D-wave heavy baryons of the $SU(3)$ flavor $\mathbf{6}_F$ representation

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(Received 8 August 2017; published 20 October 2017)

We use the method of QCD sum rules to study the D-wave charmed and bottom baryons of the $SU(3)$ flavor $\mathbf{6}_F$ and calculate their masses up to the order $\mathcal{O}(1/m_Q)$ with m_Q the heavy-quark mass. Our results suggest that the $\Xi_c(3123)$ can be well interpreted as a D-wave $\Xi_c'(\mathbf{6}_F)$ state, and it probably has a partner state close to it. Our results also suggest that there may exist as many as four D-wave Ω_c states in the energy region 3.3–3.5 GeV, and we propose to search for them in the future LHCb and BelleII experiments.

DOI: 10.1103/PhysRevD.96.074021

I. INTRODUCTION

In the past years, important experimental progress has been made in the field of heavy baryons [1], and we refer to reviews [2–7] for their recent progress. Especially, the LHCb experiment observed as many as five excited Ω_c states at the same time [8], which is probably related to the fine structure of the strong interaction. Some of these excited Ω_c states can be interpreted as P-wave states [9], inspiring us to further study the D-wave heavy baryons. Actually, in Ref. [10], we systematically studied the D-wave heavy baryons of the $SU(3)$ flavor $\bar{\mathbf{3}}_F$ using the method of QCD sum rules [11,12] in the framework of heavy-quark effective theory (HQET) [13–15], and in the present study, we shall follow the same approach to study the $SU(3)$ flavor $\mathbf{6}_F$ ones, including the D-wave Ω_c states.

There have been lots of heavy baryons observed in various experiments [1,16–18]. Among them, the $\Xi_c(3123)$ observed by the *BABAR* Collaboration [19] (but not seen in the following Belle experiment [20]) is a good candidate of the D-wave $\Xi_c'(\mathbf{6}_F)$ state and has been investigated using many phenomenological methods/models, including various quark models [21–26], the Faddeev method [27], the Regge trajectory [28], the relativistic flux tube model [29], the heavy hadron chiral perturbation theory [30], QCD sum rules [31,32], etc. More discussions can be found in Refs. [33–37], and we again refer to Refs. [2–7] for the recent progress.

We have studied the heavy baryons using the method of QCD sum rules within HQET [9,10,38,39], and in the present study, we shall further study the D-wave charmed baryons of the $SU(3)$ flavor $\mathbf{6}_F$. We shall take the $\mathcal{O}(1/m_Q)$ corrections (m_Q is the heavy-quark mass) into account during our QCD sum rule analyses and extract the chromomagnetic splitting within the same baryon multiplet. More discussions on heavy mesons and baryons containing a single heavy quark can be found in Refs. [40–54].

This paper is organized as follows. In Sec. II, we briefly introduce how we use the D-wave heavy-baryon interpolating fields to perform QCD sum rule analyses, and we suggest interested readers consult the discussion in Refs. [9,10,38,39] for details. Then, we perform numerical analyses in Sec. III and offer a short summary in Sec. IV.

II. QCD SUM RULE ANALYSES

In Ref. [10], we systematically constructed all the D-wave heavy-baryon interpolating fields but just studied the $SU(3)$ flavor $\bar{\mathbf{3}}_F$ ones using the method of QCD sum rules in the framework of HQET. In this paper, we use the same method to study the rest of the ones of the $SU(3)$ flavor $\mathbf{6}_F$, as briefly shown in Fig. 1. We suggest interested readers consult the discussion in Ref. [10] for details.

Here, we briefly explain our notations: $J^{\alpha_1\cdots\alpha_{j-1/2}}_{j,P,F,j_l,s_l,\rho-\lambda}$ denotes the D-wave heavy-baryon field belonging to the baryon multiplet $[F, j_l, s_l, \rho-\lambda]$, where j, P, and F denote

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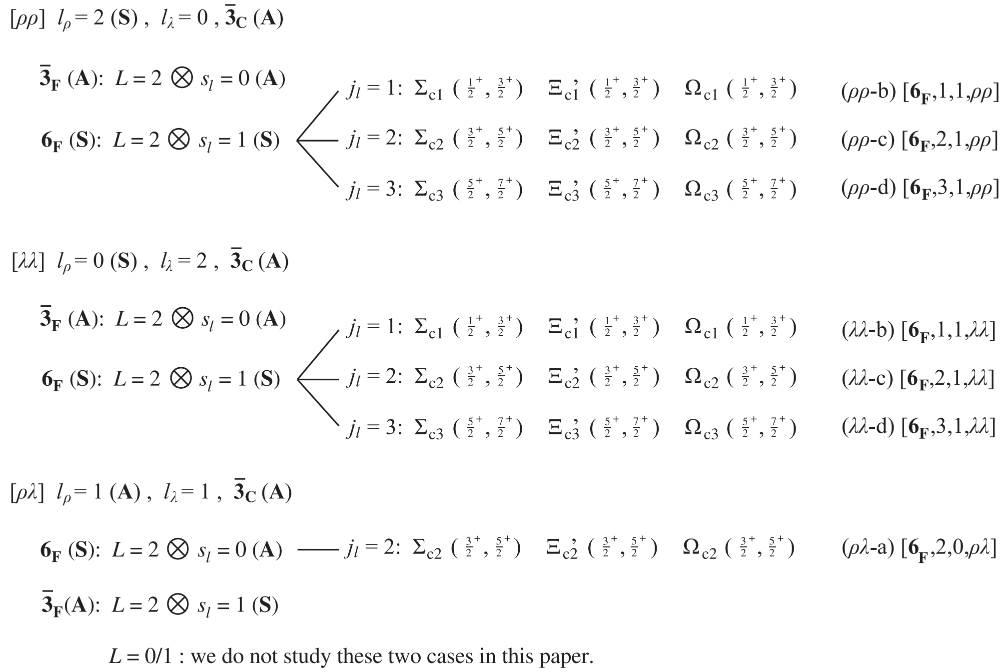

FIG. 1. The notations for the D-wave charmed baryons of the $SU(3)$ flavor $\mathbf{6}_F$. See Fig. 1 of Ref. [10] and discussions therein for details.

its total angular momentum, parity, and $SU(3)$ flavor representation ($\bar{\mathbf{3}}_F$ or $\mathbf{6}_F$); j_l and s_l are the total angular momentum and spin angular momentum of its light components, respectively; and there are three types ($\rho-\lambda$), the $\rho\rho$ type ($l_\rho = 2$ and $l_\lambda = 0$), $\lambda\lambda$ type ($l_\rho = 0$ and $l_\lambda = 2$), and $\rho\lambda$ type ($l_\rho = 1$ and $l_\lambda = 1$), where we use l_ρ to denote the orbital angular momentum between the two light quarks and l_λ to denote the orbital angular momentum between the heavy-quark and the two-light-quark systems. These parameters satisfy $L = l_\lambda \otimes l_\rho = 2$ (note that we only investigate the D-wave heavy baryons in the present study), $j_l = L \otimes s_l$, and $j = j_l \otimes s_Q = |j_l \pm 1/2|$, with $s_Q = 1/2$ the spin of the heavy quark.

The explicit forms of $J^{\alpha_1\cdots\alpha_{j-1/2}}_{j,P,F,j_l,s_l,\rho-\lambda}$ have been given in Eqs. (2–19) of Ref. [10]. We use them to perform QCD sum rule analyses by assuming their coupling to the state $|j,P,F,j_l,s_l,\rho-\lambda\rangle$ to be

$$\langle 0|J^{\alpha_1\cdots\alpha_{j-1/2}}_{j,P,F,j_l,s_l,\rho-\lambda}|j,P,F,j_l,s_l,\rho-\lambda\rangle = f_{F,j_l,s_l,\rho-\lambda} u^{\alpha_1\cdots\alpha_{j-1/2}}. \tag{1}$$

Again, we recommend interested readers consult Refs. [9,10,38,39,55–57] for details but simply list here the equation to evaluate the mass of the heavy baryon belonging to the multiplet $[F, j_l, s_l, \rho-\lambda]$:

$$m_{j,P,F,j_l,s_l,\rho-\lambda} = m_Q + \bar{\Lambda}_{F,j_l,s_l,\rho-\lambda} + \delta m_{j,P,F,j_l,s_l,\rho-\lambda}. \tag{2}$$

Here, m_Q is the heavy-quark mass; $\bar{\Lambda}_{F,j_l,s_l,\rho-\lambda} = \bar{\Lambda}_{|j_l-1/2|,P,F,j_l,s_l,\rho-\lambda} = \bar{\Lambda}_{j_l+1/2,P,F,j_l,s_l,\rho-\lambda}$ is the sum rule result obtained at the leading order; and $\delta m_{j,P,F,j_l,s_l,\rho-\lambda}$ is the sum rule result obtained at the $\mathcal{O}(1/m_Q)$ order,

$$\delta m_{j,P,F,j_l,s_l,\rho-\lambda} = -\frac{1}{4m_Q}(K_{F,j_l,s_l,\rho-\lambda} + d_M C_{mag}\Sigma_{F,j_l,s_l,\rho-\lambda}), \tag{3}$$

where $C_{mag}(m_Q/\mu) = [\alpha_s(m_Q)/\alpha_s(\mu)]^{3/\beta_0}$ with $\beta_0 = 11 - 2n_f/3$ and the coefficient $d_M \equiv d_{j,j_l}$ is defined to be

$$d_{j_l-1/2,j_l} = 2j_l + 2,$$
$$d_{j_l+1/2,j_l} = -2j_l, \tag{4}$$

which is directly related to the baryon mass splitting within the same multiplet.

In Ref. [10], we systematically studied the D-wave heavy baryons of the $SU(3)$ flavor $\bar{\mathbf{3}}_F$ using the method of QCD sum rules with HQET. In this paper, we similarly study the $SU(3)$ flavor $\mathbf{6}_F$ ones and calculate the analytical formulas for $\bar{\Lambda}_{F,j_l,s_l,\rho-\lambda}$, $K_{F,j_l,s_l,\rho-\lambda}$, and $\Sigma_{F,j_l,s_l,\rho-\lambda}$. There are altogether seven heavy-baryon multiplets of the $SU(3)$ flavor $\mathbf{6}_F$ as shown in Fig. 1, i.e., $[\mathbf{6}_F, 1, 1, \rho\rho]$, $[\mathbf{6}_F, 2, 1, \rho\rho]$, $[\mathbf{6}_F, 3, 1, \rho\rho]$, $[\mathbf{6}_F, 1, 1, \lambda\lambda]$, $[\mathbf{6}_F, 2, 1, \lambda\lambda]$, $[\mathbf{6}_F, 3, 1, \lambda\lambda]$, and $[\mathbf{6}_F, 2, 0, \rho\lambda]$. Hence, there can be as many as two $j^P = 1/2^+$ D-wave excited Σ_c states, five $j^P = 3/2^+$ ones, five $j^P = 5/2^+$ ones, and two $j^P = 7/2^+$ ones. The numbers of excited Ξ'_c and Ω_c states are the same. Recalling that the LHCb experiment observed as many as five excited Ω_c states [8], some of these D-wave heavy baryons may be observed experimentally at the same time. Theoretically, in the present study, we can only use five baryon multiplets to perform sum rule analyses because in Ref. [10] we failed to construct the currents belonging to the other two multiplets $[\mathbf{6}_F, 2, 1, \rho\rho]$ and $[\mathbf{6}_F, 2, 1, \lambda\lambda]$.

For an example, we use the charmed baryon multiplet $[\Sigma_c(\mathbf{6}_F), 1, 1, \rho\rho]$ to perform QCD sum rule analyses. It contains two charmed baryons, $\Sigma_c(1/2^+)$ and $\Sigma_c(3/2^+)$, and the relevant interpolating field is

$$\begin{aligned} J_{1/2,+,\Sigma_c,1,1,\rho\rho}(x) = \epsilon_{abc}([\mathcal{D}^t_{\mu_1}\mathcal{D}^t_{\mu_2}u^{aT}(x)]\mathbb{C}\gamma^t_{\mu_3}d^b(x) \\ - 2[\mathcal{D}^t_{\mu_1}u^{aT}(x)]\mathbb{C}\gamma^t_{\mu_3}[\mathcal{D}^t_{\mu_2}d^b(x)] \\ + u^{aT}(x)\mathbb{C}\gamma^t_{\mu_3}[\mathcal{D}^t_{\mu_1}\mathcal{D}^t_{\mu_2}d^b(x)]) \\ \times (g_t^{\mu_1\mu_3}g_t^{\mu_2\mu_4} + g_t^{\mu_2\mu_3}g_t^{\mu_1\mu_4}) \times \gamma^t_{\mu_4}\gamma_5 h_v^c(x). \end{aligned} \tag{5}$$

We can use this current to perform QCD sum rule analyses and calculate $\bar\Lambda_{\Sigma_c,1,1,\rho\rho}$, $K_{\Sigma_c,1,1,\rho\rho}$, and $\Sigma_{\Sigma_c,1,1,\rho\rho}$:

$$\begin{aligned} \Pi_{\Sigma_c,1,1,\rho\rho} &= f^2_{\Sigma_c,1,1,\rho\rho}e^{-2\bar\Lambda_{\Sigma_c,1,1,\rho\rho}/T} \\ &= \int_0^{\omega_c}\left[\frac{11}{80640\pi^4}\omega^9 - \frac{19\langle g_s^2GG\rangle}{3072\pi^4}\omega^5\right]e^{-\omega/T}d\omega, \end{aligned} \tag{6}$$

$$\begin{aligned} f^2_{\Sigma_c,1,1,\rho\rho}K_{\Sigma_c,1,1,\rho\rho}e^{-2\bar\Lambda_{\Sigma_c,1,1,\rho\rho}/T} \\ = \int_0^{\omega_c}\left[-\frac{59}{2217600\pi^4}\omega^{11} + \frac{299\langle g_s^2GG\rangle}{161280\pi^4}\omega^7\right]e^{-\omega/T}d\omega, \end{aligned} \tag{7}$$

$$\begin{aligned} f^2_{\Sigma_c,1,1,\rho\rho}\Sigma_{\Sigma_c,1,1,\rho\rho}e^{-2\bar\Lambda_{\Sigma_c,1,1,\rho\rho}/T} \\ = \int_0^{\omega_c}\left[\frac{37\langle g_s^2GG\rangle}{322560\pi^4}\omega^7\right]e^{-\omega/T}d\omega. \end{aligned} \tag{8}$$

Sum rules for Ξ'_c and Ω_c belonging to the same multiplet, $[\mathbf{6}_F, 1, 1, \rho\rho]$, as well as sum rules for the other four multiplets, $[\mathbf{6}_F, 3, 1, \rho\rho]$, $[\mathbf{6}_F, 1, 1, \lambda\lambda]$, $[\mathbf{6}_F, 3, 1, \lambda\lambda]$, and $[\mathbf{6}_F, 2, 0, \rho\lambda]$, are listed in Appendix. In the next section, we shall use these equations to perform numerical analyses.

III. NUMERICAL ANALYSES

We use the following values for various condensates and parameters to perform numerical analyses [1,58–65]:

$$\begin{aligned} \langle\bar qq\rangle &= -(0.24\pm0.01)^3\ \mathrm{GeV}^3, \\ \langle\bar ss\rangle &= 0.8\times\langle\bar qq\rangle, \\ \langle g_s^2GG\rangle &= (0.48\pm0.14)\ \mathrm{GeV}^4, \\ \langle g_s\bar q\sigma Gq\rangle &= M_0^2\times\langle\bar qq\rangle, \\ \langle g_s\bar s\sigma Gs\rangle &= M_0^2\times\langle\bar ss\rangle, \\ m_s &= 0.125\ \mathrm{GeV}, \\ M_0^2 &= 0.8\ \mathrm{GeV}^2. \end{aligned} \tag{9}$$

Besides them, we also need the *charm*-quark mass, for which we use the Particle Data Group value $m_c = 1.275\pm 0.025$ GeV [1] in the $\overline{\mathrm{MS}}$ scheme.

There are two free parameters in Eqs. (6)–(8): the Borel mass T and the threshold value ω_c. We follow Ref. [10] and use three criteria to constrain them:

(i) The first criterion requires that the high-order corrections should be less than 10%,

$$\mathrm{Convergence\,(CVG)} \equiv \left|\frac{\Pi^{\text{high-order}}_{F,j_l,s_l,\rho-\lambda}(\infty,T)}{\Pi_{F,j_l,s_l,\rho-\lambda}(\infty,T)}\right| \le 10\%, \tag{10}$$

where $\Pi^{\text{high-order}}_{F,j_l,s_l,\rho-\lambda}(\omega_c,T)$ denotes the high-order corrections, for example,

$$\Pi^{\text{high-order}}_{\Sigma_c,1,1,\rho\rho}(\omega_c,T) = \int_0^{\omega_c}\left[-\frac{19\langle g_s^2GG\rangle}{3072\pi^4}\omega^5\right]e^{-\omega/T}d\omega. \tag{11}$$

(ii) The second criterion requires that the pole contribution (PC) should be larger than 10%:

$$\mathrm{PC} \equiv \frac{\Pi_{F,j_l,s_l,\rho-\lambda}(\omega_c,T)}{\Pi_{F,j_l,s_l,\rho-\lambda}(\infty,T)} \ge 10\%. \tag{12}$$

We can use the first and second criteria together to arrive at an interval $T_{\min} < T < T_{\max}$ for a fixed threshold value ω_c.

(iii) The third criterion requires that the dependence of $m_{j,P,F,j_l,s_l,\rho-\lambda}$ on the threshold value ω_c should be weak.

Still use the baryon multiplet $[\Sigma_c(\mathbf{6}_F), 1, 1, \rho\rho]$ as an example. First, when we take $\omega_c = 3.2$ GeV, the Borel window can be evaluated to be $0.425\ \mathrm{GeV} < T < 0.487\ \mathrm{GeV}$, and the following numerical results can be obtained:

$$\begin{aligned} \bar\Lambda_{\Sigma_c,1,1,\rho\rho} &= 1.425\ \mathrm{GeV}, \\ K_{\Sigma_c,1,1,\rho\rho} &= -1.372\ \mathrm{GeV}^2, \\ \Sigma_{\Sigma_c,1,1,\rho\rho} &= 0.0091\ \mathrm{GeV}^2. \end{aligned} \tag{13}$$

Their variations are shown in Fig. 2 with respect to the Borel mass T. Inserting them into Eqs. (2) and (3), we can further obtain

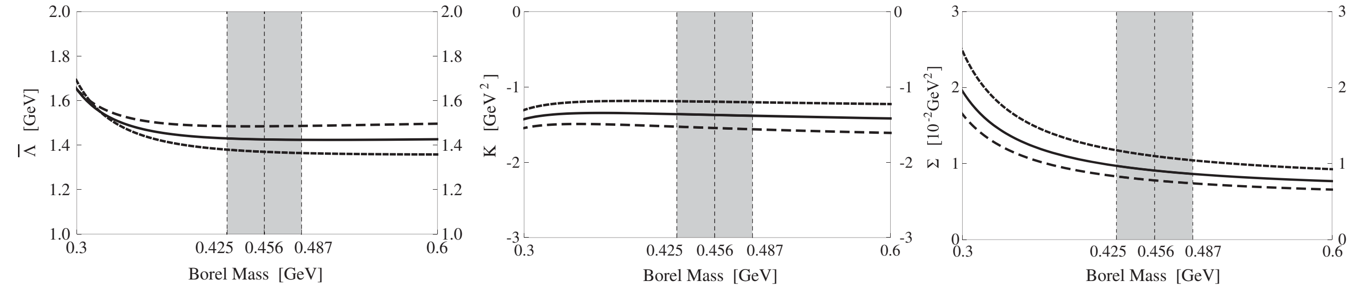

FIG. 2. Variations of $\bar{\Lambda}_{\Sigma_c,1,1,\rho\rho}$ (left), $K_{\Sigma_c,1,1,\rho\rho}$ (middle), and $\Sigma_{\Sigma_c,1,1,\rho\rho}$ (right) with respect to the Borel mass T, calculated using the charmed baryon doublet $[\Sigma_c(\mathbf{6}_F),1,1,\rho\rho]$. The short-dashed, solid, and long-dashed curves are obtained by fixing $\omega_c = 3.0$, 3.2, and 3.4 GeV, respectively.

$$
\begin{aligned}
m_{\Sigma_c(1/2^+)} &= 2.96\ \text{GeV},\\
m_{\Sigma_c(3/2^+)} &= 2.97\ \text{GeV},\\
\Delta m_{[\Sigma_c,1,1,\rho\rho]} &= 11\ \text{MeV},
\end{aligned}
\tag{14}
$$

where $m_{\Sigma_c(1/2^+)}$ and $m_{\Sigma_c(3/2^+)}$ are the masses of the $\Sigma_c(1/2^+)$ and $\Sigma_c(3/2^+)$ belonging to the baryon multiplet $[\Sigma_c(\mathbf{6}_F),1,1,\rho\rho]$ and $\Delta m_{[\Sigma_c,1,1,\rho\rho]}$ is their mass splitting. We show the variation of $m_{\Sigma_b(1/2^+)}$ with respect to the Borel mass T in the right panel of Fig. 3.

Second, we change the threshold value ω_c and redo the above process. We show the variation of $m_{\Sigma_c(1/2^+)}$ with respect to the threshold value ω_c in the left panel of Fig. 3. There are nonvanishing Borel windows as long as $\omega_c \geq 3.0$ GeV, and the ω_c dependence is weak and acceptable in the region $3.0\ \text{GeV} < \omega_c < 3.4$ GeV. The results for $\omega_c \leq 3.0$ GeV are also shown, for which cases we choose the Borel mass T when the PC defined in Eq. (12) is around 10%.

Finally, we choose our working regions to be $3.0\ \text{GeV} < \omega_c < 3.4$ GeV and $0.425\ \text{GeV} < T < 0.487$ GeV and obtain the numerical results for the baryon multiplet $[\Sigma_c(\mathbf{6}_F),1,1,\rho\rho]$,

$$
\begin{aligned}
m_{\Sigma_c(1/2^+)} &= 2.96^{+0.17}_{-0.12}\ \text{GeV},\\
m_{\Sigma_c(3/2^+)} &= 2.97^{+0.18}_{-0.13}\ \text{GeV},\\
\Delta m_{[\Sigma_c,1,1,\rho\rho]} &= 11^{+17}_{-9}\ \text{MeV},
\end{aligned}
\tag{15}
$$

where the central values correspond to $\omega_c = 3.2$ GeV and $T = 0.456$ GeV and the uncertainties come from the threshold value ω_c, the Borel mass T, the charm-quark mass m_c, and various quark and gluon condensates. We note that there are large theoretical uncertainties in our mass predictions, but the mass splitting within the same doublet is produced quite well with much less theoretical uncertainty because it does not depend much on the charm-quark mass [9,10,38,39].

To study the charmed baryon multiplets $[\Xi'_c(\mathbf{6}_F),1,1,\rho\rho]$ and $[\Omega_c(\mathbf{6}_F),1,1,\rho\rho]$, we fine tune the threshold value ω_c to be

$$
\begin{aligned}
\omega_c([\Xi'_c,1,1,\rho\rho]) &\approx 3.7\ \text{GeV},\\
\omega_c([\Omega_c,1,1,\rho\rho]) &\approx 4.2\ \text{GeV}
\end{aligned}
\tag{16}
$$

so that

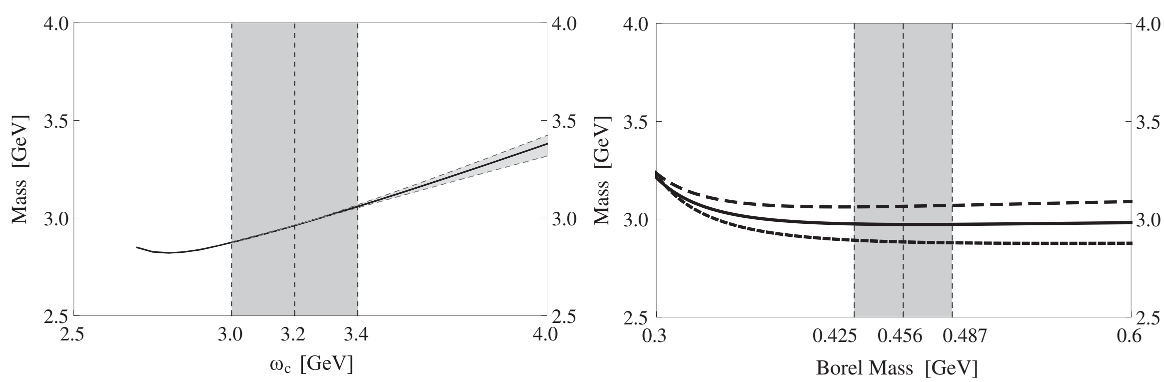

FIG. 3. Variations of $m_{\Sigma_c(1/2^+)}$ with respect to the threshold value ω_c (left) and the Borel mass T (right), calculated using the charmed baryon doublet $[\Sigma_c(\mathbf{6}_F),1,1,\rho\rho]$. The shady band in the left panel is obtained by changing T inside Borel windows. There exist nonvanishing working regions of T as long as $\omega_c \geq 3.0$ GeV, while the results for $\omega_c < 3.0$ GeV are also shown, for which cases we choose the Borel mass T when the PC defined in Eq. (12) is around 10%. In the right panel, the short-dashed, solid, and long-dashed curves are obtained by setting $\omega_c = 3.0$, 3.2, and 3.4 GeV, respectively.

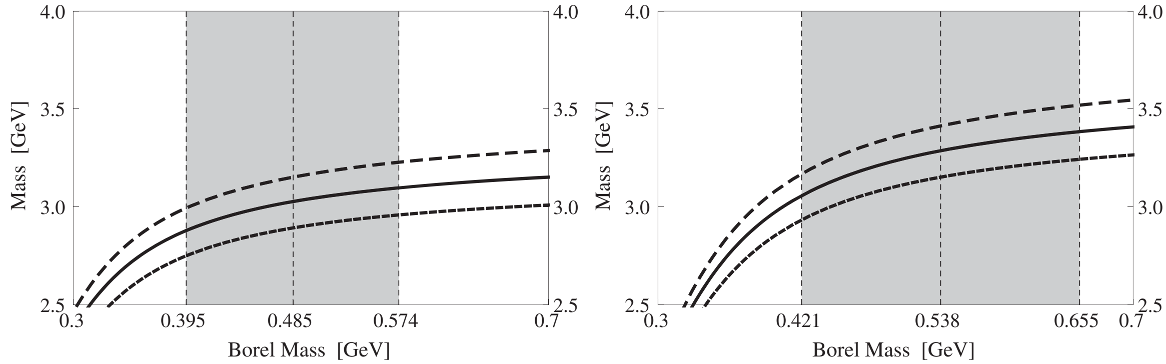

FIG. 4. Variations of $m_{\Xi_c'(1/2^+)}$ (left) and $m_{\Omega_c(1/2^+)}$ (right) with respect to the Borel mass T, calculated using the charmed baryon multiplets $[\Xi_c'(\mathbf{6}_F),1,1,\rho\rho]$ and $[\Omega_c(\mathbf{6}_F),1,1,\rho\rho]$. In the left figure, the short-dashed, solid, and long-dashed curves are obtained by setting $\omega_c = 3.5$, 3.7, and 3.9 GeV, respectively. In the left figure, the short-dashed, solid, and long-dashed curves are obtained by setting $\omega_c = 4.0$, 4.2, and 4.4 GeV, respectively.

$$\begin{aligned}&\omega_c([\Omega_c,1,1,\rho\rho]) - \omega_c([\Xi_c',1,1,\rho\rho])\\&\quad\approx \omega_c([\Xi_c',1,1,\rho\rho]) - \omega_c([\Sigma_c,1,1,\rho\rho])\\&\quad\approx 0.5\ \mathrm{GeV},\end{aligned}\tag{17}$$

the value of which is the same as those used in our previous studies on excited heavy baryons [9,10,39].

After fixing the threshold value $\omega_c([\Xi_c',1,1,\rho\rho])$ to be around 3.7 GeV, we can evaluate our working regions to be $3.5\ \mathrm{GeV} < \omega_c < 3.9\ \mathrm{GeV}$ and $0.395\ \mathrm{GeV} < T < 0.574\ \mathrm{GeV}$ and obtain the numerical results for the charmed baryon multiplet $[\Xi_c'(\mathbf{6}_F),1,1,\rho\rho]$,

$$\begin{aligned}m_{\Xi_c'(1/2^+)} &= 3.02^{+0.15}_{-0.21}\ \mathrm{GeV},\\ m_{\Xi_c'(3/2^+)} &= 3.03^{+0.15}_{-0.21}\ \mathrm{GeV},\\ \Delta m_{[\Xi_c',1,1,\rho\rho]} &= 7^{+8}_{-6}\ \mathrm{MeV},\end{aligned}\tag{18}$$

where the central values correspond to $\omega_c = 3.7$ GeV and $T = 0.485$ GeV. We show the variation of $m_{\Xi_c'(1/2^+)}$ with respect to the Borel mass T in the left panel of Fig. 4, where these curves are stable inside the Borel window $0.395\ \mathrm{GeV} < T < 0.574\ \mathrm{GeV}$. The masses of $\Xi_c'(1/2^+)$ and $\Xi_c'(3/2^+)$ are both consistent with the mass of the $\Xi_c(3123)$ observed by the *BABAR* Collaboration [19],

$$m^{\mathrm{exp}}_{\Xi_c(3123)} = 3122.9 \pm 1.3 \pm 0.3\ \mathrm{MeV},\tag{19}$$

which supports it to be a D-wave Ξ_c' state of $J^P = 1/2^+$ or $3/2^+$.

Similarly, after fixing $\omega_c([\Omega_c,1,1,\rho\rho])$ to be around 4.2 GeV, we can evaluate our working regions to be $4.0\ \mathrm{GeV} < \omega_c < 4.4\ \mathrm{GeV}$ and $0.421\ \mathrm{GeV} < T < 0.655\ \mathrm{GeV}$ and obtain the numerical results for the charmed baryon multiplet $[\Omega_c(\mathbf{6}_F),1,1,\rho\rho]$,

$$\begin{aligned}m_{\Omega_c(1/2^+)} &= 3.29^{+0.17}_{-0.25}\ \mathrm{GeV},\\ m_{\Omega_c(3/2^+)} &= 3.29^{+0.16}_{-0.25}\ \mathrm{GeV},\\ \Delta m_{[\Omega_c,1,1,\rho\rho]} &= 5^{+5}_{-4}\ \mathrm{MeV},\end{aligned}\tag{20}$$

where the central values correspond to $\omega_c = 4.2$ GeV and $T = 0.538$ GeV. We show the variation of $m_{\Omega_c(1/2^+)}$ with respect to the Borel mass T in the right panel of Fig. 4, where these curves are stable inside the Borel window $0.421\ \mathrm{GeV} < T < 0.655\ \mathrm{GeV}$.

We list the above results in Table I. Following the same procedures, we study the charmed baryon multiplets, $[\mathbf{6}_F,3,1,\lambda\lambda]$, $[\mathbf{6}_F,2,0,\rho\lambda]$, $[\mathbf{6}_F,3,1,\rho\rho]$, and $[\mathbf{6}_F,1,1,\lambda\lambda]$ and obtain the following:

(i) The baryon multiplet $[\mathbf{6}_F,3,1,\lambda\lambda]$ contains $\Sigma_c(5/2^+,7/2^+)$, $\Xi_c'(5/2^+,7/2^+)$, and $\Omega_c(5/2^+,7/2^+)$. We use them to perform sum rule analyses and obtain

$$\begin{aligned}m_{[\Sigma_c,3,1,\lambda\lambda]} &\sim 3.32^{+0.64}_{-0.24}\ \mathrm{GeV},\\ m_{[\Xi_c',3,1,\lambda\lambda]} &\sim 3.39^{+0.40}_{-0.20}\ \mathrm{GeV},\\ m_{[\Omega_c,3,1,\lambda\lambda]} &\sim 3.49^{+0.30}_{-0.19}\ \mathrm{GeV}.\end{aligned}\tag{21}$$

These values are also listed in Table I, and their variations are shown in Fig. 5 with respect to the threshold value ω_c. The masses of $\Xi_c'(5/2^+,7/2^+)$ are not far from the mass of the $\Xi_c(3123)$ [19], suggesting that the $\Xi_c(3123)$ may also be interpreted as a D-wave Ξ_c' state of $J^P = 5/2^+$ or $7/2^+$.

We note that we can only evaluate their average masses $\frac{1}{14}(6m_{\Sigma_c(5/2^+)} + 8m_{\Sigma_c(7/2^+)})$, $\frac{1}{14}(6m_{\Xi_c'(5/2^+)} + 8m_{\Xi_c'(7/2^+)})$, and $\frac{1}{14}(6m_{\Omega_c(5/2^+)} + 8m_{\Omega_c(7/2^+)})$ because their mass splittings related to the chromomagnetic interaction ($\Sigma_{\mathbf{6}_F,3,1,\lambda\lambda}$) are difficult to evaluate. More discussions can be found in Ref. [10].

(ii) The baryon multiplet $[\mathbf{6}_F,2,0,\rho\lambda]$ contains $\Sigma_c(3/2^+,5/2^+)$, $\Xi_c'(3/2^+,5/2^+)$, and $\Omega_c(3/2^+,5/2^+)$. We use them to perform sum rule analyses and obtain

TABLE I. Masses of the D-wave charmed baryons of the $SU(3)$ flavor $\mathbf{6}_F$, obtained using the charmed baryon multiplets $[\mathbf{6}_F, 1, 1, \rho\rho]$, $[\mathbf{6}_F, 3, 1, \lambda\lambda]$, and $[\mathbf{6}_F, 2, 0, \rho\lambda]$. For the charmed baryon multiplet $[\mathbf{6}_F, 3, 1, \lambda\lambda]$ containing $\Sigma_c(5/2^+, 7/2^+)$, $\Xi'_c(5/2^+, 7/2^+)$, and $\Omega_c(5/2^+, 7/2^+)$, we can only evaluate their average masses, i.e., $\frac{1}{14}(6m_{\Sigma_c(5/2^+)} + 8m_{\Sigma_c(7/2^+)})$, $\frac{1}{14}(6m_{\Xi'_c(5/2^+)} + 8m_{\Xi'_c(7/2^+)})$, and $\frac{1}{14}(6m_{\Omega_c(5/2^+)} + 8m_{\Omega_c(7/2^+)})$, as discussed in Ref. [10]. For the baryon multiplet $[\mathbf{6}_F, 2, 0, \rho\lambda]$ containing $\Sigma_c(3/2^+, 5/2^+)$, $\Xi'_c(3/2^+, 5/2^+)$, and $\Omega_c(3/2^+, 5/2^+)$, the mass differences among Σ_c, Ξ'_c, and Ω_c seem a bit large, so we do not use them to draw conclusions.

Multiplets	B	ω_c (GeV)	Working region (GeV)	$\bar\Lambda$ (GeV)	f (GeV^5)	K (GeV^2)	Σ (GeV^2)	Baryons (j^P)	Mass (GeV)	Difference (MeV)
$[\mathbf{6}_F, 1, 1, \rho\rho]$	Σ_c	3.2	$0.425 < T < 0.487$	1.425	0.079	−1.372	0.0091	$\Sigma_c(1/2^+)$	$2.96^{+0.17}_{-0.12}$	11^{+17}_{-9}
								$\Sigma_c(3/2^+)$	$2.97^{+0.18}_{-0.13}$	
	Ξ'_c	3.7	$0.395 < T < 0.574$	1.561	0.146	−0.961	0.0057	$\Xi'_c(1/2^+)$	$3.02^{+0.15}_{-0.21}$	7^{+8}_{-6}
								$\Xi'_c(3/2^+)$	$3.03^{+0.15}_{-0.21}$	
	Ω_c	4.2	$0.421 < T < 0.655$	1.761	0.274	−1.302	0.0042	$\Omega_c(1/2^+)$	$3.29^{+0.17}_{-0.25}$	5^{+5}_{-4}
								$\Omega_c(3/2^+)$	$3.29^{+0.16}_{-0.25}$	
$[\mathbf{6}_F, 3, 1, \lambda\lambda]$	Σ_c	3.1	$0.445 < T < 0.459$	1.432	0.014	−3.139	–	$\Sigma_c(5/2^+)$	$3.32^{+0.64}_{-0.24}$	–
								$\Sigma_c(7/2^+)$		
	Ξ'_c	3.3	$0.458 < T < 0.485$	1.509	0.018	−3.064	–	$\Xi'_c(5/2^+)$	$3.39^{+0.40}_{-0.20}$	–
								$\Xi'_c(7/2^+)$		
	Ω_c	3.5	$0.490 < T < 0.509$	1.612	0.024	−3.085	–	$\Omega_c(5/2^+)$	$3.49^{+0.30}_{-0.19}$	–
								$\Omega_c(7/2^+)$		
$[\mathbf{6}_F, 2, 0, \rho\lambda]$	Σ_c	3.0	$0.398 < T < 0.457$	1.336	0.022	−2.156	0.0082	$\Sigma_c(3/2^+)$	$3.02^{+0.26}_{-0.16}$	16^{+26}_{-14}
								$\Sigma_c(5/2^+)$	$3.04^{+0.28}_{-0.17}$	
	Ξ'_c	3.7	$0.505 < T < 0.543$	1.793	0.076	−2.214	0.0055	$\Xi'_c(3/2^+)$	$3.50^{+0.15}_{-0.13}$	11^{+12}_{-9}
								$\Xi'_c(5/2^+)$	$3.51^{+0.15}_{-0.13}$	
	Ω_c	4.0	$0.561 < T < 0.575$	1.986	0.115	−2.121	0.0046	$\Omega_c(3/2^+)$	$3.67^{+0.14}_{-0.14}$	9^{+9}_{-8}
								$\Omega_c(5/2^+)$	$3.68^{+0.15}_{-0.14}$	

$$
\begin{aligned}
m_{\Sigma_c(3/2^+)} &= 3.02^{+0.26}_{-0.16}\ \text{GeV},\\
m_{\Sigma_c(5/2^+)} &= 3.04^{+0.28}_{-0.17}\ \text{GeV},\\
\Delta m_{[\Sigma_c,2,0,\rho\lambda]} &= 16^{+26}_{-14}\ \text{MeV},\\
m_{\Xi'_c(3/2^+)} &= 3.50^{+0.15}_{-0.13}\ \text{GeV},\\
m_{\Xi'_c(5/2^+)} &= 3.51^{+0.15}_{-0.13}\ \text{GeV},\\
\Delta m_{[\Xi'_c,2,0,\rho\lambda]} &= 11^{+12}_{-9}\ \text{MeV},\\
m_{\Omega_c(3/2^+)} &= 3.67^{+0.14}_{-0.14}\ \text{GeV},\\
m_{\Omega_c(5/2^+)} &= 3.68^{+0.15}_{-0.14}\ \text{GeV},\\
\Delta m_{[\Omega_c,2,0,\rho\lambda]} &= 9^{+9}_{-8}\ \text{MeV}.
\end{aligned}
\tag{22}
$$

These values are also listed in Table I, and variations of $m_{\Sigma_c(3/2^+)}$, $m_{\Xi'_c(3/2^+)}$, and $m_{\Omega_c(3/2^+)}$ are shown in Fig. 6 with respect to the threshold value ω_c. However, the mass differences among $\Sigma_c(3/2^+, 5/2^+)$, $\Xi'_c(3/2^+, 5/2^+)$, and $\Omega_c(3/2^+, 5/2^+)$ seem a bit large, and we shall not use them to draw conclusions.

(iii) We use the baryon multiplet $[\mathbf{6}_F, 3, 1, \rho\rho]$ to perform sum rule analyses, but there do not exist Borel windows when $\omega_c \le 4.0$ GeV. We also use the baryon multiplet $[\mathbf{6}_F, 1, 1, \lambda\lambda]$ to perform sum rule analyses, but the obtained results depend significantly on the threshold value ω_c. Hence, these results cannot

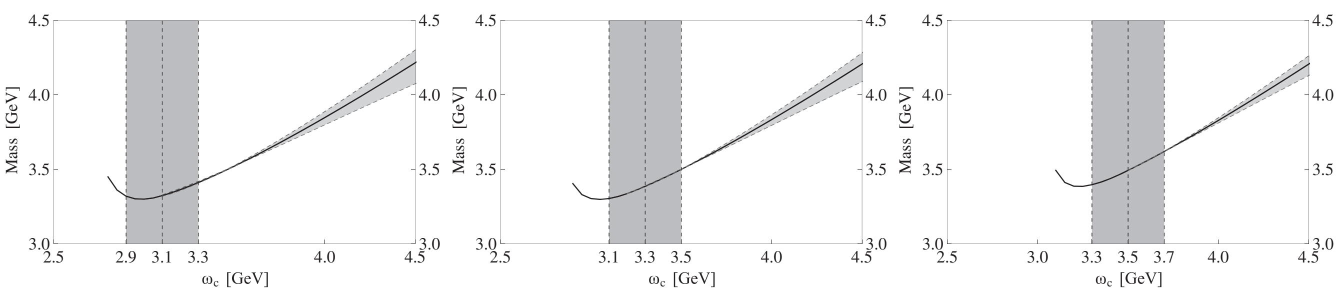

FIG. 5. Variations of $m_{[\Sigma_c,3,1,\lambda\lambda]}$ (left), $m_{[\Xi'_c,3,1,\lambda\lambda]}$ (middle), and $m_{[\Omega_c,3,1,\lambda\lambda]}$ (right) with respect to the threshold value ω_c, calculated using the charmed baryon doublet $[\mathbf{6}_F, 3, 1, \lambda\lambda]$. The shady bands in these figures are obtained by changing T inside Borel windows. There exist Borel windows as long as $\omega_c([\Sigma_c, 3, 1, \lambda\lambda]) \ge 3.1$ GeV (left), $\omega_c([\Xi'_c, 3, 1, \lambda\lambda]) \ge 3.2$ GeV (left), and $\omega_c([\Omega_c, 3, 1, \lambda\lambda]) \ge 3.5$ GeV (right). Accordingly, we choose ω_c to be around 3.1, 3.3, and 3.5 GeV in the left, middle, and right panels, respectively.

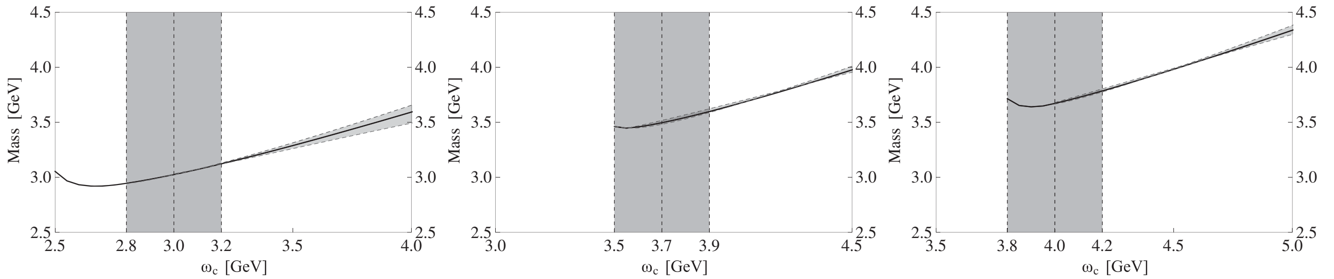

FIG. 6. Variations of $m_{\Sigma_c(3/2^+)}$ (left), $m_{\Xi'_c(3/2^+)}$ (middle), and $m_{\Omega_c(5/2^+)}$ (right) with respect to the threshold value ω_c, calculated using the charmed baryon multiplet $[\mathbf{6}_F, 2, 0, \rho\lambda]$. The shady bands in these figures are obtained by changing T inside Borel windows. There exist Borel windows as long as $\omega_c([\Sigma_c, 2, 0, \rho\lambda]) \geq 2.8$ GeV (left), $\omega_c([\Xi'_c, 2, 0, \rho\lambda]) \geq 3.6$ GeV (middle), and $\omega_c([\Omega_c, 2, 0, \rho\lambda]) \geq 4.0$ GeV (right). Accordingly, we choose ω_c to be around 3.0, 3.7, and 4.0 GeV in the left, middle, and right panels, respectively.

be interpreted well, and we shall not use them to draw conclusions.

IV. SUMMARY

In this paper, we used the method of QCD sum rules within HQET to study the *D*-wave charmed baryons of the $SU(3)$ flavor $\mathbf{6}_F$ and calculated their masses up to the order $\mathcal{O}(1/m_Q)$. We investigated five charmed baryon doublets, $[\mathbf{6}_F, 1, 1, \rho\rho]$, $[\mathbf{6}_F, 3, 1, \rho\rho]$, $[\mathbf{6}_F, 1, 1, \lambda\lambda]$, $[\mathbf{6}_F, 3, 1, \lambda\lambda]$, and $[\mathbf{6}_F, 2, 0, \rho\lambda]$ (we note that in Ref. [10] we failed to construct the currents belonging to the other two multiplets, $[\mathbf{6}_F, 2, 1, \rho\rho]$ and $[\mathbf{6}_F, 2, 1, \lambda\lambda]$). The results are summarized in Table I for the charmed baryon multiplets $[\mathbf{6}_F, 1, 1, \rho\rho]$, $[\mathbf{6}_F, 3, 1, \lambda\lambda]$, and $[\mathbf{6}_F, 2, 0, \rho\lambda]$, and those obtained using the former two multiplets ($[\mathbf{6}_F, 1, 1, \rho\rho]$ and $[\mathbf{6}_F, 3, 1, \lambda\lambda]$) are reasonable/better. We note that there are large theoretical uncertainties in our mass predictions, but the mass splittings within the same doublet are produced quite well with much less theoretical uncertainty.

Our results suggest that the $\Xi_c(3123)$ observed by the *BABAR* Collaboration [19] can be interpreted well as a *D*-wave Ξ'_c state, while its quantum numbers cannot be determined. It may belong to either the charmed baryon doublet $[\Xi'_c(\mathbf{6}_F), 1, 1, \rho\rho]$ or $[\Xi'_c(\mathbf{6}_F), 3, 1, \lambda\lambda]$, but in both cases, there would be a partner state close to it. Our results also suggest that there may exist as many as four *D*-wave Ω_c states in the energy region 3.3–3.5 GeV. They are the *D*-wave Ω_c states of $J^P = 1/2^+$, $3/2^+$, $5/2^+$, and $7/2^+$; the former two belong to the charmed baryon multiplet $[\mathbf{6}_F, 1, 1, \rho\rho]$, and the latter two belong to $[\mathbf{6}_F, 3, 1, \lambda\lambda]$. Recalling that the LHCb experiment observed as many as

TABLE II. Masses of the *D*-wave bottom baryons of the $SU(3)$ flavor $\mathbf{6}_F$, obtained using the bottom-baryon multiplets $[\mathbf{6}_F, 1, 1, \rho\rho]$, $[\mathbf{6}_F, 3, 1, \lambda\lambda]$, and $[\mathbf{6}_F, 2, 0, \rho\lambda]$.

Multiplets	B	ω_c (GeV)	Working region (GeV)	$\bar{\Lambda}$ (GeV)	f (GeV^5)	K (GeV^2)	Σ (GeV^2)	Baryons (j^P)	Mass (GeV)	Difference (MeV)
$[\mathbf{6}_F, 1, 1, \rho\rho]$	Σ_b	3.2	$0.425 < T < 0.487$	1.425	0.079	−1.372	0.0091	$\Sigma_b(1/2^+)$	$6.28^{+0.18}_{-0.12}$	2^{+4}_{-2}
								$\Sigma_b(3/2^+)$	$6.28^{+0.18}_{-0.12}$	
	Ξ'_b	3.7	$0.395 < T < 0.574$	1.561	0.146	−0.961	0.0057	$\Xi'_b(1/2^+)$	$6.39^{+0.11}_{-0.14}$	1^{+2}_{-1}
								$\Xi'_b(3/2^+)$	$6.39^{+0.11}_{-0.14}$	
	Ω_b	4.2	$0.421 < T < 0.655$	1.761	0.274	−1.302	0.0042	$\Omega_b(1/2^+)$	$6.61^{+0.12}_{-0.16}$	1^{+1}_{-1}
								$\Omega_b(3/2^+)$	$6.61^{+0.12}_{-0.16}$	
$[\mathbf{6}_F, 3, 1, \lambda\lambda]$	Σ_b	3.1	$0.445 < T < 0.459$	1.432	0.014	−3.139	⋯	$\Sigma_b(5/2^+)$	$6.38^{+0.43}_{-0.17}$	⋯
								$\Sigma_b(7/2^+)$		
	Ξ'_b	3.3	$0.458 < T < 0.485$	1.509	0.018	−3.064	⋯	$\Xi'_b(5/2^+)$	$6.45^{+0.28}_{-0.15}$	⋯
								$\Xi'_b(7/2^+)$		
	Ω_b	3.5	$0.490 < T < 0.509$	1.612	0.024	−3.085	⋯	$\Omega_b(5/2^+)$	$6.55^{+0.22}_{-0.14}$	⋯
								$\Omega_b(7/2^+)$		
$[\mathbf{6}_F, 2, 0, \rho\lambda]$	Σ_b	3.0	$0.398 < T < 0.457$	1.336	0.022	−2.156	0.0082	$\Sigma_b(3/2^+)$	$6.23^{+0.19}_{-0.13}$	3^{+6}_{-3}
								$\Sigma_b(5/2^+)$	$6.23^{+0.20}_{-0.13}$	
	Ξ'_b	3.7	$0.505 < T < 0.543$	1.793	0.076	−2.214	0.0055	$\Xi'_b(3/2^+)$	$6.69^{+0.11}_{-0.09}$	2^{+3}_{-2}
								$\Xi'_b(5/2^+)$	$6.69^{+0.09}_{-0.09}$	
	Ω_b	4.0	$0.561 < T < 0.575$	1.986	0.115	−2.121	0.0046	$\Omega_b(3/2^+)$	$6.88^{+0.09}_{-0.08}$	2^{+2}_{-2}
								$\Omega_b(5/2^+)$	$6.88^{+0.09}_{-0.08}$	

five excited Ω_c states [8] at the same time, we propose searching for these D-wave Ω_c states in the future LHCb and BelleII experiments in order to study the fine structure of the strong interaction. We note that the doubly charmed baryon Ξ_{cc}^{++} was recently discovered by the LHCb Collaboration [66] and can also be an idea platform to study the fine structure of the strong interaction [67].

Following the same procedures, we have investigated the D-wave bottom baryons of the $SU(3)$ flavor $\mathbf{6}_F$. The pole mass of the bottom quark $m_b = 4.78 \pm 0.06$ GeV [1] is used, and the obtained results are listed in Table II. We also suggest searching for them in further experiments. To end this paper, we note that not only masses but also decay and production properties are useful in understanding the heavy baryons [66], and J-PARC is planning an experimental project of such studies [68]. In the near future, the joint efforts from experimentalists and theorists will be helpful in identifying more and more charmed and bottom baryons.

ACKNOWLEDGMENTS

We thank Xian-Hui Zhong and Cheng-Ping Shen for useful discussions. This project is supported by the National Natural Science Foundation of China under Grants No. 11722540, No. 11475015, No. 11375024, No. 11222547, No. 11175073, No. 11575008, and No. 11621131001; the 973 program; the Ministry of Education of China (SRFDP under Grant No. 20120211110002 and the Fundamental Research Funds for the Central Universities); the Key Natural Science Research Program of Anhui Educational Committee (Grant No. KJ2016A774); and the National Program for Support of Top-notch Youth Professionals. A. H. is supported by Grants-in-Aid for Scientific Research [Grant No. JP17K05441(C)].

APPENDIX: OTHER SUM RULES

In this Appendix, we list the sum rules for other currents with different quark contents.

$$\begin{aligned}\Pi_{\Xi_c',1,1,\rho\rho} &= f^2_{\Xi_c',1,1,\rho\rho} e^{-2\bar\Lambda_{\Xi_c',1,1,\rho\rho}/T} \\ &= \int_{2m_s}^{\omega_c}\left[\frac{11}{80640\pi^4}\omega^9 - \frac{51m_s^2}{8960\pi^4}\omega^7 - \frac{m_s\langle\bar qq\rangle}{16\pi^2}\omega^5 + \frac{3m_s\langle\bar ss\rangle}{40\pi^2}\omega^5 - \frac{19\langle g_s^2GG\rangle}{3072\pi^4}\omega^5\right.\\ &\quad\left. + \frac{33m_s^2\langle g_s^2GG\rangle}{512\pi^4}\omega^3 - \frac{m_s\langle g_s^2GG\rangle\langle\bar qq\rangle}{16\pi^2}\omega - \frac{31m_s\langle g_s^2GG\rangle\langle\bar ss\rangle}{128\pi^2}\omega\right]e^{-\omega/T}d\omega, \end{aligned} \tag{A1}$$

$$\begin{aligned} f^2_{\Xi_c',1,1,\rho\rho}K_{\Xi_c',1,1,\rho\rho}e^{-2\bar\Lambda_{\Xi_c',1,1,\rho\rho}/T} &= \int_{2m_s}^{\omega_c}\left[-\frac{59}{2217600\pi^4}\omega^{11} + \frac{29m_s^2}{17920\pi^4}\omega^9 + \frac{m_s\langle\bar qq\rangle}{32\pi^2}\omega^7 - \frac{11m_s\langle\bar ss\rangle}{224\pi^2}\omega^7 + \frac{m_s\langle g_s\bar q\sigma Gq\rangle}{4\pi^2}\omega^5\right.\\ &\quad + \frac{69\langle g_s^2GG\rangle}{35840\pi^4}\omega^7 - \frac{2527m_s^2\langle g_s^2GG\rangle}{61440\pi^4}\omega^5 - \frac{35m_s\langle g_s^2GG\rangle\langle\bar qq\rangle}{192\pi^2}\omega^3 + \frac{839m_s\langle g_s^2GG\rangle\langle\bar ss\rangle}{2304\pi^2}\omega^3\\ &\quad\left. - \frac{19m_s\langle g_s^2GG\rangle\langle g_s\bar q\sigma Gq\rangle}{96\pi^2}\omega\right]e^{-\omega/T}d\omega, \end{aligned} \tag{A2}$$

$$f^2_{\Xi_c',1,1,\rho\rho}\Sigma_{\Xi_c',1,1,\rho\rho}e^{-2\bar\Lambda_{\Xi_c',1,1,\rho\rho}/T} = \int_{2m_s}^{\omega_c}\left[\frac{37\langle g_s^2GG\rangle}{322560\pi^4}\omega^7 - \frac{7m_s^2\langle g_s^2GG\rangle}{3840\pi^4}\omega^5 + \frac{19m_s\langle g_s^2GG\rangle\langle\bar ss\rangle}{1152\pi^2}\omega^3\right]e^{-\omega/T}d\omega. \tag{A3}$$

$$\begin{aligned}\Pi_{\Omega_c,1,1,\rho\rho} &= f^2_{\Omega_c,1,1,\rho\rho}e^{-2\bar\Lambda_{\Omega_c,1,1,\rho\rho}/T}\\ &= \int_{4m_s}^{\omega_c}\left[\frac{11}{80640\pi^4}\omega^9 - \frac{9m_s^2}{1120\pi^4}\omega^7 + \frac{9m_s^4}{80\pi^4}\omega^5 + \frac{m_s\langle\bar ss\rangle}{40\pi^2}\omega^5 - \frac{7m_s^3\langle\bar ss\rangle}{8\pi^2}\omega^3\right.\\ &\quad\left. - \frac{19\langle g_s^2GG\rangle}{3072\pi^4}\omega^5 + \frac{37m_s^2\langle g_s^2GG\rangle}{256\pi^4}\omega^3 - \frac{3m_s^4\langle g_s^2GG\rangle}{64\pi^4}\omega - \frac{39m_s\langle g_s^2GG\rangle\langle\bar ss\rangle}{64\pi^2}\omega\right]e^{-\omega/T}d\omega, \end{aligned} \tag{A4}$$

$$\begin{aligned} f^2_{\Omega_c,1,1,\rho\rho}K_{\Omega_c,1,1,\rho\rho}e^{-2\bar\Lambda_{\Omega_c,1,1,\rho\rho}/T} &= \int_{4m_s}^{\omega_c}\left[-\frac{59}{2217600\pi^4}\omega^{11} + \frac{7m_s^2}{2880\pi^4}\omega^9 - \frac{7m_s^4}{160\pi^4}\omega^7 - \frac{m_s\langle\bar ss\rangle}{28\pi^2}\omega^7 + \frac{15m_s^3\langle\bar ss\rangle}{16\pi^2}\omega^5\right.\\ &\quad + \frac{m_s\langle g_s\bar s\sigma Gs\rangle}{2\pi^2}\omega^5 + \frac{23\langle g_s^2GG\rangle}{11520\pi^4}\omega^7 - \frac{2153m_s^2\langle g_s^2GG\rangle}{30720\pi^4}\omega^5 + \frac{23m_s^4\langle g_s^2GG\rangle}{64\pi^4}\omega^3\\ &\quad\left. + \frac{149m_s\langle g_s^2GG\rangle\langle\bar ss\rangle}{384\pi^2}\omega^3 - \frac{239m_s^3\langle g_s^2GG\rangle\langle\bar ss\rangle}{192\pi^2}\omega + \frac{17m_s\langle g_s^2GG\rangle\langle g_s\bar s\sigma Gs\rangle}{48\pi^2}\omega\right]e^{-\omega/T}d\omega, \end{aligned} \tag{A5}$$

$$f^2_{\Omega_c,1,1,\rho\rho}\Sigma_{\Omega_c,1,1,\rho\rho}e^{-2\bar{\Lambda}_{\Omega_c,1,1,\rho\rho}/T} = \int_{4m_s}^{\omega_c}\left[\frac{37\langle g_s^2GG\rangle}{322560\pi^4}\omega^7 - \frac{7m_s^2\langle g_s^2GG\rangle}{1920\pi^4}\omega^5 + \frac{19m_s\langle g_s^2GG\rangle\langle\bar{s}s\rangle}{576\pi^2}\omega^3\right]e^{-\omega/T}d\omega. \tag{A6}$$

$$\Pi_{\Sigma_c,3,1,\lambda\lambda} = f^2_{\Sigma_c,3,1,\lambda\lambda}e^{-2\bar{\Lambda}_{\Sigma_c,3,1,\lambda\lambda}/T} = \int_0^{\omega_c}\left[\frac{1}{193536\pi^4}\omega^9 - \frac{13\langle g_s^2GG\rangle}{46080\pi^4}\omega^5\right]e^{-\omega/T}d\omega, \tag{A7}$$

$$f^2_{\Sigma_c,3,1,\lambda\lambda}K_{\Sigma_c,3,1,\lambda\lambda}e^{-2\bar{\Lambda}_{\Sigma_c,3,1,\lambda\lambda}/T} = \int_0^{\omega_c}\left[-\frac{41}{21288960\pi^4}\omega^{11} + \frac{353\langle g_s^2GG\rangle}{3870720\pi^4}\omega^7\right]e^{-\omega/T}d\omega. \tag{A8}$$

$$\Pi_{\Xi'_c,3,1,\lambda\lambda} = f^2_{\Xi'_c,3,1,\lambda\lambda}e^{-2\bar{\Lambda}_{\Xi'_c,3,1,\lambda\lambda}/T} = \int_{2m_s}^{\omega_c}\left[\frac{1}{193536\pi^4}\omega^9 - \frac{m_s^2}{4480\pi^4}\omega^7 - \frac{m_s\langle\bar{q}q\rangle}{480\pi^2}\omega^5 + \frac{m_s\langle\bar{s}s\rangle}{320\pi^2}\omega^5 - \frac{13\langle g_s^2GG\rangle}{46080\pi^4}\omega^5 + \frac{5m_s^2\langle g_s^2GG\rangle}{3072\pi^4}\omega^3 - \frac{m_s\langle g_s^2GG\rangle\langle\bar{s}s\rangle}{256\pi^2}\omega\right]e^{-\omega/T}d\omega, \tag{A9}$$

$$f^2_{\Xi'_c,3,1,\lambda\lambda}K_{\Xi'_c,3,1,\lambda\lambda}e^{-2\bar{\Lambda}_{\Xi'_c,3,1,\lambda\lambda}/T} = \int_{2m_s}^{\omega_c}\left[-\frac{41}{21288960\pi^4}\omega^{11} + \frac{17m_s^2}{161280\pi^4}\omega^9 + \frac{m_s\langle\bar{q}q\rangle}{960\pi^2}\omega^7 - \frac{9m_s\langle\bar{s}s\rangle}{4480\pi^2}\omega^7 + \frac{353\langle g_s^2GG\rangle}{3870720\pi^4}\omega^7 - \frac{151m_s^2\langle g_s^2GG\rangle}{122880\pi^4}\omega^5 - \frac{5m_s\langle g_s^2GG\rangle\langle\bar{q}q\rangle}{1728\pi^2}\omega^3 + \frac{31m_s\langle g_s^2GG\rangle\langle\bar{s}s\rangle}{4608\pi^2}\omega^3\right]e^{-\omega/T}d\omega. \tag{A10}$$

$$\begin{aligned}\Pi_{\Omega_c,3,1,\lambda\lambda} &= f^2_{\Omega_c,3,1,\lambda\lambda}e^{-2\bar{\Lambda}_{\Omega_c,3,1,\lambda\lambda}/T} \\ &= \int_{4m_s}^{\omega_c}\left[\frac{1}{193536\pi^4}\omega^9 - \frac{3m_s^2}{8960\pi^4}\omega^7 + \frac{3m_s^4}{640\pi^4}\omega^5 + \frac{m_s\langle\bar{s}s\rangle}{480\pi^2}\omega^5 - \frac{m_s^3\langle\bar{s}s\rangle}{24\pi^2}\omega^3 - \frac{13\langle g_s^2GG\rangle}{46080\pi^4}\omega^5 + \frac{5m_s^2\langle g_s^2GG\rangle}{1536\pi^4}\omega^3 - \frac{m_s\langle g_s^2GG\rangle\langle\bar{s}s\rangle}{128\pi^2}\omega\right]e^{-\omega/T}d\omega,\end{aligned} \tag{A11}$$

$$\begin{aligned}f^2_{\Omega_c,3,1,\lambda\lambda}K_{\Omega_c,3,1,\lambda\lambda}e^{-2\bar{\Lambda}_{\Omega_c,3,1,\lambda\lambda}/T} = \int_{4m_s}^{\omega_c}\Bigg[&-\frac{41}{21288960\pi^4}\omega^{11} + \frac{3m_s^2}{17920\pi^4}\omega^9 - \frac{27m_s^4}{8960\pi^4}\omega^7 - \frac{13m_s\langle\bar{s}s\rangle}{6720\pi^2}\omega^7 + \frac{m_s^3\langle\bar{s}s\rangle}{24\pi^2}\omega^5 \\ &+ \frac{353\langle g_s^2GG\rangle}{3870720\pi^4}\omega^7 - \frac{131m_s^2\langle g_s^2GG\rangle}{61440\pi^4}\omega^5 + \frac{7m_s^4\langle g_s^2GG\rangle}{1152\pi^4}\omega^3 \\ &+ \frac{53m_s\langle g_s^2GG\rangle\langle\bar{s}s\rangle}{6912\pi^2}\omega^3 - \frac{m_s^3\langle g_s^2GG\rangle\langle\bar{s}s\rangle}{64\pi^2}\omega\Bigg]e^{-\omega/T}d\omega.\end{aligned} \tag{A12}$$

$$\Pi_{\Sigma_c,2,0,\rho\lambda} = f^2_{\Sigma_c,2,0,\rho\lambda}e^{-2\bar{\Lambda}_{\Sigma_c,2,0,\rho\lambda}/T} = \int_0^{\omega_c}\left[\frac{1}{48384\pi^4}\omega^9 - \frac{5\langle g_s^2GG\rangle}{6912\pi^4}\omega^5\right]e^{-\omega/T}d\omega, \tag{A13}$$

$$f^2_{\Sigma_c,2,0,\rho\lambda}K_{\Sigma_c,2,0,\rho\lambda}e^{-2\bar{\Lambda}_{\Sigma_c,2,0,\rho\lambda}/T} = \int_0^{\omega_c}\left[-\frac{1}{168960\pi^4}\omega^{11} + \frac{227\langle g_s^2GG\rangle}{1451520\pi^4}\omega^7\right]e^{-\omega/T}d\omega, \tag{A14}$$

$$f^2_{\Sigma_c,2,0,\rho\lambda}\Sigma_{\Sigma_c,2,0,\rho\lambda}e^{-2\bar{\Lambda}_{\Sigma_c,2,0,\rho\lambda}/T} = \int_0^{\omega_c}\left[\frac{\langle g_s^2GG\rangle}{48384\pi^4}\omega^7\right]e^{-\omega/T}d\omega. \tag{A15}$$

$$\begin{aligned}\Pi_{\Xi'_c,2,0,\rho\lambda} &= f^2_{\Xi'_c,2,0,\rho\lambda}e^{-2\bar{\Lambda}_{\Xi'_c,2,0,\rho\lambda}/T} \\ &= \int_{2m_s}^{\omega_c}\left[\frac{1}{48384\pi^4}\omega^9 - \frac{m_s^2}{1008\pi^4}\omega^7 - \frac{m_s\langle\bar{q}q\rangle}{72\pi^2}\omega^5 + \frac{m_s\langle\bar{s}s\rangle}{48\pi^2}\omega^5 - \frac{5m_s\langle g_s\bar{q}\sigma Gq\rangle}{72\pi^2}\omega^3 - \frac{5\langle g_s^2GG\rangle}{6912\pi^4}\omega^5 + \frac{5m_s^2\langle g_s^2GG\rangle}{432\pi^4}\omega^3 - \frac{5m_s\langle g_s^2GG\rangle\langle\bar{s}s\rangle}{108\pi^2}\omega\right]e^{-\omega/T}d\omega,\end{aligned} \tag{A16}$$

$$f^2_{\Xi'_c,2,0,\rho\lambda} K_{\Xi'_c,2,0,\rho\lambda} e^{-2\bar{\Lambda}_{\Xi'_c,2,0,\rho\lambda}/T} = \int_{2m_s}^{\omega_c} \Big[-\frac{1}{168960\pi^4}\omega^{11} + \frac{137 m_s^2}{362880\pi^4}\omega^9 + \frac{37 m_s \langle\bar{q}q\rangle}{5040\pi^2}\omega^7 - \frac{17 m_s \langle\bar{s}s\rangle}{1344\pi^2}\omega^7 + \frac{m_s\langle g_s\bar{q}\sigma Gq\rangle}{15\pi^2}\omega^5 + \frac{227\langle g_s^2 GG\rangle}{1451520\pi^4}\omega^7 - \frac{3419 m_s^2\langle g_s^2 GG\rangle}{552960\pi^4}\omega^5 - \frac{13 m_s\langle g_s^2 GG\rangle\langle\bar{q}q\rangle}{648\pi^2}\omega^3 + \frac{461 m_s\langle g_s^2 GG\rangle\langle\bar{s}s\rangle}{10368\pi^2}\omega^3 - \frac{11 m_s\langle g_s^2 GG\rangle\langle g_s\bar{q}\sigma Gq\rangle}{1728\pi^2}\omega \Big] e^{-\omega/T} d\omega, \tag{A17}$$

$$f^2_{\Xi'_c,2,0,\rho\lambda} \Sigma_{\Xi'_c,2,0,\rho\lambda} e^{-2\bar{\Lambda}_{\Xi'_c,2,0,\rho\lambda}/T} = \int_{2m_s}^{\omega_c} \Big[\frac{\langle g_s^2 GG\rangle}{48384\pi^4}\omega^7 - \frac{m_s^2\langle g_s^2 GG\rangle}{2304\pi^4}\omega^5 + \frac{5 m_s\langle g_s^2 GG\rangle\langle\bar{s}s\rangle}{864\pi^2}\omega^3 \Big] e^{-\omega/T} d\omega. \tag{A18}$$

$$\Pi_{\Omega_c,2,0,\rho\lambda} = f^2_{\Omega_c,2,0,\rho\lambda} e^{-2\bar{\Lambda}_{\Omega_c,2,0,\rho\lambda}/T} = \int_{4m_s}^{\omega_c} \Big[\frac{1}{48384\pi^4}\omega^9 - \frac{m_s^2}{672\pi^4}\omega^7 + \frac{m_s^4}{48\pi^4}\omega^5 + \frac{m_s\langle\bar{s}s\rangle}{72\pi^2}\omega^5 - \frac{5 m_s^3\langle\bar{s}s\rangle}{18\pi^2}\omega^3 - \frac{5 m_s\langle g_s\bar{s}\sigma Gs\rangle}{36\pi^2}\omega^3 - \frac{5\langle g_s^2 GG\rangle}{6912\pi^4}\omega^5 + \frac{5 m_s^2\langle g_s^2 GG\rangle}{216\pi^4}\omega^3 - \frac{5 m_s\langle g_s^2 GG\rangle\langle\bar{s}s\rangle}{54\pi^2}\omega \Big] e^{-\omega/T} d\omega, \tag{A19}$$

$$f^2_{\Omega_c,2,0,\rho\lambda} K_{\Omega_c,2,0,\rho\lambda} e^{-2\bar{\Lambda}_{\Omega_c,2,0,\rho\lambda}/T} = \int_{4m_s}^{\omega_c} \Big[-\frac{1}{168960\pi^4}\omega^{11} + \frac{211 m_s^2}{362880\pi^4}\omega^9 - \frac{211 m_s^4}{20160\pi^4}\omega^7 - \frac{107 m_s\langle\bar{s}s\rangle}{10080\pi^2}\omega^7 + \frac{181 m_s^3\langle\bar{s}s\rangle}{720\pi^2}\omega^5 + \frac{2 m_s\langle g_s\bar{s}\sigma Gs\rangle}{15\pi^2}\omega^5 + \frac{227\langle g_s^2 GG\rangle}{1451520\pi^4}\omega^7 - \frac{2839 m_s^2\langle g_s^2 GG\rangle}{276480\pi^4}\omega^5 + \frac{427 m_s^4\langle g_s^2 GG\rangle}{13824\pi^4}\omega^3 + \frac{253 m_s\langle g_s^2 GG\rangle\langle\bar{s}s\rangle}{5184\pi^2}\omega^3 - \frac{263 m_s^3\langle g_s^2 GG\rangle\langle\bar{s}s\rangle}{3456\pi^2}\omega - \frac{11 m_s\langle g_s^2 GG\rangle\langle g_s\bar{s}\sigma Gs\rangle}{864\pi^2}\omega \Big] e^{-\omega/T} d\omega, \tag{A20}$$

$$f^2_{\Omega_c,2,0,\rho\lambda} \Sigma_{\Omega_c,2,0,\rho\lambda} e^{-2\bar{\Lambda}_{\Omega_c,2,0,\rho\lambda}/T} = \int_{4m_s}^{\omega_c} \Big[\frac{\langle g_s^2 GG\rangle}{48384\pi^4}\omega^7 - \frac{m_s^2\langle g_s^2 GG\rangle}{1152\pi^4}\omega^5 + \frac{5 m_s\langle g_s^2 GG\rangle\langle\bar{s}s\rangle}{432\pi^2}\omega^3 \Big] e^{-\omega/T} d\omega. \tag{A21}$$

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