Parton distribution functions with QED corrections in the valon model

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The parton distribution functions (PDFs) with QED corrections are obtained by solving the QCD ⊗ QED DGLAP evolution equations in the framework of the "valon" model at the next-to-leading-order QCD and the leading-order QED approximations. Our results for the PDFs with QED corrections in this phenomenological model are in good agreement with the newly related CT14QED global fits code [Phys. Rev. D **93**, 114015 (2016)] and APFEL (NNPDF2.3QED) program [Comput. Phys. Commun. **185**, 1647 (2014)] in a wide range of $x = [10^{-5}, 1]$ and $Q^2 = [0.283, 10^8]$ GeV^2. The model calculations agree rather well with those codes. In the latter, we proposed a new method for studying the symmetry breaking of the sea quark distribution functions inside the proton.

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I. INTRODUCTION

Nowadays, a deep knowledge of the properties of hadrons and a precise understanding of parton distributions functions (PDFs) are key ingredients in searches for new physics at the LHC (for a review, see e.g. Refs. [1,2]). Hence, reliable extraction of information on the polarized PDFs [3–5], unpolarized PDFs [6–10], and nuclear PDFs [11–13] from global QCD analyses of DIS data, as well as all related studies [14–19], provides deep understanding of the structure of hadrons in term of their quark and gluon constituents. With the advent of the electron-proton (ep) collider HERA, the kinematic range of the DIS regime has been widely extended, allowing us to achieve a much deeper understanding of the structure of nucleons.

Recently, precision achievement by ATLAS [20] at the Large Hadron Collider (LHC) on Drell-Yan processes shows that the size of the photon-induced contribution to the dileptons invariant mass is significant. The cross section of such a process, necessarily connected with the photon distribution in the proton [21–25].

It has also been shown that the precision phenomenology at the Large Hadron Collider (LHC) requires theoretical calculations which include QCD corrections and electroweak (EW) corrections [23]. An essential ingredient of these electroweak corrections is the photon parton distribution function inside the proton, $x\gamma(x, Q^2)$.

Consequently, the quantum electrodynamics (QED) and electroweak (EW) corrections are important issues for many theoretical predictions at high energy at the LHC. So, for the LHC era, the determination of the photon distribution function inside the proton has become important. Therefore, to imply the inclusion of QED corrections to perturbative evolution leads to additional partons in the proton. The photon distribution functions, which are produced by the radiation of photon from charged quarks, can be determined from the QCD ⊗ QED DGLAP evolution equations. There are some sets of the PDFs such as the MRST2004QED [26,27], NNPDF2.3QED [28], CT14QED [29] global fits codes and LUXQED code [30] that incorporated the photon contribution of the proton.

The goal of this analysis is to show how a simple phenomenological model, e.g., "valon" model, can determine the parton distribution functions in the proton. We used this model to calculate the parton distribution functions (including the photon distribution function) inside the proton. Our calculations are done in the next-to-leading-order (NLO) QCD and the leading-order (LO) QED approximations.

The organization of this paper is as follows: In Sec. II, we bring out the QCD ⊗ QED DGLAP evolution equations with suitable initial inputs. We also propose the novel method to study the symmetry breaking of the sea quark distribution functions inside the proton in Sec. III. In Sec. IV, we discuss our findings and give our conclusions on determination of the PDF with QED corrections.

II. THE QCD ⊗ QED DGLAP EVOLUTION EQUATIONS

The singlet parton distribution functions, $f_i(x, Q^2)$, obey the QCD ⊗ QED DGLAP evolution equations [31] in x space, as

$$\frac{\partial}{\partial \log Q^2}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{pmatrix} \otimes \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix}, \quad (1)$$

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and the QCD ⊗ QED DGLAP evolution equations for the nonsinglet parton distribution functions are as follows,

$$\frac{\partial f_i}{\partial \log Q^2} = P_{ii} \otimes f_i \quad i = 5, \ldots, 9, \tag{2}$$

where P_{ij} and P_{ii} are the splitting functions and are represented in Ref. [32] with details, and ⊗ denotes the convolution integral, as

$$f \otimes g = \int_x^1 \frac{dy}{y} f(y) g\left(\frac{x}{y}\right). \tag{3}$$

We utilize a PDF basis for the QCD ⊗ QED DGLAP evolution equations, defined by the following singlet and nonsinglet PDF combinations [33],

$$q^{SG}: \begin{pmatrix} f_1 = \Delta = \\ u + \bar{u} + c + \bar{c} - d - \bar{d} - s - \bar{s} - b - \bar{b} \\ f_2 = \Sigma = \\ u + \bar{u} + c + \bar{c} + d + \bar{d} + s + \bar{s} + b + \bar{b} \\ f_3 = g \\ f_4 = \gamma \end{pmatrix} \tag{4}$$

$$q^{NS}: \begin{pmatrix} f_5 = d_v = d - \bar{d} \\ f_6 = u_v = u - \bar{u} \\ f_7 = \Delta_{ds} = d + \bar{d} - s - \bar{s} \\ f_8 = \Delta_{uc} = u + \bar{u} - c - \bar{c} \\ f_9 = \Delta_{sb} = s + \bar{s} - b - \bar{b} \end{pmatrix}. \tag{5}$$

We assumed the symmetry between quark-antiquark distributions. Then the corresponding valence distributions vanish, and we have $s = \bar{s}$, $c = \bar{c}$ and $b = \bar{b}$.

It should be noted that the Q^2 dependence of the PDFs with QED corrections can be described by the QCD ⊗ QED DGLAP evolution equations. Then, the knowledge of the PDFs at fixed scale Q_0^2 is enough for us to obtain the PDFs at larger scale Q^2. There are some solutions for the DGLAP evolution equations with QED corrections based on the Laplace or Mellin transforms [32,34]. We used the solutions of these equations in Mellin space that are proposed in Ref. [34]. Here in this paper, we want to solve the QCD ⊗ QED DGLAP evolution equations using the "valon" model. To solve these integro-differential evolution equations we need suitable initial inputs that simply defined in this phenomenological model. So the next section is devoted to study the hadron structure in the valon model.

III. HADRON STRUCTURE IN THE VALON MODEL

The valon model was first proposed by R.C.Hwa [35–38] and then extended to the polarized structure of nucleon [39,40]. This model can also described the transverse structure of hadrons [41] and the double parton distribution functions (dPDFs) [42,43] very well too. In this model a hadron is viewed as a bound state of two or three valons. These "constituents-quark like" valons are defined to be the dressed valence quarks with its associated sea quarks and gluons. In the scattering process, the valons play a role similar to constituent quarks do in bound state problem. It is assumed that the valons stand between partons and hadrons and the valon distributions inside the hadron are universal and Q^2 independent. Then the proton, for example, has three valons, UUD, which carry all of the proton momentum. In the valon model, the recombination of partons into hadrons occur in two stage processes: first partons emit and absorb gluons(photons). The evolved quark-gluon(photon) cloud becomes "valons." Then, these valons recombine into hadrons. Briefly, we have

(i) At low Q^2, the internal structure of valons can not be resolved and the valons behave as constituent quarks.
(ii) At high Q^2, the internal structure of valons and the Q^2 dependence of parton distribution functions in the hadron come from solutions of the DGLAP evolution equations in each valon with suitable initial input densities in the valon.
(iii) The valon distribution functions are not depend on Q^2. They can be interpreted as the wave-function square of the constituent quarks in hadron. It also means the probability of finding a valon with momentum fraction of y of hadron momentum.

In the following subsections, we investigate the parton distribution functions with QED corrections using the valon model.

A. Parton distribution functions in the valon and proton

The valon model essentially has two steps. The first one is the solution of the DGLAP evolution equations with appropriate initial input in the valon. The second one is to convolute the parton distribution functions in the valons with the valon distribution functions, $G^p_{\mathrm{valon}}(y)$, to obtain the parton distribution functions, for example, in the proton, as follows

$$q^p(x, Q^2) = \sum_{\mathrm{valon}} \int_x^1 dy G^p_{\mathrm{valon}}(y) q^{\mathrm{valon}}\left(\frac{x}{y}, Q^2\right) \tag{6}$$

In a similar way, the proton structure function $F_2(x, Q^2)$ is a convolution of the valon distribution $G^p_{\mathrm{valon}}(y)$ and the structure function of the valon, $F_2^{\mathrm{valon}}(z, Q^2)$, then we have

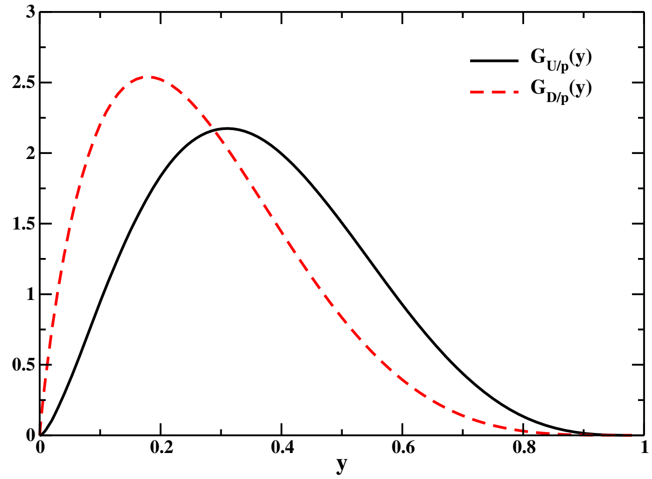

FIG. 1. The valon distribution functions at $Q = 10$ GeV for U and D valons.

$$F_2^h(x, Q^2) = \sum_{\text{valon}} \int_x^1 dy G^h_{\text{valon}}(y) F_2^{\text{valon}}\left(\frac{x}{y}, Q^2\right) \quad (7)$$

These valon distributions which means the probability of finding a valon with momentum fraction of y of the hadron momentum are given in Refs. [35–38]. The valon distribution functions which are plotted in Fig. 1 are given as follows,

$$G_{U/p}(y) = \frac{B(\alpha+1,\beta+1) y^{\alpha}(1-y)^{\alpha+\beta+1}}{B(\alpha+1,\beta+1)B(\alpha+1,\alpha+\beta+2)} \quad (8)$$

$$G_{D/p}(y) = \frac{B(\alpha+1,\alpha+1) y^{\beta}(1-y)^{2\alpha+1}}{B(\alpha+1,\beta+1)B(\alpha+1,\alpha+\beta+2)}, \quad (9)$$

where $B(m,n)$ is the beta function and $\alpha = 1.545$ and $\beta = 0.89$ [35–38]. The parton distribution functions in the valon, $q^{\text{valon}}(\frac{x}{y}, Q^2)$, come from solutions of the DGLAP evolution equations in each valon. The parton distribution functions in a hadron can be written as the convolution of the partons in the valon and the valon distribution in the hadron. The parton distribution functions in proton are obtained as

$$\begin{aligned}
u_{\text{valance}}(x, Q^2) &= 2\int_x^1 q_{\frac{\text{valance}}{\text{valon}}}\left(z = \frac{x}{y}, Q^2\right) G_{U/p}(y) dy \\
d_{\text{valance}}(x, Q^2) &= \int_x^1 q_{\frac{\text{valance}}{\text{valon}}}\left(z = \frac{x}{y}, Q^2\right) G_{D/p}(y) dy \\
\bar{q}_{\text{Sea}}(x, Q^2) &= 2\int_x^1 q_{\frac{\text{Sea}}{\text{valon}}}\left(z = \frac{x}{y}, Q^2\right) G_{U/p}(y) dy + \int_x^1 q_{\frac{\text{Sea}}{\text{valon}}}\left(z = \frac{x}{y}, Q^2\right) G_{D/p}(y) dy \\
g(x, Q^2) &= 2\int_x^1 q_{\frac{g}{\text{valon}}}\left(z = \frac{x}{y}, Q^2\right) G_{U/p}(y) dy + \int_x^1 q_{\frac{g}{\text{valon}}}\left(z = \frac{x}{y}, Q^2\right) G_{D/p}(y) dy \\
\gamma(x, Q^2) &= 2\int_x^1 q_{\frac{\gamma}{\text{valon}}}\left(z = \frac{x}{y}, Q^2\right) G_{U/p}(y) dy + \int_x^1 q_{\frac{\gamma}{\text{valon}}}\left(z = \frac{x}{y}, Q^2\right) G_{D/p}(y) dy
\end{aligned} \quad (10)$$

Here in this paper, we consider the parton distribution functions with QED corrections. Therefore, we solved the QCD ⊗ QED DGLAP evolution equations in Eqs. (1), (2). To solve these equations we need initial input densities in the valons. We work in the $\overline{\text{MS}}$ scheme with $\Lambda_{\text{QCD}} = 0.22$ GeV and $Q_0^2 = 0.283$ GeV^2. The motivation for the low value of Q_0^2 is the phenomenological consideration to requires us to choose the initial input densities as $\delta(z-1)$ at Q_0^2 (where, $z = \frac{x}{y}$). This means that at such low initial scale of Q_0^2, the nucleon can be considered as a bound state of three valence quarks which carry all of the nucleon momentum. Therefore, at this scale of Q_0^2, there is one valence quark in each valon and this valence quark carry all of the valon momentum. So we should choose the initial input densities in the valon as $q^{\text{valon}}(\frac{x}{y}, Q_0^2) = \delta(z-1)$ in z space. It is interesting to note that our choice for initial value of Q^2 is very close to the transition region reported by the CLAS Collaboration. The first moment of the proton

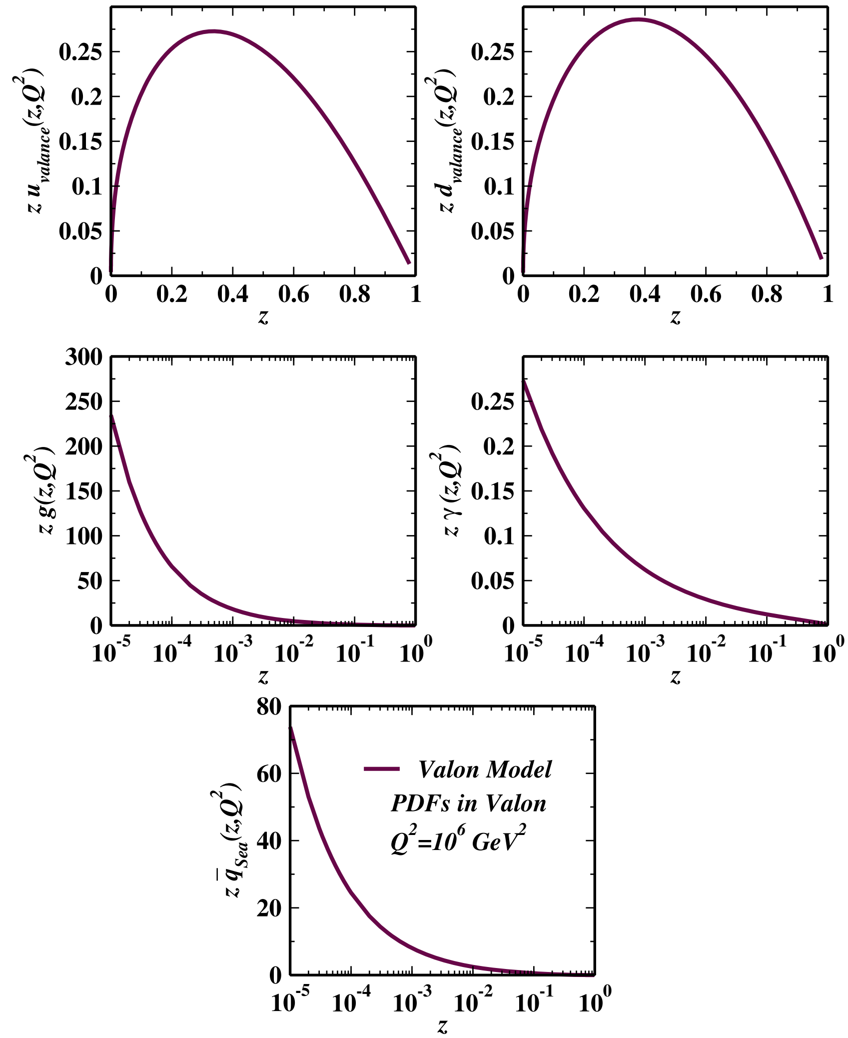

FIG. 2. The parton distribution functions inside valons at $Q^2 = 10^6$ GeV^2 in z space.

structure function has been measured at CLAS and the results show that there is a transition region around $Q^2 = 0.3$ GeV^2 [44]. To study the evolution of the parton distribution functions, we used the solutions of the QCD $\otimes$ QED DGLAP evolutions equations in Mellin space at NLO QCD and LO QED approximations [34]. Therefore, the integro-differential evolution equations reduce to sums of the parton distribution functions and pre-computable evolution kernels. Then, we choose the initial input densities (f_{i0} where $i = 1, \ldots, 9$) for Eqs. (1) and (2) in Mellin space (which is the Mellin transform of $\delta(z-1)$), as follows

$$f_{10}(N) = 1, \quad f_{20}(N) = 1, \quad f_{30}(N) = 0, \quad f_{40}(N) = 0,$$
$$f_{50}(N) = 1, \quad f_{60}(N) = 1, \quad f_{70}(N) = 1,$$
$$f_{80}(N) = 1, \quad f_{90}(N) = 0 \tag{11}$$

where the Mellin transform is defined as follows,

$$f(N) = \int_0^1 dx x^{N-1} f(x) \tag{12}$$

Our results for the parton distribution functions inside each valon are shown in Fig. 2. These plots depict $Q^2 = 10^6$ GeV^2 as a function of z. Finally, the convolution integral of Eq. (6) led us to the parton distribution functions with QED corrections inside the proton. The valance quark distribution functions are shown in Figs. 3 and 4 at $Q^2 = 10$ GeV^2 and $Q^2 = 10^6$ GeV^2 in valon model, respectively. In these figures, we compare our results with the valance quark distribution functions extracted from the CT14QED global fits code and APFEL (NNPDF2.3QED)[1] program. An excellent agreement is found for all of the flavours. Also, it is found that with an increasing in the values of Q^2,

[1]We run the *NNPDF23_nlo_as_0118_qed.LHgrid* via APFEL.

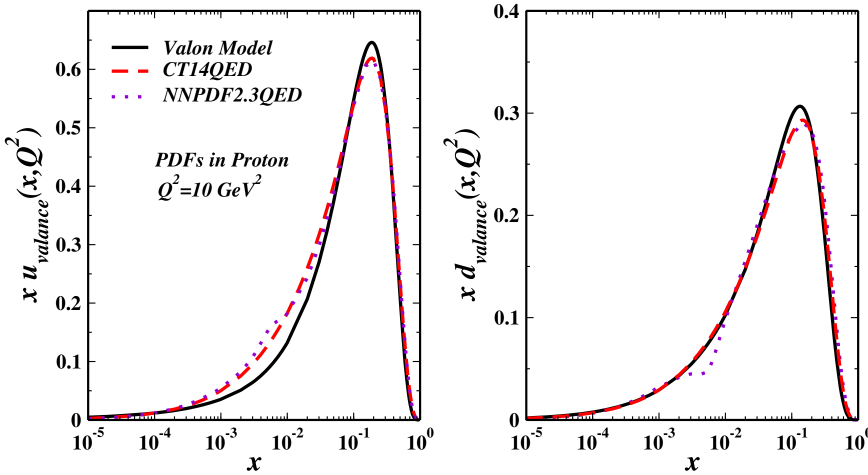

FIG. 3. The parton distribution $xu_{\text{valance}}(x, Q^2)$ and $xd_{\text{valance}}(x, Q^2)$ at $Q^2 = 10\ \text{GeV}^2$. The solid lines are our results in the valon model, dotted lines are the APFEL (NNPDF2.3QED) program [28], and dashed lines are the CT14QED code [29].

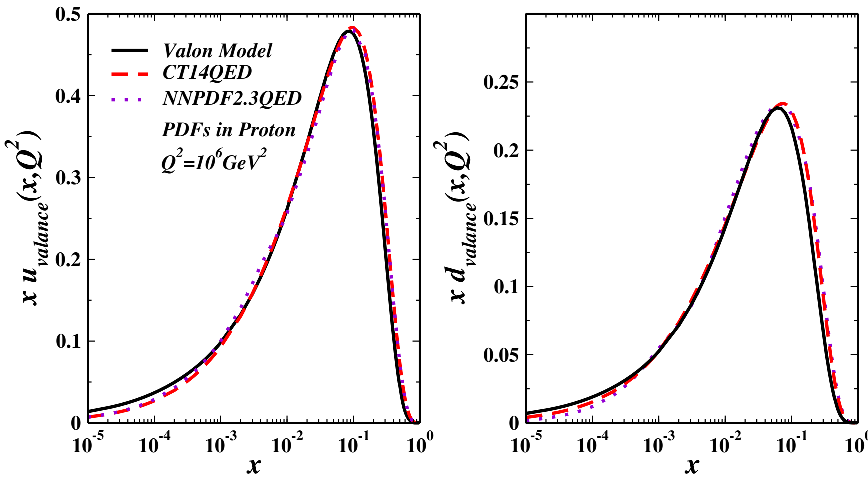

FIG. 4. The parton distribution $xu_{\text{valance}}(x, Q^2)$ and $xd_{\text{valance}}(x, Q^2)$ at $Q^2 = 10^6\ \text{GeV}^2$. The solid lines are our results in the valon model, dotted lines are the APFEL (NNPDF2.3QED) program [28], and dashed lines are the CT14QED code [29].

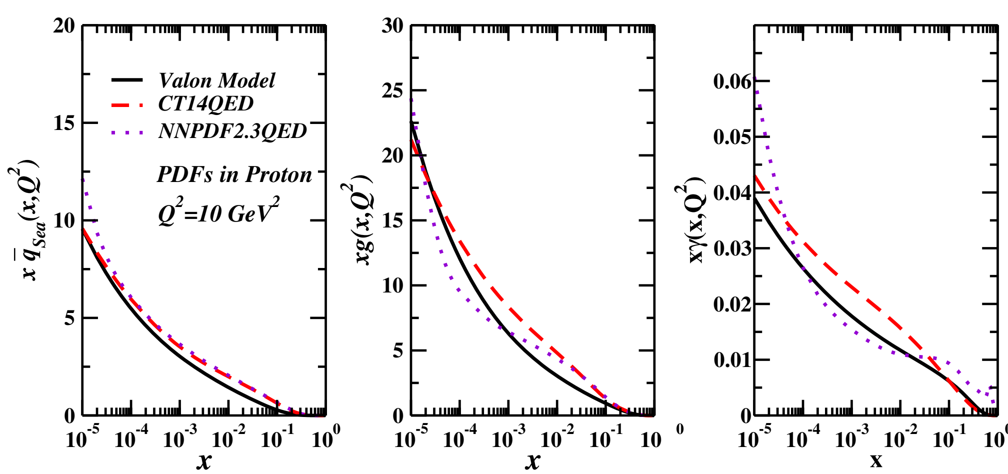

FIG. 5. The total sea quark, $\bar{q}_{\text{Sea}}(x, Q^2)$, gluon and photon distribution functions at $Q^2 = 10\ \text{GeV}^2$. The solid lines are our results in the valon model, dotted lines are the APFEL (NNPDF2.3QED) code [28], and dashed lines are the CT14QED code [29].

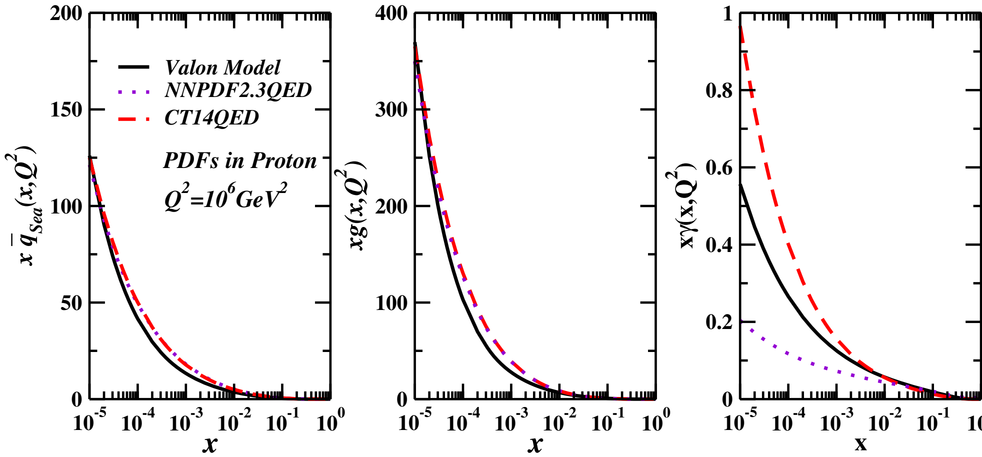

FIG. 6. The total sea quark, $\bar{q}_{\rm Sea}(x, Q^2)$, gluon and photon distribution functions at $Q^2 = 10^6$ GeV^2. The solid lines are our results in the valon model, dotted lines are the APFEL (NNPDF2.3QED) code [28], and dashed lines are the CT14QED code [29].

the valance quark distribution functions decrease for all of the values of x. Then, the contribution of photon distribution function increase with an increasing in Q^2. The total sea quark, $\bar{q}_{\rm Sea}(x, Q^2)$, gluon and photon distribution functions are plotted in Figs. 5 and 6 for two values of $Q^2 = 10$ GeV^2 and $Q^2 = 10^6$ GeV^2, respectively. Here we have good agreement with those from the CT14QED global fits code and APFEL (NNPDF2.3QED) program too.

B. Symmetry breaking in the sea quark distribution functions

In these subsections, we would like to know how we can separate the different kinds of sea quark distributions when we know the total distribution of all sea quarks. Here, we propose the new method based on the sea quark mass ratio. The total sea quark distribution functions is obtained as follows,

$$\bar{q}_{\rm Sea}(x, Q^2) = 2\bar{u}(x, Q^2) + 2\bar{d}(x, Q^2) + 2\bar{s}(x, Q^2) + 2\bar{c}(x, Q^2) + 2\bar{b}(x, Q^2) \tag{13}$$

where, we consider $\bar{u} = \bar{b}$, $s = \bar{s}$, $c = \bar{c}$ and $b = \bar{b}$. To study of the symmetry breaking of sea quark distribution functions, we use the fact that probability of finding heavier partons inside the proton is smaller than those of light partons, it means:

TABLE I. The parameters A and B for the sea quark distribution functions.

$\bar{q}_j$	A_j	B_j
$\bar{u}$	4.08513	0.5
$\bar{d}$	4.08513	0.5
$\bar{s}$	106.21330	13
$\bar{c}$	1297.02783	150
$\bar{b}$	4289.38340	450

$$\frac{q_i(x, Q^2)}{q_j(x, Q^2)} \simeq \frac{m_j}{m_i} \tag{14}$$

Therefore, we can calculate the bottom quark distribution function, as an example, as follows

$$\bar{q}_{\rm Sea}(x, Q^2) \simeq 2\frac{m_b}{m_u}\bar{b}(x, Q^2) + 2\frac{m_b}{m_d}\bar{b}(x, Q^2) + 2\frac{m_b}{m_s}\bar{b}(x, Q^2) + 2\frac{m_b}{m_c}\bar{b}(x, Q^2) + 2\bar{b}(x, Q^2) \tag{15}$$

Then, we have

$$\bar{b}(x, Q^2) \simeq \frac{\bar{q}_{\rm Sea}(x, Q^2)}{2m_b(\frac{1}{m_u} + \frac{1}{m_d} + \frac{1}{m_s} + \frac{1}{m_c} + \frac{1}{m_b})} \tag{16}$$

This leads to the following general relation:

$$\bar{q}_j(x, Q^2) = A_j \frac{\bar{q}_{\rm Sea}(x, Q^2)}{B_j} \tag{17}$$

The B parameter is constant for each kind of sea quark:

$$B_j = 2m_j\left(\frac{1}{m_u} + \frac{1}{m_d} + \frac{1}{m_s} + \frac{1}{m_c} + \frac{1}{m_b}\right) \tag{18}$$

where the j index run over all of the sea quark flavors.

The free parameter A can be extracted from experimental data. Here, we used the CT14QED PDFs set to determine this parameter for various kinds of sea quarks. The values of A and B parameters are given in Table I. The sea quark distribution functions are shown in Fig. 7; they are compared with the CT14QED and APFEL (NNPDF2.3QED) PDF sets. This figure shows that a good agreement is achieved. Figure 8 shows the sea quark and photon distribution functions in x space at fixed scale of $Q^2 = 10^6$ GeV^2. It is worth noticing

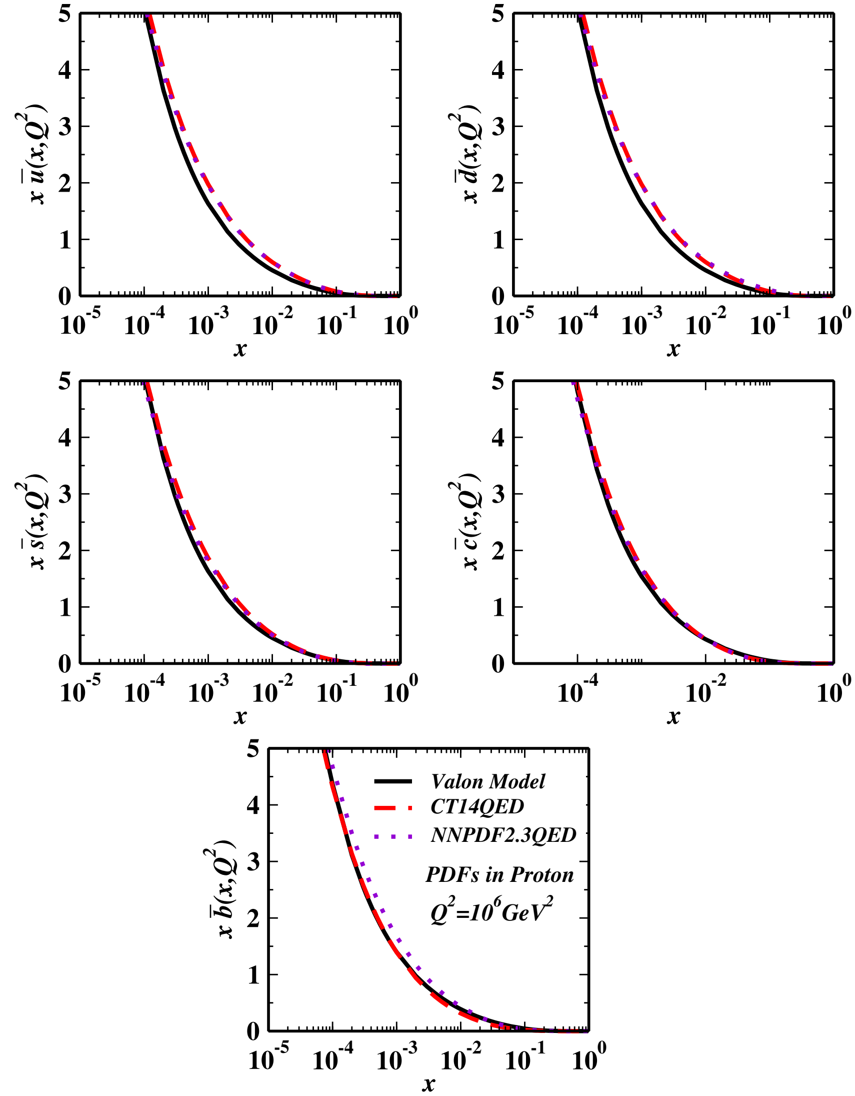

FIG. 7. The sea quark distribution functions at $Q^2 = 10^6$ GeV^2. The solid lines are our results from the valon model, dotted lines are the APFEL (NNPDF2.3QED) code [28], and dashed lines are the CT14QED code [29].

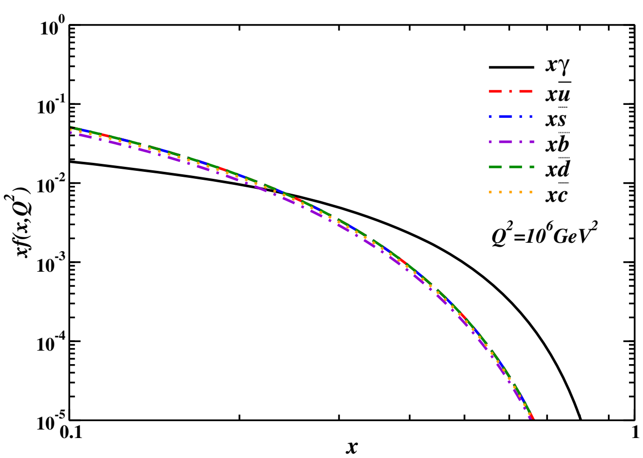

FIG. 8. The sea quark and photon distribution functions at $Q^2 = 10^6$ GeV^2 as a function of x.

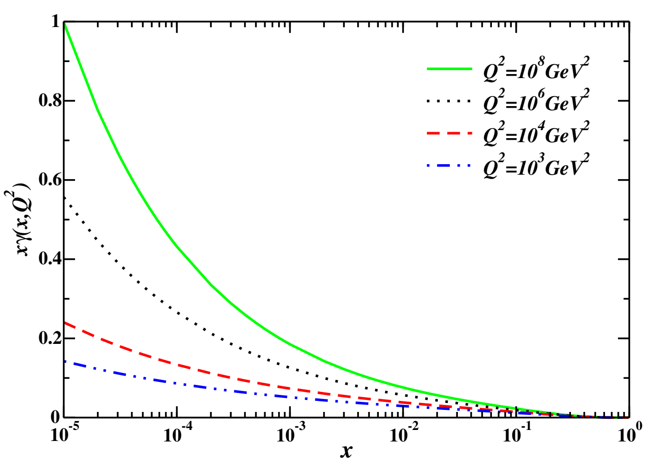

FIG. 9. The photon distribution functions at different values of Q^2 as a function of x.

that the photon distribution functions are larger than the sea quark distribution functions at large scales of energy for the large values of x. The photon distribution functions at different values of Q^2 are plotted in Fig. 9. It is obvious that the photon distribution functions become more significant at high Q^2 where more photon are produced through radiation of the quarks.

IV. Summary and conclusions

In this paper, we calculated the PDFs with QED corrections in the valon model, which have done in the next-to-leading-order QCD and the leading-order QED approximations. The QED corrections to the parton distribution functions are important especially at high Q^2 where the photons can produce more partons. In the valon model, the valance quarks inside the proton can emit and absorb gluons and photons and became valons. These valons can recombine into hadrons. The valon distribution functions are universal and Q^2 independent. The Q^2 dependence of the parton distribution functions come from the solutions of the QCD ⊗ QED DGLAP evolution equations in each valon with suitable initial inputs. We compared our QCD ⊗ QED PDFs set with those of the CT14QED global fits code and APFEL (NNPDF2.3QED) program. There is a nice agreement between them. The results show that the photon distribution functions are larger than the sea quark distribution functions at high Q^2 and high values of x. The result emphasizes that this simple phenomenological model can predict the hadron structures very well. The higher-order QCD and QED corrections can now be added to the QCD ⊗ QED DGLAP evolution equations inside valons. Therefore, in the future, we can extract the QCD ⊗ QED PDFs at N^{2}LO QCD approximation and NLO QED approximation, too.

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