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Mass generation by a Lorentz-invariant gas of spacetime defects

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We present a simple model of defects embedded in flat spacetime, where the model is designed to maintain Lorentz invariance over large length scales. Even without remnant Lorentz violation, there are still effects from these spacetime defects on the propagation of physical fields, notably mass generation for scalars and Dirac fermions.

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I. INTRODUCTION

It has been argued that spacetime over small length scales might have a nontrivial structure [1–5]. What the precise nature of this small-scale "structure" would be is, however, unclear.

It is known, for example, that static Swiss-cheese-type models affect particle propagation and experimental data strongly constrain the "holes" of such a classical spacetime [6,7]. As the static holes of such a model violate Lorentz invariance, the above-mentioned bounds strongly constrain this type of Lorentz violation; see also the related discussion in Ref. [8]. The conclusion appears to be that, if spacetime somehow has a small-scale structure, the underlying (quantum) theory manages to keep Lorentz invariance to high precision.

For this reason, it may be of interest to investigate toy models of spacetime-defects, where the models are designed to maintain Lorentz invariance on average. One class of such models involves pointlike defects, as studied in Refs. [9–11] (see also Refs. [12,13] for a general discussion). In the present article, we present one further toy model with pointlike defects and study the induced modifications of the standard particle propagation ("standard" referring to the perfect Minkowski spacetime without defects).

II. POISSON DISTRIBUTION AND LORENTZ INVARIANCE

It is a nontrivial issue to find distributions of defects over four-dimensional Minkowski spacetime, which preserve the Lorentz symmetry in the large. Let us assume, for example, that the defects are distributed over a regular hypercubic lattice in one particular reference frame. Averaged over large scales, the distribution is homogeneous. But if we go to a Lorentz-boosted frame, the density of defects will increase in the direction of the boost, while remaining constant in the perpendicular directions. Apparently, the Lorentz symmetry is broken by having a preferred reference frame in the original setup with a regular lattice.

Still, if the defects are distributed according to a Poisson process (a "sprinkling" procedure), boosts do not break Lorentz invariance [14–16]. The probability of finding n defects in a four-dimensional volume V_4 is then given by

$$P_n(V_4) = \frac{1}{n!}(\rho_d V_4)^n \exp(-\rho_d V_4). \tag{2.1}$$

The parameter ρ_d characterizes the distribution and corresponds to the average spacetime density of defects. Note that the Poisson process, for constant parameter ρ_d, depends only on the four-dimensional volume of the region considered. This implies that the probability of finding n defects contained in a region of volume V_4 is invariant under volume-preserving transformations. Since Lorentz transformations preserve the spacetime volume, the sprinkling is Lorentz invariant. Phrased in a different way, the defect distribution from the Poisson process has no built-in "structure." If present, such a built-in structure would be deformed by Lorentz contraction, just as for the regular-lattice setup discussed above.

We see immediately from the Poisson distribution (2.1) that, on average, the typical number of defects inside a region of volume V_4 is given by $\langle n\rangle_{V_4} = \rho_d V_4$ and that the fluctuations of this number are of order $\sqrt{\rho_d V_4}$, so that the relative fluctuations become irrelevant for large V_4.

III. EFFECTIVE MODEL FOR A GAS OF SPACETIME DEFECTS

A. General remarks

The explicit calculation of physical observables in a theory with finite-size spacetime defects (corresponding to,

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e.g., soliton-type solutions [17]) is prohibitively difficult. We can use, instead, a simple model with a gas of pointlike defects [9,11].

In the new model presented here, the spacetime defects are represented by randomly-positioned delta functions in a classical background Minkowski spacetime, where the delta functions are coupled to a "mediator" real scalar field $\sigma(x)$ with random charges $\epsilon_n \in \{-1, 1\}$ and a coupling constant λ. The charges of the individual defects are randomly chosen with probability $1/2$ to get charge $+1$ and probability $1/2$ to get charge -1, so that the average charge vanishes over a large enough spacetime volume. The mediator field $\sigma(x)$ is also coupled to three "physical" fields: a massless real scalar field $\phi(x)$ with a nonderivative quartic-coupling term, a massless Dirac fermion field $\psi_1(x)$ with a Yukawa coupling term, and a massless Dirac fermion field $\psi_2(x)$ with a nonrenormalizable Yukawa-type coupling term.

In short, a nontrivial spacetime with defects is modeled by a perfect classical Minkowski spacetime and an action with delta functions coupled to a real scalar field $\sigma(x)$. In turn, this mediator field $\sigma(x)$ is coupled to physical fields $\phi(x)$, $\psi_1(x)$, and $\psi_2(x)$, where all fields propagate over Minkowski spacetime.

B. Massless mediator field

The following effective action is considered:

$$S_{\text{eff}} = -\int_{\mathbb{R}^4} d^4x \bigg(\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + i\bar{\psi}_1\gamma^\mu\partial_\mu\psi_1 + i\bar{\psi}_2\gamma^\mu\partial_\mu\psi_2 + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + \lambda\sigma\left[\sum_{n=1}^{\infty}\epsilon_n\delta^{(4)}(x-x_n)\right]_{\{x_n\}\,\text{from Poisson}} + g_s\sigma^2\phi^2 + g_{f,1}\sigma\bar{\psi}_1\psi_1 + g_{f,2}\lambda\sigma^2\bar{\psi}_2\psi_2 \bigg), \tag{3.1a}$$

$$\epsilon_n \in \{-1, +1\}, \quad \text{with} \quad P_{-1} = P_{+1} = 1/2, \tag{3.1b}$$

where the Minkowski metric is given by $\eta_{\mu\nu} \equiv [\text{diag}(-1, 1, 1, 1)]_{\mu\nu}$ for standard Cartesian coordinates x^μ. The dimensionless quartic scalar coupling constant g_s is taken to be positive, so that the scalar potential is bounded from below. Throughout, we use natural units with $\hbar = 1 = c$.

In the action (3.1a), the cores of the spacetime defects are modeled by Dirac delta functions centered at the points $x_1, x_2, \ldots$ of Minkowski spacetime. As discussed in Sec. II, these points are distributed according to a Poisson process (sprinkling), in order to preserve the Lorentz symmetry. The long-range effects of the defect cores are modeled by a real scalar field σ with coupling strength λ and random charges $\epsilon_n \in \{-1, 1\}$. With equal probabilities (3.1b) for having a positive and a negative charge (no correlation between the different defects), the average charge vanishes asymptotically,

$$\lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N}\epsilon_n = 0. \tag{3.2}$$

The scalar field $\sigma(x)$ mediates between the defect cores at random positions $x = x_n$ and the physical fields $\phi(x)$ and $\psi_a(x)$, for $a = 1, 2$.

The massless scalar fields ϕ and σ in (3.1a) have mass dimension 1, the massless fermionic fields ψ_a have mass dimension $3/2$, the coupling constant λ has mass dimension -1, and the couplings g_s and $g_{f,a}$ are dimensionless. As mentioned in Sec. III A, the idea behind the action (3.1) is that a nontrivial spacetime (manifold or not) is modeled by delta functions located at the spacetime points x_n of the Minkowski manifold and by a mediator field $\sigma(x)$. The mediator field $\sigma(x)$ is coupled to the delta functions and to additional physical fields $\phi(x)$ and $\psi_a(x)$, with all fields propagating over classical Minkowski spacetime and the interaction terms given by the last three terms of the integrand of (3.1a).

In order to recover the standard perturbative results by use of Feynman diagrams, the interactions terms of (3.1a) essentially need to be "turned off" in the asymptotic regions [18]. This can be done by making the couplings in (3.1a) spacetime dependent,

$$\{\lambda(x), g_s(x), g_{f,a}(x)\} = \begin{cases} \{\bar{\lambda}, \bar{g}_s, \bar{g}_{f,a}\}, & \text{for } x \in V_{4,\text{cutoff}}, \\ \{0, 0, 0\}, & \text{otherwise}, \end{cases} \tag{3.3}$$

where the barred quantities on the right-hand side are truly constant. The spacetime volume $V_{4,\text{cutoff}}$ in (3.3) is taken to be suitably large and the behavior in the transition region can be adequately smoothed. In the following, we will keep this spacetime dependence of the couplings $\{\lambda, g_s, g_{f,a}\}$ implicit. Incidentally, restricting the sum in (3.1a) to the finite volume $V_{4,\text{cutoff}}$ makes this sum well behaved, with a finite number N of defects.

The classical solution for the mediator field σ can be easily obtained for $g_s = g_{f,a} = 0$,

$$\left.\frac{\delta S_{\text{eff}}}{\delta\sigma}\right|^{(g_s = g_{f,a} = 0)} = 0 \Rightarrow \Box\sigma(x) = \lambda\sum_n \epsilon_n\delta^{(4)}(x - x_n), \tag{3.4}$$

with the flat-spacetime d'Alembert operator $\Box \equiv \partial^\mu\partial_\mu$. The solution is given by

$$\sigma(x) = \sigma_0(x) + \lambda\sum_n \epsilon_n \int d^4x' G_0(x, x')\delta^{(4)}(x - x_n), \tag{3.5}$$

where $\sigma_0(x)$ is the free solution [corresponding to the homogeneous equation (3.4) with $\lambda = 0$] and $G_0(x, x')$ is a Green's function of the d'Alembert operator $\Box$,

$$G_0(x,x') = \frac{1}{4\pi^2|x-x'|^2}. \tag{3.6}$$

Explicitly, the solution of (3.4) takes the following form:

$$\sigma(x) = \sigma_0(x) + \lambda\sum_n \frac{\epsilon_n}{4\pi^2|x-x_n|^2} \equiv \sigma_0(x) + \sigma_1(x). \tag{3.7}$$

Let us now determine the two-point function for σ. In the quantization procedure, the free part of σ can be split in positive and negative frequency modes as usual,

$$\sigma_0(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2\omega_p}}(a_p e^{-ipx} + a_p^\dagger e^{ipx}), \tag{3.8}$$

with $p_0 = \omega_p \equiv \sqrt{p_1^2+p_2^2+p_3^2}$. If we define

$$j(x) \equiv \lambda\sum_n \epsilon_n \delta^{(4)}(x-x_n), \tag{3.9}$$

the Fourier transform takes a simple form

$$j(p) = \int d^4x e^{ipx} j(x) = \lambda\sum_n \epsilon_n e^{ipx_n}. \tag{3.10}$$

With the help of (3.10), we can expand the correction term in (3.7) as follows:

$$\sigma_1(x) = \lambda\int \frac{d^3p}{(2\pi)^3}\left(\frac{\sum_n \epsilon_n e^{ipx_n} e^{-ipx}}{2\omega_p}\mathbb{1} + \text{H.c.}\right), \tag{3.11}$$

where $\mathbb{1}$ is the identity operator. The only nonvanishing contributions to the two-point function come from terms proportional to $\langle 0|a_p a_q^\dagger|0\rangle = (2\pi^3)\delta^{(3)}(p-q)$ or proportional to $\langle 0|\mathbb{1}^2|0\rangle = 1$. The final result is given by

$$\begin{aligned}\langle 0|\sigma(x)\sigma(y)|0\rangle &= \int \frac{d^3p}{(2\pi)^3}\frac{e^{-ip(x-y)}}{2\omega_p} \\ &\quad + 4\lambda^2\sum_{m,n}\epsilon_n\epsilon_m \int \frac{d^3p}{(2\pi)^3}\frac{e^{-ip(x-x_n)}}{2\omega_p} \\ &\quad \times \int \frac{d^3q}{(2\pi)^3}\frac{e^{-iq(x_m-y)}}{2\omega_q}.\end{aligned} \tag{3.12}$$

Taking into account that the first term on the right-hand side of (3.12) corresponds to the free two-point function [denoted by $\Delta_0(x-y)$ as usual] we get

$$\begin{aligned}&\langle 0|\sigma(x)\sigma(y)|0\rangle \\ &\quad = \Delta_0(x-y) + 4\lambda^2\sum_{m,n}\epsilon_m\epsilon_n \Delta_0(x-x_n)\Delta_0(x_m-y).\end{aligned} \tag{3.13}$$

The expression (3.13) has a simple interpretation: the amplitude for the scalar field σ to propagate from a spacetime point x to a spacetime point y is given by the free amplitude plus all possible products of the free amplitude of particle propagation from x to the position of a defect x_n times the free amplitude of particle propagation from another defect x_m to y. In the random-phase approximation (see Appendix), all cross terms joining different defects in (3.13) are subdominant. The expression (3.13) then simplifies to

$$\langle 0|\sigma(x)\sigma(y)|0\rangle \approx \Delta_0(x-y) + 4\lambda^2\sum_n \Delta_0(x-x_n)\Delta_0(x_n-y). \tag{3.14}$$

This full tree-level propagator contains the free propagator and the sum of all possible insertions of a single defect.

C. Massive mediator field

For completeness, we also consider the case where the mediator field has a nonzero initial mass m_0. The massive version of the original action (3.1a) takes the following form:

$$\begin{aligned}S_{\text{eff},m_0} = -\int_{\mathbb{R}^4} d^4x \bigg(&\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + i\bar\psi_1\gamma^\mu\partial_\mu\psi_1 \\ &+ i\bar\psi_2\gamma^\mu\partial_\mu\psi_2 + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + \frac{1}{2}m_0^2\sigma^2 \\ &+ \lambda\sigma\sum_{n=1}^{\infty}\epsilon_n\delta^{(4)}(x-x_n) + g_s\sigma^2\phi^2 \\ &+ g_{f,1}\sigma\bar\psi_1\psi_1 + g_{f,2}\lambda\sigma^2\bar\psi_2\psi_2\bigg),\end{aligned} \tag{3.15}$$

with the physical fields ϕ and ψ_a still being massless, as long as interactions are neglected.

As before, the classical equation for σ can be written as

$$(\Box - m_0^2)\sigma(x) = \lambda\sum_n \epsilon_n\delta^{(4)}(x-x_n), \tag{3.16}$$

again setting $g_s = g_{f,a} = 0$. The complete solution of this equation can be split in two parts,

$$\sigma(x) = \sigma_0(x) + \lambda\sum_n \epsilon_n \int d^4x' \tilde G_0(x,x')\delta^{(4)}(x-x_n), \tag{3.17}$$

where, now, $\sigma_0(x)$ is the solution of the homogeneous equation $(\Box - m_0^2)\sigma_0(x) = 0$ and $\tilde G_0(x,x')$ is a Green's function of the operator $\Box - m_0^2$,

$$\tilde G_0(x,x') = \frac{m_0\mathcal{K}_1(m_0|x-x'|)}{4\pi^2|x-x'|}, \tag{3.18}$$

with $\mathcal{K}_\nu(z)$ the modified Bessel function of the second kind [19].

A calculation similar to the one of Sec. III B gives the following result for the two-point function:

$$\langle 0|\sigma(x)\sigma(y)|0\rangle_{m_0} = \tilde{\Delta}_0(x-y) + 4\lambda^2 \sum_{n,m} \epsilon_n \epsilon_m \tilde{\Delta}_0(x-x_n)\tilde{\Delta}_0(x_m-y), \quad (3.19)$$

where $\tilde{\Delta}_0(x-y)$ is the free massive propagator. In the random-phase approximation (see Appendix), the expression (3.19) simplifies to

$$\langle 0|\sigma(x)\sigma(y)|0\rangle_{m_0} \approx \tilde{\Delta}_0(x-y) + 4\lambda^2 \sum_{n} \tilde{\Delta}_0(x-x_n)\tilde{\Delta}_0(x_n-y), \quad (3.20)$$

which has the same structure as expression (3.14) for the massless-mediator case.

IV. MASS GENERATION FOR THE MEDIATOR FIELD

As found in Sec. III, the interaction of the mediator field $\sigma(x)$ with the delta functions leads to a nontrivial modification of the σ propagator. Let us focus on the initially massless case given by the action (3.1a). The propagator for the mediator field (3.14) can then be rewritten as follows:

$$\Delta(x,y) \approx \Delta_0(x-y) + 4\lambda^2 \Delta_1(x,y), \quad (4.1)$$

where $\Delta_0(x-y)$ corresponds to the free propagator and $\Delta_1(x,y)$ to the correction from the interactions with the delta functions (corresponding to the defect cores). We will see that this last term Δ_1 generates a nonzero mass for the σ field.

If a nonzero mass m_σ is indeed generated, then the following equation must hold:

$$(\Box_x - m_\sigma^2)\Delta(x,y) = -\delta^{(4)}(x-y), \quad (4.2)$$

for $m_\sigma^2 \neq 0$. After inserting the propagator (4.1) in (4.2), we obtain to order λ^2

$$4\lambda^2 \sum_n \delta^{(4)}(x-x_n)\Delta_0(x_n-y) - m_\sigma^2 \Delta_0(x-y) + \mathcal{O}[\lambda^4 \rho_d^2 \Delta_1(x,y)] = 0. \quad (4.3)$$

It is still not easy to interpret the first term on the left-hand side of (4.3). To do so, we can use the fact that the points x_n are distributed according to a Poisson process as discussed in Sec. II. According to (2.1), the number of defects grows with the spacetime volume, $dN \propto d^4x$, where the proportionality factor is given by the density parameter ρ_d. Furthermore, the distribution is assumed to be dense and, therefore, the characteristic distance between defects, $l_d \equiv \rho_d^{-1/4}$, is assumed to be small compared to the typical wavelengths of the fields considered. This allows us to approximate the sum in (4.3) by an integral,

$$\sum_n \to \rho_d \int_{\mathbb{R}^4} d^4x. \quad (4.4)$$

Applying (4.4) to (4.3) we obtain the result

$$m_\sigma^2 = 4\lambda^2 \rho_d, \quad (4.5)$$

in terms of the defect density ρ_d from (2.1) and the coupling constant λ from (3.1a).

The first corrections to the mass-square (4.5) will appear as loop corrections involving the dimensionless couplings g_s and $g_{f,a}$.

We conclude that, as a result of the interactions with the delta functions, the mediator field σ has acquired a mass. This result can be confirmed by working in the momentum-space representation. Start from the full propagator (3.14) and take (4.4) into account,

$$\langle 0|\sigma(x)\sigma(y)|0\rangle \approx \Delta_0(x-y) + 4\lambda^2 \rho_d \int d^4z \Delta_0(x-z)\Delta_0(z-y). \quad (4.6)$$

After shifting the z variable (in order to make explicit the dependence of the two-point function on the difference $x-y$), we can rewrite (4.6) in momentum space as follows:

$$G(p) = \frac{1}{p^2+i\epsilon} + 4\lambda^2\rho_d \int d^4z \Delta_0(z) \frac{e^{ipz}}{p^2+i\epsilon}. \quad (4.7)$$

Integration with respect to z then gives

$$G(p) = \frac{1}{p^2+i\epsilon} - \frac{1}{p^2+i\epsilon} 4\lambda^2\rho_d \frac{1}{p^2+i\epsilon}. \quad (4.8)$$

The expression (4.8) can be rewritten to quadratic order in λ as follows:

$$G(p) = \frac{1}{p^2 - 4\lambda^2\rho_d + i\epsilon} + \mathcal{O}[\lambda^4\rho_d^2(p^2+i\epsilon)^{-3}]. \quad (4.9)$$

The mass-square term (4.5) for σ appears in this representation as a pole in the momentum-space propagator. In the limit $\lambda \to 0$ (no long-range effects of the defect cores) and/or in the limit $\rho_d \to 0$ (vanishing density of defect cores), the mediator field remains massless.

For the initially massive case (3.15), the pole in the σ-propagator is already shifted by the mass-square m_0^2 and an effective mass-square $m_\sigma^2 = m_0^2 + 4\lambda^2\rho_d$ is obtained.

V. PHYSICAL FIELDS

A. Setup

The massless physical fields ϕ and ψ_a only "feel" the presence of the defect cores by interaction with the mediator field σ. From now on, we work with the effective action obtained from (3.1a) for a vanishing initial σ mass. We can write the obtained effective action at order λ^2 as follows

$$S_{\mathrm{eff},\lambda^2} = -\int_{\mathbb{R}^4} d^4x \bigg(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + i\bar{\psi}_1\gamma^\mu\partial_\mu\psi_1 + i\bar{\psi}_2\gamma^\mu\partial_\mu\psi_2 + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + 2\lambda^2\rho_d\sigma^2 + g_s\sigma^2\phi^2 + g_{f,1}\sigma\bar{\psi}_1\psi_1 + g_{f,2}\lambda\sigma^2\bar{\psi}_2\psi_2 + \mathcal{O}[\lambda^4\rho_d^2]\bigg), \tag{5.1}$$

where the delta functions have produced a mass term for the σ field as calculated in Sec. IV.

B. Physical scalar field ϕ

We now ask what happens to the massless physical field ϕ by its interaction with the mediator field σ. The self-energy for the scalar field is given by Fig. 1. With appropriate regularization, the last term in Fig. 1 corresponds the counterterm of Fig. 2. This counterterm cancels the divergence coming from the one-loop integral.

Consider the Pauli–Villars (PV) regularization method [20,21]. For the 1-loop diagram of Fig. 1, the divergent integral is

$$I(m_\sigma^2) \equiv g_s \int \frac{d^4k_E}{(2\pi)^4}\frac{1}{k_E^2 + m_\sigma^2}, \tag{5.2}$$

where k_E is the Euclidean momentum. We now define the PV-regularized integral by

$$I_{\mathrm{PV}}(m_\sigma^2) = I(m_\sigma^2) - I(\Lambda^2) - (m_\sigma^2 - \Lambda^2)I'(\Lambda^2), \tag{5.3}$$

with the PV regulator mass Λ and $I'(x) \equiv dI(x)/dx$. Evaluating (5.3) gives

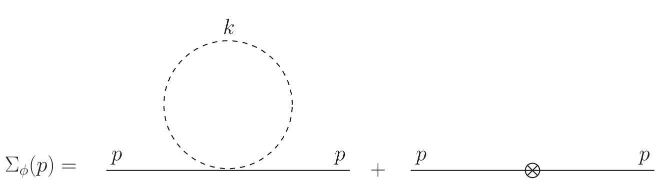

FIG. 1. Self-energy contribution for the physical scalar ϕ. The dashed propagator corresponds to the mediator scalar field σ and the counterterm is given in Fig. 2.

FIG. 2. Counterterm for Fig. 1.

$$I_{\mathrm{PV}}(m_\sigma^2) = -g_s\frac{m_\sigma^2}{(4\pi)^2} + g_s\frac{\Lambda^2}{(4\pi)^2}\left(1 - \log\frac{\Lambda^2}{m_\sigma^2}\right). \tag{5.4}$$

The counterterm of Fig. 2 cancels exactly the Λ terms of (5.4),

$$\delta_\phi = 0, \tag{5.5a}$$

$$\delta_{m_\phi} = g_s\frac{\Lambda^2}{(4\pi)^2}\left(1 - \log\frac{\Lambda^2}{m_\sigma^2}\right). \tag{5.5b}$$

As a result, a nonzero mass for ϕ is generated at one-loop level,

$$M_\phi^2\Big|^{(\mathrm{PV\,reg})} = \lim_{p\to 0}\Sigma_\phi(p)\Big|^{(\mathrm{PV\,reg})} = g_s\frac{m_\sigma^2}{(4\pi)^2}, \tag{5.6}$$

with $m_\sigma^2 = 4\lambda^2\rho_d$ from (4.5). The point-splitting and dimensional-regularization methods [21] give a similar result for the generated scalar mass-square, with the same parametric dependence $g_s m_\sigma^2$.

The generated mass-square for the scalar field as given by (5.6) depends linearly on both ρ_d and λ^2. This implies that, in order to give a nonzero mass to the scalar field, both the presence of defect cores ($\rho_d \neq 0$) and the interaction of defect cores with the mediator field ($\lambda \neq 0$) are essential.

Two general remarks are in order. First, the underlying nontrivial spacetime produces not only the model (3.1) but also the required counterterms such as (5.5). If a single energy scale E_{foam} (equal or not equal to the Planck energy $E_P \equiv G^{-1/2}$) sets the parameters $\lambda \sim 1/E_{\mathrm{foam}}$ and $\rho_d \sim (E_{\mathrm{foam}})^4$, then it is also to be expected that $\Lambda \sim E_{\mathrm{foam}}$, and the Pauli–Villars "regularization" is no longer a mere mathematical device but is rooted in physical reality. The generated mass-square (5.6) can be very much smaller than $(E_{\mathrm{foam}})^2$ if $g_s \ll 1$.

Second, the following simple question arises: is it not possible that the generated mass-square (5.6) gets absorbed in the square of the renormalized mass and that, thereby, the effects from our spacetime defects become invisible? In general, this is certainly possible, but not for the setup of the theory as outlined in Sec. III. Specifically, the coupling λ is taken to vanish far out, according to (3.3). This means that the constant renormalized mass relevant to the infinite-volume perfect Minkowski spacetime is unaffected by spacetime defects, as their effects ($\bar{\lambda} \neq 0$) are confined to the finite volume $V_{4,\mathrm{cutoff}}$.

C. Physical fermion field ψ_1

The interaction of the massless fermionic field ψ_1 with σ gives rise to different effects compared to those of the scalar field ϕ. The self-energy for the fermion including the counterterm is given by Figs. 3 and 4.

Consider again Pauli–Villars regularization. The unsubtracted regularized integral from the 1-loop diagram of Fig. 3 gives

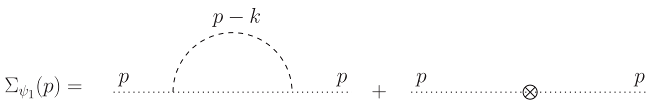

FIG. 3. Self-energy contribution for the physical fermion ψ_1. The dashed propagator corresponds to the mediator scalar field σ and the counterterm is given in Fig. 4.

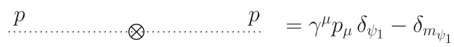

FIG. 4. Counterterm for Fig. 3.

$$\Sigma_{\psi_1}(p)\big|^{(\text{PV reg, unsubtr})} = -\frac{g_{f,1}^2}{16\pi^2}\not{p}\log\left(\frac{\Lambda^2}{m_\sigma^2}\right) - \frac{g_{f,1}^2}{8\pi^2}\not{p}\int_0^1 dzz\log\left(\frac{m_\sigma^2}{m_\sigma^2 - p^2(1-z)}\right), \tag{5.7}$$

with the Pauli–Villars regulator Λ and the Feynman slash notation $\not{p} \equiv \gamma^\mu p_\mu$. The required counterterm is then

$$\delta_{\psi_1} = \frac{g_{f,1}^2}{16\pi^2}\not{p}\log\left(\frac{\Lambda^2}{m_\sigma^2}\right), \tag{5.8a}$$

$$\delta_{m_{\psi_1}} = 0. \tag{5.8b}$$

This results in

$$M_{\psi_1}\Big|^{(\text{PV reg})} = \lim_{p\to 0}\Sigma_{\psi_1}(p)\Big|^{(\text{PV reg})} = 0. \tag{5.9}$$

The point-splitting and dimensional-regularization methods also give a vanishing generated mass for ψ_1.

Hence, the fermion ψ_1 does not get a mass due to the interaction with the mediator field σ, at least at the 1-loop level and under the assumption that σ does not acquire a vacuum expectation value. Expanding on this last point, the effective action (5.1) is invariant under the following axial transformation:

$$\psi_1(x) \to \exp[i(\pi/2)\gamma_5]\psi_1(x), \tag{5.10a}$$

$$\sigma(x) \to -\sigma(x). \tag{5.10b}$$

If unbroken, the axial symmetry (5.10) of the effective action (5.1) rules out a direct (or generated) mass term $M\bar{\psi}_1\psi_1$.

D. Physical fermion field ψ_2

The interaction of the massless fermionic field ψ_2 with σ differs from that of ψ_1 with σ and is, in fact, similar to the interaction of the scalar field ϕ with σ.

The self-energy for the ψ_2 fermion including the counterterm is given by Figs. 5 and 6. It is now clear that the ψ_2 self-energy of Fig. 5 has the same structure as the ϕ self-energy of Fig. 1. In other words, the divergent integral for the ψ_2 self-energy is proportional to (5.2).

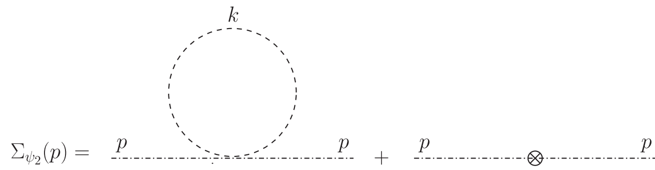

FIG. 5. Self-energy contribution for the physical fermion ψ_2. The dashed propagator corresponds to the mediator scalar field σ and the counterterm is given in Fig. 6.

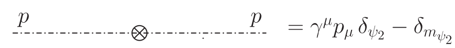

FIG. 6. Counterterm for Fig. 5.

Consider again Pauli–Villars regularization and take over the relevant results from Sec. V B. The counterterm is then given by

$$\delta_{\psi_2} = 0, \tag{5.11a}$$

$$\delta_{m_{\psi_2}} = g_{f,2}\lambda\frac{\Lambda^2}{(4\pi)^2}\left(1 - \log\frac{\Lambda^2}{m_\sigma^2}\right), \tag{5.11b}$$

and, adapting the constants in (5.6), the following generated mass is obtained:

$$M_{\psi_2}\Big|^{(\text{PV reg})} = \lim_{p\to 0}\Sigma_{\psi_2}(p)\Big|^{(\text{PV reg})} = g_{f,2}\lambda\frac{m_\sigma^2}{(4\pi)^2}, \tag{5.12}$$

with $m_\sigma^2 = 4\lambda^2\rho_d$ from (4.5). The point-splitting and dimensional-regularization methods give a similar result for the generated fermion mass, with the same parametric dependence $g_{f,2}\lambda m_\sigma^2$.

As the ψ_2 interaction term of the effective action (5.1) involves a factor σ^2 [instead of the single factor σ of the ψ_1 interaction term], the axial transformation (5.10), with ψ_1 replaced by ψ_2, no longer leaves the action (5.1) invariant. Hence, there is no axial symmetry for the ψ_2 field to exclude the appearance of a ψ_2 mass term.

VI. DISCUSSION

It has become clear over the last years that, if a "quantum spacetime foam" somehow results in an effective classical spacetime manifold with small-scale structure, this effective manifold must be Lorentz-invariant to high precision (at the 10^{-15} level in the photon sector [7]). In the present article, we have, therefore, investigated a model of spacetime defects which is Lorentz-invariant over large enough spacetime volumes. Even though there is no apparent Lorentz violation in this model, there may still be nontrivial effects for the propagation of particles. The quantities that feel the effects of this small-scale structure must be themselves Lorentz-invariant, an obvious example being mass. Indeed, we have found a generated mass for both a scalar field and a Dirac fermion field, as long as there is no effective axial symmetry of the toy model considered. It is, in fact, possible to keep the Dirac fermion field massless, if an effective axial symmetry is built into the toy model.

Incidentally, the mass generation found here is not entirely surprising, as mass generation is known to occur for non-Minkowskian manifolds [22]. The manifold considered in Ref. [22] has nontrivial topology at large length scales (manifold $\mathbb{R}^3 \times S^1$), whereas we are interested in nontrivial topology at small length scales (cf. the discussion in Ref. [9]).

Assuming our results to apply to the Higgs scalar boson of the standard model of elementary particle physics (with a Higgs mass around 125 GeV [23]), we have from (5.6) the following upper bound:

$$g_s \frac{4\lambda^2 \rho_d}{(4\pi)^2} \ll (125\ \text{GeV})^2, \tag{6.1}$$

where the defect density ρ_d is defined by (2.1) and the dimensional coupling constant λ and the positive dimensionless coupling constant g_s are defined by (3.1a). We have used a strong inequality in (6.1), in order to make sure that the spacetime defects, if present, do not upset the standard Higgs mechanism for mass generation of gauge bosons and fermions.

In Sec. V B, we already mentioned the possibility that a single energy scale E_{foam} controls the small-scale structure of spacetime and, therefore, sets the parameters of our model (3.1a),

$$\lambda \sim 1/E_{\text{foam}}, \tag{6.2a}$$

$$\rho_d \sim (E_{\text{foam}})^4. \tag{6.2b}$$

With the scenario (6.2) and $g_s \sim 10^{-2}$, bound (6.1) gives

$$E_{\text{foam}} \ll 8\ \text{TeV}\left(\frac{10^{-2}}{g_s}\right)^{1/2}. \tag{6.3}$$

In this case, the picture is that the spacetime defects have an effective size [of order $1/m_\sigma \sim \lambda^{-1}(\rho_d)^{-1/2}$] and typical distance between neighboring defects [of order $(\rho_d)^{-1/4}$] with the same order of magnitude, $1/E_{\text{foam}}$. This single length scale is, however, very much larger than the Planck length $1/E_P \approx 1.62\times 10^{-35}\,\text{m}$, with $E_P \equiv G^{-1/2} \approx 1.22\times 10^{16}\,\text{TeV}$.

Scenarios different from (6.2) are also possible. One scenario has, for example, $\lambda \sim 1/E_P$ and $\rho_d \ll (10^{-2}/g_s)(8\ \text{TeV})^2(E_P)^2$. Ultimately, only the derivation of our model (3.1) from the underlying spacetime (assuming the toy model to be relevant at all) can decide between the different scenarios.

From a general perspective, it may be of interest to have found another possible origin of mass, barring questions of naturalness and the unknown nature of quantum spacetime. The toy model considered here is rather simple in that it only gives mass to scalars and Dirac fermions. More difficult would be spacetime-defect mass generation for the Weyl fermions and the gauge bosons of a chiral gauge theory (the type of theory relevant to the standard model). For the Weyl fermions, we may consider replacing the single real scalar field $\sigma(x)$ of our model by a complex scalar field $\Sigma(x)$ in an appropriate representation of the gauge group and using this scalar Σ in a gauge-invariant Yukawa-type coupling term. For the gauge bosons, perhaps the spacetime-defect mechanism can be merged with a modified version of the Higgs mechanism.

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APPENDIX: RANDOM-PHASE APPROXIMATION

Let us investigate a double momentum integral of the following form:

$$\int d^4p\, d^4q\, f(p,q) J(p) J^\star(q), \tag{A1}$$

where $J(p)$ is defined by

$$J(p) \equiv \sum_n \epsilon_n e^{ipx_n}, \tag{A2}$$

with random defect positions $x_n \in \mathbb{R}^4$ and random defect charges $\epsilon_n \in \{-1,+1\}$ as discussed in Secs. II and III A, respectively. The defect index n runs over positive integers,

$$n \in \mathcal{N} \equiv \{1,2,3,\ldots,N\}, \tag{A3}$$

where N is taken to infinity at the end of the calculation, together with the volume $V_{4,\text{cutoff}}$ discussed in Sec. III B.

Now consider the product of two J's at different momenta,

$$J(p)J^\star(q) = \sum_{m,n} \epsilon_m \epsilon_n e^{ipx_n - iqx_m}, \tag{A4}$$

where the indices m and n run over the set $\mathcal{N}$ as given by (A3). The product (A4) can be split in two parts, a single sum and a double sum,

$$J(p)J^\star(q) = \sum_n e^{ix_n(p-q)} + \sum_{m\neq n} \epsilon_m \epsilon_n e^{ipx_n - iqx_m}. \tag{A5}$$

It is instructive to rewrite the double sum of (A5) as follows

$$\sum_{m\neq n} \epsilon_n \epsilon_m e^{ipx_n - iqx_m} = \sum_{m\neq n} e^{ipx_n - iqx_m + i\pi g_{m,n}}, \tag{A6}$$

with

$$g_{m,n} \equiv \frac{1}{2}(1 - \epsilon_n \epsilon_m) \in \{0, 1\}. \tag{A7}$$

Hence, the double sum (A6) is a sum of pure random phases, even for the case $p \to q$. For $p = q = 0$, the norm of this double sum is of order $\sqrt{N}$, and the same holds for generic values of p and q. The norm of the single sum of (A5) has a maximum value N for $p = q$.

Let us briefly recapitulate. As discussed in Sec. III A, the charges of individual defects are chosen randomly and the average charge approaches zero in the limit of an infinite number of defects ($N \to \infty$). The random distribution of defect charges ϵ_n also makes that the double sum on the right-hand side of (A5) scatters around zero for a large number of defects ($N \gg 1$), with a spread proportional to $\sqrt{N}$. Note that the single sum on the right-hand side of (A5) is independent of the charge distribution, being proportional to $\epsilon_n^2 = 1$, and takes the value N for $p = q$. For further discussion and a numerical example, see Sec. IV of Ref. [9].

With the double sum on the right-hand side of (A5) being subdominant, we can approximate the product (A4) by

$$J(p)J^\star(q) \approx \sum_n e^{ix_n(p-q)}, \tag{A8}$$

where the sum approaches the value N as $p \to q$. If we now take into account that the points x_n are randomly distributed by a Poisson sprinkling process, there is only a significant contribution to (A8) if $p \approx q$. This allows us to replace the sum (A8) by a delta function, which becomes exact under the identification (4.4). We, finally, have

$$J(p)J^\star(q) \approx \rho_d \int d^4z e^{iz(p-q)} = (2\pi)^4 \rho_d \delta^{(4)}(p-q), \tag{A9}$$

which simplifies the original double integral (A1).

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