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Exploring the quark flavor puzzle within the three-Higgs doublet model

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We extend the standard model with two extra Higgs doublets. Making use of a symmetry principle, we present flavor symmetries based on cycle groups Z_N that oblige each Higgs doublet to contribute to the mass of only one generation. The Higgs doublets couple to the fermions with different strengths and in this way accommodate the quark mass hierarchy. We systematically search for all charge configurations that naturally lead to the alignment in flavor space of the quark sectors, resulting in a Cabibbo-Kobayashi-Maskawa matrix near to the identity, determined by the quark mass hierarchy, and with the correct overall phenomenological features. The minimal realization is by the group Z_7. We show that only a limited number of solutions exist and that any accidental global symmetry that may occur together with the discrete symmetry is necessarily anomalous. A phenomenological study of each class of solutions concerning predictions to the flavor-changing neutral current phenomena is also performed; for some solutions, it is possible to obtain realistic quark masses and mixing, while the flavor-violating neutral Higgs are light enough to be accessible at the LHC.

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I. INTRODUCTION

The discovery of the Higgs boson in 2012 at the LHC has attested the success of the standard model (SM) in describing the observed fermions and their interactions. However, there exist many theoretical issues or open questions that have no satisfactory answer. In particular, the observed flavor pattern lacks of a definitive explanation, i.e., the quark Yukawa coupling matrices Y_u and Y_d, which in the SM reproduce the six quark masses, three mixings angles, and a complex phase to account for CP-violation phenomena, are general complex matrices, not constrained by any gauge symmetry.

Experimentally, the flavor puzzle is very intricate. First, there is the quark mass hierarchy in both sectors. Second, the mixings in the SM, encoded in the Cabibbo-Kobayashi-Maskawa (CKM) unitary matrix, turns out to be close to the identity matrix. If one takes also the lepton sector into account, the hierarchy there is even more puzzling [1]. On the other hand, in the SM, there is in general no connection between the quark mass hierarchy and the CKM mixing pattern. In fact, if one considers the Extreme Chiral Limit, where the quark masses of the first two generations are set to zero, the mixing does not necessarily vanish [2], and one concludes that the CKM matrix V being close to the identity matrix has nothing to do with the fact that the quark masses are hierarchical. Indeed, in order to have $V \approx \mathbf{1}$, one must have a definite alignment of the quark mass matrices in the flavor space, and to explain this alignment, a flavor symmetry or some other mechanism is required [2].

Among many attempts made in the literature to address the flavor puzzle, extensions of the SM with new Higgs doublet are particularly motivating. This is due to fact that the number of Higgs doublets is not constrained by the SM symmetry. Moreover, the addition of scalar doublets gives rise to new Yukawa interactions, and as a result, it provides a richer framework in approaching the theory of flavor. On the other hand, any new extension of the Higgs sector must be very much constrained, since it naturally leads to flavor-changing neutral currents (FCNCs). At tree level, in the SM, all the flavor-changing transitions are mediated through charged weak currents, and the flavor mixing is controlled by the CKM matrix [3,4]. If new Higgs doublets are added, one expects large FCNC effects already present at tree level. Such effects have not been experimentally observed, and they constrain severely any model with extra Higgs doublets, unless a flavor symmetry suppresses or avoids large FCNCs [5].

Minimal flavor-violating models [6–11] are examples of a multi-Higgs extension in which FCNCs are present at tree level but their contributions to FCNC phenomena involve only off-diagonal elements of the CKM matrix or their products. The first consisted of this kind were proposed by

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Branco, Grimus, and Lavoura (BGL) [12] and subsisted of the SM with two-Higgs doublets together with the requirement of an additional discrete symmetry. BGL models are compatible with lower neutral Higgs masses, and FCNCs occur at tree level, with the new interactions entirely determined in terms of the CKM matrix elements.

The goal of this paper is to generalize the previous BGL models and to, systematically, search for patterns in which a discrete flavor symmetry naturally leads to the alignment of the flavor space of both the quark sectors. Although the quark mass hierarchy does not arise from the symmetry, the effect of both is such that the CKM matrix is near to the identity and has the correct overall phenomenological features, determined by the quark mass hierarchy [13]. To do this, we extend the SM with two extra Higgs doublets to a total of three Higgs ϕ_a. The choice for discrete symmetries is to avoid the presence of Goldstone bosons that appear in the context of any global continuous symmetry, when the spontaneous electroweak symmetry breaking occurs. For the sake of simplicity, we restrict our search to the family group Z_N and demand that the resulting up-quark mass matrix M_u is diagonal. This is to say that, due to the expected strong up-quark mass hierarchy, we only consider those cases in which the contribution of the up-quark mass matrix to quark mixing is negligible.

If one assumes that all Higgs doublets acquire vacuum expectation values with the same order of magnitude, then each Higgs doublet must couple to the fermions with different strengths. Possibly, one could obtain similar results, assuming that the vacuum expectation values (VEVs) of the Higgs have a definite hierarchy instead of the couplings, but this is not considered here. Combining this assumption with the symmetry, we obtain the correct ordered hierarchical pattern if the coupling with ϕ_3 gives the strength of the third generation, the coupling with ϕ_2 gives the strength of the second generation, and the coupling with ϕ_1 gives the strength of the first generation. Therefore, from our point of view, the three Higgs doublets are necessary to ensure that there exist three different coupling strengths, one for each generation, to guarantee simultaneously a hierarchical mass spectrum and a CKM matrix that has the correct overall phenomenological features, e.g., $|V_{cb}|^2 + |V_{ub}|^2 = O(m_s/m_b)^2$, and denoted here by $V \approx \mathbf{1}$.

Indeed, our approach is within the BGL models and such that the FCNC flavor structure is entirely determined by CKM. Through the symmetry, the suppression of the most dangerous FCNCs, by combinations of the CKM matrix elements and light quark masses, is entirely natural.

The paper is organized as follows. In the next section, we present our model and classify the patterns allowed by the discrete symmetry in combination with our assumptions. In Sec. III, we give a brief numerical analysis of the phenomenological output of our solutions. In Sec. IV, we examine the suppression of scalar-mediated FCNCs in our framework for each pattern. Finally, in Sec. V, we present our conclusions.

II. THE MODEL

We extend the Higgs sector of the SM with two extra new scalar doublets, yielding a total of three scalar doublets, as ϕ_1, ϕ_2, ϕ_3. As was mentioned in the Introduction, the main idea for having three Higgs doublets is to implement a discrete flavor symmetry that leads to the alignment of the flavor space of the quark sectors. The quark mass hierarchy does not arise from the symmetry, but together with the symmetry, the effect of both is such that the CKM matrix is near to the identity and has the correct overall phenomenological features, determined by the quark mass hierarchy.

Let us start by considering the most general quark Yukawa coupling Lagrangian invariant in our setup,

$$-\mathcal{L}_{\mathrm{Y}} = (\Omega_a)_{ij}\bar{Q}_{Li}\tilde{\phi}_a u_{R_j} + (\Gamma_a)_{ij}\bar{Q}_{Li}\phi_a d_{R_j} + \mathrm{H.c.}, \tag{1}$$

with the Higgs labeling $a = 1, 2, 3$ and i and j being just the usual flavor indices identifying the generations of fermions. In the above Lagrangian, one has three Yukawa coupling matrices, Ω_1, Ω_2, and Ω_3, for the up-quark sector and three Yukawa coupling matrices, Γ_1, Γ_2, and Γ_3, for the down sector, corresponding to each of the Higgs doublets, ϕ_1, ϕ_2, and ϕ_3. Assuming that only the neutral components of the three Higgs doublets acquire a VEV, the quark masses M_u and M_d are then easily generated as

$$M_u = \Omega_1\langle\phi_1\rangle^* + \Omega_2\langle\phi_2\rangle^* + \Omega_3\langle\phi_3\rangle^*, \tag{2a}$$

$$M_d = \Gamma_1\langle\phi_1\rangle + \Gamma_2\langle\phi_2\rangle + \Gamma_3\langle\phi_3\rangle, \tag{2b}$$

where VEVs $\langle\phi_i\rangle$ are parametrized as

$$\langle\phi_1\rangle = \frac{v_1}{\sqrt{2}}, \qquad \langle\phi_2\rangle = \frac{v_2 e^{i\alpha_2}}{\sqrt{2}}, \qquad \langle\phi_3\rangle = \frac{v_3 e^{i\alpha_3}}{\sqrt{2}}, \tag{3}$$

with v_1, v_2, and v_3 being the VEV moduli and α_2 and α_3 being just complex phases. We have chosen the VEV of ϕ_1 to be real and positive, since this is always possible through a proper gauge transformation. As stated, we assume that the moduli of VEVs v_i are of the same order of magnitude, i.e.,

$$v_1 \sim v_2 \sim v_3. \tag{4}$$

Each of the ϕ_a couples to the quarks with a coupling $(\Omega_a)_{ij}, (\Gamma_a)_{ij}$, which we take be of the same order of magnitude, unless some element vanishes by imposition of the flavor symmmetry. In this sense, each ϕ_a and (Ω_a, Γ_a) will generate its own respective generation; i.e., our model is such that, by imposition of the flavor symmmetry, ϕ_3, Ω_3, and Γ_3 will generate m_t, respectively m_b; that ϕ_2, Ω_2, and Γ_2 will generate m_c, respectively m_s; and that

ϕ_1, Ω_1, and Γ_1 will generate m_u, respectively m_d. Generically, we have

$$v_1|(\Omega_1)_{ij}|\sim m_u,\quad v_2|(\Omega_2)_{ij}|\sim m_c,\quad v_3|(\Omega_3)_{ij}|\sim m_t, \tag{5a}$$

$$v_1|(\Gamma_1)_{ij}|\sim m_d,\quad v_2|(\Gamma_2)_{ij}|\sim m_s,\quad v_3|(\Gamma_3)_{ij}|\sim m_b, \tag{5b}$$

which together with Eq. (4) implies a definite hierarchy among the nonvanishing Yukawa coupling matrix elements:

$$|(\Omega_1)_{ij}| \ll |(\Omega_2)_{ij}| \ll |(\Omega_3)_{ij}|, \tag{6a}$$

$$|(\Gamma_1)_{ij}| < |(\Gamma_2)_{ij}| \ll |(\Gamma_3)_{ij}|. \tag{6b}$$

Next, we focus on the required textures for the Yukawa coupling matrices Ω_a and Γ_a that naturally lead to a hierarchical mass quark spectrum and at the same time to a realistic CKM mixing matrix. These textures must be reproduced by our choice of the flavor symmetry. As referred to in the Introduction, we search for quark mass patterns in which the mass matrix M_u is diagonal. Therefore, one derives from Eqs. (2a) and (6a) the following textures for Ω_a:

$$\Omega_1 = \begin{pmatrix} \mathsf{x} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \Omega_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mathsf{x} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$\Omega_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathsf{x} \end{pmatrix}. \tag{7}$$

The entry x means a nonzero element. In this case, the up-quark masses are given by $m_u = v_1|(\Omega_1)_{11}|$, $m_c = v_2|(\Omega_2)_{22}|$ and $m_t = v_3|(\Omega_3)_{33}|$.

Generically, the down-quark Yukawa coupling matrices must have the following indicative textures:

$$\Gamma_1 = \begin{pmatrix} \mathbf{x} & \mathbf{x} & \mathbf{x} \\ \mathsf{x} & \mathsf{x} & \mathsf{x} \\ \mathsf{x} & \mathsf{x} & \mathsf{x} \end{pmatrix}, \qquad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ \mathbf{x} & \mathbf{x} & \mathbf{x} \\ \mathsf{x} & \mathsf{x} & \mathsf{x} \end{pmatrix},$$
$$\Gamma_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \mathbf{x} & \mathbf{x} & \mathbf{x} \end{pmatrix}. \tag{8}$$

We distinguish rows with bold $\mathbf{x}$ in order to indicate that it is mandatory that at least one of matrix elements within that row must be nonvanishing. Rows denoted with x may be set to zero, without modifying the mass matrix hierarchy. These textures ensure that not only is the mass spectrum hierarchy respected but it also leads to the alignment of the flavor space of both the quark sectors [13] and to a CKM matrix $V \approx \mathbf{1}$. For instance, if one were to not have a vanishing, or comparatively very small, (1,3) entry in the Γ_2, this would not necessarily spoil the scale of m_s, but it would dramatically change the predictions for the CKM mixing matrix.

To force the Yukawa coupling matrices Ω_a and Γ_a to have the indicative forms outlined in Eqs. (7) and (8), we introduce a global flavor symmetry. Since any global continuous symmetry leads to the presence of massless Goldstone bosons after the spontaneous electroweak breaking, one should instead consider a discrete symmetry. Among many possible discrete symmetry constructions, we restrict our searches to the case of cycle groups Z_N. Thus, we demand that any quark or boson multiplet χ transforms according to Z_N as

$$\chi \to \chi' = e^{i\mathcal{Q}(\chi)\frac{2\pi}{N}}\chi, \tag{9}$$

where $\mathcal{Q}(\chi) \in \{0, 1, \ldots, N\}$ is the Z_N charge attributed for the multiplet χ.

We have chosen the up-quark mass matrix M_u to be diagonal. This restricts the flavor symmetry Z_N. We have found that, in order to ensure that all Higgs doublet charges are different, and to have appropriate charges for fields Q_{Li} and u_{Ri}, we must have $N \geq 7$. We simplify our analysis by fixing $N = 7$ and choose

$$\mathcal{Q}(Q_{Li}) = (0, 1, -2), \tag{10a}$$

$$\mathcal{Q}(u_{Ri}) = (0, 2, -4). \tag{10b}$$

In addition, we may also fix

$$\mathcal{Q}(Q_{Li}) = \mathcal{Q}(\phi_i). \tag{11}$$

It turns out that these choices do not restrict the results, i.e., the possible textures that one can have for the Γ_i matrices. Other choices would only imply that we reshuffle the charges of the multiplets.

With the purpose of enumerating the different possible textures for the Γ_i matrices implementable in Z_7, we write down the charges of the trilinears $\mathcal{Q}(\bar{Q}_{Li}\phi_a d_{Rj})$ corresponding to each ϕ_a as

$$\mathcal{Q}(\bar{Q}_{Li}\phi_1 d_{Rj}) = \begin{pmatrix} d_1 & d_2 & d_3 \\ d_1 - 1 & d_2 - 1 & d_3 - 1 \\ d_1 + 2 & d_2 + 2 & d_3 + 2 \end{pmatrix}, \tag{12a}$$

$$\mathcal{Q}(\bar{Q}_{Li}\phi_2 d_{Rj}) = \begin{pmatrix} d_1 + 1 & d_2 + 1 & d_3 + 1 \\ d_1 & d_2 & d_3 \\ d_1 + 3 & d_2 + 3 & d_3 + 3 \end{pmatrix}, \tag{12b}$$

$$\mathcal{Q}(\bar{Q}_{Li}\phi_3 d_{Rj}) = \begin{pmatrix} d_1 - 2 & d_2 - 2 & d_3 - 2 \\ d_1 - 3 & d_2 - 3 & d_3 - 3 \\ d_1 & d_2 & d_3 \end{pmatrix}, \tag{12c}$$

where $d_i \equiv \mathcal{Q}(d_{Ri})$. One can check that, in order to have viable solutions, one must vary the values of $d_i \in \{0, 1, -2, -3\}$.

We summarize in Table I all the allowed textures for the Γ_a matrices and the resulting M_d mass matrix texture, excluding all cases that are irrelevant, e.g., matrices that have too much texture zeros and are singular, or matrices that do not accommodate CP violation. It must be stressed that these are the textures obtained by the different charge configurations that one can possibly choose. However, if one assumes a definite charge configuration, then the entire texture, M_d and M_u, and the respective phenomenology are fixed. As stated, the list of textures in Table I remains unchanged even if one chooses any other set than in Eqs. (10) and (11). As stated, all patterns presented here are of the minimal flavor-violation type [6–11].

Pattern I in the table was already considered in Ref. [14] in the context of Z_8. We discard patterns IV, VII, and X because, contrary to our starting point, at least one of three nonzero couplings with ϕ_1 will turn out be of the same order as the larger coupling with ϕ_2 in order to meet the phenomenological requirements of the CKM matrix.

TABLE I. The table shows the viable configurations for the right-handed down-quark d_{Ri} and their corresponding Γ_1, Γ_2, Γ_3, and M_d matrices. It is understood that, for each pattern and coupling, the parameters expressed here by the same symbol are in fact different but denote the same order of magnitude (or possibly smaller). For example, in pattern I, coupling Γ_1, the three δ, δ, and δ stand for δ_1, δ_2, and δ_3. The same applies to the ε's and c's. For patterns IV, VII, and X, which will be excluded, one of the couplings in Γ_1 turns out to be much larger.

Pattern	$\mathcal{Q}(d_{Ri})$	Γ_1	Γ_2	Γ_3	M_d
I	(0,0,0)	$\begin{pmatrix} \delta & \delta & \delta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ \varepsilon & \varepsilon & \varepsilon \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ c & c & c \end{pmatrix}$	$\begin{pmatrix} \delta & \delta & \delta \\ \varepsilon & \varepsilon & \varepsilon \\ c & c & c \end{pmatrix}$
II	(0,0,1)	$\begin{pmatrix} \delta & \delta & 0 \\ 0 & 0 & \delta \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ \varepsilon & \varepsilon & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$	$\begin{pmatrix} \delta & \delta & 0 \\ \varepsilon & \varepsilon & \delta \\ c & c & 0 \end{pmatrix}$
III	$(0, 0, -3)$	$\begin{pmatrix} \delta & \delta & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ \varepsilon & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$	$\begin{pmatrix} \delta & \delta & 0 \\ \varepsilon & \varepsilon & 0 \\ c & c & \varepsilon \end{pmatrix}$
IV	$(0, 0, -2)$	$\begin{pmatrix} \delta & \delta & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ \varepsilon & \varepsilon & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$	$\begin{pmatrix} \delta & \delta & 0 \\ \varepsilon & \varepsilon & 0 \\ c & c & \varepsilon \end{pmatrix}$
V	(0,1,0)	$\begin{pmatrix} \delta & 0 & \delta \\ 0 & \delta & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ \varepsilon & 0 & \varepsilon \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ c & 0 & c \end{pmatrix}$	$\begin{pmatrix} \delta & 0 & \delta \\ \varepsilon & \delta & \varepsilon \\ c & 0 & c \end{pmatrix}$
VI	$(0, -3, 0)$	$\begin{pmatrix} \delta & 0 & \delta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ \varepsilon & 0 & \varepsilon \\ 0 & \varepsilon & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ c & 0 & c \end{pmatrix}$	$\begin{pmatrix} \delta & 0 & \delta \\ \varepsilon & 0 & \varepsilon \\ c & \varepsilon & c \end{pmatrix}$
VII	$(0, -2, 0)$	$\begin{pmatrix} \delta & 0 & \delta \\ 0 & 0 & 0 \\ 0 & \varepsilon & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ \varepsilon & 0 & \varepsilon \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ c & 0 & c \end{pmatrix}$	$\begin{pmatrix} \delta & 0 & \delta \\ \varepsilon & 0 & \varepsilon \\ c & \varepsilon & c \end{pmatrix}$
VIII	(1,0,0)	$\begin{pmatrix} 0 & \delta & \delta \\ \delta & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon & \varepsilon \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & c & c \end{pmatrix}$	$\begin{pmatrix} 0 & \delta & \delta \\ \delta & \varepsilon & \varepsilon \\ 0 & c & c \end{pmatrix}$
IX	$(-3, 0, 0)$	$\begin{pmatrix} 0 & \delta & \delta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon & \varepsilon \\ \varepsilon & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & c & c \end{pmatrix}$	$\begin{pmatrix} 0 & \delta & \delta \\ 0 & \varepsilon & \varepsilon \\ \varepsilon & c & c \end{pmatrix}$
X	$(-2, 0, 0)$	$\begin{pmatrix} 0 & \delta & \delta \\ 0 & 0 & 0 \\ \varepsilon & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon & \varepsilon \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & c & c \end{pmatrix}$	$\begin{pmatrix} 0 & \delta & \delta \\ 0 & \varepsilon & \varepsilon \\ \varepsilon & c & c \end{pmatrix}$

Notice also that the structure of other M_d's cannot be trivially obtained, e.g., from pattern I, by a transformation of the right-handed down-quark fields.

Our symmetry model may be extended to the charged leptons and neutrinos, e.g., in the context of type-1 seesaw. Choosing for the lepton doublets L_i the charges $\mathcal{Q}(L_i) = (0, -1, 2)$, opposite to the Higgs doublets in Eq. (11), and e.g., for the charges $\mathcal{Q}(e_{Ri}) = (0, -2, 4)$ of the right-handed fields e_{Ri}, we force the charged lepton mass matrix to be diagonal. Then, for the right-handed neutrinos ν_{Ri}, choosing $\mathcal{Q}(\nu_{Ri}) = (0, 0, 0)$, we obtain for the neutrino Dirac mass matrix a pattern similar to pattern I. Of course, for this case, the heavy right-handed neutrino Majorana mass matrix is totally arbitrary. In other cases, i.e., for other patterns and charges, in particular for the right-handed neutrinos, we could introduce scalar singlets with suitable charges, which would then lead to certain heavy right-handed neutrino Majorana mass matrices.

Next, we address an important issue of the model, namely, whether accidental $U(1)$ symmetries may appear in the Yukawa sector or in the potential. One may wonder whether a continuous accidental $U(1)$ symmetry could arise, once the Z_7 is imposed at the Lagrangian level in Eq. (1). This is indeed the case; i.e., for all realizations of Z_7, one has the appearance of a global $U(1)_X$. However, any consistent global $U(1)_X$ must obey to the anomaly-free conditions of global symmetries [15], which read for the anomalies $SU(3)^2 \times U(1)_X$, $SU(2)^2 \times U(1)_X$ and $U(1)_Y^2 \times U(1)_X$ as

$$A_3 \equiv \frac{1}{2}\sum_{i=1}^{3}(2X(Q_{Li}) - X(u_{Ri}) - X(d_{Ri})) = 0, \tag{13a}$$

$$A_2 \equiv \frac{1}{2}\sum_{i=1}^{3}(3X(Q_{Li}) + X(\ell_{Li})) = 0, \tag{13b}$$

$$A_1 \equiv \frac{1}{6}\sum_{i=1}^{3}(X(Q_{Li}) + 3X(\ell_{Li}) - 8X(u_{Ri}) - 2X(d_{Ri}) - 6X(e_{Ri})) = 0, \tag{13c}$$

where $X(\chi)$ is the $U(1)_X$ charge of the fermion multiplet χ. We have properly shifted the Z_7 charges in Eq. (10) and in Table I so that $X(\chi) = \mathcal{Q}(\chi)$, apart from an overall $U(1)_X$ convention. In general, to test those conditions, one needs to specify the transformation laws for all fermionic fields. Looking at Table I, we derive that all the cases, except the first case corresponding to $d_i = (0, 0, 0)$, violate the condition given in Eq. (13a) that depends only on colored fermion multiplets. In the case $d_i = (0, 0, 0)$, if one assigns the charged lepton charges as $X(\ell_{Li}) = X(Q_{Li})$, one concludes that the condition given in Eq. (13b) is violated. One then concludes that the global $U(1)_X$ symmetry is anomalous and therefore only the discrete symmetry Z_7 persists.

We also comment on the scalar potential of our model. The most general scalar potential with three scalars invariant under Z_7 reads as

$$V(\phi) = \sum_i [-\mu_i^2 \phi_i^\dagger \phi_i + \lambda_i (\phi_i^\dagger \phi_i)^2] + \sum_{i<j} [+C_i (\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) + \bar{C}_i |\phi_i^\dagger \phi_j|^2], \tag{14}$$

where the constants μ_i^2, λ_i, C_i, and $\bar{C}_i$ are taken real for $i, j = 1, 2, 3$. Analyzing the potential above, one sees that it gives rise to the accidental global continuous symmetry $\phi_i \to e^{i\alpha_i}\phi_i$, for arbitrary α_i, which upon spontaneous symmetry breaking leads to a massless neutral scalar, at tree level. Introducing soft-breaking terms like $m_{ij}^2 \phi_i^\dagger \phi_j +$ H.c. can erase the problem. Another possibility without spoiling the Z_7 symmetry is to add new scalar singlets so that the coefficients m_{ij}^2 are effectively obtained once the scalar singlets acquire VEVs.

III. NUMERICAL ANALYSIS

In this section, we give the phenomenological predictions obtained by the patterns listed in Table I. Note that, although these patterns arise directly from the chosen discrete charge configuration of the quark fields, one may further preform a residual flavor transformation of the right-handed down-quark fields, resulting in an extra zero entry in M_d. Taking this into account, all the parameters in each pattern may be uniquely expressed in terms of down-quark masses and the CKM matrix elements V_{ij}. This follows directly from the diagonalization equation of M_d,

$$V^\dagger M_d W = \mathrm{diag}(m_d, m_s, m_b) \Rightarrow M_d = V diag(m_d, m_s, m_b) W^\dagger, \tag{15}$$

with V being the CKM mixing matrix, since M_u is diagonal. Because of the zero entries in M_d, it is easy to extract the right-handed diagonalization matrix W, completely in terms of the down-quark masses and the V_{ij}. Thus, all parameters, modulo the residual transformation of the right-handed down-quark fields, are fixed; i.e., all parameters in each pattern may be uniquely expressed in terms of down-quark masses and the CKM matrix elements V_{ij}, including the right-handed diagonalization matrix W of M_d. More precisely, all matrix elements of V are written in terms of Wolfenstein real parameters λ, A, $\bar{\rho}$, and $\bar{\eta}$, defined in terms of rephasing invariant quantities as

$$\lambda \equiv \frac{|V_{us}|}{\sqrt{|V_{us}|^2 + |V_{ud}|^2}}, \qquad A \equiv \frac{1}{\lambda}\left|\frac{V_{cb}}{V_{us}}\right|, \tag{16a}$$

$$\bar{\rho} + i\bar{\eta} \equiv -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}, \tag{16b}$$

and $\mathrm{diag}(m_d, m_s, m_b)$ in Eq. (15),

$$\begin{array}{lcl} \sqrt{\frac{m_d}{m_s}} = \sqrt{\frac{k_d}{k_s}}\lambda & & m_d = k_d\lambda^4 m_b \\ & \Rightarrow & \\ \frac{m_s}{m_b} = k_s\lambda^2 & & m_s = k_s\lambda^2 m_b, \end{array} \tag{17}$$

with, phenomenologically, k_d and k_s being factors of order 1. Writing $W^\dagger$ in Eq. (15) as $W^\dagger = (v_1, v_2, v_3)$, with the v_i vectors formed by the ith column of $W^\dagger$, we find, e.g., for pattern II,

$$v_3 = \frac{1}{n_3}\begin{pmatrix} \frac{m_d}{m_b}V_{11} \\ \frac{m_s}{m_b}V_{12} \\ V_{13} \end{pmatrix} \times \begin{pmatrix} \frac{m_d}{m_b}V_{31} \\ \frac{m_s}{m_b}V_{32} \\ V_{33} \end{pmatrix}, \tag{18}$$

where n_3 is the norm of the vector obtained from the external product of the two vectors. Taking into account the extra freedom of transformation of the right-handed fields, we may choose $M_{31}^d = 0$, corresponding to $c_1 = 0$ in Table I, and we conclude that

$$v_1 = \frac{1}{n_1}\begin{pmatrix} \frac{m_d}{m_b}V_{31} \\ \frac{m_s}{m_b}V_{32} \\ V_{33} \end{pmatrix} \times v_3^*. \tag{19}$$

Obviously, then $v_2 = \frac{1}{n_2} v_1^* \times v_3^*$. This process is replicated for all patterns. Thus, V and W are entirely expressed in terms of Wolfenstein parameters and k_d and k_s of Eq. (17). These two matrices will be used later to compute the patterns of the FCNCs in Table III. Indeed, in this way, we find, e.g., for pattern II, in leading order order,

$$M_d = m_b\begin{pmatrix} -k_d\lambda^3 & (\bar{\rho} - i\bar{\eta})A\lambda^3 & 0 \\ -k_d\lambda^2 & A\lambda^2 & -k_s\lambda^3 \\ 0 & 1 & 0 \end{pmatrix}, \tag{20}$$

which corresponds to the expected power series in which the couplings in Γ_1 to the first Higgs ϕ_1 are comparatively smaller than the couplings in Γ_2, and these smaller than the couplings in Γ_3. Similar results are obtained for all patterns

TABLE II. A numerical example of a Yukawa coupling configuration for each pattern that gives the correct hierarchy among the quark masses and mixing.

Pattern	$v_1 Y_1$	$v_2 Y_2$	$v_3 Y_3$	M_d
I	$\begin{pmatrix} 0.00277 & 0.0124 & 0.0101e^{1.907i} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.0537 & 0.119 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2.86 \end{pmatrix}$	$\begin{pmatrix} 0.00277 & 0.00124 & 0.0101e^{1.907i} \\ 0 & 0.0537 & 0.119 \\ 0 & 0 & 2.86 \end{pmatrix}$
II	$\begin{pmatrix} 0.0123 & 0.0101e^{-1.235i} & 0 \\ 0 & 0 & 0.012 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0.0524 & 0.119 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2.86 & 0 \end{pmatrix}$	$\begin{pmatrix} 0.0123 & 0.0101e^{-1.235i} & 0 \\ 0.0524 & 0.119 & 0.012 \\ 0 & 2.86 & 0 \end{pmatrix}$
III	$\begin{pmatrix} 0.0127 & 0.0102e^{-1.253i} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0.0523 & 0.120 & 0 \\ 0 & 0 & 0.295 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2.844 & 0 \end{pmatrix}$	$\begin{pmatrix} 0.0127 & 0.0102e^{-1.253i} & 0 \\ 0.0523 & 0.120 & 0 \\ 0 & 2.844 & 0.295 \end{pmatrix}$
V	$\begin{pmatrix} 0.0127 & 0 & 0.0101e^{-1.234i} \\ 0 & 0.0117 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0.0524 & 0 & 0.112 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2.86 \end{pmatrix}$	$\begin{pmatrix} 0.0127 & 0 & 0.0101e^{-1.234i} \\ 0.0524 & 0.0117 & 0.112 \\ 0 & 0 & 2.86 \end{pmatrix}$
VI	$\begin{pmatrix} 0.0127 & 0 & 0.0102e^{-1.253i} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0.0523 & 0 & 0.120 \\ 0 & 0.295 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2.844 \end{pmatrix}$	$\begin{pmatrix} 0.0127 & 0 & 0.0102e^{-1.253i} \\ 0.0523 & 0 & 0.120 \\ 0 & 0.295 & 2.844 \end{pmatrix}$
VIII	$\begin{pmatrix} 0 & 0.0127 & 0.0102e^{1.907i} \\ 0.0117 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.0524 & 0.119 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2.86 \end{pmatrix}$	$\begin{pmatrix} 0 & 0.0127 & 0.0102e^{1.907i} \\ 0.0117 & 0.0524 & 0.119 \\ 0 & 0 & 2.86 \end{pmatrix}$
IX	$\begin{pmatrix} 0 & 0.0127 & 0.0101e^{-1.253i} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.0523 & 0.120 \\ 0.295 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2.844 \end{pmatrix}$	$\begin{pmatrix} 0 & 0.0127 & 0.0101e^{-1.253i} \\ 0 & 0.0523 & 0.120 \\ 0.295 & 0 & 2.844 \end{pmatrix}$

in Table I, except for patterns IV, VII, and X, in which, e.g., for pattern IV, we find that the coupling in $(\Gamma_1)_{33}$ is proportional to λ, which is too large and contradicts our initial assumption that all couplings in Γ_1 to the first Higgs ϕ_1 must be smaller than the couplings in Γ_2 to the second Higgs ϕ_2. Therefore, we exclude patterns IV, VII, and X.

We give in Table II a numerical example of a Yukawa coupling configuration for each pattern. We use the following quark running masses at the electroweak scale M_Z,

$$m_u = 1.3^{+0.4}_{-0.2}\ \text{MeV}, \qquad m_d = 2.7 \pm 0.3\ \text{MeV},$$
$$m_s = 55^{+5}_{-3}\ \text{MeV}, \tag{21a}$$

$$m_c = 0.63 \pm 0.03\ \text{GeV}, \qquad m_b = 2.86^{+0.05}_{-0.04}\ \text{GeV},$$
$$m_t = 172.6 \pm 1.5\ \text{GeV}, \tag{21b}$$

which were obtained from a renormalization group equation evolution at four-loop level [16], which, taking into account all experimental constrains [17], implies

$$\lambda = 0.2255 \pm 0.0006, \qquad A = 0.818 \pm 0.015, \tag{22a}$$

$$\bar{\rho} = 0.124 \pm 0.024, \qquad \bar{\eta} = 0.354 \pm 0.015. \tag{22b}$$

IV. PREDICTIONS OF FLAVOR-CHANGING NEUTRAL CURRENTS

In the SM, FCNCs are forbidden at tree level, both in the gauge and the Higgs sectors. However, by extending the SM field content, one obtains Higgs flavor-violating neutral couplings [18]. In terms of the quark mass eigenstates, the Yukawa couplings to the Higgs neutral fields are

$$\begin{aligned}-\mathcal{L}_{\text{Neutral Yukawa}} = {}& \frac{H_0}{v}(\overline{d_L} D_d d_R + \overline{u_L} D_u u_R) \\ &+ \frac{1}{v'}\overline{d_L} N_1^d (R_1 + iI_1) d_R \\ &+ \frac{1}{v'}\overline{u_L} N_1^u (R_1 - iI_1) u_R \\ &+ \frac{1}{v''}\overline{d_L} N_2^d (R_2 + iI_2) d_R \\ &+ \frac{1}{v''}\overline{u_L} N_2^u (R_2 - iI_2) u_R + h.c,\end{aligned} \tag{23}$$

where the $N_i^{u,d}$ are the matrices that give the strength and the flavor structure of the FCNC,

$$N_1^d = \frac{1}{\sqrt{2}} V^\dagger (v_2 \Gamma_1 - v_1 e^{i\alpha_2} \Gamma_2) W, \tag{24a}$$

$$N_2^d = \frac{1}{\sqrt{2}} V^\dagger \left(v_1 \Gamma_1 + v_2 e^{i\alpha_2} \Gamma_2 - \frac{v_1^2 + v_2^2}{v_3} e^{i\alpha_3} \Gamma_3 \right) W, \tag{24b}$$

$$N_1^u = \frac{1}{\sqrt{2}} (v_2 \Omega_1 - v_1 e^{-i\alpha_2} \Omega_2), \tag{24c}$$

$$N_2^u = \frac{1}{\sqrt{2}} \left(v_1 \Omega_1 + v_2 e^{-i\alpha_2} \Omega_2 - \frac{v_1^2 + v_2^2}{v_3} e^{-i\alpha_3} \Omega_3 \right). \tag{24d}$$

Since in our case the N_i^u are diagonal, there are no flavor-violating terms in the up sector. Therefore, the analysis of the FCNC resumes only to the down-quark sector. One can use the equations of the mass matrices presented in Eq. (2) to simplify the Higgs-mediated FCNC matrices for the down sector:

TABLE III. For all allowed patterns, we find that the matrices $N_1^d - D_d$ and N_2^d are proportional to the following patterns, where λ is the Cabibbo angle.

Pattern	$(N_1^d - D_d)\sim$	$N_2^d\sim$
I	$\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^7 & \lambda^4 & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda^4 & \lambda^7 & \lambda^3 \\ \lambda^9 & \lambda^2 & \lambda^2 \\ \lambda^7 & \lambda^4 & 1 \end{pmatrix}$
II	$\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^5 & \lambda^4 & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda^4 & \lambda^7 & \lambda^3 \\ \lambda^9 & \lambda^2 & \lambda^2 \\ \lambda^7 & \lambda^4 & 1 \end{pmatrix}$
III	$\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$
IV	$\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$
V	$\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^5 & \lambda^4 & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda^4 & \lambda^7 & \lambda^3 \\ \lambda^7 & \lambda^2 & \lambda^2 \\ \lambda^5 & \lambda^4 & 1 \end{pmatrix}$
VI	$\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$
VII	$\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$
VIII	$\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^5 & \lambda^4 & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda^4 & \lambda^7 & \lambda^3 \\ \lambda^7 & \lambda^2 & \lambda^2 \\ \lambda^5 & \lambda^4 & 1 \end{pmatrix}$
IX	$\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$
X	$\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$

$$N_1^d = \frac{v_2}{v_1} D_d - \frac{v_2}{\sqrt{2}} \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) e^{i\alpha_2} V^\dagger \Gamma_2 W - \frac{v_2 v_3}{v_1 \sqrt{2}} e^{i\alpha_3} V^\dagger \Gamma_3 W \tag{25a}$$

$$N_2^d = D_d - \frac{v^2}{v_3 \sqrt{2}} e^{i\alpha_3} V^\dagger \Gamma_3 W. \tag{25b}$$

To satisfy experimental constraints arising from $K^0 - \overline{K^0}$, $B^0 - \overline{B^0}$, and $D^0 - \overline{D^0}$, the off-diagonal elements of the Yukawa interactions N_1^d and N_2^d must be highly suppressed [19,20]. For each of our ten solutions in Table I, we summarize in Table III all FCNC patterns, for each solution, and for $v_1 = v_2 = v_3$ and $\alpha_2 = \alpha_3 = 0$. These patterns are of the BGL type, since in Eq. (25) all matrices can be expressed in terms of the CKM mixing matrix elements and the down-quark masses. As explained, to obtain these patterns, we express the CKM matrix V and the matrix W in terms of Wolfenstein parameters.

The tree-level Higgs-mediated $\Delta S = 2$ amplitude must be suppressed. This can always be achieved if one chooses the masses of the flavor-violating neutral Higgs scalars sufficiently heavy. However, from the experimental point of view, it would be interesting to have these masses as low as possible. Therefore, we also estimate the lower bound of these masses by considering the contribution to $B^0 - \overline{B^0}$ mixing. We choose this mixing, since for our patterns the (3,1) entry of the matrix N_1^d is less suppressed in certain cases and would require very heavy flavor-violating neutral Higgs. The relevant quantity is the off-diagonal matrix element M_{12}, which connects the B meson with the corresponding antimeson. This matrix element, $M_{12}^{\rm NP}$, receives contributions [19] both from a SM box diagram and a tree-level diagram involving the FCNC,

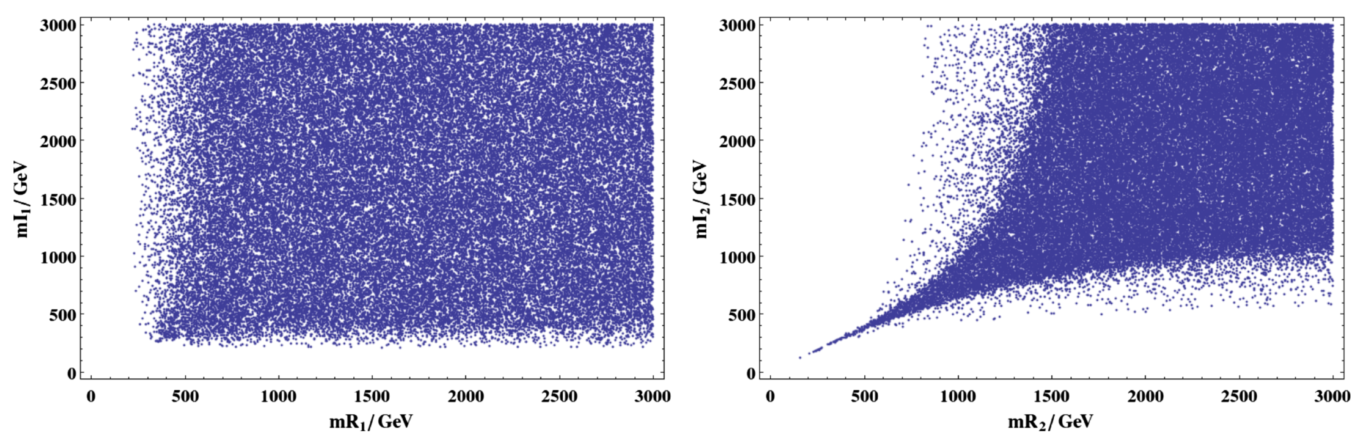
$$M_{12} = M_{12}^{\rm SM} + M_{12}^{\rm NP}, \tag{26}$$

where the New Physics (NP) short-distance tree-level contribution to the meson-antimeson contribution is

$$M_{12}^{\rm NP} = \sum_i^2 \frac{f_B^2 m_B}{96 v^2 m_{R_i}^2} \left\{ \left[\left(1 + \left(\frac{m_B}{m_d + m_b} \right)^2 \right) (a_i^R)_{12} \right] - \left[\left(1 + 11 \left(\frac{m_B}{m_d + m_b} \right)^2 \right] \right) (b_i^R)_{12} \right\} + \sum_i^2 \frac{f_B^2 m_B}{96 v^2 m_{I_i}^2} \left\{ \left[\left(1 + \left(\frac{m_B}{m_d + m_b} \right)^2 \right) (a_i^I)_{12} \right] - \left[\left(1 + 11 \left(\frac{m_B}{m_d + m_b} \right)^2 \right] \right) (b_i^I)_{12} \right\} \tag{27}$$

with $v^2 = v_1^2 + v_2^2 + v_2^2$ and

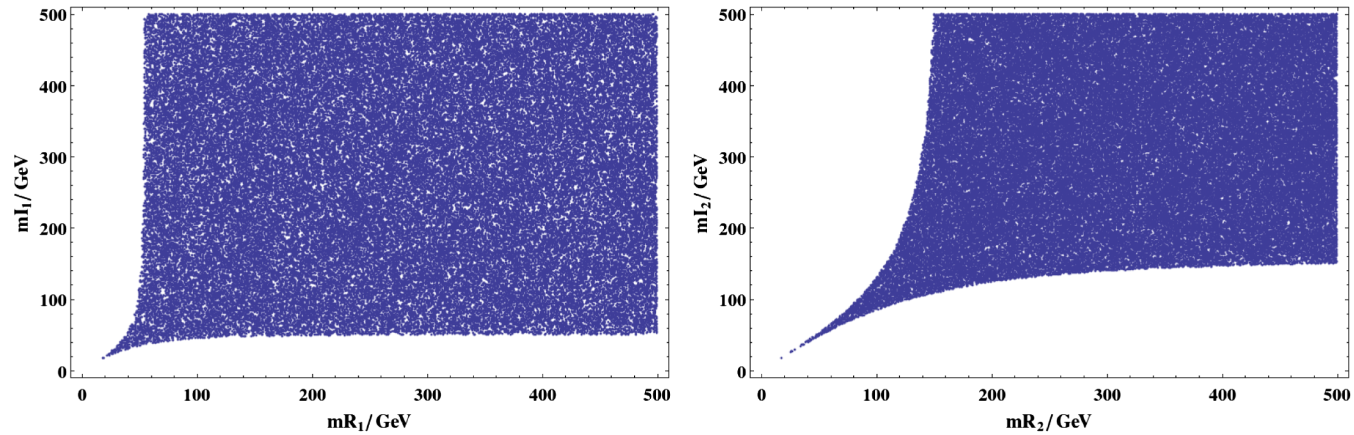
$$\begin{matrix} (a_i^R)_{12} = [(N_i^d)_{31}^* + (N_i^d)_{13}]^2 \\ (a_i^I)_{12} = -[(N_i^d)_{31}^* - (N_i^d)_{13}]^2 \end{matrix}, \qquad \begin{matrix} (b_i^R)_{12} = [(N_i^d)_{31}^* - (N_i^d)_{13}]^2 \\ (b_i^I)_{12} = -[(N_i^d)_{31}^* + (N_i^d)_{13}]^2 \end{matrix}, \qquad i = 1, 2. \tag{28}$$



(a) Estimate of the lower bound for the flavour-violating Higgs masses for R_1 and I_1.

(b) Estimate of the lower bound for the flavour-violating Higgs masses for R_2 and I_2.

FIG. 1. Lower bound for the flavor-violating Higgs masses for case III.



(a) Estimate of the lower bound for the flavour-violating Higgs masses for R_1 and I_1.

(b) Estimate of the lower bound for the flavour-violating Higgs masses for R_2 and I_2.

FIG. 2. Lower bound for the flavor-violating Higgs masses for case VIII.

To obtain a conservative measure, we have tentatively expanded the original expression in Ref. [19] and, for the three-Higgs case, included all neutral Higgs mass eigenstates.

Adopting as input values the Particle Data Group experimental determinations of f_B, m_B, and Δm_B and considering a common VEV for all Higgs doublets, we impose the inequality $M_{12}^{\rm NP} < \Delta m_B$. The following plots show an estimate of the lower bound for the flavor-violating Higgs masses for two different patterns. We plot two masses chosen from the set $(m_1^R, m_2^R, m_1^I, m_2^I)$, while the other two are varied over a wide range. In Fig. 1, we illustrate these lower bounds for pattern III, which are restricted by the (3,1) entry of N_1^d matrix and suppressed by a factor of λ. For pattern VIII, in Fig. 2, we find the flavor-violating neutral Higgs to be much lighter and possibly accessible at the LHC.

V. CONCLUSIONS

We have presented a model based on the SM with three Higgs and an additional flavor discrete symmetry. We have shown that there exist flavor discrete symmetry configurations that lead to the alignment of the quark sectors. By allowing each scalar field to couple to each quark generation with a distinctive scale, one obtains the quark mass hierarchy, and although this hierarchy does not arise from the symmetry, the effect of both is such that the CKM matrix is near to the identity and has the correct overall phenomenological features. In this context, we have obtained seven solutions fulfilling these requirements, with the additional constraint of the up-quark mass matrix being diagonal and real.

We have also verified if accidental $U(1)$ symmetries may appear in the Yukawa sector or in the potential, particularly the case in which a continuous accidental $U(1)$ symmetry could arise, once the Z_7 is imposed at the Lagrangian level. This was indeed the case; however, we have shown that the anomaly-free conditions of global symmetries were violated. Thus, the global $U(1)_X$ symmetry is anomalous, and therefore only the discrete symmetry Z_7 persists.

As in this model new Higgs doublets are added, one expects large FCNC effects, already present at tree level. However, such effects have not been experimentally observed. We show that for certain specific implementations of the flavor symmetry it is possible to suppress the FCNC effects and to ensure that the flavor-violating neutral Higgs are light enough to be accessible at the LHC. Indeed, in this respect, our model is a generalization of the BGL models for the three-Higgs doublet model, since the FCNC flavor structure is entirely determined by CKM.

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