Study of B_c decays into charmonia and D mesons

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In the wake of recent measurements of the decays $B_c^+ \to J/\psi D_s^+$ and $B_c^+ \to J/\psi D_s^{*+}$ performed by the LHCb and ATLAS collaborations, we recalculate their branching fractions in the framework of the covariant confined quark model. We compare the obtained results with available experimental data, our previous findings, and numbers from other approaches.

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I. INTRODUCTION

Recently the ATLAS Collaboration reported on the measurement of the various branching fractions of the decays $B_c^+ \to J/\psi D_s^+$ and $B_c^+ \to J/\psi D_s^{*+}$ [1]. The first observations of these decays have been performed by the LHCb Collaboration [2]. In view of these developments, we decided to recalculate the amplitudes and branching fractions within the covariant confined quark model. Our previous study of exclusive semileptonic and nonleptonic decays of the B_c meson was done more than ten years ago within a relativistic constituent quark model [3–6]. The modern approach with embedded infrared confinement [for short, covariant confined quark model (CCQM)] is a successor of the previous approach. Due to the confinement feature it has a wider region of applications.

Many facets of the B_c production were discussed in theoretical papers by Likhoded and his co-authors; see, e.g. [7–9]. The decay properties of the above processes were studied in various theoretical approaches [10–21]. The decays $B_c^+ \to J/\psi D_s^+$ and $B_c^+ \to J/\psi D_s^{*+}$ proceed via $b \to c\bar{c}s$ transition which is theoretically described by the effective Hamiltonian with the relevant Wilson coefficients. The physical amplitudes are described by color-enhanced, color-suppressed, and annihilation diagrams. The two first diagrams are factorized into the leptonic decay part and the transition of the B_c meson into charmonium or D meson. The theoretical description of this transition gives the most sizable uncertainties to the predicted physical observables.

In Ref. [10] heavy quark effective theory in combination with a suitable Bethe-Salpeter kernel was used to evaluate the form factors. The form factors were computed in [11] as an overlap integral of the meson wave functions obtained using a QCD relativistic potential model. In Refs. [12,13] the B_c decays were studied in the framework of QCD sum rules. Semileptonic and nonleptonic decays of the B_c meson to charmonium and a D meson were studied in the framework of the relativistic quark model in [14]. The decay form factors were expressed through the overlap integrals of the meson wave functions in the whole accessible kinematical range. Decays $B_c \to J/\psi + n\pi$ were considered in [15]. Using existing parametrizations for $B_c \to J/\psi$ form factors and $W \to n\pi$ spectral functions, branching fractions and transferred momentum distributions have been calculated. An analysis of the B_c form factors in the Wirbel-Stech-Bauer framework has been performed in [16]. Branching ratios of two body decays of B_c meson to pseudoscalar and vector mesons were obtained. In Ref [17] form factors for the transitions $B_c \to J/\psi$ and $B_c \to \psi(2S)$ were calculated within the light-front quark model (LFQM) numerically. Then the partial widths of the semileptonic and nonleptonic decays were determined. A systematic investigation of the two-body nonleptonic decays $B_c \to J/\psi(\eta_C) + P(V)$ was performed in [18] by employing the perturbative QCD approach based on the k_T factorization. The exclusive nonleptonic $B_c \to VV$ decays were studied in [19] within the factorization approximation, in the framework of the relativistic independent quark model, based on a confining potential in the scalar-vector harmonic form. In the recent paper [20] the form factors of the transition of B_c meson into S-wave charmonium were investigated within the nonrelativistic QCD effective theory. The next-to-leading order relativistic corrections to the form factors were obtained.

II. EFFECTIVE HAMILTONIAN AND MATRIX ELEMENT

The effective Hamiltonian describing the B_c nonleptonic decays into charmonium and $D(D_s)$ meson is given by (see, Ref. [22])

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TABLE I. Values of the Wilson coefficients.

C_1	C_2	C_3	C_4	C_5	C_6
−0.2632	1.0111	−0.0055	−0.0806	0.0004	0.0009

$$\begin{aligned}
\mathcal{H}_{\mathrm{eff}} &= -\frac{G_F}{\sqrt{2}} V_{cb} V_{cq}^{\dagger} \sum_{i=1}^{6} C_i \mathcal{O}_i,\\
\mathcal{O}_1 &= (\bar{c}_{a_1} b_{a_2})_{V-A} (\bar{q}_{a_2} c_{a_1})_{V-A},\\
\mathcal{O}_2 &= (\bar{c}_{a_1} b_{a_1})_{V-A}, (\bar{q}_{a_2} c_{a_2})_{V-A},\\
\mathcal{O}_3 &= (\bar{q}_{a_1} b_{a_1})_{V-A} (\bar{c}_{a_2} c_{a_2})_{V-A},\\
\mathcal{O}_4 &= (\bar{q}_{a_1} b_{a_2})_{V-A} (\bar{c}_{a_2} c_{a_1})_{V-A},\\
\mathcal{O}_5 &= (\bar{q}_{a_1} b_{a_1})_{V-A} (\bar{c}_{a_2} c_{a_2})_{V+A},\\
\mathcal{O}_6 &= (\bar{q}_{a_1} b_{a_2})_{V-A} (\bar{c}_{a_2} c_{a_1})_{V+A},
\end{aligned} \tag{1}$$

where the subscript $V-A$ refers to the usual left-chiral current $O^{\mu} = \gamma^{\mu}(1-\gamma^5)$ and $V+A$ to the usual right-chiral one $O_{+}^{\mu} = \gamma^{\mu}(1+\gamma^5)$. The a_i denote the color indices. The quark q stands for either s or d.

The calculation of the matrix elements of the four-quark operators corresponds to the use of the naive factorization approach, which is affected by an intrinsic uncertainty that is difficult to assess. The numerical values of the Wilson coefficients are taken from Ref. [23]. They were computed at the matching scale $\mu_0 = 2M_W$ at the NNLO precision and run down to the hadronic scale $\mu_b = 4.8$ GeV. They are listed in Table I.

Since the numerical values of the C_5 and C_6 are negligibly small, we drop the contribution from those operators.

By using the Fierz transformation one can check that $\mathcal{O}_3 = \mathcal{O}_1$ and $\mathcal{O}_4 = \mathcal{O}_2$. Then the calculation of the matrix elements describing the nonleptonic decays of the B_c meson into charmonium and $D(D_s)$ meson is straightforward. Pictorial representation of the matrix elements is shown in Fig. 1.

The combinations of the Wilson coefficients appear as $a_1 = C_2 + C_4 + \xi(C_1 + C_3)$ and $a_2 = C_1 + C_3 + \xi(C_2 + C_4)$ with $\xi = 1/N_c$. In the numerical calculations we set the color-suppressed parameter ξ to zero. Then the Wilson coefficients are equal to

$$a_1 = C_2 + C_4 = 0.93, \quad \text{and} \quad a_2 = C_1 + C_3 = -0.27 \tag{2}$$

which should be compared with the old ones $a_1 = 1.14$ and $a_2 = -0.20$ used in our previous paper [3].

One has to note that the signs in front of the leptonic decay constants f_D and f_{η_c} should be opposite to those defined in their leptonic decays. It comes from the observation that the meson momentum flows in the opposite direction in the case of the nonleptonic decays as compared with the case of the leptonic decays.

III. INVARIANT AND HELICITY AMPLITUDES

The invariant form factors for the semileptonic B_c decay into the hadron with spin $S = 0$, 1 are defined by

$$\mathcal{M}_{S=0}^{\mu} = P^{\mu} F_{+}(q^2) + q^{\mu} F_{-}(q^2), \tag{3}$$

$$\begin{aligned}
\mathcal{M}_{S=1}^{\mu} = \frac{1}{m_1+m_2} \epsilon_{\nu}^{\dagger} \{ &-g^{\mu\nu} Pq A_0(q^2) + P^{\mu} P^{\nu} A_{+}(q^2)\\
&+ q^{\mu} P^{\nu} A_{-}(q^2) + i\varepsilon^{\mu\nu\alpha\beta} P_{\alpha} q_{\beta} V(q^2) \},
\end{aligned} \tag{4}$$

where $P = p_1 + p_2$ and $q = p_1 - p_2$. Here p_1 is the momentum of the ingoing meson with a mass m_1 (B_c) and p_2 is the momentum of the outgoing meson with a mass m_2. It is convenient to express all physical observables through the helicity form factors H_m. The helicity form factors H_m can be written in terms of the invariant form factors in the following way [6]:

Spin $S = 0$:

$$\begin{aligned}
H_t &= \frac{1}{\sqrt{q^2}} \{ (m_1^2 - m_2^2) F_{+} + q^2 F_{-} \}, \qquad H_{\pm} = 0,\\
H_0 &= \frac{2m_1 |\mathbf{p_2}|}{\sqrt{q^2}} F_{+}.
\end{aligned} \tag{5}$$

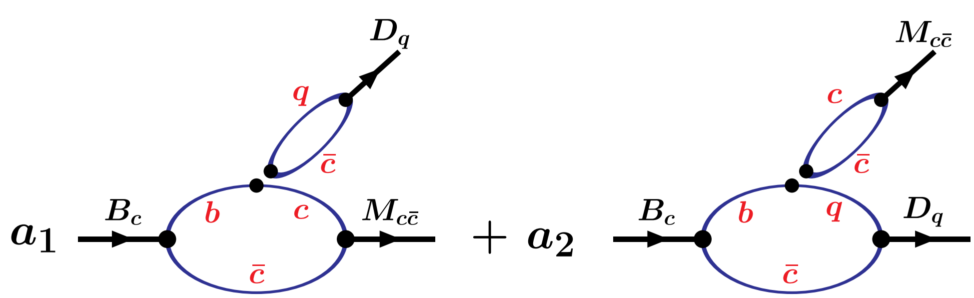

FIG. 1. Pictorial representation of the matrix elements of the nonleptonic B_c decays.

Spin $S = 1$:

$$H_t = \frac{1}{m_1+m_2}\frac{m_1|\mathbf{p_2}|}{m_2\sqrt{q^2}}\{(m_1^2-m_2^2)(A_+-A_0)+q^2A_-\},$$
$$H_\pm = \frac{1}{m_1+m_2}\{-(m_1^2-m_2^2)A_0 \pm 2m_1|\mathbf{p_2}|V\},$$
$$H_0 = \frac{1}{m_1+m_2}\frac{1}{2m_2\sqrt{q^2}}\{-(m_1^2-m_2^2)(m_1^2-m_2^2-q^2)A_0 + 4m_1^2|\mathbf{p_2}|^2A_+\}. \tag{6}$$

Here $|\mathbf{p_2}| = \lambda^{1/2}(m_1^2, m_2^2, q^2)/(2m_1)$ is the momentum of the outgoing meson in the B_c rest frame.

The nonleptonic B_c decay widths in terms of the helicity amplitudes are given by

$$\Gamma(B_c\to\eta_c D_q) = N_W\{a_1 f_{D_q^-}m_{D_q^-}H_t^{B_c\to\eta_c}(m_{D_q^-}^2) + a_2 f_{\eta_c}m_{\eta_c}H_t^{B_c\to D_q^-}(m_{\eta_c}^2)\}^2,$$
$$\Gamma(B_c\to\eta_c D_q^*) = N_W\{a_1 f_{D_q^{*-}}m_{D_q^{*-}}H_0^{B_c\to\eta_c}(m_{D_q^{*-}}^2) - a_2 f_{\eta_c}m_{\eta_c}H_t^{B_c\to D_q^{*-}}(m_{\eta_c}^2)\}^2,$$
$$\Gamma(B_c\to J/\psi D_q) = N_W\{-a_1 f_{D_q^-}m_{D_q^-}H_t^{B_c\to J/\psi}(m_{D_q^-}^2) + a_2 f_{J/\psi}m_{J/\psi}H_0^{B_c\to D_q^-}(m_{J/\psi}^2)\}^2,$$
$$\Gamma(B_c\to J/\psi D_q^*)) = N_W\sum_{i=0,\pm}\{a_1 f_{D^{*-}}m_{D_q^{*-}}H_i^{B_c\to J/\psi}(m_{D_q^{*-}}^2) + a_2 f_{J/\psi}m_{J/\psi}H_i^{B_c\to D_q^{*-}}(m_{J/\psi}^2)\}^2,$$

where we use the short notation

$$N_W \equiv \frac{G_F^2}{16\pi}\frac{|\mathbf{p_2}|}{m_1^2}|V_{cb}V_{cq}^\dagger|^2.$$

IV. FORM FACTORS

We calculate the relevant hadronic form factors in the framework of the covariant confined quark model [24].

The starting point of the CCQM is the effective Lagrangian describing coupling of the given hadron with its interpolating quark current. In particular, the coupling of a meson M to its constituent quarks q_1 and $\bar{q}_2$ is given by the Lagrangian

$$\mathcal{L}_{\rm int}(x) = g_M M(x)\cdot J_M(x) + \text{H.c.},$$
$$J_M(x) = \int dx_1\int dx_2 F_M(x;x_1,x_2)\bar{q}_2(x_2)\Gamma_M q_1(x_1), \tag{7}$$

where g_M denotes the coupling strength of the meson with its constituent quarks, and the Dirac matrix Γ_M projects onto the relevant meson state, i.e., $\Gamma_M = I$ for a scalar meson, $\Gamma_M = \gamma^5$ for a pseudoscalar meson, and $\Gamma_M = \gamma^\mu$ for a vector meson. The vertex function F_M is chosen in the translational invariant form

$$F_M(x;x_1,x_2) = \delta(x - w_1x_1 - w_2x_2)\Phi_M((x_1-x_2)^2),$$
$$\Phi_M((x_1-x_2)^2) = \int\frac{d^4\ell}{(2\pi)^4}e^{-i\ell(x_1-x_2)}\tilde{\Phi}_M(-\ell^2), \quad \text{where}$$
$$\tilde{\Phi}_M(-\ell^2) = e^{\ell^2/\Lambda_M^2}. \tag{8}$$

Here $w_i = m_{q_i}/(m_{q_1}+m_{q_2})$ so that $w_1+w_2=1$, and the parameter Λ_M characterizes the meson size. The matrix elements of the physical processes are defined by the appropriate S-matrix elements with the S-matrix being constructed by using the interaction Lagrangian given by Eq. (7). The S-matrix elements in the momentum space are described by a set of Feynman diagrams which are presented as convolution of quark propagators and vertex functions. The free local fermion propagator is used for the constituent quark:

$$S_q(k) = \frac{1}{m_q - \not{k} - i\epsilon} = \frac{m_q+\not{k}}{m_q^2-k^2-i\epsilon} \tag{9}$$

with an effective constituent quark mass m_q. The coupling strength g_M is determined by the so-called compositeness condition which was discussed in our previous paper in great details; see, e.g. Refs. [24–26]. The infrared cutoff parameter λ is introduced on the last step of calculations which effectively guarantees the confinement of quarks within hadrons. This method is quite general and can be used for diagrams with an arbitrary number of loops and propagators. In the CCQM the infrared cutoff parameter λ is taken to be universal for all physical processes.

The model parameters are determined by fitting calculated quantities of basic processes to available experimental data or lattice simulations. In this paper we will use the updated least-squares fit performed in Refs. [27–29]. All necessary details of the calculations of the leptonic decay constants and hadronic form factors may be found in our recent publications [30–32].

The fitted values of the meson size parameters are given in Table II.

TABLE II. Values of the meson size parameters in GeV.

Λ_{B_c}	Λ_{η_c}	$\Lambda_{J/\psi}$	Λ_D	Λ_{D^*}	Λ_{D_s}	$\Lambda_{D_s^*}$
2.73	3.87	1.74	1.6	1.53	1.75	1.56

TABLE III. The calculated values of leptonic decay constants in MeV.

f_{B_c}	f_{η_c}	$f_{J/\psi}$	f_D	f_{D^*}	f_{D_s}	$f_{D_s^*}$
489	628	415	206	244	257	272

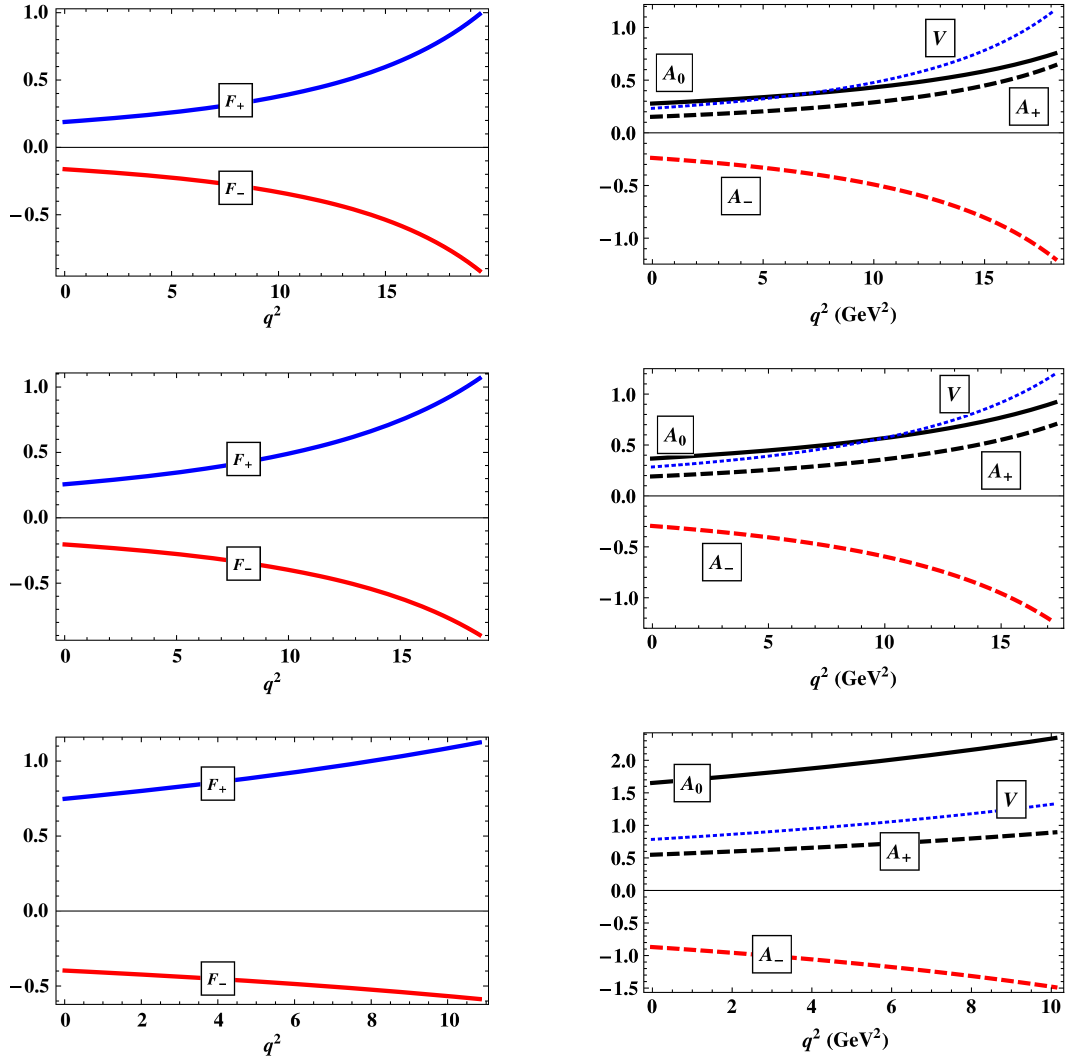

FIG. 2. Left: the form factors $F_+(q^2)$ and $F_-(q^2)$ for $B_c \to D, D_s, \eta_c$ transitions (from top to bottom). Right: the form factors A_0, A_-, A_+ and V for $B_c \to D^*, D_s^*, J/\psi$ transitions (from top to bottom).

The calculated values of leptonic decay constants are given in Table III. Note that the decay constant f_{η_c} was calculated by using the size parameter Λ_{η_c} which was obtained from fitting the branching ratio of the η_c meson two-photon decay to its experimental value given in PDG [33].

TABLE IV. $q^2 = 0$ results for the various form factors.

	$B_c \to D$	$B_c^+ \to D_s$	$B_c \to \eta_c$
$F_+(0)$	0.186	0.254	0.74
$F_-(0)$	−0.160	−0.202	−0.39
	$B_c \to D^*$	$B_c \to D_s^*$	$B_c \to J/\psi$
$A_0(0)$	0.276	0.365	1.65
$A_+(0)$	0.151	0.190	0.55
$A_-(0)$	−0.236	−0.293	−0.87
$V(0)$	0.230	0.282	0.78

The form factors are calculated in the full kinematical region of momentum transfer squared. The curves are depicted in Fig. 2.

The values of the form factors at maximum recoil ($q^2 = 0$) are given in Table IV.

V. NUMERICAL RESULTS

We are aiming to compare our results with those obtained by the ATLAS [1] and LHCb [2] collaborations. They reported the results of measurements of the ratios of the branching fractions:

$$\mathcal{R}_{D_s^+/\pi^+} = \frac{\mathcal{B}_{B_c^+\to J/\psi D_s^+}}{\mathcal{B}_{B_c^+\to J/\psi\pi^+}}, \qquad \mathcal{R}_{D_s^{*+}/\pi^+} = \frac{\mathcal{B}_{B_c^+\to J/\psi D_s^{*+}}}{\mathcal{B}_{B_c^+\to J/\psi\pi^+}},$$
$$\mathcal{R}_{D_s^{*+}/D_s^+} = \frac{\mathcal{B}_{B_c^+\to J/\psi D_s^{*+}}}{\mathcal{B}_{B_c^+\to J/\psi D_s^+}} \tag{10}$$

TABLE V. Values of the CKM-matrix elements.

$\lvert V_{ud}\rvert$	$\lvert V_{us}\rvert$	$\lvert V_{cd}\rvert$	$\lvert V_{cs}\rvert$	$\lvert V_{cb}\rvert$	$\lvert V_{ub}\rvert$
0.974	0.225	0.220	0.995	0.0405	0.00409

TABLE VI. Values of meson masses in GeV.

m_{B_c}	m_{η_c}	$m_{J/\psi}$	m_D	m_{D^*}	m_{D_s}	$m_{D_s^*}$
6.275	2.983	3.097	1.869	2.010	1.968	2.112

and the transverse polarization fraction in $B_c^+ \to J/\psi D_s^{*+}$ decay which is determined to be

$$\frac{\Gamma_{++}}{\Gamma} = \frac{\Gamma_{++}(B_c^+ \to J/\psi D_s^{*+})}{\Gamma(B_c^+ \to J/\psi D_s^{*+})}. \tag{11}$$

First, we show the input parameters used in calculations. The central values of the CKM-matrix elements are taken from the PDG [33] and shown in Table V. The central values of the relevant meson masses are taken from the PDG [33] and shown in Table VI.

In Table VII we show the values of branching fractions obtained in this work for two different sets of the Wilson coefficients. One can see the difference is almost a factor of 2 between them. Note that the values obtained with the old set $a_1 = 1.14, a_2 = -0.20$ are very close to the predictions given in our previous paper [3].

We also calculate the widths of the decays $B_c \to M_{c\bar{c}}\pi$ to be able to compare with available experimental data. Their analytical expressions are given by

$$\Gamma(B_c \to \pi^+ M_{\bar{c}c}) = \frac{G_F^2}{16\pi}\frac{|\mathbf{p_2}|}{m_1^2}|V_{cb}V_{ud}^\dagger a_1 f_\pi m_\pi|^2 (H_t^{B_c\to M_{\bar{c}c}}(m_\pi^2))^2, \tag{12}$$

where $M_{c\bar{c}} = J/\psi$ or η_c.

TABLE VII. Branching ratios (in %) of nonleptonic B_c decays obtained in this work for two different sets of the Wilson coefficients.

Mode	$a_1 = +0.93$, $a_2 = -0.27$	$a_1 = +1.14$, $a_2 = -0.20$	[3]
$B_c \to \eta_c D_s$	0.22	0.50	0.44
$B_c \to \eta_c D_s^*$	0.22	0.42	0.37
$B_c \to J/\psi D_s$	0.10	0.22	0.34
$B_c \to J/\psi D_s^*$	0.41	0.78	0.97
$B_c \to \eta_c D$	0.0073	0.016	0.019
$B_c \to \eta_c D^*$	0.0098	0.019	0.019
$B_c \to J/\psi D$	0.0035	0.0074	0.015
$B_c \to J/\psi D^*$	0.017	0.031	0.045

TABLE VIII. Comparison of the results for the ratios of branching fractions with those of ATLAS and LHCb collaborations and theoretical predictions. Abbreviations not already defined in the text denote the following: RCQM: relativistic constituent quark model; QCD PM: QCD potential model; QCD SR: QCD sum rules; BSW RQM; Wirbel-Stech-Bauer quark model; pQCD: perturbative QCD; and RIQM: relativistic independent quark model.

$\mathcal{R}_{D_s^+/\pi^+}$	$\mathcal{R}_{D_s^{*+}/\pi^+}$	$\mathcal{R}_{D_s^{*+}/D_s^+}$	$\Gamma_{\pm\pm}/\Gamma$	Ref.
3.8 ± 1.2	10.4 ± 3.5	$2.8^{+1.2}_{-0.9}$	0.38 ± 0.24	ATLAS [1]
2.90 ± 0.62	$\cdots$	2.37 ± 0.57	0.52 ± 0.20	LHCb [2]
1.29 ± 0.26	5.09 ± 1.02	3.96 ± 0.80	0.46 ± 0.09	CCQM
2.0	5.7	2.9	$\cdots$	RCQM [3]
2.6	4.5	1.7	$\cdots$	QCD PM [11]
1.3	5.2	3.9	$\cdots$	QCD SR [12]
2.2	$\cdots$	$\cdots$	$\cdots$	BSW RQM [16]
2.06 ± 0.86	$\cdots$	3.01 ± 1.23	$\cdots$	LFQM [17]
$3.45^{+0.49}_{-0.17}$	$\cdots$	$2.54^{+0.07}_{-0.21}$	0.48 ± 0.04	pQCD [18]
$\cdots$	$\cdots$	$\cdots$	0.410	RIQM [19]

However, the ratio of the branching fractions is insensitive to the choice of the Wilson coefficients:

$$R_{D_s^{*+}/D_s^+} = \frac{B(B_c^+ \to J/\psi D_s^{*+})}{B(B_c^+ \to J/\psi D_s^+)} = \begin{cases} 3.55 & (a_1 = 1.14, a_2 = -0.20) \\ 3.96 & (a_1 = 0.93, a_2 = -0.27) \end{cases} \tag{13}$$

Finally, we compare our results with available experimental data and the results obtained in other approaches. For this purpose, we take the table from the paper [1] and add our numbers see Table VIII.

One can see that the results of our calculations for the ratios $\mathcal{R}_{D_s^{*+}/D_s^+}$ and $\Gamma_{\pm\pm}/\Gamma$ are consistent with measurements and other approaches. The results for the ratios $\mathcal{R}_{D_s^+/\pi^+}$ and $\mathcal{R}_{D_s^{*+}/\pi^+}$ are smaller than the measured values but the discrepancies do not exceed two standard deviations.

VI. SUMMARY

We performed the calculations of the B_c meson nonleptonic decays: $B_c^+ \to J/\psi\pi^+$, $B_c^+ \to M_{c\bar{c}}D_q^{(*+)}$ where $M_{c\bar{c}} = J/\psi$ or η_c, $D_q^{(*+)} = D_q^{*+}$ or D_q^+, and $q = s, d$.

We compared the obtained results for several ratios of branching fractions with those measured by the ATLAS and LHCb collaborations and other theoretical approaches.

We found that our prediction for the ratios $\mathcal{R}_{D_s^{*+}/D_s^+}$ and $\Gamma_{\pm\pm}/\Gamma$ are consistent with measurements and other approaches. The results for the ratios $\mathcal{R}_{D_s^+/\pi^+}$ and $\mathcal{R}_{D_s^{*+}/\pi^+}$ are smaller than the measured values but the discrepancies do not exceed two standard deviations.

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