Triply heavy $QQ\bar{Q}\bar{q}$ tetraquark states

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Within the framework of QCD sum rules, we have investigated the tetraquark states with three heavy quarks. We systematically construct the interpolating currents for the possible $cc\bar{c}\bar{q}$, $cc\bar{b}\bar{q}$, $bc\bar{b}\bar{q}$, $bb\bar{b}\bar{q}$ tetraquark states with quantum numbers $J^P = 0^+$ and $J^P = 1^+$. Using these interpolating currents, we have calculated the two-point correlation functions and extracted the mass spectra for the above tetraquark states. We also discuss the decay patterns of these tetraquarks, and notice that the $cc\bar{c}\bar{q}$, $cc\bar{b}\bar{q}$, $bc\bar{b}\bar{q}$ may decay quickly with a narrow width due to their mass spectra. The $bb\bar{b}\bar{q}$ tetraquarks are expected to be very narrow resonances since their OZI (Okubo-Zweig-Iizuka)-allowed decay modes are kinematically forbidden. These states may be searched for in the final states with a B meson plus a light meson or photon.

I. INTRODUCTION

The existence of multiquark states was first suggested at the birth of the quark model [1,2]. In the past fifty years, the search of the multiquark matter has been an extremely intriguing research issue, which shall provide important hints to the understanding of the nonperturbative QCD [3–9].

Tetraquarks are composed of diquarks and antidiquarks. They are compact hadron states bound by colored force between four quarks. The light tetraquark configurations $q\bar{q}q\bar{q}$ and $q\bar{q}s\bar{s}$ were proposed to explain the scalar mesons below 1 GeV [10–13]. In the heavy sector, the exotic charmed mesons $D_{s0}^*(2317)$ [14] and $D_{s1}(2460)$ [15] were studied as the singly charmed-strange tetraquark states in Refs. [16–18]. Very recently, the D0 Collaboration reported a narrow structure in the $B_s^0\pi^\pm$ invariant mass spectrum [19]. This charged $X(5568)$ meson, if it really exists, could be a good candidate for a tetraquark state consisting of four different flavor quarks $su\bar{d}\bar{b}$ (or $sd\bar{u}\bar{b}$). Various theoretical configurations have been proposed to investigate the nature of $X(5568)$, such as the diquark-antidiquark tetraquark state [20–24], hadron molecule [25,26], and so on. The detailed introduction of this state can be found in the recent review paper [27].

The hidden flavor tetraquarks are the most extensively studied ones in the past decade, due to the plenty of the charmonium-like and bottomonium-like states observed in experiments [3,7–9,28]. Most of these XYZ states do not fit into the quark model spectrum easily and their decay final products contain a heavy quark-antiquark pair. Especially for the charged Z_c (Z_b) states, they contain a $c\bar{c}$ ($b\bar{b}$) pair plus at least a pair of light quark-antiquark states. They were considered as very good tetraquark candidates with two heavy quarks [3]. However, some of these XYZ mesons can also be interpreted as loosely bound molecular states, especially for those near-threshold resonances [3,9]. Sometimes, it is not so easy to distinguish a loosely hadron molecule from a compact tetraquark state.

The doubly hidden flavor $QQ\bar{Q}\bar{Q}$ system consists of four heavy quarks. Such a four-quark system favors the compact tetraquark configuration more than the hadron molecule because the binding force comes from the short-range gluon exchange and the light mesons cannot be exchanged between two quarkonium states. Recently, several collaborations reported the observations of the J/ψ pairs at LHCb [29], D0 [30], and CMS [31], the $\Upsilon(1S)$ pair at CMS [32], and the simultaneous $J/\psi\Upsilon(1S)$ events at DØ [33] and CMS [34]. These observations have trigged theoretical discussions of the $QQ\bar{Q}\bar{Q}$ tetraquark systems [35–39]. It is natural to investigate the existence of the triply heavy $QQ\bar{Q}\bar{q}$ tetraquark states [40] in the same schemes. Actually, the associated production of bottomonia and open charm hadrons $\Upsilon D/D_s$ in pp collisions was recently reported by the LHCb Collaboration [41]. In this work, we

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shall study the mass spectra of the possible triply heavy $Q Q \bar{Q} \bar{q}$ tetraquark states in the QCD sum rule method.

This paper is organized as follows. In Sec. II, we construct the triply heavy tetraquark interpolating currents and introduce the formalism of QCD sum rules. The spectral densities for these currents are then evaluated and collected in the Appendix. In Sec. III, we perform numerical analyses for the tetraquark mass sum rules and extract the mass spectra for the $c c \bar{c} \bar{q}$, $c c \bar{b} \bar{q}$, $b c \bar{b} \bar{q}$, $b b \bar{b} \bar{q}$ systems. We also discuss the possible decay patterns of these tetraquark states. The last section is a brief summary.

II. FORMALISM

The starting point of the QCD sum rule [42–44] is the interpolating current which couples to the hadrons with the same quantum numbers. We construct the interpolating currents for the triply heavy tetraquarks with quantum numbers $J^P = 0^+$,

$$
\begin{aligned}
J_1 &= Q_{1a}^T C \gamma_5 Q_{2b} (\bar{Q}_{3a} \gamma_5 C \bar{q}_b^T + \bar{Q}_{3b} \gamma_5 C \bar{q}_a^T), \\
J_2 &= Q_{1a}^T C \gamma_\mu Q_{2b} (\bar{Q}_{3a} \gamma^\mu C \bar{q}_b^T + \bar{Q}_{3b} \gamma^\mu C \bar{q}_a^T), \\
J_3 &= Q_{1a}^T C \gamma_5 Q_{2b} (\bar{Q}_{3a} \gamma_5 C \bar{q}_b^T - \bar{Q}_{3b} \gamma_5 C \bar{q}_a^T), \\
J_4 &= Q_{1a}^T C \gamma_\mu Q_{2b} (\bar{Q}_{3a} \gamma^\mu C \bar{q}_b^T - \bar{Q}_{3b} \gamma^\mu C \bar{q}_a^T), \quad (1)
\end{aligned}
$$

where Q denotes the heavy quark c or b and q denotes the light quark u or d. The currents J_1 and J_2 are color symmetric $[\mathbf{6}_c]_{Q_1 Q_2} \otimes [\bar{\mathbf{6}}_c]_{\bar{Q}_3 \bar{q}}$, and the currents J_3 and J_4 are color antisymmetric $[\bar{\mathbf{3}}_c]_{Q_1 Q_2} \otimes [\mathbf{3}_c]_{\bar{Q}_3 \bar{q}}$.

Note that the currents J_2 and J_3 vanish in the $Q Q \bar{Q}' \bar{q}$ systems when the two heavy quarks in the diquark $Q Q$ have the same flavors (cc or bb), due to Fermi statistics [45,46].

The interpolating currents of the tetraquarks with quantum numbers $J^P = 1^+$ are

$$
\begin{aligned}
J_{1\mu} &= Q_{1a}^T C \gamma_5 Q_{2b} (\bar{Q}_{3a} \gamma_\mu C \bar{q}_b^T + \bar{Q}_{3b} \gamma_\mu C \bar{q}_a^T), \\
J_{2\mu} &= Q_{1a}^T C \gamma_\mu Q_{2b} (\bar{Q}_{3a} \gamma_5 C \bar{q}_b^T + \bar{Q}_{3b} \gamma_5 C \bar{q}_a^T), \\
J_{3\mu} &= Q_{1a}^T C \gamma_5 Q_{2b} (\bar{Q}_{3a} \gamma_\mu C \bar{q}_b^T - \bar{Q}_{3b} \gamma_\mu C \bar{q}_a^T), \\
J_{4\mu} &= Q_{1a}^T C \gamma_\mu Q_{2b} (\bar{Q}_{3a} \gamma_5 C \bar{q}_b^T - \bar{Q}_{3b} \gamma_5 C \bar{q}_a^T). \quad (2)
\end{aligned}
$$

The currents $J_{2\mu}$ and $J_{3\mu}$ also vanish in the $Q Q \bar{Q}' \bar{q}$ systems.

In this work, we shall discuss the $c c \bar{c} \bar{q}$, $c c \bar{b} \bar{q}$, $b c \bar{b} \bar{q}$, $b b \bar{b} \bar{q}$ tetraquark systems in the framework of the QCD

sum rules. The correlation function for the scalar currents reads

$$
\Pi(q) = i \int d^4 x e^{i q \cdot x} \langle 0 | T[j(x) j^\dagger(0)] | 0 \rangle, \quad (3)
$$

and that for the vector currents

$$
\begin{aligned}
\Pi_{\mu\nu}(q) &= i \int d^4 x e^{i q \cdot x} \langle 0 | T[j_\mu(x) j_\nu^\dagger(0)] | 0 \rangle \\
&= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \Pi_1(q^2) + \frac{q_\mu q_\nu}{q^2} \Pi_0(q^2) \quad (4)
\end{aligned}
$$

where $\Pi_1(q^2)$ and $\Pi_0(q^2)$ are the vector and scalar current polarization functions, respectively. They can be extracted by the projection,

$$
\Pi_1(q^2) = -\frac{1}{3} \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \Pi_{\mu\nu}(q), \quad (5)
$$

$$
\Pi_0(q^2) = \frac{q^\mu q^\nu}{q^2} \Pi_{\mu\nu}(q). \quad (6)
$$

At the hadron level, the correlation function can be expressed with the spectral function through the dispersion relation

$$
\Pi(q^2) = (q^2)^N \int_0^\infty ds \frac{\rho(s)}{s^N (s - q^2 - i\epsilon)} + \sum_{k=0}^{N-1} b_n(q^2)^k, \quad (7)
$$

where $\rho(s)$ is the spectral representation which is related to the imaginary part of the correlation function $\rho(s) \equiv \text{Im}\Pi(s)/\pi$. In the QCD sum rule framework, $\rho(s)$ is always assumed to be

$$
\begin{aligned}
\rho(s) &= f_X^2 \delta(s - m_X^2) \langle 0 | \eta | n \rangle \langle n | \eta^\dagger | 0 \rangle \\
&= f_X^2 \delta(s - m_X^2) + \text{continuum}, \quad (8)
\end{aligned}
$$

where m_X is the mass of the lowest-lying resonance X and f_X is the coupling constant.

At the QCD level, the correlation function can be calculated via the operator product expansion (OPE) method [47]. To get the Wilson coefficients we will use the propagator for the light quarks

$$
\begin{aligned}
iS^q(x) &= \frac{i\delta_{ab}\slashed{x}}{2\pi^2(x^2)^{\frac{D}{2}}} - \frac{\delta_{ab} m_q}{4\pi^2(x^2)^{\frac{D}{2}-1}} - \frac{\delta_{ab}}{12}\langle\bar{q}q\rangle + \frac{i\delta_{ab}\slashed{x}}{48}m_q\langle\bar{q}q\rangle - \frac{x^2\delta_{ab}}{192}\langle\bar{q}Gq\rangle + \frac{i\delta_{ab}x^2\slashed{x}}{1152}m_q\langle\bar{q}Gq\rangle \\
&\quad - g_s\frac{it_{ab}^N}{32\pi^2(x^2)^{\frac{D}{2}-1}}(\sigma^{\alpha\beta}\slashed{x} + \slashed{x}\sigma^{\alpha\beta}) - \frac{t_{ab}^N\sigma_{\alpha\beta}}{192}\langle\bar{q}Gq\rangle + \frac{it_{ab}^N}{768}(\sigma_{\alpha\beta}\slashed{x} + \slashed{x}\sigma_{\alpha\beta})m_q\langle\bar{q}Gq\rangle, \quad (9)
\end{aligned}
$$

and that for the heavy quarks

$$
\begin{aligned}
iS^Q(p,m) =& \frac{i(\not{p}+m)}{(p^2-m^2+i\epsilon)}\delta^{ab} \\
&- G_{\mu\nu}^c \frac{i}{4}(t^c)^{ab}\frac{\sigma^{\mu\nu}(\not{p}+m)+(\not{p}+m)\sigma^{\mu\nu}}{(p^2-m+i\epsilon)^2} \\
&+ \frac{i}{12}\delta^{ab}\langle g^2 G_{\alpha\beta}^a G_{\alpha\beta}^a\rangle m \frac{p^2+m\not{p}}{(p^2-m^2+i\epsilon)^4}.
\end{aligned} \quad (10)
$$

The propagator of the light quark is presented in coordinate space, while the propagator of the heavy quark is presented in momentum space. These two forms are related by Fourier transformation,

$$
S(x) = \int \frac{d^4p}{(2\pi)^4}e^{-ip\cdot x}S(p). \quad (11)
$$

In order to suppress the contribution of the higher states and remove the unknown subtraction terms in Eq. (7), we perform the Borel transformation to the correlation function. The Borel transformation is defined as

$$
\hat{B}[f(q^2)] = \lim_{\substack{-q^2,n\to\infty \\ -q^2/n\equiv M_B^2}} \frac{1}{n!}(-q^2)^{n+1}\left(\frac{d}{dq^2}\right)^n f(q^2). \quad (12)
$$

The correlation function after the Borel transformation can be expressed as

$$
\hat{B}\Pi(q^2) = \frac{1}{\pi}\int ds e^{-s/M_B^2}\text{Im}\Pi(s). \quad (13)
$$

Comparing the correlation function at both the OPE side and phenomenological side and using quark-hadron duality, one gets

$$
f_X^2 e^{-m_X^2/M_B^2} = \int_<^{s_0} dx e^{-s/M_B^2}\rho(s), \quad (14)
$$

where s_0 is the threshold parameter and M_B is the Borel parameter. The expressions of the spectral functions $\rho(s)$ are very complicated and lengthy. We only present the expressions of the spectral functions for J_3 and $J_{3\mu}$ in the Appendix. Using the sum rule equation (14), the mass of the resonance can be extracted as

$$
m^2 = \frac{\int_<^{s_0} ds s\rho^{\text{OPE}}(s)e^{-\frac{s}{M_B^2}}}{\int_<^{s_0} ds \rho^{\text{OPE}}(s)e^{-\frac{s}{M_B^2}}}. \quad (15)
$$

As a function of s_0 and M_B, we need to find suitable working regions in the parameter space (s_0, M_B^2) to extract the hadron mass. We will discuss the details in the next section.

III. NUMERICAL ANALYSIS

We will use the following parameters [43,48–50] in the numerical analysis:

$$
\begin{aligned}
m_c(m_c) = \bar{m}_c &= (1.275 \pm 0.025)\text{ GeV}, \\
m_b(m_b) = \bar{m}_b &= (4.18 \pm 0.03)\text{ GeV}, \\
\langle\bar{q}q\rangle &= -(0.24 \pm 0.01)^3\text{ GeV}^3, \\
\langle\bar{q}g_s\sigma\cdot Gq\rangle &= -M_0^2\langle\bar{q}q\rangle, \\
M_0^2 &= (0.8 \pm 0.2)\text{ GeV}^2, \\
\langle g_s^2 GG\rangle &= (0.48 \pm 0.14)\text{ GeV}^4.
\end{aligned} \quad (16)
$$

To determine the Borel parameter, we first require that the contribution of the quark condensate term be less than 30% of the contribution of the perturbation term. Such a limitation ensures the convergence of the OPE series and leads to the lower bound on M_B. We then require the following pole contribution to be larger than 10%:

$$
PC(s_0, M_B^2) = \frac{\int_<^{s_0} ds\rho^{\text{OPE}}(s)e^{-\frac{s}{M_B^2}}}{\int_<^{\infty} ds\rho^{\text{OPE}}(s)e^{-\frac{s}{M_B^2}}}, \quad (17)
$$

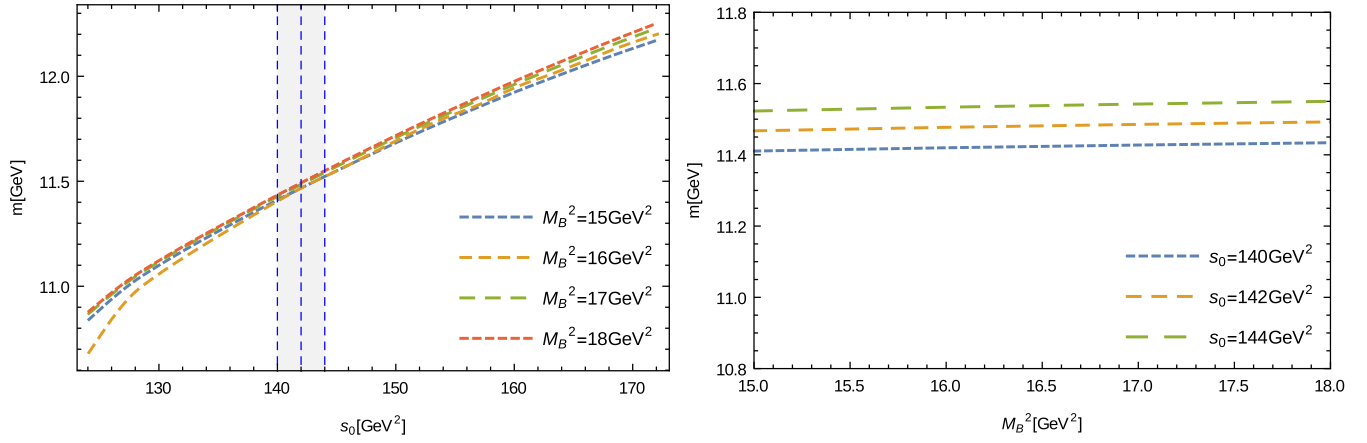which gives the upper bound on M_B. An optimal value for the threshold parameter s_0 is determined by requiring the M_B dependence of the hadron mass m as less as possible. These criteria lead to suitable working regions for s_0 and M_B.

We take the current $J_3^{bc\bar{c}\bar{d}}$ as an example to illustrate the mass sum rule analyses. We show the hadron mass as a function of the continuum threshold s_0 and Borel parameter M_B^2 in Fig. 1. From the left graph of Fig. 1, the M_B dependance of the extracted mass becomes very weak for 140 GeV$^2 \leq s_0 \leq$ 144 GeV^2. In this region, we plot the hadron mass m as a function of the Borel parameter M_B in the right graph of Fig. 1. The mass curves are very stable in such a Borel window. Accordingly, we extract the tetraquark mass as

$$
m = 11.5 \pm 0.3\text{ GeV}, \quad (18)
$$

where the error comes from the errors of the input parameters m_b, m_c, the condensates $\langle\bar{q}q\rangle$, $\langle g_s^2 GG\rangle$, $\langle\bar{q}g_s\sigma\cdot Gq\rangle$, and the uncertainties of M_B and s_0.

For the other tetraquark systems, we can perform similar mass sum rule analyses and extract their masses. The extracted masses for the $cc\bar{b}\bar{q}$, $cc\bar{c}\bar{q}$, $bc\bar{b}\bar{q}$, $bb\bar{b}\bar{q}$ tetraquarks with $J^P = 0^+$ and $J^P = 1^+$ are listed in Table I and Table II, respectively. For the $cc\bar{c}\bar{q}$ tetraquarks, it is shown that the interpolating currents J_1 and J_4 lead to the same numerical results of the hadron mass, although they have totally different diquark-antidiquark color structures. In principle, the color antitriplet diquark is the attractive

FIG. 1. The dependence of the extracted mass on the threshold parameter s_0 and M_B for the current $J_3^{bc\bar{b}q}$.

channel while the color sextet diquark is the repulsive one in the one-gluon exchange model. However, the interaction between the color 6 and $\bar{6}$ diquarks will be attractive when they form a color singlet tetraquark. In other words, the sum of the repulsion between the two quarks within the diquark/antidiquark and the strong attraction between the 6 and $\bar{6}$ diquarks leads to a net attraction, which is roughly the same as the total attraction in the $3–\bar{3}$ channel. Therefore, we obtain roughly equal masses of the particles associated to J_1 and J_4 within errors.

In the following, we discuss the decay patterns of the possible $cc\bar{b}\bar{q}$, $cc\bar{c}\bar{q}$, $bc\bar{b}\bar{q}$, $bb\bar{b}\bar{q}$ tetraquark states using the mass spectra obtained in the previous section. We will only consider the two-body hadronic decays. In fact, these tetraquark states may decay into meson pairs easily if the kinematics allows.

Considering the conservations of the spin-parity, we list the possible decay modes for the $cc\bar{c}\bar{q}$, $cc\bar{b}\bar{q}$, $bc\bar{b}\bar{q}$, $bb\bar{b}\bar{q}$ tetraquarks in Table III. From Tables I and II, the masses of the $cc\bar{c}\bar{q}$, $cc\bar{b}\bar{q}$, $bc\bar{b}\bar{q}$ tetraquarks are higher than the corresponding meson-meson thresholds. They will decay easily through rearrangement or the so-called fall-apart

mechanism. Since the decay momentum is not very large, these tetraquarks are not expected to be very broad resonances. Thus, it may be difficult and still possible to observe them experimentally. However, the situation is different for the $bb\bar{b}\bar{q}$ tetraquark systems. Note that the extracted masses for both the scalar and axial-vector triply bottomed tetraquarks are lower than the open bottom

TABLE II. The extracted masses for various tetraquarks with $J^P = 1^+$.

System	Current	s_0 (GeV^2)	$[M_{B,\min}^2, M_{B,\max}^2]$	m (GeV)
$bc\bar{b}\bar{q}$	$j_{1\mu}$	142	[15, 18]	11.5 ± 0.5
	$j_{2\mu}$	136	[15, 18]	11.3 ± 0.4
	$j_{3\mu}$	144	[15, 18]	11.6 ± 0.4
	$j_{4\mu}$	140	[15, 18]	11.4 ± 0.5
$cc\bar{b}\bar{q}$	$j_{1\mu}$	71	[8, 11]	8.1 ± 0.3
	$j_{4\mu}$	73	[8, 11]	8.2 ± 0.3
$cc\bar{c}\bar{q}$	$j_{1\mu}$	30	[5, 8]	5.1 ± 0.2
	$j_{4\mu}$	30	[5, 8]	5.1 ± 0.2
$bb\bar{b}\bar{q}$	$j_{1\mu}$	189	[16, 19]	13.5 ± 0.4
	$j_{4\mu}$	189	[16, 19]	13.5 ± 0.4

TABLE I. The extracted masses for various tetraquarks with $J^P = 0^+$.

System	Current	s_0 (GeV^2)	$[M_{B,\min}^2, M_{B,\max}^2]$	m (GeV)
$bc\bar{b}\bar{q}$	j_1	133	[15, 18]	11.4 ± 0.6
	j_2	136	[15, 18]	11.3 ± 0.3
	j_3	142	[15, 18]	11.5 ± 0.3
	j_4	141	[15, 18]	11.4 ± 0.4
$cc\bar{b}\bar{q}$	j_1	70	[8, 11]	8.0 ± 0.3
	j_4	73	[8, 11]	8.2 ± 0.3
$cc\bar{c}\bar{q}$	j_1	29	[5, 8]	5.1 ± 0.2
	j_4	30	[5, 8]	5.1 ± 0.2
$bb\bar{b}\bar{q}$	j_1	189	[16, 19]	13.5 ± 0.4
	j_4	198	[16, 19]	13.7 ± 0.3

TABLE III. Decay modes of the tetraquark states with different quantum numbers and flavor contents.

J^P	Flavor content	S-wave decay
0^+	$cc\bar{c}\bar{q}$	$(J/\psi D^*), (\eta_c D)$
	$cc\bar{b}\bar{q}$	$(B_c D)$
	$bc\bar{b}\bar{q}$	$(BB_c), (\Upsilon D^*), (\eta_b D)$
	$bb\bar{b}\bar{q}$	$\cdots$
1^+	$cc\bar{c}\bar{q}$	$(J/\psi D^*), (\eta_c D^*), (J/\psi D)$
	$cc\bar{b}\bar{q}$	$(B_c D^*)$
	$bc\bar{b}\bar{q}$	$(\Upsilon D^*), (\eta_b D^*), (\Upsilon D)$
	$bb\bar{b}\bar{q}$	$\cdots$

thresholds $T_{\eta_b B} = 14.68$ GeV and $T_{\eta_b B^*} = 14.72$ GeV. These tetraquark states cannot decay into a bottomonia plus a $B^{(*)}$ meson, which implies that the $bb\bar{b}\bar{q}$ states should be stable.

IV. DISCUSSIONS AND CONCLUSIONS

In the framework of QCD sum rules, we have investigated the mass spectra of the triply heavy tetraquark states with $J^P = 0^+$ and $J^P = 1^+$. We first construct the interpolating currents of the $cc\bar{c}\bar{q}$, $cc\bar{b}\bar{q}$, $bc\bar{b}\bar{q}$, $bb\bar{b}\bar{q}$ systems with quantum numbers $J^P = 0^+, 1^+$. Using these interpolating currents, we have calculated the two-point correlation functions and spectral densities. Then we perform numerical analyses for the mass sum rules and extract the mass spectra of various triply heavy tetraquark states. Accordingly, we have also discussed the possible decay patterns of these tetraquark states.

As shown in Tables I and II, the masses of the $cc\bar{c}\bar{q}$, $cc\bar{b}\bar{q}$, $bc\bar{b}\bar{q}$ tetraquarks seem higher than some two-meson thresholds. They may decay easily into the two-body final states through the fall-apart mechanism. However, these results are much lower than those obtained in the framework of the color-magnetic interaction [40]. Because of the limited phase space, these states might not be very broad. In contrast, the $bb\bar{b}\bar{q}$ tetraquark states lie below the bottomonia plus $B^{(*)}$

thresholds and their OZI-allowed strong decays are kinematically forbidden. It is very interesting to note that some $bb\bar{q}\bar{q}$ states also lie below the two bottom meson threshold and are probably stable [45,46,51–53].

However, the heavy quark pair annihilation and light quark pair creation processes $bb\bar{b}\bar{q} \to (b\bar{q}) + (q\bar{q})$ are still possible. Such characteristic B meson plus a light meson modes will contribute significantly to the $bb\bar{b}\bar{q}$ decay width. These $bb\bar{b}\bar{q}$ tetraquark states may also decay via electromagnetic and weak interactions. They can decay into $\bar{B}^{(*)}\gamma$ through the $b\gamma_\mu\bar{b} \to \gamma$ photon production process. The weak decay $bb\bar{b}\bar{q} \to J/\psi \Upsilon K$ is also allowed, although the phase space is limited. If such $bb\bar{b}\bar{q}$ states do exist, they may be detected in the final states with B meson and other light mesons or photon. With the running of LHC at 13 TeV and the forthcoming BelleII, searching for such triply heavy tetraquark states will probably become feasible in the near future.

APPENDIX: THE SPECTRAL DENSITY FUNCTIONS

For the interpolating current J_3, the spectral functions are

$$\rho^{\mathrm{OPE}}(s) = \rho^{\mathrm{pert}}(s) + \rho^{\langle \bar{q}q \rangle}(s) + \rho^{\langle GG \rangle}(s) + \rho^{\langle \bar{q}Gq \rangle}(s) \tag{A1}$$

$$
\begin{aligned}
\rho^{\mathrm{pert}}(s) = &\frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{pert}}(s) \\
= &\int_0^1 d\alpha \int_0^{1-\alpha} d\beta \int_0^{1-\alpha-\beta} d\gamma\, \theta\left(s - \frac{m_1^2}{\alpha} - \frac{m_2^2}{\beta} - \frac{m_2^2}{\gamma}\right) \frac{-\alpha\beta}{256\pi^6}\left(\frac{m_1^2}{\alpha} + \frac{m_2^2}{\beta} + \frac{m_2^2}{\gamma} - s\right)^2 \\
&\times \frac{1}{(\alpha+\beta-1)}\left\{ 3\gamma(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1)\left(\frac{m1^2}{\alpha} + \frac{m2^2}{\beta} + \frac{m3^2}{\gamma} - s\right)^2 \right. \\
&- 2s\left(\frac{m1^2}{\alpha} + \frac{m2^2}{\beta} + \frac{m3^2}{\gamma} - s\right)[5\alpha^4 + \alpha^3(13\beta + 12\gamma - 13) + \alpha^2(11\beta^2 + \beta(30\gamma - 22) + 6\gamma^2 - 23\gamma + 11) \\
&+ \alpha(3\beta^3 + 3\beta^2(8\gamma - 3) + \beta(12\gamma^2 - 38\gamma + 9) - 6\gamma^2 + 14\gamma - 3) + \gamma(6\beta^3 + 3\beta^2(2\gamma - 5) - 6\beta(\gamma - 2) + 2\gamma - 3)] \\
&+ \left. 6s^2(\alpha^2 + \alpha(\beta + \gamma - 1) + \beta\gamma)(2\alpha^2 + \alpha(3\beta + \gamma - 3) + \beta^2 + \beta(\gamma - 2) + 1) \right\} \\
&+ \int d\alpha \int d\beta \int d\gamma\, \frac{1}{128\pi^6} m1\, m2\left(\frac{m1^2}{\alpha} + \frac{m2^2}{\beta} + \frac{m3^2}{\gamma} - s\right) \\
&\times \left[2(\alpha+\beta+\gamma-1)\gamma\left(\frac{m1^2}{\alpha} + \frac{m2^2}{\beta} + \frac{m3^2}{\gamma} - s\right) \right. \\
&- \left. \frac{3s(\alpha^2 + \alpha(\beta + \gamma - 1) + \beta\gamma)(2\alpha^2 + \alpha(3\beta + \gamma - 3) + \beta^2 + \beta(\gamma - 2) + 1)}{(\alpha+\beta-1)^2} \right]^2
\end{aligned} \tag{A2}
$$

$$\rho^{\langle\bar{q}q\rangle}(s) = \frac{1}{\pi}\text{Im}\Pi^{\langle\bar{q}q\rangle}(q^2)$$

$$= \langle\bar{q}q\rangle\int_0^1 d\alpha\int_0^{1-\alpha}d\beta\,\theta\left(s-\frac{m_1^2}{\alpha}-\frac{m_2^2}{\beta}-\frac{m_3^2}{1-\alpha-\beta}\right)\left[\frac{m_1m_2m_3}{16\pi^4}\left(\frac{m_1^2}{\alpha}+\frac{m_2^2}{\beta}+\frac{m_3^2}{1-\alpha-\beta}-s\right)\right.$$

$$\left.-\frac{m_3[(\alpha+\beta-1)(\alpha m_2^2+\beta m_1^2-\alpha\beta s)-\alpha\beta m_3^2][(\alpha+\beta-1)(\alpha m_2^2+\beta m_1^2-2\alpha\beta s)-\alpha\beta m_3^2]}{16\pi^4\alpha\beta(1-\alpha-\beta)^2}\right]$$

(A3)

$$\rho^{\langle GG\rangle}(s) = \int_0^1 d\alpha\int_0^{1-\alpha}d\beta\int_0^{1-\alpha-\beta}d\gamma\,\theta\left(s-\frac{m_1^2}{\alpha}-\frac{m_2^2}{\beta}-\frac{m_2^2}{\gamma}\right)\frac{-\langle g_s^2 GG\rangle}{3072\pi^6}\left\{-\frac{4\gamma m1^3 m2(\alpha+\beta+\gamma-1)}{\alpha^3}\right.$$

$$-\frac{4\gamma m1m2^3(\alpha+\beta+\gamma-1)}{\beta^3}-\frac{4m1m2m3^2(\alpha+\beta+\gamma-1)}{\gamma^2}$$

$$+\frac{1}{\alpha^3(\alpha+\beta-1)^2}2m1^2[6\gamma(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1)(\beta m1^2+\alpha m2^2)+6\alpha\beta m3^2(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1)$$

$$-\alpha\beta s(5\alpha^4+\alpha^3(13\beta+18\gamma-13)+\alpha^2(11\beta^2+\beta(48\gamma-22)+12\gamma^2-41\gamma+11)$$

$$+\alpha(3\beta^3+\beta^2(42\gamma-9)+\beta(24\gamma^2-74\gamma+9)-18\gamma^2+32\gamma-3)$$

$$+\gamma(12\beta^3+3\beta^2(4\gamma-11)-6\beta(3\gamma-5)+8\gamma-9))]$$

$$-\frac{1}{\alpha^3\beta^3\gamma(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1)}3m2(2\alpha^3\gamma+\alpha^2(\beta^2+2(\gamma-1)\gamma)-2\alpha\beta(\gamma-1)\gamma+2\beta^2\gamma(\beta+\gamma-1))$$

$$\times[2\gamma(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1)(\beta m1^2+\alpha m2^2)+2\alpha\beta m3^2(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1)-\alpha\beta s$$

$$\times(2\alpha^4+5\alpha^3(\beta+\gamma-1)+\alpha^2(4\beta^2+\beta(13\gamma-8)+3\gamma^2-10\gamma+4)$$

$$+\alpha(\beta^3+\beta^2(11\gamma-3)+3\beta(2\gamma^2-6\gamma+1)-4\gamma^2+7\gamma-1)+\gamma(3\beta^3+\beta^2(3\gamma-8)+\beta(7-4\gamma)+2(\gamma-1)))]$$

$$+\left[\frac{s^2(2\alpha^4+\alpha^3(5\beta+3\gamma-5)+\alpha^2(4\beta^2+\beta(7\gamma-8)+(\gamma-2)^2)+\alpha(\beta^3+\beta^2(5\gamma-3)+\beta(2\gamma^2-6\gamma+3)+\gamma-1)+\beta\gamma(\beta^2+\beta(\gamma-2)+1))}{(\alpha+\beta-1)^2}\right.$$

$$+\frac{s}{\alpha\beta\gamma(\alpha+\beta-1)^2}(-\gamma(\beta(m1^2-\alpha s)+\alpha m2^2)-\alpha\beta m3^2)$$

$$\times(5\alpha^4+\alpha^3(13\beta+12\gamma-13)+\alpha^2(11\beta^2+\beta(30\gamma-22)+6\gamma^2-23\gamma+11)$$

$$+\alpha(3\beta^3+3\beta^2(8\gamma-3)+\beta(12\gamma^2-38\gamma+9)-6\gamma^2+14\gamma-3)+\gamma(6\beta^3+3\beta^2(2\gamma-5)-6\beta(\gamma-2)+2\gamma-3))$$

$$+\frac{3(\alpha+\beta+\gamma-1)(\gamma(\beta(m1^2-\alpha s)+\alpha m2^2)+\alpha\beta m3^2)^2}{\alpha^2\beta^2\gamma}\left]+\frac{2}{(\alpha+\beta-1)^2}\left(\frac{m2^2}{\beta^3}+\frac{m3^2}{\gamma^3}\right)\right.$$

$$\times[6\gamma(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1)(\beta m1^2+\alpha m2^2)+6\alpha\beta m3^2(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1)$$

$$-\alpha\beta s(5\alpha^4+\alpha^3(13\beta+18\gamma-13)+\alpha^2(11\beta^2+\beta(48\gamma-22)+12\gamma^2-41\gamma+11)$$

$$+\alpha(3\beta^3+\beta^2(42\gamma-9)+\beta(24\gamma^2-74\gamma+9)-18\gamma^2+32\gamma-3)+\gamma(12\beta^3+3\beta^2(4\gamma-11)-6\beta(3\gamma-5)+8\gamma-9))]\bigg\}$$

$$+\int_0^1 d\alpha\int_0^{1-\alpha}d\beta\int_0^{1-\alpha-\beta}d\gamma\,\delta\left(s-\frac{m_1^2}{\alpha}-\frac{m_2^2}{\beta}-\frac{m_3^2}{1-\alpha-\beta}\right)\frac{1}{1536\pi^6\alpha^3\beta^3\gamma^3(\alpha+\beta-1)^2}$$

$$\times\{(\gamma^3(\beta^3 m1^2+\alpha^3 m2^2)+\alpha^3\beta^3 m3^2)(m1m2+\alpha\beta s)[2\alpha^4+\alpha^3(5\beta+3\gamma-5)+\alpha^2(4\beta^2+\beta(7\gamma-8)+(\gamma-2)^2)$$

$$+\alpha(\beta^3+\beta^2(5\gamma-3)+\beta(2\gamma^2-6\gamma+3)+\gamma-1)+\beta\gamma(\beta^2+\beta(\gamma-2)+1)]\}$$

(A4)

$$\rho^{\langle \bar{q}Gq \rangle}(s) = \frac{1}{\pi} \text{Im}\Pi^{\langle \bar{q}Gq \rangle}(q^2)$$

$$= \langle \bar{q}Gq \rangle \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \frac{-m3s(m1m2 + \alpha\beta s)}{64\pi^4} \delta\left(s - \frac{m_1^2}{\alpha} - \frac{m_2^2}{\beta} - \frac{m_3^2}{1-\alpha-\beta}\right)$$

$$+ \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \theta\left(s - \frac{m_1^2}{\alpha} - \frac{m_2^2}{\beta} - \frac{m_3^2}{1-\alpha-\beta}\right) \frac{-m3}{64\pi^4(\alpha+\beta-1)^2}[-2\beta m1^2(3\alpha^2 + \alpha(6\beta - 7) + 3\beta^2 - 7\beta + 4)$$

$$+ m1m2(2\alpha + 2\beta - 3)(\alpha + \beta - 1) + \alpha(\beta(2m3^2(3\alpha + 3\beta - 4) + 3s(4\alpha + 4\beta - 5)(\alpha + \beta - 1))$$

$$- 2m2^2(3\alpha^2 + \alpha(6\beta - 7) + 3\beta^2 - 7\beta + 4))]. \tag{A5}$$

For the interpolating current $J_{3\mu}$, the spectral functions are

$$\rho^{\text{OPE}}(s) = \rho^{\text{pert}}(s) + \rho^{\langle \bar{q}q \rangle}(s) + \rho^{\langle GG \rangle}(s) + \rho^{\langle \bar{q}Gq \rangle}(s) \tag{A6}$$

$$\rho^{\text{pert}}(s) = \frac{1}{\pi} \text{Im}\Pi^{\text{pert}}(s)$$

$$= \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \int_0^{1-\alpha-\beta} d\gamma \theta\left(s - \frac{m_1^2}{\alpha} - \frac{m_2^2}{\beta} - \frac{m_2^2}{\gamma}\right) \frac{-\alpha\beta}{256\pi^6} \left(\frac{m_1^2}{\alpha} + \frac{m_2^2}{\beta} + \frac{m_2^2}{\gamma} - s\right)^2 \frac{1}{(\alpha+\beta-1)}$$

$$\times \left\{ 3\gamma(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1)\left(\frac{m1^2}{\alpha} + \frac{m2^2}{\beta} + \frac{m3^2}{\gamma} - s\right)^2 \right.$$

$$- 2s\left(\frac{m1^2}{\alpha} + \frac{m2^2}{\beta} + \frac{m3^2}{\gamma} - s\right)[5\alpha^4 + \alpha^3(13\beta + 12\gamma - 13) + \alpha^2(11\beta^2 + \beta(30\gamma - 22) + 6\gamma^2 - 23\gamma + 11)$$

$$+ \alpha(3\beta^3 + 3\beta^2(8\gamma - 3) + \beta(12\gamma^2 - 38\gamma + 9) - 6\gamma^2 + 14\gamma - 3) + \gamma(6\beta^3 + 3\beta^2(2\gamma - 5) - 6\beta(\gamma - 2) + 2\gamma - 3)]$$

$$\left. + 6s^2(\alpha^2 + \alpha(\beta + \gamma - 1) + \beta\gamma)(2\alpha^2 + \alpha(3\beta + \gamma - 3) + \beta^2 + \beta(\gamma - 2) + 1) \right\}$$

$$+ \int d\alpha \int d\beta \int d\gamma \frac{1}{128\pi^6} m1m2\left(\frac{m1^2}{\alpha} + \frac{m2^2}{\beta} + \frac{m3^2}{\gamma} - s\right)$$

$$\times \left[2(\alpha+\beta+\gamma-1)\gamma\left(\frac{m1^2}{\alpha} + \frac{m2^2}{\beta} + \frac{m3^2}{\gamma} - s\right) \right.$$

$$\left. - \frac{3s(\alpha^2 + \alpha(\beta + \gamma - 1) + \beta\gamma)(2\alpha^2 + \alpha(3\beta + \gamma - 3) + \beta^2 + \beta(\gamma - 2) + 1)}{(\alpha+\beta-1)^2} \right]^2 \tag{A7}$$

$$\rho^{\langle \bar{q}q \rangle}(s) = \frac{1}{\pi} \text{Im}\Pi^{\langle \bar{q}q \rangle}(q^2)$$

$$= \langle \bar{q}q \rangle \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \theta\left(s - \frac{m_1^2}{\alpha} - \frac{m_2^2}{\beta} - \frac{m_3^2}{1-\alpha-\beta}\right) \left[\frac{m_1 m_2 m_3}{16\pi^4} \left(\frac{m_1^2}{\alpha} + \frac{m_2^2}{\beta} + \frac{m_3^2}{1-\alpha-\beta} - s\right) \right.$$

$$\left. - \frac{m_3[(\alpha+\beta-1)(\alpha m_2^2 + \beta m_1^2 - \alpha\beta s) - \alpha\beta m_3^2][(\alpha+\beta-1) \quad (\alpha m_2^2 + \beta m_1^2 - 2\alpha\beta s) - \alpha\beta m_3^2]}{16\pi^4\alpha\beta(1-\alpha-\beta)^2} \right] \tag{A8}$$

$$\rho^{\langle GG\rangle}(s) = \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \int_0^{1-\alpha-\beta} d\gamma \,\theta\left(s - \frac{m_1^2}{\alpha} - \frac{m_2^2}{\beta} - \frac{m_2^2}{\gamma}\right) \frac{-\langle g_s^2 GG\rangle}{3072\pi^6}$$

$$\times \left\{ -\frac{4\gamma m1^3 m2(\alpha+\beta+\gamma-1)}{\alpha^3} - \frac{4\gamma m1 m2^3(\alpha+\beta+\gamma-1)}{\beta^3} - \frac{4 m1 m2 m3^2(\alpha+\beta+\gamma-1)}{\gamma^2} \right.$$

$$+\frac{1}{\alpha^3(\alpha+\beta-1)^2} 2 m1^2 [6\gamma(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1)(\beta m1^2 + \alpha m2^2) + 6\alpha\beta m3^2(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1)$$

$$-\alpha\beta s(5\alpha^4 + \alpha^3(13\beta+18\gamma-13) + \alpha^2(11\beta^2 + \beta(48\gamma-22) + 12\gamma^2 - 41\gamma + 11)$$

$$+\alpha(3\beta^3 + \beta^2(42\gamma-9) + \beta(24\gamma^2 - 74\gamma + 9) - 18\gamma^2 + 32\gamma - 3) + \gamma(12\beta^3 + 3\beta^2(4\gamma-11) - 6\beta(3\gamma-5) + 8\gamma - 9))]$$

$$-\frac{1}{\alpha^3\beta^3\gamma(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1)} 3 m2(2\alpha^3\gamma + \alpha^2(\beta^2 + 2(\gamma-1)\gamma) - 2\alpha\beta(\gamma-1)\gamma + 2\beta^2\gamma(\beta+\gamma-1))$$

$$\times[2\gamma(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1)(\beta m1^2 + \alpha m2^2) + 2\alpha\beta m3^2(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1) - \alpha\beta s$$

$$\times(2\alpha^4 + 5\alpha^3(\beta+\gamma-1) + \alpha^2(4\beta^2 + \beta(13\gamma-8) + 3\gamma^2 - 10\gamma + 4)$$

$$+\alpha(\beta^3 + \beta^2(11\gamma-3) + 3\beta(2\gamma^2 - 6\gamma + 1) - 4\gamma^2 + 7\gamma - 1) + \gamma(3\beta^3 + \beta^2(3\gamma-8) + \beta(7-4\gamma) + 2(\gamma-1)))]$$

$$+\left[\frac{\begin{aligned}&s^2(2\alpha^4 + \alpha^3(5\beta+3\gamma-5) + \alpha^2(4\beta^2 + \beta(7\gamma-8) + (\gamma-2)^2) + \alpha(\beta^3 + \beta^2(5\gamma-3) + \beta(2\gamma^2 - 6\gamma + 3) + \gamma - 1) + \\ &\qquad \beta\gamma(\beta^2 + \beta(\gamma-2) + 1))\end{aligned}}{(\alpha+\beta-1)^2}\right.$$

$$+\frac{s}{\alpha\beta\gamma(\alpha+\beta-1)^2}(-\gamma(\beta(m1^2 - \alpha s) + \alpha m2^2) - \alpha\beta m3^2)$$

$$\times(5\alpha^4 + \alpha^3(13\beta+12\gamma-13) + \alpha^2(11\beta^2 + \beta(30\gamma-22) + 6\gamma^2 - 23\gamma + 11)$$

$$+\alpha(3\beta^3 + 3\beta^2(8\gamma-3) + \beta(12\gamma^2 - 38\gamma + 9) - 6\gamma^2 + 14\gamma - 3) + \gamma(6\beta^3 + 3\beta^2(2\gamma-5) - 6\beta(\gamma-2) + 2\gamma - 3))$$

$$\left.+\frac{3(\alpha+\beta+\gamma-1)(\gamma(\beta(m1^2 - \alpha s) + \alpha m2^2) + \alpha\beta m3^2)^2}{\alpha^2\beta^2\gamma}\right] + \frac{2}{(\alpha+\beta-1)^2}\left(\frac{m2^2}{\beta^3} + \frac{m3^2}{\gamma^3}\right)$$

$$\times[6\gamma(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1)(\beta m1^2 + \alpha m2^2) + 6\alpha\beta m3^2(\alpha+\beta-1)^2(\alpha+\beta+\gamma-1)$$

$$-\alpha\beta s(5\alpha^4 + \alpha^3(13\beta+18\gamma-13) + \alpha^2(11\beta^2 + \beta(48\gamma-22) + 12\gamma^2 - 41\gamma + 11)$$

$$\left.+\alpha(3\beta^3 + \beta^2(42\gamma-9) + \beta(24\gamma^2 - 74\gamma + 9) - 18\gamma^2 + 32\gamma - 3) + \gamma(12\beta^3 + 3\beta^2(4\gamma-11) - 6\beta(3\gamma-5) + 8\gamma - 9))]\right\}$$

$$+\int_0^1 d\alpha \int_0^{1-\alpha} d\beta \int_0^{1-\alpha-\beta} d\gamma\,\delta\left(s - \frac{m_1^2}{\alpha} - \frac{m_2^2}{\beta} - \frac{m_3^2}{1-\alpha-\beta}\right) \frac{1}{1536\pi^6\alpha^3\beta^3\gamma^3(\alpha+\beta-1)^2}$$

$$\times\{(\gamma^3(\beta^3 m1^2 + \alpha^3 m2^2) + \alpha^3\beta^3 m3^2)(m1 m2 + \alpha\beta s)[2\alpha^4 + \alpha^3(5\beta+3\gamma-5) + \alpha^2(4\beta^2 + \beta(7\gamma-8) + (\gamma-2)^2)$$

$$+\alpha(\beta^3 + \beta^2(5\gamma-3) + \beta(2\gamma^2 - 6\gamma + 3) + \gamma - 1) + \beta\gamma(\beta^2 + \beta(\gamma-2) + 1)]\} \tag{A9}$$

$$\rho^{\langle\bar{q}Gq\rangle}(s) = \frac{1}{\pi}\text{Im}\Pi^{\langle\bar{q}Gq\rangle}(q^2)$$

$$= \langle\bar{q}Gq\rangle \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \frac{-m3 s(m1 m2 + \alpha\beta s)}{64\pi^4}\delta\left(s - \frac{m_1^2}{\alpha} - \frac{m_2^2}{\beta} - \frac{m_3^2}{1-\alpha-\beta}\right)$$

$$+\int_0^1 d\alpha \int_0^{1-\alpha} d\beta\,\theta\left(s - \frac{m_1^2}{\alpha} - \frac{m_2^2}{\beta} - \frac{m_3^2}{1-\alpha-\beta}\right) \frac{-m3}{64\pi^4(\alpha+\beta-1)^2}[-2\beta m1^2(3\alpha^2 + \alpha(6\beta-7) + 3\beta^2 - 7\beta + 4)$$

$$+ m1 m2(2\alpha + 2\beta - 3)(\alpha+\beta-1) + \alpha(\beta(2 m3^2(3\alpha + 3\beta - 4) + 3s(4\alpha + 4\beta - 5)(\alpha+\beta-1))$$

$$- 2 m2^2(3\alpha^2 + \alpha(6\beta-7) + 3\beta^2 - 7\beta + 4))]. \tag{A10}$$

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