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Hairy black-hole solutions in generalized Proca theories

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We present a family of exact black-hole solutions on a static spherically symmetric background in second-order generalized Proca theories with derivative vector-field interactions coupled to gravity. We also derive nonexact solutions in power-law coupling models including vector Galileons and numerically show the existence of regular black holes with a primary hair associated with the longitudinal propagation. The intrinsic vector-field derivative interactions generally give rise to a secondary hair induced by nontrivial field profiles. The deviation from General Relativity is most significant around the horizon and hence there is a golden opportunity for probing the Proca hair by the measurements of gravitational waves (GWs) in the regime of strong gravity.

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I. INTRODUCTION

The no-hair conjecture of black holes (BHs) [1] was originally suggested by the existence of uniqueness theorems for Schwarzschild, Reissner-Nordström (RN), and Kerr solutions in General Relativity (GR) [2–4]. However, there are several assumptions for proving the absence of hairs besides mass, charge, and angular momentum in the form of no-hair theorems. One of such assumptions for a scalar field ϕ is that the standard canonical term $\nabla_\mu\phi\nabla^\mu\phi/2$ is the only field derivative in the action [5]. Hence, the no-hair theorem of Ref. [5] loses its validity for theories containing noncanonical kinetic terms.

There are theories with noncanonical scalars with nonlinear derivative interactions-like Galileons [6,7] and its extension to Horndeski theories [8,9]. In shift-symmetric Horndeski theories, a no-hair theorem for static and spherically symmetric BHs was proposed [10] by utilizing the regularity of a Noether current on the horizon. A counterexample of a hairy BH evading one of the conditions discussed in Ref. [10] was advocated for the scalar field linearly coupled to a Gauss-Bonnet term [11]. For a time-dependent scalar with nonminimal derivative coupling to the Einstein tensor, there is also a stealth Schwarzschild solution with a nontrivial field profile [12].

For a massless vector field in GR, the static and spherically symmetric BH solution is described by the RN metric with mass M and charge Q. The introduction of a vector-field mass breaks the $U(1)$ gauge symmetry, which allows the propagation of the longitudinal mode. For this massive Proca field, Bekenstein showed [13] that a static BH does not have a vector hair. The vector field A^μ vanishes throughout the BH exterior from the requirement that a physical scalar constructed from A^μ is bounded on a nonsingular horizon. In this case, the static and spherically symmetric BH solution is described by the Schwarzschild metric with mass M.

The Bekenstein's no hair theorem [13] cannot be applied to the massive vector field with nonlinear derivative interactions. In Refs. [14–17] the action of generalized Proca theories was constructed by demanding the condition that the equations of motion are up to second order to avoid the Ostrogradsky instability. An exact static and spherically symmetric BH solution with the Abelian vector hair[1] was found in Ref. [20] for the Lagrangian $\mathcal{L} = (M_{\rm pl}^2/2)R - F_{\mu\nu}F^{\mu\nu}/4 + \beta_4 G^{\mu\nu}A_\mu A_\nu$ with the specific coupling $\beta_4 = 1/4$, where $M_{\rm pl}$ is the reduced Planck mass, R and $G_{\mu\nu}$ are the Ricci scalar and Einstein tensor respectively, and $F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu}$ (a semicolon represents a covariant derivative) is the field strength. This is a stealth Schwarzschild solution containing mass M alone with a temporal vector component $A_0 = P + Q/r$ and a nonvanishing longitudinal component A_1, where r is the distance from the center of spherical symmetry. Unlike the RN solution present for the massless vector in GR, the Proca hair P is physical but P as well as the electric charge Q does not appear in metrics.

The exact BH solutions studied in Ref. [20] have been extended to nonasymptotically flat solutions [21,22], rotating solutions [21], and solutions for $\beta_4 \neq 1/4$ [22,23]. All these studies considered only the $\beta_4 G^{\mu\nu}A_\mu A_\nu$ coupling. It is crucial to investigate whether the self-derivative interactions of generalized Proca theories give rise to compelling new hairy BH solutions. In this letter we provide a

[1]In this letter we focus on the Abelian vector field, but there are hairy BH solutions for non-Abelian Yang-Mills fields [18]. A complex Abelian vector field can also give rise to hairy Kerr solutions [19].

systematic prescription for constructing new BH solutions in generalized Proca theories on the static and spherically symmetric background given by the line element

$$ds^2 = -f(r)dt^2 + h^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (1)$$

with the vector field $A_\mu = (A_0(r), A_1(r), 0, 0)$, where $f(r)$, $h(r)$, $A_0(r)$, and $A_1(r)$ are arbitrary functions of r. We will derive exact solutions under the condition of a constant norm of the vector field $A_\mu A^\mu =$ constant. We also numerically obtain hairy BH solutions for power-law coupling models including vector Galileons. Unlike scalar-tensor theories in which hairy BH solutions exist only for restrictive cases, we will show that the presence of a temporal vector component besides a longitudinal scalar mode gives rise to a bunch of hairy BH solutions in broad classes of models.

The generalized Proca theories are given by the action

$$S = \int d^4x\sqrt{-g}\left(F + \sum_{i=2}^{6}\mathcal{L}_i\right), \quad (2)$$

where $F = -F_{\mu\nu}F^{\mu\nu}/4$, and [14,17]

$$\mathcal{L}_2 = G_2(X), \qquad \mathcal{L}_3 = G_3(X)A^\mu{}_{;\mu},$$
$$\mathcal{L}_4 = G_4(X)R + G_{4,X}[(A^\mu{}_{;\mu})^2 - A_{\nu;\mu}A^{\mu;\nu}] - 2g_4(X)F,$$
$$\mathcal{L}_5 = G_5(X)G_{\mu\nu}A^{\nu;\mu} - \frac{G_{5,X}}{6}[(A^\mu{}_{;\mu})^3 - 3A^\mu{}_{;\mu}A_{\sigma;\rho}A^{\rho;\sigma} + 2A_{\sigma;\rho}A^{\rho;\nu}A_\nu{}^{;\sigma}] - g_5(X)\tilde{F}^{\alpha\mu}\tilde{F}^\beta_\mu A_{\beta;\alpha},$$
$$\mathcal{L}_6 = G_6(X)L^{\mu\nu\alpha\beta}A_{\nu;\mu}A_{\beta;\alpha} + \frac{G_{6,X}}{2}\tilde{F}^{\alpha\beta}\tilde{F}^{\mu\nu}A_{\mu;\alpha}A_{\nu;\beta}. \quad (3)$$

The functions $G_{2,3,4,5,6}$ and $g_{4,5}$ depend on $X = -A_\mu A^\mu/2$, with the notation $G_{i,X} = \partial G_i/\partial X$. The vector field A^μ has nonminimal couplings with the Ricci scalar R, the Einstein tensor $G_{\mu\nu}$, and the double dual Riemann tensor $L^{\mu\nu\alpha\beta} = \mathcal{E}^{\mu\nu\rho\sigma}\mathcal{E}^{\alpha\beta\gamma\delta}R_{\rho\sigma\gamma\delta}/4$, where $\mathcal{E}^{\mu\nu\rho\sigma}$ is the Levi-Civita tensor and $R_{\rho\sigma\gamma\delta}$ is the Riemann tensor. The dual strength tensor $\tilde{F}^{\mu\nu}$ is defined by $\tilde{F}^{\mu\nu} = \mathcal{E}^{\mu\nu\alpha\beta}F_{\alpha\beta}/2$. The Lagrangians containing g_4, g_5, G_6 correspond to intrinsic vector modes that vanish in the scalar limit $A^\mu \to \pi^{;\mu}$. Throughout the analysis we take into account the Einstein-Hilbert term $M_{\rm pl}^2/2$ in $G_4(X)$.

II. EXACT SOLUTIONS

The exact solution of Ref. [20] was found for the model $G_4(X) = M_{\rm pl}^2/2 + \beta_4 X$ with $\beta_4 = 1/4$. For this solution there are two relations

$$f = h, \qquad X = X_c, \quad (4)$$

where X_c is a constant. On using these conditions for the vector field A_μ, it follows that

$$A_1 = \pm\sqrt{A_0^2 - 2fX_c}/f. \quad (5)$$

Introducing the tortoise coordinate $dr_* = dr/f(r)$, the scalar product $A_\mu dx^\mu$ reduces to $A_0 du_\pm$ around the horizon, where $u_\pm = t \pm r_*$. The advanced and retarded null coordinates u_+ and u_- are regular at the future and past event horizons, respectively. Hence the regularity of solutions at the corresponding (future or past) horizon is ensured for each branch of (5), which is analogous to the case of shift-symmetric scalar-tensor theories [12]. We will search for exact solutions by imposing the two conditions (4).

Provided the condition $G_{4,XX}(X_c) = 0$ is satisfied for the quartic-order coupling $G_4(X)$, the equation of motion for A_1 reduces to $G_{4,X}(rf' + f - 1)A_1 = 0$, where a prime represents the derivative with respect to r. As long as $G_{4,X} \neq 0$, there are two branches characterized by $rf' + f - 1 = 0$ or $A_1 = 0$. The first gives rise to the stealth Schwarzschild solution $f = h = 1–2M/r$ found in Ref. [20]. In this case the temporal vector component obeys $A_0'' + 2A_0'/r = 0$, whose integrated solution is given by $A_0 = P + Q/r$. Since the constant P is independent of M and Q, it is regarded as a primary hair [24]. The other two independent equations are satisfied for $G_{4,X}(X_c) = 1/4$ and $X_c = P^2/2$, so the longitudinal mode (5) reads $A_1 = \pm\sqrt{2P(MP+Q)r + Q^2}/(r - 2M)$. A concrete model satisfying the above mentioned conditions is

$$G_4(X) = G_4(X_c) + \frac{1}{4}(X - X_c) + \sum_{n=3}b_n(X - X_c)^n, \quad (6)$$

where b_n's are constants. The model $G_4(X) = M_{\rm pl}^2/2 + X/4$ corresponds to the special case of Eq. (6).

Besides the nonvanishing A_1 solution there exists another branch $A_1 = 0$ for the couplings $G_i(X)$ with even i-index, in which case the relation $A_0^2(r) = 2f(r)X_c$ holds from Eq. (5). For the quartic coupling $G_4(X)$ the equation for A_0 can be satisfied under the conditions $G_{4,X}(X_c) = 0$ and $2rff'' - rf'^2 + 4ff' = 0$. The latter leads to the solution $f = (C - M/r)^2$ with two integrated constants C and M. For the consistency with the other two equations of motion, we require that $C = 1$ and $G_4(X_c) = X_c/2$. Hence we obtain the extremal RN BH solution

$$f = h = \left(1 - \frac{M}{r}\right)^2, \quad A_0 = P - \frac{PM}{r}, \quad A_1 = 0, \quad (7)$$

where $P = \pm\sqrt{2X_c}$. An explicit model realizing this solution is

$$G_4(X) = \frac{X_c}{2} + \sum_{n=2}b_n(X - X_c)^n. \quad (8)$$

For the metric (7), P depends on M by reflecting the fact that the charge $Q = -PM$ has a special relation with the mass M. Hence the Proca hair is of the secondary type.

For the cubic coupling $G_3(X)$ the equation for A_1 reads

$$G_{3,X}[f^2(rf'+4f)A_1^2+rA_0(2fA_0'-f'A_0)]=0, \quad (9)$$

so there are two branches satisfying (i) $G_{3,X}(X_c)=0$ or (ii) $G_{3,X}(X_c)\neq 0$. For the branch (i) the consistency with the other equations requires that $2(rf'+f-1)M_{\rm pl}^2+r^2A_0'^2=0$ and $A_0''+2A_0'/r=0$, so the integrated solutions are of the RN forms:

$$f=h=1-\frac{2M}{r}+\frac{Q^2}{2M_{\rm pl}^2r^2}, \qquad A_0=P+\frac{Q}{r}, \quad (10)$$

with the nonvanishing longitudinal mode (5). This exact solution can be realized by the model

$$G_3(X)=G_3(X_c)+\sum_{n=2}b_n(X-X_c)^n. \quad (11)$$

Unlike the RN solution in GR with $G_3(X)=0$, P in Eq. (10) has the meaning of the primary hair with the nonvanishing longitudinal mode (5). The branch (ii) corresponds to the case in which the terms in the square bracket of Eq. (9) vanishes. On using Eq. (5) and imposing the asymptotically flat boundary condition $f\to 1$ for $r\to\infty$, we obtain the extremal RN BH solution (7) with $P=\pm\sqrt{2}M_{\rm pl}$.

For the quintic coupling $G_5(X)$ the temporal component obeys $A_0''+2A_0'/r=0$ under the conditions (4), so the resulting solution is $A_0=P+Q/r$. Imposing the condition $G_{5,X}(X_c)=0$ further, the equation for A_1 reduces to $(A_0A_0'-X_cf')A_1^2G_{5,XX}=0$ and hence there are two branches satisfying (i) $A_0A_0'=X_cf'$ or (ii) $A_1=0$. For the branch (i), the resulting solutions are given by the RN solutions (10) with the particular relations $P=-2MM_{\rm pl}^2/Q$ and $X_c=M_{\rm pl}^2$. The longitudinal mode (5) reduces to

$$A_1=\pm\frac{2M_{\rm pl}^3\sqrt{2(2M^2M_{\rm pl}^2-Q^2)}r^2}{Q[2M_{\rm pl}^2r(2M-r)-Q^2]}, \quad (12)$$

whose existence requires the condition $2M^2M_{\rm pl}^2>Q^2$. Since P depends on M and Q, the Proca hair P is secondary. This exact solution can be realized by the model

$$G_5(X)=G_5(X_c)+\sum_{n=2}b_n(X-M_{\rm pl}^2)^n. \quad (13)$$

Another branch $A_1=0$ is the special case of Eq. (12), i.e., $Q^2=2M^2M_{\rm pl}^2$, under which the solution is given by the extremal RN BH solution (7) with $P=\pm\sqrt{2}M_{\rm pl}$.

The sixth-order coupling $G_6(X)$ has the two branches (i) $A_1=0$ or (ii) $A_0'=0$. For the branch (i) there exists an exact solution if the two conditions $G_6(X_c)=0$ and $G_{6,X}(X_c)=0$ hold. This is the extremal RN BH solution (7) with $X_c=M_{\rm pl}^2$ and $P=\pm\sqrt{2}M_{\rm pl}$, which can be realized for the model

$$G_6(X)=\sum_{n=2}b_n(X-M_{\rm pl}^2)^n. \quad (14)$$

The branch (ii) corresponds to $A_0=$ constant, in which case we obtain the stealth Schwarzschild solution $f=h=1$–$2M/r$. This exists for general couplings $G_6(X)$ with arbitrary values of A_1. Since we are now imposing the second condition of Eq. (4), the longitudinal mode is fixed to be $A_1=\pm\sqrt{r[(A_0^2-2X_c)r+4MX_c]}/(r-2M)$.

III. POWER-LAW COUPLINGS

So far we have imposed the conditions (4) to derive exact solutions, but we will also study BH solutions for the power-law models

$$G_i(X)=\tilde{\beta}_iX^n, \qquad g_j(X)=\tilde{\gamma}_jX^n, \quad (15)$$

where n is a positive integer, and $\tilde{\beta}_i$ and $\tilde{\gamma}_j$ are coupling constants[2] with $i=3$, 4, 5, 6 and $j=4$, 5.

Let us begin with the cubic vector-Galileon interaction $G_3(X)=\beta_3X$. Then, the longitudinal mode obeys

$$A_1=\pm\sqrt{\frac{rA_0(f'A_0-2fA_0')}{fh(rf'+4f)}}. \quad (16)$$

Around the horizon characterized by the radius r_h, we expand f, h, A_0 in the forms

$$f=\sum_{i=1}^{\infty}f_i(r-r_h)^i, \qquad h=\sum_{i=1}^{\infty}h_i(r-r_h)^i,$$
$$A_0=a_0+\sum_{i=1}^{\infty}a_i(r-r_h)^i, \quad (17)$$

where f_i, h_i, a_0, a_i are constants. To recover the RN solutions of the form $f=h=(r-r_h)(r-\mu r_h)/r^2$ in the limit $\beta_3\to 0$, where the constant μ is in the range $0<\mu<1$ so that $r=r_h$ corresponds to the outer horizon, we choose

$$f_1=h_1=(1-\mu)/r_h. \quad (18)$$

Taking the positive branch of A_1 with $a_0>0$ and picking up linear-order terms in β_3, the effect of the coupling β_3 starts to appear at second order of $(r-r_h)^i$, such that

[2] For the dimensionless coupling constants, we use the notations β_i and γ_j in the following.

$$a_1 = \frac{M_{\rm pl}\sqrt{2\mu}}{r_h}, \qquad a_2 = -\frac{M_{\rm pl}\sqrt{2\mu}}{r_h^2} + \alpha_2\beta_3,$$
$$f_2 = \frac{2\mu - 1}{r_h^2} + \mathcal{F}_2\beta_3, \qquad h_2 = \frac{2\mu - 1}{r_h^2} + \mathcal{H}_2\beta_3, \quad (19)$$

where α_2, $\mathcal{F}_2$, $\mathcal{H}_2$ depend on the three parameters (h_1, r_h, a_0). The coupling β_3 induces the difference between the metrics f and h. The leading-order longitudinal mode around $r = r_h$ is given by $A_1 = a_0/[f_1(r - r_h)]$, so the scalar product $A_\mu dx^\mu$ becomes $A_\mu dx^\mu \simeq a_0 du_+$, which is regular at the future horizon $r = r_h$.

We also search for asymptotic flat solutions at spatial infinity ($r \to \infty$) by expanding f, h, A_0 in the forms

$$f = 1 + \sum_{i=1}^{\infty} \frac{\tilde{f}_i}{r^i}, \qquad h = 1 + \sum_{i=1}^{\infty} \frac{\tilde{h}_i}{r^i},$$
$$A_0 = P + \sum_{i=1}^{\infty} \frac{\tilde{a}_i}{r^i}. \quad (20)$$

For the cubic Galileon, the asymptotic solution for A_1 reduces to $A_1 = \sum_{i=2}^{\infty} \tilde{b}_i/r^i$, where the first-order coefficient $\tilde{b}_1$ vanishes from the background equations of motion. The iterative solutions are given by $f=1-2M/r-P^2M^3/(6M_{\rm pl}^2 r^3)+\mathcal{O}(1/r^4)$, $h=1-2M/r-P^2M^2/(2M_{\rm pl}^2 r^2)-P^2M^3/(2M_{\rm pl}^2 r^3)+\mathcal{O}(1/r^4)$, and $A_0=P-PM/r-PM^2/(2r^2)+\mathcal{O}(1/r^3)$, where we have set $\tilde{f}_1 = \tilde{h}_1 = -2M$. The coefficient $\tilde{b}_2$ and the coupling β_3 begin to appear at the orders of $1/r^4$ and $1/r^5$, respectively, in f, h, A_0.

In Fig. 1 we plot one example of numerically integrated solutions outside the horizon derived by using the boundary conditions (17)–(19) around $r = r_h$. The solutions in the two asymptotic regimes smoothly join each other without any discontinuity. As estimated above, the longitudinal mode behaves as $A_1 \propto (r - r_h)^{-1}$ for $r \simeq r_h$ and $A_1 \propto r^{-2}$ for $r \gg r_h$. Since the time t can be reparametrized such that f shifts to 1 at spatial infinity, we have performed this rescaling of f after solving the equations of motion up to $r = 2 \times 10^7 r_h$. In Fig. 1 the difference between h and f manifests itself in the regime of strong gravity with the radius $r \lesssim 100 r_h$.

Since the two asymptotic solutions discussed above are continuous, the three parameters $(\tilde{b}_2, M, P)$ appearing in the expansion (20) with $A_1 = \sum_{i=2}^{\infty} \tilde{b}_i/r^i$ are related to the three parameters (h_1, r_h, a_0) arising in the expansion (17), as $\tilde{b}_2 = \tilde{b}_2(h_1, r_h, a_0)$, $M = M(h_1, r_h, a_0)$, and $P = P(h_1, r_h, a_0)$. Since $\tilde{b}_2$ is not fixed by the two parameters M and P alone, this is regarded as a primary hair.

For the cubic interaction $G_3(X) = \beta_3 M_{\rm pl}^2 (X/M_{\rm pl}^2)^n$ with $n \geq 2$, there is the nonvanishing A_1 branch satisfying the relation (16). In this case, the property of two asymptotic solutions (17) and (20) is similar to that discussed for $n = 1$. The solutions are also regular throughout the BH exterior with the difference between f and h induced by β_3. There is also another branch obeying

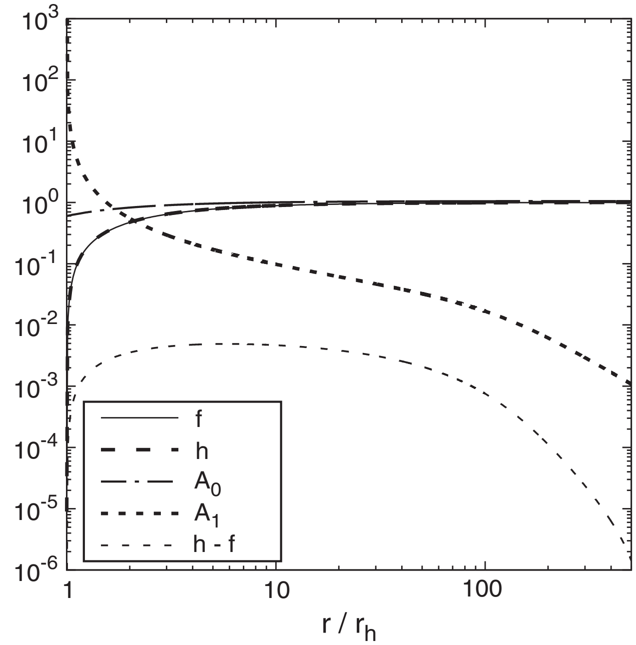

FIG. 1. Numerical solutions of f, h, A_0, A_1, $h - f$ outside the horizon for the cubic vector-Galileon model $G_3(X) = \beta_3 X$ with $\beta_3 = 10^{-3}/(r_h M_{\rm pl})$. The boundary conditions around the horizon are chosen to satisfy Eqs. (17)–(19) with $\mu = 0.1$ and $a_0 = 0.6 M_{\rm pl}$ at $r = 1.001 r_h$.

$$A_1 = \pm\sqrt{A_0^2/(fh)}, \quad (21)$$

for which the resulting solutions correspond to the RN solutions (10). Indeed, this exact solution is the special case of the model (11) with $X_c = 0$ and $G_3(X_c) = 0$.

Let us proceed to the quartic coupling $G_4(X) = \beta_4 M_{\rm pl}^2 (X/M_{\rm pl}^2)^n$ with $n \geq 2$. In general, we have two branches characterized by (i) $A_1 \neq 0$ or (ii) $A_1 = 0$. For $n \geq 3$ there exists the nonvanishing A_1 branch (21) with the RN solutions (10). Another nonvanishing A_1 branch gives rise to hairy BH solutions with $f \neq h$. Indeed, the solutions around the horizon are given by the expansion (17) with the coupling β_4 appearing at second order ($i = 2$) in f, h and at first order ($i = 1$) in A_0. They are characterized by the three parameters (h_1, r_h, a_0) under the condition (18). The solutions expanded at spatial infinity are the RN solutions (10) with corrections induced by β_4. If $n = 2$, for example, such corrections to f, h, A_0 arise at second order in $1/r^2$, e.g., $\delta f = 3P^2Q^2(5P^2 - 8M_{\rm pl}^2)\beta_4/(4M_{\rm pl}^6 r^2)$, $\delta h = 3P^2Q^2(11P^2 - 16M_{\rm pl}^2)\beta_4/(4M_{\rm pl}^6 r^2)$, and $\delta A_0 = PQ^2(3P^2 - 4M_{\rm pl}^2)\beta_4/(M_{\rm pl}^4 r^2)$, respectively. The longitudinal mode behaves as $A_1 \propto (r - r_h)^{-1}$ for $r \simeq r_h$ and $A_1 \propto r^{-1/2}$ for $r \gg r_h$. Numerically we confirmed that the solution around $r = r_h$ smoothly connects to that in the asymptotic regime $r \gg r_h$, so the parameters (P, Q, M) are related to (h_1, r_h, a_0). Therefore, the Proca hair P is of the primary type.

For the second branch $A_1 = 0$, the solutions (17) around $r = r_h$ are subject to the constraint $a_0 = 0$. Hence they are expressed in terms of the two parameters (h_1, r_h) with the coupling β_4 appearing at the order of $(r - r_h)^3$ in f, h, A_0 for $n = 2$. At spatial infinity, the effect of β_4 works as corrections to the RN solutions (10). For $n = 2$ the leading-order corrections to f, h, A_0 are given, respectively, by $\delta f = -4P^3(MP + Q)\beta_4/(M_{\rm pl}^4 r)$, $\delta h = 3P^4 Q^2 \beta_4/(4M_{\rm pl}^6 r^2)$, and $\delta A_0 = -P^3 Q(2MP + Q)\beta_4/(2M_{\rm pl}^4 r^2)$. The matching of two asymptotic solutions has been also confirmed numerically, so (P, M, Q) depend on the two parameters (h_1, r_h) alone. Hence P corresponds to the secondary hair.

The sixth-order coupling $G_6(X) = (\beta_6/M_{\rm pl}^2)(X/M_{\rm pl}^2)^n$ with the power $n \geq 0$ has the branch satisfying $A_1^2/A_0^2 = (3h - 1)/[fh\{(2n + 1)h - 1\}]$ besides $A_0' = 0$, $A_1 = 0$, and $A_1 = \pm\sqrt{A_0^2/(fh)}$ (the last one is present for $n \geq 3$). However, the first one does not exist in the region $1/(2n + 1) < h < 1/3$ outside the horizon. Since the second and fourth branches correspond to the Schwarzschild and RN solutions, respectively, the branch $A_1 = 0$ alone leads to the solutions with $f \neq h$. The $U(1)$-invariant interaction derived by Horndeski [25] corresponds to $n = 0$, in which case the coupling β_6 appears in the expansion (17) around $r = r_h$ at second order for f, h and at first order for A_0, with a_0 unfixed. For $n \geq 1$ the effect of β_6 arises at $n + 1$ order in Eq. (17), with $a_0 = 0$. At spatial infinity, the leading-order corrections to the RN solutions (10) read $\delta f = -P^{2n} Q^2 \beta_6/(2^{1+n} M_{\rm pl}^{4+2n} r^4)$, $\delta h = (2n - 1)MP^{2n} Q^2 \beta_6/(2^{1+n} M_{\rm pl}^{4+2n} r^5)$, and $\delta A_0 = -MP^{2n} Q\beta_6/(2^n M_{\rm pl}^{2+2n} r^4)$, which match with those derived by Horndeski in the $U(1)$-invariant case ($n = 0$) [26]. For $n \geq 0$, the numerically integrated solutions are regular throughout the horizon exterior with the difference between f and h. When $n = 0$, P has no physical meaning due to the $U(1)$ gauge symmetry, so there are two physical hairs M and Q related to the parameters h_1 and r_h around the horizon. For $n \geq 1$ the Proca hair P is secondary, which reflects the fact that (P, M, Q) depend on (h_1, r_h) alone.

The quintic coupling $G_5(X) = \beta_5(X/M_{\rm pl}^2)^n$ does not lead to regular solutions with $A_1 \neq 0$ due to the divergence at $h = 1/(2n + 1)$. For the intrinsic vector-mode couplings $g_4(X) = \gamma_4(X/M_{\rm pl}^2)^n$ and $g_5(X) = (\gamma_5/M_{\rm pl}^2)(X/M_{\rm pl}^2)^n$ with $n \geq 1$, there are hairy regular BH solutions with $f \neq h$ characterized by $A_1 = 0$ and $A_1 = \pm\sqrt{A_0^2/[(1 + 2n)fh]}$, respectively. The couplings γ_4 and γ_5 give rise to corrections to the RN solutions (10), where the near-horizon expansion (17) can be expressed in terms of the two parameters h_1 and r_h with $a_0 = 0$. In this case the Proca hair P is dependent on M and Q at spatial infinity, so it is of the secondary type.

IV. CONCLUSIONS

We have systematically constructed new exact BH solutions under the conditions (4) and also obtained a family of hairy numerical BH solutions with $f \neq h$ for the power-law models (15). For the cubic and quartic couplings $G_3(X) = \tilde{\beta}_3 X^n$ and $G_4(X) = \tilde{\beta}_4 X^n$, there exist nonvanishing A_1 branches with the primary Proca hair with the difference between f and h manifesting around the horizon. For the intrinsic vector-mode couplings $G_6(X) = \tilde{\beta}_6 X^n$, $g_4(X) = \tilde{\gamma}_4 X^n$, $g_5(X) = \tilde{\gamma}_5 X^n$ with $n \geq 1$, there are regular BH solutions (RN solutions with corrections induced by the couplings) characterized by the secondary Proca hair P.

Since astronomical observations of BHs have increased their accuracies, there will be exciting possibilities for probing deviations from GR in the foreseeable future, e.g., in the measurements of innermost stable circular orbits. GWs emitted from quasicircular BH binaries can generally place tight bounds on modified gravitational theories with large deviations from GR in the regime of strong gravity [27]. The future GW measurements will be able to measure the Proca charge P through the corrections to the Schwarzschild or RN metrics and the precise determination of polarizations. The existence of such a new vector hair will shed new light on the construction of unified theories connecting gravitational theories with particle theories. Our analysis in the strong gravity regime is also complementary to the cosmological analysis with the late-time acceleration [28] and the solar-system constraints [29]. The combination of them will allow us to probe vector-tensor theories in all scales in astrophysics and cosmology.

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[1] R. Ruffini and J. A. Wheeler, Phys. Today **24**, 30 (1971).
[2] W. Israel, Phys. Rev. **164**, 1776 (1967).
[3] B. Carter, Phys. Rev. Lett. **26**, 331 (1971).
[4] S. W. Hawking, Commun. Math. Phys. **25**, 152 (1972).
[5] J. D. Bekenstein, Phys. Rev. D **51**, R6608 (1995).
[6] A. Nicolis, R. Rattazzi, and E. Trincherini, Phys. Rev. D **79**, 064036 (2009).
[7] C. Deffayet, G. Esposito-Farese, and A. Vikman, Phys. Rev. D **79**, 084003 (2009).
[8] G. W. Horndeski, Int. J. Theor. Phys. **10**, 363 (1974).
[9] C. Deffayet, X. Gao, D. A. Steer, and G. Zahariade, Phys. Rev. D **84**, 064039 (2011).
[10] L. Hui and A. Nicolis, Phys. Rev. Lett. **110**, 241104 (2013).
[11] T. P. Sotiriou and S. Y. Zhou, Phys. Rev. Lett. **112**, 251102 (2014).
[12] E. Babichev and C. Charmousis, J. High Energy Phys. 08 (2014) 106.
[13] J. D. Bekenstein, Phys. Rev. D **5**, 1239 (1972).
[14] L. Heisenberg, J. Cosmol. Astropart. Phys. 05 (2014) 015.
[15] G. Tasinato, J. High Energy Phys. 04 (2014) 067.
[16] E. Allys, P. Peter, and Y. Rodriguez, J. Cosmol. Astropart. Phys. 02 (2016) 004.
[17] J. B. Jimenez and L. Heisenberg, Phys. Lett. B **757**, 405 (2016).
[18] R. Bartnik and J. Mckinnon, Phys. Rev. Lett. **61**, 141 (1988); M. S. Volkov and D. V. Galtsov, JETP Lett. **50**, 346 (1989); P. Bizon, Phys. Rev. Lett. **64**, 2844 (1990).
[19] C. Herdeiro, E. Radu, and H. Runarsson, Classical Quantum Gravity **33**, 154001 (2016).
[20] J. Chagoya, G. Niz, and G. Tasinato, Classical Quantum Gravity **33**, 175007 (2016).
[21] M. Minamitsuji, Phys. Rev. D **94**, 084039 (2016).
[22] E. Babichev, C. Charmousis, and M. Hassaine, J. High Energy Phys. 05 (2017) 114.
[23] J. Chagoya, G. Niz, and G. Tasinato, Classical Quantum Gravity **34**, 165002 (2017).
[24] C. A. R. Herdeiro and E. Radu, Int. J. Mod. Phys. D **24**, 1542014 (2015).
[25] G. W. Horndeski, J. Math. Phys. (N.Y.) **17**, 1980 (1976).
[26] G. W. Horndeski, Phys. Rev. D **17**, 391 (1978).
[27] K. Yagi, N. Yunes, and T. Tanaka, Phys. Rev. Lett. **109**, 251105 (2012).
[28] A. De Felice, L. Heisenberg, R. Kase, S. Mukohyama, S. Tsujikawa, and Y.-l. Zhang, J. Cosmol. Astropart. Phys. 06 (2016) 048.
[29] A. De Felice, L. Heisenberg, R. Kase, S. Tsujikawa, Y.-l. Zhang, and G. B. Zhao, Phys. Rev. D **93**, 104016 (2016).