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Gravitational catalysis of merons in Einstein-Yang-Mills theory

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We construct regular configurations of the Einstein-Yang-Mills theory in various dimensions. The gauge field is of meron-type: it is proportional to a pure gauge (with a suitable parameter λ determined by the field equations). The corresponding smooth gauge transformation cannot be deformed continuously to the identity. In the three-dimensional case we consider the inclusion of a Chern-Simons term into the analysis, allowing λ to be different from its usual value of $1/2$. In four dimensions, the gravitating meron is a smooth Euclidean wormhole interpolating between different vacua of the theory. In five and higher dimensions smooth meron-like configurations can also be constructed by considering warped products of the three-sphere and lower-dimensional Einstein manifolds. In all cases merons (which on flat spaces would be singular) become regular due to the coupling with general relativity. This effect is named "gravitational catalysis of merons".

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I. INTRODUCTION

The existence of topological solitons is one of the most important nonperturbative effects in field theory [1]. These non-trivial topological objects are believed to play a fundamental role in the color confinement problem (for a detailed review, see [2]) which is one of the "big" open issues in gauge field theory. A very important class of topological solitons is the Euclidean one (namely, regular solutions of the Euclidean theory). Euclidean topological solitons are especially relevant as they play a very important role at quantum level as nontrivial saddle points of the path integral. The most important Euclidean solutions are instantons (which are local regular minima of the Euclidean action) and sphalerons [which are saddle points with one-or a finite number of-unstable mode(s)]. Unfortunately, analytic solutions are available only in special cases (in particular, when suitable Bogomol'nyi–Prasad–Sommerfield bounds can be saturated). In the case of instantons of Yang-Mills theory in 4 dimensions the saturation of the bound is equivalent to the self-duality condition. From the point of view of gravitational back-reaction, instantons are not very interesting as the self-duality condition implies that the energy-momentum tensor of the self-dual instanton vanishes so that it does not back-reacts on the metric at semiclassical level. From the Yang-Mills point of view, a very important type of Euclidean configurations are the so-called merons, first introduced in [3]. Merons are gauge fields interpolating between different topological sectors.[1] In particular, instantons can be interpreted as merons bound states [4–7]. It is commonly accepted that merons are quite relevant configurations from the point of view of the confinement problem (see, for instance, [2,6]). In flat Euclidean spaces, merons are usually singular. Hence, on flat Euclidean spaces, a single "isolated" meron gives a vanishing contribution to the path integral as its Euclidean action is divergent. It is well known that merons are relevant only as "building blocks" of the instantons in the usual cases.

It is quite obvious that in many physically relevant situations the coupling with Einstein gravity[2] cannot be neglected (this is the case for instance in early cosmology [8] when topological solitons are believed to play a fundamental role). Consequently, a very important question arises: Is it still true that merons are necessarily singular even when the coupling with general relativity is taken into account? Indeed, due to the reasons mentioned above, whether or not merons are singular[3] can have a big influence on our understanding of the confinement problem. A first hint that the coupling of merons with general relativity can change the "flat" picture quite considerably can be found (with Lorentzian signature) in [9,10] where it has been shown that the singularity of the simplest meron can be hidden behind a black hole horizon.

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[1] One of the results of the present paper, as it will be explained in the next sections, is to construct a quite remarkable and concrete confirmation of this interpretation in the gravitating case.
[2] Or, at the very least, the non-vanishing curvature of space-time.
[3] Hence, whether or not merons can give a finite contribution to the semiclassical path integral through the corresponding saddle points.

A further very important situation where topological solitons play a fundamental role is in three Euclidean dimensions. The interest of the 3-dimensional case lies in the fact that difficult non-perturbative questions are easier to understand in three-dimensional Yang–Mills theory than in the four dimensions. Despite being simpler than QCD, three-dimensional Yang-Mills theory possesses local interacting degrees of freedom. A further benefit of three-dimensional Yang-Mills theory is that it is a good approximation of high temperature QCD[4] Last but not least, the Chern–Simons term can be included [11,12], leading to a mass for the gauge field which is of topological origin. The inclusion of the Chern-Simons term is not only a nice theoretical exercise since it can be shown that such a term appears upon integrating out the fermions (see, for instance [13,14]; a detailed review is [15]). Moreover, the nonperturbative features of topologically massive Yang-Mills theory in three dimensions are in a very good agreement with the expected confinement picture [16].

Very deep open issues related to three-dimensional topologically massive Yang-Mills theory are related to the following fact. Such a theory in a suitable range of parameters (see [16]) is confining. Standard arguments (see [2]) suggest that regular nontrivial Euclidean saddle points of the path integral must play a fundamental role to understand confinement. However, in three Euclidean dimensions, it is not possible to construct the usual self-dual Yang-Mills instantons (since one would need the four-dimensional Levi-Civita ε-symbol). In fact, as it will be discussed in the next sections, although there are no self-dual instantons in three Euclidean dimensions one can still construct regular smooth gravitating merons.

In general, it is very difficult to analyze the gravitational properties of topologically nontrivial configurations. Due to the difficulties in constructing analytic regular configurations of the four-dimensional Einstein-Yang-Mills system many of the available results are numerical (see, for instance, [17–21]).

The first aim of the present paper is to show that, nevertheless, it is possible to construct analytic regular solutions corresponding to gravitating merons in various dimensions in Euclidean Einstein-Yang-Mills theory. In order to achieve this goal two techniques are combined. The first technique is based on the $SU(2)$-valued generalized hedgehog ansatz (introduced in [22–37]), which works both for the Skyrme model and for the Yang-Mills-Higgs system. The second is based on the Cho approach [38–42].

The second aim is to show the coupling with Einstein gravity can change quite dramatically the usual physical interpretation of merons. In the three-dimensional case, we construct regular gravitating meron-like configurations and include a Chern-Simons term into the analysis. Due to the fact that in three dimensions it is not possible to define self-dual configurations, the regular Euclidean saddle points constructed here are likely to play a fundamental role to understand the non-perturbative features of the theory. In the four-dimensional case, we construct different regular gravitating meron-like configurations. Such configurations can be seen as smooth Euclidean wormholes interpolating between different vacua of the theory. Euclidean wormholes [43–55] (see, for a recent view on this topic, [56]) can be defined as extrema of the action in Euclidean quantum gravity connecting distant regions. It is widely recognized that such configurations can have quite remarkable physical consequences (as discussed in details in the above references). In five dimensions we construct regular meron-like configurations that generalize the three-dimensional result previously found for $\lambda = 1/2$. The metric is given by the a two-dimensional constant curvature space times the three-sphere. This result can be further extended to arbitrary higher dimensions. In dimension $D > 6$ the metric turns out to be given by the warped product of the three-sphere and any solution of the $D - 3$-dimensional Einstein equations in vacuum with an effective cosmological constant.

This paper is organized as follows: in Sec. II, meron-like configurations within the Euclidean Einstein-Yang-Mills theory are introduced. In Sec. III, we present a general ansatz to construct merons and Einstein-Yang-Mills equations are discussed. In Sec. IV the solutions are constructed. First, three-dimensional smooth regular gravitating merons are considered, the effects of the Chern-Simons term are included and the corresponding Euclidean action is computed. In four-dimensional case, smooth and regular gravitating merons are presented and their interpretation as Euclidean wormholes is discussed. Finally, we construct regular meron-like configurations in five and higher dimensions. In Sec. V, some conclusions are drawn.

II. THE SYSTEM

We consider the Euclidean Einstein-Yang-Mills system in D dimensions with cosmological constant. The action of the system is

$$S = S_{\mathrm{G}} + S_{\mathrm{SU(2)}}, \tag{1}$$

where the gravitational action S_{G} and the gauge field action $S_{\mathrm{SU(2)}}$ are given by

$$S_{\mathrm{G}} = -\frac{1}{16\pi G}\int d^D x\sqrt{g}(R - 2\Lambda), \tag{2}$$

$$S_{\mathrm{SU(2)}} = -\frac{1}{8e^2}\int d^D x\sqrt{g}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}), \tag{3}$$

where R is the Ricci scalar, G is Newton's constant, Λ is the cosmological constant, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$ is the field strength associated to the gauge field A_μ, and e is the Yang-Mills coupling constant. In our conventions $c = \hbar = 1$. The resulting N-dimensional Einstein equations are

[4] In which case the mass gap plays the role of the magnetic mass.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (4)$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the stress-energy tensor of the Yang-Mills field

$$T_{\mu\nu} = \frac{2}{\sqrt{g}}\frac{\delta S_{SU(2)}}{\delta g^{\mu\nu}} = -\frac{1}{2e^2}\mathrm{Tr}\left(F_{\mu\alpha}F_{\nu\beta}g^{\alpha\beta} - \frac{1}{4}g_{\mu\nu}F^{\rho\sigma}F_{\rho\sigma}\right). \quad (5)$$

The Yang-Mills equations are given by

$$\mathrm{YM}^{\mu} = \nabla_{\nu}F^{\mu\nu} + [A_{\nu}, F^{\mu\nu}] = 0, \quad (6)$$

where ∇^{μ} is the Levi-Civita covariant derivative. The connection $A_{\mu} = A^A_{\mu} t_A$ takes values on the $SU(2)$ algebra, whose generators are defined as

$$t_A = i\sigma_A, \qquad A = 1, 2, 3, \quad (7)$$

σ_A being the Pauli matrices.

Meron-like configurations as well as their important role in the non-perturbative sector of Yang-Mills theory have been extensively discussed in the literature (see, for instance, [3–7]). All the most important examples can be written in the following form[5][7]

$$A_{\mu} = \lambda U^{-1}\partial_{\mu}U, \qquad \lambda \neq 0, 1. \quad (8)$$

As it will be shown in the following, the Yang-Mills equations fix the parameter λ. Therefore, our definition of meron in the present paper will be a regular configuration of the form in Eq. (8) constructed with a topologically nontrivial $SU(2)$ map $U(x^{\mu})$. Note that the definition of meron in Eq. (8) works both with Euclidean and with Lorentzian signature. Although we will focus in this work mainly on the Euclidean case, many of the present results can be easily extended to the Lorentzian case.

We adopt the standard parametrization of the $SU(2)$-valued scalar $U(x^{\mu})$

$$U^{\pm 1}(x^{\mu}) = Y^0(x^{\mu})\mathbf{I} \pm Y^A(x^{\mu})t_A,$$
$$(Y^0)^2 + Y^A Y_A = 1, \quad (9)$$

where $\mathbf{I}$ is the 2×2 identity. The last equality implies that (Y^0, Y^A) is a unit vector in a three sphere, which is naturally accounted for by writing

$$Y^0 = \cos\alpha, \qquad Y^A = n^A \cdot \sin\alpha,$$
$$n^1 = \sin\Theta\cos\Phi, \qquad n^2 = \sin\Theta\sin\Phi, \qquad n^3 = \cos\Theta. \quad (10)$$

As it will be explained in the next sections, the ansatz for the α, Θ, and Φ functions will be chosen in order to have a nonvanishing winding number.

III. ANSATZ

For our purposes it will be convenient to introduce the left-invariant Maurer-Cartan forms on $SU(2)$, which can be defined in terms of the Euler angles $x^i = (\psi, \theta, \varphi)$ by

$$\Gamma_1 = \frac{1}{2}(\sin\psi d\theta - \sin\theta\cos\psi d\varphi),$$
$$\Gamma_2 = \frac{1}{2}(-\cos\psi d\theta - \sin\theta\sin\psi d\varphi),$$
$$\Gamma_3 = \frac{1}{2}(d\psi + \cos\theta d\varphi),$$
$$0 \leq \psi < 4\pi, \qquad 0 \leq \theta < \pi, \qquad 0 \leq \varphi < 2\pi.$$

We will consider a D-dimensional euclidean space-time of the form

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} = \gamma_{ab}(z)dz^a dz^b + \rho(z)^2\sum_{i=1}^{3}\Gamma_i \otimes \Gamma_i, \quad (11)$$

where we have split the coordinates as $x^{\mu} = (z^a, x^i)$, $a = 1, \ldots, d = D - 3$, γ_{ab} is a d-dimensional metric and $\rho(z)$ is a warping factor depending on the coordinates z^a only.

As it has been discussed in [30,36,37], the following choice for the functions in (10) is suitable for the class of metrics (11):

$$\Phi = \frac{\psi + \varphi}{2}, \qquad \tan\Theta = \frac{\cot(\frac{\theta}{2})}{\cos(\frac{\psi-\varphi}{2})}, \qquad \tan\alpha = \frac{\sqrt{1+\tan^2\Theta}}{\tan(\frac{\psi-\varphi}{2})}. \quad (12)$$

It is easy to verify directly that *in any background metric of the form in* Eq. (11), a meron ansatz of the form in Eqs. (8), (10) and (12) *identically satisfies the Lorentz gauge condition* (something which simplifies considerably the Yang-Mills equation):

$$\nabla^{\mu}A_{\mu} = 0. \quad (13)$$

It is also worth to emphasize that the present ansatz is topologically nontrivial as it has a nontrivial winding number along the $z^a = \mathrm{const}$ hypersurfaces of the metric in Eq. (11):

[5]It is more common to use the 't Hooft symbol (which is a Levi-Civita ε-tensor in which some of the indices are internal while other are space-time indices). On flat spaces, the usual notation is equivalent to the one in Eq. (8). On curved spaces the notation in Eq. (8) is much more convenient as it avoids the problem to properly define the 't Hooft symbol on curved spaces.

$$W = -\frac{1}{24\pi^2}\int_{S^3} \mathrm{tr}[(U^{-1}dU)^3] = -\frac{1}{2\pi^2}\int \sin^2\alpha \sin\Theta d\alpha d\Theta d\Phi = 1. \tag{14}$$

Hence, the present configuration cannot be deformed continuously to the trivial vacuum.

A. Yang-Mills equations

In the coordinates $x^\mu = (z^a, x^i)$, the gauge potential is split in two parts $A_\mu = \{A_a, A_i\}$. The ansatz in Eqs. (8), (10) and (12) leads to the following form for A_i

$$\begin{aligned} A_\psi &= -\frac{\lambda}{2}(\sin\theta\cos\varphi t_1 + \sin\theta\sin\varphi t_2 - \cos\theta t_3),\\ A_\theta &= \frac{\lambda}{2}(\sin\varphi t_1 - \cos\varphi t_2),\\ A_\varphi &= \frac{\lambda}{2}t_3, \end{aligned} \tag{15}$$

while the components A_a identically vanish

$$A_a = 0.$$

As the connection is time independent, the non-Abelian "electric" field vanishes and this meron-like configuration is purely "magnetic." In fact, the nonvanishing space-time components of the field strength are

$$\begin{aligned} F_{\psi\theta} &= -\frac{\lambda(\lambda-1)}{2}(\cos\theta\cos\varphi t_1 + \cos\theta\sin\varphi t_2 + \sin\theta t_3),\\ F_{\psi\varphi} &= \frac{\lambda(\lambda-1)}{2}\sin\theta(\sin\varphi t_1 - \cos\varphi t_2),\\ F_{\theta\varphi} &= \frac{\lambda(\lambda-1)}{2}(\cos\varphi t_1 + \sin\varphi t_2) \end{aligned} \tag{16}$$

and the left-hand sides of Yang-Mills equations (6) become,

$$\begin{aligned} \mathrm{YM}^\psi &= \frac{8\lambda(\lambda-1)}{\rho^4\sin\theta}(2\lambda-1)(\cos\varphi t_1 + \sin\varphi t_2),\\ \mathrm{YM}^\theta &= \frac{8\lambda(\lambda-1)(2\lambda-1)}{\rho^4}(-\sin\varphi t_1 + \cos\varphi t_2),\\ \mathrm{YM}^\varphi &= -\frac{8\lambda(\lambda-1)(2\lambda-1)}{\rho^4\sin\theta}\\ &\quad\times(\cos\theta\cos\varphi t_1 + \cos\theta\sin\varphi t_2 + \sin\theta t_3),\\ \mathrm{YM}^a &= 0. \end{aligned} \tag{17}$$

Therefore, the Yang-Mills equations are identically satisfied for

$$\lambda = \frac{1}{2}. \tag{18}$$

This is the standard value of λ for meronic configurations (8). As we will show, in three-dimensions it is possible to find a different result for λ when a Chern-Simons term is included in the action for the SU(2) gauge field. For $D > 3$ however, $\lambda = \frac{1}{2}$ will be assumed.

B. Einstein equations

In (11), the metric $g_{\mu\nu}$ splits as $g_{ij} = \rho(z)^2 h_{ij}(x)$, $g_{ab} = \gamma_{ab}(z)$, $g_{ia} = 0$, where h_{ij} is the metric of the three sphere in the coordinates x^i,

$$\sum_{i=1}^{3}\Gamma_i \otimes \Gamma_i = h_{ij}dx^i dx^j = \frac{1}{4}(d\psi^2 + 2\cos\theta d\psi d\varphi + d\theta^2 + d\varphi^2).$$

The Ricci tensor and the Ricci scalar are then given by

$$\begin{aligned} R_{ij} &= 2h_{ij}\left(1 - \tilde\nabla_a\rho\tilde\nabla^a\rho - \frac{1}{2}\rho\tilde\nabla^2\rho\right),\\ R_{ia} &= 0,\\ R_{ab} &= \tilde R_{ab} - \frac{3}{\rho}\tilde\nabla_b\tilde\nabla_a\rho,\\ R &= \tilde R + \frac{6}{\rho^2}(1 - \tilde\nabla_a\rho\tilde\nabla^a\rho - \rho\tilde\nabla^2\rho). \end{aligned} \tag{19}$$

where $\tilde R_{ab}$, $\tilde R$ and $\tilde\nabla$ denote the Ricci tensor, the Ricci scalar and the covariant derivative associated to the metric γ_{ab} respectively. Therefore, the Einstein tensor takes the form

$$\begin{aligned} G_{ij} &= h_{ij}\left(\tilde\nabla_a\rho\tilde\nabla^a\rho + 2\rho\tilde\nabla^2\rho - \frac{\rho^2}{2}\tilde R - 1\right),\\ G_{ia} &= 0,\\ G_{ab} &= \tilde R_{ab} - \frac{1}{2}\gamma_{ab}\tilde R\\ &\quad + \frac{3}{\rho^2}[\gamma_{ab}(\tilde\nabla_c\rho\tilde\nabla^c\rho + \rho\tilde\nabla^2\rho - 1) - \rho\tilde\nabla_b\tilde\nabla_a\rho]. \end{aligned} \tag{20}$$

The stress-energy tensor (5) for the meron field is given by

$$\begin{aligned} T_{ij} &= \frac{2\lambda^2(\lambda-1)^2}{e^2\rho^2}h_{ij},\\ T_{ia} &= 0,\\ T_{ab} &= -\frac{6\lambda^2(\lambda-1)^2}{e^2\rho^4}\gamma_{ab}, \end{aligned} \tag{21}$$

and therefore Einstein equations (4) yield

$$\tilde\nabla_a\rho\tilde\nabla^a\rho + 2\rho\tilde\nabla^2\rho - \frac{\rho^2}{2}\tilde R + \Lambda\rho^2 - 1 = \frac{16\pi G}{e^2\rho^2}\lambda^2(\lambda-1)^2, \tag{22}$$

$$\frac{\rho^2}{3}\tilde{G}_{ab} + \gamma_{ab}\left(\tilde{\nabla}_c\rho\tilde{\nabla}^c\rho + \rho\tilde{\nabla}^2\rho + \frac{\Lambda\rho^2}{3} - 1\right) - \rho\tilde{\nabla}_b\tilde{\nabla}_a\rho = -\frac{16\pi G}{e^2\rho^2}\lambda^2(\lambda-1)^2\gamma_{ab}. \quad (23)$$

where we have defined $\tilde{G}_{ab} = \tilde{R}_{ab} - \frac{1}{2}\gamma_{ab}\tilde{R}$ as the Einstein tensor associated to the metric γ_{ab}. Notice that the (i,j) components of the field equations reduce into a single equation (22). It should be emphasized that the same reduction of the (i,j) components of the field equations hold even with the Gauss-Bonnet term on the left-hand side of the field equations, which is given by

$$H_{\mu\nu} = 2(RR_{\mu\nu} - 2R_{\mu\rho}R^{\rho}{}_{\nu} - 2R^{\rho\sigma}R_{\mu\rho\nu\sigma} + R_{\mu}{}^{\rho\alpha\beta}R_{\nu\rho\alpha\beta}) - \frac{1}{2}g_{\mu\nu}(R^2 - 4R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}). \quad (24)$$

This can be easily shown by observing the following equations

$$R_{ijkm} = \rho^2(1 - \tilde{\nabla}_a\rho\tilde{\nabla}^a\rho)(h_{ik}h_{jm} - h_{im}h_{jk}), \quad (25)$$

$$R_{ijab} = R_{iabc} = R_{aijk} = 0, \quad (26)$$

$$R_{iajb} = -\rho h_{ij}\tilde{\nabla}_a\tilde{\nabla}_b\rho, \quad (27)$$

from which one has $H_{ia} = 0$, and $H_{ij} \propto h_{ij}$. To solve the Einstein-Yang-Mills equations for meron configurations with the Gauss-Bonnet term is a valuable task in its own right. In this paper, however, we focus only on the Einstein-Hilbert action for the gravity sector, and the issues related to the Gauss-Bonnet gravity will be studied in a separate paper.

IV. SOLUTIONS

A. $D = 3$

In three dimensions $\gamma_{ab} = 0$ and $\rho = \rho_0$ is constant. Therefore the metric (11) is simply given by

$$ds^2 = \rho_0^2\sum_{i=1}^{3}\Gamma_i \otimes \Gamma_i = \frac{\rho_0^2}{4}(d\tau^2 + 2\cos\theta d\tau d\varphi + d\theta^2 + d\varphi^2), \quad (28)$$

where we have considered $\psi = \tau$ as the Euclidean time. In this case, Einstein equations (22) yield one single algebraic equation for ρ_0,

$$\Lambda\rho_0^2 - 1 = \frac{16\pi G}{e^2\rho_0^2}\lambda^2(\lambda-1)^2, \quad (29)$$

which can be solved for $\Lambda > 0$. As Yang-Mills equations (17) require $\lambda = 1/2$, Eq. (29) fixes ρ_0 to be

$$\rho_0^2 = \frac{1}{2\Lambda}\left(1 \pm \sqrt{1 + \frac{4\pi G\Lambda}{e^2}}\right). \quad (30)$$

The meronic configuration in this case is defined on the three-sphere with overall fact ρ_0 and it is regular and smooth everywhere.

1. Chern-Simons term

In the three-dimensional case it is possible to find a more general meron-like solution by adding a Chern-Simons term to the action (3) and considering

$$S_{\mathrm{SU}(2)} = -\frac{1}{8e^2}\int d^Dx\sqrt{g}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + S_{CS},$$

where the Chern-Simons action for the $SU(2)$ valued gauge field is given by

$$S_{CS} = \frac{k}{2e^2}\int \mathrm{Tr}\left[AdA + \frac{2}{3}A^3\right], \quad (31)$$

and k is related to the Chern-Simons level[6] This modification leads to the Yang-Mills-Chern-Simons equations

$$\mathrm{YMCS}^{\mu} = \nabla_{\nu}F^{\mu\nu} + [A_{\nu}, F^{\mu\nu}] + k\epsilon^{\nu\rho\sigma}F_{\rho\sigma} = 0. \quad (32)$$

Using (17) and (16) it is straightforward to check that

$$\mathrm{YMCS}^{\psi} = \frac{8\lambda(\lambda-1)}{\rho^4\sin\theta}(2\lambda - 1 + k\rho_0)(\cos\varphi t_1 + \sin\varphi t_2)$$
$$\mathrm{YMCS}^{\theta} = \frac{8\lambda(\lambda-1)(2\lambda-1+k\rho_0)}{\rho^4}(-\sin\varphi t_1 + \cos\varphi t_2),$$
$$\mathrm{YMCS}^{\varphi} = -\frac{8\lambda(\lambda-1)(2\lambda-1+k\rho_0)}{\rho^4\sin\theta} \times(\cos\theta\cos\varphi t_1 + \cos\theta\sin\varphi t_2 + \sin\theta t_3), \quad (33)$$

which leads to

$$\lambda = \frac{1}{2}(1 - k\rho_0) \quad (34)$$

(note that in the Einstein-Yang-Mills case $(k = 0)$ we get the usual "meronic" value $\lambda = 1/2$).

Due to its topological nature, the Chern-Simons term does not contribute to the energy-momentum tensor (21). This means that, when (31) is included in the gauge field action, the only modification in the Einstein equations (29) is the value of λ. In this case we obtain

[6]There are two possible conventions for the Chern-Simons level k: we will comment on them in the following sections.

$$\rho_0^2\Lambda - 1 = \frac{\pi G}{e^2\rho_0^2}(1 - k^2\rho_0^2)^2, \tag{35}$$

which can be solved for $\Lambda > 0$ to give

$$\rho_0^2 = \frac{e^2 - 2\pi G k^2 \pm e\sqrt{e^2 + 4\pi G(\Lambda - k^2)}}{2e^2\Lambda - 2\pi G k^4}. \tag{36}$$

Note that for ρ_0^2 to be positive, one of the following conditions must hold:

(i) $e^2 + 4\pi G(\Lambda - k^2) > 0, \quad 2\pi G k^2 > e^2, \quad e^2\Lambda < \pi G k^4$,

(ii) $e^2\Lambda > \pi G k^4$,

(iii) $e^2 + 4\pi G(\Lambda - k^2) = 0$,
$(e^2 - 2\pi G k^2)(e^2\Lambda - \pi G k^4) > 0$,

(iv) $e^2\Lambda = \pi G k^4, \quad k^2/\Lambda < 2$.

2. Imaginary coupling

In order for the theory to have a well-defined Lorentzian continuation, the Euclidean Chern-Simons term must have imaginary coupling ($k \to ik$, $k \in \mathbb{N}$, $i^2 = -1$). In this case the solutions look very similar with the difference that the meron parameter λ is not real anymore:

$$\lambda = \frac{1}{2}(1 - ikR_0), \qquad R_0 \in \mathbb{R}.$$

These configurations represent complex saddle points of the Einstein-Yang-Mills-Chern-Simons action. In recent years, it has been shown in many nontrivial examples (see [57] and references therein; for detailed reviews see [58–60]) that nontrivial complex saddle points are necessary to give a consistent nonperturbative definition of the path integral. In particular, when such complex saddles are not included in the analysis, inconsistencies appear. Hence, the present results strongly suggest that these gravitating merons are relevant building blocks to get a consistent path-integral in the Einstein-Yang-Mills-Chern-Simons case.

3. Euclidean action

Also in the three-dimensional case the non-perturbative nature of this configurations is apparent as they depend on $1/e^2$. In particular, the classical Euclidean action I_E corresponding to the set of solutions can be easily computed to give:

$$I_E = h\left(\frac{1}{e^2}, \Lambda, G, k\right) = \frac{\pi\rho_0}{4G}(\rho_0^2\Lambda - 3) + \frac{12\pi^2}{e^2\rho_0}\lambda^2(\lambda - 1)^2 - \frac{4\pi^2 k}{e^2}\lambda^2(\lambda + 3). \tag{37}$$

The obvious relevance of this result is that, at semiclassical level, the contribution of this configuration to the path-integral is proportional to Z_E,

$$Z_E \approx \exp[-I_E]. \tag{38}$$

Therefore, gravitating merons play an important role in the nonperturbative sector of the theory. This is especially important in the three-dimensional case in which self-dual instantons do not exist and, consequently, these Euclidean smooth regular (and with finite actions) configurations can be quite relevant.

It is also worthwhile to emphasize the remarkable effect of the Chern-Simons term which supports the existence of gravitating merons with $\lambda \neq 1/2$. To the best of the authors' knowledge, these are the first examples of smooth merons with this characteristic. Due to the fact that the Chern-Simons term can arise upon integrating over Fermionic degrees of freedom, it is natural to wonder whether one could construct merons with $\lambda \neq 1/2$ even with Fermionic matter fields. We hope to come back on this very interesting question in a future publication. As it has been already emphasized, in the case in which the Chern-Simons coupling is taken as ik with $k \in \mathbb{R}$, the present configurations have to be considered as smooth regular complex saddle points. Correspondingly, the Euclidean action also gets a nontrivial imaginary part. These configurations have to be properly analyzed using resurgence techniques (following [57–60]). We hope to come back on this issue in a future publication.

As far as the evaluation of the Euclidean action of the four dimensional solutions is concerned, it involves some subtleties. The reason is that, in the presence of a negative cosmological constant, one needs to include suitable boundary terms to obtain a finite results. The construction of these boundary terms when topologically nontrivial non-Abelian gauge fields are present has not been discussed in details in the literature. We hope to come back to this interesting issue in a future publication.

B. $D = 4$

In four dimensions we consider only one extra coordinate $z = r$ in (11) and for simplicity we will just take $\gamma_{rr} = 1$. The metric then takes the form

$$ds^2 = dr^2 + \frac{\rho^2(r)}{4}(d\tau^2 + 2\cos\theta d\tau d\varphi + d\theta^2 + d\varphi^2). \tag{39}$$

where again we have considered $\psi = \tau$ as the Euclidean time. Einstein equations (22) and (23) are reduced to two ordinary differential equations

$$\rho'^2 + 2\rho\rho'' + \Lambda\rho^2 - 1 = \frac{\pi G}{e^2\rho^2}, \tag{40}$$

$$\rho'^2 + \frac{\Lambda}{3}\rho^2 - 1 = -\frac{\pi G}{e^2\rho^2}, \tag{41}$$

where we have already replaced (18). If we plug the Eq. (41) into (40), then we have a single ordinary differential equation of $\rho(r)$,

$$\rho\rho'' + \frac{\Lambda}{3}\rho^2 - \frac{\pi G}{e^2\rho^2} = 0. \tag{42}$$

When the cosmological constant Λ is positive, there does not exist real solution to this equation. Now let us examine the cases of zero and negative cosmological constants. Similar results have been discussed in [43–47].

Case 1: $\Lambda = 0$

When $\Lambda = 0$, the solution to (42) is,

$$\rho(r) = \frac{1}{e}\sqrt{a(r+b)^2 + \frac{\pi G e^2}{a}}, \tag{43}$$

where a and b are integration constants. This solution satisfies the Eqs. (40) and (41) if

$$a = e^2.$$

Thus the solution for vanishing cosmological constant is,

$$\rho(r) = \sqrt{\frac{\pi G}{e^2} + (r+b)^2}. \tag{44}$$

Hence, these configurations can be interpreted as smooth asymptotically flat Euclidean wormholes sourced by merons. The size of the throat is proportional to $1/e^2$ thus showing explicitly that the "opening of the throat" is a nonperturbative phenomenon. Moreover, the fact that such Euclidean wormholes are sourced by Yang-Mills merons (which, by themselves, represent tunneling between different Gribov vacua [7]) sheds considerable light on the physical interpretation of these Euclidean wormholes. Indeed, the solution is smooth and regular everywhere, the gauge field is regular and the scale factor ρ is smooth and non-vanishing. In particular, both asymptotic regions (corresponding to $r \to \pm\infty$) are flat (the wormhole throat being at $r = -b$). Similar Euclidean wormhole solutions have been studied in [43–46]. Examples of Euclidean wormholes embedded in higher dimensional theories as well as including the explicit presence of axionic fields have been worked out in [48–56].

Case 2: $\Lambda < 0$

When $\Lambda < 0$, the solution to (42) is,

$$\rho(r) = \frac{1}{4e}\left[2C_1\left(\left(\frac{64\pi G e^2}{C_1^2} + C_2^2\right)\exp\left(2\sqrt{\frac{-\Lambda}{3}}r\right)\right.\right.$$
$$\left.\left. - \frac{3}{4\Lambda}\exp\left(-2\sqrt{\frac{-\Lambda}{3}}r\right) + \sqrt{\frac{3}{-\Lambda}}C_2\right)\right]^{1/2}, \tag{45}$$

where C_1 and C_2 are integration constants. The above solution is real whenever C_1 is positive. In addition, this solution satisfies the Eqs. (40) and (41) if C_1 and C_2 are related by

$$C_1 C_2 = -4e^2\sqrt{\frac{3}{-\Lambda}}.$$

With these conditions, we have the solution $\rho(r)$ given by,

$$\rho(r) = \frac{1}{4e}\left[\frac{2}{C_1}\left(64\pi G e^2 - \frac{48e^4}{\Lambda}\right)\exp\left(2\sqrt{\frac{-\Lambda}{3}}r\right)\right.$$
$$\left. - \frac{3C_1}{2\Lambda}\exp\left(-2\sqrt{\frac{-\Lambda}{3}}r\right) + \frac{24e^2}{\Lambda}\right]^{1/2}. \tag{46}$$

Let us notice that the argument of the square root is positive definite, and its minimum value

$$\rho_{\min} = \frac{\sqrt{6}}{2\sqrt{-\Lambda}}\left[\sqrt{1 - \frac{4\pi G\Lambda}{3e^2}} - 1\right]^{1/2} \tag{47}$$

occurs at

$$r = \frac{1}{2}\sqrt{\frac{3}{-\Lambda}}\mathrm{Log}\left(\frac{\sqrt{3}C_1}{8e\sqrt{3e^2 - 4\pi G\Lambda}}\right), \tag{48}$$

if the right-hand side of (48) is positive. Therefore, if we choose a sufficiently small positive constant C_1, then the corresponding solution is regular and smooth everywhere for any $r \in \mathbb{R}$. In these cases both asymptotic regions (namely, $r \to \pm\infty$) are (the Euclidean version of) AdS.

Thus, both in *Case 1* and in *Case 2* described above the gravitating merons can be interpreted as smooth Euclidean wormholes interpolating between the vacua of the theory. It is also worthwhile to emphasize that also in this case the (size of the) wormhole throat is nonperturbative in the Yang-Mills coupling e^2 (as it depends on $1/e^2$: see Eqs. (44) and (47)). Consequently, the present configurations will be relevant in the nonperturbative sector of Einstein-Yang-Mills theory.

C. $D = 5$

Solutions with constant ρ analogous to the three-dimensional one (30) previously constructed cannot be generalized to four dimensions, as in that case the Eqs. (22) and (23) do not admit solutions for constant ρ. For $D > 4$, however, the warping factor ρ can be taken as a constant ρ_0. In five dimensions we can consider coordinates $z^a = (\tau, r)$, were τ is the Euclidean time and r a radial coordinate. The simplest solutions of the form (11) can be obtained by considering a two-dimensional metric γ_{ab} with constant curvature $\tilde{R} = K$ and

$$\gamma_{ab} = \begin{pmatrix} r^2 & 0 \\ 0 & \frac{1}{1-\frac{K}{2}r^2} \end{pmatrix}.$$

In that case, Einstein equations (22) and (23) take the form

$$\left(\frac{K}{2}-\Lambda\right)\rho_0^2+1+\frac{\pi G}{e^2\rho_0^2}=0, \tag{49}$$

$$\frac{\Lambda\rho_0^2}{3}-1+\frac{\pi G}{e^2\rho_0^2}=0. \tag{50}$$

Equation (49) fixes K in terms of ρ_0, Λ, G and e,

$$K=2\Lambda-\frac{2}{\rho_0^2}\left(1+\frac{\pi G}{e^2\rho_0^2}\right),$$

while Eq. (50) determines ρ_0^2:

(i) For $\Lambda>0$ and $\frac{4\pi G\Lambda}{3e^2}\leq 1$,

$$\rho_0^2=\frac{3}{2\Lambda}\left[1\pm\sqrt{1-\frac{4\pi G\Lambda}{3e^2}}\right]. \tag{51}$$

(ii) For $\Lambda=0$,

$$\rho_0^2=\frac{\pi G}{e^2}. \tag{52}$$

(iii) For $\Lambda<0$,

$$\rho_0^2=\frac{3}{2\Lambda}\left[1-\sqrt{1-\frac{4\pi G\Lambda}{3e^2}}\right]. \tag{53}$$

As in the three-dimensional case, one could be tempted to add a five-dimensional Chern-Simons term to the Yang-Mills actions (3). However the five-dimensional Chern-Simons equations are proportional to $\epsilon^{\mu\nu\rho\sigma\lambda}F_{\nu\rho}F_{\sigma\lambda}$ which vanishes for the meron field-strength (16). The same argument holds in higher odd-dimensional cases.

D. Higher dimensions

Solutions of the form (51) can be easily extended to arbitrarily higher dimensions. In fact, for $\rho=\rho_0$ a constant, and γ_{ab} a d-dimensional metric. Einstein equations (22) and (23) reduce in general to

$$\left(\frac{\tilde{R}}{2}-\Lambda\right)\rho_0^2+1+\frac{\pi G}{e^2\rho_0^2}=0 \tag{54}$$

$$\tilde{G}_{ab}+\left[\Lambda+\frac{3}{\rho_0^2}\left(\frac{\pi G}{e^2\rho_0^2}-1\right)\right]\gamma_{ab}=0. \tag{55}$$

The first equation implies that the Ricci tensor $\tilde{R}$ for the metric γ_{ab} is constant, while the second equation can be written as the Einstein equations for γ_{ab} with an effective cosmological constant:

$$\tilde{\Lambda}=\Lambda+\frac{3}{\rho_0^2}\left(\frac{\pi G}{e^2\rho_0^2}-1\right).$$

This means that in any dimension $D=d+3$ with $d>2$, the metric γ_{ab} is an Einstein manifold with cosmological constant $\tilde{\Lambda}$, i.e.,

$$\tilde{R}_{ab}=\frac{2\tilde{\Lambda}}{d-2}\gamma_{ab},$$

which means that the Ricci scalar $\tilde{R}$ is given by

$$\tilde{R}=\frac{2d}{d-2}\left[\Lambda+\frac{3}{\rho_0^2}\left(\frac{\pi G}{e^2\rho_0^2}-1\right)\right].$$

Plugging this back in Eq. (54) we find ρ_0^2 to be

(i) For $\Lambda>0$ and $\frac{4\pi G\Lambda(2d-1)}{e^2(d+1)^2}\leq 1$,

$$\rho_0^2=\frac{d+1}{2\Lambda}\left[1\pm\sqrt{1-\frac{4\pi G\Lambda(2d-1)}{e^2(d+1)^2}}\right]. \tag{56}$$

(ii) For $\Lambda=0$,

$$\rho_0^2=\frac{\pi G(2d-1)}{e^2(d+1)}. \tag{57}$$

(iii) For $\Lambda<0$,

$$\rho_0^2=\frac{d+1}{2\Lambda}\left[1\pm\sqrt{1-\frac{4\pi G\Lambda(2d-1)}{e^2(d+1)^2}}\right]. \tag{58}$$

The fact that any d-dimensional Einstein manifold with cosmological constant $\tilde{\Lambda}$ and constant provides a solution for γ_{ab} is very interesting. In higher dimensions one could use different known solutions plus the three-sphere to construct Euclidean geometries supporting meron-like configurations of the form (15). One interesting example would be, for instance, to use the Euclidean Schwarzschild-AdS or Euclidean Kerr-AdS black holes in four dimensions as the metric γ_{ab}, to form a seven-dimensional black brane with three compact dimensions. It would be also very interesting to construct solutions with a nonconstant and regular warp factor. This task, however, is quite nontrivial and it is likely that some extra ingredients are required to achieve it. We hope to come back to this issue in a future publication.

V. CONCLUSIONS

Analytic smooth configurations of Euclidean Einstein-Yang-Mills system have been constructed. The ansatz for the gauge field is of meron-type: it is proportional to a pure gauge (with a suitable parameter λ which is determined by solving the field equations). The smooth gauge transformation used to construct the meron cannot be deformed continuously to the identity as it possesses a nonvanishing winding number. In the three dimensional case, the solution is smooth and the spatial geometry is a three-sphere. The effects of the inclusion of a Chern-Simons term can be studied explicitly. Interestingly enough, one of the effects of the Chern-Simons term is that, unlike what happens in the pure Yang-Mills case, the parameter λ is in general different from $1/2$: the value of λ in the 3D Yang-Mills-Chern-Simons case depends explicitly on the Chern-Simons coupling. In dimensions greater than three, one gets $\lambda = 1/2$. In four dimensions the corresponding geometry can be interpreted as a smooth Euclidean wormhole interpolating between different vacua of the theory (thus, extending the usual flat interpretation of merons). In five dimensions regular meron-like configurations have been found, where the metric is given by the three-sphere times a constant curvature space. This last result can be extended to arbitrary higher dimensions where the metric is given by the warped product the three-sphere with any solution of the $(D-3)$-dimensional Einstein equations in vacuum with an effective cosmological constant. In all theses cases, the coupling of the meron with general relativity "regularizes" the configurations. Namely, Yang-Mills configurations (which on flat spaces would be singular) become regular when the coupling with general relativity is considered. This remarkable effect could be named gravitational catalysis of merons. One of the consequences of this fact is that, while in the flat case the Euclidean action of merons is divergent (so that a single meron gives vanishing contribution to the semi-classical path integral), gravitating merons can be smooth and regular and, consequently, they can give a nonvanishing contribution to the semiclassical path integral (as the present examples clearly show). In Cho's approach we can express the vacuum potential $\Omega_\mu = U^{-1}\partial_\mu U$ explicitly with $\hat{n} = (n_1, n_2, n_3)$, and express the ansatz (11) solely by $\hat{n}$. With this we can obtain the same result using $\hat{n}$ [38–42]. A very interesting issue (to which we hope to come back in a future publication) is the resurgence analysis (along the lines of [57]) of the complex regular meron-like saddle points which appear in the Einstein-Yang-Mills-Chern-Simons case when the Chern-Simons coupling constant is taken as ik.

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