

Leggett-Garg K_3 quantity discriminates between Dirac and Majorana neutrinos

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The K_3 quantity, introduced in a context of the Leggett-Garg inequality violation, is studied for the neutrino oscillations in matter with a phenomenologically modeled dissipative environment. It is shown that the K_3 function acquires different values depending on whether the neutrino is a Dirac or Majorana particle, provided that there is a dissipative interaction between matter and neutrinos. The difference occurs for various matter densities and can serve as a potential quantifier verifying the neutrino nature. Moreover, working within a phenomenological model one can suggest the values of the matter density and dissipation for which the difference is the most visible. There exist also special conditions in which the violation of the Leggett-Garg inequality, to a different extent for both kinds of neutrino, is observed.

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I. INTRODUCTION

For a long time there were two central controversies in neutrino physics. The first was the (already solved) problem of neutrino masses. Neutrinos are massive [1,2]. The second (still alive) concerns neutrino nature which can be either Dirac, with particles and antiparticles being different, or Majorana, with particles and antiparticles being the same as it is in the case of photons. Measuring properties of such fragile and weakly interacting particles as neutrinos is a challenge for experimentalists. One can invoke here a short but embarrassing history of faster-than-light neutrino anomaly. That is why the proposal of a potentially helpful quantifier credibly indicating desired neutrino's property is of interest to both theoretical and experimental physicists.

In the literature one can find numerous proposals for solving the Dirac-Majorana neutrino controversy [3]. In particular, interferometric pattern [4,5] or geometric phases of various type [6,7] can serve as a quantifiers of Majorana and Dirac neutrino's difference.

In 1964 Bell proposed a way to experimentally exclude deterministic local (hidden variable theories) as a possible interpretation of quantum mechanics [8]. His celebrated inequality has been later verified in the experiments performed on the pairs of entangled photons [9,10] which provided then quite convincing evidence against the hidden local-variable theories. Quantum entanglement has been also studied in a context of oscillating neutrino systems [11]. In

1984 another test of the quantum mechanics principles has been developed [12]: while the Bell inequality questioned the correlations between spatially separated systems, the Leggett-Garg inequalities (LGI) put the limits on the time correlations. They were derived based on the assumptions of *macroscopic realism* (MR) and *noninvasive measurability* (NIM); the first of which presupposes that the system can be only in one of the available sets of states, whereas the second one claims that it is possible to perform a measurement without disturbing a system [13,14]. Just as the Bell inequalities, Leggett-Garg inequalities have been reported to be violated in the variety of systems (see for instance [15,16]). Especially interesting in terms of a study of the Leggett-Garg inequalities seems to be the neutrinos oscillations, for which the coherence length (the distance beyond which the interference of different massive neutrinos is not observable [17]) is much bigger than for the other systems. Such research has already been made both from the theoretical [18,19] and experimental points of view—cf. Ref. [20] reporting violation of the LGI in neutrino systems based upon the MINOS experiment's data.

Our aim in this work is essentially more modest: instead of searching for violations of neutrino's macro-realism our primary aim is to present a quantifier which, being (at least in principle) measurable, can discriminate between Dirac and Majorana neutrinos.

In this work, complementary to recent studies of neutrinos conducted from the perspective of quantum information [21], we propose such a quantifier. We show that the K_3 correlator, used in the simplest class of LGIs calculated for neutrinos oscillating in matter in a presence of decohering interaction, has different value for both kinds of

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neutrinos. A possible source of such a decoherence in neutrino oscillations can originate from quantum gravity effects as suggested in Refs. [22–24].

Instead of complex modeling, following [25], in this work we adopt Kossakowski-Lindblad-Gorini-type phenomenological equations. This way of describing the open quantum systems is particularly useful in the quantum optics [26]. It is nevertheless very general as it can be applied to any completely positive dynamics of quantum systems [27]. Although in some cases of open quantum dynamics one can directly relate the parameters of the Lindblad equation to the microscopic system-plus-environment [27], in a case of the neutrino oscillations our modeling is purely phenomenological—the effect of decoherence is encoded in a Kossakowski matrix describing the nonunitary character of quantum dynamics.

Furthermore, we show that a certain form of the Kossakowski matrix describing decoherence is a *sine qua non* condition for usefulness of the K_3 function in determining the neutrino's nature. We also theoretically investigate the behavior of the LGI violation for oscillating neutrinos treated as an open quantum system undergoing the dissipation and decoherence while interacting with an environment.

The paper is organized as follows: first (in Sec. II) for self-consistency of the presentation we review dissipative dynamics of the neutrino oscillations following [7,28], then in Sec. III, we introduce the notion of the Leggett-Garg inequalities [13] simultaneously providing the conditions for which the K_3 correlator allows to discriminate between Dirac and Majorana neutrinos. Finally, we summarize our work.

II. DISSIPATIVE NEUTRINO OSCILLATIONS IN MATTER

As the neutrino of the given flavor $\alpha = \{e, \mu, \tau\}$ is produced at some point, it propagates further as the coherence mixture of 3 massive states. Mathematically, it means that neutrino flavor states $\{|\nu_e\rangle, |\nu_\mu\rangle, |\nu_\tau\rangle\}$ can be considered the linear combinations of the massive states $\{|1\rangle, |2\rangle, |3\rangle\}$. The correspondence between them is expressed via unitary transformation sometimes referred to as the lepton mixing matrix U [17]:

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U \begin{pmatrix} |1\rangle \\ |2\rangle \\ |3\rangle \end{pmatrix}. \quad (1)$$

In this paper, we limit ourselves only to the two neutrino oscillations case (between e and μ). Since the third neutrino effectively decouples it seems to be a quite reasonable assumption [29]. Then U matrix can be parametrized by one mixing angle $\theta_{12} = \theta$ and in the most general case by one CP -violating phase ϕ :

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = U \begin{pmatrix} |1\rangle \\ |2\rangle \end{pmatrix}, \quad U = \begin{pmatrix} \cos\theta & \sin\theta e^{i\phi} \\ -\sin\theta & \cos\theta e^{i\phi} \end{pmatrix}. \quad (2)$$

At this point, it is necessary to emphasize the importance of the phase ϕ . The neutrino is the only massive and electrically neutral elementary fermion. Its nature is therefore ambiguous: neutrino can be Dirac or Majorana ([30,31]). In the Dirac case, the phase ϕ is not physical and can be easily removed by rephasing the neutrino fields occurring in the weak lepton charged current. On the other hand, in the Majorana case ϕ remains and is considered as a real and measurable quantity.

Let us now consider Hamiltonian H describing neutrino propagating in matter, which is divided into kinetic (H_0) and interaction (H_{int}) parts:

$$H = H_0 + H_{\text{int}}. \quad (3)$$

In the ultrarelativistic limit energy of an individual neutrino E_i can be expressed as [17]:

$$E_i = E + \frac{1}{2} \frac{m_i^2}{E^2}, \quad (4)$$

where E is an average energy of a neutrino. Then H_0 can be, in the mass basis, expressed as function of $\Delta m_{21}^2 = m_2^2 - m_1^2$ [29]:

$$H_0 = \begin{pmatrix} E + \frac{1}{2} \frac{\Delta m_{21}^2}{4E} & 0 \\ 0 & E - \frac{1}{2} \frac{\Delta m_{21}^2}{4E} \end{pmatrix}. \quad (5)$$

The potential part arises due to the coherent forward scattering of the electron neutrinos with the electrons (charged current) and all kinds of neutrinos with the electrons, neutrons and protons (neutral current). Since the contribution from the neutral current is the same for all flavors, the real influence on the neutrino oscillation comes only from the charged current V_{CC} :

$$\begin{aligned} H_{\text{int}} &= U^\dagger \begin{pmatrix} V_{CC} & 0 \\ 0 & 0 \end{pmatrix} U \\ &= \frac{V_{CC}}{2} \begin{pmatrix} 1 + \cos 2\theta & e^{-i\phi} \sin 2\theta \\ e^{i\phi} \sin 2\theta & 1 - \cos 2\theta \end{pmatrix}, \end{aligned} \quad (6)$$

where U is the mixing matrix given in Eq. (2).

It can be easily seen that the Hamiltonian given in Eq. (3) does not allow for any discrimination between Dirac and Majorana neutrinos: its transformation to the flavor basis makes the phase ϕ disappear. However, the treatment of the neutrinos propagating in matter as an open quantum system can dramatically change the situation.

In general, the Markovian evolution of a system interacting with an external environment can be described by the

Lindblad-Kossakowski master equation, which in the Heisenberg picture reads as [26,27,32,33]:

$$\frac{dA(t)}{dt} = L^*[A(t)],$$

$$L^*[A(t)] = i[H, A] + \frac{1}{2} \sum_{i,j=0}^{n^2-1} c_{ij} (F_i^\dagger [A, F_j] + [F_i^\dagger, A] F_j), \quad (7)$$

where A stand for an observable and $F = \{F_0, F_1, \dots, F_{n^2-1}\}$ denotes a set of the matrices creating a basis in the n -dimensional space.

Note that the right-hand side of Eq. (7) is split into two parts. In the first part one recognizes the ordinary Heisenberg equation, whereas the second part is responsible for additional effects connected with an interaction of a system with an environment. As for the latter part, in order to assure the trace- and hermicity-preserving evolution, the coefficients c_{ij} must obey the following conditions [26,34]:

$$|c_{ij}| \leq \frac{1}{2} (c_{ii} + c_{jj}), \quad i, j = 1, \dots, n^2 - 1. \quad (8)$$

Neutrino during its propagation interacts weakly with matter. It is therefore subjected to the decoherence and dissipation effects, which justifies the application of Eq. (7). Since our system is two-dimensional, the master equation is presented as ($\hbar = 1$):

$$L^*(A) = i[H, A] + \sum_{i,j=0}^3 c_{ij} \left(\sigma_j^\dagger A \sigma_i - \frac{1}{2} \{ \sigma_j^\dagger, \sigma_i \} A \right), \quad (9)$$

where σ_i denotes one of the set of matrices ($\mathbb{1}_2, \sigma_x, \sigma_y, \sigma_z$) for $i = 0, 1, 2, 3$, respectively.

Assuming the Hamiltonian given in Eq. (3) one can easily simplify the form of Eq. (9) by rearranging it to the Schrödinger-like form:

$$\frac{dA(t)}{dt} = (\mathcal{H} + \mathcal{D})A(t) = \mathcal{H}_{\text{eff}}A(t), \quad (10)$$

where

$$A(t) = \sum_{i=0}^3 a_i \sigma_i = \begin{pmatrix} a_0(t) \\ a_1(t) \\ a_2(t) \\ a_3(t) \end{pmatrix}$$

is a vector represented in the basis of the sigma matrices. Let us notice that Eq. (10) is nothing more than a convenient way of rewriting the Lindblad master equation Eq. (7).

As for the matrix \mathcal{H}_{eff} , it consists of two parts. The dissipative part \mathcal{D} can be parametrized with six real independent parameters [28]:

$$\mathcal{D} = -2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & b & \alpha & \beta \\ 0 & c & \beta & \gamma \end{pmatrix}, \quad (11)$$

where elements of \mathcal{D} have got the following correspondence with the $[c_{ij}]$ matrix elements:

$$\begin{aligned} \mathcal{D}_{11} &= c_{22} + c_{33}, & \mathcal{D}_{22} &= c_{11} + c_{33}, \\ \mathcal{D}_{33} &= c_{11} + c_{22}, & \mathcal{D}_{12} &= \mathcal{D}_{21} = -c_{12}, \\ \mathcal{D}_{13} &= \mathcal{D}_{31} = -c_{13}, & \mathcal{D}_{23} &= \mathcal{D}_{32} = -c_{23}. \end{aligned} \quad (12)$$

Subsequently, the direct calculation of the \mathcal{H} matrix elements reveals that:

$$\begin{aligned} \mathcal{H}_{12} &= -\mathcal{H}_{21} = -\frac{\Delta m_{21}^2}{2E} + V_{CC} \cdot \cos 2\theta, \\ \mathcal{H}_{13} &= -\mathcal{H}_{31} = -V_{CC} \cdot \sin \phi \sin 2\theta, \\ \mathcal{H}_{23} &= -\mathcal{H}_{32} = V_{CC} \cdot \cos \phi \sin 2\theta. \end{aligned} \quad (13)$$

The remaining elements in the \mathcal{H} and \mathcal{D} matrices are all equal to 0.

It is now easy to solve Eq. (10), for which the formal solution has an exponential form:

$$A(t) = e^{\mathcal{H}_{\text{eff}} t} A(0). \quad (14)$$

Let us notice that in principle it is possible to derive the coefficients \mathcal{D}_{ij} starting from microscopic description of a system including its environment [27]. However, for the systems, which require descriptions based on the relativistic field theory this task is often very challenging [35].

III. LEGGETT-GARG K_3 FUNCTION FOR THE NEUTRINO

The Leggett-Garg inequalities are usually constructed for the dichotomic (i.e. whose eigenvalues are either +1 or -1) observable \hat{Q} for which the quantity defined as:

$$C_{ij} = \langle \hat{Q}(t_i) \hat{Q}(t_j) \rangle \quad (15)$$

constitutes a benchmark of the correlations between its measurements performed in the different times $t_i < t_j$ [13]. Then, the Leggett-Garg parameter K_3 :

$$K_3 = C_{21} + C_{32} - C_{13}, \quad (16)$$

defined for 3 times $t_3 > t_2 > t_1$ is lower and upper-bounded under the MR and NIM assumptions [13]:

$$-3 \leq K_3 \leq 1, \quad (17)$$

which is known as the simplest from the family of the Leggett-Garg inequalities.

It is shown that for a 2-level system (the kind of which is considered in this paper) the correlation function C_{ij} can be expressed as the expectation value of the symmetrized product [13,36,37]:

$$C_{ij} = \frac{1}{2} \langle \phi | \{ \hat{Q}(t_i), \hat{Q}(t_j) \} | \phi \rangle, \quad (18)$$

where $\{\hat{A}, \hat{B}\}$ states for the anticommutator of the operators \hat{A} and \hat{B} . Then, representing an observable \hat{Q} in the basis of sigma matrices as $\hat{Q}(t_i) = \vec{q}(t_i) \cdot \vec{\sigma}$, $\vec{\sigma} = (\sigma_0 = \mathbb{1}_2, \sigma_1, \sigma_2, \sigma_3)$ one arrives to the simplified formula for the coefficient C_{ij} [20]:

$$C_{ij} = \vec{q}(t_i) \cdot \vec{q}(t_j). \quad (19)$$

It is important to point out that in the above-introduced formulas we made use of the *stationarity* of the system [no explicit time dependence in the Lindblad equation governing time evolution of the neutrino system Eq. (10)], according to which the values of the correlation functions C_{ij} are not the functions of the specific times t_i and t_j but rather the time interval $\tau = t_i - t_j$. Such a requirement accounts to the formal fulfillment of the NIM condition [38].

In order to observe the behavior of the LGI for the neutrinos propagating in the interactive environment, it is convenient to choose $Q(0) = \sigma_z$ as a dichotomic observable. Note that, since

$$Q|\nu_e\rangle = |\nu_e\rangle, \quad Q|\nu_\mu\rangle = -|\nu_\mu\rangle, \quad (20)$$

it measures the neutrino flavor as projected on the z-axis [20].

Making use of the general solution [Eq. (14)] to the Lindblad-Kossakowski master equation [Eq. (9)] for $A(t) = Q(t)$ we are able to examine the variability of the parameter K_3 [Eq. (16)] as a function of different quantities. In particular, we focus on a difference

$$\Delta K_3 = K_3(\phi = 0) - K_3(\phi) \quad (21)$$

between the K_3 function calculated for the Dirac neutrino, corresponding to vanishing CP -violating phase $\phi = 0$, and the Majorana one with $\phi \neq 0$. Nonvanishing ΔK_3 is a potential quantifier, a hallmark, indicating different behavior of these two types of neutrinos.

An influence of decoherence for the properties of ΔK_3 for equal time intervals τ and for different values of the off-diagonal elements $c_{12} = c_{21}$, with remaining parameters (such as E and V_{CC}) fixed, is presented in Fig. 1. As we set $\hbar = 1$ all other parameters are expressed in suitable powers of eV . In particular, in our calculations we set $\tau = 0.1$ and

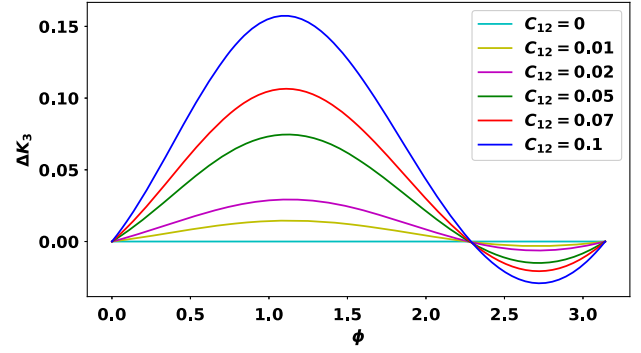


FIG. 1. The difference ΔK_3 as a function of time for Dirac and Majorana neutrinos versus ϕ for different values of $c_{12} = c_{21}$, fixed potential $V_{CC} = 2$ and $c_{11} = c_{22} = c_{33} = 0.1$ with $E = 1$.

assume the experimental values of the solar mixing angle $\theta_{12} = 0.187\pi$ and $\Delta m_{21}^2 \approx 7.54 \times 10^{-5}$ [39]. Furthermore, the values of the $[c_{ij}]$ Kossakowski matrix, rather than derived from any fundamental properties of the system, such as the coupling with the well-described thermal environment [27,40], result from a phenomenological modeling [7,28]. Our choice of the nonvanishing c_{ij} , yet the simplest possible, has no qualitative influence on the presented results. Increasing either the number or the value [provided that Eq. (8) is satisfied] of nonvanishing matrix elements c_{ij} simply lowers the amplitude of ΔK_3 .

Let us explicitly state here the central result of our work: nonvanishing of any of the off-diagonal elements of the Kossakowski matrix $c_{ij} \neq 0$ is *necessary* for $\Delta K_3 \neq 0$. In other words, any “*off-diagonal decoherence*” is a *sine qua non* condition for ΔK_3 to be able to discriminate the Dirac and Majorana natures of the oscillating neutrinos.

In Fig. 2 we present the parameter ΔK_3 as function of ϕ influenced by different values of V_{CC} quantifying interaction of neutrinos with the normal matter. The results of the numerical tests for Dirac and Majorana neutrinos bring us to a conclusion that there is an optimal value of V_{CC} (in the case of parameters chosen here $V_{CC} \approx 2$) for which the

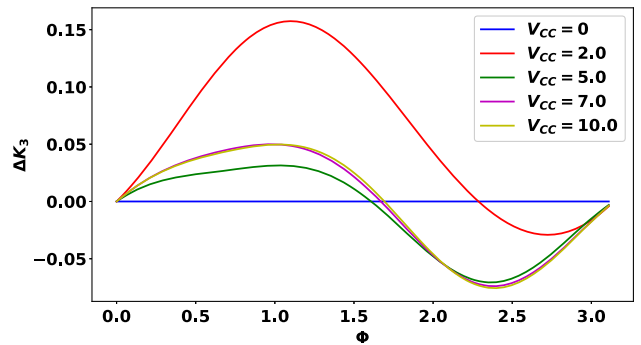


FIG. 2. The difference ΔK_3 as a function of time for Dirac and Majorana neutrinos for different values of potential V_{CC} , given energy $E = 1$ and elements of the Kossakowski matrix $c_{12} = c_{21} = c_{11} = c_{22} = c_{33} = 0.1$ with $E = 1$.

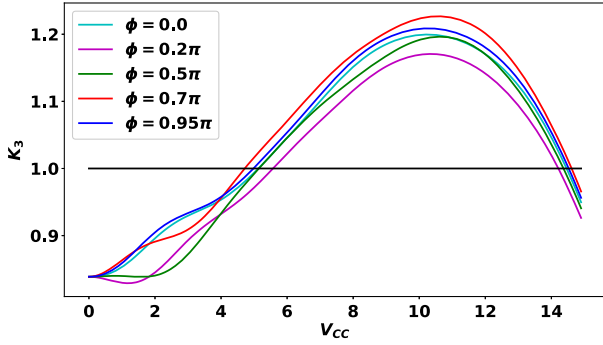


FIG. 3. The dependence of the coefficient K_3 on the potential V_{CC} in the dissipative ($c_{12} = c_{21} = c_{11} = c_{22} = c_{33} = 0.01$) environment with $E = 1$.

difference ΔK_3 is maximal and allows for the most efficient discrimination between Dirac and Majorana neutrino's nature. For our parameters the difference is the most apparent for $V_{CC} \approx 2$, whereas is hardly visible for $V_{CC} < 0.6$ and $V_{CC} > 7.0$. As the potential V_{CC} describes the interaction of neutrinos with matter one can conjecture that this type of interaction and, in particular, a presence of matter, can enhance the possible implications of the neutrino's nature at least on certain measurable properties.

In studies of the K_3 correlator, it is natural to investigate the possibility of violation of the LGI Eq. (17) in dissipative neutrino systems. We investigate whether the condition $K_3 > 1$ may be satisfied more likely either for Dirac or for Majorana neutrinos. The first observation is that for both nondissipative environments (with $c_{ij} = 0$) and the diagonal Kossakowski matrix, there is no possibility of distinguishing them from Dirac and Majorana neutrinos solely upon studies of LGIs' violation. On the other hand, from Fig. 3 one infers that for *off*-diagonal decoherence there exists a certain value of the *CP*-violating phase ϕ for which the value of K_3 is maximized. Similarly, there is also an optimal value of V_{CC} , in our case $V_{CC} \approx 10$, such that the violation of the LGI is most significant. Let us notice,

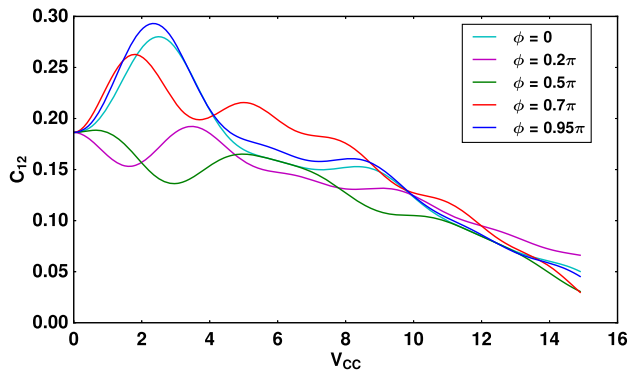


FIG. 4. The dependence of the correlation function C_{ij} Eq. (18) on the potential V_{CC} in the dissipative ($c_{12} = c_{21} = c_{11} = c_{22} = c_{33} = 0.01$) environment with $E = 1$.

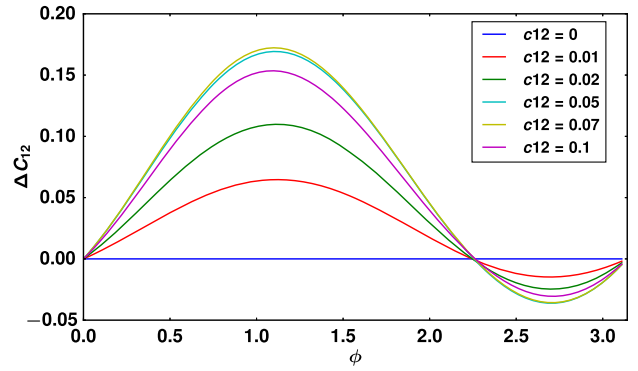


FIG. 5. The difference of the correlation function $\Delta C_{ij} = C_{ij}(\phi = 0) - C_{ij}(\phi)$ for Dirac and Majorana neutrinos for different values of potential V_{CC} , given energy $E = 1$ and elements of the Kossakowski matrix $c_{12} = c_{21} = c_{11} = c_{22} = c_{33} = 0.1$ with $E = 1$.

however, that the value of V_{CC} which leads to a maximal violation of the LGI is different compared to that which results in a maximal value of ΔK_3 discussed previously. According to the results of our numerical analysis not reproduced here this property is valid universally, i.e. for any choice of parameters of the system.

Let us notice that both the K_3 LG-correlator Eq. (16) and the difference ΔK_3 in Eq. (21) are formed by a very special combination of correlation functions C_{ij} defined in Eq. (18). It is worth mentioning that all the properties of the LG-correlators which seem to be useful for distinguishing Dirac and Majorana neutrinos essentially originate from the properties of C_{ij} and $\Delta C_{ij} = C_{ij}(\phi = 0) - C_{ij}(\phi)$ as presented in Fig. 4 and Fig. 5 respectively. However, the additional physical content present in the LG-correlator K_3 supported by recent experimental efforts on studies of a LGI violation in neutrino systems [20] is encouraging to relate this phenomenon—quantified by K_3 rather than C_{ij} —and the investigation of the neutrino's nature.

Eventually, let us notice that, contrary to V_{CC} , an energy E in Eq. (5)—one of these parameters which are crucial for the properties of oscillating neutrinos—affects neither ΔK_3 nor K_3 . The chosen value of E in Fig. 1, Fig. 2 and Fig. 3 is completely arbitrary. In other words, one cannot either make Dirac and Majorana neutrinos distinguishable or improve their distinguishability solely via setting a value of E in a similar manner as it can be done with V_{CC} .

IV. CONCLUSIONS

Determination of the neutrino nature, if it is a Dirac or Majorana particle, is one of the most important, still unsolved, problems in studies of these fascinating particles. There are various problems arising in this context. First of all, the measurement of any of the neutrinos' properties is a hard task due to a well-known weakness of the neutrino's interactions. Secondly, one needs to find and measure an

observable, which would allow for the establishment of the neutrino nature. In this theoretical work we attempt to contribute to this second aspect. We show that the K_3 correlator studied recently in a context of violation of the Leggett-Garg inequalities takes different values for Dirac and Majorana neutrinos under certain conditions which were identified in our work. We studied neutrino's oscillation in a presence of decoherence and matter using a phenomenological approach based on the Kossakowski-Lindblad-Gorini completely positive dynamics [27].

The main conclusion of our modeling is that the difference in K_3 occurs provided that (i) there is a certain type of decoherence represented by an *off*-diagonal Kossakowski matrix and (ii) the neutrino interacts with a matter, i.e. $V_{CC} \neq 0$ in Eq. (6). These two conditions are necessary for making the difference ΔK_3 a nonvanishing and potentially useful candidate for a neutrino's nature quantifier.

We also studied the difference in a possible violation of the LGI for Dirac and Majorana neutrinos. We concluded that the same two conditions as formulated above are valid also for the $K_3 > 1$ Leggett-Garg condition. However, there is a crucial difference: the value of V_{CC} necessary for violation of the LGI ($K_3 > 1$) is one order of magnitude larger than the corresponding value necessary for $\Delta K_3 \neq 0$. As the value of V_{CC} is related to density of matter affecting

neutrino's oscillations, this observation may be potentially interesting and useful for experimentalists.

Let us emphasize here that credibility of any quantitative results obtained via any phenomenological model are questionable as long as the model is not confronted with a huge amount of experimental data. As neutrino physics suffers from a shortage of experimental data *quantitative* predictions presented in our work require considerable caution in interpretation. One can not exclude a very pessimistic scenario such that the proposed quantifier discriminating neutrinos nature works in a tailored range of parameters which is essentially inaccessible in any direct experiment and that our predictions are of purely theoretical character.

Mysterious neutrinos, more than any other "fundamental" particles, attract physicists outside the particle physics community. We hope that our results are not only a modest contribution to physics of neutrinos *per se* but also indicate some unusual properties of neutrinos shedding light on even more unusual properties of the quantum world.

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