

Higgsploding universeValentin V. Khoze^{*} and Michael Spannowsky[†]*Institute for Particle Physics Phenomenology, Department of Physics, Durham University,
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Higgspllosion is a dynamical mechanism that introduces an exponential suppression of quantum fluctuations beyond the Higgspllosion energy scale E_* and further guarantees perturbative unitarity in multi-Higgs production processes. By calculating the Higgspllosion scale for spin 0, 1/2, 1 and 2 particles at leading order, we argue that Higgspllosion regulates all n -point functions, thereby embedding the standard model of particle physics and its extensions into an asymptotically safe theory. There are no Landau poles and the Higgs self-coupling stays positive. Asymptotic safety is of particular interest for theories of particle physics that include quantum gravity. We argue that in a Higgsploding theory one cannot probe shorter and shorter length scales by increasing the energy of the collision beyond the Higgspllosion energy and there is a minimal length set by $r_* \sim 1/E_*$ that can be probed. We further show that Higgspllosion is consistent and not in conflict with models of inflation and the existence of axions. There is also a possibility of testing Higgspllosion experimentally at future high energy experiments.

DOI: [10.1103/PhysRevD.96.075042](https://doi.org/10.1103/PhysRevD.96.075042)**I. INTRODUCTION**

Higgspllosion and Higgsplersion [1] are two intertwined mechanisms which aim to explain why physical scattering processes involving many scalars are not bound to violate unitarity, while solving at the same time the hierarchy problem of the Higgs boson.

It was shown in several independent calculations at leading order [2–6], one-loop resummed [5,7,8] or using a semiclassical approach [9–11], that the rate for a transition $h^* \rightarrow n \times h$ grows factorially with n . Based on these results it was expected that either the cross section of physical processes, e.g. in proton collisions of $pp \rightarrow n \times h$, has to grow as well, thereby violating perturbative unitarity in tree-level processes above certain critical values of collision energy and multiplicity and pointing to a validity limit of the standard model of particle physics, or that a nontrivial energy-dependent form factor had to emerge in a strongly coupled perturbation theory or even nonperturbatively. A second shortcoming of the standard model, identified early on, is that the Higgs boson is known to receive quadratic contributions from quantum corrections $m_h^2 \simeq m_0^2 + \delta m_{\text{new}}^2$. Thus, in order to obtain the physical mass $m_h \simeq 125$ GeV, the bare parameter of the theory m_0 has to be increasingly precisely tuned, depending on the hierarchy between the electroweak and new physics scale. With the discovery of an elementary scalar particle [12,13], both of these problems cannot be ignored if the standard model is not to be fundamentally modified at energy scales of $\mathcal{O}(100)$ TeV, i.e., energy scales possibly within reach of a future hadron collider [14].

In [1] we argued that the factorial growth of the amplitude $h^* \rightarrow n \times h$ or $X \rightarrow n \times h$ beyond a Higgspllosion scale E_* can be directly related to the exponential growth of the imaginary part of the width of h and X using the optical theorem, thus avoiding violation of perturbative unitarity in the production of many Higgs bosons and suppressing high-scale contributions of X to the mass of the Higgs boson. Hence, the Higgs boson can provide a self-consistent solutions to the problems it introduced to the standard model.

Assuming Higgspllosion and Higgsplersion were realized in nature, the question this paper aims to address is what are their phenomenological consequences and in how far is it possible to address and solve other fundamental questions in high-energy physics, beyond providing a solution [1] to the hierarchy or fine-tuning problem of the Higgs boson in the standard model. We will describe the scope of Higgspllosion, pointing out that not only the Higgs boson and heavy resonances that can decay into many Higgs bosons higgsplode at a scale E_* , but the same fate can affect all particles, in particular all standard model particles. We further show that Higgspllosion can provide a self-consistent picture of nature up to very high energy scales and a rich phenomenology with direct implications to high-scale physics.

We will discuss the quantum field theoretical consequences of Higgspllosion and Higgsplersion in Sec. II. By explicitly calculating the effect of Higgspllosion based on leading-order amplitudes for the Higgs boson (spin-0), the top quark (spin-1/2), a vector boson (spin-1) and the graviton (spin-2) we show that all standard model particles higgsplode and when this happens, all n -point functions are rendered finite. Further, not only high loop momenta are suppressed but in general all processes with propagators with high p^2 , leaving in collisions at very high energies

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only t -channel processes while rendering s -channel processes negligible. We also argue that in the higgsplosion theories one cannot probe distances shorter than a certain minimal scale r_* that is set by the inverse energy of Higgsplosion E_* . Above this energy, the theory screens the processes that attempt to probe shorter length scales. This is the effect of Higgspersion. In parallel we argue that higgsplosion theories are asymptotically safe: all coupling constants reach finite values in the UV regime above E_* , there are no Landau poles and the Higgs self-coupling λ remains positive. In Sec. III we outline the consequences of Sec. II on some aspects of particle phenomenology. We discuss the running of the coupling constants and show that Higgsplosion can provide an embedding of the standard model into an asymptotically-safe framework. We also explain that Higgsplosion is consistent and not in conflict neither with inflation nor with the existence of axions. In Sec. IV we offer a brief summary and conclusions.

II. QUANTUM FIELD THEORY IN PRESENCE OF HIGGSPLOSION

A. The interplay between Higgsplosion and Higgspersion in physical processes

Higgsplosion [1] denotes the rapid decay of a heavy or highly energetic resonance X into multiple Higgs bosons. The initial state X can in the simplest case be the Higgs boson itself, which we denote as h^* to emphasise that it is a highly virtual state with the momentum $p^2 \gg M_h^2$, or equivalently it can be another standard model degree of freedom with the energy sufficiently high, $\sqrt{p^2} = \sqrt{s} > nM_h$, so that its decay into final states involving a large number $n \gtrsim 100$ of Higgs bosons is kinematically possible. Importantly, the state X can also represent a very heavy new physics state with the mass $M_X \gg nM_h$, which can then decay into multiple Higgs bosons already at momentum scales $\sqrt{p^2} = \sqrt{s} > nM_h$ that are below its mass shell $\ll M_X$. The latter case would correspond to a heavy state which potentially decays much before it can be realized as an on-shell particle state, and this setup is important for addressing the hierarchy problem by taming the radiative corrections from the virtual loops of X contributing to the Higgs mass parameter, see Ref. [1] and Sec. II C below.

In several independent calculations at leading order [2–6], one-loop resummed [5,7,8] or using a semiclassical approach [9–11], it was shown that the rate for a transition $h^* \rightarrow n \times h$ grows factorially with n , where the state's total and partial widths are respectively given by

$$\begin{aligned} \Gamma(p^2) &= \sum_{n=2}^{\infty} \Gamma_n(p^2), \\ \Gamma_n(p^2) &= \frac{1}{2M_h} \int \frac{1}{n!} d\Pi_n |\mathcal{M}(h^* \rightarrow n \times h)|^2 \end{aligned} \quad (2.1)$$

with $|\mathcal{M}|^2$ being the scattering amplitude squared which is integrated over the n -particle phase space $d\Pi_n$ including the bosonic symmetry factor $\frac{1}{n!}$, and M is the mass. With the recent calculation of [11] the parametric dependence of the Higgs boson's partial width on the number of final state Higgs bosons n , their average kinetic energy $\varepsilon = (\sqrt{p^2} - nM_h)/(nM_h)$ and the Higgs self coupling λ can be expressed by

$$\begin{aligned} \Gamma_n(p^2) &\propto \mathcal{R}(\lambda; n, \varepsilon) \\ &= \exp \left[\frac{\lambda n}{\lambda} \left(\log \frac{\lambda n}{4} + 3.02 \sqrt{\frac{\lambda n}{4\pi}} - 1 \right) \right. \\ &\quad \left. + \frac{3}{2} \left(\log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} \varepsilon \right]. \end{aligned} \quad (2.2)$$

Here we defined the dimensionless variable $\mathcal{R}(\lambda; n, \varepsilon)$ or for brevity \mathcal{R}_n which is the rescaled n -particle decay rate $\mathcal{R}_n(p^2) = \Gamma_n(p^2)/M_h = -\text{Im}\Sigma(p^2)/M_h^2$.

The expression (2.2) was derived in the combined weak-coupling–large multiplicity limit, $\lambda \rightarrow 0$, $n \rightarrow \infty$, in the regime where the final state particles are assumed to be nonrelativistic, $\varepsilon \ll 1$, and the effective coupling parameter λn is large, $\lambda n \gg 1$. The characteristic exponential factor in (2.2) has a semiclassical origin and it was argued in [5,9] that it is not affected by the choice of X in the initial state in so far as X is coupled to Higgs bosons.

Using the optical theorem, the decay width of X can be directly related to the imaginary part of its self-energy, which results in a rapidly growing imaginary part of the denominator of its dressed propagator

$$\Delta_X(p) = \frac{i}{p^2 - M_X^2 - i\text{Im}\Sigma(p^2)} = \frac{i}{p^2 - M_X^2 + iM_X\Gamma(p^2)}. \quad (2.3)$$

Due to the Higgsplosion effect it was believed for a long time that the cross-section of the physical processes, such as the gluon fusion $gg \rightarrow n \times h$ going through an intermediate virtual Higgs boson(s) produced in the s -channel, $gg \rightarrow h^* \rightarrow n \times h$, was bound to grow factorially with n and would violate perturbative unitarity at energy scales of $\mathcal{O}(100)$ TeV for $n\lambda \gg 1$ (unless large quantum corrections at strong effective coupling $n\lambda \gg 1$ or nonperturbative physics would introduce a nontrivial suppression factor).

However, including the dressed propagators for intermediate h^* in this process results in Higgspersion, i.e. a well-behaved cross section for arbitrary n up to very high energies [1]

$$\sigma_{gg \rightarrow n \times h}^{\Delta} \sim y_t^2 m_t^2 \log^4 \left(\frac{m_t}{\sqrt{p^2}} \right) \times \frac{1}{p^4 + M_h^4 \mathcal{R}^2} \times \mathcal{R}_n, \quad (2.4)$$

and thus

$$\sigma_{gg \rightarrow n \times h} \sim \begin{cases} \mathcal{R}: & \text{for } \mathcal{R} \lesssim 1 \\ 1/\mathcal{R} \rightarrow 0: & \text{for } \mathcal{R} \gg 1 \quad \text{at } p^2 \rightarrow \infty. \end{cases} \quad (2.5)$$

Hence, by avoiding a breakdown of perturbative unitarity in multiboson production, the theory can retain consistency and predictivity to much higher, technically even unlimited, energy scales.

Let us summarize what we mean by Higgsplosion:

- (1) Higgsplosion is triggered by the rapid exponential growth of the n -particle decay rate $\Gamma_n \propto \mathcal{R}_n \rightarrow \infty$ with n and $\sqrt{s} \sim n$, as in (2.2). The Higgsplosion energy E_* is where the rate changes from $\mathcal{R}_n \ll 1$ to $\mathcal{R}_n \gg 1$. It is a new dynamically generated non-perturbative scale in the theory, $E_* = \text{const}_* M/\lambda$, where M_h and λ are the mass and the coupling of the elementary scalars—the Higgs bosons produced in the final state, and const_* is the calculable constant factor which is typically $\gg 1$.
- (2) The initial building blocks of Higgsplosion are the Dyson-resummed propagators for all degrees of freedom X in the theory, given by (2.3). They are obtained by summing up the geometric progression for 2-point functions over all self-energy insertions. The self-energy $\Sigma_X(p^2)$ is a function of the momentum of the propagator and it is represented by its large imaginary part due to Higgsplosion.¹ Because of the rapid increase of $\text{Im}\Sigma_X(p^2)$ the propagators in (2.3) vanish for momenta exceeding the scale $p^2 > E_*^2$. This is the result of highly energetic modes becoming unstable against the multiparticle decays, and these modes at momentum scales above E_* lose their particle interpretation. The correct interpretation of the vanishing propagators is that the highly energetic field theoretical degrees of freedom become composite states made out of large numbers of relatively soft Higgs bosons and are no longer described by an individually propagating particle degree of freedom. The dynamical scale of compositeness or classicalization, here an interaction and virtuality dependent experimentally unresolvable minimal length scale, is for each particle i set by $E_*^i \sim 1/r_i$.
- (3) Technically, $\text{Im}\Sigma_X(p^2)$ entering the expressions for Dyson-resummed propagators (2.3), is obtained from the decay rates Γ_n using the optical theorem, and hence is determined in the physical domain $p^2 = s > 0$. However the underlying concept of compositeness should not depend on whether the momentum scale at which we probe the particle is

¹Below, in Sec. II B we will explain the rationale why $\text{Im}\Sigma_X(p^2)$ is expected to higgsplode, based on perturbative calculations at leading order.

time-like or space-like. If this is true that a given degree of freedom becomes composite in the physical domain of positive $p^2 > E_*^2$, the same should equally apply at space-like momenta, and only $|p^2| > E_*^2$ matters. Hence in constructing the formalism for computing quantum (loop) corrections in higgsplooding theories we will describe the non-perturbative effect of compositeness caused by Higgsplosion in terms of the propagators

$$\Delta_X(p) = \frac{i}{p^2 - M_X^2} K(p^2/E_*^2), \quad (2.6)$$

where $K(x)$ is the nonperturbative Higgsplosion (or more precisely Higgspersion) formfactor,

$$K(x) = \begin{cases} 1: & \text{for } x < 1 \\ 0: & \text{for } x > 1. \end{cases} \quad (2.7)$$

In the physical domain and for $0 < p^2 \leq E_*^2$ there is little difference between the propagators (2.3) and (2.7). Due to the sharp exponential rise of the decay rate \mathcal{R} with energy in (2.2), both propagators are sharply cut off at or just above $p^2 = E_*^2$ and are vanishing above this value. At $p^2 < E_*^2$, the multiparticle contribution to the decay rate is exponentially suppressed, to a very high accuracy it is zero, and both expressions are correctly described by the bare propagators. (One can always add the additional non-higgsplooding width effects to the denominators of both expressions if required.)

- (4) The dressed propagators (2.6) are then used as the *input* into the computation of n -point functions with $n \geq 3$. All quantum contributions to the n -point vertices are obtained in our formalism by computing loop effects that involve these dressed propagators and ordinary renormalized vertices. This is done order by order in the loop expansion, where the internal lines in the loops are given by the dressed propagators in the form (2.6) treated as the input. Integrations over the loop momenta can now also be carried out in Euclidean space if desired.
- (5) It then follows from this formalism that the theory is made UV-finite by Higgsplosion and the couplings are asymptotically safe, as will be explained in section II.D. There are no Landau poles and the Higgs self-coupling cannot become negative and hence the electroweak vacuum is stable.
- (6) Finally it is also easy to check [1], using the same method, that the real part of the self-energy is also UV-finite and that even the finite fine-tuning of the quantum corrections to Higgs mass parameter from integrating out a very heavy X state, $M_X \gg E_*$, is reduced by many orders of magnitude from $\delta M_h^2 \sim M_X^2$ to $\delta M_h^2 \sim E_*^4/M_X^2$. This solves the hierarchy problem.

B. Higgsplosion of the self-energy

The Higgsplosion effect becomes operative when the imaginary part of the self-energy $\Sigma_X(p^2)$ for a given field theoretical degree of freedom X becomes exponentially large, i.e. when the external momentum p approaches the critical energy scale E_* of Higgsplosion. Specifically,

$$\text{Im}\Sigma_X(p^2) \sim \mathcal{R}_X, \quad \text{where} \quad \begin{cases} \mathcal{R}_X \ll 1: & \text{for } p^2 < E_*^2 \\ \mathcal{R}_X \gg 1: & \text{for } p^2 \gtrsim E_*^2. \end{cases} \quad (2.8)$$

The value of the Higgsplosion scale E_* for X in general depends on the nature and strength of interactions between X and the Higgs bosons.

To study the implications and extend of Higgsplosion, we will now consider different choices for X , first by taking it to be the Higgs boson itself; second another light degree of freedom (for example the top quark (spin-1/2), a vector boson (spin-1) or a graviton (spin-2)); and finally a heavy degree of freedom with the mass much greater than the electro-weak scale and possibly unstable to decay into multiple Higgses.

1. Higgsplosion in the self-energy of the Higgs boson

We first take X to be the Higgs field itself and recall the rationale for the Higgsplosion of $\text{Im}\Sigma_h(p^2)$ at $p^2 = E_*^2$. The main point we want to emphasise here is that the dominant contribution to the higgsplooding self-energy, or equivalently, the multiparticle decay rate Γ_n , comes from summing over the interference terms between different diagrammatic contributions to the amplitudes. In particular, each amplitude with n Higgs bosons in the final state

contains of the order of $n!$ terms. The decay rate or the imaginary part of Σ arises from squaring the amplitude, then dividing by the symmetry factor of $n!$ and integrating over the phase space. This implies that there are $\sim n! \times n! \times \frac{1}{n!}$ terms. This results in $\Gamma_n \sim n!$. Clearly, this factorial growth of the rate is entirely due to the interference terms (i.e. all the cross terms) in the product of two amplitudes. If, on the other hand, one would decide to neglect all the cross terms in the product of two amplitudes, each of which contains $n!$ terms, $\mathcal{A}_n \sim n!$, one would get the total of only a single factor of $n!$ which is then cancelled by the $1/n!$ symmetry factor. In other words, schematically we have,

$$\begin{aligned} \text{Im}\Sigma_n &\sim \frac{1}{n!} (\mathcal{A}_n)^2 \\ &\sim \begin{cases} \frac{1}{n!} \times n! \times n! \sim n!: & \text{all terms included} \\ \frac{1}{n!} \times n! \sim 1: & \text{no interference terms.} \end{cases} \end{aligned} \quad (2.9)$$

We thus are led to a retrospectively obvious conclusion that Higgsplosion is a result of taking into account all interference effects between individual diagrams. These diagrams are sketched in Fig. 1 and correspond to the sum of all possible combinations of n_1 , n_2 , \tilde{n}_1 and \tilde{n}_2 , where $n_1 + n_2 = \tilde{n}_1 + \tilde{n}_2 = n$. This is to be compared with the diagrams depicted in Fig. 2, where the cross terms between the \mathcal{A}_{n_1} and \mathcal{A}_{n_2} subamplitudes on the left and on the right of the cut were not included. As a result, the diagrams in Fig. 2 are subleading relative to those in Fig. 1, and do not lead to Higgsplosion.

What does lead to Higgsplosion is the correct accounting of the interference effects in the product of the two

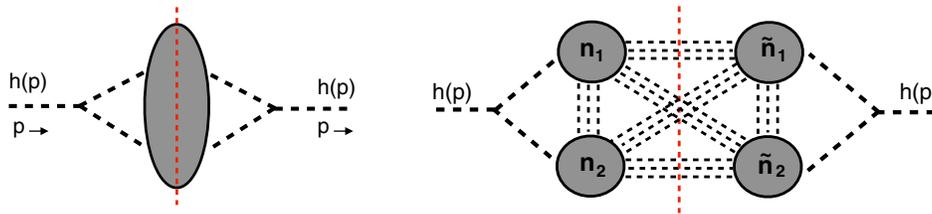


FIG. 1. Dominant contributions to the self-energy of the Higgs boson from interference terms between subamplitudes for all possible combinations of n_1 , n_2 , \tilde{n}_1 and \tilde{n}_2 , where $n_1 + n_2 = \tilde{n}_1 + \tilde{n}_2 = n$. Such diagrams contain only multiparticle cuts in the “ t -channel.”

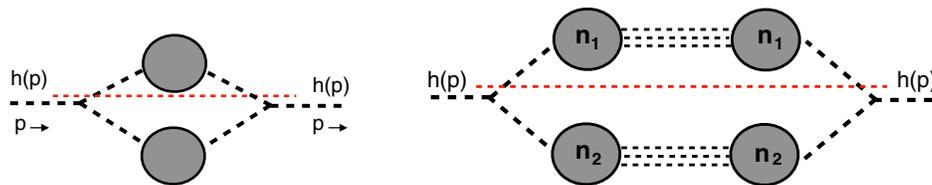


FIG. 2. Subleading contributions to $\Sigma_h(p^2)$ with no interference between the sub-amplitudes. These non-interference diagrams can be cleanly separated into mutually independent dressed propagators in the loop. These diagrams are 2-particle reducible in the “ t -channel.”

amplitudes. For reader's convenience and for future reference we will now also present a more technical rendering of the above $n!$ -counting argument for Higgsplosion from interference, based on the technique of generating n -point amplitudes from classical solutions. (Readers already familiar with this argument can directly skip to the next section II B 2.)

A more technical argument: $\mathcal{A}_{h^ \rightarrow n \times h}$ from classical solutions.*—At tree-level, all n -point scattering amplitudes for an off-shell field h^* to produce n Higgs particles,

$$h^* \rightarrow n \times h, \quad (2.10)$$

can be obtained from solving the Euler-Lagrange equations and following the generating functions formalism of Brown [2]. For simplicity, as in Ref. [1], we will assume a simplified model description of the Higgs boson in terms of a single real scalar field h with the VEV v and the self-coupling λ ,

$$\mathcal{L} = \frac{1}{2} \partial^\mu h \partial_\mu h - \frac{\lambda}{4} (h^2 - v^2)^2. \quad (2.11)$$

According to [2], since the final state in (2.10) contains only the outgoing particles, the solution $h_{\text{cl}}(x)$ relevant to the problem at hand should contain only the positive frequency modes, $e^{+inM_h t}$ where $M_h = \sqrt{2\lambda}v$ is the Higgs boson mass. This specifies the initial conditions, or equivalently the analytic structure of the solution—its time-dependence is described by the complex variable z ,

$$z(t) = z_0 e^{iM_h t}, \quad M_h = \sqrt{2\lambda}v \quad (2.12)$$

on which the configuration h_{cl} depends holomorphically, so that there is no dependence on the complex-conjugate variable z^* ,

$$h_{\text{cl}}(\vec{x}, t) = v + \sum_{n=1}^{\infty} d_n(\vec{x}) z(t)^n, \quad (2.13)$$

and $d_n(\vec{x})$ are the coefficients of the Taylor expansion in powers of z . Next we consider the simplest kinematical configuration, where all the final state particles are produced at their mass threshold (i.e. with vanishing spacial momenta). In this case, the classical solution in question, h_{cl} , is uniform in space and the Euler-Lagrange equation becomes an ordinary differential equation,

$$d_t^2 h = -\lambda h^3 + \lambda v^2 h, \quad (2.14)$$

with the initial conditions, $h_{\text{cl}} = v + z + \mathcal{O}(z^2)$. The coefficients d_n of the Taylor expansion of the classical solution now become space-independent with $d_1 = 1$ and the

solution is uniquely specified. Its analytic form is remarkably simple [2],

$$h_{\text{cl}}(t) = v \frac{1 + \frac{z(t)}{2v}}{1 - \frac{z(t)}{2v}}, \quad (2.15)$$

and its Taylor expansion reads,

$$h_{\text{cl}}(t) = v + z + \sum_{n=2}^{\infty} d_n z^n, \quad (2.16)$$

with $d_n = (2v)^{1-n}$,
for $n = 1, \dots, \infty$.

The presence of the singularity of (2.15) at $z = 2v$ is the consequence of the finite radius of convergence of the Taylor expansion (2.15). The classical solution h_{cl} defines the generating functional for the tree-level scattering amplitudes. All n -point tree-level amplitudes at threshold are found by differentiating h_{cl} with respect to z , [2]

$$\mathcal{A}_{h^* \rightarrow n \times h} = \left(\frac{\partial}{\partial z} \right)^n h_{\text{cl}} \Big|_{z=0} = n! d_n = n! (2v)^{1-n}. \quad (2.17)$$

The expression (2.17) is an exact result and it makes it clear that the $1^* \rightarrow n$ -point amplitudes evaluated on the n -particle mass thresholds grow factorially with the number of particles in the final state. The $n!$ behavior is the consequence of coherently adding contributions from the order- $n!$ of Feynman diagrams contributing to these amplitudes.

A remarkable fact that plays an important role in Higgsplosion is that the $n!$ growth of the n -point amplitudes (2.17) continues to persist in the more general kinematics when the external lines are taken off the mass threshold, and furthermore when the leading order quantum corrections from the resummed loops are taken into account. For the model (2.11) the result is

$$\mathcal{A}_{h^* \rightarrow n \times h}(p_1 \dots p_n) = n! (2v)^{1-n} \exp \left[-\frac{7}{6} n \varepsilon + \frac{\sqrt{3}}{8\pi} n \lambda n \right]. \quad (2.18)$$

The above expression is derived in the nonrelativistic limit, where ε denotes the kinetic energy per particle per mass in the final state, $\varepsilon = (E - nM_h)/(nM_h)$, and is taken to be small, $\varepsilon \ll 1$. Hence the first term in the exponent,

$$-\frac{7}{6} n \varepsilon(p_1 \dots p_n) = -\frac{71}{62} \frac{1}{M_h^2} \sum_{i=1}^n \vec{p}_i^2, \quad (2.19)$$

describes the amplitude dependence on the momenta of nonrelativistic particles in the final state; and the second

term is the resummed leading-order loop-level correction. The expression in the exponent (over n) would also contain higher-order corrections in ε and in higher powers of λn which we have neglected. The details of the derivation of (2.18) and an overview can be found in Ref. [6] and a selection of the earlier fundamental papers on n -point amplitudes in a scalar QFT is [2–5,7–9,15,16].

One can now proceed to square the amplitudes (2.18) and integrate them over the n -particle phase space in the non-relativistic approximation, as was done in e.g. [6,9], ultimately providing the foundation of the Higgspllosion phenomenon, as explained in Ref. [1] and analysed further in [11]. The main point for us here is that Higgspllosion is driven by the total of $n!$ interference terms in the product of the amplitudes, as signified by the upper line in (2.9).

These considerations imply that the effect of Higgspllosion does not arise from the diagrams with cleanly separated (i.e. mutually independent) dressed propagators in the loop. These are precisely the diagrams in Fig. 2 which neglect all the interference effects between the upper and the lower half. The Higgspllosion effect cannot be derived from diagrams containing only the dressed propagators connected by bare vertices, i.e. by working order by order in the loop expanded perturbation theory. What does generate the Higgspllosion are the fully interacting diagrams in Fig. 1, with no easily separable dressed propagators and the entire interaction represented by the left diagram in Fig. 1.

2. Higgspllosion in the self-energy of other light degrees of freedom

Let us now consider the self-energy of other standard model degrees of freedom. More generally, we assume that the field X has a mass much smaller than the Higgspllosion scale, it can for example be of the order of the electroweak scale, or even lighter, and that it interacts with the Higgs sector. Does the imaginary part of $\Sigma_X(p^2)$ become large and higgsplodes at some high critical energy scale E_* ?

For concreteness we first consider here the case of the top quark, $X = t$, but the same qualitative conclusion can immediately be drawn for all standard model particle (such as the electroweak vector bosons, gluons and fermions) as

well as other not-too-heavy BSM degrees of freedom coupled to the Higgs boson.

Higgspllosion in the self-energy of the top.—For the case of the t quark we concentrate on its self-energy $\Sigma_t(p^2)$ and consider the Yukawa interactions, $y_t \bar{t} t h$ as well as the Higgs self-interactions. A priori it may be tempting to organise perturbative contributions to $\Sigma_t(p^2)$ in terms of a loop assembled two or more mutually independent dressed propagators of the Higgs field and of the top quark, connected by bare vertices. The leading-order one-loop contribution involving the Higgs and the top dressed propagators is shown on the left in Fig. 3. This amplitude receives a contribution $\sim \mathcal{R}_n$ from Higgspllosion and two contributions $\sim 1/\mathcal{R}_n$ due to Higgspllosion and is thus negligible for $p^2 > E_*^2$.

However, following the discussion above, we know that such diagrams ignore the interference terms between the sub-amplitudes of the top and the bottom parts of the diagram. To achieve the Higgspllosion effect one should instead consider the more general diagram shown in Fig. 3 on the right, which accounts for the contributions of all cross-terms in the product of individual sub-amplitudes; in this sense they are similar to the Higgs self-energy contributions on Fig. 1. Pictorially there is certainly not much difference between the multiple Higgs emissions from the top quark internal line and the Higgs internal line, as Fig. 4 indicates. Hence it is entirely likely (and expected) that the imaginary part of the top quark self energy will also higgsplode in analogy with the pure Higgs case.

To demonstrate the connection between Higgspllosion for the top quark self-energy and the original Higgs field Higgspllosion, let us compute the tree-level amplitude $\mathcal{A}_{t^* \rightarrow n \times h + t}$ for the top-quark-initiated process depicted in the plot on the right in Fig. 4. We will first compute the generating functional of all such amplitudes on the multi-particle mass-threshold by solving the classical equation for the top quark field $\psi_{\text{cl}}(t)$

$$\left(i\gamma^0 d_0 - \frac{m_t}{v} h_{\text{cl}} \right) \psi_{\text{cl}} = 0, \quad (2.20)$$

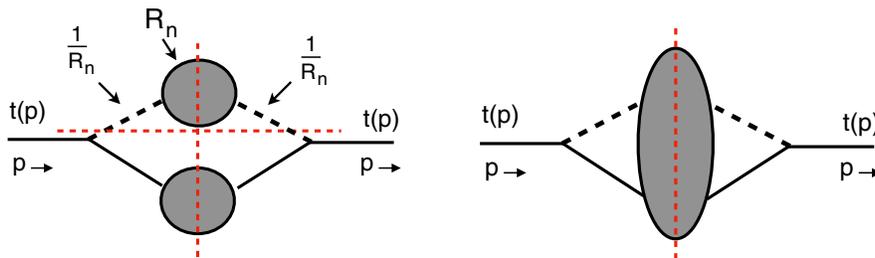


FIG. 3. Contributions to $\Sigma_t(p^2)$ from mutually independent dressed propagators in the loop. These sub-processes do not contribute to Higgspllosion and correspond to 2-particle reducible diagrams in the t -channel. On the right, dominant contributions to the self-energy of the top quark come from the interference terms between the sub-amplitudes. Such diagrams contain only multi-particle cuts in the t -channel.

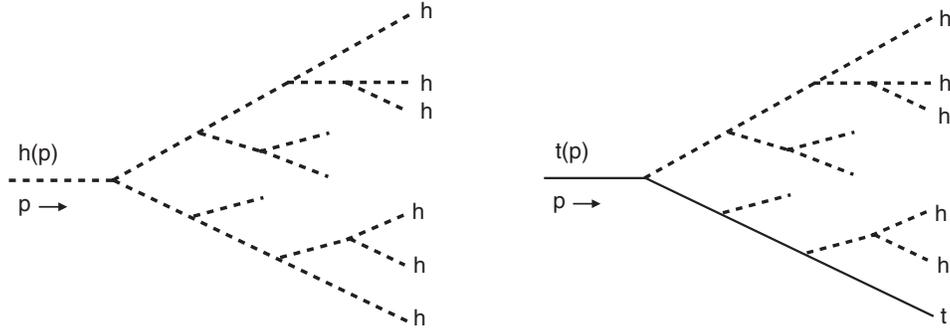


FIG. 4. Not much difference pictorially between emitting multiple Higgses from the top or from the Higgs internal line

in the background Higgs field $h_{\text{cl}}(t)$ given by the Brown's solution (2.15) that follows from the scalar field Lagrangian, ignoring the backreaction from the $t\bar{t}$ pairs ψ and $\bar{\psi}$ in

$$\mathcal{L} = \frac{1}{2} \partial^\mu h \partial_\mu h - \frac{\lambda}{4} (h^2 - v^2)^2 + \bar{\psi} \left(i\gamma^\mu d_\mu - \frac{m_t}{v} h \right) \psi. \quad (2.21)$$

The general procedure is to solve the coupled Euler-Lagrange equations for the Higgs field $h_{\text{cl}}(t)$ and for the top quark field $\psi_{\text{cl}}(t)$, both represented in the form of the double Taylor expansion in terms of the complex variables z and ξ^2 ,

$$h_{\text{cl}}(t) = v + \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} d_{n,2k} z^n \xi^{2k}, \quad (2.22)$$

with $d_{0,0} = 0, \quad d_{1,0} = 1,$

$$\psi_{\text{cl}}(t) = \xi \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} b_{n,2k} z^n \xi^{2k}, \quad \text{with } b_{0,0} = 1, \quad (2.23)$$

where

$$z(t) = z_0 e^{iM_h t}, \quad \text{and } \xi(t) = \xi_0 e^{im_t t}. \quad (2.24)$$

m_t is the top mass and the factors of ξ account for the production of top quarks. For example, in the fermionic generating function (2.23) the factors of $\xi \xi^{2k}$ correspond to a single top plus k additional $t\bar{t}$ pairs in the final state produced from the virtual incoming top quark. The amplitudes initiated by the virtual Higgs or by the virtual top are obtained by differentiating respectively $h_{\text{cl}}(z, \xi)$ or $\psi_{\text{cl}}(z, \xi)$ with respect to the variables z and ξ . The same idea was used previously in the gauge-Higgs system in Refs. [6,17] for computations of amplitudes containing Higgses and vector bosons in the final state. Here we are doing the same for the Higgs- $t\bar{t}$ system (2.21).

However for our present purpose, which is to account for the amplitudes with a *single* top plus multiple Higgses in the final state, cf. the plot on the right in Fig. 4, we can

neglect the effects of the additional $t\bar{t}$ pair production, by setting $k = 0$ in the sums in (2.22)–(2.23). Hence, as stated earlier, it is sufficient to solve the Dirac equation (2.20) in the background of the already determined Higgs solution (2.15) or (2.16),

$$h_{\text{cl}} = v \left(1 + 2 \sum_{n=1}^{\infty} (2v)^{-n} z^n \right), \quad (2.25)$$

and search for the top-quark solution in the form,

$$\psi_{\text{cl}} = \xi \left(1 + \sum_{n=1}^{\infty} b_n z^n \right). \quad (2.26)$$

The more general procedure of solving the coupled system in (2.22)–(2.23) can also be carried out, as in [17], but we will not pursue it further here.

We can now solve the classical equation (2.20) for the fermionic generating functional ψ_{cl} recursively and determine the Taylor expansion coefficients b_n in (2.26). To this end we first act on the equation (2.20) from the left by the operator $(i\gamma^0 d_0 + \frac{m_t}{v} h_{\text{cl}})$ to obtain,

$$\left(-d_t^2 - \left(\frac{m_t}{v} h_{\text{cl}} \right)^2 - i\gamma^0 \frac{m_t}{v} (d_t h_{\text{cl}}) \right) \psi_{\text{cl}} = 0. \quad (2.27)$$

Finally rewriting the last term in the brackets in the equation above with the help of the original equation (2.20), we obtain the equation for ψ_{cl} which contains no γ matrices

$$\left(-d_t^2 - \left(\frac{m_t}{v} h_{\text{cl}} \right)^2 + \frac{(d_t h_{\text{cl}})}{h_{\text{cl}}} d_t \right) \psi_{\text{cl}} = 0. \quad (2.28)$$

We now define the mass ratio parameter

$$\rho := m_t / M_h \approx 1.38, \quad (2.29)$$

and introduce the rescaled dimensionless variable $t = M_h t$, so that $z = z_0 e^{it}$ and $\xi = \xi_0 e^{i\rho t}$, and the dimensionless field variable $\phi_{\text{cl}}(z)$,

$$h_{\text{cl}} = v(1 + 2\phi_{\text{cl}}), \quad \text{where } \phi_{\text{cl}}(z) = \sum_{n=1}^{\infty} \left(\frac{z}{2v} \right)^n. \quad (2.30)$$

While the Taylor coefficients of the scalar field $\phi_{\text{cl}}(z)$ are already fixed $= 1/(2v)^n$ by the known Higgs solution in (2.30), the coefficients b_n defining the fermionic function $\psi_{\text{cl}}(z)$ in (2.26) are still to be determined by solving the Eq. (2.28) which takes the form,

$$\left(-d_t^2 - \rho^2(1 + 2\phi_{\text{cl}})^2 + \frac{(2d_t\phi_{\text{cl}})}{1 + 2\phi_{\text{cl}}}d_t\right) \sum_{n=0}^{\infty} b_n e^{i(\rho+n)t} = 0. \quad (2.31)$$

We solved this equation by iterations using *Mathematica*, first setting $b_0 = 1$ and determining all $b_{n>1}$. For the first 50 coefficients $b_n := \tilde{b}_n/(2v)^n$ [using the numerical value of the mass ratio given in (2.29)] we have:

$$\begin{aligned} \tilde{b}_n = \{ & 1, 2.75, 5.17, 8.19, 11.8, 15.9, 20.6, 25.7, 31.3, 37.4, 44., 51., 58.5, 66.4, 74.7, 83.4, 92.6, 102., 112., 122., 133., \\ & 144., 156., 168., 180., 192., 205., 219., 232., 247., 261., 276., 291., 306., 322., 338., 355., 371., 389., 406., 424., \\ & 442., 460., 479., 498., 518., 538., 558., 578., 599., 620., 641., \dots \} \end{aligned} \quad (2.32)$$

The tree-level scattering amplitudes (or more precisely, the currents) for the process $t^* \rightarrow n \times h + t$ are then given by

$$\mathcal{A}_{t^* \rightarrow n \times h + t} = \frac{\partial}{\partial \xi} \left(\frac{\partial}{\partial z} \right)^n \psi_{\text{cl}} \Big|_{z=0} = n! b_n = n! \tilde{b}_n / (2v)^n, \quad (2.33)$$

with the coefficients \tilde{b}_n given in (2.32). Restoring the kinematic dependence due to n nonrelativists momenta of the Higgs bosons in the final state, as in (2.18), we have

$$\mathcal{A}_{t^* \rightarrow n \times h + t}(p_1, \dots, p_{n+1}) = n! \tilde{b}_n (2v)^{-n} e^{-(7/6)n\epsilon}. \quad (2.34)$$

This amplitude retains the factorial growth with the number of the Higgs bosons in the final state and in view of the coefficient values in (2.18) with $\tilde{b}_n > 1$, the amplitude initiated by the top-quark line is not inferior to the n -point amplitude for the pure Higgs production process in (2.18),

$$\mathcal{A}_{n^* \rightarrow n \times h}(p_1, \dots, p_n) = n! (2v)^{1-n} e^{-(7/6)n\epsilon}. \quad (2.35)$$

Based on these tree-level considerations, and admittedly not having attempted to add and re-sum higher order quantum corrections involving top quark loops, we conclude that the Higgsplosion of the top quark self-energy $\text{Im}\Sigma_t(p^2)$ is as likely as the Higgsplosion of the Higgs boson $\text{Im}\Sigma_h(p^2)$ at $p^2 = E_*^2$.

Higgsplosion in the self-energy of vector bosons.—As another example of higgsplooding the SM degrees of freedom, one can consider the amplitudes involving the Higgs as well as the weak-sector massive vector bosons. Tree-level amplitudes for multiple Higgs bosons and longitudinal components of W 's and Z 's were already considered in Refs. [6,17] on and off the multiparticle thresholds. The formalism is very similar and involves solving the

time-dependent classical equations for the gauge-Higgs system,

$$-d_t^2 h = \lambda h^3 - \lambda v^2 h + \frac{g^2}{4} (A_L^a)^2 h, \quad (2.36)$$

$$-d_t^2 A_L^a = \frac{g^2}{4} h^2 A_L^a. \quad (2.37)$$

The classical solutions for the Higgs field, h_{cl} , and for the longitudinal components of the vector boson fields $A_{L\text{cl}}^a$ are represented as double Taylor expansions in terms of the z and the w^a variables,

$$z(t) = z_0 e^{iM_V t}, \quad \text{and} \quad w^a(t) = w_0^a e^{iM_V t}, \quad (2.38)$$

where M_V is the vector bosons mass.

$$h_{\text{cl}}(z, w^a) = v + 2v \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} d_{n,2k} \left(\frac{z}{2v} \right)^n \left(\frac{w^a w^a}{(2v)^2} \right)^k, \quad (2.39)$$

$$A_{L\text{cl}}^a(z, w^a) = w^a \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} a_{n,2k} \left(\frac{z}{2v} \right)^n \left(\frac{w^a w^a}{(2v)^2} \right)^k, \quad (2.40)$$

with the lowest-order Taylor coefficients $d_{0,0} = 0$ and $a_{0,0} = 1$.

The amplitudes involving vector bosons and Higgs bosons in the final state on the multiparticle thresholds are given by the following expressions in terms of the Taylor expansion coefficients $d_{n,2k}$ and $a_{n,2k}$,

$$\mathcal{A}_{h^* \rightarrow n \times h + m \times Z_L} = (2v)^{1-n-m} n! m! d_{n,m}, \quad (2.41)$$

and for the longitudinal Z decaying into n Higgses and $m + 1$ vector bosons we have,

$$\mathcal{A}_{Z_L^* \rightarrow n \times h + (m+1) \times Z_L} = \frac{1}{(2v)^{n+m}} n!(m+1)! a_{n,m}, \quad (2.42)$$

and obtained by differentiating the classical generating functions in (2.39) and (2.40) with respect to the variables z and w^a where $a = 1, 2, 3$ is the isospin index.

The coefficients $d_{n,m}$ and $a_{n,m}$ were computed in Ref. [17] by solving the classical equations above for the given ratio of the vector to the Higgs boson masses,

$$a_n = \{1, 0.718, 0.678, 0.652, 0.633, 0.617, 0.605, 0.594, 0.585, 0.577, 0.57, 0.564, 0.558, 0.553, 0.548, 0.543, 0.539, 0.535, 0.531, 0.528, 0.524, 0.521, 0.518, 0.516, 0.513, 0.51, 0.508, 0.505, 0.503, 0.501, 0.499, 0.497, 0.495, 0.493, 0.491, 0.489, 0.488, 0.486, 0.484, 0.483, 0.48, 0.479, 0.477, 0.476, 0.474, 0.481, 0.48, 0.479, 0.477, 0.476, 0.474, 0.473, 0.472, 0.471, 0.469, 0.468, 0.467, 0.466, 0.465, 0.464, 0.463, \dots\} \quad (2.45)$$

Given the ~ 1 values of the coefficients in (2.45), the amplitudes (2.44) with a single vector boson line are numerically very similar to the pure Higgs amplitudes in (2.35). These considerations imply that the Higgspllosion should also occur in the self-energy of the vector boson propagator with the Higgspllosion scale of the same (or similar) magnitude as E_* in the pure Higgs case.

While the calculation of higher order quantum corrections to the tree-level amplitudes involving interactions with the top quark and the vector bosons have not been carried out so far, which could certainly change the scale where Higgspllosion occurs quantitatively, it is nevertheless a self-consistent assumption to conjecture that the self-energy will higgsplode for all light degrees of freedom coupled to the Higgs boson.

Higgspllosion in the self-energy of the graviton.—We start with the Lagrangian

$$\sqrt{-g}\mathcal{L} = \sqrt{-g} \left(-\frac{M_{\text{Pl}}^2}{2} R + \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{GF}} \right), \quad (2.46)$$

where the first term is the Einstein-Hilbert Lagrangian of gravity, the second term is the coupling of gravity to the Higgs field h , and the final term is the gauge-fixing term for gravity. R is the scalar curvature and M_{Pl} is the reduced Planck mass, which is often re-written as $M_{\text{Pl}} = 2/\kappa$, where $\kappa^2 = 32\pi G$ and G is the Newton's constant $G = (1.22 \times 10^{19} \text{ GeV})^{-1}$. The graviton field $g_{\mu\nu}$ is defined as the fluctuation of the metric tensor around the Minkowski space metric, $g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa\chi_{\mu\nu}(x)$ and we use the convention $\eta_{\mu\nu} = (+1, -1, -1, -1)$. We can now simplify this Lagrangian by retaining only the terms linear and quadratic in the graviton field $\chi_{\mu\nu}$, which takes the form (cf. e.g. the lecture notes [18]),

$$\kappa := \frac{g}{2\sqrt{2}\lambda} = \frac{M_V}{M_h} \approx 0.64. \quad (2.43)$$

In particular, for the simplest case of $m = 0$ of no vector boson pairs present in the final state, the amplitudes for the $Z_L^* \rightarrow n \times h + Z_L$ process are given by

$$\mathcal{A}_{Z_L^* \rightarrow n \times h + Z_L} = \frac{1}{(2v)^n} n! a_n e^{-(7/6)n}, \quad (2.44)$$

with the first 50+ coefficients $a_n = a_{n,0}$ given by

$$\mathcal{L} = \frac{1}{2} \chi_{\mu\nu} P^{\mu\nu\alpha\beta} \partial^2 \chi_{\alpha\beta} - \frac{\kappa}{2} \chi^{\mu\nu} T_{\mu\nu}. \quad (2.47)$$

Here

$$P^{\mu\nu\alpha\beta} = \frac{1}{2} (\eta^{\mu\alpha} \eta^{\nu\beta} + \eta^{\mu\beta} \eta^{\nu\alpha} - \eta^{\mu\nu} \eta^{\alpha\beta}), \quad (2.48)$$

and $T_{\mu\nu}$ is the energy-momentum tensor of the Higgs field,

$$T_{\mu\nu} = \partial_\mu h \partial_\nu h - \eta_{\mu\nu} \mathcal{L}(h) \quad (2.49)$$

where $\mathcal{L}(h)$ is our scalar-field Lagrangian (2.11). To describe the tree-level process where the incoming graviton decays into a multiparticle final state made entirely out of Higgs bosons (and no additional gravitons), $\chi_{\mu\nu} \rightarrow n \times h$, it is sufficient to solve the linearized equation for the graviton field in the background of the classical Higgs solution. The graviton equation reads [18],

$$\partial^2 P^{\mu\nu\alpha\beta} \chi_{\alpha\beta} = \frac{\kappa}{2} T^{\mu\nu}(h_{\text{cl}}). \quad (2.50)$$

Furthermore, when we restrict the particles in the final state to be on the multiparticle threshold, the equation becomes an ordinary differential equation with respect to time, $\partial^2 \rightarrow d_t^2$, and for the background Higgs field we can as before use the already known analytic expression (2.15).

There is an additional simplification, namely it is easy to check that when evaluated on the classical Higgs configuration (2.15), the energy-momentum tensor becomes,

$$T_{00} = \mathcal{H}(h_{\text{cl}}) = \frac{1}{2} (d_t h_{\text{cl}})^2 + \frac{\lambda}{4} (h_{\text{cl}}^2 - v^2)^2 = 0, \quad (2.51)$$

$$T_{ij} = \delta_j^i \mathcal{L}(h_{\text{cl}}) = -\delta_j^i \frac{\lambda}{2} (h_{\text{cl}}^2 - v^2)^2. \quad (2.52)$$

Hence we search for the solution of

$$d_{\tilde{\chi}_{\text{cl}}}^2 = -\frac{\kappa\lambda}{4}(h_{\text{cl}}^2 - v^2)^2 \quad (2.53)$$

where $\tilde{\chi}_{\text{cl}} := (P\chi_{\text{cl}})^{ii}(t)$ with no sum over i . We are looking for the solution of the form of the Taylor expansion in the variable $z = z_0 e^{iM_n t}$,

$$\tilde{\chi}_{\text{cl}}(z) = \frac{v^2}{M_{\text{Pl}}} \sum_{n=2}^{\infty} g_n \left(\frac{z}{v}\right)^n, \quad (2.54)$$

and the Taylor coefficients g_n can be determined analytically by solving (2.53) by iterations. Working in units of $v = 1$ we find,

$$g_n = \frac{1}{n^2} (h_{\text{cl}}^2 - 1)^2 \Big|_{z^n} = \frac{8n^2 - 1}{3 \cdot 2^n n}. \quad (2.55)$$

In summary, we have for the generating function of all graviton decay amplitudes into n Higgs bosons the following expression,

$$\tilde{\chi}_{\text{cl}}(z) = \frac{8}{3} \frac{v^2}{M_{\text{Pl}}} \sum_{n=2}^{\infty} \frac{n^2 - 1}{n} \left(\frac{z}{2v}\right)^n. \quad (2.56)$$

This is to be compared with the pure Higgs solution in (2.16). In terms of the h_{cl} Taylor coefficients $d_n = (2v)^{1-n}$ we now have the graviton Taylor coefficients (including now all dimensional factors),

$$g_n = \frac{v}{M_{\text{Pl}}} \frac{4n^2 - 1}{3n} d_n. \quad (2.57)$$

As the n -point amplitude is given by

$$\mathcal{A}_{\text{graviton} \rightarrow n \times h} = n! g_n \quad (2.58)$$

we see that the graviton decay rate \mathcal{R}_n will be suppressed by a relative constant factor of $\frac{v}{M_{\text{Pl}}}$ times n^2 . This suppression is by a constant, i.e. energy and n -independent factor, so it will not prevent Higgspllosion, but will result in the considerably higher Higgspllosion scale E_* for the graviton relative to Higgses.

3. Heavy degrees of freedom

The behavior of the self-energy of the degrees of freedom with masses $M_X \gg E_*$ will depend on whether the field under consideration is stable or not, in other words whether the heavy X can decay into light degrees of freedom $X \rightarrow \text{Light} + nh$, or if it is required that $X \rightarrow X + nh$. One can consider the following possibilities:

- (1) For a heavy scalar field X which can decay into multiple Higgs bosons alone, $X \rightarrow nh$, the discussion follows Sec. 5 of [1]. The main point is that

there is no difference between X or h on the external lines of the self-energy. The result is that $\text{Im}\Sigma_X(p^2)$ higgsplodes.²

- (2) For a bosonic or fermionic X which decays as $X \rightarrow \text{Light} + nh$ the situation is very similar to the top quark self-energy. Here X can for example be a very heavy 10^{11-13} GeV sterile neutrino which first decays into a light neutrino and the Higgs, and subsequently into many Higgses. The self-energy of such X is also expected to higgsplode in direct analogy to the top quark (or other SM fermions).
- (3) For stable heavy degrees of freedom X the story is different because the remaining X in the decay process $X \rightarrow X + nh$ will carry away the momentum of the order of the mass of X , thus depleting the energy left for the Higgspllosion into multiple Higgs bosons. Thus Higgspllosion could occur only at energies $E_*^X \simeq M_X + E_*^{nh}$. If $M_X \gg E_*^{nh}$ such states will reintroduce a hierarchy problem for the Higgs boson, and hence should be avoided in model building.

C. Solving the hierarchy problem

Here we are interested in assessing the contributions to the Higgs self-energy $\Sigma_h(p^2)$ at low energy scales, of the order of the measured Higgs mass and much below the Higgspllosion scale, $p^2 \sim M_h^2 \ll E_*$. Clearly the multi-particle Higgs production relevant for Higgspllosion is impossible at such low scales and hence the imaginary part of $\Sigma_h(p^2)$ plays no role, we are interested predominantly in how big or small its real part is after the integration over the loop momenta.

The whole point and the origin of the hierarchy problem for the Higgs mass for such low values of the external momenta $p^2 \sim M_h^2$ of the $\text{Re}\Sigma_h(p^2)$ is that the super-heavy degrees of freedom propagating in the loops contributing to $\Sigma_h(p^2)$, do not decouple because of the UV divergencies in the integration over the loop momenta. Thus to address the hierarchy problem it is sufficient to concentrate only on the UV-divergent and/or M_X -sensitive contributions to the self-energy of the Higgs.

The UV-sensitive diagrams contributing to the self-energy at small external momenta are conceptually different from the higgsploding contributions which were UV-finite at tree level and computed at $p^2 = E_*^2 \gg M_h$. In the case at hand one needs to address the cases with the smallest numbers of propagators (and hence the vertices) in the loops—after all we are only after the UV-divergent contributions. In this case the correct procedure is the expansion of the self-energy in the number of loops, each with the minimal number of internal propagators. The new

²In fact, the exponential factor in (2.2) is the same for the incoming X in the process $X \rightarrow nh$ and for the virtual Higgs h^* in the process $h^* \rightarrow nh$.

input from the Higgsploding is that the internal propagators $\Delta_X(p)$ are the dressed propagators (2.6). It then immediately follows that all the UV divergencies and even the finite terms are cut-off by the values of the loop momenta approaching the Higgsploding scales of the relevant self-energy factors in the dressed propagators. For example, following [1] for the case of the heavy scalar X interacting with the Higgs sector via

$$\mathcal{L}_X = \frac{1}{2} \partial^\mu X \partial_\mu X - \frac{1}{2} M_X^2 X^2 - \frac{\lambda_P}{4} X^2 h^2 - \mu X h^2, \quad (2.59)$$

the 1-loop radiative correction to the Higgs mass parameter is UV-finite and $\ll M_X^2$,

$$\begin{aligned} \Delta M_h^2 &\sim \lambda_P \int \frac{d^4 p}{16\pi^4} \frac{1}{M_X^2 - p^2 + i\text{Im}\Sigma_X(p^2)} \\ &\propto \lambda_P \frac{E_*^2}{M_X^2} \ll \lambda_P M_X^2. \end{aligned} \quad (2.60)$$

In general the computation of the real part of $\Sigma_h(p^2)$ at $p^2 \simeq M_h^2$ proceeds as explained in Sec. 5 of our original Higgsploding paper [1]. We note that the diagrammatic technique employed now involves dressed propagators and bare vertices and is conceptually different to the diagrams in Fig. 2 that contributed to the Higgsploding of the self-energy discussed in the preceding sections. However, there is no contradiction. In addressing the Hierarchy we are working in a different regime, where the external momenta are much smaller than the Higgsploding scale and we are tracing what used to be the UV-divergent contributions that arose from integrations over the loop momenta. Such diagrams are correctly accounted by the loop diagrams with the minimal numbers of dressed propagators in the loops, and in presence of Higgsploding, the loop momenta are dynamically cut off at E_* . Hence there are no contributions to M_h^2 proportional to neither the Λ_{UV}^2 nor the $\sim M_X^2$ factors. The radiative corrections to the Higgs mass squared are cut off at the much lower scales set by E_* , as indicated in (2.60), thus solving the hierarchy problem.

D. UV finiteness of n -point functions and asymptotic safety

As we already noted in Sec. II A Higgsploding is triggered in a given QFT if and when the multi-particle decay widths Γ_n of all degrees of freedom become exponentially large above a certain dynamically generated nonperturbative scale E_* , and exhibit the behavior described by (2.8).

While a fully nonperturbative self-consistent formalism is currently lacking, in this paper we would like to advocate a simple diagrammatic approach for computing quantum effects in a higgsploding QFT based on a resummed perturbation theory. The two building blocks are (I) the

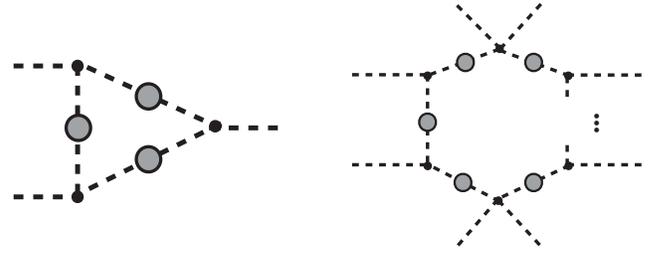


FIG. 5. One loop contributions to the three-point (left) and n -point (right) Green functions. The grey blobs represent dressed propagators and the black dots are the microscopic 3- and 4-point interaction vertices.

dressed propagators (2.6) that include the Higgspersion formfactor for all field theoretic degrees of freedom present in the problem, and (II) the bare vertices that are read directly from the microscopic Lagrangian. The renormalized vertices which depend on the RG scale μ are then obtained in the standard way from computing the n -point one-particle irreducible LSZ-amputated Green functions G_n . These computations are performed order by order in the loop expansion, with the only difference from the usual approach that one is required to use the dressed propagators (2.6) on all internal lines. The leading order one-loop contributions to the 3-point and the n -point vertices are shown in Fig. 5.

This way of computing quantum effects in a higgsploding QFT leads to a powerful conclusion that all momenta of virtual particles propagating in the loops are effectively cut off at the Higgsploding scale E_* . Integrations over the loop momenta are convergent, all the contributions to the n -point functions are UV finite and quantum fluctuations are damped above E_* .

There is an interesting parallel between this approach and Polchinski's implementation of the Wilson approach to renormalization [19,20] presented in Ref. [21] for a massive ϕ^4 theory. In the construction of [21] the UV cutoff is implemented by multiplying the propagators by a formfactor $K(p^2/\Lambda_0^2)$ which is equal to 1 for momenta $p^2 \leq \Lambda_0^2$ and rapidly vanishes for $p^2 > \Lambda_0^2$. What defines the theory with the (large) UV cutoff Λ_0 is the Lagrangian with the modified propagator and bare vertices. When the cutoff is lowered from Λ_0 to Λ_R , one is required to integrate out the high momentum components of the field. This is implemented by changing the form factor in the propagator to $K(p^2/\Lambda_R^2)$ and integrating out the modes with $p^2 > \Lambda_R^2$. This generates new effective interactions and expresses them in terms of the couplings at the scale Λ_R . The analogy of our method for computing the n -point functions with the approach of [21] is that the theory with a large UV cutoff is defined by the modified propagators and bare vertices. The momentum modes above the cutoff are switched off in both cases simply by the fact that the modified propagators vanish. In the case of Higgsploding, what we referred to as

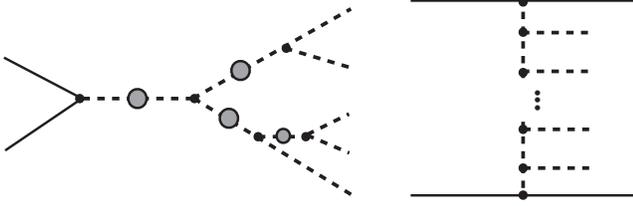


FIG. 6. s -channel (left) and t -channel (right) interaction diagrams.

the large UV cutoff is the dynamically generated Higgsplosion scale E_* , and the original propagators are modified by the self-energy $\Sigma(p^2)$ contributions (2.3). The theory with momenta above the Higgsplosion scale is the theory above the UV cutoff; its propagators vanish so it has no propagating degrees of freedom left, but its vertices are the usual bare vertices fixed at the scale E_* . There are no quantum fluctuations and no running above the scale E_* .

The coupling constants in a higgsplooding theory receive no quantum corrections from the modes above E_* . Hence the running couplings become flat for the values of the RG scale $\mu > E_*$. Below the Higgsplosion scale, $\mu < E_*$, the couplings exhibit the usual logarithmic running with μ , but above E_* , their beta functions vanish and the couplings stay constant with the values determined at E_* . This amounts to the asymptotically safe theory.

E. High-energy scattering with Higgspersion

Before concluding this section, let us pose the following question: is it true that due to Higgsplosion and Higgspersion, all scattering processes beyond the Higgsplosion scale become non-interacting? The couplings stay constant, while all scatterings cross-sections nevertheless vanish exponentially as dictated by (2.5). Is this a consistent picture?

The main point is whether or not the internal propagators appearing in the diagrams contributing to the high-energy scattering process involve momenta that can exceed the Higgsplosion scale E_* . For the high-energy processes dominated by the s -channel exchanges, such as the diagram on the left of Fig. 6, the presence of at least one propagator with $p^2 \geq E_*^2$ is unavoidable when the total $\sqrt{s} \geq E_*$. These are the processes we are considering in this paper and they indeed shut down beyond the Higgsplosion scale due to the Higgspersion of the dressed s -channel propagator, where $\Delta(p^2) \sim 1/\mathcal{R}(p^2) \rightarrow 0$. A physically intuitive meaning of this effect is that the propagating degree of freedom simply disappears from the spectrum, it is no longer an individual particle but a manifestation of multi-Higgs radiation.

On the other hand, not all of the contributing diagrams to high energy $\sqrt{s} \geq E_*$ scattering are of this form. The diagram on the right of Fig. 6 represents a t -channel process. The transverse momentum is not required to be

large or anywhere near the scale E_* . Hence the Higgsplosion/Higgspersion effects will be absent in such processes, and their contributions to the scattering processes will survive even at high energies.

In summary, Higgspersion shuts down the processes which probe shorter and shorter distances at higher and higher energy. This is not happening because the couplings vanish but because the internal propagators with momenta above E_* in fact turn off and disappear. The dynamically generated Higgsplosion scale sets the minimal length scale $r_* \sim 1/E_*$ that can be probed at any arbitrary high energy.³

In this sense Higgsplosion provides a dynamical realisation of the idea of classicalization [22,23], where the role of the classicalization radius is played by the Higgsplosion scale r_* . It also resounds the importance of multi-regge kinematics for high-energy scatterings as introduced by Balitsky, Fadin, Kuraev and Lipatov [24–27].

III. PHENOMENOLOGY AND THE EARLY UNIVERSE WITH HIGGSPLOSION

Many parts of models of the early Universe are relying on finite temperature effects and quantum fluctuations, all of which could receive corrections from the Higgsplosion mechanism, e.g. inflation [28–31], reheating [32], the cosmological microwave background [33,34], black hole formation [35–37] and the vacuum energy density during the evolution of the Universe [38]. As these phenomena are not entirely independent and are deserving of a detailed investigation in their own right, a full study of the early history of the Universe in presence of Higgsplosion is beyond the scope of this work. However, in this section we briefly comment on whether and how radically Higgsplosion would change the standard big bang model of the early Universe.

A. Higgsplosion and the running of gauge and gravity couplings

General relativity is inherently difficult to reconcile with the quantum field theoretical description of the standard model. While the quantum theory of the standard model is predictive to all orders in perturbation theory, loop corrections to gravity can only be taken into account order by order and have to be treated in the context of an effective field theory with expansion parameter E^2/M_{Pl}^2 . One way of addressing this problem is the concept of asymptotic

³In the analysis of Higgsplosion for the $gg \rightarrow h^* \rightarrow n \times h$ processes, it is the Higgspersion effect in the first Higgs propagator of the most energetic state h^* that turns off the cross section and restores unitarity. The effects of the Dyson resummation of subsequent intermediate Higgs propagators are irrelevant at energies just above the Higgsplosion threshold. This is because after each 2-point or 3-point splitting of the virtual Higgs state h^* , the energies/virtualities carried by the emerging propagators are 1/2 or 1/3 of the initial energy; they fall below the Higgsplosion scale, and their self-energy insertions are irrelevant.

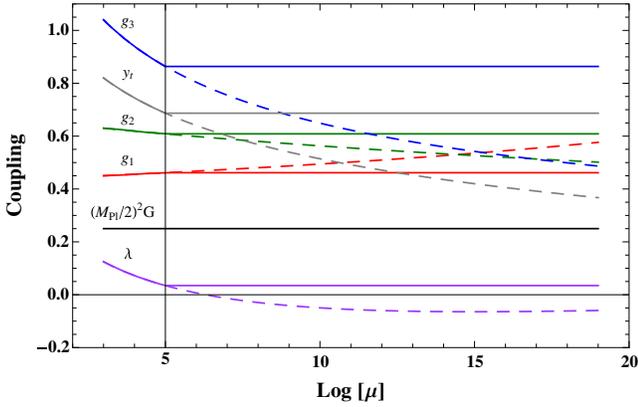


FIG. 7. Two-loop running of standard model couplings with Higgspllosion (solid lines) and without (dashed lines). We assumed the Higgspllosion scale to be uniformly at $E_* = 10^5$ GeV.

safety [39–44] which ensures that quantum field theories remain fundamental and predictive up to highest energies. This scenario indicates that a quantum theory of gravity can be renormalizable on a nonperturbative level, despite being perturbatively nonrenormalizable. In gravity asymptotic safety aims to provide a path-integral framework where the metric field is the carrier of the fundamental degrees of freedom in the classical and quantum regime of the theory. Thus, the quantum field theoretical description of gravity can be extended to infinitely large energy scales. A realisation of asymptotic safety requires that the beta-functions of all couplings g_i vanish at fix points g_i^* , i.e. $\beta(g_i^*) = 0$. The number of parameters g_i defines the dimensionality of the ultraviolet critical surface formed by all trajectories attracted to the fixed point.

In Fig. 7 we show the impact of Higgspllosion on the running of the Yukawa, scalar, gauge and gravity couplings. For the standard model couplings we use 2-loop running as implemented in SARAH [45,46], while for gravity we show the classical value normalized to $(M_{\text{Pl}}/2)^2$, as we do not want to speculate about the theory that governs quantum gravity in the UV. The couplings can have quantitatively different scales μ for $\beta(g_i) = 0$, depending on the masses, couplings and stability of the degrees of freedom that drive their running. For simplicity we chose $E_* = 10^5$ GeV for all couplings.

The whole SM exhibits a fix point with a finite-dimensional critical surface, as required for an asymptotically safe theory. Further, neither are there UV Landau poles associated with any of the gauge groups, Yukawa couplings or scalar interactions. In particular, if the Higgspllosion scale for all particles is below $\sim 10^6$ GeV, the Higgs potential remains stable on cosmological time scales.

While embedding the standard model into an asymptotically safe theory, i.e. a theory free of Landau poles and free of a hierarchy problem due to Higgs-gravity interactions, has been a challenge [47–49], within the

Higgspllosion framework, following arguments of Sec. II, this is automatically realized for the minimal standard model and for most of its proposed extensions. In the standard model without Higgspllosion one expects gravity to give rise to a fine-tuning problem when $\delta M_h^2 \gtrsim M_h^2$, where $\delta M_h^2 \sim l G_N \Lambda_G^4$ with $l \sim (4\pi)^{-4}$, i.e. around the scale $\Lambda_G \simeq 10^{11}$ GeV. However, with the graviton Higgspllosion scale E_*^G being much smaller than Λ_G , as calculated in Sec. II B 2, the gravitational contributions to the fine-tuning of the Higgs mass is cut off at $\delta M_h^2 \sim l G_N (E_*^G)^4 \ll M_h^2$. Hence, in the Higgspllosion scenario, there is no introduction of a hierarchy problem due to gravity and the need to construct a scenario that softens gravity in the UV is absent.

B. Higgspllosion during inflation

In the standard big bang cosmology inflation was proposed to solve simultaneously the flatness, isotropy, homogeneity, horizon and relics problems [28–31]. Thus inflation is the most popular theory of the early universe, in full agreement with observations, including the recent data from Planck satellite [50], which favour a simple inflationary scenario with only one slow rolling scalar field.

While an exhaustive discussion of inflation is beyond the scope of this article, we want to investigate if Higgspllosion can be reconciled with inflation or if one or the other has to be abandoned. We thus focus on a scenario with a non-minimally coupled scalar singlet field S to gravity [51] (see also [52–55]) as an example to show that Inflation can be incorporated into the Higgspllosion framework. A non-minimal inflaton coupling to gravity allows for a tensor-scalar ratio of $r_{0.05} \gtrsim -0.004$ well in agreement with current limits of $r_{0.05} < 0.07$ [56]. We take the relevant Jordan frame Lagrangian to be

$$\mathcal{L} = \sqrt{-g} \left[-\frac{M_{\text{Pl}} + \xi_s S^2}{2} R + \partial_\mu H^\dagger \partial^\mu H + (\partial_\mu S)^2 - V(H, S) \right] \quad (3.1)$$

with the Higgs doublet $H = (\phi^+, 1/\sqrt{2}(h + i\phi^0))^T$. The inflaton's nonminimal coupling term to gravity $\xi_s S^2 R/2$ should have a large parameter $\xi_s \sim 10^4$. The tree-level two-field scalar potential is

$$V(H, S) = -\mu_h H^\dagger H + \lambda_h (H^\dagger H)^2 - \frac{1}{2} \mu_s^2 S^2 + \frac{1}{4} \lambda_s S^4 + \frac{1}{2} \lambda_{sh} H^\dagger H S^2. \quad (3.2)$$

To bound the potential from below we take all λ_i to be positive.

The inflaton develops a vacuum expectation value v_s during inflation. Thus, during inflation, the mass of both the inflaton and the Higgs boson in the inflaton background are large

$$M_h \simeq \sqrt{\frac{\lambda_{Sh}}{2}} S(x) \simeq \frac{M_{\text{Pl}}}{\sqrt{\xi_s}}. \quad (3.3)$$

As h is of the order of the mass of the inflation S , or even heavier, the inflaton cannot higgsplode during inflation. Phenomenologically, inflation within the singlet extended standard model can remain unaffected by Higgspllosion.

The picture can change during reheating, where the inflaton oscillates around the potential's minimum, and with it the Higgs mass varies. One could imagine that reheating becomes more efficient if Higgspllosion sets in and that the mechanism is different from standard resonant reheating, which is associated with growing classical instabilities. A detailed study of reheating seems warranted but is beyond the scope of this work.

C. Axions

The strong CP problem, the discrepancy between the theoretically allowed value of the sum of the QCD topological angle and the quark mass phase $\theta = \theta_0 + \arg \det M_q$ and its experimentally observed size of less than $\mathcal{O}(10^{-10})$, provides a motivation to augment the SM by an additional pseudo-Nambu-Goldstone boson a of a spontaneously broken $U(1)_{PQ}$ symmetry [57–63]. The axion's Lagrangian below the Peccei Quinn (PQ) breaking scale can be written as

$$\begin{aligned} \mathcal{L}_a = & \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a} \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{a}{4} g_{a\gamma\gamma}^0 F_{\mu\nu} \tilde{F}^{\mu\nu} \\ & + \frac{\partial_\mu a}{2f_a} g_q \bar{q} \gamma^\mu \gamma_5 q \end{aligned} \quad (3.4)$$

where the axion decay constant f_a is the order parameter associated with the breaking of $U(1)_{PQ}$ via $a(x) \rightarrow a(x) + \alpha f_a$ and the dual gluon field strength $\tilde{G}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}$ (analogously for $\tilde{F}^{\mu\nu}$).

In the context of Higgspllosion, the QCD-axion provides a well-defined framework to answer the question if light degrees of freedom are allowed, or if they would lower the Higgspllosion scale enough to render this mechanism incompatible with the existence of very light and weakly coupled scalars. The only free parameter that is defining the axion's interactions and mass is f_a . At next-to-leading order⁴ the axion mass and self-interaction are respectively calculated to be [64]

$$m_a \simeq \frac{5.7 \times 10^{15} \text{ eV}}{f_a} \quad (3.5)$$

⁴While it is conceptually inconsistent to use a next-to-leading order result calculated without Higgspllosion, we do not expect a large quantitative effects from discarding the high loop momenta in the calculation of m_a and λ_a .

and

$$\lambda_a \equiv \left. \frac{\partial^4 V(a)}{\partial a^4} \right|_{a=0} \simeq -0.346 \frac{m_a^2}{f_a^2}. \quad (3.6)$$

We can now estimate the Higgspllosion scale where α_s stops running due to axion contributions in the Higgspllosion of the gluon. A crude approximation, assuming $\lambda_a n_a \gtrsim 20$ results in Higgspllosion, gives a Higgspllosion scale E_* of

$$E_*^{\text{Axion}} \simeq 60 \frac{f_a^2}{m_a}. \quad (3.7)$$

If we require the axion's Higgspllosion⁵ scale to be above the Higgs boson's Higgspllosion scale, i.e. $E_*^{\text{Axion}} > 10^5 \text{ GeV}$, we find a limit $f_a \gtrsim 2.1 \text{ GeV}$, which is easily achievable. Such a bound is far below existing experimental limits of $f_a \gtrsim 10^8 - 10^{17} \text{ GeV}$ [65–68].

Thus, the existence of QCD axions is not in conflict with Higgspllosion and the axion's contribution to the Higgspllosion of SM light degrees of freedom is negligible for sufficiently large f_a .

IV. DISCUSSION AND CONCLUSIONS

The discovery of the Higgs boson has unravelled an extraordinary building block in our understanding of elementary particle physics: the first elementary scalar particle. An immense effort is being devoted to determining its precise properties and in using it as a vehicle to uncover answers to thus far inexplicable observations in nature. Yet, the Higgs boson's hierarchy problem and its peculiar contribution to $h^* \rightarrow n \times h$ transition amplitudes have been puzzling for a long time and have provided motivation to extend the standard model of particle physics by novel degrees of freedom and interactions. In [1] we have proposed two intertwined mechanisms, Higgspllosion and Higgspllosion, to address both of these issues. In this paper work we have extended the discussion and interpretation of Higgspllosion and Higgspllosion and have outlined some phenomenological, i.e. experimentally testable, consequences in case these mechanisms are realised in nature.

Our findings are

- (1) All particles of the standard model and its extensions higgsplode at individually different scales. The particle's Higgspllosion scale depends on its interaction strength with the Higgs boson, its mass and whether it is stable or not.
- (2) The Higgspllosion scale establishes a radius $E_* \sim 1/r_*$ that cannot be probed further inside by

⁵To avoid confusion we will refrain from calling the rapid increase of the transition amplitude in the process $g \rightarrow gn \times a$ axionpllosion, and will just refer to it as the axion's Higgspllosion process.

increasing momenta. Thus, interaction processes with propagators with $p^2 \geq E_*^2$, i.e. all s -channel processes shut down, pronouncing the importance of t -channel interactions, e.g. multi-Regge kinematics, in high-energy scatterings. E_* effectively defines a virtuality and interaction dependent compositeness or classicalization scale.

- (3) Not only tree-level interactions are higgspered, but n -point functions to all loop orders are cut-off as well at $p^2 = E_*^2$. Hence, beyond the Higgspllosion scale the beta functions of all couplings vanish, i.e. $\beta(g_i^*) = 0$, and the standard model becomes asymptotically safe and free of Landau poles.
- (4) If additional scalars are in the theory that are either heavy, have a small mass ratio to the Higgs boson, or are feeble coupled they will not higgsplode or provide a significant contribution to the Higgspllosion of other particles. Thus, inflation and the

existence of QCD axions are not in conflict with the Higgspllosion mechanism.

Arguably the effects of Higgspllosion and Higgsperion open up to a radical re-examination of the standard wisdom associated with the UV behavior in quantum field theory, its phenomenological applications, and its probes of nature. Many fundamental questions are as yet left unanswered and we believe that this offers potential for exciting discoveries and warrant further studies in this area.

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