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Big bang nucleosynthesis with stable ^{8}Be and the primordial lithium problem

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A change in the fundamental constants of nature or plasma effects in the early universe could stabilize ^{8}Be against decay into two ^{4}He nuclei. Coc *et al.* examined the former effect on big bang nucleosynthesis as a function of B_8, the mass difference between two ^{4}He nuclei and a single ^{8}Be nucleus, and found no effects for $B_8 \leq 100$ keV. Here we examine stable ^{8}Be with larger B_8 and also allow for a variation in the rate for $^4\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^8\mathrm{Be}$ to determine the threshold for interesting effects. We find no change to standard big bang nucleosynthesis for $B_8 < 1$ MeV. For $B_8 \gtrsim 1$ MeV and a sufficiently large reaction rate, a significant fraction of ^{4}He is burned into ^{8}Be, which fissions back into ^{4}He when B_8 assumes its present-day value, leaving the primordial ^{4}He abundance unchanged. However, this sequestration of ^{4}He results in a decrease in the primordial ^{7}Li abundance. Primordial abundances of ^{7}Li consistent with observationally inferred values can be obtained for reaction rates similar to those calculated for the present-day (unbound ^{8}Be) case. Even for the largest binding energies and largest reaction rates examined here, only a small fraction of ^{8}Be is burned into heavier elements, consistent with earlier studies. There is no change in the predicted deuterium abundance for any model we examined.

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I. INTRODUCTION

Big bang nucleosynthesis (BBN) has long served as a useful constraint on the physics of the early universe. In particular, any change in the fundamental constants of nature could significantly alter BBN, allowing constraints to be placed on such models [1–15]. For a review of BBN with time-varying constants, see Ref. [16].

A particularly interesting possibility is that an appropriate change in the constants of nature might allow for the stability of ^{8}Be, which normally spontaneously fissions into $^4\mathrm{He} + {}^4\mathrm{He}$ with a very short lifetime. Coc *et al.* [14] investigated the effects of stable ^{8}Be on BBN. More recently, Adams and Grohs [17] examined stellar evolution with stable ^{8}Be. Their goal was not to constrain such a model, but rather to refute anthropic arguments for the fine-tuning needed to allow the triple-α reaction to proceed by showing that stable ^{8}Be could provide an acceptable alternative pathway for the production of heavier elements. A completely different mechanism to stabilize ^{8}Be has been suggested by Yao *et al.* [18]. They proposed that plasma effects could stabilize ^{8}Be in the early universe, obviating the need for new physics.

To keep our results as general as possible, we will not assume a particular model for stable ^{8}Be, but will instead treat the ^{8}Be binding energy as a free parameter. Following Ref. [14], we define the mass difference between a single ^{8}Be nucleus and two ^{4}He nuclei to be

$$B_8 = 2M(^4\mathrm{He}) - M(^8\mathrm{Be}). \tag{1}$$

Present-day measurements give $B_8 = -0.092$ MeV. However, if the constants of nature during BBN were sufficiently different so as to make B_8 positive, then ^{8}Be would be stable, significantly altering the reaction network; a similar effect might occur due to plasma effects in the early universe. Coc *et al.* examined BBN for $B_8 \leq 100$ keV and found no significant effect on any of the resulting element abundances. Here, we extend this calculation to larger values of B_8. Given uncertainties in the nuclear rates when the constants of nature are allowed to change, we also parametrize the rate for $^4\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^8\mathrm{Be}$ in terms of an overall multiplicative factor. In the next section, we present our calculations of the primordial element abundances and give the results of these calculations. We discuss our results in Sec. III. We find that BBN can be significantly altered for $B_8 \sim 1$–3 MeV, with a large reduction in the ^{7}Li abundance, while the predicted deuterium and ^{4}He abundances are unchanged. The nuclear fusion rates necessary to achieve this are similar to those calculated for the present-day ^{8}Be binding energy.

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II. CALCULATION OF ELEMENT ABUNDANCES

Consider first the standard model for BBN. (For recent reviews, see Refs. [19,20].) In the first stage of BBN, the weak interactions interconvert protons and neutrons, maintaining a thermal equilibrium ratio:

$$n + \nu_e \leftrightarrow p + e^-,$$
$$n + e^+ \leftrightarrow p + \bar{\nu}_e,$$
$$n \leftrightarrow p + e^- + \bar{\nu}_e, \tag{2}$$

while a thermal abundance of deuterium is maintained via

$$n + p \leftrightarrow \mathrm{D} + \gamma. \tag{3}$$

After the weak reactions drop out of thermal equilibrium at $T \sim 0.8$ MeV, free neutron decay continues until $T \sim 0.1$ MeV, when the thermal equilibrium abundance of deuterium becomes large enough to allow rapid fusion into heavier elements. Almost all of the remaining neutrons end up bound into ^{4}He, with a small fraction remaining behind in the form of deuterium. There is also some production of ^{7}Li via

$$^4\mathrm{He} + {}^3\mathrm{H} \to {}^7\mathrm{Li} + \gamma, \tag{4}$$

$$^4\mathrm{He} + {}^3\mathrm{He} \to {}^7\mathrm{Be} + \gamma, \tag{5}$$

where the ^{7}Be decays into ^{7}Li via electron capture at the beginning of the recombination era [21].

The element abundances produced in BBN depend on the baryon/photon ratio η, which can be independently determined from the cosmic microwave background (CMB). We adopt a value of $\eta = 6.1 \times 10^{-10}$, consistent with recent results from Planck [22]. This value of η yields predicted abundances of D and ^{4}He consistent with observations. Recent observational estimates of D/H include those of the Particle Data Group [23]: $\mathrm{D/H} = (2.53 \pm 0.04) \times 10^{-5}$ and Cooke *et al.* [24]: $\mathrm{D/H} = (2.547 \pm 0.033) \times 10^{-5}$. The primordial ^{4}He abundance, designated Y_p, is not as well established. Izotov *et al.* [25] give $Y_p = 0.2551 \pm 0.0022$, while Aver *et al.* [26] give $Y_p = 0.2449 \pm 0.0040$. The Particle Data Group limit is [23] $Y_p = 0.2465 \pm 0.0097$. Given these discrepant estimates, a safe limit on ^{4}He is $Y_p = 0.25 \pm 0.01$. As noted, both the deuterium and ^{4}He abundances are consistent with the predictions of standard BBN with the CMB value for η.

The same cannot be said for the ^{7}Li abundance. The primordial lithium abundance is estimated to be [23]

$$^7\mathrm{Li/H} = (1.6 \pm 0.3) \times 10^{-10}. \tag{6}$$

However, standard BBN with $\eta \sim 6 \times 10^{-10}$ predicts a primordial value for ^{7}Li/H that is roughly three times higher than this observationally inferred value. For this value of η, most of the primordial ^{7}Li is produced in the form of ^{7}Be, which decays into ^{7}Li much later, as noted above. This discrepancy between the predicted and observationally inferred primordial ^{7}Li abundances has been dubbed the "lithium problem," and it remains unresolved at present (for a further discussion, see Ref. [27]). Hypothetical changes in the constants of nature have been invoked previously as a possible solution of the lithium problem [11].

In standard BBN, ^{8}Be is excluded from the reaction network, as it undergoes spontaneous fission,

$$^8\mathrm{Be} \to {}^4\mathrm{He} + {}^4\mathrm{He}, \tag{7}$$

with a lifetime $\sim 10^{-16}$ sec; the energy liberated in this fission is $-B_8$. Here we assume that during the era of BBN, $B_8 > 0$, so that ^{8}Be is stable. Coc *et al.* [14] examine a specific model for time variation of the fundamental constants, in which all of the binding energies can be calculated as functions of the change in the nucleon-nucleon interactions. A similar approach is taken by Epelbaum *et al.* [28]. Adams and Grohs [17] take a more general approach and discuss several ways in which changes in the fundamental constants might alter B_8. Since we are interested in isolating the particular effects of stable ^{8}Be, we shall adopt the latter approach and treat B_8 as a free parameter. The major limitation of our treatment is that we do not consider changes in the other nuclear binding energies; these require the assumption of a specific model like the one in Ref. [14]. We discuss this issue further in Sec. III.

Given the existence of stable ^{8}Be, the primary new reactions of importance are

$$^4\mathrm{He} + {}^4\mathrm{He} \to {}^8\mathrm{Be} + \gamma, \tag{8}$$

and

$$^8\mathrm{Be} + {}^4\mathrm{He} \to {}^{12}\mathrm{C} + \gamma, \tag{9}$$

along with the corresponding reverse reactions. Estimated rates for these two reactions have been derived by Nomoto *et al.* [29], Langanke *et al.* [30] and Descouvement and Baye [31]. Adams and Grohs [17] use the nonresonant reactions from Ref. [29], while Coc *et al.* [11] derived their own expressions for these rates based on a particular model for changes in the nuclear interaction strength. There are, of course, large uncertainties in any calculation of this kind. For example, the nonresonant rate calculated in Ref. [29] is not a direct-capture rate; instead, it represents the low-energy wing of the resonance at the current ^{8}Be binding energy. Any change in B_8 might have profound effects on this rate. On the other hand, the calculation of Ref. [14] assumes a particular model for changes in the nuclear interaction strength.

We have chosen to parametrize the uncertainty in this calculation by expressing the rate for reaction (8) in terms of the standard expression for charged-particle interactions along with an overall multiplicative constant, which we allow to vary. This allows us to the determine the threshold for interesting effects, which can then be compared (at least in order of magnitude) to previous estimates for the rate. For reaction (9), which is less important for our results, we follow Ref. [17] and use the nonresonant rate from Ref. [29].

Recall that for a charged-particle reaction like reaction (8), the cross section as a function of center-of-mass energy E can be written as

$$\sigma(E) = S(E)E^{-1}\exp(-2\pi\eta), \tag{10}$$

where η is the Sommerfeld parameter, $\eta = Z_1 Z_2 e^2/\hbar v$, with Z_1 and Z_2 the charges on the incoming nuclei and v their relative velocities. If the reaction is nonresonant, then $S(E)$ is generally a slowly varying function of E (see, e.g., Ref. [32] for a pedagogical discussion). The standard procedure is to expand $S(E)$ in a power series around $E = 0$ and convolve the cross section with the thermal distribution of nuclei. For reaction (8) we obtain an expression of the form [33]:

$$N_A\langle\alpha\alpha\rangle = T_9^{-2/3}\exp(-13.489T_9^{-1/3}) \times \sum_{N=0}^{5} F_N T_9^{N/3}\ \mathrm{cm^3\,sec^{-1}mole^{-1}}, \tag{11}$$

where T_9 is the temperature in units of 10^9 K, and N_A is Avogadro's number. In this expression for the reaction rate, the F_N are functions of $S(E)$ and its first and second derivatives at $E = 0$ [34]. For the purposes of this study, we take only the constant term F_0 in Eq. (11), and we ignore all of the higher powers of $T_9^{1/3}$ so that

$$N_A\langle\alpha\alpha\rangle = F_0 T_9^{-2/3}\exp(-13.489T_9^{-1/3})\ \mathrm{cm^3\,sec^{-1}\,mole^{-1}}. \tag{12}$$

Effectively, this amounts to treating $S(E)$ as a constant as $E \to 0$. We do not claim that this is likely to be the most accurate description of the form for the reaction rate. However, it gives a simple one-parameter model for this rate that can be compared (at least at the order-of-magnitude level) with other expressions for the cross section. As noted earlier, for reaction (9), we simply use the nonresonant rate of Nomoto *et al.* [29]. As we will see, this process has little impact on our final results.

The reverse reaction rates can be calculated from the forward rates using detailed balance. For reactions of the form $i + j \to k + \gamma$, we have (see, e.g., Ref. [33])

$$\langle\sigma v\rangle_{k\gamma} = \frac{1}{1+\delta_{ij}}\left(\frac{A_i A_j}{A_k}\right)^{3/2}\frac{1.0\times 10^{10}\ \mathrm{g/cm^3}}{\rho_B} \times T_9^{3/2}\exp(-11.605Q^*/T_9)\langle\sigma v\rangle_{ij}, \tag{13}$$

where we have used the fact that all of the nuclei in reactions (8) and (9) are spin singlet states, and Q^* denotes the Q values for reactions (8)–(9) when B_8 is allowed to vary from its present value. The present-day Q values for these two reactions are, respectively, $Q_{\alpha\alpha} = -0.092$ MeV and $Q_{\alpha^8\mathrm{Be}} = 7.27$ MeV. When we allow the binding energy of ^{8}Be to change, the new Q values become $Q^*_{\alpha\alpha} = B_8$ and $Q^*_{\alpha^8\mathrm{Be}} = 7.27\ \mathrm{MeV} - B_8$.

We expect reactions (8) and (9) to be the most important new pathways for the buildup of heavier elements when ^{8}Be is stable. However, we have also examined the effects of the following reactions:

$$^7\mathrm{Be} + n \leftrightarrow {}^8\mathrm{Be} + \gamma, \tag{14}$$

$$^7\mathrm{Li} + p \leftrightarrow {}^8\mathrm{Be} + \gamma, \tag{15}$$

$$^7\mathrm{Be} + {}^2\mathrm{H} \leftrightarrow {}^8\mathrm{Be} + p, \tag{16}$$

$$^7\mathrm{Li} + {}^2\mathrm{H} \leftrightarrow {}^8\mathrm{Be} + n, \tag{17}$$

$$^8\mathrm{Be} + n \leftrightarrow {}^9\mathrm{Be} + \gamma, \tag{18}$$

$$^8\mathrm{Li} + p \leftrightarrow {}^8\mathrm{Be} + n, \tag{19}$$

$$^8\mathrm{B} + n \leftrightarrow {}^8\mathrm{Be} + p, \tag{20}$$

$$^9\mathrm{Be} + p \leftrightarrow {}^8\mathrm{Be} + {}^2\mathrm{H}, \tag{21}$$

$$^{11}\mathrm{B} + p \leftrightarrow {}^8\mathrm{Be} + {}^4\mathrm{He}, \tag{22}$$

$$^{11}\mathrm{C} + n \leftrightarrow {}^8\mathrm{Be} + {}^4\mathrm{He}. \tag{23}$$

To get a rough estimate of the effect of these reactions, we simply used the rates for the corresponding 2α reactions. We calculated the element abundances both with and without reactions (14)–(23). Over our parameter range of interest, we found no significant difference in the predicted element abundances when we included these additional reactions, in agreement with the earlier results of Coc *et al.* [14].

We calculated the primordial element abundances using the AlterBBN computer code [35], stripping out the triple-α reaction and replacing it with reactions (8) and (9). We allowed B_8 to vary up to 3 MeV, and we examined F_0 from 10^9 to 10^{12}. Our results for ^{4}He and ^{8}Be are displayed in Fig. 1 for $F_0 = 10^{11}$, and Fig. 2 gives the ^{7}Li abundance as a function of both B_8 and F_0.

Note first that the primordial ^{2}H abundance (not displayed) is completely insensitive to B_8 even for the largest values of F_0 we examined. This makes sense, as this abundance is determined by the rate of deuterium burning into heavier elements, which is unaffected by helium burning into beryllium. This implies that the excellent

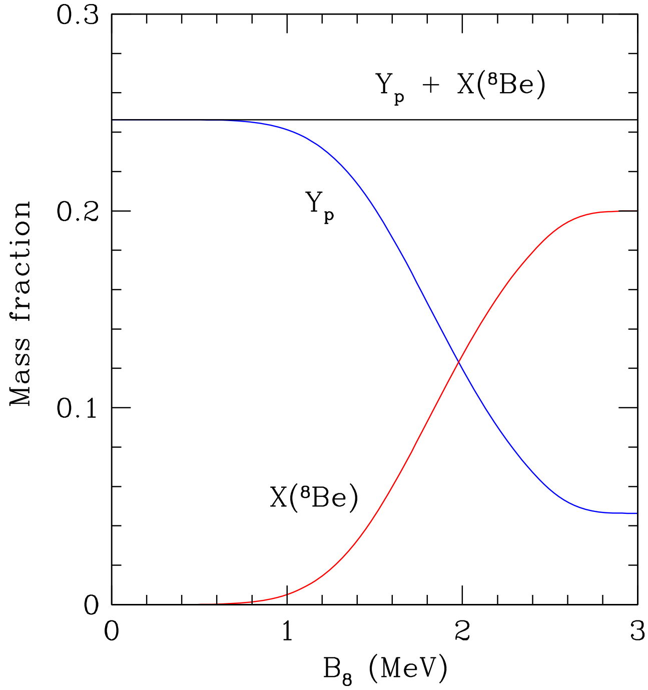

FIG. 1. The primordial mass fractions of ^{4}He (blue) and ^{8}Be (red) along with the sum of the ^{4}He and ^{8}Be mass fractions (black) as a function of B_8, the mass difference between two ^{4}He nuclei and a single ^{8}Be nucleus, for $F_0 = 1.0 \times 10^{11}$, where F_0 parametrizes the $^4\mathrm{He} + {}^4\mathrm{He}$ rate in Eq. (12).

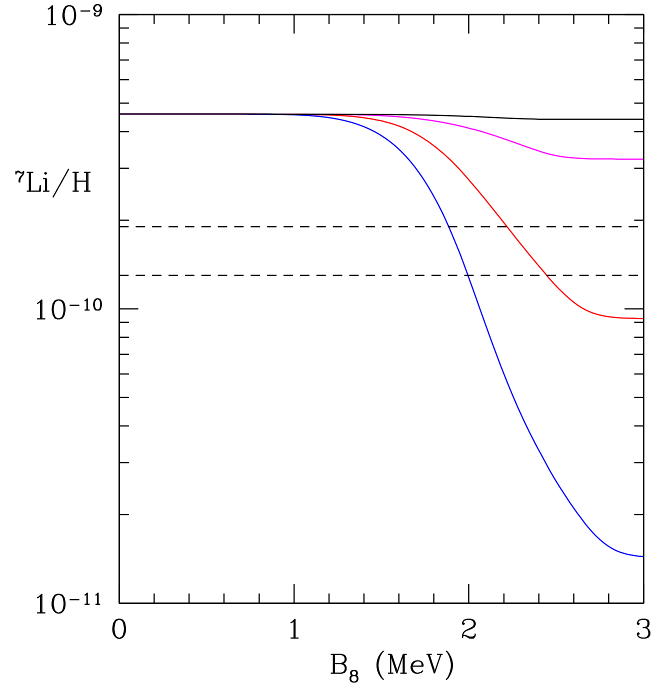

FIG. 2. The abundance (relative to hydrogen) of ^{7}Li, as a function of B_8, the mass difference between two ^{4}He nuclei and a single ^{8}Be nucleus, for (top to bottom) $F_0 = 1.0 \times 10^9$ (black), 1.0×10^{10} (magenta), 1.0×10^{11} (red), 1.0×10^{12} (blue), where F_0 parametrizes the $^4\mathrm{He} + {}^4\mathrm{He}$ rate in Eq. (12). The ^{7}Li abundance is the sum of the primordial ^{7}Li and ^{7}Be abundances, as the latter decays into the former. Horizontal dashed lines give the range for the observationally inferred value of ^{7}Li/H.

agreement between the observed and predicted abundances of ^{2}H is preserved (although see the discussion in Sec. III regarding the deuterium binding energy).

We also find essentially no change in any of the element abundances for small binding energies. Our results agree with Ref. [14], who found no significant change in the primordial element abundances for B_8 as large as 100 keV. We can extend this conclusion to larger values of the binding energy: we find no discernable changes in element abundances for B_8 as large as 600 keV, and significant changes only occur for $B_8 > 1$ MeV.

In Fig. 1, we show Y_p (the ^{4}He mass fraction) and the ^{8}Be mass fraction as a function of B_8 for $F_0 = 10^{11}$. The results for our other values of F_0 are qualitatively similar. As B_8 increases from 1 to 3 MeV, there is a sharp reduction in Y_p and a corresponding increase in the ^{8}Be abundance. Naively, one might expect that this reduction in Y_p to values far below the value estimated from observations would rule out this model, but this is not the case. We know that B_8 must assume its present-day negative value at some time after BBN. When this occurs, ^{8}Be will no longer be stable and will fission back into ^{4}He. Thus, the present-day mass fraction of ^{4}He will be given by the sum of the ^{4}He and ^{8}Be mass fractions. We have plotted this sum in Fig. 1. Only an infinitesimal fraction of ^{8}Be is burned into heavier nuclides, so this sum is constant and equal to its value at $B_8 = 0$. Thus, ^{4}He, like ^{2}H, is essentially unaffected by stable ^{8}Be even for very large binding energies and large rates for reaction (8).

However, large values of B_8 do have an important effect on the ^{7}Li abundance, which is displayed, relative to hydrogen, in Fig. 2. The value of ^{7}Li/H is clearly very sensitive to both the ^{8}Be binding energy and the rate of reaction (8). For $F_0 \leq 1.0 \times 10^9$, there is essentially no effect on the lithium abundance. Significant reduction begins to occur at $F_0 = 1.0 \times 10^{10}$ and $B_8 \geq 2$ MeV. Lithium abundances in agreement with the observations can be achieved as F_0 increases from 1.0×10^{10} to 1.0×10^{11}, and for larger F_0, there is a narrow range of values for B_8 for which the predicted primordial lithium abundance agrees with the observationally inferred value. As in standard BBN, we find that most of the ^{7}Li, at our chosen value of η, is produced in the form of ^{7}Be. The physical mechanism for this decrease is the sequestration of ^{4}He in the form of ^{8}Be as seen in Fig. 1. This decrease in the ^{4}He abundance during BBN then inhibits reactions (4)–(5).

The CNO elements are produced in very small amounts in standard BBN, with typical abundances relative to hydrogen of CNO/H $\sim 10^{-15}$–10^{-14} [36]. A larger primordial production of CNO elements would be interesting, as the results of Ref. [37] suggest that the value of CNO/H begins to affect the first generation of stars (population III) when CNO/H increases above 10^{-11}. However, our results agree with those of Ref. [14]; even for the largest B_8 and F_0 values we examined, we see no significant primordial production of CNO elements.

III. DISCUSSION

We find that BBN with stable ^{8}Be can begin to produce interesting changes in the final element abundances for $B_8 \gtrsim 1$ MeV. The deuterium and ^{4}He abundances are unchanged, although the latter is sequestered in the form of ^{8}Be until B_8 drops below zero at late times. This sequestration leads to a reduction in the ^{7}Li abundance and can push it into a regime consistent with observations for a sufficiently large $^4\mathrm{He} + {}^4\mathrm{He}$ rate. We can compare the value of F_0 needed to produce this reduction in the lithium abundance with the nonresonant cross section of Ref. [29]. For $T_9 \sim 1$–0.1, the prefactor in Ref. [29] corresponding to our F_0 lies between 4×10^{11} and 2×10^{10}. As we have already noted, it is not at all clear that the rates of Ref. [29] can be extrapolated to a model with large binding energies for B_8. However, this comparison with Ref. [29] does indicate that the reaction rates examined here are not completely unreasonable.

A value of $B_8 \sim 1$ MeV is larger than has been considered in previous BBN calculations. In the context of plasma effects, it requires a very large Debye mass ($m_D \sim 3$ MeV) if one simply extrapolates the linear approximation of Yao *et al.* [18]. This is larger than the value of m_D predicted from plasma effects during BBN, although there are some uncertainties in these calculations [18]. Such a large value of B_8 can be more plausibly achieved in the context of time variation of the fundamental constants. A value of $B_8 \sim 1$ MeV can be obtained with a change in the strong coupling constant of $\sim$15%, or changes in the quark masses or fine structure constant by a similar amount [17,28].

The major caveat in this discussion is that we have limited our analysis to changes in the ^{8}Be binding energy alone. This was intentional, as we wished to isolate the effects of large changes in this binding energy in a model-independent way. A realistic model would result in changes to all of the nuclear binding energies, as in Ref. [14]. In changing the other nuclear binding energies, the one likely to have the largest impact is deuterium [6–8,11]. In the model presented in Ref. [14], $\sim$1 MeV values of B_8 would result in a 50% increase in the deuterium binding energy. A larger deuterium binding energy would result in an earlier onset of nuclear fusion, leading to more ^{7}Li, and potentially canceling the reduction in ^{7}Li noted here. However, all of these conclusions depend on the particular model invoked to alter the nuclear binding energies. A systematic estimate of the effects of changing other binding energies can be found in Ref. [12]. It is possible that the plasma effects proposed by Yao *et al.* [18] would have a much larger effect on the ^{8}Be binding energy than on the other nuclear binding energies, since these plasma effects are sensitive to the existence of the 92 keV resonance in ^{8}Be.

Our results indicate that it is difficult to produce significant abundances of CNO elements in BBN even with MeV-scale binding energies for ^{8}Be. In that regard, the famous "mass gap" at $A = 8$ is misleading; the failure to produce heavier elements in the early universe is a result of the lower densities and shorter times for nuclear fusion than prevail in stars [14]. This analysis ignores the possibility that, for large values of B_8 and F_0, the buildup of a large mass fraction of ^{8}Be might allow the reaction $^8\mathrm{Be} + {}^8\mathrm{Be} \to {}^{16}\mathrm{O} + \gamma$ to compete with reaction (9) as a mechanism for the production of the CNO elements, but that seems unlikely in view of the large Coulomb barrier. Of course, these results are also sensitive to the assumed rate for $^8\mathrm{Be} + {}^4\mathrm{He}$; a rate that diverges from that of Ref. [29] could alter our conclusions regarding the CNO elements.

This work is admittedly speculative; our goal was to establish a threshold on the ^{8}Be binding energy and the $^4\mathrm{He} + {}^4\mathrm{He}$ reaction rate that would produce a reduction in the primordial lithium abundance. While the possibility of solving the lithium problem through a change in the constants of nature, including the binding energies of the light nuclei, is not new [11], the sequestration of ^{4}He during BBN noted here represents a qualitatively new mechanism to achieve this.

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