# Three-body systems with open flavor heavy mesons

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We solve the Faddeev equations for the bound state problem of three particles to study different threebody systems containing open flavor heavy mesons:  $\overline{D}NN$ , BNN,  $\overline{D}\overline{D}N$  and BBN. We use a coupledchannel formalism considering the *S*-wave spin excitations of the constituent hadrons:  $\overline{D}^*$ ,  $B^*$ , and  $\Delta$  when deemed relevant. Our study comes motivated by the recent prediction of deeply bound states in different two-body subsystems participating in the aforementioned three-body states. The existence of three-body bound states with open flavor heavy mesons would open the door to exotic nuclei with heavy flavors. Our results point against such possibility.

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# I. INTRODUCTION

The possible existence of exotic deeply bound states containing open flavor heavy mesons has been recently revived by lattice QCD calculations [1,2]. Using the HAL QCD method to extract the potential between two mesons, the lattice QCD simulations of Ref. [1] obtained attractive S-wave phase shifts for different isoscalar two-meson systems with open-flavor heavy mesons. Although no bound states or resonances were reported at the pion masses used in Ref. [1],  $m_{\pi} = 410-700 \text{ MeV}/c^2$ , the authors noticed that the attraction becomes more prominent as the pion mass decreases, particularly prominent in the isoscalar state  $\bar{u} \, \bar{d} \, cc$  with quantum numbers  $i^p = 1^+$ . Very recently, the lattice QCD simulations of Ref. [2] using nonrelativistic QCD to simulate the bottom quarks, reported an unambiguous signal for a strong-interactionstable isoscalar  $ud\bar{b}\bar{b}$  bound state with quantum numbers  $j^p = 1^+$  with a binding energy of 189(10) MeV. Similar findings were reported in Ref. [3] based on different constituent quark models within a hyperspherical harmonic formalism. The same results were later on obtained by means of the solution of the Lippmann-Schwinger equation in a coupled-channel approach [4]. Although expectations are less clear for the charm sector, theoretical models also suggest the existence of a doubly charm isoscalar fourquark bound state with quantum numbers  $j^p = 1^+$  [5]. The possible existence of such states due to the asymmetry of the masses of the constituents was already anticipated in simple potential model calculations in Ref. [6].

Although not so clear as the systems mentioned above, there are predictions about possible bound states of N's and  $\Delta$ 's with  $\overline{D}$  or *B* mesons. The effective Lagrangian approach to the BN system of Ref. [7] in a  $BN - B^*N$ coupled-channel calculation based on heavy quark symmetry with light quark chiral dynamics, finds a bound state with quantum numbers  $(i)j^p = (0)1/2^-$  and a binding energy of 9.4 MeV. In Ref. [8] the authors revisited the same system with the inclusion of short-range interactions, finding that the binding energy increases up to 19–23 MeV. depending on the interacting model used. A bound state with the same quantum numbers,  $(i)j^p = (0)1/2^-$ , and a binding energy of 1.4 MeV was found in the  $\overline{D}N$  system. These systems were also studied in constituent quark model approaches [9,10], highlighting the importance of coupled channel effects for the bottom sector.<sup>1</sup>

Finally, it is well known the general attractive character of the S-wave interaction of a two-baryon system with N's and  $\Delta$ 's generating bound or quasibound states in the NN [11], N $\Delta$  [12,13] and  $\Delta\Delta$  [14] systems. With all these ingredients together, one may wonder if few-body systems containing N's,  $\Delta$ 's,  $\overline{D}$ 's and B's could enhance the attraction of the different two-body subsystems generating three-body bound states. If this were the case one may think about the possible existence of bound states with a larger number of constituents. Thus, our aim in this paper is to

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<sup>&</sup>lt;sup>1</sup>Note that going from the charm to the bottom sector, there is a factor 3 in the mass difference between the pseudoscalar and vector open flavor heavy mesons, which is directly translated to the mass difference between thresholds:  $M(\bar{D}^*N) - M(\bar{D}N) =$  141 MeV, whereas  $M(B^*N) - M(BN) =$  45 MeV, what makes the coupled-channel effect much more important in the bottom sector.

study different three-body systems containing open flavor heavy mesons:  $\overline{D}NN$ , BNN,  $\overline{D}\overline{D}N$ , and BBN looking for bound states. Unlike the DNN and  $\overline{B}NN$  systems, the states studied in this work would be stable against the strong interaction, because the lower thresholds  $\pi Y_c N$  and  $\pi Y_b N$ are closed.

### **II. FORMALISM**

Let us start by describing the solution of the Faddeev equations for the bound-state problem of a three-hadron system with two identical fermions or two identical bosons. The first case would apply to the  $\overline{D}NN$  and BNN systems, while the latter would represent the  $\overline{D} \ \overline{D} N$  and BBN states. The Faddeev equations for the bound-state problem in the case of three particles with total isospin *I* and total spin *J*, if one restricts to configurations where all three particles are in *S*-wave states, are

$$T_{i;IJ}^{i_i j_i}(p_i q_i) = \sum_{j \neq i} \sum_{i_j j_j} h_{ij;IJ}^{i_i j_i j_j} \frac{1}{2} \int_0^\infty q_j^2 dq_j$$
  
  $\times \int_{-1}^1 d\cos\theta t_{i;i_i j_i}(p_i, p_i'; E - q_i^2/2\nu_i)$   
  $\times \frac{1}{E - p_j^2/2\mu_j - q_j^2/2\nu_j} T_{j;IJ}^{i_j j_j}(p_j q_j),$ (1)

where  $t_{i;i_ij_i}$  stands for the two-body amplitude of the pair jk with isospin  $i_i$  and spin  $j_i$ .  $p_i$  is the momentum of the pair jk and  $q_i$  the momentum of particle i with respect to the pair jk.  $\mu_i$  and  $\nu_i$  are the corresponding reduced masses,

$$\mu_i = \frac{m_j m_k}{m_j + m_k}, \qquad \nu_i = \frac{m_i (m_j + m_k)}{m_i + m_j + m_k}, \qquad (2)$$

and the momenta  $p'_i$  and  $p_j$  in Eq. (1) are given by,

$$p'_{i} = \sqrt{q_{j}^{2} + \frac{\mu_{i}^{2}}{m_{k}^{2}}q_{i}^{2} + 2\frac{\mu_{i}}{m_{k}}q_{i}q_{j}\cos\theta},$$
  
$$p_{j} = \sqrt{q_{i}^{2} + \frac{\mu_{j}^{2}}{m_{k}^{2}}q_{j}^{2} + 2\frac{\mu_{j}}{m_{k}}q_{i}q_{j}\cos\theta}.$$
(3)

 $h_{ij;IJ}^{i_ij_i;i_jj_j}$  are the spin-isospin coefficients,

$$\begin{aligned} h_{ij;IJ}^{i_ij_i;i_jj_j} &= (-)^{i_j + \tau_j - I} \sqrt{(2i_i + 1)(2i_j + 1)} W(\tau_j \tau_k I \tau_i; i_i i_j) \\ &\times (-)^{j_j + \sigma_j - J} \sqrt{(2j_i + 1)(2j_j + 1)} W(\sigma_j \sigma_k J \sigma_i; j_i j_j), \end{aligned}$$

$$(4)$$

where *W* is the Racah coefficient and  $\tau_i$ ,  $i_i$ , and *I* ( $\sigma_i$ ,  $j_i$ , and *J*) are the isospins (spins) of particle *i*, of the pair *jk*, and of the three-body system. If one expands the two-body

amplitudes in terms of Legendre polynomials as described in Ref. [15], Eq. (1) takes the form,

$$T_{i;IJ}^{ni_i j_i}(q_i) = \sum_{j \neq i} \sum_{mi_j j_j} \int_0^\infty dq_j A_{ij;IJ}^{ni_i j_i;mi_j j_j}(q_i, q_j; E) T_{j;IJ}^{ms_j i_j}(q_j),$$
(5)

with

$$A_{ij;IJ}^{ni_{i}j_{i};mi_{j}j_{j}}(q_{i},q_{j};E) = h_{ij;IJ}^{i_{i}j_{i};i_{j}j_{j}} \sum_{r} \tau_{i;i_{i}j_{i}}^{nr} (E - q_{i}^{2}/2\nu_{i}) \frac{q_{j}^{2}}{2} \\ \times \int_{-1}^{1} d\cos\theta \frac{P_{r}(x_{i}')P_{m}(x_{j})}{E - p_{j}^{2}/2\mu_{j} - q_{j}^{2}/2\nu_{j}},$$
(6)

and

$$\tau_{i;i_ij_i}^{nr}(e) = \frac{2n+1}{2} \frac{2r+1}{2} \int_{-1}^{1} dx_i \\ \times \int_{-1}^{1} dx'_i P_n(x_i) t_{i;i_ij_i}(x_i, x'_i; e) P_r(x'_i).$$
(7)

The three amplitudes  $T_{1;IJ}^{ri_1j_1}(q_1)$ ,  $T_{2;IJ}^{mi_2j_2}(q_2)$ , and  $T_{3;IJ}^{ni_3j_3}(q_3)$ in Eq. (5) are coupled together. The number of coupled equations can be reduced, however, since two of the particles are identical. The reduction procedure for the case where one has two identical particles has been described before [16,17] and will not be repeated here. With the assumption that particles 2 and 3 are the identical ones, the three equations (5) reduce to a single equation for the amplitude  $T_2$ ,

$$T_{2;IJ}^{ni_2j_2}(q_2) = \sum_{mi_3j_3} \int_0^\infty dq_3 K_{IJ}^{ni_2j_2;mi_3j_3}(q_2, q_3; E) T_{2;IJ}^{mi_3j_3}(q_3),$$
(8)

where

$$K_{IJ}^{ni_{2}j_{2};mi_{3}j_{3}}(q_{2},q_{3};E) = \pm (-)^{\sigma_{1}+\sigma_{3}-j_{2}+\tau_{1}+\tau_{3}-i_{2}}A_{23;IJ}^{ni_{2}j_{2};mi_{3}j_{3}}(q_{2},q_{3};E) + 2\sum_{r_{i}j_{1}}\int_{0}^{\infty} dq_{1}A_{31;IJ}^{ni_{2}j_{2};r_{i}j_{1}}(q_{2},q_{1};E)A_{13;IJ}^{ri_{1}j_{1};mi_{3}j_{3}}(q_{1},q_{3};E).$$

$$(9)$$

The + or – sign in Eq. (9) correspond to the case when the two identical particles are bosons or fermions, respectively, while the quantum numbers  $i_1j_1$  (the identical pair) in the last term of the equation are those allowed by the Pauli principle in each case.

TABLE I. Two-body  $\overline{D}N$  channels with a N as spectator  $(i_{\overline{D}}, s_{\overline{D}})_N$ ,  $\overline{D}^*N$  channels with a N as spectator  $(i_{\overline{D}^*}, s_{\overline{D}^*})_N$ , NN channels with a  $\overline{D}$  as spectator  $(i_N, s_N)_{\overline{D}}$ , and NN channels with a  $\overline{D}^*$  as spectator  $(i_N, s_N)_{\overline{D}^*}$ , that contribute to a given  $\overline{D}NN$  or  $\overline{D}^*NN$  state with total isospin I and spin J. The same channels would contribute to the BNN and  $B^*NN$  systems.

I	J	$(i_{ar D}, j_{ar D})_N$	$(i_{ar D^*},j_{ar D^*})_N$	$(i_N,j_N)_{ar D}$	$(i_N,j_N)_{ar D^*}$
1/2	0	(0, 1/2), (1, 1/2)	(0, 1/2), (1, 1/2)	(1,0)	(0,1)
1/2	1	(0, 1/2), (1, 1/2)	(0, 1/2), (1, 1/2), (0, 3/2), (1, 3/2)	(0,1)	(0,1),(1,0)
1/2	2	•••	(0, 3/2), (1, 3/2)		(0,1)
3/2	0	(1, 1/2)	(1, 1/2)	(1,0)	
3/2	1	(1, 1/2)	(1, 1/2), (1, 3/2)		(1,0)
3/2	2	•••	(1,3/2)		•••

### **III. RESULTS**

Let us first of all discuss the systems with baryon number two:  $\overline{D}NN$  and BNN. The two-body subsystems containing the heavy meson,  $\overline{D}N$  and BN, do not present quarkantiquark annihilation complications that may obscure the predictions of a particular model under some nonconsidered dynamical effects. Besides, they contain a heavy antiquark, what makes the interaction rather simple. As said above, there are different models in the literature dealing with the  $\overline{D}N$  and BN interactions based on effective Lagrangian approaches [7,8] or constituent quark models [9,10]. It is worth to note the general agreement that the (i, j) =(0, 1/2) is the most attractive channel among those not containing  $\Delta$  isobars, that will be discussed further below. The smaller binding energy or unbound nature obtained in quark-model calculations is a remnant of quark antisymmetry effects, as it was shown in Fig. 10 of Ref. [9]. In this work we make use of the two-body quark-model based interactions [9,10,18]. They have the advantage of providing parameter-free predictions for the interaction in a barvonmeson system with charm -1 or bottom +1, starting from a coherent description of the NN system [18]. Besides, due to the existence of identical light quarks in the two hadrons, the quark-Pauli effects appearing in some particular channels are fully considered. They give rise to short-range repulsion in some particular channels due to lacking degrees of freedom to accommodate the light quarks.

It is also worthwhile to note that the increase of the hadron masses for heavy flavor meson reduces the kinetic energy of the system due to the larger reduced mass, and one would expect a priori to get more stable states. However for those quantum numbers where several channels contribute the coupled channel dynamics may result in no so simple considerations. The two-body thresholds and the hadron-hadron interactions are modified when changing the flavor of the heavy quark. This is due to the degeneracy of pseudoscalar and vector open-flavor mesons for large heavy quark masses, as predicted by the chromomagnetic interaction [19]. This effect is the responsible that the coupled channel dynamics is more involved and predictions in the bottom sector might differ significantly from those on the charm sector [20]. Thus, similarly to the two-body problem, the pattern of states may change when moving among different flavor sectors.

We show in Table I the two-body channels of the different two-body subsystems, either  $\overline{D}N$ ,  $\overline{D}^*N$ , and NN or BN,  $B^*N$ , and NN, that contribute to a given



FIG. 1. (a) Fredholm determinant for the J = 0 and J = 1 (I, J)  $\overline{D}NN$  channels. (b) Fredholm determinant for the J = 2  $(I, J)\overline{D}^*NN$  channels.



FIG. 2. (a) Fredholm determinant for the J = 0 and J = 1 (I, J) BNN channels. (b) Fredholm determinant for the J = 2 (I, J) $B^*NN$  channels.

three-body  $\overline{D}NN$  ( $\overline{D}^*NN$ ) or BNN ( $B^*NN$ ) state, (I, J). Note that J = 2 channels can only be reached through a vector meson  $\overline{D}^*$  or  $B^*$ . All allowed internal couplings between the different two-body subsystems contributing to a three-body channel have been considered. From this table one can construct the basis wave functions used for the three-body calculation.

The results are presented in Fig. 1 for the DNN and  $D^*NN$  systems, and in Fig. 2 for the BNN and  $B^*NN$  systems. As can be seen none of the three-body channels is a candidate to lodge a bound state or a resonance. The Fredholm determinant is flat or it increases when approaching the threshold, indicating a general repulsive system. The situation is not improved when going from the charm to the bottom sector. The change in the hadron masses reduces the kinetic energy contribution and the Fredholm determinant becomes smaller, although the character of the interaction is not modified.

We have also looked for metastable states that may take advantage of stronger interactions in the baryonic sector containing  $\Delta$ 's. For this purpose we have singled out a three-body system containing the quantum numbers of the asymptotic  $\Delta\Delta$  state bound by nearly 100 MeV,  $(i) i^p =$  $(0)3^+$  [14]. This state was predicted by Dyson and Xuong [21] already in 1964 and later on also by Goldman et al. [22], who called it the *inevitable dibaryon*  $d^*$ , due to its unique symmetry features. The resonant structure was advertised in the NN scattering data in the quark-model calculation of Ref. [23]. The attractive character of the  $\Delta\Delta$ interaction might be combined with the attraction of the  $\overline{D}^*\Delta$  interaction in the (i, j) = (1, 5/2) channel [9]. Thus, we have considered the most promising candidate to show a bound state, the  $\overline{D}^*\Delta\Delta$  (*I*, *J*) = (1/2, 4) channel, that due to angular momentum selection rules is not coupled to lower channels containing N's or pseudoscalar  $\overline{D}$  mesons. We show in Table II the  $\Delta \Delta$  and  $\bar{D}^* \Delta$  two-body channels contributing to such state.

The Fredholm determinant for this state is shown in Fig. 3. As can be seen, although both two-body subsystems show a bound state, the three-body system is not bound. We have done several trials, as for example neglecting the  $(i, j) = (2, 5/2)\overline{D}^*\Delta$  two-body channel, which is strongly repulsive due to Pauli blocking effects [9], and the threebody system remains unbound. Similarly, if the  $\bar{D}^*$  is replaced by a  $B^*$  meson, the system is not bound. When moving to the bottom sector, the two-body state showing the strongest attraction is the (i, j) = (2, 3/2), due to  $B\Delta - B^*\Delta$  coupled channel effects [10]. However, all three-body channels containing this two-body state, as for example the three-body system with quantum number (I, J) = (1/2, 3), may decay strongly to the lower  $B^*N\Delta$ three-body system. Thus, one has a  $B^*N\Delta - B\Delta\Delta - B^*\Delta\Delta$ coupled channel problem, the lowest threshold  $B^*N\Delta$ being 247 MeV below the attractive coupled channel system  $B\Delta\Delta - B^*\Delta\Delta$  state. Besides, there contribute  $N\Delta$  channels that are strongly repulsive [18], minimizing any possibility of binding.

A similar study of the possible existence of exotic dibaryons with a heavy antiquark in the three-body systems:  $\overline{D}NN$ ,  $\overline{D}^*NN$ , BNN and  $B^*NN$  has been done in Ref. [24]. The authors used a one-pion exchange potential between the heavy meson and the nucleon, and the Argonne  $v'_8$  potential for the nucleon-nucleon interaction [25]. The three-body problem is solved by means of a powerful Gaussian expansion method [26]. The authors obtained a bound state in the (I, J) = (1/2, 0)

TABLE II. Different two-body channels contributing to the  $(I, J) = (1/2, 4)\bar{D}^*\Delta\Delta$  channel.

Interacting pair	(i, j)	Spectator
ΔΔ	(0,3)	$ar{D}^*$
$ar{D}^*\Delta$	(1, 5/2) (2, 5/2)	Δ



FIG. 3. Fredholm determinant for the  $(I, J) = (1/2, 4)\overline{D}^*\Delta\Delta$  system.

channel, with a binding energy of 5.2 MeV in the charm sector and 26.2 MeV in the bottom sector. They also reported Feshbach resonances in the (I, J) = (1/2, 1)channel for both heavy flavor sectors. The main responsible for the bound states is the tensor force of the one-pion exchange interaction between the heavy meson and the nucleon. To understand the difference between our conclusions and those of Ref. [24] one might have a look to Figs. 4, 7 and 10 of Ref. [9]. In Fig. 4 one observes how there are important differences between a hadronic potential, without quark-exchange effects (quoted as Dir in the figure) and the contributions coming from quark-exchange effects (quoted as Ex). Such a difference was highlighted in Fig. 10 obtaining a bound state when quark-exchange effects are not considered, in agreement with Refs. [7,8]. Thus, as mentioned above, the smaller binding energy or unbound nature obtained in quark-model calculations is a remnant of quark antisymmetry effects. Besides, in Fig. 7 of Ref. [9], one observes how the coupling between the different two-body systems with a nucleon and a heavy meson, the responsible for the binding in the three-body systems of Ref. [24], is generally not too strong. Such a coupling comes reinforced by the vicinity of the different two-body thresholds as it is clearly illustrated in Fig. 3 of Ref. [20]. Therefore, this comparison makes evident the great importance that quark-exchange effects may have in the system under study and it also represents a sharp example of a system where the quark-exchange dynamics may have observable consequences. A future effort in the study of two and three-body systems with heavy open flavor mesons will provide us with evidence to learn about the importance of quark-exchange dynamics.

Once the possible existence of bound states in systems containing two baryons and a heavy meson was explored, we were motivated by the lattice QCD prediction of a deeply bound state in a system coupled to the quantum numbers of two open flavor heavy mesons [1,2]. The isoscalar  $j^p = 1^+$  state predicted by lattice QCD calculations in the doubly bottom sector and its partner in the charm sector, appear as  $BB^* - B^*B^*$  and  $\overline{D}\overline{D}^* - \overline{D}^*\overline{D}^*$ coupled channel states [3–5]. Having in mind the attractive character of the  $N\bar{D}$  and NB interactions in some particular channels [7–10], we have explored the possibility of finding bound states of this exotic doubly heavy flavor state in the presence of nucleons. For this purpose we have selected the three-body channel with quantum numbers (I, J) = (1/2, 3/2), that it is not coupled to the lower  $\overline{D} \, \overline{D} \, N$  (BBN) channel.

We have indicated in Table III the two-body channels contributing to the system studied. This channel contains in the charm sector a  $\overline{D}\overline{D}^* - \overline{D}^*\overline{D}^*$  bound state of 16.65 MeV. In the bottom sector it presents a  $BB^* - B^*B^*$  bound state of 179.2 MeV, close to the lattice QCD results. The Fredholm determinant of the three-body system is shown in Fig. 4 by the solid line. As it can be seen there is no any signal of a possible bound state or resonance. We have performed a calculation neglecting the repulsive channels appearing in Table III, thus just considering the  $N\overline{D}(i, j) =$ (0, 1/2) channel and the  $\overline{D}\overline{D}^*$  and  $\overline{D}^*\overline{D}^*$  (i, j) = (0, 1)



FIG. 4. (a) Fredholm determinant for the  $(I, J) = (1/2, 3/2) \overline{D}\overline{D}^*N - \overline{D}^*\overline{D}^*N$  state. (b) Fredholm determinant for the  $(I, J) = (1/2, 3/2) BB^*N - B^*B^*N$  state. See text for details.

TABLE III. Different two-body channels contributing to the  $(I, J) = (1/2, 3/2) \overline{D}\overline{D}^*N - \overline{D}^*\overline{D}^*N$ . The same channels would contribute to the  $BB^*N - B^*B^*N$  system.

Interacting pair	(i, j)	Spectator	
$ar{D}ar{D}^*$	(0,1) (1,1)	N	
$ar{D}^*ar{D}^*$	(0,1) (1,2)	Ν	
ND	(0, 1/2) (1, 1/2)	$ar{D}^*$	
$Nar{D}^*$	(0, 3/2) (1, 3/2)	$\bar{D}$	
$N\bar{D}^*$	(0, 1/2) (1, 1/2) (0, 3/2) (1, 3/2)	$ar{D}^*$	

channels. The results are shown by the dashed line in Fig. 4, and although the system becomes less repulsive is still far from being bound.

Let us finally note the intricate bound state three-body problem. The presence of a third particle generates additional two-body channels and thresholds contributing to the threebody systems that might avoid the existence of three-body bound states. It seems that when the internal two-body thresholds of the three-body system are far away, they conspire against the possible binding of the three-body system. This seems to be the reason why in spite of having a large  $BB^* - B^*B^*$  binding, we do not get a  $BB^*N - B^*B^*N$ bound state. This was already observed in our previous studies of the  $N\Delta\Delta$  and  $NN\Delta$  three-body systems [13,27].

### **IV. SUMMARY**

In summary, we have solved the Faddeev equations for the bound state problem of three-body systems containing open flavor heavy mesons. In particular, we have studied systems with two identical baryons or two identical mesons. We have singled out three-body systems with two-body subsystems presenting deeply bound states. The existence of these states stems from different models, as it can be lattice QCD studies, effective Lagrangian approaches or constituent quark models. In spite of the presence of strongly attractive two-body subsystems, the three-body systems remain always unbound. In fact, they present a rather important repulsive character. Thus, our results point against the existence of exotic nuclei containing open flavor heavy mesons.

It is expected that in the near future the existence of exotic nuclei with a variety of flavors could be explored at hadron facilities such as the planned installation of a 50 GeV high-intensity proton beam at Japan Proton Accelerator Research Complex (J-PARC) [28,29]. Particularly interesting is also the possibility of producing exotic hadrons in high-energy heavy-ion collision [30,31]. A Super B collider offers similar possibilities [32]. Future experimental results will help to scrutinize among the different models, and in this way to improve our phenomenological understanding of QCD in the highly nonperturbative low-energy regime. The three-body systems studied in this work provide a nice opportunity to discriminate between predictions based on quark-exchange dynamics or hadronic models. This challenge could only be achieved by means of a cooperative experimental and theoretical effort.

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