Selected strong decays of $\eta(2225)$ and $\phi(2170)$ as $\Lambda\bar{\Lambda}$ bound states

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(Received 29 May 2017; published 26 October 2017)

The strong decays of the two resonances $\eta(2225)$ and $\phi(2170)$ are discussed for selected decay channels. The two resonances are considered as the $\Lambda\bar{\Lambda}$ bound states in the molecular scenario. The phenomenological hadronic molecular approach is employed for the calculation of respective decay modes using effective Lagrangians. Our results show that the decay modes $\eta(2225) \to K^*K$ and $\phi(2175) \to KK$ dominate over the partial decay widths of $\eta(2275) \to VV(\phi\phi, \omega\omega, K^*K^*)$ and $\phi(2175) \to VS(\omega\sigma, K^*K_0^*(800), \phi f_0(980))$ due to phase space and couplings.

DOI: 10.1103/PhysRevD.96.074027

I. INTRODUCTION

Recently, the BESIII Collaboration performed a partial wave analysis of the decay process $J/\psi \to \gamma\phi\phi$ and confirmed the existence of the $\eta(2225)$ state, which has a mass of 2216^{+4+21}_{-5-11} MeV and a width of 185^{+12+43}_{-14-17} MeV [1]. The quantum numbers of $\eta(2225)$ were assigned to be $I^G(J^{PC}) = 0^+(0^{-+})$. There are only a few theoretical studies on $\eta(2225)$ in the literature. In Refs. [2,3] the strong decays of $\eta(2225)$ as a conventional $s\bar{s}$ state together with its partners were investigated in the framework of the quark-pair creation model, and the 4^1S_0 $s\bar{s}$ assignment was favored for the $\eta(2225)$ state. An alternative interpretation of $\eta(2225)$ as a bound state of $\Lambda\bar{\Lambda}(^1S_0)$ has been proposed in the one-boson exchange model in Ref. [4]. Conversely, the $\phi(2170)$ state with $I^G(J^{PC}) = 0^-(1^{--})$, denoted previously as $Y(2175)$, has been considered using different physical interpretations. The mass and width of the $\phi(2170)$ state are 2180 ± 10 MeV and 83 ± 12 MeV, respectively [5]. We also quote a recent result of the BESIII Collaboration [6] for the mass $2135 \pm 8 \pm 9$ MeV and for the width $104 \pm 24 \pm 12$ MeV. Taking into account information about production and decays of the $Y(4260)$ state [7], $\phi(2170)$ might be its strange partner. Possible interpretations include a traditional $s\bar{s}$ state [8–11], hybrid state [8,12], tetraquark state [13–15], $\Lambda\bar{\Lambda}(^3S_1)$ bound state [4,16], and $\phi K\bar{K}$ resonance state [17,18].

In the traditional quark model the total decay width of $\eta(2225)$ can be described well by considering it as the 4^1S_0 state [2]. However, when assigning $\phi(2170)$ as the 3^3S_1 or 2^3D_1 state, then it will result in a much larger decay width [8,10,19] than observed. Moreover, the very small mass difference between these two states can hardly be explained within the quark potential model, in which the mass of the $4S$ state should be much higher than that of the $3S$ state, even if the spin fine splitting is taken into account [20]. The interpretation of $\phi(2170)$ as the 4^3S_1 state also causes the reversal of the fine structure [11]. Considering that the masses of $\eta(2225)$ and $\phi(2170)$ are very close to the $\Lambda\bar{\Lambda}$ threshold, it also seems natural that $\eta(2225)$ and $\phi(2170)$ are considered as the $\Lambda\bar{\Lambda}(^1S_0)$ and $\Lambda\bar{\Lambda}(^3S_1)$ bound states, respectively [4]. Within the one-boson exchange model the mass of the $\Lambda\bar{\Lambda}(^1S_0)$ state is slightly higher than that of the $\Lambda\bar{\Lambda}(^3S_1)$ state, which is in good agreement with experimental data [1,5]. Besides the mass spectrum, it is natural to examine the strong decay behavior within the same framework. Note that "deuteronlike" states near the respective baryon-antibaryon threshold were originally discussed in the context of the nucleon-antinucleon system. There the notion of so-called quasinuclear $N\bar{N}$ bound states, weak composites of $N\bar{N}$, and their properties was intensely pursued to explain resonance structures observed in $N\bar{N}$ annihilation reactions. For one of our contributions to this topic see, for example, Ref. [21].

In this paper, we present a study of selected strong decay modes of the $\eta(2225)$ and $\phi(2170)$ states. We employ a hadronic molecular scenario [22–24] by taking the two resonances as weakly bound states of $\Lambda\bar{\Lambda}$ in a phenomenological Lagrangian approach. It should be mentioned that the approach is based on the compositeness condition [25–29]—a powerful method in quantum field theory for the study of composite bound states (hadrons, glueballs, hybrids, hadronic atoms and molecules, multiquark states), which was extensively used in Refs. [26–29] and [22–24]. In particular, the compositeness condition gives an equation for the coupling constant of the bound state with its constituents where the mass of the bound state is the input parameter. We suppose that our analyses of the $\eta(2225)$ and $\phi(2170)$ strong decays are useful for running and future experiments.

This paper is organized as follows. In Sec. II we briefly show our formalism, the calculations for the couplings of $\eta(2225)\Lambda\bar{\Lambda}$ and $\phi(2175)\Lambda\bar{\Lambda}$, and the matrix elements for the transitions of $\eta(2225)\to VV$ (vector-vector mesons), $\eta(2225)\to VP$ (vector-pseudoscalar mesons), $\phi(2175)\to VS$ (vector-scalar mesons), and $\phi(2175)\to PP$ (pseudoscalar-pseudoscalar mesons) in the hadronic molecular scenario. In Sec. III we present an application of our approach to the selected strong decays of $\eta(2225)$ and $\phi(2170)$ states. A short summary is given in Sec. IV.

II. APPROACH

In our numerical calculation, we use the following spin-parity quantum numbers for the $\eta(2225)$ and $\phi(2170)$ states $J^{PC}=0^{-+}$ and $J^{PC}=1^{--}$, respectively. Since the masses of the Λ baryon $[I(J^P)=0(1/2^+)]$ and the $\Lambda\bar{\Lambda}$ system are 1115.683 ± 0.006 MeV and about 2232 MeV, respectively, the discussed $\eta(2215)$ and $\phi(2175)$ resonances are about 16 MeV and 57 MeV below the threshold of the $\Lambda\bar{\Lambda}$ system. We consider the states $\eta(2225)$ and $\phi(2170)$ as weakly bound states of Λ and $\bar{\Lambda}$ in the hadronic molecular scenario. For this purpose we employ our phenomenological Lagrangian approach to describe these resonances. The interaction Lagrangians, describing the couplings of the $\eta(2225)$ and $\phi(2170)$ $\Lambda\bar{\Lambda}$ baryonium states with the constituents, read

$$\mathcal{L}_\eta(x)=g_\eta\eta(x)\int d^4y\Phi(y^2)\bar{\Lambda}(x+y/2)i\gamma_5\Lambda(x-y/2), \tag{1}$$

$$\mathcal{L}_\phi(x)=g_\phi\phi^\mu(x)\int d^4y\Phi(y^2)\bar{\Lambda}(x+y/2)\gamma_\mu\Lambda(x-y/2). \tag{2}$$

Here $\Phi(y^2)$ is a phenomenological correlation function describing the distribution of Λ and $\bar{\Lambda}$ constituents in the $\eta(2225)$ and $\phi(2170)$ states. To produce ultraviolet-finite Feynman diagrams, the Fourier transform of the correlation function $\Phi(y^2)$ should vanish sufficiently fast in the ultraviolet region of the Euclidean space. We use the Gaussian form for the correlation function

$$\tilde{\Phi}(p_E^2)\doteq\exp(-p_E^2/\Lambda_H^2),\qquad H=\eta(2225),\phi(2170), \tag{3}$$

where p_E is the Euclidean Jacobi momentum and Λ_H is a free size parameter, which has a value of about 1 GeV.

The couplings of g_η and g_ϕ with the Λ and $\bar{\Lambda}$ constituents are calculated from the compositeness condition (see Refs. [22–29])

$$Z_H=1-\Sigma'_H(m_H^2)\equiv0, \tag{4}$$

where Σ'_H is the derivative of the mass operator in the case of $\eta(2225)$ and of the transverse part of the mass operator $\Sigma^T_{\phi(2170)}$ in the case of the $\phi(2170)$ state, respectively. Note that the compositeness condition gives the relation between the coupling constant g_H of the bound state with their constituents and its mass m_H.

The quantity $Z_H^{1/2}$ is the matrix element between a physical particle state and the corresponding bare state. The compositeness condition $Z_H=0$ enables one to represent a bound state by introducing a hadronic field interacting with its constituents so that the renormalization factor is equal to zero. This does not mean that we can solve the QCD bound state equations but we are able to show that the condition $Z_H=0$ provides an effective and self-consistent way to describe the coupling of a hadron to its constituents. In particular, the compositeness condition gives an equation for the coupling constant of the bound state with its constituents where the mass of the bound state is the input parameter. One starts with an effective interaction Lagrangian written down in terms of quark and hadron variables. Then, by using Feynman rules, the S-matrix elements describing hadron-hadron interactions are given in terms of a set of quark level Feynman diagrams.

Decomposition of the $\phi(2170)$ mass operator in the transverse $\Sigma^T_{\phi(2170)}$ and longitudinal $\Sigma^L_{\phi(2170)}$ parts reads

$$\Sigma^{\mu\nu}_\phi(p)=g_\perp^{\mu\nu}\Sigma^T_\phi(p^2)+\frac{p^\mu p^\nu}{p^2}\Sigma^L_\phi(p^2), \tag{5}$$

where $g_\perp^{\mu\nu}=g^{\mu\nu}-p^\mu p^\nu/p^2$. The corresponding Feynman diagrams describing the mass operators of the $\eta(2225)$ and $\phi(2170)$ states are shown in Fig. 1.

The expressions for the mass operators of $\eta(2225)$ and $\phi(2170)$ are given by

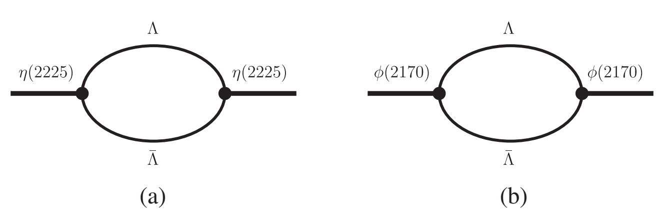

FIG. 1. Mass operators of $\eta(2225)$ and $\phi(2170)$.

$$\Sigma_\eta(p^2) = g_\eta^2 \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}^2(-k^2) \mathrm{Tr}[\gamma^5 S_\Lambda(k+p/2)\gamma^5 S_\Lambda(k-p/2)], \tag{6}$$

$$\Sigma_\phi^{\mu\nu}(p) = g_\phi^2 \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}^2(-k^2) \mathrm{Tr}[\gamma^\mu S_\Lambda(k+p/2)\gamma^\nu S_\Lambda(k-p/2)], \tag{7}$$

where $S_\Lambda(k) = 1/(m_\Lambda - \not{k})$ is the free Λ spin-1/2 baryon propagator with m_Λ being the mass of the Λ hyperon.

The expressions for the coupling constants g_H are given by

$$\frac{g_H^2}{4\pi^2} = \frac{1}{I_H}, \tag{8}$$

where I_H is the structure integral

$$I_H = \frac{1}{2}\int_0^\infty \frac{d\alpha d\beta}{\Delta^3} u_H e^{-w_H}, \qquad \Delta = 1+\alpha+\beta \tag{9}$$

and

$$\begin{aligned} w_H &= \frac{2m_\Lambda^2}{\Lambda_H^2}(\alpha+\beta) - \frac{m_H^2}{2\Lambda_H^2}\frac{\alpha+\beta+4\alpha\beta}{1+\alpha+\beta}, \\ u_\eta &= \frac{m_\Lambda^2}{\Lambda_H^2}(\alpha+\beta+2\alpha\beta) + \frac{1+4(\alpha+\beta)+12\alpha\beta}{2\Delta} + \frac{m_\eta^2}{4\Lambda_H^2}\frac{(1+2\alpha)(1+2\beta)(\alpha+\beta+4\alpha\beta)}{\Delta^2}, \\ u_\phi &= \frac{m_\Lambda^2}{\Lambda_H^2}(\alpha+\beta+2\alpha\beta) + \frac{1+3(\alpha+\beta)+8\alpha\beta}{2\Delta} + \frac{m_\phi^2}{4\Lambda_H^2}\frac{(1+2\alpha)(1+2\beta)(\alpha+\beta+4\alpha\beta)}{\Delta^2}. \end{aligned} \tag{10}$$

The use of the central values of the $\eta(2225)$ and $\phi(2170)$ masses $m_{\eta(2225)} = 2221$ MeV and $m_{\phi()} = 2188$ MeV in Eqs. (8) gives predictions for the $g_{\eta(2225)}$ and $g_{\phi(2170)}$ couplings.

In this paper we calculate some selected strong two-body decays $\eta(2225) \to VV$ and $\phi(2170) \to VS(PP)$, which are described by the Feynman diagrams shown in Fig. 2. For the additional hadronic interaction vertices the empirical meson-baryon form factors $\mathcal{F}(x-y)$ are employed. Those effective Lagrangians are

$$\mathcal{L}_{K^*\Lambda N}(x) = -g_{K^*\Lambda N} K^{*\mu}(x)\bar{\Lambda}(x)\gamma_\mu \int d^4y \mathcal{F}_N(x-y) N(y) + \mathrm{H.c.}, \tag{11}$$

$$\mathcal{L}_{\omega\Lambda\Lambda}(x) = -g_{\omega\Lambda\Lambda}\omega^\mu(x)\bar{\Lambda}(x)\gamma_\mu \int d^4y \mathcal{F}_\Lambda(x-y)\Lambda(y) + \mathrm{H.c.}, \tag{12}$$

$$\mathcal{L}_{\phi\Lambda\Lambda}(x) = -g_{\phi\Lambda\Lambda}\phi^\mu(x)\bar{\Lambda}(x)\gamma_\mu \int d^4y \mathcal{F}_\Lambda(x-y)\Lambda(y) + \mathrm{H.c.}, \tag{13}$$

$$\mathcal{L}_{a_0(980)\Lambda\Sigma}(x) = g_{a_0(980)\Lambda\Sigma} a_0(x)\bar{\Lambda}(x) \int d^4y \mathcal{F}_\Sigma(x-y)\Sigma(y) + \mathrm{H.c.}, \tag{14}$$

$$\mathcal{L}_{K_0^*(800)\Lambda N}(x) = g_{K_0^*(800)\Lambda N} K_0^*(x)\bar{\Lambda}(x) \int d^4y \mathcal{F}_N(x-y) N(y) + \mathrm{H.c.}, \tag{15}$$

$$\mathcal{L}_{\sigma\Lambda\Lambda}(x) = g_{\sigma\Lambda\Lambda}\sigma(x)\bar{\Lambda}(x) \int d^4y \mathcal{F}_\Lambda(x-y)\Lambda(y) + \mathrm{H.c.}, \tag{16}$$

$$\mathcal{L}_{f_0(980)\Lambda\Lambda}(x) = g_{f_0(980)\Lambda\Lambda} f_0(x)\bar{\Lambda}(x) \int d^4y \mathcal{F}_\Lambda(x-y)\Lambda(y) + \mathrm{H.c.}, \tag{17}$$

$$\mathcal{L}_{K\Lambda N}(x) = g_{K\Lambda N} K(x)\bar{\Lambda}(x) i\gamma_5 \int d^4y \mathcal{F}_N(x-y) N(y) + \mathrm{H.c.} \tag{18}$$

In the case of vector meson-baryon couplings we restrict to the minimal coupling—leading-order contribution in the inverse baryon mass expansion; i.e., we neglect the nonminimal couplings (or ignore the tensor coupling in the VBB interaction as in [4]). We fix meson-nucleon couplings using $SU(3)$ symmetry predictions and phenomenological constraints [4,30],

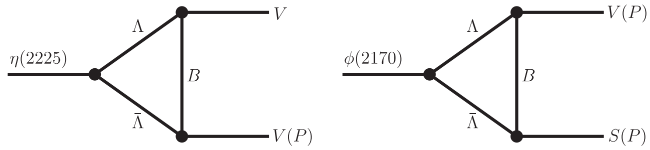

FIG. 2. Feynman diagrams describing decays $\eta(2225) \to VV(VP)$ and $\phi(2170) \to VS(PP)$.

$$
\begin{aligned}
g_{K^*\Lambda N} &= -\frac{1}{\sqrt{3}}(2\alpha_V + 1) g_{\rho NN},\\
g_{\omega\Lambda\Lambda} &= \frac{2}{3}(5\alpha_V - 2) g_{\rho NN},\\
g_{K_0^*\Lambda N} &= -\frac{1}{\sqrt{3}}(2\alpha_S + 1) g_{a_0(980)NN},\\
g_{\sigma\Lambda\Lambda} &= \frac{2}{3}(5\alpha_S - 2) g_{a_0(980)NN},\\
g_{K\Lambda N} &= -\frac{1}{\sqrt{3}}(2\alpha_P + 1) g_{\pi NN},
\end{aligned} \tag{19}
$$

where $\alpha_V = \alpha_S = 1$ and $\alpha_P = 0.4$. The set of numerical values of the meson-baryon couplings is listed in Table I [4,30]. Here we employ the monopole-type form factor $\tilde{\mathcal{F}}_B(q^2)$ (in momentum space) of the form

TABLE I. Effective meson-baryon couplings.

$g_{K^*\Lambda N}$	$g_{K\Lambda N}$	$g_{\omega\Lambda\Lambda}$	$g_{\phi\Lambda\Lambda}$	$g_{K_0^*(800)\Lambda N}$	$g_{\sigma\Lambda\Lambda}$	$g_{f_0(980)\Lambda\Lambda}$
−9.153	−13.926	10.569	5.284	−5.710	6.593	3.296

$$
\begin{aligned}
\mathcal{F}_B(x) &= \int d^4x e^{-iqx} \tilde{\mathcal{F}}_B(q^2),\\
\tilde{\mathcal{F}}_B(q^2) &= \frac{\Lambda_B^2 - M_B^2}{\Lambda_B^2 - q^2}
\end{aligned} \tag{20}
$$

proposed in Ref. [31] and extensively used in literature [31–37] with M_B being the exchange baryon mass and Λ_B being the cutoff parameter for the exchange momentum. According to the discussion in the literature [31–37], we choose $\Lambda_B = M_B + \alpha\Lambda_{\mathrm{QCD}}$ with $\Lambda_{\mathrm{QCD}} = 220$ MeV. These form factors are necessary to be consistent with the phenomenological Lagrangians utilized before in [4].

Now it is straightforward to write down the matrix elements for the discussed two-body transition,

$$
\begin{aligned}
\mathcal{M}^{\alpha\beta}_{\eta(2225)\to VV} \epsilon^*_\alpha(q_1)\epsilon^*_\beta(q_2) &= 2\epsilon^*_\alpha(q_1)\epsilon^*_\beta(q_2) g_\eta g^2_{V\Lambda B} \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}(-k^2) \tilde{\mathcal{F}}^2_B((k+p/2-q_1)^2)\\
&\quad \times \mathrm{Tr}[\gamma^\alpha S_\Lambda(k+p/2) i\gamma^5 S_\Lambda(k-p/2)\gamma^\beta S_B(k+p/2-q_1)]\\
&= \frac{g_{\eta VV}}{m_{\eta(2225)}} \epsilon^{\mu\nu\alpha\beta} q_{1\mu} q_{2\nu} \epsilon^*_\alpha(q_1)\epsilon^*_\beta(q_2),\\
\mathcal{M}^{\alpha}_{\eta(2225)\to VP} \epsilon^*_\alpha(q_1) &= \epsilon^*_\alpha(q_1) g_\eta g_{V\Lambda B} g_{P\Lambda B} \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}(-k^2) \tilde{\mathcal{F}}^2_B((k+p/2-q_1)^2)\\
&\quad \times \mathrm{Tr}[\gamma^\alpha S_\Lambda(k+p/2) i\gamma^5 S_\Lambda(k-p/2) i\gamma^5 S_B(k+p/2-q_1)]\\
&= g_{\eta VP} q_2^\alpha \epsilon^*_\alpha(q_1)
\end{aligned} \tag{21}
$$

for the $\eta(2225) \to VV$ and $\eta(2225) \to VP$ decays and

$$
\begin{aligned}
\mathcal{M}^{\mu\alpha}_{\phi(2170)\to VS} \epsilon_\mu(p)\epsilon^*_\alpha(q_1) &= \epsilon_\mu(p)\epsilon^*_\alpha(q_1) g_\phi g_{S\Lambda B} g_{V\Lambda B} \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}(-k^2) \tilde{\mathcal{F}}^2_B((k+p/2-q_1)^2)\\
&\quad \times \mathrm{Tr}[S_\Lambda(k+p/2)\gamma^\mu S_\Lambda(k-p/2)\gamma^\alpha S_B(k+p/2-q_1)]\\
&= m_{\phi(2170)} \left(g^{\mu\alpha} g_{\phi VS} + \frac{q_1^\mu q_2^\alpha}{m^2_{\phi(2170)}} f_{\phi VS} \right) \epsilon_\mu(p)\epsilon^*_\alpha(q_1),
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
\mathcal{M}^{\mu}_{\phi(2170)\to PP} \epsilon_\mu(p) &= 2\epsilon_\mu(p) g_\phi g_{P\Lambda B} g_{P\Lambda B} \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}(-k^2) \tilde{\mathcal{F}}^2_B((k+p/2-q_1)^2)\\
&\quad \times \mathrm{Tr}[S_\Lambda(k+p/2)\gamma^\mu S_\Lambda(k-p/2) i\gamma^5 S_B(k+p/2-q_1) i\gamma^5]\\
&= g_{\phi PP}(q_1 - q_2)^\mu \epsilon_\mu(p)
\end{aligned} \tag{23}
$$

for the $\phi(2170) \to VS$ and $\phi(2170) \to PP$ decays, where p and q_1, q_2 are the momenta of initial and final particles; $g_{\eta VV}$ and $g_{\phi VS}$, $f_{\phi VS}$, $g_{\phi PP}$ are dimensionless couplings of $\eta(2225)$ and $\phi(2170)$ with final mesons, respectively; $\epsilon^*_\mu(p)$, $\epsilon^*_\alpha(q_1)$, and $\epsilon^*_\beta(q_2)$ are the polarization vectors of the $\phi(2170)$ state and produced vector mesons, respectively; $S_B(k)$ is the free spin-1/2 baryon propagator.

Two-body strong decay widths are calculated according to the formulas

$$\Gamma(\eta(2225)\to VV) = \frac{g^2_{\eta VV}}{64\pi} m_{\eta(2225)} \left(1 - \frac{4m_V^2}{m^2_{\eta(2225)}}\right)^{3/2},$$

$$\Gamma(\eta(2225)\to VP) = \frac{g^2_{\eta VP}}{8\pi}\frac{|\mathbf{q_1}|_\eta^3}{m_V^2},$$

$$\Gamma(\phi(2170)\to VS) = \frac{g^2_{\phi VS}}{24\pi}|\mathbf{q_1}|_\phi\left[3 + \frac{|\mathbf{q_1}|_\phi^2}{m_V^2} + \frac{m^2_{\phi(2170)} + m_V^2 - m_S^2}{m^2_{\phi(2170)}}\frac{|\mathbf{q_1}|_\phi^2}{m_V^2}R + \frac{|\mathbf{q_1}|_\phi^4}{m^2_{\phi(2170)}m_V^2}R^2\right],$$

$$R = \frac{f_{\phi VS}}{g_{\phi VS}},$$

$$\Gamma(\phi(2170)\to PP) = \frac{g^2_{\phi PP}}{96\pi} m_{\phi(2170)} \left(1 - \frac{4m_P^2}{m^2_{\phi(2170)}}\right)^{3/2}. \quad (24)$$

Here $|\mathbf{q_1}|_\eta = \lambda(m^2_{\eta(2225)}, m_V^2, m_P^2)/(2m_{\eta(2225)})$ and $|\mathbf{q_1}|_\phi = \lambda(m^2_{\phi(2170)}, m_V^2, m_S^2)/(2m_{\phi(2170)})$ are the 3-momenta of the decay products in the center of mass frame and $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the Källen kinematical triangle function.

III. RESULTS AND DISCUSSIONS

In Fig. 3 we show the dependence of the couplings g_H, $H = \eta(2225), \phi(2170)$ on the cutoff parameter Λ_H [see Eq. (8)]. When Λ_H is varied in the region of (0.8–1.2 GeV), the two resulting couplings are not too sensitive to the model parameter Λ_H. The variations of the dimensionless couplings are $(3.4 \to 3.2)$ and $(5.8 \to 5.1)$, respectively. According to our previous calculations in the context of XYZ resonances and to the deuteron system, a typical value of $\Lambda_H \sim 1$ GeV is often employed. Thus, in this calculation we get $g_H = 3.282$ and 5.356 for $\eta(2225)$ and $\phi(2170)$, respectively. To make detailed calculations for the decay processes of Fig. 2, the couplings of the effective Lagrangians in Eqs. (11)–(18) are needed. We take these from Refs. [4,30] as listed in Table I.

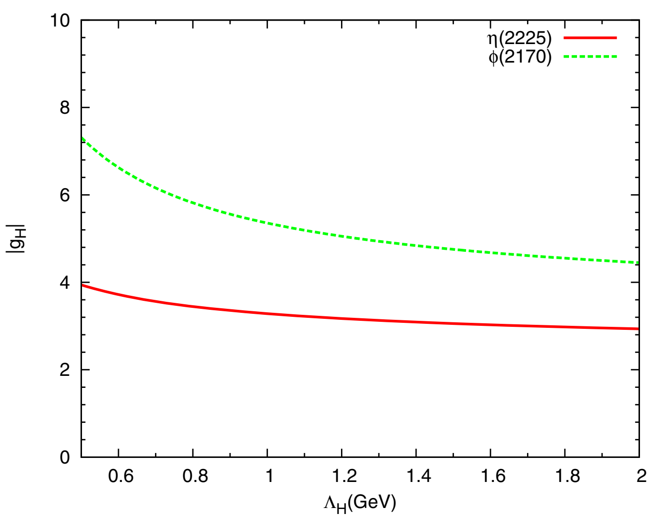

FIG. 3. The couplings of $\eta(2225)$ (red-solid line) and $\phi(2170)$ (green-dashed line) versus the parameter Λ_H of the correlation function.

It should be reiterated that additional phenomenological form factors $\tilde{\mathcal{F}}$ in the matrix elements of Eqs. (21) and (22) are introduced, which contain a free parameter α. This parameter is fixed from data on the total widths of $\eta(2225)$ and $\phi(2170)$ [5]: $\Gamma_{\eta(2225)} = 185^{+40}_{-20}$ MeV and $\Gamma_{\phi(2170)} = 83 \pm 12$ MeV. In particular, an increase of the parameter α leads to an increase of the partial widths of $\eta(2225)$ and $\phi(2170)$. We compare the sum of the partial modes of $\eta(2225)$ and $\phi(2170)$, which include the dominant channels $\eta(2225)\to K^*K$, $\eta(2225)\to K^*K^*$, and $\eta(2225)\to \omega\omega$ in the case of $\eta(2225)$ and $\phi(2170)\to KK$ in the case of $\phi(2170)$, with total widths of these states. Using data on the widths of the $\eta(2225)$ and $\phi(2170)$ states we found that in the case of $\eta(2225)$ the parameter α is constrained as $0.91 \le \alpha \le 1.08$, while in the case of $\phi(2170)$ the parameter α is constrained as $0.85 \le \alpha \le 1.0$. In both cases the lower and upper limits for the α correspond to the lower and upper limits for the sum of the partial decays modes, respectively. Therefore, taking into account the two above constraints for α we finally conclude that from data on the total widths of $\eta(2225)$ and $\phi(2170)$ the parameter α should be varied in the region $0.91 \le \alpha \le 1.0$.

Table II summarizes the numerical results for the partial decay widths of the two resonances including the variation of parameter α from 0.91 to 1.0. We compare our predictions for the sum of partial widths with data for

TABLE II. Numerical results for the $\eta(2225)$ and $\phi(2170)$ decay widths (in MeV).

Modes	Γ [MeV]		
$\eta(2225)$ decay	This work	$^3P_0$ model within $s\bar{s}$ [2]	Data [5]
K^*K	71.1–87.3	9.1	…
$\phi\phi$	1.1–1.3	12.6	Seen
$\omega\omega$	53.6–63.3	0	…
K^*K^*	37.1–43.7	0.5	…
Total	162.9–195.6	22.2	185^{+40}_{-20}
$\phi(2170)$ decay	This work	$^3P_0$ model within $s\bar{s}$ [10]	Data [5]
KK	73.8–87.7	…	…
$\phi f_0(980)$	0.25–0.3	<10	Seen
$\omega\sigma$	4.2–4.9	…	…
$K^*K_0^*(800)$	1.8–2.1	…	…
Total	80.1–95		83 ± 12

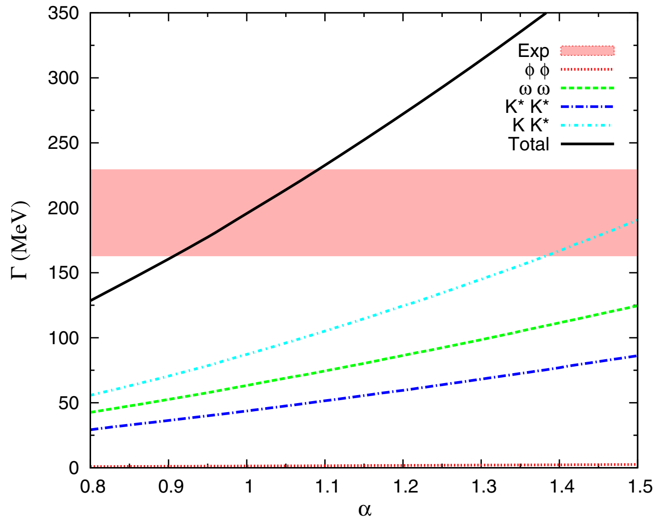

FIG. 4. $\eta(2225) \to VP(VV)$ decays and their sum in dependence on α and comparison with data for $\Gamma_{\eta(2225)}$.

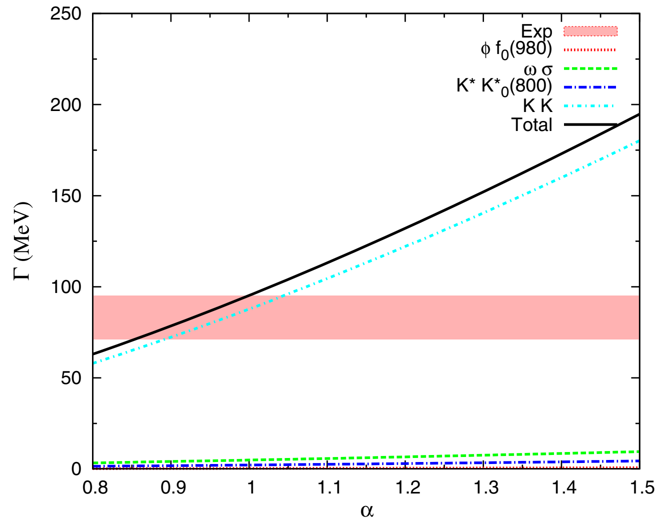

FIG. 5. $\phi(2170) \to VS(PP)$ decays in dependence on α and comparison with data for $\Gamma_{\phi(2170)}$.

the total widths of $\eta(2225)$ and $\phi(2170)$. Also we present a comparison of partial widths with available calculations in the $^3P_0$ model using the $s\bar{s}$ interpretation [2,10]. The much larger decay widths of the $\eta(2225) \to K^*K$, $\eta(2225) \to \omega\omega$, and $\eta(2225) \to K^*K^*$ channels compared to $\eta(2225) \to \phi\phi$ are due to the phase space and particularly to the couplings. A similar feature occurs for the $\phi(2170)$ state, for which the decay $\phi(2170) \to KK$ dominates over the others because of the phase space and relatively big coupling constant $g_{K\Lambda N}$. We see that for $\eta(2225)$, the $\omega\omega$ channel dominates for the $\Lambda\bar{\Lambda}$ bound state, while it is a Okubo-Zweig-Iizuka-forbidden mode within the $s\bar{s}$ interpretation. For $\phi(2175)$, the $^3P_0$ model calculations in the literature usually neglect its SV modes and give a rather larger total decay width, which disfavor the $s\bar{s}$ interpretation. These differences can help us to distinguish the $\Lambda\bar{\Lambda}$ bound state and $s\bar{s}$ interpretation.

As an independent check of consistency of our results we would like to compare our result for the decay width $\Gamma(\phi(2170) \to \phi(1020) + f_0(980)) = 0.25–0.3$ MeV with data $\Gamma(\phi(2170) \to \phi(1020) + f_0(980)) = 0.1–1$ MeV, which can be extracted using experimental results for $\Gamma(\phi(2170) \to \phi(1020) + f_0(980))\Gamma(\phi(2170) \to e^+e^-)/\Gamma_{\text{tot}} = (2.3 \pm 0.3 \pm 0.3)$ eV and typical values for the branching of $\text{Br}(\phi(2170) \to e^+e^-) = 10^{-6} - 10^{-5}$ deduced using known data for other ω states.

For convenience, in Figs. 4 and 5 we also display the dependence of the partial widths of $\eta(2225)$ and $\phi(2170)$ and their sums on the parameter α varied in the wide region $0.8 \leq \alpha \leq 1.5$ and compare the total width with the data. Again, one can see that data on the total decays of the $\eta(2225)$ and $\phi(2170)$ states give strong constraint on the parameter α: $0.91 \leq \alpha \leq 1$.

IV. SUMMARY

We have employed the hadronic molecular scenario for the two resonances $\eta(2225)$ and $\phi(2170)$ considering them as weakly bound $\Lambda\bar{\Lambda}$ states. A phenomenological effective Lagrangian approach is applied for some selected partial decay widths. Our numerical results show that the $\Lambda\bar{\Lambda}$ scenario gives a reasonable description of the partial decay widths of the $\eta(2225)$ and $\phi(2170)$ states showing that the modes $\eta(2225) \to VP$ and $\phi(2170) \to PP$ modes are, respectively, dominant. Moreover, together with the study of the mass spectrum of the two resonances in Ref. [4], we conclude that the $\Lambda\bar{\Lambda}$ baryonium interpretations for the two resonances might be possible. Using data on the total widths of the $\eta(2225)$ and $\phi(2170)$ states we derive the constraint on the parameter α in the phenomenological form factor controlling the off-shell behavior of the exchanged baryon between the produced two final mesons: $0.91 \leq \alpha \leq 1$. For these values of the α parameter our predictions for the partial decay widths of $\eta(2225)$ and $\phi(2170)$ are shown in Table II. Here we studied selected decay modes of the $\eta(2225)$ and $\phi(2170)$ states and included the dominant decay modes $\eta(2225) \to VV$ and $\phi(2170) \to PP$. There are, of course, many other channels, such as $\eta(2225) \to N\bar{N}, PV, PS$ and $\phi(2170) \to N\bar{N}, PV, SS$, which contribute a full coupled-channel calculation and will be studied elsewhere.

ACKNOWLEDGMENTS

We thank Bing-Song Zou, Fei Huang, and Yin Huang for useful discussions. This work is supported by the National Natural Sciences Foundations of China under the Grants No. 11475192, No. 11565007, No. 11635009, No. 11705056, and No. 11521505 and by the fund of Sino-German CRC 110 “Symmetries and the Emergence of

Structure in QCD" project by NSFC under Grant No. 11621131001, China Postdoctoral Science Foundation under Grant No. 2016M601133. This work was funded by the German Bundesministerium für Bildung und Forschung (BMBF) under Project 05P2015—ALICE at High Rate (BMBF-FSP 202): "Jet- and fragmentation processes at ALICE and the parton structure of nuclei and structure of heavy hadrons," by CONICYT (Chile) PIA/Basal FB0821, by Tomsk State University Competitiveness Improvement Program and the Russian Federation program "Nauka" (Contract No. 0.1764.GZB.2017) and by Tomsk Polytechnic University Competitiveness Enhancement Program (Grant No. VIU-FTI-72/2017). Y. B. D. also thanks the Alexander von Humboldt foundation for support and the Institute of Theoretical Physics, Tuebingen University, Germany, for hospitality.

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