# Strong decays of exotic and nonexotic heavy baryons in the chiral quark-soliton model

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In the large  $N_c$  limit both heavy and light baryons are described by the universal mean field, which allows us to relate the properties of heavy baryons to light ones. With the only input from the decays of light octet baryons (due to the universality of the chiral mean field), excellent description of strong decays of both charm and bottom sextets is obtained. The parameter-free prediction for the widths of exotic antidecapentaplet  $(\overline{15})$  baryons is also made. The exotic heavy baryons should be anomalously narrow despite the large phase space available. In particular, the widths of  $\Omega_c(3050)$  and  $\Omega_c(3119)$ , interpreted as members of 15-plet, are very small: 0.48 MeV and 1.12 MeV respectivly. This result is in very good agreement with the measurements of the LHCb Collaboration and provides natural and parameter-free explanation of the LHCb observation that  $\Omega_c(3050)$  and  $\Omega_c(3119)$  have anomalously small widths among five recently observed states.

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### I. INTRODUCTION

In a recent paper [\[1\]](#page-6-0) we have proposed to interpret two narrow  $\Omega_c$  resonances reported recently by the LHCb Collaboration [\[2\]](#page-6-1) as exotic pentaquark states belonging to the SU(3) representation  $\overline{15}$  (antidecapentaplet) [\[3\]](#page-6-2). To this end we have used the chiral quark soliton model  $(\chi$ QSM) [\[4\]](#page-6-3) (for a review see Ref. [\[5\]](#page-6-4) and references therein) modified in the spirit of heavy quark symmetry [\[6](#page-6-5)–8] to accommodate one heavy quark [\[3,9\].](#page-6-2) The  $\chi$ QSM is based on an old argument by Witten [\[10\],](#page-6-6) which says that in the limit of a large number of colors ( $N_c \rightarrow \infty$ ),  $N_c$ relativistic valence quarks generate chiral mean fields represented by a distortion of a Dirac sea that in turn interact with the valence quarks themselves. In this way a self-consistent configuration called a soliton is formed. The mean fields exhibit so-called hedgehog symmetry, which means that neither quark spin  $(S_q)$  nor quark isospin  $(T_q)$  are "good" quantum numbers. Instead a grand spin  $K = S_q + T_q$  is a good quantum number.

In order to project out spin and isospin quantum numbers one has to rotate the soliton, both in flavor and configuration spaces. These rotations are then subsequently quantized semiclassically and the collective Hamiltonian is computed. The model predicts rotational baryon spectra that satisfy the following selection rules:

- (i) allowed SU(3) representations must contain states with hypercharge  $Y' = N_c/3$ ,
- (ii) the isospin T' of the states with  $Y' = N_c/3$  couples with the soliton spin  $J$  to a singlet:  $T' + J = 0$ .

In the case of light positive parity baryons the lowest allowed representations are 8 of spin  $1/2$ , 10 of spin 3/2, and also exotic pentaquark representation  $\overline{10}$  of spin 1/2 with the lightest state corresponding to the putative  $\Theta^+(1540)$ . Chiral models in general predict that pentaquarks are light [\[11,12\]](#page-6-7) and—in some specific models—narrow [\[12\]](#page-6-8).

In order to construct a heavy baryon in the  $\chi$ QSM we have to strip off one light quark from the valence level and quantize the soliton with a new constraint  $Y'=(N_c - 1)/3$ , which modifies the above selection rules in the following way:

- (i) allowed SU(3) representations must contain states with hypercharge  $Y'=(N_c - 1)/3$ ,
- (ii) the isospin T' of the states with  $Y'=(N_c-1)/3$ couples with the soliton spin  $J$  to a singlet:  $T' + J = 0.$

This  $N_c - 1$  light quark configuration is then coupled to a heavy quark Q to form a color singlet. The lowest allowed SU(3) representations are shown in Fig. [1.](#page-1-0) They correspond to the soliton in representation in  $\bar{3}$  of spin 0 and to 6 of spin 1. Therefore the baryons constructed from such a soliton and a heavy quark form an SU(3) antitriplet of spin  $1/2$  and two sextets of spin  $1/2$  and  $3/2$  that are

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<span id="page-1-0"></span>

FIG. 1. Lowest lying SU(3) flavor representations allowed by the constraint  $Y' = 2/3$ . The first *exotic* representation,  $\overline{15}$ contains the putative pentaquark states  $\Omega_c$  with  $\Omega_c^0$  marked in red.

subject to a hyperfine splitting. The next allowed representation of the rotational excitations corresponds to the exotic  $\overline{15}$  of spin 0 or spin 1 [\[1\].](#page-6-0) The spin 1 soliton has lower mass and when it couples with a heavy quark it forms spin  $1/2$  or  $3/2$  exotic multiplets that should be hyperfine split similarly to the ground state sextets by ∼70 MeV. In Ref. [\[1\]](#page-6-0) we have proposed to interpret two LHCb states:  $\Omega_c^0(3050)$  and  $\Omega_c^0(3119)$  as  $1/2^+$  and  $3/2^+$  pentaquarks<br>belonging to the SU(3)  $\overline{15}$ , As can be seen from Eig. 1 they belonging to the SU(3)  $\overline{15}$  $\overline{15}$  $\overline{15}$ . As can be seen from Fig. 1 they belong to the isospin triplets, and therefore should have charged partners of the same mass, which allows, in principle, for rather straightforward experimental verification of this interpretation.

In the present paper we calculate strong decay widths of nonexotic and exotic heavy quark baryons (both charm and bottom) in an approach proposed many years ago by Adkins et al. [\[13\]](#page-6-9) and expanded in Ref. [\[12\]](#page-6-8), which is based on the Goldberger-Treiman relation where strong decay constants are expressed in terms of the axial-vector current couplings. We show that by fixing these axial-vector current couplings from the hyperon decays in the light sector, we can predict strong decay widths of heavy baryons with only one free parameter related to the modification of these couplings due to the fact that the soliton is constructed from  $N_c - 1$  rather than  $N_c$  light quarks. We test our approach against experimentally known charm and bottom baryon decay widths, and then show that the decay widths of  $\Omega_c^0(3050)$  and  $\Omega_c^0(3119)$  are small and compatible with the LHCb measurements. Overall agreecompatible with the LHCb measurements. Overall agreement of the predicted decay widths with experiment is more than satisfactory and exceeds the expected model accuracy, which is believed to be at the level of 10%–20%.

The rotational states described above correspond to positive parity. Negative parity states generated by the soliton configurations with one light quark excited to the valence level from the Dirac sea have been discussed in Ref. [\[1\].](#page-6-0) It has been shown that the remaining three LHCb  $\Omega_c^0$  states can be accommodated in such an approach. The formalism used in the present paper has to be rather strongly modified to describe negative parity state decay widths, and we plan to address this issue in a separate publication.

Other authors have also considered a possibility that at least some of the LHCb  $\Omega_c$  states may be interpreted as pentaquarks [14–[17\]](#page-6-10) in different variants of the quark (or quark-diquark) models. However, the states considered in these papers have negative parity and are isospin singlets. A simpler interpretation that they are p-wave or radial and p-wave excitations of the ss-diquark c-quark system has been put forward in Refs. [\[18](#page-6-11)–23]. Both radial and p-wave excitations have been also studied in the framework of the QCD sum rules [\[24](#page-6-12)–28], in a phenomenological approach [\[29\]](#page-7-0) and on the lattice [\[30\].](#page-7-1) A nonperturbative holographic QCD has been applied to investigate heavy, regular, and exotic baryons in Refs. [\[31\]](#page-7-2). The variety of quantum number assignments proposed in the above references has been possible because all the above approaches (including the one in the present paper) suffer from systematic uncertainties that exceed tiny experimental errors of the heavy baryon masses. An ongoing analysis of spin and parity of the LHCb  $\Omega_c$  states is therefore of utmost importance to discriminate different theoretical models.

The paper is organized as follows. In Sec. [II](#page-1-1) we describe briefly the formalism for description of the baryon strong decays in the  $\chi$ QSM. In the following Sec. [III](#page-4-0) we test the mean-field picture of heavy baryons against the known strong decays of ground-state baryon sextets. In Sec. [IV](#page-5-0) the parameter-free prediction for decays of exotic  $\overline{15}$ -plet baryons is made. Particular attention is paid to strong decays of  $\Omega_c(3050)$  and  $\Omega_c(3119)$ , which we interpret as members of the exotic antidecapentaplet. Conclusions are presented in Sec. [V.](#page-5-1)

## <span id="page-1-1"></span>II. HEAVY BARYONS AND THEIR STRONG DECAYS IN THE CHIRAL QUARK-SOLITON MODEL

<span id="page-1-2"></span>In our picture the  $\bar{\bf 3}$ , sextet, and exotic  $\bar{\bf 15}$  baryons shown in Fig. [1](#page-1-0) are rotational excitations of the meson mean field, which is essentially the same as for light baryons. The corresponding wave function of the light sector is given in terms of the Wigner rotational  $D(A)$  matrices,

$$
\Psi_{(B; -Y'SS_3)}^{(\mathcal{R};B)}(A) = \langle A|\mathcal{R}, B, (-Y', S, S_3) \rangle \n= \sqrt{\dim(\mathcal{R})} (-)^{S_3 - Y'/2} D_{(Y,T,T_3)(Y',S,-S_3)}^{(\mathcal{R})*}(A),
$$
\n(1)

where  $R$  denotes the SU(3) representation of the light sector,  $B = (Y, T, T_3)$  stands for the SU(3) quantum numbers of a baryon in question, and the second index of the D function,  $(Y', S, -S_3)$ , corresponds to the soliton<br>spin For the heavy baryons  $Y' - (N - 1)/3$ ,  $A(t)$  denotes spin. For the heavy baryons  $Y'=(N_c - 1)/3$ . A $(t)$  denotes relative configuration space—SU(3) group space rotation matrix.

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<span id="page-2-0"></span>The total wave function of a heavy baryon of spin  $J$  is constructed by coupling [\(1\)](#page-1-2) to a heavy quark ket  $|1/2, s_3\rangle$ , with a pertinent SU(2) Clebsch-Gordan coefficient,

$$
|\mathcal{R}, B, J, J_3\rangle = \sum_{S_3, s_3} \begin{pmatrix} S & 1/2 & J \\ S_3 & s_3 & J_3 \end{pmatrix}
$$

$$
\times |1/2, s_3\rangle |\mathcal{R}, B, (-Y', S, S_3)\rangle. \quad (2)
$$

The masses of various multiplets are obtained by sandwiching the collective Hamiltonian between the wave function [\(2\);](#page-2-0) see for details Refs. [\[1,9\].](#page-6-0) To calculate the decays of the heavy baryons one has to sandwich the corresponding decay operator between the wave functions [\(2\)](#page-2-0). Following Ref. [\[12\]](#page-6-8) we use in this paper the decay operator describing the emission of a p-wave pseudoscalar meson  $\varphi$ , as in the case of regular baryons, with possible rescaling of the coefficients  $G_i$ (see below),

<span id="page-2-1"></span>
$$
\mathcal{O}_{\varphi} = \frac{3}{M_1 + M_2} \sum_{i=1,2,3} \left[ G_0 D_{\varphi i}^{(8)} - G_1 d_{ibc} D_{\varphi b}^{(8)} \hat{S}_c \right] - G_2 \frac{1}{\sqrt{3}} D_{\varphi 8}^{(8)} \hat{S}_i \right] p_i.
$$
 (3)

We are considering decays  $B_1 \rightarrow B_2 + \varphi$ , where  $M_{1,2}$ denote masses of the initial and final baryons respectively and  $p_i$  is the c.m. momentum of the outgoing meson of mass  $m$ ,

$$
|\vec{p}| = p = \frac{1}{2M_1} \sqrt{(M_1^2 - (M_2 + m)^2)(M_1^2 - (M_2 - m)^2)}.
$$
\n(4)

The decay width is related to the matrix element of  $\mathcal{O}_{\varphi}$  squared, summed over the final and averaged over the initial spin and isospin denoted as  $[...]^2$ ; see the appendix of Ref. [12] for details of the corresponding appendix of Ref. [\[12\]](#page-6-8) for details of the corresponding calculations,

$$
\Gamma_{B_1 \to B_2 + \varphi} = \frac{1}{2\pi} \overline{\langle B_2 | \mathcal{O}_{\varphi} | B_1 \rangle^2} \frac{M_2}{M_1} p. \tag{5}
$$

Here factor  $M_2/M_1$  is the same as in heavy baryon chiral perturbation theory (HBChPT); see, e.g., Ref. [\[32\].](#page-7-3) This factor arises because in the heavy quark effective theory and in HBChPT the velocities of  $B_1$  and  $B_2$  are the same up to corrections of order  $O(1/m<sub>O</sub>)$ , which we neglect.

The pseudoscalar meson-baryon couplings can be related to the transition  $(B_1 \rightarrow B_2)$  axial-vector constants with the help of the Goldberger-Treiman relation; see Ref. [\[33\]](#page-7-4) for the derivation in the case of heavy baryons.

Using the Goldberger-Treiman relation we obtain for the couplings  $G_{0,1,2}$ 

$$
\{G_0, G_1, G_2\} = \frac{M_1 + M_2}{2F_\varphi} \frac{1}{3} \{-a_1, a_2, a_3\},\tag{6}
$$

<span id="page-2-2"></span>where constants<sup>1</sup>  $a_{1,2,3}$  enter the definition of the axialvector current [\[34\]](#page-7-5) and have been extracted from the semileptonic decays of the baryon octet in Ref. [\[35\],](#page-7-6)

$$
a_1 = -3.509 \pm 0.011,
$$
  
\n
$$
a_2 = 3.437 \pm 0.028,
$$
  
\n
$$
a_3 = 0.604 \pm 0.030.
$$
\n(7)

<span id="page-2-5"></span>For the decay constants  $F_{\varphi}$  we have chosen the convention in which  $F_\pi = 93$  MeV and  $F_K = 1.2 F_\pi = 112$  MeV.

The final formula for the decay width in terms of axialvector constants  $a_{1,2,3}$  reads as follows:

$$
\Gamma_{B_1 \to B_2 + \varphi} = \frac{1}{72\pi} \frac{p^3}{F_{\varphi}^2} \frac{M_2}{M_1} G_{\mathcal{R}_1 \to \mathcal{R}_2}^2
$$
\n
$$
\times 3 \frac{\dim \mathcal{R}_2}{\dim \mathcal{R}_1} \begin{bmatrix} 8 & \mathcal{R}_2 & \mathcal{R}_1 \\ 01 & Y'S_2 & Y'S_1 \end{bmatrix}^2
$$
\n
$$
\times \begin{bmatrix} 8 & \mathcal{R}_2 & \mathcal{R}_1 \\ Y_{\varphi} T_{\varphi} & Y_2 T_2 & Y_1 T_1 \end{bmatrix}^2.
$$
\n(8)

<span id="page-2-4"></span>Here  $\mathcal{R}_{1,2}$  are the SU(3) representations of the initial and final baryons;  $[\dots]$ ...] are the SU(3) isoscalar factors. The decay constants  $G_{\mathcal{R}_1 \to \mathcal{R}_2}$  are calculated from the matrix elements of [\(3\)](#page-2-1) as

$$
\overline{\mathbf{15}}_1 \rightarrow \overline{\mathbf{3}}_0 \quad G_{\overline{\mathbf{3}}} = -a_1 - \frac{1}{2} a_2 = 0.44,
$$
  

$$
\overline{\mathbf{15}}_1 \rightarrow \mathbf{6}_1 \quad G_6 = -a_1 - \frac{1}{2} a_2 - a_3 = -0.16,
$$
  

$$
\mathbf{6}_1 \rightarrow \overline{\mathbf{3}}_0 \quad H_{\overline{\mathbf{3}}} = -a_1 + \frac{1}{2} a_2 = 3.88,
$$
 (9)

<span id="page-2-3"></span>where numerical values have been calculated with the help of Eq. [\(7\).](#page-2-2) With these definitions of the couplings the formulas for the decay widths averaged over the initial isospin and summed over the final isospin read as follows:

<sup>&</sup>lt;sup>1</sup>For the reader's convenience we give the relations of the constants  $a_{1,2,3}$  to nucleon axial charges in the chiral limit:  $g_A = \frac{7}{30}(-a_1 + \frac{1}{2}a_2 + \frac{1}{14}a_3), \quad g_A^{(0)} = \frac{1}{2}a_3, \quad g_A^{(8)} = \frac{1}{10\sqrt{3}}(-a_1 +$  $\frac{1}{2}a_2+\frac{1}{2}a_3$ ).

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$$
\Gamma_{\Sigma(\mathbf{6}_{1})\to\Lambda(\bar{3}_{0})+\pi} = \frac{1}{72\pi} \frac{p^{3}}{F_{\pi}^{2}} \frac{M_{\Lambda(\bar{3}_{0})}}{M_{\Sigma(\mathbf{6}_{1})}} H_{\bar{3}}^{2} \frac{3}{8},
$$
\n
$$
\Gamma_{\Xi(\mathbf{6}_{1})\to\Xi(\bar{3}_{0})+\pi} = \frac{1}{72\pi} \frac{p^{3}}{F_{\pi}^{2}} \frac{M_{\Xi(\bar{3}_{0})}}{M_{\Xi(\mathbf{6}_{1})}} H_{\bar{3}}^{2} \frac{9}{32},
$$
\n
$$
\Gamma_{\Omega(\overline{15}_{1})\to\Xi(\bar{3}_{0})+K} = \frac{1}{72\pi} \frac{p^{3}}{F_{K}^{2}} \frac{M_{\Xi(\bar{3}_{0})}}{M_{\Omega(\overline{15}_{1})}} G_{\bar{3}}^{2} \frac{3}{10},
$$
\n
$$
\Gamma_{\Omega(\overline{15}_{1})\to\Omega(\mathbf{6}_{1})+\pi} = \frac{1}{72\pi} \frac{p^{3}}{F_{\pi}^{2}} \frac{M_{\Omega(\mathbf{6}_{1})}}{M_{\Omega(\overline{15}_{1})}} G_{\mathbf{6}}^{2} \frac{4}{15},
$$
\n
$$
\Gamma_{\Omega(\overline{15}_{1})\to\Xi(\mathbf{6}_{1})+K} = \frac{1}{72\pi} \frac{p^{3}}{F_{K}^{2}} \frac{M_{\Xi(\mathbf{6}_{1})}}{M_{\Omega(\overline{15}_{1})}} G_{\mathbf{6}}^{2} \frac{2}{15}.
$$
\n(10)

When we need a decay width for a specific isospin combination, the widths of Eqs. [\(10\)](#page-2-3) have to be multiplied by a pertinent SU(2) Clebsch-Gordan coefficient.

Note that in the  $\chi$ QSM couplings [\(7\)](#page-2-2) are expressed in terms of the inertia parameters [\[34\]](#page-7-5),

$$
a_1 = A_0 - \frac{B_1}{I_1}
$$
,  $a_2 = 2\frac{A_2}{I_2}$ ,  $a_3 = 2\frac{A_1}{I_1}$ . (11)

Since all inertia parameters scale as  $N_c$ , we see that formally  $a_1$  contains both leading and the first subleading term in  $N_c$ , whereas  $a_{2,3}$  scale as  $N_c^0$ . This can be the best seen in the nonrelativistic (NR) limit for small soliton size, where [\[36\]](#page-7-7)

<span id="page-3-0"></span>
$$
A_0 \to -N_c
$$
,  $\frac{B_1}{I_1} \to 2$ ,  $\frac{A_2}{I_2} \to 2$ ,  $\frac{A_1}{I_1} \to 1$  (12)

<span id="page-3-2"></span>or

$$
a_1 \to -(N_c + 2), \qquad a_2 \to 4, \qquad a_3 \to 2. \tag{13}
$$

Remember that in this limit the axial-vector coupling constant

$$
g_A = \frac{7}{30} \left( -a_1 + \frac{1}{2} a_2 + \frac{1}{14} a_3 \right) \to \frac{5}{3},\tag{14}
$$

which is equal to the naive quark model result for  $g_A$ , whereas for the phenomenological values [\(7\)](#page-2-2)  $g_A = 1.23$ .

In the case of heavy baryons all inertia parameters should be rescaled by approximately<sup>2</sup>  $(N_c - 1)/N_c$ , which does not change the scaling of their ratios; however it does change the value of  $a_1$  (strictly speaking the  $A_0$  part of  $a_1$ ). Therefore for heavy baryons we should use

$$
A_0 \to \tilde{A}_0 = \frac{N_c - 1}{N_c} A_0.
$$
 (15)

Unfortunately from the fits to the experimental data we do not know separately the values of  $A_0$  and  $B_1/I_1$ . We know only these values in the NR limit [\(12\).](#page-3-0) Making the rather bold assumption that NR relation  $B_1 = 2A_1$  holds also for realistic soliton sizes, we approximate  $B_1/I_1 \sim a_3$ , which gives  $A_0 = a_1 + a_3$ . Therefore, following Eq. [\(15\),](#page-3-1) we make the following replacement:

<span id="page-3-3"></span>
$$
a_1 \rightarrow \tilde{a}_1 = \left[\frac{N_c - 1}{N_c}(a_1 + a_3) - a_3\right] \sigma.
$$
 (16)

Here  $\sigma$  is a correction factor that takes into account possible deviations from the assumption that  $B_1 = 2A_1$  and possible deviation of the rescaling factor from  $(N_c - 1)/N_c$ . The parameter  $\sigma$  characterizes the modification of the mean field when one goes from  $N_c$  quarks in light baryons to  $N_c - 1$  quarks in heavy baryons. In the ideal case  $\sigma$  should be close to unity; however in practice, as we see in the following, a 15% correction is required to get satisfactory description of the decay widths. Such small modification of the mean field is fully compatible with the expected size of  $1/N_c$  corrections.

It is interesting to calculate the decay constants [\(9\)](#page-2-4) in the nonrelativistic quark model limit [\(13\)](#page-3-2) with  $a_1 \rightarrow -(N_c + 1)$ . One can see that in this limit  $G_6 = 0$ . This means that the decay channels of the putative heavy pentaquarks to the sextets should be strongly suppressed. This situation is very similar to the suppression of  $\Theta^+(1530)$  decay width [\[12\].](#page-6-8)

One should remember that in the large  $N_c$  limit flavor representations of the light sector should be generalized [\[37](#page-7-8)–39], and in the present case the standard generalization takes the following form [\[40\]](#page-7-9):

$$
\begin{aligned}\n\text{"}\bar{\mathbf{3}}\text{"} &= (0, q + 1), \\
\text{"}\mathbf{6}\text{"} &= (2, q), \\
\text{"}\bar{\mathbf{15}}\text{"} &= (1, q + 2),\n\end{aligned} \tag{17}
$$

with  $q = (N_c - 3)/2$ . With this generalization the pertinent  $SU(3)$  Clebsch-Gordan coefficients acquire  $N_c$  dependence. For example

$$
G_{\bar{3}} = -a_1 - \frac{N_c - 1}{4}a_2,\tag{18}
$$

which means that  $G_3$  is suppressed in the large  $N_c$  NR limit [\(13\)](#page-3-2), since the leading  $N_c$  terms cancel out. This cancellation is similar as in the case of high-dimensional exotic representations of light baryons [\[39\]](#page-7-10). We see that large  $N_c$ together with the arguments based on the nonrelativistic limit explains the numerical hierarchy of the decay

<span id="page-3-1"></span><sup>&</sup>lt;sup>2</sup>Strictly speaking rescaling by a factor  $(N_c - 1)/N_c$  should only for quantities dominated by valence levels. As the work well only for quantities dominated by valence levels. As the contribution of the sea quarks to some quantities can be sizeable one may expect 10%–20% variations of that rescaling factor.

<span id="page-4-0"></span>couplings [\(9\).](#page-2-4) Full  $N_c$  dependence of the mass splittings and decay widths is discussed elsewhere [\[40\]](#page-7-9).

## III. DECAY WIDTHS OF SEXTET HEAVY BARYONS

A number of decay widths are measured for heavy baryons. In Ref. [\[32\]](#page-7-3) decays of charm sextet  $\Sigma_c$ , both of spin  $1/2$  and  $3/2$ , have been fitted with the help of the formula analogous to [\(10\)](#page-2-3) in terms of a single coupling  $q_2$ . The updated phenomenological value is presently  $g_2$  = 0.56 with a 5% error. Coupling  $g_2$  can be expressed in terms of  $H_{\bar{3}}$ ,

$$
|g_2| = \frac{1}{4\sqrt{3}} H_{\bar{3}} = \frac{1}{4\sqrt{3}} \left( -\tilde{a}_1 + \frac{1}{2} a_2 \right). \tag{19}
$$

To fit the phenomenological value of 0.56 we need a correction factor  $\sigma = 0.85$ . Other than that there is no extra freedom and all decay widths are genuine predictions of the present model.

Two comments are in order here. Decay widths of different  $T_3$  states show small isospin violations, which are mainly due to a small phase volume and hence due to the sensitivity to the mass difference of  $\pi^{\pm} - \pi^{0}$ . Decay couplings are calculated in the isospin symmetric limit. Secondly, different decays of particles within the same initial and final SU(3) multiplets ( $\mathcal{R}_1 \rightarrow \mathcal{R}_2$ ) are related by the SU(3) symmetry, which is more general than the present

<span id="page-4-1"></span>TABLE I. Charm sextet baryon decay widths in MeV. Experimental data are taken from Particle Data Group [\[41\].](#page-7-11)

$1.89^{+0.09}_{-0.18}$ 1.93 $1 \quad \Sigma_c^{++}(\mathbf{6}_1, 1/2) \rightarrow \Lambda_c^{+}(\mathbf{3}_0, 1/2) + \pi^{+}$ <4.6 2 $\Sigma_c^+(\mathbf{6}_1, 1/2) \to \Lambda_c^+(\mathbf{3}_0, 1/2) + \pi^0$ 2.24 $1.83^{+0.11}_{-0.19}$ 1.90 3 $\Sigma_c^0(6_1, 1/2) \rightarrow \Lambda_c^+(\bar{3}_0, 1/2) + \pi^-$ $14.78_{-0.19}^{+0.30}$ 4 $\Sigma_c^{++}(\mathbf{6}_1, 3/2) \rightarrow \Lambda_c^{+}(\mathbf{3}_0, 1/2) + \pi^{+}$ 14.47 $\Sigma_c^+({\bf 6}_1,3/2) \to \Lambda_c^+({\bf \bar 3}_0,1/2) + \pi^0$ <17 5 15.02 $15.3^{+0.4}_{-0.5}$ $\Sigma_c^0(6_1,3/2) \to \Lambda_c^+(\bar{3}_0,1/2) + \pi^-$ 6 14.49 2.35 7 $\Xi_c^+({\bf 6}_1,3/2)\to \Xi_c({\bf \bar 3}_0,1/2)+\pi$ 8 2.53 $\Xi_c^0(6_1,3/2) \to \Xi_c(\bar{3}_0,1/2) + \pi$	#	Decay	This work	Exp.
				$2.14 \pm 0.19$
				$2.35 \pm 0.22$

<span id="page-4-2"></span>TABLE II. Bottom sextet baryon decay widths in MeV. Experimental data are taken from Particle Data Group [\[41\].](#page-7-11)



model.  $\chi$ QSM relates couplings of different SU(3) multiplets, like  $H_3$ ,  $G_3$ , or  $G_6$ , which are expressed as different

combinations of  $a_{1,2,3}$  [\(7\).](#page-2-2) Below, in Tables [I](#page-4-1) and [II](#page-4-2), we list our results and the experimental values both for the charm and bottom baryons. Note that in the case of  $\Xi_{Q}^{q}(\mathbf{6}_{1}, 3/2)$  (where q<br>denotes the pertinent charge) decays to  $\Xi_{q}^{q}(\mathbf{3}, 1/2)$ denotes the pertinent charge) decays to  $\Xi_{Q}(\bar{3}_0, 1/2)$ <br>+π the width is summed with the Clebsch-Gordan coef- $+\pi$  the width is summed with the Clebsch-Gordan coefficients  $\frac{1}{3} \left( \Xi_Q^q(\bar{\mathbf{3}}_0, 1/2) + \pi^0 \right) + \frac{2}{3} \left( \Xi_Q^{q+1}(\bar{\mathbf{3}}_0, 1/2) + \pi^{\pm} \right).$ When no charges are specified the width is averaged over initial and summed over final isospin.

We see remarkably good agreement of the  $\gamma$ OSM results with the experimental widths for both charm and bottom baryons. To better illustrate this we have plotted in Figs. [2](#page-4-3) and [3](#page-4-4) the results collected in Tables [I](#page-4-1) and [II](#page-4-2). We stress that

<span id="page-4-3"></span>

FIG. 2. Decay widths of the charm baryons. Numbers on the horizontal axis label the decay modes as listed in Tables [I,](#page-4-1) [III,](#page-5-2) and [IV.](#page-5-3) Red full circles correspond to our theoretical predictions. Dark green triangles correspond to the experimental data [\[41\].](#page-7-11) Data for decays 4–7 of  $\Sigma_c(6_1, 3/2)$  (down triangles) have been divided by a factor of 5 to fit within the plot area. Widths of two LHCb [\[2\]](#page-6-1)  $\Omega_c$  states that we interpret as pentaquarks are plotted as black full squares.

<span id="page-4-4"></span>

FIG. 3. Decay widths of the bottom baryons. Numbers on the horizontal axis label the decay modes as listed in Table [II](#page-4-2). Red full circles correspond to our theoretical predictions. Dark green triangles correspond to the experimental data [\[41\].](#page-7-11) Data for decays 6 and 7 of  $\Xi_b(6_1, 3/2)$  (down triangles) have been multiplied by a factor of 5 to be better visible on the plot.

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for the calculation of the heavy baryon widths essentially we did not need any new parameter—everything was fixed in the light baryon sector. The scaling factor  $\sigma$  introduced in Eq. [\(16\)](#page-3-3) is not really a new parameter; it rather characterizes the ∼15% modification of the universal meson mean field due to  $1/N_c$  corrections.

### <span id="page-5-0"></span>IV. DECAY WIDTHS OF EXOTIC ANTIDECAPENTAPLET BARYONS

With all decay constants fixed [\(7\)](#page-2-2) from the decays of light baryons we can now predict the widths of the putative pentaquark  $\Omega_c(3050)$  and  $\Omega_c(3119)$  states that are col-lected in Tables [III](#page-5-2) and [IV.](#page-5-3) Note that the exotic  $\Omega_c$ 's from  $\overline{15}$  have the isospin one and hence the decay mode to  $\Omega_c(6) + \pi$  is allowed. However, the corresponding decay constant  $G_6 = -\tilde{a}_1 - \frac{1}{2}a_2 - a_3$  is 0 in the nonrelativistic<br>small soliton size limit, analogously as for the correspond small soliton size limit, analogously as for the corresponding coupling of the light pentaquark  $\Theta^+$ . Therefore, we expect that the  $\Omega_c(6) + \pi$  decay mode should be strongly suppressed.

Note that the kaon momentum in the decay of  $\Omega_c(3050)$ ,  $p<sub>K</sub> = 275$  MeV, is quite close to the pion momentum in the decay of  $\Delta$ ,  $p_{\pi} = 228$  MeV, yet the  $\Delta$  decay width is 2 orders of magnitude larger than the one of  $\Omega_c(3050)$ . From Tables [III](#page-5-2) and [IV](#page-5-3) we see that, despite the large phase volume available, the predicted decay widths are very small and are in agreement with the small (∼1 MeV) decay widths observed by the LHCb collaboration (see also Fig. [2\)](#page-4-3). Note that  $\Omega_c(3050)$  and  $\Omega_c(3119)$  are the narrowest states among five LHCb  $\Omega_c$ 's and our mean-field picture gives a natural, parameter-free explanation of this

<span id="page-5-2"></span>TABLE III.  $\Omega_c(\overline{\bf 15}_1, 1/2)$  partial and total decay widths in MeV. Experimental value is from the LHCb measurement [\[2\].](#page-6-1)

Decay	This work	Exp.
	0.339	.
$\Omega_c(\overline{\bf 15}_1, 1/2) \rightarrow \Omega_c({\bf 6}_1, 1/2) + \pi$	0.097	.
$\Omega_c(\overline{\bf 15}_1, 1/2) \rightarrow \Omega_c({\bf 6}_1, 3/2) + \pi$	0.045	
Total	0.48	$0.8 \pm 0.2 \pm 0.1$
	$\Omega_c(\overline{\bf 15}_1, 1/2) \to \Xi_c(\bar{\bf 3}_0, 1/2) + K$	

<span id="page-5-3"></span>TABLE IV.  $\Omega_c(\overline{\bf 15}_1, 3/2)$  partial and total decay widths in MeV. Experimental value is from the LHCb measurement [\[2\].](#page-6-1)



<span id="page-5-4"></span>TABLE V. Predictions in MeV for the partial and total decay widths of explicitly exotic  $\Xi_c^{3/2}(\overline{\bf 15}_1, J)$ .

Decay	$J = 1/2$	$J = 3/2$
$\Xi_c^{3/2}(\overline{\bf 15}_1, J) \to \Xi_c(\bar{\bf 3}_0, 1/2) + \pi$	1.67	2.49
$\Xi_c^{3/2}(\overline{\bf 15}_1, J) \rightarrow \Xi_c({\bf 6}_1, 1/2) + \pi$	0.045	0.079
$\Xi_c^{3/2}(\overline{\bf 15}_1, J) \rightarrow \Xi_c({\bf 6}_1, 3/2) + \pi$	0.022	0.046
$\Xi_c^{3/2}(\overline{\bf 15}_1, J) \to \Sigma_c({\bf 6}_1, 1/2) + K$	.	0.019
Total	1.74	2.64

observation. In the large  $N_c$  nonrelativistic limit discussed at the end of Sec. [II](#page-1-1) the decay constant to  $6$  is strongly suppressed, whereas the decay constant to  $\bar{3}$  is suppressed in the leading order of  $N_c$ .

The results in this section also imply that other members of the exotic antidecapentaplet are expected to be anomalously narrow. All their partial decay widths can be easily computed in our model with the help of general formula [\(8\)](#page-2-5). As an illustration we quote here the result for the decay widths of other explicitly exotic members of the antidecapentaplet,  $\Xi_c^{3/2-}$  and  $\Xi_c^{3/2++}$ , which have the minimal quark content (cdds $\bar{u}$ ) and (cuusd). The masses of these states are predicted in Ref. [\[1\]](#page-6-0) to be 2931 and 3000 MeV for the  $J^P = 1/2^+$  and  $J^P = 3/2^+$  multiplets, respectively. The predictions for the partial widths<sup>3</sup> of exotic  $\Xi_c^{3/2}$  are given in Table [V.](#page-5-4) We see that, indeed, the widths are anomalously small. Interesting is that the width of the isospin-1/2  $\Xi_c$  from 15-plet is even smaller (<1 MeV), with the dominant decay mode,  $\Lambda_c + K$ .

#### V. CONCLUSIONS

<span id="page-5-1"></span>In the large  $N_c$  limit both heavy and light baryons are described by the universal mean field. This allows us to relate the properties of heavy baryons to the light ones. The goal of the present paper was twofold: first, to test the mean-field picture of baryons against the data on strong decays of sextets of charm and bottom baryons. Second, to make predictions for the decay widths of exotic antidecapentaplet  $(\overline{15})$  baryons.

With the only input from the decays of light octet baryons (due to the universality of the chiral mean field) we have obtained excellent description of strong decays of both charm and bottom sextets. The agreement is illustrated in Fig. [2](#page-4-3) for charm baryon decays and in Fig. [3](#page-4-4) for bottom decays. We have also shown that going from  $N_c$  light quarks in light baryons to  $N_c - 1$  light quarks in heavy baryons the mean field is modified by about 15%. That moderate modification is an agreement with expected size of  $1/N_c$  corrections.

<sup>&</sup>lt;sup>3</sup>Note that the decay  $\Xi_c^{3/2} \to \Lambda_c + K$  is forbidden by the spin symmetry. isospin symmetry.

Given the excellent agreement of our calculation with the measured widths of ground-state heavy baryons we made parameter-free predictions for the decays of exotic 15-plet baryons. We have shown that the widths of  $\overline{15}$  baryons must be anomalously small, due to essentially the same mechanism as in the case of narrow antidecuplet light baryons. In particular, for  $\Omega_c(3050)$  and  $\Omega_c(3119)$ , which we interpreted in Ref. [\[1\]](#page-6-0) as belonging to the antidecapentaplet, we obtained widths of 0.48 and 1.12 MeV correspondingly. Experimentally, these two (among five) states have the smallest widths [\[2\],](#page-6-1) which are in agreement with our parameter-free calculations; see Tables [III](#page-5-2) and [IV](#page-5-3) and Fig. [2](#page-4-3). The parametrical suppression of the pertinent decay constants has been discussed at the end of Sec. [II](#page-1-1). We have shown that theoretical arguments based on large  $N_c$  and NR limits explain the numerical hierarchy of the decay couplings [\(9\)](#page-2-4).

For the complete description of the LHCb  $\Omega_c$  states, estimates of strong decays for negative parity baryons are needed. In the  $\chi$ QSM, negative parity baryons correspond to the configuration with one quark excited from the Dirac see to the empty valence level (see Ref. [\[1\]](#page-6-0)). Therefore one expects that the decay operator will depend on a new parameter related to this transition, similarly to the case of mass splittings for these states. Furthermore, negative parity particles will have s-wave and/or d-wave decays, which are not possible if the parity is positive. Finally, for heavier states, decays to light baryons and heavy mesons are possible. Such decays require new theoretical treatment within the framework of the  $\gamma$ QSM since the Goldberger-Treiman relation is not directly applicable in this case. All these issues require further study; therefore we have not attempted to address them in the present paper.

The results of the present study reinforce our conclu-sions from Ref. [\[1\]](#page-6-0) that the two narrowest  $\Omega_c^0$  states reported recently in the LHCb collaboration in Ref. [\[2\]](#page-6-1) correspond to the exotic SU(3) multiplet, namely the antidecapentaplet (15). As seen from Fig. [1](#page-1-0) these states belong to the isospin triplet, rather than the singlet. Therefore this quantum number assignment can be relatively easily verified experimentally.

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