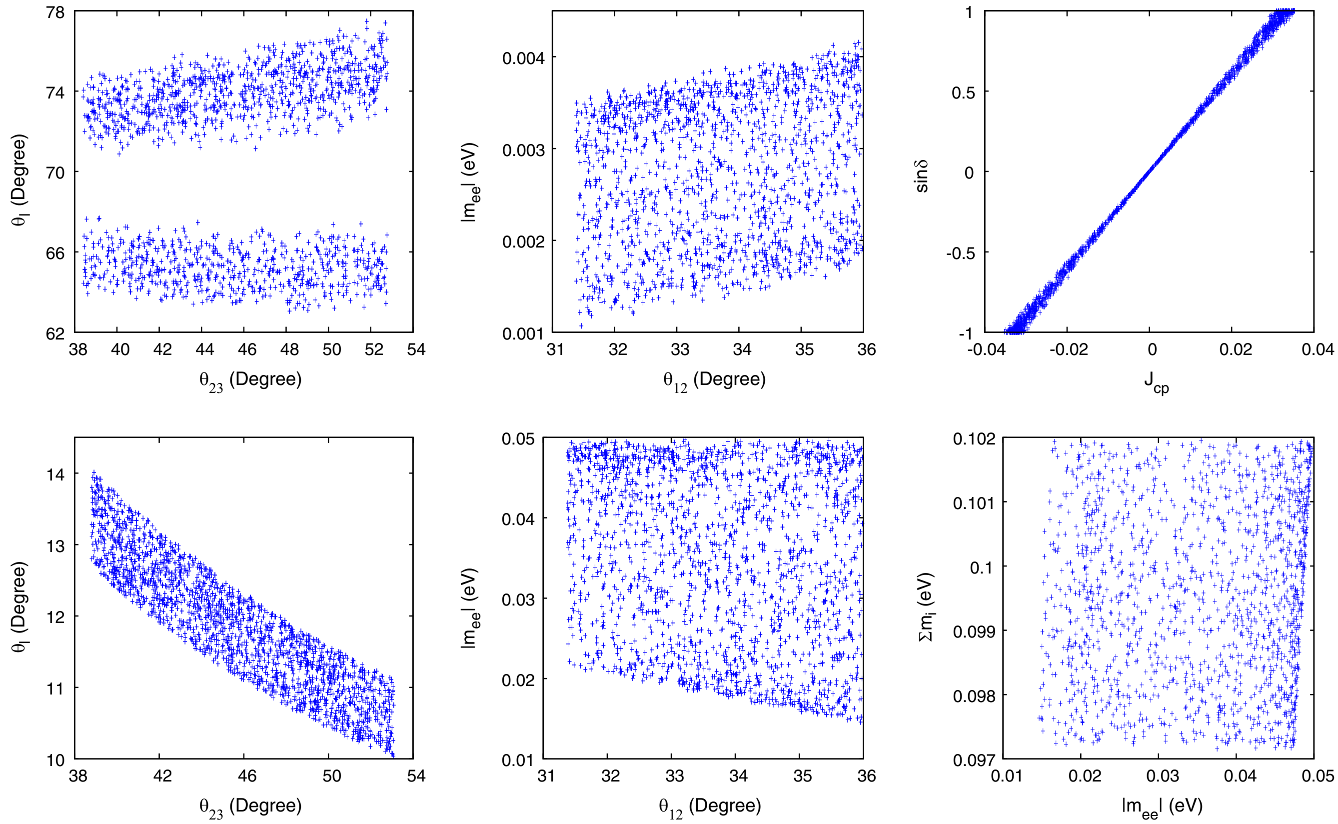

FIG. 3. Correlation plots between different parameters for NO (upper panel) and IO (lower panel) in texture IV-(D).

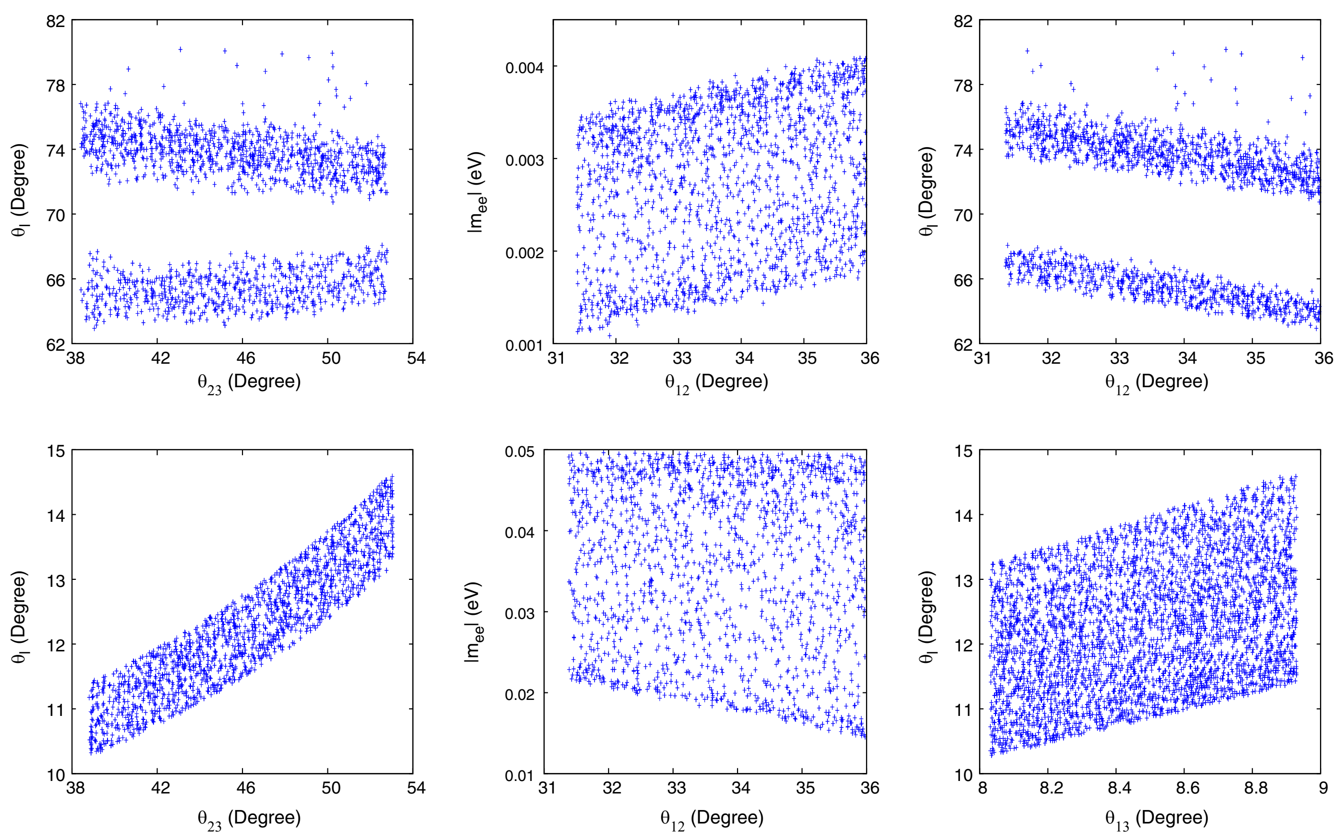

FIG. 4. Correlation plots between different parameters for NO (upper panel) and IO (lower panel) in texture IV-(E).

except for NO of texture IV-(F) for which $\sin\delta$ is bounded by (−0.6–0.6). The Jarlskog CP invariant parameter J_{CP} varies in the range (−0.02–0.02) for NO in texture IV-(F) whereas for other viable textures the range is (−0.036–0.036). There is a strong correlation between the charged lepton mixing angle θ_l and $(\theta_{12}, \theta_{23})$ for both mass orderings for all allowed textures of this class. Similar correlations are, also, present for $|m_{ee}|$ with θ_{12} and θ_{23} as shown in Figs. 3, 4 and 5. For textures IV-(D), IV-(E) and IV-(F), the effective Majorana mass $|m_{ee}|$ is highly constrained to lie in the ranges (0.001–0.0045) eV for NO and (0.014–0.05) eV for IO. For viable textures of Class-IV, the charged lepton correction θ_l is very large for normal mass ordering. For inverted mass ordering θ_l is small and the results are in agreement with Ref. [36], where charged lepton corrections were taken to be of the order of the Cabibbo angle. The sum of neutrino masses $\sum m_i$ varies in the range (0.057–0.061) eV for NO and (0.097–0.102) eV for IO. Textures IV-(A), IV-(B) and IV-(C) of Class-IV are phenomenologically incompatible with the 3σ neutrino oscillation data.

C. Class-V

In this class, there are three possible texture structures viz. $M_{\nu 17}H_{l1}, M_{\nu 17}H_{l2}$ and $M_{\nu 17}H_{l3}$. H_l and M_ν for this class have the following form:

$$H_{l1} = \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}, \qquad H_{l2} = \begin{pmatrix} \times & 0 & \times \\ 0 & m_\mu^2 & 0 \\ \times & 0 & \times \end{pmatrix},$$

$$H_{l3} = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix}, \tag{52}$$

$$M_{\nu 17} = \begin{pmatrix} \Delta & \times & \times \\ \times & \Delta & \times \\ \times & \times & \Delta \end{pmatrix} \equiv P_\nu \begin{pmatrix} a & \sqrt{ab} & -\sqrt{ad} \\ \sqrt{ab} & b & \sqrt{bd} \\ -\sqrt{ad} & \sqrt{bd} & d \end{pmatrix} P_\nu^T$$
$$= P_\nu M_{\nu 17}^r P_\nu^T \tag{53}$$

where P_ν is a diagonal phase matrix. The real symmetric matrix $M_{\nu 17}^r$ can be diagonalized by the orthogonal matrix $O_{\nu 17}$:

$$M_{\nu 17}^r = O_{\nu 17} M_{\nu 17}^D O_{\nu 17}^T. \tag{54}$$

$O_{\nu 17}$ is given by

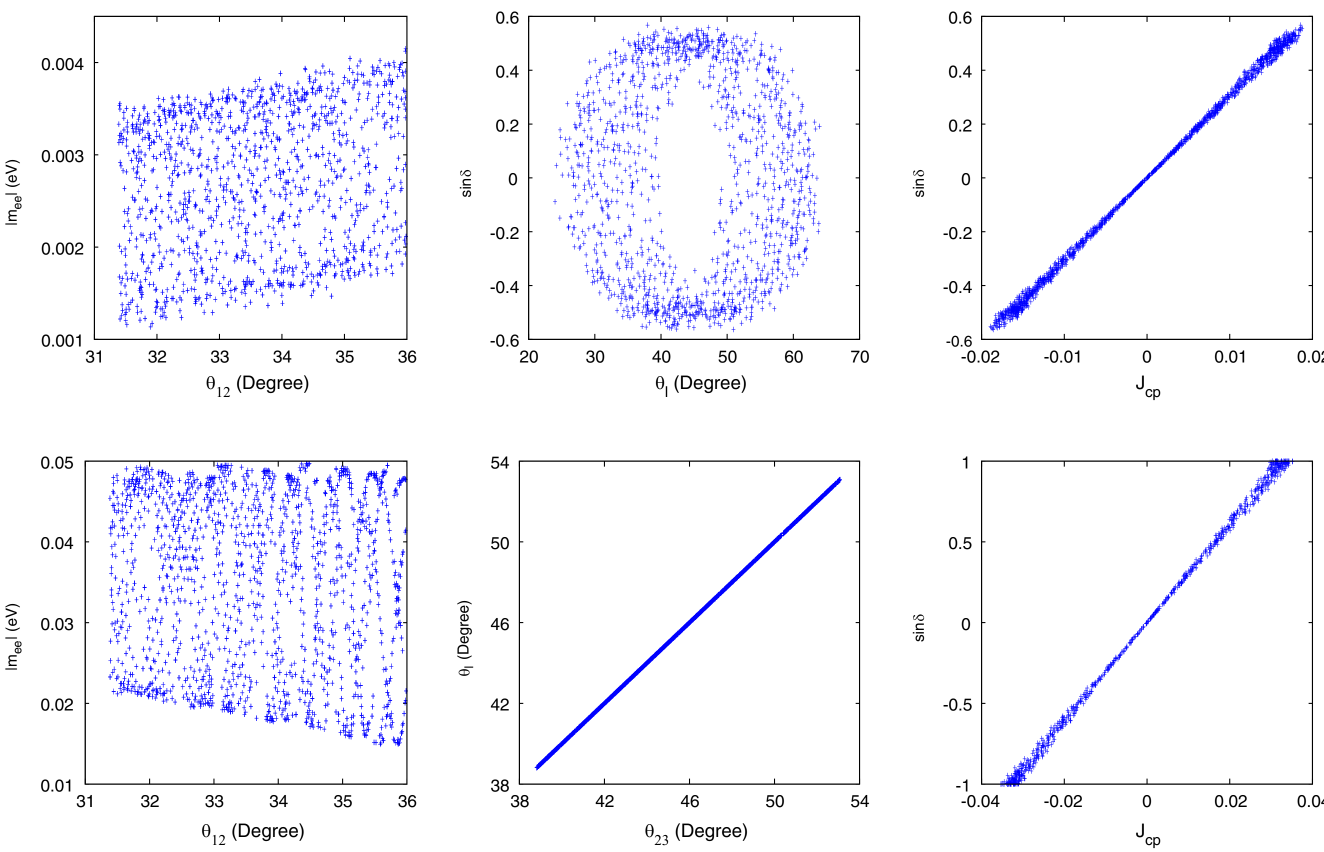

FIG. 5. Correlation plots between different parameters for NO (upper panel) and IO (lower panel) in texture IV-(F).

$$O_{\nu 17} = \begin{pmatrix} \frac{\sqrt{ad}(m_1-2b)}{(a+b-m_1)m_1\sqrt{x+1}} & \frac{\sqrt{ad}(m_2-2b)}{(a+b-m_2)m_2\sqrt{y+1}} & \frac{\sqrt{ad}(m_3-2b)}{(a+b-m_3)m_3\sqrt{z+1}} \\ \frac{\sqrt{bd}(2a-m_1)}{(a+b-m_1)m_1\sqrt{x+1}} & \frac{\sqrt{bd}(2a-m_2)}{(a+b-m_2)m_2\sqrt{y+1}} & \frac{\sqrt{bd}(2a-m_3)}{(a+b-m_3)m_3\sqrt{z+1}} \\ \frac{1}{\sqrt{x+1}} & \frac{1}{\sqrt{y+1}} & \frac{1}{\sqrt{z+1}} \end{pmatrix} \tag{55}$$

where

$$x = \frac{bd(2a-m_1)^2}{m_1^2(a+b-m_1)^2} + \frac{ad(m_1-2b)^2}{m_1^2(a+b-m_1)^2}, \qquad y = \frac{bd(2a-m_2)^2}{m_2^2(a+b-m_2)^2} + \frac{ad^2(m_2-2b)^2}{m_2^2(a+b-m_2)^2},$$
$$z = \frac{bd(2a-m_3)^2}{m_3^2(a+b-m_3)^2} + \frac{ad(m_3-2b)^2}{m_3^2(a+b-m_3)^2} \tag{56}$$

and the parameters a, b are related to the neutrino masses m_1, m_2 and m_3 as

$$a = \frac{-d^2 + d(m_1+m_2+m_3) + \sqrt{d(d(-d+m_1+m_2+m_3)^2 + m_1m_2m_3)}}{2d},$$
$$b = -\frac{d^2 - d(m_1+m_2+m_3) + \sqrt{d(d(-d+m_1+m_2+m_3)^2 + m_1m_2m_3)}}{2d}.$$

Three vanishing diagonal cofactors of M_ν relate the mass eigenvalues as $m_1m_2 + m_2m_3 + m_1m_3 = 0$. All texture structures of Class-V are inconsistent with the present neutrino oscillation data at the 3σ level.

Texture structures III-(A) to III-(G) and V-(A) to V-(C) cannot simultaneously satisfy the experimental constraints on the mass-squared differences and mixing angles and thus are inconsistent with neutrino oscillation data. In Table III we have summarized the parameter space for the three mixing angles associated with each disallowed texture. One can see that the three mixing angles cannot simultaneously have values lying within their experimental 3σ ranges for the disallowed textures.

TABLE III. Parameter space for neutrino oscillation parameters for phenomenologically disallowed textures of classes III and V. 3σ in the table denotes that the mixing angle lies within the 3σ experimental range.

	NO			IO		
Texture	θ_{13}	θ_{12}	θ_{23}	θ_{13}	θ_{12}	θ_{23}
III-(A)	3σ	16°–20°	3σ	79°–90°	45°–90°	3σ
III-(B)	3σ	45°–46°	3σ	<7.5°	44°–46°	3σ
III-(C)	3σ	45°–46°	3σ	<7.5°	44°–46°	3σ
III-(D)	3σ	>50°	3σ	3σ	>50°	>83°
III-(E)	3σ	3σ	16°–20°	3σ	44°–47°	<08°
III-(F)	3σ	3σ	70°–74°	3σ	44°–47°	>82°
III-(G)	3σ	>50°	3σ	3σ	>49°	<7°
V-(A)	53°–57°	3σ	3σ	35°–36°	3σ	3σ
V-(B)	>20°	3σ	3σ	>20°	3σ	3σ
V-(C)	>13°	3σ	3σ	>15°	3σ	3σ

D. Neutrino mass matrices with four vanishing cofactors

Another possibility for lepton mass matrices includes four vanishing cofactors in the neutrino mass matrices with five nonzero elements in the charged lepton mass matrices. These textures have a total of eight degrees of freedom and such textures should have the same predictability as the two-texture-zero neutrino mass matrices in the flavor basis. There are a total of 15 possible structures for M_ν having four vanishing cofactors. In a 3×3 complex symmetric matrix, the vanishing of any set of four cofactors leads to either the vanishing of the fifth or all six cofactors, simultaneously. A neutrino mass matrix where all six cofactors vanish leads to two degenerate neutrino masses which is incompatible with the experimental data. The remaining possible structures of M_ν which have five vanishing cofactors and nondegenerate mass eigenvalues are given below:

Class-VII

$$M_{\nu 21} = \begin{pmatrix} \Delta & \Delta & \Delta \\ \Delta & \Delta & \Delta \\ \Delta & \Delta & \times \end{pmatrix}, \qquad M_{\nu 22} = \begin{pmatrix} \Delta & \Delta & \Delta \\ \Delta & \times & \Delta \\ \Delta & \Delta & \Delta \end{pmatrix},$$
$$M_{\nu 23} = \begin{pmatrix} \times & \Delta & \Delta \\ \Delta & \Delta & \Delta \\ \Delta & \Delta & \Delta \end{pmatrix}. \tag{57}$$

The charged lepton mass matrices with five nonzero matrix elements are of the following form:

$$M_l = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ \times & 0 & \times \end{pmatrix} \tag{58}$$

with all possible reorderings of rows and columns of M_l. The Hermitian products $H_l = M_l M_l^\dagger$, whose diagonalization gives the charged lepton mixing matrix V_l, are given below:

$$H_{l1} = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}, \qquad H_{l2} = \begin{pmatrix} \times & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix},$$
$$H_{l3} = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \tag{59}$$

$$H_{l4} = \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}, \qquad H_{l5} = \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix},$$
$$H_{l6} = \begin{pmatrix} \times & \times & \times \\ \times & \times & 0 \\ \times & 0 & \times \end{pmatrix}, \tag{60}$$

$$H_{l7} = \begin{pmatrix} \times & \times & \Delta \\ \times & \times & \times \\ \Delta & \times & \times \end{pmatrix}, \qquad H_{l8} = \begin{pmatrix} \times & \Delta & \times \\ \Delta & \times & \times \\ \times & \times & \times \end{pmatrix},$$
$$H_{l9} = \begin{pmatrix} \times & \times & \times \\ \times & \times & \Delta \\ \times & \Delta & \times \end{pmatrix} \tag{61}$$

where Δ at the ij position represents the vanishing cofactors corresponding to the ij elements.

If M_ν is one of Eq. (57) and H_l is one of Eq. (59), the lepton mixing matrix U has one of its elements equal to zero which is inconsistent with current experimental data and, hence, the charged lepton mass matrices listed in Eq. (59) are phenomenologically ruled out for four vanishing cofactors in M_ν. Therefore, we focus on other forms of H_l given in Eqs. (60) and (61). There are 18 possible combinations of M_ν and H_l but not all are independent as interchanging rows and columns of M_ν is equivalent to reordering the rows and columns of M_l. Table IV gives independent combinations of M_ν and H_l along with their viabilities. The neutrino mass matrices for this class cannot be diagonalized directly due to the presence of a nonremovable phase and, hence, we diagonalize the Hermitian product $M_\nu M_\nu^\dagger$, which gives the neutrino mixing matrix V_ν. The neutrino mass matrix M_ν can be written as

TABLE IV. Possible independent structures of H_l, M_ν and their viability.

Class	Texture	H_l M_ν	Viability NO	Viability IO
VII	VII-(A)	H_{l4}, $M_{\nu21}$ ~ H_{l5}, $M_{\nu23}$ ~ H_{l6}, $M_{\nu22}$	√	√
	VII-(B)	H_{l4}, $M_{\nu22}$ ~ H_{l5}, $M_{\nu21}$ ~ H_{l6}, $M_{\nu23}$	√	√
	VII-(C)	H_{l4}, $M_{\nu23}$ ~ H_{l5}, $M_{\nu22}$ ~ H_{l6}, $M_{\nu21}$	√	√
	VII-(D)	H_{l7}, $M_{\nu21}$ ~ H_{l8}, $M_{\nu23}$ ~ H_{l9}, $M_{\nu22}$	×	×
	VII-(E)	H_{l7}, $M_{\nu22}$ ~ H_{l8}, $M_{\nu21}$ ~ H_{l9}, $M_{\nu23}$	√	√
	VII-(F)	H_{l7}, $M_{\nu23}$ ~ H_{l8}, $M_{\nu22}$ ~ H_{l9}, $M_{\nu23}$	√	√

$$M_{\nu21} = \begin{pmatrix} \Delta & \Delta & \Delta \\ \Delta & \Delta & \Delta \\ \Delta & \Delta & \times \end{pmatrix} \equiv \begin{pmatrix} Ae^{i\phi_1} & Be^{i\phi_3} & 0 \\ Be^{i\phi_3} & De^{i\phi_2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{62}$$

and the corresponding Hermitian matrix is given by

$$M_{\nu21}M_{\nu21}^\dagger = \begin{pmatrix} a & be^{i\phi} & 0 \\ be^{-i\phi} & d & 0 \\ 0 & 0 & 0 \end{pmatrix} = P_\nu^\dagger \begin{pmatrix} a & b & 0 \\ b & d & 0 \\ 0 & 0 & 0 \end{pmatrix} P_\nu \tag{63}$$

where

$$a = A^2 + B^2,$$
$$be^{i\phi} = ABe^{-i(\phi_3-\phi_1)} + BDe^{i(\phi_3-\phi_2)},$$
$$d = B^2 + D^2 \tag{64}$$

and $P_\nu = \mathrm{diag}(e^{i\phi}, 1, 1)$. For NO, the neutrino masses are $m_1 = 0$, $m_2 = \sqrt{\Delta m_{21}^2}$, $m_3 = \sqrt{\Delta m_{31}^2}$ and for IO $m_1 = \sqrt{\Delta m_{23}^2 - \Delta m_{21}^2}$, $m_2 = \sqrt{\Delta m_{23}^2}$, $m_3 = 0$. The orthogonal matrix $O_{\nu21}$ for NO and IO is given by

$$O_{\nu21|NO} = \begin{pmatrix} 0 & -\frac{\sqrt{d-m_2^2}}{\sqrt{m_3^2-m_2^2}} & \frac{\sqrt{m_3^2-d}}{\sqrt{m_3^2-m_2^2}} \\ 0 & \frac{\sqrt{m_3^2-d}}{\sqrt{m_3^2-m_2^2}} & \frac{\sqrt{d-m_2^2}}{\sqrt{m_3^2-m_2^2}} \\ 1 & 0 & 0 \end{pmatrix} \quad \text{and}$$
$$O_{\nu21|IO} = \begin{pmatrix} -\frac{\sqrt{d-m_1^4}}{\sqrt{m_2^4-m_1^4}} & \frac{\sqrt{m_2^4-d}}{\sqrt{m_2^4-m_1^4}} & 0 \\ \frac{\sqrt{m_2^4-d}}{\sqrt{m_2^4-m_1^4}} & \frac{\sqrt{d-m_1^4}}{\sqrt{m_2^4-m_1^4}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{65}$$

where $m_2^2 < d < m_3^2$ for NO and $m_1^2 < d < m_2^2$ for IO.

The mixing matrix for $M_{\nu21}$ is given by

$$V_{\nu21} = P_\nu O_{\nu21}. \tag{66}$$

For texture $M_{\nu 22}$, which is related to $M_{\nu 21}$ by permutation symmetry, $M_{\nu 22} \to S_{23} M_{\nu 21} S_{23}^T$, the neutrino mixing matrix is given by

$$V_{\nu 22} = S_{23} P_\nu O_{\nu 21}. \tag{67}$$

For $M_{\nu 23}$, the neutrino mixing matrix is given by

$$V_{\nu 23} = S_{123} P_\nu O_{\nu 21}. \tag{68}$$

The charged lepton mixing matrix for structures given in Eqs. (60) and (61) can be parametrized as

$$V_l = P_l O_l, \quad \text{with}$$
$$O_l = \begin{pmatrix} \bar{c}_{12}\bar{c}_{13} & \bar{c}_{13}\bar{s}_{12} & \bar{s}_{13} \\ -\bar{c}_{23}\bar{s}_{12} - \bar{c}_{12}\bar{s}_{13}\bar{s}_{23} & \bar{c}_{12}\bar{c}_{23} - \bar{s}_{12}\bar{s}_{13}\bar{s}_{23} & \bar{c}_{13}\bar{s}_{23} \\ \bar{s}_{12}\bar{s}_{23} - \bar{c}_{12}\bar{c}_{23}\bar{s}_{13} & -\bar{c}_{23}\bar{s}_{12}\bar{s}_{13} - \bar{c}_{12}\bar{s}_{23} & \bar{c}_{13}\bar{c}_{23} \end{pmatrix} \tag{69}$$

where $\bar{s}_{ij} = \sin\chi_{ij}$, $\bar{c}_{ij} = \cos\chi_{ij}$ and $P_l = \mathrm{diag}(e^{i\phi'}, 1, e^{i\phi''})$ is the unitary phase matrix.

For the charged lepton mass matrix H_{l4}, a zero at the (1,3) position implies

$$m_e^2 O_{l11} O_{l31} + m_\mu^2 O_{l12} O_{l32} + m_\tau^2 O_{l13} O_{l33} = 0 \tag{70}$$

which gives

$$\sin\chi_{13} = \frac{(m_e^2 - m_\mu^2)\sin\chi_{12}\cos\chi_{12}\tan\chi_{23})}{m_e^2\cos^2\chi_{12} + m_2^2\sin^2\chi_{12} - m_\tau^2}. \tag{71}$$

For the charged lepton mass matrix H_{l7}, a vanishing cofactor at the (1,3) position implies

$$H_l|_{21} H_l|_{32} - H_l|_{22} H_l|_{31} = 0 \tag{72}$$

which gives

$$\sin\chi_{13} = \frac{m_\tau^2(m_e^2 - m_\mu^2)\sin 2\chi_{12}\tan\chi_{23}}{m_\tau^2(m_e^2 - m_\mu^2)\cos 2\chi_{12} + m_e^2(2m_\mu^2 - m_\tau^2) - m_\mu^2 m_\tau^2}. \tag{73}$$

The PMNS mixing matrix for this class is given by

$$U = V_l^\dagger V_\nu = O_l^T P_l^\dagger P_\nu O_{\nu|NO(IO)} = O_l^T P O_{\nu|NO(IO)}, \tag{74}$$

where $P = \mathrm{diag}(e^{i(\phi-\phi')}, 1, e^{-i\phi''})$. The orthogonal matrices O_ν and O_l are given in Eqs. (65) and (69).

In our numerical analysis, the parameters χ_{12}, χ_{23}, d, ϕ, ϕ' and ϕ'' have been generated randomly and 3σ experimental constraints on oscillation parameters have been used. We found that all textures of this class are viable except VII-(D) which is unable to fit neutrino oscillation data for both mass orderings. For texture VII-(D), θ_{12} becomes too large in the case of NO and θ_{13} becomes very small for IO. The predictions of all viable textures VII-(A), VII-(B), VII-(C), VII-(E) and VII-(F) for the oscillation parameters are very similar for NO and IO. The Jarlskog CP invariant parameter J_{cp} for this class varies in the range (−0.04–0.04) and $\sin\delta$ spans the range (−1–1) for both mass orderings. The ranges for $|m_{ee}|$ are (0.0032–0.0042) eV and (0.01–0.05) eV for NO and IO, respectively. The parameter $\sum m_i$ lies within the ranges (0.057–0.0605) eV and (0.097–0.102) eV, for NO and IO, respectively.

E. Symmetry realization

It has been shown in Ref. [37] that vanishing cofactors and texture zeros in the neutrino mass matrix can be realized with an extended scalar sector by means of discrete Abelian symmetries. We present a type-I seesaw realization of three vanishing cofactors using discrete Abelian flavor symmetries in the nonflavor basis.

The Abelian group Z_6 can be used for symmetry realization of mass matrix textures of Class-III. To obtain texture structure III-(H), one of the simplest possibilities is to have the following structures for M_D, M_R and M_l:

$$M_D = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \qquad M_R = \begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & 0 \end{pmatrix}, \quad \text{and}$$
$$M_l = \begin{pmatrix} \times & \times & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}. \tag{75}$$

Here, M_D is diagonal and texture zeros in M_R propagate as vanishing cofactors in the effective neutrino mass matrix M_ν. In addition to the SM left-handed $SU(2)_L$ lepton doublets $D_{lL} = \binom{\nu_{lL}}{l_L}$, $(l = e, \mu, \tau)$ and the right-handed charged lepton $SU(2)_L$ singlets l_R, we introduce three right-handed neutrinos ν_{lR}. In the scalar sector, we need two $SU(2)_L$ Higgs doublets ϕ and two scalar singlets χ.

We consider the following transformation properties of various fields under Z_6 for texture III-(H):

$$\begin{aligned} &\bar{D}_{eL} \to \omega^5 \bar{D}_{eL}, && e_R \to \omega^2 e_R, && \nu_{eR} \to \omega \nu_{eR}, \\ &\bar{D}_{\mu L} \to \bar{D}_{\mu L}, && \mu_R \to \omega \mu_R, && \nu_{\mu R} \to \omega^5 \nu_{\mu R}, \\ &\bar{D}_{\tau L} \to \omega^3 \bar{D}_{\tau L}, && \tau_R \to \omega^3 \tau_R, && \nu_{\tau R} \to \omega^2 \nu_{\tau R} \end{aligned} \tag{76}$$

where $\omega = e^{\frac{2\pi i}{6}}$ is the generator of the Z_6 group. The bilinears $\bar{D}_{lL} l_R, \bar{D}_{lL}\nu_{lR}$ and $\nu_{lR}\nu_{lR}$ relevant for M_l, M_D and M_R, respectively, transform as

$$\bar{D}_{lL}l_R \sim \begin{pmatrix} \omega & 1 & \omega^2 \\ \omega^2 & \omega & \omega^3 \\ \omega^5 & \omega^4 & 1 \end{pmatrix}, \quad \bar{D}_{lL}\nu_{lR} \sim \begin{pmatrix} 1 & \omega^4 & \omega \\ \omega & \omega^5 & \omega^2 \\ \omega^4 & \omega^2 & \omega^5 \end{pmatrix} \quad \text{and}$$

$$\nu_{lR}\nu_{lR} \sim \begin{pmatrix} \omega^2 & 1 & \omega^3 \\ 1 & \omega^4 & \omega \\ \omega^3 & \omega & \omega^4 \end{pmatrix}. \tag{77}$$

For M_R, the (1,2) element is invariant under Z_6 and hence the corresponding mass term is directly present in the Lagrangian without any scalar field. However, the (1,1) and (2,3) matrix elements require the presence of two scalar singlets χ_1 and χ_2 which transform under Z_6 as $\chi_1 \to \omega^4\chi_1$ and $\chi_2 \to \omega^5\chi_2$, respectively. The other entries of M_R remain zero in the absence of any further scalar singlets. To achieve a nondiagonal charged lepton mass matrix M_l, two scalar Higgs doublets are needed which transform under Z_6 as $\phi_1 \to \phi_1$, $\phi_2 \to \omega^5\phi_2$. A diagonal M_D is obtained since the scalar Higgs doublets $\tilde{\phi}_j (\equiv \imath\sigma_2\phi_j^*)$ transform under Z_6 as $\tilde{\phi}_1 \to \tilde{\phi}_1$ and $\tilde{\phi}_2 \to \omega\tilde{\phi}_2$. Thus, the Z_6 invariant Yukawa Lagrangian for texture III-(H) is given by

$$\begin{aligned}\mathcal{L}_Y = {} & -Y^l_{ee}\bar{D}_{eL}\phi_2 e_R - Y^l_{e\mu}\bar{D}_{eL}\phi_1\mu_R - Y^l_{\mu\mu}\bar{D}_{\mu L}\phi_2\mu_R \\ & - Y^l_{\tau\tau}\bar{D}_{\tau L}\phi_1\tau_R - Y^D_{ee}\bar{D}_{eL}\tilde{\phi}_1\nu_{eR} - Y^D_{\mu\mu}\bar{D}_{\mu L}\tilde{\phi}_2\nu_{\mu R} \\ & - Y^D_{\tau\tau}\bar{D}_{\tau L}\tilde{\phi}_2\nu_{\tau R} + \frac{Y^R_{ee}}{2}\nu^T_{eR}C^{-1}\nu_{eR}\chi_1 \\ & + \frac{M^R_{e\mu}}{2}(\nu^T_{eR}C^{-1}\nu_{\mu R} + \nu^T_{\mu R}C^{-1}\nu_{eR}) \\ & + \frac{Y^R_{\mu\tau}}{2}(\nu^T_{\mu R}C^{-1}\nu_{\tau R} + \nu^T_{\tau R}C^{-1}\nu_{\mu R})\chi_2 + \text{H.c.}\end{aligned} \tag{78}$$

For texture III-(I), the structures for M_D, M_R, M_l are given by

$$M_D = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad M_R = \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix}, \quad M_l = \begin{pmatrix} \times & 0 & \times \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix} \tag{79}$$

and the leptonic fields are required to transform as

$$\begin{aligned} &\bar{D}_{eL} \to \omega^5\bar{D}_{eL}, \quad && e_R \to \omega^2 e_R, \quad && \nu_{eR} \to \omega\nu_{eR}, \\ &\bar{D}_{\mu L} \to \omega^3\bar{D}_{\mu L}, && \mu_R \to \omega^3\mu_R, && \nu_{\mu R} \to \omega^2\nu_{\mu R}, \\ &\bar{D}_{\tau L} \to \bar{D}_{\tau L}, && \tau_R \to \omega\tau_R, && \nu_{\tau R} \to \omega^5\nu_{\tau R} \end{aligned} \tag{80}$$

under Z_6. The bilinears $\bar{D}_{lL}l_R, \bar{D}_{lL}\nu_{lR}$ and $\nu_{lR}\nu_{lR}$ corresponding to M_l, M_D and M_R transform as

$$\bar{D}_{lL}l_R \sim \begin{pmatrix} \omega & \omega^2 & 1 \\ \omega^5 & 1 & \omega^4 \\ \omega^2 & \omega^3 & \omega \end{pmatrix}, \quad \bar{D}_{lL}\nu_{lR} \sim \begin{pmatrix} 1 & \omega & \omega^4 \\ \omega^4 & \omega^5 & \omega^2 \\ \omega & \omega^2 & \omega^5 \end{pmatrix} \quad \text{and}$$

$$\nu_{lR}\nu_{lR} \sim \begin{pmatrix} \omega^2 & \omega^3 & 1 \\ \omega^3 & \omega^4 & \omega \\ 1 & \omega & \omega^4 \end{pmatrix}. \tag{81}$$

The two scalar fields required for nonzero elements in M_R, transform as $\chi_1 \to \omega^4\chi_1$, $\chi_2 \to \omega^5\chi_2$ under Z_6. The desired form of M_l requires the Higgs fields to transform as $\phi_1 \to \phi_1$ and $\phi_2 \to \omega^5\phi_2$. The scalar Higgs doublets acquire nonzero vacuum expectation values (VEVs) at the electroweak scale, while scalar singlets acquire VEVs at the seesaw scale.

Similarly, the symmetry realization for Class-IV can also be achieved using the Z_6 group. The transformation properties of leptonic and scalar fields under the Z_6 group are given in Table V.

For four vanishing cofactors in M_ν and five nonzero elements in M_l, the symmetry realization of textures can be achieved by the cyclic group Z_9. Table VI depicts the transformation properties of leptonic and scalar fields under the Z_9 group for Class-VII.

TABLE V. Transformation properties of lepton and scalar fields under Z_6 for Class-IV.

Model	M_l, M_R, M_D	$\bar{D}_{eL}, \bar{D}_{\mu L}, \bar{D}_{\tau L}$	e_R, μ_R, τ_R	$\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$	ϕ_1, ϕ_2, ϕ_3	χ
IV-(D)	$\begin{pmatrix} \times & \times & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}$	$1, \omega, \omega^2$	$\omega^2, \omega, \omega^4$	$1, \omega, \omega^4$	$1, \omega^4, \omega^5$	ω^4
IV-(E)	$\begin{pmatrix} \times & 0 & \times \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}$	$1, \omega^2, \omega$	$\omega^2, \omega^4, \omega$	$1, \omega, \omega^4$	$1, \omega^4, \omega^5$	ω^4
IV-(F)	$\begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & 0 & \times \end{pmatrix}, \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \begin{pmatrix} 0 & 0 & \times \\ \times & 0 & \times \\ 0 & 0 & \times \end{pmatrix}$	$\omega^2, 1, \omega$	$\omega^4, \omega^2, \omega$	$1, \omega, \omega^4$	$1, \omega^4, \omega^5$	ω^4

TABLE VI. Transformation properties of lepton and scalar fields under Z_9 for Class-VII.

Model	M_l, M_R, M_D	$\bar{D}_{eL}, \bar{D}_{\mu L}, \bar{D}_{\tau L}$	e_R, μ_R, τ_R	$\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$	ϕ_1, ϕ_2, ϕ_3	χ_1, χ_2
VII-(A)	$\begin{pmatrix} \times & \times & 0 \\ 0 & \times & \times \\ 0 & 0 & \times \end{pmatrix}, \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & 0 \end{pmatrix}$	$\omega^2, \omega^4, \omega^6$	$\omega^7, \omega^5, \omega^3$	$1, \omega^4, \omega^7$	$1, \omega^2, \omega^3$	ω, ω^4
VII-(B)	$\begin{pmatrix} \times & \times & 0 \\ 0 & \times & \times \\ 0 & 0 & \times \end{pmatrix}, \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \begin{pmatrix} 0 & \times & \times \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$	$\omega^5, \omega^6, \omega^4$	$\omega^4, \omega, \omega^3$	$1, \omega^4, \omega^7$	$1, \omega^2, \omega^3$	ω, ω^4
VII-(C)	$\begin{pmatrix} \times & \times & 0 \\ 0 & \times & \times \\ 0 & 0 & \times \end{pmatrix}, \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & 0 & \times \end{pmatrix}$	$\omega^6, \omega^5, \omega^4$	$1, \omega, \omega^2$	$1, \omega^4, \omega^7$	$1, \omega^2, \omega^3$	ω, ω^4
VII-(E)	$\begin{pmatrix} \times & \times & 0 \\ 0 & \times & 0 \\ 0 & \times & \times \end{pmatrix}, \begin{pmatrix} \times & \times & 0 \\ 0 & \times & 0 \\ 0 & \times & \times \end{pmatrix}, \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & 0 \\ \times & 0 & \times \end{pmatrix}$	$\omega^4, \omega, \omega^2$	$\omega^2, \omega^5, \omega^7$	$1, \omega^4, \omega^7$	$1, \omega^2, \omega^3$	ω, ω^4
VII-(F)	$\begin{pmatrix} \times & \times & 0 \\ 0 & \times & 0 \\ 0 & \times & \times \end{pmatrix}, \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ \times & 0 & \times \\ 0 & 0 & \times \end{pmatrix}$	$\omega, \omega^2, \omega^4$	$\omega^8, \omega^5, \omega^2$	$1, \omega^4, \omega^7$	$1, \omega^2, \omega^3$	ω, ω^4

F. Conclusion

We have investigated some new texture structures for lepton mass matrices. We have studied texture structures with three (four) vanishing cofactors in the neutrino mass matrix M_ν with four (five) nonzero elements in the charged lepton mass matrix M_l. There are three possible structures for H_l and 20 possible structures of M_ν grouped into Classes-I, II, III, IV, V and VI, for three vanishing cofactors in M_ν and four nonzero elements in M_l. It was found that among the six classes only Classes-III and -IV are phenomenologically viable. We also found that there are five viable textures having four vanishing cofactors in M_ν with five nonzero elements in M_l. By using the recent global neutrino oscillation data and data from cosmological experiments, a systematic phenomenological analysis has been done for each viable texture. We, also, presented the symmetry realization for the allowed texture structures using discrete Abelian symmetries in the framework of the type-I seesaw mechanism.

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