

Derivation of the gap and Bethe-Salpeter equations at large N_c limit and symmetry preserving truncations

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We develop a framework for deriving Dyson-Schwinger equations (DSEs) and the Bethe-Salpeter equation (BSE) in QCD at the large N_c limit. The starting point is a modified form (with auxiliary fields) of the QCD generating functional. This framework provides a natural order-by-order truncation scheme for DSEs and the BSE, and the kernels of the equations up to any order are explicitly given. Chiral symmetry (at the chiral limit) is preserved in any-order truncation, so it exemplifies the symmetry preserving truncation scheme. It provides a method to study DSEs and BSE beyond the rainbow-ladder truncation and is especially useful to study contributions from non-Abelian dynamics (those arising from gluon self-interactions). We also derive the equation for the quark-ghost scattering kernel and discuss the Slavnov-Taylor identity connecting the quark-gluon vertex, the quark propagator, and the quark-ghost scattering kernel.

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I. INTRODUCTION

The standard form of a QCD generating functional is the basis for perturbative expansion, while alternate forms may have advantages in nonperturbative studies. For example, In Ref. [1], Cahill and Roberts introduced a bilocal action to study meson properties and the bag model. The action was obtained by taking an approximation that only the gluon propagator was retained. So, it is not QCD but a model that is usually called the global color model (GCM) (for a review, see Ref. [2]). It is possible to generalize their method to obtain a bilocal action that is equivalent to the QCD action, and the GCM action may be served as the leading order of this new bilocal action. In the present paper, we shall employ a bilocal action that was first introduced in Ref. [3] and was used to study chiral perturbation theory therein. With this action, various problems can be addressed. In this work, we generalize the method used in Ref. [3] by introducing external sources for single quark fields, gluon fields, and ghost fields to study Dyson-Schwinger equations (DSEs) and the Bethe-Salpeter equation (BSE). In this approach, the parameter N_c in the action is extracted and shown explicitly, which is

especially beneficial for large N_c expansion. In the present work, we concentrate on the large N_c limit, although higher-order corrections can be taken into account in principle. We shall derive DSEs and the BSE and discuss their truncations. Since all equations are derived from the same generating functional, consistency among these equations is easily maintained when truncations are made. In particular, chiral symmetry (at chiral limit) is preserved automatically in truncated DSEs and the BSE up to any order. This is a more elegant approach than truncating the DSEs and the BSE separately and discussing their consistency thereafter.

DSEs and the BSE are important tools in low-energy QCD and hadron physics. Particularly, the meson spectrum can be calculated with the BSE combined with the gap equation, i.e., the DSE for the quark propagator. It is well known that these equations must be truncated in practical calculations. The simplest truncation for the gap equation and the meson BSE is the rainbow-ladder (RL) truncation [4–8]. In this truncation, the full quark-gluon vertex is replaced with the bare vertex γ^μ (up to a color matrix), and the BSE kernel takes the ladder approximation. Although many achievements have been made based on the RL truncation, recent research has indicated the necessity of going beyond the RL truncation. For instance, the spectra for light scalar and axial-vector mesons calculated under the RL truncation are relatively poor compared to those for

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light pseudoscalar and vector mesons [9–11]. It is believed that the reason for this is that in the pseudoscalar and vector channels the higher-order corrections to the RL truncation largely cancel, while in the scalar and axial-vector channels, the corrections do not typically cancel [12,13]. So, in recent years, a lot of effort has been made to aim at going beyond the RL truncation in DSEs and the BSE [14–20].

Chiral symmetry breaking plays an important role in the low-energy QCD, so the truncated DSEs and BSE should reflect this feature properly. It is realized that chiral symmetry imposes an important connection between the integration kernel of the gap equation and the BSE kernel. This connection guarantees that the pion state as a solution of the BSE for a quark-antiquark pair is automatically the Goldstone particle when chiral symmetry is spontaneously broken (in the chiral limit) [21,22]. In this respect, the symmetry preserving truncation scheme is proposed and requires the approximation made in the BSE kernel to be consistent with the truncation made in the gap equation such that chiral symmetry is preserved in the chiral limit [22]. In this scheme, the well-known RL truncation is just its lowest-order truncation.

To go beyond RL truncation, one can make use of the DSE for the quark-gluon vertex (QGV). This equation explicitly shows how the strong interaction makes corrections to the bare quark-gluon vertex. At one-loop level, keeping all the propagators dressed, the QGV has two triangle Feynman diagrams contributions, which are usually served as the next-to-leading-order correction. One may continue to consider higher-loop contributions and improve the truncation gradually. This method of going beyond the RL truncation has a benefit that the effects of QCD dynamics in the quark condensate and physical observables can be directly tested. Especially, the gluon self-interactions, which are typical non-Abelian dynamics, can be directly tested [15,23].

Another way of going beyond the RL truncation makes use of the Slavnov-Taylor identity (STI) of the QGV. The STI relates the QGV to the quark propagator and the quark-ghost scattering kernel, and thus it can be used to model the QGV with the other two Green's functions [24–26]. Because this method escapes from the seemingly endless truncations using the QGV DSE and the STI is automatically fulfilled, it becomes one of the major focuses in the studies beyond the RL truncation. However, due to the appearance of the quark-ghost scattering kernel, the STI of the QGV is more complicated than its counterpart in QED, i.e., the Ward-Takahashi identity (WTI) for the fermion-photon vertex. In QED, the fermion-photon vertex may be modeled fully by the fermion propagators, such as the Ball-Chiu (BC) vertex [27] and the Curtis-Pennington (*CP*) vertex [28], and a closed form of DSEs arises accordingly; while in QCD, the additional Green's function makes the modeling of the QGV more complicated and introduces new inputs. So some authors use the BC vertex or *CP*

vertex in studying QCD DSEs and sacrifices the non-Abelian feature of the STI. Another problem of this method is that the STI or the WTI in QED only fixes the longitudinal part of the vertex, leaving the transverse part undetermined. In this direction, some efforts have been made [29,30].

Since the study of DSEs and the BSE beyond the RL truncation is one of the major directions in hadron physics, we develop a framework for studying the quark DSE and the meson BSE in an order-by-order truncation scheme. We shall derive a bilocal form of the QCD generating functional first, then make the large N_c expansion. After taking the large N_c limit, the DSEs and BSE will be derived, and the truncation scheme will be presented. It will be clear that in this truncation scheme the leading order, i.e., RL truncation, is just the Abelian approximation, which means all the higher-order corrections are due to non-Abelian-type dynamics of QCD. Thus, the large N_c limit extracts the non-Abelian dynamics in the DSEs and is especially useful for testing the effects of gluon self-interactions. On the other hand, from direct analysis of the Feynman diagrams, we know that at the large N_c limit, mesons are free particles [31,32], and this will simplify the corresponding BSE. Furthermore, if we reformulate QCD in loop space [33], the DSE for Wilson loops becomes a closed equation, in contrast to ordinary DSEs and BSE, which have an infinite tower of coupled equation groups and then need truncation. Although we still do not fully understand why DSEs in loop space are closed, this feature must have some impacts on ordinary DSEs and makes large N_c limit a special approximation for ordinary DSEs.

In our approach, the higher-order corrections contribute through higher-loop diagrams in the equations. This way of going beyond the RL truncation was studied previously in a number of works as mentioned before. Some of the works concentrated on the next-leading-order corrections without giving explicit forms for higher orders [15,16,23,34]; some of them only considered Abelian-type diagrams, and the non-Abelian contributions were absorbed into parameters [14]. For the former case, because the QGV DSE may be written in different forms [19], the truncated gap equations arising from different forms will actually be different in higher orders, which causes an ambiguity. So, a systematic way to give a unique order-by-order improvement in the truncated gap equation is called for. Our method presented here provides such a scheme under the large N_c limit, in which each order is just an integration of the connected gluon Green's functions in the corresponding order. Moreover, considering large N_c expansion, $1/N_c$ corrections and higher-order contributions can also be taken into account systematically in principle. So, eventually, we could have a systematic way to approach the fundamental theory, i.e., QCD. For the latter case, it is already known that non-Abelian corrections in the next leading order are dominant compared to Abelian contributions [14,16,23,34]. And we shall show that this is true up to any order. So, considering

non-Abelian-type contributions in higher orders is of more significance. Our truncation scheme does exactly this job. In addition, as indicated in Ref. [13], the H-type diagrams in the Bethe-Salpeter (BS) kernel are important because a ladder-like BS kernel with numbers of crossed-box diagrams is insufficient to preserve the Ward-Green-Takahashi identities in general. The H-type diagrams can only originate from non-Abelian diagrams.

Our framework treats the gauge sector and the fermion sector separately, and its focus is the quark DSE, the QGV DSE, and the meson BSE. We also derived the equation for the quark-ghost scattering kernel, and then all the Green's functions included in the QGV STI were expressed in the same framework. We hope this can be useful to verify the STI or shed some light on the modeling of the QGV using the STI.

The remaining part of this paper is organized as follows. We start with developing the generating functional for deriving the DSEs in Sec. II. The gap equation and the QGV DSE at large N_c are then derived in Sec. III. The meson BSE is derived from the same generating functional in Sec. IV, and the symmetry preserving truncation scheme is illustrated. Section V is devoted to the discussion on the STI. A summary follows in Sec. VI.

II. GENERATING FUNCTIONAL AT LARGE N_c LIMIT

Consider a QCD-type gauge theory with $SU(N_c)$ gauge symmetry. Let $A_\mu^i (i = 1, 2, \dots, N_c^2 - 1)$ be the gauge field and $\psi_\alpha^{a\eta}$ be the fermion field, where $\alpha (\alpha = 1, 2, \dots, N_c)$ is the color index, η is Lorentz spinor index and $a (a = 1, 2, \dots, N_f)$ is the flavor index. For convenience, we simply call $\psi_\alpha^{a\eta}$ the ‘‘quark field’’ and A_μ^i the ‘‘gluon field.’’ Let us introduce local external sources \mathcal{I}_i^μ for gauge field A_μ^i , $\bar{\mathcal{I}}_\alpha^{a\eta}$ for $\psi_\alpha^{a\eta}$, $\mathcal{I}_\alpha^{a\eta}$ for $\bar{\psi}_\alpha^{a\eta}$, and $J_{\sigma\rho}$ for the composite quark fields $\bar{\psi}^\sigma\psi^\rho$, where σ and ρ are short notations for the spinor and flavor indices. The external source J can be decomposed into scalar, pseudoscalar, vector, axial-vector, and tensor parts,

$$J(x) = -s(x) + ip(x)\gamma_5 + \not{p}(x) + \not{d}(x)\gamma_5 + \sigma_{\mu\nu}\bar{t}^{\mu\nu}(x), \quad (1)$$

where $s(x)$, $p(x)$, $v_\mu(x)$, and $a_\mu(x)$ are Hermitian matrices and the quark masses have been absorbed into the definition of $s(x)$. The vector and axial-vector sources $\not{p}(x)$, $\not{d}(x)$, and $\bar{t}^{\mu\nu}(x)$ are taken to be traceless.

We start from constructing the generating functional

$$\begin{aligned} Z[J, \mathcal{I}, \bar{\mathcal{I}}, I] &= \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \exp i \int d^4x \{ \mathcal{L}(\psi, \bar{\psi}, A_\mu) + \bar{\psi} J \psi + \mathcal{I}_i^\mu A_\mu^i + \bar{\mathcal{I}} \psi + \bar{\psi} I \} \\ &= \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left\{ i \int d^4x \{ \bar{\psi} (i\partial + J) \psi + \bar{\mathcal{I}} \psi + \bar{\psi} I \} \right\} \\ &\quad \times \int \mathcal{D}A_\mu \Delta_F(A_\mu) \exp \left\{ i \int d^4x \left[\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 + \mathcal{I}_i^\mu A_\mu^i \right] \right\}, \end{aligned} \quad (2)$$

where $\mathcal{L}_G(A) = -\frac{1}{4} A_{\mu\nu}^i A^{i\mu\nu}$ is the gluon kinetic energy term, $\mathcal{I}_i^\mu \equiv \mathcal{I}_i^\mu - g\bar{\psi} \frac{\lambda_i}{2} \gamma^\mu \psi$ absorb the quark current (or gauge interaction term) into the external source for the gauge field, $-\frac{1}{2\xi} [F^i(A_\mu)]^2$ is the gauge-fixing term, and $\Delta_F(A_\mu)$ is the Fadeev-Popov determinant. The traditional QCD generating functional will be arrived at by taking $N_c = 3$ and the limit $J(x) \rightarrow -M$ with M the quark mass matrix. We introduce the external source J for composite quark fields to keep the generating functional's potential for further use, and it will not harm the current study.

Let us first consider the integration over $\mathcal{D}A_\mu$ for a given configuration of ψ and $\bar{\psi}$; i.e., the current \mathcal{I}_i^μ serves as an effective external source in the integration over $\mathcal{D}A_\mu$. The result of such an integration can be formally written as

$$\begin{aligned} \int \mathcal{D}A_\mu \Delta_F(A_\mu) \exp \left\{ i \int d^4x \left[\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 + \mathcal{I}_i^\mu A_\mu^i \right] \right\} &= \exp i \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n \frac{i^n}{n!} G_{\mu_1 \cdots \mu_n}^{i_1 \cdots i_n}(x_1, \dots, x_n) \\ &\quad \times \mathcal{I}_{i_1}^{\mu_1}(x_1) \cdots \mathcal{I}_{i_n}^{\mu_n}(x_n), \end{aligned} \quad (3)$$

where $G_{\mu_1 \cdots \mu_n}^{i_1 \cdots i_n}$ is the full connected n-gluon Green's function without inner quark loops. Precisely, it is defined as

$$\begin{aligned} i^n G_{\mu_1 \cdots \mu_n}^{i_1 \cdots i_n}(x_1, \dots, x_n) &\equiv i^n \langle 0 | T [A_{\mu_1}^{i_1}(x_1) \cdots A_{\mu_n}^{i_n}(x_n)] | 0 \rangle_{\text{connected, pure YM}} \\ &= \frac{\delta^n}{\delta \mathcal{I}_{i_1}^{\mu_1}(x_1) \cdots \delta \mathcal{I}_{i_n}^{\mu_n}(x_n)} (-i) \ln \int \mathcal{D}A_\mu \Delta_F(A_\mu) \exp \left\{ i \int d^4x \left[\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 + \mathcal{I}_i^\mu A_\mu^i \right] \right\} \Big|_{\mathcal{I}_i^\mu(x)=0}. \end{aligned} \quad (4)$$

Note that if the gauge interaction is not non-Abelian but Abelian, then only the two-gluon Green's function (here and later on, we omit "without inner quark loops" for convenience) is nonzero due to the absence of self-interactions among the gauge fields. Hence, we define "Abelian approximation" as only keeping the two-gluon Green's functions in the result and dropping three-point and higher-point gluon Green's functions.

The source terms in Eq. (3) can be written explicitly as

$$\begin{aligned}
& \int \mathcal{D}A_\mu \Delta_F(A_\mu) \exp \left\{ i \int d^4x \left[\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 + \mathcal{I}_i^\mu A_\mu^i \right] \right\} \\
&= \exp i \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n \frac{i^n}{n!} G_{\mu_1 \cdots \mu_n}^{i_1 \cdots i_n}(x_1, \dots, x_n) \\
&\quad \times \left[\left[-g\bar{\psi}_{\alpha_1}^{a_1}(x_1) \left(\frac{\lambda_{i_1}}{2} \right)_{\alpha_1 \beta_1} \gamma^{\mu_1} \psi_{\beta_1}^{a_1}(x_1) \right] \cdots \left[-g\bar{\psi}_{\alpha_n}^{a_n}(x_n) \left(\frac{\lambda_{i_n}}{2} \right)_{\alpha_n \beta_n} \gamma^{\mu_n} \psi_{\beta_n}^{a_n}(x_n) \right] \right. \\
&\quad + n \mathcal{I}_{i_1}^{\mu_1}(x_1) \left[-g\bar{\psi}_{\alpha_2}^{a_2}(x_2) \left(\frac{\lambda_{i_2}}{2} \right)_{\alpha_2 \beta_2} \gamma^{\mu_2} \psi_{\beta_2}^{a_2}(x_2) \right] \cdots \left[-g\bar{\psi}_{\alpha_n}^{a_n}(x_n) \left(\frac{\lambda_{i_n}}{2} \right)_{\alpha_n \beta_n} \gamma^{\mu_n} \psi_{\beta_n}^{a_n}(x_n) \right] \\
&\quad + n(n-1) \mathcal{I}_{i_1}^{\mu_1}(x_1) \mathcal{I}_{i_2}^{\mu_2}(x_2) \left[-g\bar{\psi}_{\alpha_3}^{a_3}(x_3) \left(\frac{\lambda_{i_3}}{2} \right)_{\alpha_3 \beta_3} \gamma^{\mu_3} \psi_{\beta_3}^{a_3}(x_3) \right] \cdots \left[-g\bar{\psi}_{\alpha_n}^{a_n}(x_n) \left(\frac{\lambda_{i_n}}{2} \right)_{\alpha_n \beta_n} \gamma^{\mu_n} \psi_{\beta_n}^{a_n}(x_n) \right] \\
&\quad \left. + \cdots + \mathcal{I}_{i_1}^{\mu_1}(x_1) \cdots \mathcal{I}_{i_n}^{\mu_n}(x_n) \right], \tag{5}
\end{aligned}$$

By Fierz reordering, we can diagonalize the color indices of the quark fields. For the source independent terms, we have

$$\begin{aligned}
& \int d^4x_2 \cdots d^4x_n G_{\mu_1 \cdots \mu_n}^{i_1 \cdots i_n}(x_1, \dots, x_n) \left[-g\bar{\psi}_{\alpha_1}^{a_1}(x_1) \left(\frac{\lambda_{i_1}}{2} \right)_{\alpha_1 \beta_1} \gamma^{\mu_1} \psi_{\beta_1}^{a_1}(x_1) \right] \cdots \left[-g\bar{\psi}_{\alpha_n}^{a_n}(x_n) \left(\frac{\lambda_{i_n}}{2} \right)_{\alpha_n \beta_n} \gamma^{\mu_n} \psi_{\beta_n}^{a_n}(x_n) \right] \\
&= \int d^4x_2 \cdots d^4x_n d^4x'_1 \cdots d^4x'_n (-1)^n g^{2n-2} \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n) \bar{\psi}_{\alpha_1}^{\sigma_1}(x_1) \psi_{\alpha_1}^{\rho_1}(x'_1) \cdots \bar{\psi}_{\alpha_n}^{\sigma_n}(x_n) \psi_{\alpha_n}^{\rho_n}(x'_n), \tag{6}
\end{aligned}$$

where $\bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n)$ is an extended Green's function containing $2n$ space-time points. Using the relations listed in Appendix A, one can check that the extended two-point Green's function is

$$\begin{aligned}
\bar{G}_{\rho_1 \rho_2}^{\sigma_1 \sigma_2}(x_1, x'_1, x_2, x'_2) &= -\frac{1}{2} G_{\mu_1 \mu_2}(x_1, x_2) \left[(\gamma^{\mu_1})_{\sigma_1 \rho_2} (\gamma^{\mu_2})_{\sigma_2 \rho_1} \delta(x'_1 - x_2) \delta(x'_2 - x_1) \right. \\
&\quad \left. + \frac{1}{N_c} (\gamma^{\mu_1})_{\sigma_1 \rho_1} (\gamma^{\mu_2})_{\sigma_2 \rho_2} \delta(x'_1 - x_1) \delta(x'_2 - x_2) \right], \tag{7}
\end{aligned}$$

where σ, ρ 's are combined spinor and flavor indices.

Similarly, we introduce extended Green's functions \tilde{G} for linear source-dependent terms, which satisfy

$$\begin{aligned}
& \int d^4x_2 \cdots d^4x_n G_{\mu_1 \cdots \mu_n}^{i_1 \cdots i_n}(x_1, \dots, x_n) \mathcal{I}_{i_1}^{\mu_1}(x_1) \left[-g\bar{\psi}_{\alpha_2}^{a_2}(x_2) \left(\frac{\lambda_{i_2}}{2} \right)_{\alpha_2 \beta_2} \gamma^{\mu_2} \psi_{\beta_2}^{a_2}(x_2) \right] \cdots \left[-g\bar{\psi}_{\alpha_n}^{a_n}(x_n) \left(\frac{\lambda_{i_n}}{2} \right)_{\alpha_n \beta_n} \gamma^{\mu_n} \psi_{\beta_n}^{a_n}(x_n) \right] \\
&= \int d^4x_2 \cdots d^4x_n d^4x'_1 \cdots d^4x'_n (-1)^{n-1} g^{2n-3} \mathcal{I}_{i_1}^{\mu_1}(x_1) (\lambda_{i_1})_{\alpha\beta} \tilde{G}_{\mu_1, \rho \rho_3 \cdots \rho_n}^{\sigma \sigma_3 \cdots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n) \\
&\quad \times \bar{\psi}_{\alpha}^{\sigma}(x'_1) \psi_{\beta}^{\rho}(x'_2) \bar{\psi}_{\alpha_3}^{\sigma_3}(x_3) \psi_{\alpha_3}^{\rho_3}(x'_3) \cdots \bar{\psi}_{\alpha_n}^{\sigma_n}(x_n) \psi_{\alpha_n}^{\rho_n}(x'_n). \tag{8}
\end{aligned}$$

Now, Eq. (2) can be written as

$$\begin{aligned}
Z[J, \mathcal{I}, \bar{I}, I] = & \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp i \left\{ \int d^4x \{ \bar{\psi}(i\partial + J)\psi + \bar{I}\psi + \bar{\psi}I \} + \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n d^4x'_1 \cdots d^4x'_n \right. \\
& \times \left[\frac{(-i)^n (g^2)^{n-1}}{n!} \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n) \bar{\psi}_{\alpha_1}^{\sigma_1}(x_1) \psi_{\alpha_1}^{\rho_1}(x'_1) \cdots \bar{\psi}_{\alpha_n}^{\sigma_n}(x_n) \psi_{\alpha_n}^{\rho_n}(x'_n) \right. \\
& + \frac{i(-i)^{n-1} g^{2n-3}}{(n-1)!} \mathcal{I}_i^\mu(x_1) (\lambda_i)_{\alpha\beta} \tilde{G}_{\mu, \rho_3 \cdots \rho_n}^{\sigma_3 \cdots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n) \bar{\psi}_{\alpha}^{\sigma}(x'_1) \psi_{\beta}^{\rho}(x'_2) \bar{\psi}_{\alpha_3}^{\sigma_3}(x_3) \psi_{\alpha_3}^{\rho_3}(x'_3) \cdots \bar{\psi}_{\alpha_n}^{\sigma_n}(x_n) \psi_{\alpha_n}^{\rho_n}(x'_n) \\
& \left. \left. + O(\mathcal{I}^2) \right] \right\}. \tag{9}
\end{aligned}$$

To integrate out the quark fields ψ and $\bar{\psi}$, we introduce a bilocal auxiliary field $\Phi^{(an)(b\zeta)}(x, x')$ by inserting into (9) the following constant:

$$\int \mathcal{D}\Phi \delta(N_c \Phi^{(an)(b\zeta)}(x, x') - \bar{\psi}_\alpha^{an}(x) \psi_\alpha^{b\zeta}(x')). \tag{10}$$

We see from (6) that the bilocal auxiliary field $\Phi^{(an)(b\zeta)}(x, x')$ embodies the bilocal composite fields $\bar{\psi}_\alpha^{an}(x) \psi_\alpha^{b\zeta}(x')$, which reflects the meson fields. Inserting (10) into (9), we get

$$\begin{aligned}
Z[J, \mathcal{I}, \bar{I}, I] = & \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}\Phi \delta(N_c \Phi^{(an)(b\zeta)}(x, x') - \bar{\psi}_\alpha^{an}(x) \psi_\alpha^{b\zeta}(x')) \exp i \left\{ \int d^4x \{ \bar{\psi}(i\partial + J)\psi + \bar{I}\psi + \bar{\psi}I \} \right. \\
& + \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n d^4x'_1 \cdots d^4x'_n \\
& \times \left[N_c \frac{(-i)^n (N_c g^2)^{n-1}}{n!} \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n) \Phi^{\sigma_1 \rho_1}(x_1, x'_1) \cdots \Phi^{\sigma_n \rho_n}(x_n, x'_n) \right. \\
& + \frac{i(-i)^{n-1} g^{2n-3} N_c^{n-2}}{(n-1)!} \mathcal{I}_i^\mu(x_1) (\lambda_i)_{\alpha\beta} \tilde{G}_{\mu, \rho_3 \cdots \rho_n}^{\sigma_3 \cdots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n) \bar{\psi}_\alpha^{\sigma}(x'_1) \psi_\beta^{\rho}(x'_2) \\
& \left. \left. \times \Phi^{\sigma_3 \rho_3}(x_3, x'_3) \cdots \Phi^{\sigma_n \rho_n}(x_n, x'_n) + O(\mathcal{I}^2) \right] \right\}. \tag{11}
\end{aligned}$$

The δ function in (11) can be further expressed in the Fourier representation:

$$\delta(N_c \Phi(x, x') - \bar{\psi}(x) \psi(x')) \sim \int \mathcal{D}\Pi e^{i \int d^4x d^4x' \Pi(x, x') \cdot (N_c \Phi(x, x') - \bar{\psi}(x) \psi(x'))}.$$

The generating functional then becomes

$$\begin{aligned}
Z[J, \mathcal{I}, \bar{I}, I] = & \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}\Phi \mathcal{D}\Pi \exp i \left\{ \int d^4x \{ \bar{\psi}(i\partial + J - \Pi)\psi + \bar{I}\psi + \bar{\psi}I \} + \int d^4x d^4x' N_c \Phi^{\sigma\rho}(x, x') \Pi^{\sigma\rho}(x, x') \right. \\
& + \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n d^4x'_1 \cdots d^4x'_n N_c \frac{(-i)^n (N_c g^2)^{n-1}}{n!} \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n) \Phi^{\sigma_1 \rho_1}(x_1, x'_1) \cdots \Phi^{\sigma_n \rho_n}(x_n, x'_n) \\
& \left. + \int d^4x_1 d^4x'_1 d^4x'_2 \bar{\psi}_\alpha^{\sigma}(x'_1) \tilde{\mathcal{I}}_{\alpha\beta}^\mu(x_1) \Delta_{\Phi, \mu}^{\sigma\rho}(x_1, x'_1, x'_2) \psi_\beta^{\rho}(x'_2) + O(\mathcal{I}^2) \right\}, \tag{12}
\end{aligned}$$

where $\tilde{\mathcal{I}}_{\alpha\beta}^\mu(x_1) \equiv \mathcal{I}_i^\mu(\lambda_i)_{\alpha\beta}$ and

$$\begin{aligned} \Delta_{\Phi,\mu}^{\sigma\rho}(x_1, x'_1, x'_2) &\equiv \int d^4x_2 \cdots d^4x_n d^4x'_3 \cdots d^4x'_n \frac{i(-i)^{n-1} g^{2n-3} N_c^{n-2}}{(n-1)!} \tilde{G}_{\mu,\rho\rho_3 \cdots \rho_n}^{\sigma\sigma_3 \cdots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n) \\ &\times \Phi^{\sigma_3\rho_3}(x_3, x'_3) \cdots \Phi^{\sigma_n\rho_n}(x_n, x'_n). \end{aligned} \quad (13)$$

Integrating out the ψ and $\bar{\psi}$ fields leads us to

$$\begin{aligned} Z[J, \mathcal{I}, \bar{I}, I] &= \int \mathcal{D}\Phi \mathcal{D}\Pi \exp i \left\{ -i \text{Tr}' \ln [i\partial + J - \Pi + \tilde{\mathcal{I}}\Delta_\Phi] - \bar{I} [i\partial + J - \Pi + \tilde{\mathcal{I}}\Delta_\Phi]^{-1} I \right. \\ &+ \int d^4x d^4x' N_c \Phi^{\sigma\rho}(x, x') \Pi^{\sigma\rho}(x, x') + N_c \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n d^4x'_1 \cdots d^4x'_n \frac{(-i)^n (N_c g^2)^{n-1}}{n!} \\ &\left. \times \tilde{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n) \Phi^{\sigma_1\rho_1}(x_1, x'_1) \cdots \Phi^{\sigma_n\rho_n}(x_n, x'_n) + O(\mathcal{I}^2) \right\}, \end{aligned} \quad (14)$$

where Tr' is the functional trace with respect to the space-time, color, spinor, and flavor indices. Expanding the first two terms on the exponential with respect to $\tilde{\mathcal{I}}$, we arrive at

$$\begin{aligned} Z[J, \mathcal{I}, \bar{I}, I] &= \int \mathcal{D}\Phi \mathcal{D}\Pi \exp i \left\{ -i N_c \text{Tr} \ln [i\partial + J - \Pi] - \bar{I} [i\partial + J - \Pi]^{-1} I + \bar{I} [i\partial + J - \Pi]^{-1} \tilde{\mathcal{I}} \Delta_\Phi [i\partial + J - \Pi]^{-1} I \right. \\ &+ \int d^4x d^4x' N_c \Phi^{\sigma\rho}(x, x') \Pi^{\sigma\rho}(x, x') + N_c \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n d^4x'_1 \cdots d^4x'_n \frac{(-i)^n (N_c g^2)^{n-1}}{n!} \\ &\left. \times \tilde{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n) \Phi^{\sigma_1\rho_1}(x_1, x'_1) \cdots \Phi^{\sigma_n\rho_n}(x_n, x'_n) + O(\mathcal{I}^2) \right\}. \end{aligned} \quad (15)$$

The difference between Tr and Tr' is that Tr does not include a trace of the color index.

Taking the large N_c limit, we obtain

$$\begin{aligned} Z'[J, \mathcal{I}, \bar{I}, I] &\equiv \lim_{N_c \rightarrow \infty} Z[J, \mathcal{I}, \bar{I}, I] \\ &= \text{const} \times \exp i \left\{ -i N_c \text{Tr} \ln [i\partial + J - \Pi_c] - \bar{I} [i\partial + J - \Pi_c]^{-1} I \right. \\ &+ \bar{I} [i\partial + J - \Pi_c]^{-1} \tilde{\mathcal{I}} \Delta_{\Phi_c} [i\partial + J - \Pi_c]^{-1} I \\ &+ \int d^4x d^4x' N_c \Phi_c^{\sigma\rho}(x, x') \Pi_c^{\sigma\rho}(x, x') + N_c \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n d^4x'_1 \cdots d^4x'_n \frac{(-i)^n (N_c g^2)^{n-1}}{n!} \\ &\left. \times \tilde{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n) \Phi_c^{\sigma_1\rho_1}(x_1, x'_1) \cdots \Phi_c^{\sigma_n\rho_n}(x_n, x'_n) + O(\mathcal{I}^2) \right\}, \end{aligned} \quad (16)$$

where O_c is the expectation value of O , i.e., $O_c \equiv \frac{\int \mathcal{D}\Phi \mathcal{D}\Pi O e^{iS}}{\int \mathcal{D}\Phi \mathcal{D}\Pi e^{iS}}$, and iS is the exponent in Eq. (15). In this paper, whenever the large N_c limit is taken, it is implied that only leading terms in $1/N_c$ expansion are retained in the n-gluon Green's functions \tilde{G} and \tilde{G} .

III. DERIVING THE GAP EQUATION AND THE QGV DSE

The exponential of the generating functional given in Eq. (16) is also the effective action for Π_c and Φ_c ; so, Π_c and Φ_c satisfy stationary equations, which give

$$\begin{aligned}
\frac{-i\delta \ln Z'[J, \mathcal{I}, \bar{I}, I]}{\delta \Pi_c^{\sigma\rho}(x, y)} &= N_c [i[i\partial + J - \Pi_c]^{-1, \rho, \sigma}(y, x) + \Phi_c^{\sigma\rho}(x, y)] + [[i\partial + J - \Pi_c]^{-1} \bar{I} [i\partial + J - \Pi_c]^{-1}]^{\sigma\rho}(x, y) \\
&\quad - [[i\partial + J - \Pi_c]^{-1} \tilde{\mathcal{I}} \Delta_{\Phi_c} [i\partial + J - \Pi_c]^{-1} \bar{I} [i\partial + J - \Pi_c]^{-1}]^{\sigma\rho}(x, y) \\
&\quad - [[i\partial + J - \Pi_c]^{-1} \bar{I} [i\partial + J - \Pi_c]^{-1} \tilde{\mathcal{I}} \Delta_{\Phi_c} [i\partial + J - \Pi_c]^{-1}]^{\sigma\rho}(x, y) \\
&= 0
\end{aligned} \tag{17}$$

and

$$\begin{aligned}
\frac{-i\delta \ln Z'[J, \mathcal{I}, \bar{I}, I]}{\delta \Phi_c^{\sigma\rho}(x, y)} &= N_c \left[\Pi_c^{\sigma\rho}(x, y) + \sum_{n=2}^{\infty} \int d^4 x_2 \cdots d^4 x_n d^4 x'_2 \cdots d^4 x'_n \frac{(-i)^n (N_c g^2)^{n-1}}{(n-1)!} \tilde{G}_{\rho\rho_2 \cdots \rho_n}^{\sigma\sigma_2 \cdots \sigma_n}(x, y, x_2, x'_2, \dots, x_n, x'_n) \right. \\
&\quad \left. \times \Phi_c^{\sigma_2 \rho_2}(x_2, x'_2) \cdots \Phi_c^{\sigma_n \rho_n}(x_n, x'_n) \right] + \bar{I} [i\partial + J - \Pi_c]^{-1} \tilde{\mathcal{I}} \frac{\delta \Delta_{\Phi_c}}{\delta \Phi_c^{\sigma\rho}(x, y)} [i\partial + J - \Pi_c]^{-1} I \\
&= 0,
\end{aligned} \tag{18}$$

up to $O(\mathcal{I}^1)$ order. Switching off all external sources, the above equations are reduced to

$$\Phi_c^{\sigma\rho}(x, y) = -i[i\partial - M - \Pi_c]^{-1, \rho, \sigma}(y, x), \tag{19}$$

$$\begin{aligned}
\Pi_c^{\sigma\rho}(x, y) &= - \sum_{n=2}^{\infty} \int d^4 x_2 \cdots d^4 x_n d^4 x'_2 \cdots d^4 x'_n \frac{(-i)^n (N_c g^2)^{n-1}}{(n-1)!} \tilde{G}_{\rho\rho_2 \cdots \rho_n}^{\sigma\sigma_2 \cdots \sigma_n}(x, y, x_2, x'_2, \dots, x_n, x'_n) \\
&\quad \times \Phi_c^{\sigma_2 \rho_2}(x_2, x'_2) \cdots \Phi_c^{\sigma_n \rho_n}(x_n, x'_n).
\end{aligned} \tag{20}$$

The first equation says that $i\Phi_c$ is just the quark propagator and Π_c is the quark self-energy (up to a color factor), which can be checked by evaluating the functional derivative with respect to I and \bar{I} :

$$\begin{aligned}
\langle 0 | T[\psi_\beta^\rho(y) \bar{\psi}_\alpha^\sigma(x)] | 0 \rangle_c &= i \frac{\delta^2 \ln Z'[J, \mathcal{I}, \bar{I}, I]}{\delta \bar{I}_\beta^\rho(y) \delta (-I_\alpha^\sigma(x))} \Big|_{J=-M, \mathcal{I}=\bar{I}=I=0} \\
&= \delta_{\alpha\beta} [i\partial - M - \Pi_c]^{-1, \rho, \sigma}(y, x).
\end{aligned} \tag{21}$$

In deriving this equation, one should notice that, due to Eqs. (17) and (18), only those terms with external sources explicitly would contribute, i.e.,

$$\frac{\delta \ln Z'}{\delta (-I_\alpha^\sigma(x))} = \frac{\delta \ln Z'}{\delta \Phi_c} \frac{\delta \Phi_c}{\delta (-I_\alpha^\sigma(x))} + \frac{\delta \ln Z'}{\delta \Pi_c} \frac{\delta \Pi_c}{\delta (-I_\alpha^\sigma(x))} + \frac{\partial \ln Z'}{\partial (-I_\alpha^\sigma(x))} = \frac{\partial \ln Z'}{\partial (-I_\alpha^\sigma(x))}. \tag{22}$$

Substituting Eq. (19) into Eq. (20), we obtain

$$\begin{aligned}
[-i\Phi_c^{-1} - i\partial + M]^{\rho\sigma}(y, x) &= \sum_{n=2}^{\infty} \int d^4 x_2 \cdots d^4 x_n d^4 x'_2 \cdots d^4 x'_n \frac{(-i)^n (N_c g^2)^{n-1}}{(n-1)!} \tilde{G}_{\rho\rho_2 \cdots \rho_n}^{\sigma\sigma_2 \cdots \sigma_n}(x, y, x_2, x'_2, \dots, x_n, x'_n) \\
&\quad \times \Phi_c^{\sigma_2 \rho_2}(x_2, x'_2) \cdots \Phi_c^{\sigma_n \rho_n}(x_n, x'_n).
\end{aligned} \tag{23}$$

Then, we recognize that this is just the Dyson-Schwinger equation for the quark propagator.

Using the generating functional, we can also derive the quark-gluon three-point Green's function:

$$\begin{aligned} \langle 0|T[\psi_\beta^\rho(y)\bar{\psi}_\alpha^\sigma(x)A_\mu^i(z)]|0\rangle_c &= \frac{\delta^3 \ln Z'[J, \mathcal{I}, \bar{I}, I]}{\delta\bar{l}_\beta^\rho(y)\delta(-I_\alpha^\sigma(x))\delta\mathcal{I}_i^\mu(z)} \Big|_{J=-M; \mathcal{I}=\bar{I}=I=0} \\ &= i(\lambda_i)_{\beta\alpha} \int d^4z_1 d^4z_2 [i\partial - M - \Pi_c]^{-1, \rho\rho'}(y, z_1) \Delta_{\Phi_c, \mu}^{\rho'\sigma'}(z, z_1, z_2) [i\partial - M - \Pi_c]^{-1, \sigma'\sigma}(z, x). \end{aligned} \quad (24)$$

The quark-gluon three-point Green's function relates to the quark-gluon vertex $\Gamma_{\mu, \alpha\beta}^i$ as

$$\langle 0|T[\psi_\beta^\rho(y)\bar{\psi}_\alpha^\sigma(x)A_\mu^i(z)]|0\rangle_c = i \int d^4x' d^4y' d^4z' G_{\mu\nu}^{ij}(z, z') \Phi_c^{\rho\rho'}(y, y') \Gamma_{j, \beta\alpha}^{\nu, \rho'\sigma'}(z', y', x') \Phi_c^{\sigma'\sigma}(x', x). \quad (25)$$

Combining Eq. (25) with Eqs. (24) and (19), we obtain

$$\begin{aligned} - \int d^4z' G_{\mu\nu}(z, z') \Gamma_{i, \beta\alpha}^{\nu, \rho'\sigma'}(z', y', x') &= (\lambda_i)_{\beta\alpha} \Delta_{\Phi_c, \mu}^{\rho'\sigma'}(z, y', x') \\ &= (\lambda_i)_{\beta\alpha} \sum_{n=2}^{\infty} \int d^4x_2 \cdots d^4x_n d^4x'_3 \cdots d^4x'_n \frac{i(-i)^{n-1} g^{2n-3} N_c^{n-2}}{(n-1)!} \\ &\quad \times \tilde{G}_{\mu, \sigma'\rho_3 \cdots \rho_n}^{\rho'\sigma_3 \cdots \sigma_n}(z, y', x_2, x', x_3, x'_3 \cdots, x_n, x'_n) \Phi_c^{\sigma_3\rho_3}(x_3, x'_3) \cdots \Phi_c^{\sigma_n\rho_n}(x_n, x'_n). \end{aligned} \quad (26)$$

We have derived the expression of the QGV at the large N_c limit; now, the standard form of the gap equation can be written as

$$[-i\Phi_c^{-1} - i\partial + M]^{\rho\sigma}(x, z) \delta_{\alpha\beta} = -g \left(\frac{\lambda_i}{2} \right) \gamma_{\beta\beta'}^\mu \int d^4y' d^4z' G_{\mu\nu}(z, z') \Phi_c^{\rho'\sigma'}(y', z) \Gamma_{i, \beta'\alpha}^{\nu, \rho'\sigma'}(z', y', x). \quad (27)$$

Using the Eqs. (6) and (8), we find the relation between \tilde{G} and \bar{G} :

$$\begin{aligned} &\delta(\bar{x}_1 - x_2) \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n) \\ &= \int d^4\bar{x}_2 \left\{ -\delta(x'_1 - \bar{x}_2) \delta(x'_2 - x_1) \tilde{G}_{\mu_1, \rho_1 \rho_3 \cdots \rho_n}^{\sigma_2 \sigma_3 \cdots \sigma_n}(x_1, \bar{x}_1, x_2, \bar{x}_2, \dots, x_n, x'_n) \gamma_{\sigma_1 \rho_2}^{\mu_1} \right. \\ &\quad \left. - \frac{1}{N_c} \delta(x'_1 - x_1) \delta(\bar{x}_2 - x'_2) \tilde{G}_{\mu_1, \rho_2 \rho_3 \cdots \rho_n}^{\sigma_2 \sigma_3 \cdots \sigma_n}(x_1, \bar{x}_1, x_2, \bar{x}_2, \dots, x_n, x'_n) \gamma_{\sigma_1 \rho_1}^{\mu_1} \right\}. \end{aligned} \quad (28)$$

With this relation, one can check that Eqs. (23) and (27) are indeed the same.

We have derived the gap equation and the DSE for the QGV in our framework; now, we can discuss truncations for these equations. As a benefit, truncations can be made either through the QGV DSE and the gap equation directly or through the generating functional $-i \ln Z'$. Truncating the generating functional guarantees all the equations derived from it are consistent with each other, and any linearly represented symmetry conserved in the truncated generating functional is also conserved by these equations. The last term on the exponential of Eq. (16) is an infinite summation of integrations. Recalling that each extended $2n$ -point Green's function corresponds to the connected n -gluon Green's function (without quark loops), we find that this summation is an expansion with respect to the number

of external legs of the connected gluon Green's functions. So, a natural way to approximate the generating functional is to keep finite orders of this expansion. [Because of Eq. (28), the Δ_Φ term should be approximated accordingly.] The lowest order is to keep only the two-gluon Green's function, i.e., the Abelian approximation. Making use of Eqs. (26) and (28), one can find that

$$\begin{aligned} \Gamma_{i, \beta\alpha}^{\nu, \rho'\sigma'}(z', y', x') &\stackrel{\text{large } N_c; \text{ Abelian approximation}}{=} \\ &= -g \left(\frac{\lambda_i}{2} \right)_{\beta\alpha} \gamma_{\rho'\sigma'}^\nu \delta(z' - y') \delta(z' - x'). \end{aligned} \quad (29)$$

Therefore, Abelian approximation at the large N_c limit is just the well-known rainbow approximation in which the quark-gluon vertex takes its bare form. The next-to-leading

order is to keep up to three-gluon Green's functions in the expansion, which already gives corrections to the bare QGV. Continuing to add higher-point gluon Green's functions, then we have an order-by-order truncation scheme. All the correction terms to the bare QGV are due to non-Abelian-type interactions, so it is useful for testing non-Abelian dynamics in a beyond-rainbow approximation.

In this framework outlined before, the gauge sector and the fermion sector are treated differently. Pure gluon Green's functions have already been introduced in the generating functional. It is convenient to derive any-point quark Green's functions in our method, but we can only derive the equations for one-gluon-plus-n-quark Green's functions, such as the quark-gluon three-point Green's function discussed before. The reason is that our current

form of the generating functional does not give the explicit form for terms including two and higher gluon external sources \mathcal{I} . The positive side of this method is that one can concentrate on the fermion sector and treat the gauge sector as inputs, which can be extracted from studies on the corresponding pure Yang-Mills theory.

IV. MESON BETHE-SALPETER EQUATION AND CHIRAL SYMMETRY PRESERVING TRUNCATIONS

We have shown the DSEs derivation and their truncations at the large N_c limit in our framework; now, we turn to discuss the meson Bethe-Salpeter equation. To derive the meson BSE, we derive the four-point quark Green's function first. Noticing Eq. (22), one can directly have

$$\begin{aligned}
 & \langle 0 | T [\psi_{\beta_1}^{\rho_1}(y_1) \bar{\psi}_{\alpha_1}^{\sigma_1}(x_1) \psi_{\beta_2}^{\rho_2}(y_2) \bar{\psi}_{\alpha_2}^{\sigma_2}(x_2)] | 0 \rangle_c \\
 &= -i \frac{\delta^4 \ln Z' [J, \mathcal{I}, \bar{I}, I]}{\delta \bar{I}_{\beta_1}^{\rho_1}(y_1) \delta I_{\alpha_1}^{\sigma_1}(x_1) \delta \bar{I}_{\beta_2}^{\rho_2}(y_2) \delta I_{\alpha_2}^{\sigma_2}(x_2)} \Big|_{\text{external sources vanishing (e.s.v.)}} \\
 &= \int d^4 x' d^4 y' \left\{ \delta_{\alpha_2 \beta_2} [i\partial - M - \Pi_c]^{-1, \rho_2 \sigma'}(y_2, x') \frac{\delta \Pi_c^{\sigma' \rho'}(x', y')}{\delta \bar{I}_{\beta_1}^{\rho_1}(y_1) \delta I_{\alpha_1}^{\sigma_1}(x_1)} \Big|_{\text{e.s.v.}} [i\partial - M - \Pi_c]^{-1, \rho' \sigma_2}(y', x_2) \right. \\
 & \quad \left. - \delta_{\alpha_2 \beta_1} [i\partial - M - \Pi_c]^{-1, \rho_1 \sigma'}(y_1, x') \frac{\delta \Pi_c^{\sigma' \rho'}(x', y')}{\delta \bar{I}_{\beta_2}^{\rho_2}(y_2) \delta I_{\alpha_1}^{\sigma_1}(x_1)} \Big|_{\text{e.s.v.}} [i\partial - M - \Pi_c]^{-1, \rho' \sigma_2}(y', x_2) \right\}, \quad (30)
 \end{aligned}$$

where we have used the equations

$$\frac{\delta \Pi_c^{\sigma \rho}(x, y)}{\delta I_{\alpha}^{\sigma}(z)} \Big|_{\text{e.s.v.}} = \frac{\delta \Pi_c^{\sigma \rho}(x, y)}{\delta \bar{I}_{\alpha}^{\sigma}(z)} \Big|_{\text{e.s.v.}} = 0. \quad (31)$$

For convenience, we reduce the notation according to

$$\sigma_i \equiv (\sigma_i, x_i), \quad \rho_i \equiv (\rho_i, y_i) \quad (32)$$

$$\sigma'_i \equiv (\sigma'_i, z_i), \quad \rho'_i \equiv (\rho'_i, z'_i) \quad (33)$$

$$\frac{\delta \Pi_c^{\sigma \rho}(x, y)}{\delta \bar{I}_{\beta_1}^{\rho_1}(y_1) \delta I_{\alpha_1}^{\sigma_1}(x_1)} \Big|_{\text{e.s.v.}} \equiv \delta \Pi_{c \rho_1 \sigma_1, \beta_1 \alpha_1}^{\sigma \rho} \quad (34)$$

$$\frac{\delta \Phi_c^{\sigma \rho}(x, y)}{\delta \bar{I}_{\beta_1}^{\rho_1}(y_1) \delta I_{\alpha_1}^{\sigma_1}(x_1)} \Big|_{\text{e.s.v.}} \equiv \delta \Phi_{c \rho_1 \sigma_1, \beta_1 \alpha_1}^{\sigma \rho}. \quad (35)$$

With these simplified notations, Eq. (30) can be rewritten as

$$\begin{aligned}
 & \langle 0 | T [\psi_{\beta_1}^{\rho_1} \bar{\psi}_{\alpha_1}^{\sigma_1} \psi_{\beta_2}^{\rho_2} \bar{\psi}_{\alpha_2}^{\sigma_2}] | 0 \rangle_c = \delta_{\alpha_2 \beta_2} [i\partial - M - \Pi_c]^{-1, \rho_2, \sigma'} \delta \Pi_{c \rho_1 \sigma_1, \beta_1 \alpha_1}^{\sigma' \rho'} [i\partial - M - \Pi_c]^{-1, \rho' \sigma_2} \\
 & \quad - \delta_{\alpha_2 \beta_1} [i\partial - M - \Pi_c]^{-1, \rho_1, \sigma'} \delta \Pi_{c \rho_2 \sigma_1, \beta_2 \alpha_1}^{\sigma' \rho'} [i\partial - M - \Pi_c]^{-1, \rho' \sigma_2}. \quad (36)
 \end{aligned}$$

The exact four-point quark Green's function (without taking the large N_c limit) satisfies an inhomogeneous Bethe-Salpeter equation; however, the lhs of Eq. (36) is not the corresponding Green's function at the large N_c limit because it has four free color indices. Only after we extract the colorless part (for instance, timing $\delta_{\alpha_1 \beta_1} \delta_{\alpha_2 \beta_2}$ then summing over all the color indices) do we then obtain a meaningful Green's function under the large N_c limit. This colorless part of the Green's

function should satisfy an inhomogeneous Bethe-Salpeter equation. Since mesons only appear in color singlet channels, this is sufficient to derive the homogeneous BSE for mesons.

We may proceed by timing $\delta_{\alpha_1 \beta_1} \delta_{\alpha_2 \beta_2}$ then summing over all the color indices in Eq. (36); however, there is a simpler way for our purpose once we notice the relationship between the $\delta \Phi_{\rho_1 \sigma_1, \beta_1 \alpha_1}^{\sigma \rho}$ and the four-point quark Green's function. We have

$$\begin{aligned}\Phi_c^{\sigma\rho}[I, \bar{I}] &= \frac{-\delta_{\alpha\beta}}{N_c} \langle 0 | T[\psi_\beta^\rho \bar{\psi}_\alpha^\sigma] | 0 \rangle_f \\ &= \frac{-\delta_{\alpha\beta}}{N_c} \frac{\int [d\phi] \psi_\beta^\rho \bar{\psi}_\alpha^\sigma e^{iS[\phi, I, \bar{I}]}}{\int [d\phi] e^{iS[\phi, I, \bar{I}]}} ,\end{aligned}\quad (37)$$

where $\langle \dots \rangle_f$ denotes the full Green's function, which is different from the connected Green's function; $S[\phi, I, \bar{I}]$ is the traditional QCD action with ϕ denoting all the basic fields in the QCD Lagrangian (we have omitted the irrelevant external sources). Then,

$$\begin{aligned}\left. \frac{\delta \Phi_c^{\sigma\rho}}{\delta \bar{I}_{\beta_1}^{\rho_1} \delta I_{\alpha_1}^{\sigma_1}} \right|_{\bar{I}=I=0} &= \frac{-\delta_{\alpha\beta}}{N_c} \frac{\delta^2}{\delta \bar{I}_{\beta_1}^{\rho_1} \delta I_{\alpha_1}^{\sigma_1}} \left[\frac{\int [d\phi] \psi_\beta^\rho \bar{\psi}_\alpha^\sigma e^{iS[\phi, I, \bar{I}]}}{\int [d\phi] e^{iS[\phi, I, \bar{I}]}} \right] \Big|_{\bar{I}=I=0} \\ &= \frac{-\delta_{\alpha\beta}}{N_c} \left\{ \frac{1}{Z[0]} \int [d\phi] \psi_\beta^\rho \bar{\psi}_\alpha^\sigma i\psi_{\beta_1}^{\rho_1} (-i\bar{\psi}_{\alpha_1}^{\sigma_1}) e^{iS[\phi]} - \frac{\int [d\phi] \psi_\beta^\rho \bar{\psi}_\alpha^\sigma e^{iS[\phi]} \int [d\phi] i\psi_{\beta_1}^{\rho_1} (-i\bar{\psi}_{\alpha_1}^{\sigma_1}) e^{iS[\phi]}}{Z[0]} \right\} \\ &= \frac{-\delta_{\alpha\beta}}{N_c} \{ \langle 0 | T[\psi_\beta^\rho \bar{\psi}_\alpha^\sigma \psi_{\beta_1}^{\rho_1} \bar{\psi}_{\alpha_1}^{\sigma_1}] | 0 \rangle_f - \langle 0 | T[\psi_\beta^\rho \bar{\psi}_\alpha^\sigma] | 0 \rangle_f \langle 0 | T[\psi_{\beta_1}^{\rho_1} \bar{\psi}_{\alpha_1}^{\sigma_1}] | 0 \rangle_f \} .\end{aligned}\quad (38)$$

Since it is closely related to the four-point quark Green's function, it may satisfy an inhomogeneous Bethe-Salpeter equation, and we found this is indeed the case. Taking derivatives on Eq. (17) with respect to $\bar{I}I$, we obtain

$$\begin{aligned}0 &= N_c [i\not{\partial} - M - \Pi_c]^{-1, \rho, \sigma'} \delta \Pi_{c\rho_1\sigma_1\beta_1\alpha_1}^{\sigma\rho} [i\not{\partial} - M - \Pi_c]^{-1, \rho', \sigma} + \delta \Phi_{c\rho_1\sigma_1\beta_1\alpha_1}^{\sigma\rho} \\ &\quad + \delta_{\alpha_1\beta_1} [i\not{\partial} - M - \Pi_c]^{-1, \rho, \sigma_1} [i\not{\partial} - M - \Pi_c]^{-1, \rho_1, \sigma} .\end{aligned}\quad (39)$$

Similarly, from Eq. (18), we have

$$\delta \Pi_{c\rho_1\sigma_1\beta_1\alpha_1}^{\sigma\rho} = - \sum_{n=2}^{\infty} \frac{(-i)^n (N_c g^2)^{n-1}}{(n-2)!} \bar{G}_{\rho\rho_2\rho_3'\dots\rho_n'}^{\sigma\sigma_2\sigma_3'\dots\sigma_n'} \delta \Phi_{c\rho_1\sigma_1\beta_1\alpha_1}^{\sigma_2\rho_2} \Phi_c^{\sigma_3\rho_3'} \dots \Phi_c^{\sigma_n\rho_n'} .\quad (40)$$

Inserting Eq. (40) into Eq. (39), we immediately arrive at an inhomogeneous Bethe-Salpeter equation:

$$\begin{aligned}\delta \Phi_{c\rho_1\sigma_1\beta_1\alpha_1}^{\sigma\rho} &= i [i\not{\partial} - M - \Pi_c]^{-1, \rho, \sigma'} \sum_{n=2}^{\infty} \frac{(-i)^n (N_c g^2)^{n-1}}{(n-2)!} \bar{G}_{\rho'\rho_2\rho_3'\dots\rho_n'}^{\sigma\sigma_2\sigma_3'\dots\sigma_n'} \delta \Phi_{c\rho_1\sigma_1\beta_1\alpha_1}^{\sigma_2\rho_2} \Phi_c^{\sigma_3\rho_3'} \dots \Phi_c^{\sigma_n\rho_n'} [i\not{\partial} - M - \Pi_c]^{-1, \rho', \sigma} \\ &\quad - \frac{\delta_{\alpha_1\beta_1}}{N_c} [i\not{\partial} - M - \Pi_c]^{-1, \rho, \sigma_1} [i\not{\partial} - M - \Pi_c]^{-1, \rho_1, \sigma} .\end{aligned}\quad (41)$$

Now, we can use the standard technique to extract the pole contribution from $\delta \Phi_{c\rho_1\sigma_1\beta_1\alpha_1}^{\sigma\rho}$ and obtain the homogenous Bethe-Salpeter equation for mesons,

$$\begin{aligned}\chi_{P,S}^{\rho\sigma} &= i [i\not{\partial} - M - \Pi_c]^{-1, \rho, \sigma'} \sum_{n=2}^{\infty} \frac{(-i)^n (N_c g^2)^{n-1}}{(n-2)!} \bar{G}_{\rho'\rho_2\rho_3'\dots\rho_n'}^{\sigma\sigma_2\sigma_3'\dots\sigma_n'} \Phi_c^{\sigma_3\rho_3'} \dots \Phi_c^{\sigma_n\rho_n'} [i\not{\partial} - M - \Pi_c]^{-1, \rho', \sigma} \chi_{P,S}^{\rho_2\sigma_2} \\ &= -i [i\not{\partial} - M - \Pi_c]^{-1, \rho, \sigma'} K_{\sigma_2\rho_2}^{\sigma'\rho'} [i\not{\partial} - M - \Pi_c]^{-1, \rho', \sigma} \chi_{P,S}^{\rho_2\sigma_2} ,\end{aligned}\quad (42)$$

where the BS kernel $K_{\sigma_2\rho_2}^{\sigma'\rho'}$ is defined as

$$K_{\sigma_2\rho_2}^{\sigma'\rho'} \equiv - \sum_{n=2}^{\infty} \frac{(-i)^n (N_c g^2)^{n-1}}{(n-2)!} \bar{G}_{\rho'\rho_2\rho_3'\dots\rho_n'}^{\sigma\sigma_2\sigma_3'\dots\sigma_n'} \Phi_c^{\sigma_3\rho_3'} \dots \Phi_c^{\sigma_n\rho_n'} = \frac{\delta \Pi_c^{\sigma'\rho'}}{\delta \Phi_c^{\sigma_2\rho_2}} .\quad (43)$$

From Eq. (43), we see that the BS kernel can be obtained by breaking each of the quark propagators in the quark self-energy. Once a truncation is made in the gap equation, the BS kernel should be truncated accordingly. This relation is also shown in studies under symmetry preserving

truncations [13,14], so our method exemplifies the chiral symmetry preserving truncation scheme. The lowest order in this truncation scheme is the RL truncation. To go beyond the RL truncation, one can include the three-gluon Green's function contribution, which gives quark-gluon

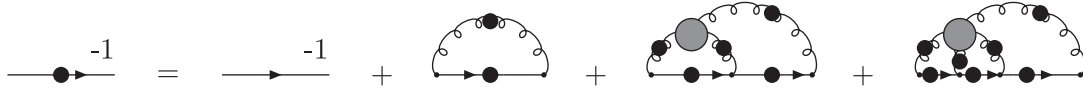


FIG. 1. The gap equation up to the NNL-order truncation. The black circles indicate the propagators are fully dressed. The gray circles are connected Green's functions. The dots are bare vertices.

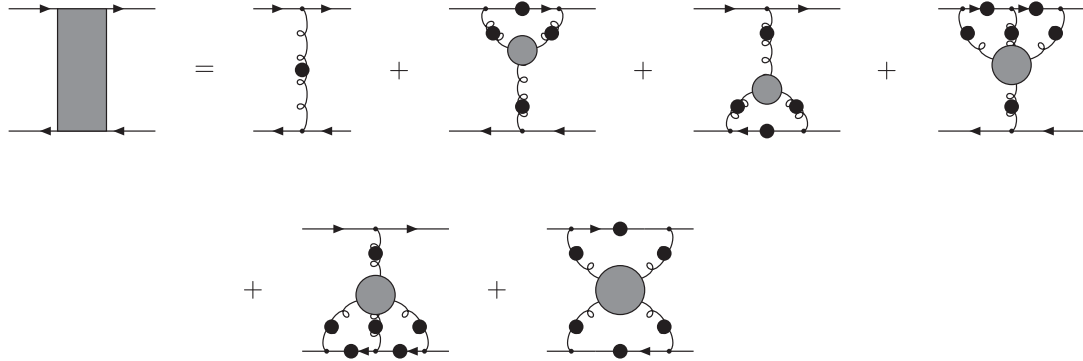


FIG. 2. The meson BS kernel up to the NNL-order truncation. The gray rectangle is the BS kernel. The black circles indicate the propagators are fully dressed. The gray circles are connected Green's functions. The dots are bare vertices.

vertex corrections in the gap equation and in the BSE. When the connected four-gluon Green's functions contributions are included, the gap equation still receives QGV corrections, while the BSE receives corrections due to H-shape diagrams, which may be important in scalar and axial-vector channels. The diagrammatic expressions for the gap equation and the BSE truncated up to the next-to-next-to-leading (NNL) order are shown in Figs. 1 and 2, respectively.

As indicated in the Introduction, RL truncation gives poor results in some channels, such as the scalar and axial-vector channels. The truncation scheme presented here allows us to make improvements in these channels. A few

works have studied the impacts of going beyond the RL truncation by taking into account the three-gluon self-interaction [15–17,23], which corresponds to the next-to-leading-order corrections in our scheme. It was found that at this order considerable corrections already appear in the scalar and axial-vector channels, and improvements were found in the axial-vector channel [15–17].

Our framework automatically provides symmetry preserving truncations. The reason can be understood if we write the meson BSE in another form. Actually, Eq. (42) can also be written in terms of derivatives of the effective action $\Gamma[\bar{\Phi}, \bar{\Pi}]$ as

$$\left\{ \bar{\Phi}^{-1, \rho_2 \sigma'} \frac{\delta^2 \Gamma}{\delta \bar{\Pi}^{\sigma' \rho'} \delta \bar{\Phi}^{\sigma_1 \rho_1}} \bar{\Phi}^{-1, \rho' \sigma_2} + i \frac{\delta^2 \Gamma}{\delta \bar{\Phi}^{\sigma_2 \rho_2} \delta \bar{\Phi}^{\sigma_1 \rho_1}} \right\} \Big|_{\bar{\Phi}=\Phi_c, \bar{\Pi}=\Pi_c} \chi_{P,S}^{\rho_2 \sigma_2} = 0, \quad (44)$$

where $\Gamma[\bar{\Phi}, \bar{\Pi}]$ is just $\frac{-i}{N_c} \ln Z' |_{\text{e.s.v.}}$ with Φ_c and Π_c replaced with arbitrary bilocal functions $\bar{\Phi}$ and $\bar{\Pi}$, respectively, i.e.,

$$\Gamma[\bar{\Phi}, \bar{\Pi}] = -i \text{Tr} \ln [i \not{\partial} - M - \bar{\Pi}] + \bar{\Phi}^{\sigma \rho} \bar{\Pi}^{\sigma \rho} + \sum_{n=2}^{\infty} \frac{(-i)^n (N_c g^2)^{n-1}}{n!} \bar{G}_{\rho_1 \dots \rho_n}^{\sigma_1 \dots \sigma_n} \bar{\Phi}^{\sigma_1 \rho_1} \dots \bar{\Phi}^{\sigma_n \rho_n}. \quad (45)$$

Equation (44) is just the analog of Eq. (3) in Ref. [21]. We have seen that the BSE and the DSEs can all be derived from the same generating functional, so symmetries respected by the generating functional will be automatically respected by the equations derived from it. Especially, truncations can be made in the generating functional $-i \ln Z'$, then all the equations derived from this generating functional will be consistent with each other and respect symmetries retained in the truncated generating functional. This is why we automatically have a chiral symmetry preserving truncation scheme.

For the phenomenological studies focusing on meson properties under RL truncation, the gluon propagator is usually treated as an input, i.e., given as a model, by fitting lattice results, etc.. It avoids complications in dealing with too many coupled equations and thus is very useful for practical use. In our approach, the gauge sector and fermion sector of QCD are treated differently at the very beginning, so it retains the merit just mentioned. In this respect, our approach can be viewed as an extension from considering only the gluon propagator's effects to considering higher-order gluon Green's functions' effects on the fermion

sector. It results in truncations beyond the RL approximation on one hand and completes the gauge sector because of the inclusion of the gluon self-interactions on the other.

All the higher-order terms in our truncation scheme originate from non-Abelian-type dynamics, i.e., gluon self-interactions and gluon-ghost interactions. Remember that we have taken the large N_c limit, which means that non-Abelian-type contributions are always dominant compared to Abelian-type contributions up to any order (beyond RL truncation).

V. SLAVNOV-TAYLOR IDENTITY FOR THE QUARK-GLUON VERTEX

The Slavnov-Taylor identity for the QGV provides an important constraint among the quark propagator, the QGV, and the quark-ghost scattering kernel, which reflects the

Becchi-Rouet-Stora (BRS) symmetry of QCD. In principle, a proper truncation should guarantee solutions of truncated DSEs satisfy STIs. However, it is not easy to maintain this requirement in practice. To see the explicit form of the STI of the QGV in our formalism, we derive the quark-ghost scattering kernel in this section. Now, we need to express the Fadeev-Popov determinant in terms of ghost fields $\bar{\phi}_i(x)$ and $\phi_i(x)$; then, we have a ghost term in the QCD Lagrangian:

$$\mathcal{L}_{\text{ghost}} = -(\partial_\mu \bar{\phi}^i) D_{ij}^\mu \phi^j. \quad (46)$$

Using the BRS symmetry of the theory, one can arrive at a STI relating the quark-gluon three-point Green's function to the quark antiquark-ghost antighost four-point Green's function [35],

$$0 = \omega \left\{ \frac{i}{2} g \lambda_{\alpha\gamma}^k \langle 0 | T [\phi_k(x) \psi_\gamma(x) \bar{\psi}_\beta(y) \bar{\phi}_i(z)] | 0 \rangle_f - \frac{i}{2} g \lambda_{\gamma\beta}^k \langle 0 | T [\psi_\alpha(x) \bar{\psi}_\gamma(y) \phi_k(y) \bar{\phi}_i(z)] | 0 \rangle_f + \frac{1}{\xi} \langle 0 | T [\psi_\alpha(x) \bar{\psi}_\beta(y) \partial_\mu A_i^\mu(z)] | 0 \rangle_f \right\}. \quad (47)$$

We shall see that this is just the STI for the QGV. Let us introduce the quark-ghost scattering kernels $H_{\alpha\beta}^i$ and $\bar{H}_{\alpha\beta}^i$ according to

$$H_{\alpha\gamma}^k(p, q, r) S_{\gamma\beta}(f) \tilde{D}_{k'i}(q^2) \equiv \frac{1}{2} \lambda_{\alpha\gamma}^k \int d^4x d^4y e^{-ip \cdot x} e^{-ir \cdot y} \langle 0 | T [\phi_k(x) \psi_\gamma(x) \bar{\psi}_\beta(y) \bar{\phi}_i(0)] | 0 \rangle_f \quad (48)$$

and

$$S_{\alpha\gamma}(-\not{p}) \bar{H}_{\gamma\beta}^k(-r, -q, -p) \tilde{D}_{k'i}(q^2) \equiv \frac{1}{2} \lambda_{\gamma\beta}^k \int d^4x d^4y e^{-ip \cdot x} e^{-ir \cdot y} \langle 0 | T [\psi_\alpha(x) \bar{\psi}_\gamma(y) \phi_k(y) \bar{\phi}_i(0)] | 0 \rangle_f, \quad (49)$$

where $q = -p - r$ and $\tilde{D}_{ij}(q^2)$ is the ghost propagator. With these definitions, Eq. (47) becomes (after taking Fourier transformation)

$$g H_{\alpha\gamma}^k(p, q, r) S_{\gamma\beta}(f) \tilde{D}_{k'i}(q^2) - g S_{\alpha\gamma}(-\not{p}) \bar{H}_{\gamma\beta}^k(-r, -q, -p) \tilde{D}_{k'i}(q^2) = \frac{1}{\xi} S_{\alpha\gamma}(-\not{p}) [-i g \Gamma_{\nu,\gamma\alpha}^j(q, p, r)] S_{\alpha\beta}(f) G_{ji}^{\nu\mu}(q) q_\mu. \quad (50)$$

Since for the gluon propagator $G_{ji}^{\nu\mu}(q)$ the nontransverse part equals that of a free gluon propagator, the equation can be reduced. After extracting the color structure, we obtain

$$S^{-1}(-\not{p}) H(p, q, r) - \bar{H}(-r, -q, -p) S^{-1}(-\not{f}) = -\frac{1}{q^2} q^\nu \Gamma_\nu(q, p, r) \tilde{D}^{-1}(q^2), \quad (51)$$

which is just the usual form of the STI of the QGV. In QED, ghost fields decouple from other fields, i.e., $H = \bar{H} = 1$ and $\tilde{D}^{-1}(q^2) = -q^2$, and then the corresponding Ward-Takahashi identity in QED follows:

$$S^{-1}(-\not{p}) - S^{-1}(-\not{f}) = q^\nu \Gamma_\nu(q, p, r). \quad (52)$$

Without taking the large N_c limit, the Slavnov-Taylor identities are of course satisfied. Since the BRS symmetry is independent of the $SU(N_c)$ group parameter N_c , the Slavnov-Taylor identities should still hold at the large N_c limit. So, in our formalism, the solutions of the gap equation, the QGV DSE, and the quark-ghost scattering kernel (untruncated equations) satisfy Eq. (51). To give an explicit form of the quark-ghost scattering kernel, or equivalently the quark antiquark-ghost antighost four-point Green's function, we need to introduce external

sources for ghost fields. The appearance of these external sources will largely complicate the form of the generating functional. We put these materials in Appendix B. After a lengthy amount of work, we obtain the generating functional with ghost external sources (up to $\bar{\eta}\eta$ order) at the large N_c limit as shown in Eq. (B16).

Now, the quark antiquark-ghost antighost four-point Green's function can be derived directly by calculating

$\frac{-i\delta^4 \ln Z'}{\delta\bar{\eta}^j \delta(-\eta^i) \delta\bar{l} \delta(-l_\alpha)}$. Eventually, we obtain

$$\begin{aligned} & \langle 0|T[\phi_j(\tilde{y})\bar{\phi}_i(\tilde{x})\psi_\rho^\rho(y)\bar{\psi}_\alpha^\sigma(x)]|0\rangle_c \\ &= \int d^4x' d^4y' \left[-(i\partial - M - \Pi_c)_{\rho\rho'}^{-1}(y, y') \frac{\delta^2 \Pi_c^{\rho'\sigma'}(y', x')}{\delta\bar{\eta}^j(\tilde{y})\delta(-\eta^i(\tilde{x}))} (i\partial - M - \Pi_c)_{\sigma'\sigma}^{-1}(x', x) \right] \delta_{\alpha\beta} \\ &+ \int d^4x' d^4y' [(i\partial - M - \Pi_c)_{\rho\rho'}^{-1}(y, y') (\Delta_{\Phi_c}^{b\rho'\sigma'}(\tilde{x}, \tilde{y}; y', x') [\lambda^i \lambda^j]_{\beta\alpha} \\ &+ \Delta_{\Phi_c}^{c\rho'\sigma'}(\tilde{x}, \tilde{y}; y', x') [\lambda^j \lambda^i]_{\beta\alpha}) (i\partial - M - \Pi_c)_{\sigma'\sigma}^{-1}(x', x)]. \end{aligned} \quad (53)$$

The full Green's function is

$$\begin{aligned} & \langle 0|T[\phi_j(\tilde{y})\bar{\phi}_i(\tilde{x})\psi_\rho^\rho(y)\bar{\psi}_\alpha^\sigma(x)]|0\rangle_f \\ &= i\langle 0|T[\phi_j(\tilde{y})\bar{\phi}_i(\tilde{x})\psi_\rho^\rho(y)\bar{\psi}_\alpha^\sigma(x)]|0\rangle_c - \langle 0|T[\psi_\rho^\rho(y)\bar{\psi}_\alpha^\sigma(x)]|0\rangle_c \langle 0|T[\phi_j(\tilde{y})\bar{\phi}_i(\tilde{x})]|0\rangle_c \\ &= \int d^4x' d^4y' \left[-(i\partial - M - \Pi_c)_{\rho\rho'}^{-1}(y, y') \frac{i\delta^2 \Pi_c^{\rho'\sigma'}(y', x')}{\delta\bar{\eta}^j(\tilde{y})\delta(-\eta^i(\tilde{x}))} (i\partial - M - \Pi_c)_{\sigma'\sigma}^{-1}(x', x) \right] \delta_{\alpha\beta} \\ &+ \int d^4x' d^4y' [(i\partial - M - \Pi_c)_{\rho\rho'}^{-1}(y, y') (i\Delta_{\Phi_c}^{b\rho'\sigma'}(\tilde{x}, \tilde{y}; y', x') [\lambda^i \lambda^j]_{\beta\alpha} \\ &+ i\Delta_{\Phi_c}^{c\rho'\sigma'}(\tilde{x}, \tilde{y}; y', x') [\lambda^j \lambda^i]_{\beta\alpha}) (i\partial - M - \Pi_c)_{\sigma'\sigma}^{-1}(x', x)] \\ &- \delta_{\alpha\beta} \delta_{ij} (i\partial - M - \Pi_c)_{\rho\sigma}^{-1}(y, x) \tilde{D}(\tilde{x}, \tilde{y}). \end{aligned} \quad (54)$$

Taking Eqs. (24) and (54) into Eq. (47), we obtain

$$\begin{aligned} & \lambda_{\alpha\beta}^i \left(\frac{ig}{2} \right) \int d^4x' d^4y' (i\partial - M - \Pi_c)_{\rho\rho'}^{-1}(x, y') \left\{ -\frac{i\delta^2 \Pi_c^{\rho'\sigma'}(y', x')}{\delta\bar{\eta}^j(x)\delta(-\eta^i(z))} + \frac{i\delta^2 \Pi_c^{\rho'\sigma'}(y', x')}{\delta\bar{\eta}^j(y)\delta(-\eta^i(z))} \right. \\ &+ 2N_c (i\Delta_{\Phi_c}^{c\rho'\sigma'}(z, x; y', x') - i\Delta_{\Phi_c}^{b\rho'\sigma'}(z, y; y', x')) \\ &+ (i\partial - M - \Pi_c)_{\rho'\sigma'}(y', x') (\tilde{D}(z, x) - \tilde{D}(z, y)) \left. \right\} (i\partial - M - \Pi_c)_{\sigma'\sigma}^{-1}(x', y) \\ &= \lambda_{\alpha\beta}^i \int d^4x' d^4y' (i\partial - M - \Pi_c)_{\rho\rho'}^{-1}(x, y') \left[-\frac{1}{\xi} \frac{\partial}{\partial z^\mu} \Delta_{\Phi_c}^{\mu\rho'\sigma'}(z, y', x') \right] (i\partial - M - \Pi_c)_{\sigma'\sigma}^{-1}(x', y). \end{aligned} \quad (55)$$

This is the STI for the QGV in our formalism.

The undetermined function $\frac{\delta^2 \Pi_c^{\rho'\sigma'}(y', x')}{\delta\bar{\eta}^j(x)\delta(-\eta^i(z))}$ satisfies coupled integral equations, which, in a compact form, reads

$$\begin{aligned} \frac{\delta^2 \Pi_c}{\delta\bar{\eta}^j \delta(-\eta^i)} &= i \frac{\delta_{ij}}{2N_c} \text{Tr} \left[(i\partial - M - \Pi_c)^{-1} \left(\frac{\delta \Delta_{\Phi_c}^b}{\delta \Phi_c} + \frac{\delta \Delta_{\Phi_c}^c}{\delta \Phi_c} \right) \right] \\ &- \frac{\delta_{ij}}{N_c} \frac{\delta \Delta_{\Phi_c}^a}{\delta \Phi_c} - \frac{\delta \Delta_{\Phi_c}^d}{N_c \delta \Phi_c} \langle \bar{\psi} \lambda^i \psi \rangle \langle \bar{\psi} \lambda^j \psi \rangle + \sum_{n=2}^{\infty} \frac{(-i)^n (N_c g^2)^{n-1}}{(n-2)!} \bar{G}_{n-1} \frac{\delta^2 \Phi_c}{\delta\bar{\eta}^j \delta(-\eta^i)} \Phi_c \cdots \Phi_c, \end{aligned} \quad (56)$$

$$\begin{aligned} \frac{\delta^2 \Phi_c}{\delta \bar{\eta}^j \delta(-\eta^i)} &= -i(i\partial - M - \Pi_c)^{-1} \frac{\delta^2 \Pi_c}{\delta \bar{\eta}^j \delta(-\eta^i)} (i\partial - M - \Pi_c)^{-1} \\ &+ i \frac{\delta_{ij}}{2N_c} \text{Tr}[(i\partial - M - \Pi_c)^{-1} (\Delta_{\Phi_c}^b + \Delta_{\Phi_c}^c)]. \end{aligned} \quad (57)$$

$\Delta_{\Phi_c}^a, \Delta_{\Phi_c}^b, \Delta_{\Phi_c}^c$, and $\Delta_{\Phi_c}^d$ on the lhs of Eq. (55) are integrations of two-ghost+n-gluon Green's functions, while $\Delta_{\Phi_c}^u$ on the rhs of Eq. (55) is an integration of n-gluon Green's functions. These Green's functions are not independent. Actually, they satisfy their own DSEs and STIs derived from the pure Yang-Mills part of the theory.

We have given the expression of the quark antiquark-ghost antighost four-point Green's function [Eq. (54)], with which the quark-ghost scattering kernel is defined, and the STI for the quark-gluon vertex in coordination space [Eq. (55)] in our formulism. As discussed before, the solutions of untruncated DSEs at the large N_c limit should satisfy Eq. (55). A more interesting case arises when a truncation is made in the DSEs. The simplest truncation is the Abelian approximation (rainbow approximation). In this case, the ghost fields can decouple from other fields, so the QGV STI reduces to the WTI, and one can check that the gauge-fermion vertex WTI is not satisfied under the rainbow approximation. In principle, the STI for the QGV at higher-order truncations can be tested using Eq. (55). Unfortunately, we cannot do it due to the complicated forms of relevant Green's functions. However, we hope the discussion in this section can reveal some hints in the studies of the QGV's STI.

VI. SUMMARY

We introduced an alternate form for the QCD generating functional, which is a generalization of the one used in a previous study. This form has the power to address various nonperturbative problems in QCD. Specifically, we employed it to study Dyson-Schwinger equations and Bethe-Salpeter equation and their truncations. The large N_c expansion was taken, and we concentrated on the large N_c limit. Under the large N_c limit, a systematic order-by-order truncation scheme with all higher-order terms explicitly given was proposed. One benefit of this framework is that one can make truncations in the generating functional from which all the equations are derived. So, any linearly presented symmetry preserved in the truncated generating functional can automatically transmit to the DSEs

and the BSE. To be specific, truncations can be made by keeping finite terms in the expansion of the generating functional with respect to the number of external legs of connected gluon Green's functions. With any such truncation, the chiral symmetry is conserved (in the chiral limit). So, the truncated DSEs and BSE preserve chiral symmetry, and the truncation scheme proposed here is actually a symmetry preserving truncation scheme. Another benefit is that our truncation scheme avoids ambiguities appearing in the methods making direct use of the QGV DSE.

Since the explicit forms of the DSEs and the BSE are established at the large N_c limit, the integration kernels suffer from corrections of $1/N_c$ order. The positive side of taking the large N_c limit is that all terms in the integration kernels making corrections to the RL truncation are non-Abelian type, i.e., they have no counterparts in QED, which means that non-Abelian-type contributions are always dominant compared to Abelian-type contributions up to any order (beyond RL truncation), and it is especially useful for testing non-Abelian dynamics. In this framework, the H-shape diagram in the BSE kernel, which is considered important in the scalar and axial-vector channels, will appear when connected four-gluon Green's functions are present. Meanwhile, the four-gluon self-interaction appears at the same order, so truncation made up to this order (i.e., keep the gluon propagator term, the three-gluon self-interaction term, and the connected four-gluon Green's functions term in the kernels) is of great interest.

To study the QGV STI, we derived the quark-ghost scattering kernel. Ghost fields are explicitly shown in the generating functional, and the external sources are introduced. Although we cannot verify the STI directly due to the complicated form of the quark-ghost scattering kernel, we hope those discussions could shed some light on further studies and on the modeling of the QGV using the STI.

ACKNOWLEDGMENTS

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APPENDIX A: FIERZ REORDERING

In this Appendix, we give a detailed discussion on the Fierz reordering. To obtain Eq. (7), we note that

$$\begin{aligned}
& G_{\mu_1\mu_2}^{i_1i_2}(x_1, x_2) \left[-g\bar{\psi}_{\alpha_1}^{a_1}(x_1) \left(\frac{\lambda_{i_1}}{2}\right)_{\alpha_1\beta_1} \gamma^{\mu_1} \psi_{\beta_1}^{a_1}(x_1) \right] \left[-g\bar{\psi}_{\alpha_2}^{a_2}(x_2) \left(\frac{\lambda_{i_2}}{2}\right)_{\alpha_2\beta_2} \gamma^{\mu_2} \psi_{\beta_2}^{a_2}(x_2) \right] \\
&= G_{\mu_1\mu_2}(x_1, x_2) [-g\bar{\psi}_{\alpha_1}^{a_1}(x_1) \gamma^{\mu_1} \psi_{\beta_1}^{a_1}(x_1)] [-g\bar{\psi}_{\alpha_2}^{a_2}(x_2) \gamma^{\mu_2} \psi_{\beta_2}^{a_2}(x_2)] \frac{1}{2} \left(\delta_{\alpha_1\beta_2} \delta_{\alpha_2\beta_1} - \frac{1}{N_c} \delta_{\alpha_1\beta_1} \delta_{\alpha_2\beta_2} \right) \\
&= \int d^4x'_1 d^4x'_2 g^2 G_{\mu_1\mu_2}(x_1, x_2) \left[-\frac{1}{2} (\gamma^{\mu_1})_{\sigma_1\rho_2} (\gamma^{\mu_2})_{\sigma_2\rho_1} \delta(x'_1 - x_2) \delta(x'_2 - x_1) \right. \\
&\quad \left. - \frac{1}{2N_c} (\gamma^{\mu_1})_{\sigma_1\rho_1} (\gamma^{\mu_2})_{\sigma_2\rho_2} \delta(x'_1 - x_1) \delta(x'_2 - x_2) \right] \bar{\psi}_{\alpha_1}^{\sigma_1}(x_1) \psi_{\alpha_1}^{\rho_1}(x'_1) \bar{\psi}_{\alpha_2}^{\sigma_2}(x_2) \psi_{\alpha_2}^{\rho_2}(x'_2). \tag{A1}
\end{aligned}$$

In obtaining the above result, we have used the relations

$$G_{\mu_1\mu_2}^{i_1i_2}(x_1, x_2) = \delta^{i_1i_2} G_{\mu_1\mu_2}(x_1, x_2), \tag{A2}$$

$$\left(\frac{\lambda_i}{2}\right)_{\alpha_1\beta_1} \left(\frac{\lambda_i}{2}\right)_{\alpha_2\beta_2} = \frac{1}{2} \left(\delta_{\alpha_1\beta_2} \delta_{\alpha_2\beta_1} - \frac{1}{N_c} \delta_{\alpha_1\beta_1} \delta_{\alpha_2\beta_2} \right). \tag{A3}$$

Hence, the extended two-point Green's function is

$$\begin{aligned}
\bar{G}_{\rho_1\rho_2}^{\sigma_1\sigma_2}(x_1, x'_1, x_2, x'_2) &= -\frac{1}{2} G_{\mu_1\mu_2}(x_1, x_2) \left[(\gamma^{\mu_1})_{\sigma_1\rho_2} (\gamma^{\mu_2})_{\sigma_2\rho_1} \delta(x'_1 - x_2) \delta(x'_2 - x_1) \right. \\
&\quad \left. + \frac{1}{N_c} (\gamma^{\mu_1})_{\sigma_1\rho_1} (\gamma^{\mu_2})_{\sigma_2\rho_2} \delta(x'_1 - x_1) \delta(x'_2 - x_2) \right]. \tag{A4}
\end{aligned}$$

For the three-point functions, we need the $SU(N_c)$ relations

$$\begin{aligned}
f_{ijk} &= -2i \left(\left[\frac{\lambda_i}{2}, \frac{\lambda_j}{2} \right] \frac{\lambda_k}{2} \right) = \frac{i}{4} (\lambda_j)_{\alpha\beta} (\lambda_i)_{\beta\gamma} (\lambda_k)_{\gamma\alpha} - \frac{i}{4} (\lambda_i)_{\alpha\beta} (\lambda_j)_{\beta\gamma} (\lambda_k)_{\gamma\alpha} \\
d_{ijk} &= -2i \left(\left\{ \frac{\lambda_i}{2}, \frac{\lambda_j}{2} \right\} \frac{\lambda_k}{2} \right) = \frac{1}{4} (\lambda_j)_{\alpha\beta} (\lambda_i)_{\beta\gamma} (\lambda_k)_{\gamma\alpha} + \frac{1}{4} (\lambda_i)_{\alpha\beta} (\lambda_j)_{\beta\gamma} (\lambda_k)_{\gamma\alpha}, \tag{A5}
\end{aligned}$$

then

$$f_{ijk}(\lambda_j)_{\alpha_2\beta_2} (\lambda_k)_{\alpha_3\beta_3} = i[\delta_{\alpha_3\beta_2} (\lambda_i)_{\alpha_2\beta_3} - \delta_{\alpha_2\beta_3} (\lambda_i)_{\alpha_3\beta_2}] \tag{A6}$$

$$d_{ijk}(\lambda_j)_{\alpha_2\beta_2} (\lambda_k)_{\alpha_3\beta_3} = \delta_{\alpha_3\beta_2} (\lambda_i)_{\alpha_2\beta_3} + \delta_{\alpha_2\beta_3} (\lambda_i)_{\alpha_3\beta_2} - \frac{2}{N_c} \delta_{\alpha_3\beta_3} (\lambda_i)_{\alpha_2\beta_2} - \frac{2}{N_c} \delta_{\alpha_2\beta_2} (\lambda_i)_{\alpha_3\beta_3} \tag{A7}$$

and

$$f_{ijk}(\lambda_i)_{\alpha_1\beta_1} (\lambda_j)_{\alpha_2\beta_2} (\lambda_k)_{\alpha_3\beta_3} = 2i(\delta_{\alpha_3\beta_2} \delta_{\alpha_2\beta_1} \delta_{\alpha_1\beta_3} - \delta_{\alpha_3\beta_1} \delta_{\alpha_1\beta_2} \delta_{\alpha_2\beta_3}) \tag{A8}$$

$$\begin{aligned}
d_{ijk}(\lambda_i)_{\alpha_1\beta_1} (\lambda_j)_{\alpha_2\beta_2} (\lambda_k)_{\alpha_3\beta_3} &= 2(\delta_{\alpha_3\beta_2} \delta_{\alpha_2\beta_1} \delta_{\alpha_1\beta_3} + \delta_{\alpha_3\beta_1} \delta_{\alpha_1\beta_2} \delta_{\alpha_2\beta_3}) \\
&\quad - \frac{2}{N_c} \delta_{\alpha_1\beta_1} \delta_{\alpha_2\beta_3} \delta_{\alpha_3\beta_2} - \frac{2}{N_c} \delta_{\alpha_1\beta_3} \delta_{\alpha_2\beta_2} \delta_{\alpha_3\beta_1} - \frac{2}{N_c} \delta_{\alpha_1\beta_2} \delta_{\alpha_2\beta_1} \delta_{\alpha_3\beta_3} + \frac{4}{N_c^2} \delta_{\alpha_1\beta_1} \delta_{\alpha_2\beta_2} \delta_{\alpha_3\beta_3}. \tag{A9}
\end{aligned}$$

The three-point Green's function can be written as

$$G_{\mu_1\mu_2\mu_3}^{i_1i_2i_3}(x_1, x_2, x_3) = gf_{i_1i_2i_3} G_{\mu_1\mu_2\mu_3}^{(0)}(x_1, x_2, x_3) + gd_{i_1i_2i_3} G_{\mu_1\mu_2\mu_3}^{(1)}(x_1, x_2, x_3). \tag{A10}$$

Combining them together, we get

$$\begin{aligned}
& \tilde{G}_{\rho_1\rho_2\rho_3}^{\sigma_1\sigma_2\sigma_3}(x_1, x'_1, x_2, x'_2, x_3, x'_3) \\
&= \frac{i}{4} G_{\mu_1\mu_2\mu_3}^{(0)}(x_1, x_2, x_3) [\gamma_{\sigma_1\rho_1}^{\mu_1} \gamma_{\sigma_2\rho_2}^{\mu_2} \gamma_{\sigma_3\rho_3}^{\mu_3} \delta(x'_1 - x_3) \delta(x'_2 - x_1) \delta(x'_3 - x_2) - \gamma_{\sigma_1\rho_3}^{\mu_1} \gamma_{\sigma_2\rho_1}^{\mu_2} \gamma_{\sigma_3\rho_2}^{\mu_3} \delta(x'_1 - x_2) \delta(x'_2 - x_3) \delta(x'_3 - x_1)] \\
&+ \frac{1}{4} G_{\mu_1\mu_2\mu_3}^{(1)}(x_1, x_2, x_3) \left[\gamma_{\sigma_1\rho_3}^{\mu_1} \gamma_{\sigma_2\rho_1}^{\mu_2} \gamma_{\sigma_3\rho_2}^{\mu_3} \delta(x'_1 - x_2) \delta(x'_2 - x_3) \delta(x'_3 - x_1) + \gamma_{\sigma_1\rho_2}^{\mu_1} \gamma_{\sigma_2\rho_3}^{\mu_2} \gamma_{\sigma_3\rho_1}^{\mu_3} \delta(x'_1 - x_3) \delta(x'_2 - x_1) \delta(x'_3 - x_2) \right. \\
&+ \frac{2}{N_c} \gamma_{\sigma_1\rho_1}^{\mu_1} \gamma_{\sigma_2\rho_3}^{\mu_2} \gamma_{\sigma_3\rho_2}^{\mu_3} \delta(x'_1 - x_1) \delta(x'_2 - x_3) \delta(x'_3 - x_2) + \frac{2}{N_c} \gamma_{\sigma_1\rho_3}^{\mu_1} \gamma_{\sigma_2\rho_2}^{\mu_2} \gamma_{\sigma_3\rho_1}^{\mu_3} \delta(x'_1 - x_3) \delta(x'_2 - x_2) \delta(x'_3 - x_1) \\
&\left. + \frac{2}{N_c} \gamma_{\sigma_1\rho_2}^{\mu_1} \gamma_{\sigma_2\rho_1}^{\mu_2} \gamma_{\sigma_3\rho_3}^{\mu_3} \delta(x'_1 - x_2) \delta(x'_2 - x_1) \delta(x'_3 - x_3) + \frac{4}{N_c^2} \gamma_{\sigma_1\rho_1}^{\mu_1} \gamma_{\sigma_2\rho_2}^{\mu_2} \gamma_{\sigma_3\rho_3}^{\mu_3} \delta(x'_1 - x_1) \delta(x'_2 - x_2) \delta(x'_3 - x_3) \right]. \quad (A11)
\end{aligned}$$

In general, $\tilde{G}_{\rho_1 \dots \rho_n}^{\sigma_1 \dots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n)$, has following properties:

- (i) The basic term is $\gamma_{\sigma_1\rho_1}^{\mu_1} \delta(x_1 - x'_1) \dots \gamma_{\sigma_n\rho_n}^{\mu_n} \delta(x_n - x'_n)$ with some nonlocal coefficient that depends on indices μ_1, \dots, μ_n and space-time coordinates x_1, \dots, x_n .
- (ii) Different terms correspond to different arrangements of number $1, \dots, n$ to i_1, \dots, i_n .
- (iii) There is no $\gamma_{ii}^{\mu} \delta(x_i - x'_i)$ factor.

For the extended Green's functions \tilde{G}_μ 's, the two-point and three-point Green's functions are, respectively,

$$\begin{aligned}
& \tilde{G}_{\mu_1, \rho}^{\sigma}(x_1, x'_1, x_2, x'_2) \\
&= \frac{1}{2} \gamma_{\sigma\rho}^{\mu_2} G_{\mu_1\mu_2}(x_1, x_2) \delta(x'_1 - x_2) \delta(x'_2 - x_2) \quad (A12)
\end{aligned}$$

and

$$\begin{aligned}
& \tilde{G}_{\mu_1, \rho\rho_3}^{\sigma\sigma_3}(x_1, x'_1, x_2, x'_2, x_3, x'_3) \\
&= \frac{1}{4} \left[i G_{\mu_1\mu_2\mu_3}^{(0)}(x_1, x_2, x_3) (-\gamma_{\sigma_1\rho_3}^{\mu_2} \gamma_{\sigma_3\rho}^{\mu_3} \delta(x'_1 - x_2) \delta(x'_2 - x_3) \delta(x'_3 - x_2) + \gamma_{\sigma_3\rho}^{\mu_2} \gamma_{\sigma_1\rho_3}^{\mu_3} \delta(x'_1 - x_3) \delta(x'_2 - x_2) \delta(x'_3 - x_3)) \right. \\
&+ G_{\mu_1\mu_2\mu_3}^{(1)}(x_1, x_2, x_3) (-\gamma_{\sigma_1\rho_3}^{\mu_2} \gamma_{\sigma_3\rho}^{\mu_3} \delta(x'_1 - x_2) \delta(x'_2 - x_3) \delta(x'_3 - x_2) - \gamma_{\sigma_3\rho}^{\mu_2} \gamma_{\sigma_1\rho_3}^{\mu_3} \delta(x'_1 - x_3) \delta(x'_2 - x_2) \delta(x'_3 - x_3) \\
&\left. - \frac{4}{N_c} \gamma_{\sigma\rho}^{\mu_2} \gamma_{\sigma_3\rho_3}^{\mu_3} \delta(x'_1 - x_2) \delta(x'_2 - x_2) \delta(x'_3 - x_3)) \right]. \quad (A13)
\end{aligned}$$

APPENDIX B: THE GENERATING FUNCTIONAL WITH GHOST EXTERNAL SOURCES

We rewrite the original generating functional with ghost fields written explicitly and introducing external sources for them:

$$\begin{aligned}
& Z[J, \mathcal{I}, \bar{I}, I, \bar{\eta}^i, \eta^i] \\
&= \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left\{ i \int d^4x \{ \bar{\psi} (i\partial + J) \psi + \bar{I} \psi + \bar{\psi} I \} \right\} \\
&\times \int \mathcal{D}A_\mu \mathcal{D}\phi \mathcal{D}\bar{\phi} \exp \left\{ i \int d^4x \left[\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 - (\partial_\mu \bar{\phi}^i) D_{ij}^\mu \phi^j + \mathcal{I}_i^\mu A_\mu^i + \bar{\eta}^i \phi^i + \bar{\phi}^i \eta^i \right] \right\}. \quad (B1)
\end{aligned}$$

Formally integrating out the gauge fields and ghost fields, we obtain

$$\begin{aligned}
& \int \mathcal{D}A_\mu \mathcal{D}\phi \mathcal{D}\bar{\phi} \exp \left\{ i \int d^4x \left[\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 - (\partial_\mu \bar{\phi}^i) D_{ij}^\mu \phi^j + \mathcal{I}_i^\mu A_\mu^i + \bar{\eta}^i \phi^i + \bar{\phi}^i \eta^i \right] \right\} \\
&= \exp i \sum_{n=2}^{\infty} \int d^4x_1 \dots d^4x_n \frac{i^n}{n!} G_{\mu_1 \dots \mu_n}^{i_1 \dots i_n}(x_1, \dots, x_n) [\eta^i, \bar{\eta}^j] \mathcal{I}_{i_1}^{\mu_1}(x_1) \dots \mathcal{I}_{i_n}^{\mu_n}(x_n), \quad (B2)
\end{aligned}$$

where $G_{\mu_1 \dots \mu_n}^{i_1 \dots i_n}(x_1, \dots, x_n) [\eta^i, \bar{\eta}^j]$ is connected n-point gluon Green's function depending on $\eta^i, \bar{\eta}^j$. Then, the color structure of these Green's functions involves $\eta^i, \bar{\eta}^j$. For example, the two-point Green's function $G_{\mu\nu}^{ij}$ is no longer proportional to δ^{ij} ; it also includes terms proportional to $\eta^i \bar{\eta}^j$.

Note that the derivation of the STI for the quark-gluon vertex only needs a bilinear term of ghost sources; therefore, we do not need to give the general expression of (B2), and instead we only keep those terms with ghost sources at most bilinear. In this simplified case, the color structure of $G_{\mu_1 \dots \mu_n}^{i_1 \dots i_n}(x_1, \dots, x_n)[\eta^i, \bar{\eta}^j]$ can be explicitly figured out:

$$\begin{aligned}
& \int \mathcal{D}A_\mu \mathcal{D}\phi \mathcal{D}\bar{\phi} \exp \left\{ i \int d^4x \left[\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 - (\partial_\mu \bar{\phi}^i) D_{ij}^\mu \phi^j + \mathcal{I}_i^\mu A_\mu^i + \bar{\eta}^i \phi^i + \bar{\phi}^i \eta^i \right] \right\} \\
&= \int \mathcal{D}A_\mu \Delta_F(A_\mu) \exp \left\{ i \int d^4x \left[\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 + \mathcal{I}_i^\mu A_\mu^i \right] - \bar{\eta} (\partial_\mu D^\mu)^{-1} \eta \right\} \\
&= \int \mathcal{D}A_\mu \Delta_F(A_\mu) \exp \left[i \int d^4x \left(\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 + \mathcal{I}_i^\mu A_\mu^i \right) \right] \\
&\quad \times \left\{ 1 - \frac{\int d^4x d^4y \bar{\eta}^i(x) \eta^j(y) \int \mathcal{D}A_\mu \Delta_F(A_\mu) (\partial_\mu D^\mu)^{-1, ij}(x, y) \exp[i \int d^4x (\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 + \mathcal{I}_i^\mu A_\mu^i)]}{\int \mathcal{D}A_\mu \Delta_F(A_\mu) \exp[i \int d^4x (\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 + \mathcal{I}_i^\mu A_\mu^i)]} + O((\eta \bar{\eta})^2) \right\} \\
&= \int \mathcal{D}A_\mu \Delta_F(A_\mu) \exp \left[i \int d^4x \left(\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 + \mathcal{I}_i^\mu A_\mu^i \right) \right] \\
&\quad \times \exp \left\{ - \frac{\int d^4x d^4y \bar{\eta}^i(x) \eta^j(y) \int \mathcal{D}A_\mu \Delta_F(A_\mu) (\partial_\mu D^\mu)^{-1, ij}(x, y) \exp[i \int d^4x (\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 + \mathcal{I}_i^\mu A_\mu^i)]}{\int \mathcal{D}A_\mu \Delta_F(A_\mu) \exp[i \int d^4x (\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 + \mathcal{I}_i^\mu A_\mu^i)]} + O((\eta \bar{\eta})^2) \right\}. \tag{B3}
\end{aligned}$$

We define

$$\begin{aligned}
& \frac{\int d^4x d^4y \bar{\eta}^i(x) \eta^j(y) \int \mathcal{D}A_\mu \Delta_F(A_\mu) (\partial_\mu D^\mu)^{-1, ij}(x, y) \exp[i \int d^4x (\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 + \mathcal{I}_i^\mu A_\mu^i)]}{\int \mathcal{D}A_\mu \Delta_F(A_\mu) \exp[i \int d^4x (\mathcal{L}_G(A) - \frac{1}{2\xi} [F^i(A_\mu)]^2 + \mathcal{I}_i^\mu A_\mu^i)]} \\
&= i \sum_{n=0}^{\infty} \int d^4x d^4y d^4x_1 \dots d^4x_n \frac{i^n}{n!} \mathcal{G}_{\mu_1 \dots \mu_n}^{(n) i, j; i_1 \dots i_n}(x, y; x_1, \dots, x_n) \bar{\eta}^i(x) \eta^j(y) \mathcal{I}_{i_1}^{\mu_1}(x_1) \dots \mathcal{I}_{i_n}^{\mu_n}(x_n), \tag{B4}
\end{aligned}$$

where $\mathcal{G}^{(n)}$ is just the connected ghost-antighost+n-gluon Green's function. Since $\mathcal{I}_i^\mu \equiv \mathcal{I}_i^\mu - g\bar{\psi} \frac{\lambda_i}{2} \gamma^\mu \psi$, we can expand the \mathcal{I} 's at each term and obtain terms of \mathcal{I}^0 order, \mathcal{I}^1 order, \dots , \mathcal{I}^n order. For our purpose, we only need to give explicit expressions of the \mathcal{I}^0 -order terms, which read

$$i \sum_{n=0}^{\infty} \int d^4x d^4y d^4x_1 \dots d^4x_n \frac{i^n}{n!} \mathcal{G}_{\mu_1 \dots \mu_n}^{(n) i, j; i_1 \dots i_n}(x, y; x_1, \dots, x_n) \bar{\eta}^i(x) \eta^j(y) \left(-g\bar{\psi} \frac{\lambda_{i_1}}{2} \gamma^{\mu_1} \psi(x_1) \right) \dots \left(-g\bar{\psi} \frac{\lambda_{i_n}}{2} \gamma^{\mu_n} \psi(x_n) \right). \tag{B5}$$

To do Fierz rearrangement, we need to show color factors explicitly. In general, terms including $\mathcal{G}^{(2)}$ have two free color indices of the adjoint representation i, j and four free color indices of the fundamental representation $\alpha, \beta, \delta, \gamma$, so the most general forms of color factors include

$$\delta_{ij} \delta_{\alpha\beta} \delta_{\delta\gamma}, \quad \delta_{\alpha\beta} [\lambda^i \lambda^j]_{\delta\gamma}, \quad \delta_{\alpha\beta} [\lambda^j \lambda^i]_{\delta\gamma}, \quad \lambda_{\alpha\beta}^i \lambda_{\delta\gamma}^j. \tag{B6}$$

For higher-order $\mathcal{G}^{(n)}$'s ($n > 2$), the forms of color factors would be different from them by multiplying $\delta_{\alpha_1\beta_1} \dots$. For example, $\mathcal{G}^{(3)}$ should have $\delta_{ij} \delta_{\alpha\beta} \delta_{\delta\gamma} \delta_{\alpha_1\beta_1}$, $\delta_{\alpha\beta} [\lambda^i \lambda^j]_{\delta\gamma} \delta_{\alpha_1\beta_1}$, $\delta_{\alpha\beta} [\lambda^j \lambda^i]_{\delta\gamma} \delta_{\alpha_1\beta_1}$, and $\lambda_{\alpha\beta}^i \lambda_{\delta\gamma}^j \delta_{\alpha_1\beta_1}$. So, $\mathcal{G}^{(n)}$ decomposes into

$$\mathcal{G}^{(n)} = \mathcal{G}^{(na)} [\delta_{ij} \delta_{\alpha\beta} \delta_{\delta\gamma} \delta_{\alpha_1\beta_1} \dots] + \mathcal{G}^{(nb)} [\delta_{\alpha\beta} [\lambda^i \lambda^j]_{\delta\gamma} \delta_{\alpha_1\beta_1} \dots] + \mathcal{G}^{(nc)} [\delta_{\alpha\beta} [\lambda^j \lambda^i]_{\delta\gamma} \delta_{\alpha_1\beta_1} \dots] + \mathcal{G}^{(nd)} [\lambda_{\alpha\beta}^i \lambda_{\delta\gamma}^j \delta_{\alpha_1\beta_1} \dots]. \tag{B7}$$

So, we can write

$$\begin{aligned}
& i \sum_{n=0}^{\infty} \int d^4x d^4y d^4x_1 \cdots d^4x_n \frac{i^n}{n!} \mathcal{G}_{\mu_1 \cdots \mu_n}^{(n) i, j; i_1 \cdots i_n}(x, y; x_1, \dots, x_n) \bar{\eta}^i(x) \eta^j(y) \times \left(-g\bar{\psi} \frac{\lambda_{i_1}}{2} \gamma^{\mu_1} \psi(x_1) \right) \cdots \left(-g\bar{\psi} \frac{\lambda_{i_n}}{2} \gamma^{\mu_n} \psi(x_n) \right) \\
& = i \left\{ \int d^4x d^4y d^4x_1 \Delta_{\Phi}^a(x, y) \bar{\eta}^i(x) \eta^j(y) + \int d^4x d^4y d^4z_1 d^4z_1' \Delta_{\Phi}^{b\sigma\rho}(x, y; z_1, z_1') \bar{\psi}^{\sigma}(z_1) \tilde{\eta}(x) \tilde{\eta}(y) \psi^{\rho}(z_1') \right. \\
& \quad - \int d^4x d^4y d^4z_1 d^4z_1' \Delta_{\Phi}^{c\sigma\rho}(x, y; z_1, z_1') \bar{\psi}^{\sigma}(z_1) \tilde{\eta}(y) \tilde{\eta}(x) \psi^{\rho}(z_1') \\
& \quad \left. + \int d^4x d^4y d^4z_1 d^4z_1' d^4z_2 d^4z_2' \Delta_{\Phi}^{d\sigma\rho; \sigma'\rho'}(x, y; z_1, z_1', z_2, z_2') \frac{1}{N_c^2} \bar{\psi}^{\sigma}(z_1) \tilde{\eta}(x) \psi^{\rho}(z_1') \bar{\psi}^{\sigma'}(z_2) \tilde{\eta}(y) \psi^{\rho'}(z_2') \right\}, \quad (\text{B8})
\end{aligned}$$

where $\tilde{\eta}_{\alpha\beta} = \eta^i \lambda_{\alpha\beta}^i$, $\tilde{\eta}_{\alpha\beta} = \bar{\eta}^i \lambda_{\alpha\beta}^i$, and

$$\Delta_{\Phi}^a(x, y) = \sum_{n=0}^{\infty} \int d^4x_1 d^4x_1' \cdots d^4x_n d^4x_n' \frac{(-1)^n i^n N_c^n}{n!} \mathcal{G}_{\rho_1 \cdots \rho_n}^{(na) \sigma_1 \cdots \sigma_n}(x, y; x_1, x_1' \cdots, x_n, x_n') \Phi^{\sigma_1 \rho_1}(x_1, x_1') \cdots \Phi^{\sigma_n \rho_n}(x_n, x_n'), \quad (\text{B9})$$

$$\begin{aligned}
\Delta_{\Phi}^{b\sigma\rho}(x, y; z_1, z_1') & = \sum_{n=0}^{\infty} \int d^4x_1 d^4x_1' \cdots d^4x_n d^4x_n' \frac{(-1)^n i^{n+1} N_c^n}{(n+1)!} \mathcal{G}_{\rho\rho_1 \cdots \rho_n}^{((n+1)b) \sigma \sigma_1 \cdots \sigma_n}(x, y; z_1, z_1', x_1, x_1' \cdots, x_n, x_n') \\
& \quad \times \Phi^{\sigma_1 \rho_1}(x_1, x_1') \cdots \Phi^{\sigma_n \rho_n}(x_n, x_n'), \quad (\text{B10})
\end{aligned}$$

$$\begin{aligned}
\Delta_{\Phi}^{c\sigma\rho}(x, y; z_1, z_1') & = \sum_{n=0}^{\infty} \int d^4x_1 d^4x_1' \cdots d^4x_n d^4x_n' \frac{(-1)^n i^{n+1} N_c^n}{(n+1)!} \mathcal{G}_{\rho\rho_1 \cdots \rho_n}^{((n+1)c) \sigma \sigma_1 \cdots \sigma_n}(x, y; z_1, z_1', x_1, x_1' \cdots, x_n, x_n') \\
& \quad \times \Phi^{\sigma_1 \rho_1}(x_1, x_1') \cdots \Phi^{\sigma_n \rho_n}(x_n, x_n'), \quad (\text{B11})
\end{aligned}$$

and

$$\begin{aligned}
\Delta_{\Phi}^{d\sigma\rho; \sigma'\rho'}(x, y; z_1, z_1', z_2, z_2') & = \sum_{n=0}^{\infty} \int d^4x_1 d^4x_1' \cdots d^4x_n d^4x_n' \frac{(-1)^n i^{n+2} N_c^{n+2}}{(n+2)!} \\
& \quad \times \mathcal{G}_{\rho\rho' \rho_1 \cdots \rho_n}^{((n+2)c) \sigma \sigma' \sigma_1 \cdots \sigma_n}(x, y; z_1, z_1', z_2, z_2', x_1, x_1' \cdots, x_n, x_n') \Phi^{\sigma_1 \rho_1}(x_1, x_1') \cdots \Phi^{\sigma_n \rho_n}(x_n, x_n'). \quad (\text{B12})
\end{aligned}$$

Now, the generating functional can be written as

$$\begin{aligned}
Z[J, \mathcal{I}, \bar{I}, I, \bar{\eta}^i, \eta^i] & = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}\Phi \mathcal{D}\Pi \exp i \left\{ \int d^4x \{ \bar{\psi}(i\partial + J - \Pi)\psi + \bar{I}\psi + \bar{\psi}I \} + \int d^4x d^4x' N_c \Phi^{\sigma\rho}(x, x') \Pi^{\sigma\rho}(x, x') \right. \\
& \quad + \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n d^4x_1' \cdots d^4x_n' N_c \frac{(-i)^n (N_c g^2)^{n-1}}{n!} \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x_1', \dots, x_n, x_n') \\
& \quad \times \Phi^{\sigma_1 \rho_1}(x_1, x_1') \cdots \Phi^{\sigma_n \rho_n}(x_n, x_n') \\
& \quad \left. + \int d^4x_1 d^4x_1' d^4x_2' \bar{\psi}_{\alpha}^{\sigma}(x_1') \tilde{\mathcal{I}}_{\alpha\beta}^{\mu}(x_1) \Delta_{\Phi, \mu}^{\sigma\rho}(x_1, x_1', x_2') \psi_{\beta}^{\rho}(x_2') + \mathcal{O}(\mathcal{I}^2) \right\} \\
& \quad \times \exp i \left\{ \int d^4x d^4y \Delta_{\Phi}^a(x, y) \bar{\eta}^i(x) \eta^j(y) + \int d^4x d^4y d^4z_1 d^4z_1' \Delta_{\Phi}^{b\sigma\rho}(x, y; z_1, z_1') \bar{\psi}^{\sigma}(z_1) \tilde{\eta}(x) \tilde{\eta}(y) \psi^{\rho}(z_1') \right. \\
& \quad - \int d^4x d^4y d^4z_1 d^4z_1' \Delta_{\Phi}^{c\sigma\rho}(x, y; z_1, z_1') \bar{\psi}^{\sigma}(z_1) \tilde{\eta}(y) \tilde{\eta}(x) \psi^{\rho}(z_1') \\
& \quad \left. + \int d^4x d^4y d^4z_1 d^4z_1' d^4z_2 d^4z_2' \Delta_{\Phi}^{d\sigma\rho; \sigma'\rho'}(x, y; z_1, z_1', z_2, z_2') \bar{\psi}^{\sigma}(z_1) \tilde{\eta}(x) \psi^{\rho}(z_1') \bar{\psi}^{\sigma'}(z_2) \tilde{\eta}(y) \psi^{\rho'}(z_2') \right\}. \quad (\text{B13})
\end{aligned}$$

Introducing

$$\int \mathcal{D}\Phi_{\bar{\eta}} \mathcal{D}\Phi_{\eta} \delta(N_c \Phi_{\bar{\eta}}^{\sigma\rho}(x, x'; y) - \bar{\psi}^{\sigma}(x) \tilde{\eta}(y) \psi^{\rho}(x')) \delta(N_c \Phi_{\eta}^{\sigma\rho}(x, x'; y) - \bar{\psi}^{\sigma}(x) \tilde{\eta}(y) \psi^{\rho}(x')), \quad (\text{B14})$$

where

$$\delta(N_c \Phi_{\bar{\eta}}^{\sigma\rho}(x, x'; y) - \bar{\psi}^{\sigma}(x) \tilde{\eta}(y) \psi^{\rho}(x')) \sim \int \mathcal{D}\Pi_{\bar{\eta}} e^{i \int d^4x d^4x' d^4y (N_c \Phi_{\bar{\eta}}^{\sigma\rho}(x, x'; y) - \bar{\psi}^{\sigma}(x) \tilde{\eta}(y) \psi^{\rho}(x')) \cdot \Pi_{\bar{\eta}}^{\sigma\rho}(x, x'; y)},$$

and following the procedure similar to that in Sec. II, we obtain

$$\begin{aligned} & Z[J, \mathcal{I}, \bar{I}, I, \bar{\eta}^i, \eta^i] \\ &= \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}\Phi \mathcal{D}\Pi \mathcal{D}\Phi_{\bar{\eta}} \mathcal{D}\Phi_{\eta} \mathcal{D}\Pi_{\bar{\eta}} \mathcal{D}\Pi_{\eta} \\ &\times \exp i \left\{ \int d^4x d^4y \{ \bar{\psi} [(i\partial + J - \Pi) \delta(x - y) + \tilde{\mathcal{I}} \Delta_{\Phi} + \Delta_{\Phi}^b \tilde{\eta} \tilde{\eta} - \Delta_{\Phi}^c \tilde{\eta} \tilde{\eta} + \tilde{\eta} \Pi_{\bar{\eta}} + \Pi_{\eta} \tilde{\eta}] \psi \right. \\ &+ \int d^4x \{ \bar{I} \psi + \bar{\psi} I \} + \int d^4x d^4x' N_c \Phi^{\sigma\rho}(x, x') \Pi^{\sigma\rho}(x, x') + \int d^4x d^4y \Delta_{\Phi}^a(x, y) \bar{\eta}^i(x) \eta^i(y) \\ &+ N_c \int d^4x d^4x' d^4y [\Phi_{\bar{\eta}}^{\sigma\rho}(x, x', y) \Pi_{\bar{\eta}}^{\sigma\rho}(x, x'; y) + \Phi_{\eta}^{\sigma\rho}(x, x', y) \Pi_{\eta}^{\sigma\rho}(x, x'; y)] \\ &+ \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n d^4x'_1 \cdots d^4x'_n N_c \frac{(-i)^n (N_c g^2)^{n-1}}{n!} \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n) \Phi^{\sigma_1 \rho_1}(x_1, x'_1) \cdots \Phi^{\sigma_n \rho_n}(x_n, x'_n) \\ &\left. + \int d^4x d^4y d^4z_1 d^4z'_1 d^4z_2 d^4z'_2 \Delta_{\Phi}^{d\sigma\rho; \sigma'\rho'}(x, y; z_1, z'_1, z_2, z'_2) \Phi_{\bar{\eta}}^{\sigma\rho}(z_1, z'_1; y) \Phi_{\eta}^{\sigma'\rho'}(z_2, z'_2; x) \right\}. \quad (\text{B15}) \end{aligned}$$

Integrating out ψ and $\bar{\psi}$, keeping up to $\eta\bar{\eta}$ -order terms, we obtain

$$\begin{aligned} & Z'[J, \mathcal{I}, \bar{I}, I, \bar{\eta}^i, \eta^i] \equiv \lim_{N_c \rightarrow \infty} Z[J, \mathcal{I}, \bar{I}, I, \bar{\eta}^i, \eta^i] \\ &= \exp i \left\{ -i N_c \text{Tr} \ln [(i\partial + J - \Pi_c)] - i \text{Tr}' [(i\partial + J - \Pi_c)^{-1} (\Delta_{\Phi_c}^b \tilde{\eta} \tilde{\eta} - \Delta_{\Phi_c}^c \tilde{\eta} \tilde{\eta})] \right. \\ &+ i \text{Tr}' [(i\partial + J - \Pi_c)^{-1} \tilde{\eta} \Pi_{\bar{\eta}_c} (i\partial + J - \Pi_c)^{-1} \tilde{\eta} \Pi_{\eta_c}] \\ &- \bar{I} [(i\partial + J - \Pi_c)^{-1}] I + \bar{I} [(i\partial + J - \Pi_c)^{-1} (\tilde{\mathcal{I}} \Delta_{\Phi_c} + \Delta_{\Phi_c}^b \tilde{\eta} \tilde{\eta} - \Delta_{\Phi_c}^c \tilde{\eta} \tilde{\eta}) (i\partial + J - \Pi_c)^{-1}] I \\ &- \bar{I} [(i\partial + J - \Pi_c)^{-1} \tilde{\eta} \Pi_{\bar{\eta}_c} (i\partial + J - \Pi_c)^{-1} \Pi_{\eta_c} \tilde{\eta} (i\partial + J - \Pi_c)^{-1}] I \\ &- \bar{I} [(i\partial + J - \Pi_c)^{-1} \Pi_{\eta_c} \tilde{\eta} (i\partial + J - \Pi_c)^{-1} \tilde{\eta} \Pi_{\bar{\eta}_c} (i\partial + J - \Pi_c)^{-1}] I \\ &+ \int d^4x d^4y \Delta_{\Phi_c}^a(x, y) \bar{\eta}^i(x) \eta^i(y) + N_c \int d^4x d^4x' \Phi_c^{\sigma\rho}(x, x') \Pi_c^{\sigma\rho}(x, x') \\ &+ N_c \int d^4x d^4x' d^4y [\Phi_{\bar{\eta}_c}^{\sigma\rho}(x, x', y) \Pi_{\bar{\eta}_c}^{\sigma\rho}(x, x'; y) + \Pi_{\eta_c}^{\sigma\rho}(x, x'; y) \Phi_{\eta_c}^{\sigma\rho}(x, x', y)] \\ &+ N_c \sum_{n=2}^{\infty} \int d^4x_1 d^4x'_1 \cdots d^4x_n d^4x'_n \frac{(-i)^n (N_c g^2)^{n-1}}{n!} \\ &\times \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x'_1, \dots, x_n, x'_n) \Phi_c^{\sigma_1 \rho_1}(x_1, x'_1) \cdots \Phi_c^{\sigma_n \rho_n}(x_n, x'_n) \\ &\left. + \int d^4x d^4y d^4z_1 d^4z'_1 d^4z_2 d^4z'_2 \Delta_{\Phi_c}^{d\sigma\rho; \sigma'\rho'}(x, y; z_1, z'_1, z_2, z'_2) \Phi_{\bar{\eta}_c}^{\sigma\rho}(z_1, z'_1; y) \Phi_{\eta_c}^{\sigma'\rho'}(z_2, z'_2; x) \right\}, \quad (\text{B16}) \end{aligned}$$

where Φ_{η_c} , $\Phi_{\bar{\eta}_c}$, Π_{η_c} , and $\Pi_{\bar{\eta}_c}$ are the expectation values of fields Φ_{η} , $\Phi_{\bar{\eta}}$, Π_{η} , and $\Pi_{\bar{\eta}}$, respectively.

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