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$N = 4$ supersymmetric BMS_3 algebras from asymptotic symmetry analysis

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We consider three dimensional $N = 4$ flat supergravity, with an Abelian R-symmetry enhancing the gravitational phase space. We obtain the field configuration whose asymptotic symmetries at null infinity coincide with the centrally extended $N = 4$ super-Bondi-Metzner-Sachs (BMS) algebra. The killing spinors for this generic configuration are obtained together with the energy bounds imposed by supersymmetry. It is explicitly shown that the same algebra can be obtained as a flat (AdS radius $\to \infty$) limit of the combined (2,0) and (0,2) sectors of AdS supergravity.

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I. INTRODUCTION AND SUMMARY

Supergravity theories in $2 + 1$ dimensions have many interesting features which have no equivalent in their higher-dimensional counterparts. It is well known for instance, that no local degrees of freedom exist in the bulk and that it is not possible to define the linear momentum or the supercharges for any solution at spatial infinity [1–3]. One can only define the energy and the angular momentum there associated to asymptotic time translations and spatial rotations. However, the scenario changes at the null infinity. Almost half a century ago, in their seminal works [4,5], Bondi, van der Burg, Metzner and independently Sachs first introduced the symmetries of 4D flat space times at their null infinity, named BMS symmetry. Later, in [1,6,7], it has been shown that the asymptotic structure for flat three-dimensional gravity at their null infinity is also much richer: it consists of an infinite dimensional symmetry whose generators, supertranslation and superrotation generators, act on the boundary coordinates.

A similar symmetry enhancement also takes place when one considers the asymptotic algebra of symmetries of three dimensional AdS (super)gravity. In their seminal paper [8], Brown and Henneaux showed that, upon imposing suitable boundary conditions for the fields in asymptotically AdS_3 space, the symmetry enhances from SO(2,2) to the infinite dimensional conformal algebra in two dimensions. This is connected to the enhancement of the flat asymptotic algebra, as the latter corresponds to a well defined flat space limit of the AdS algebra [9–14].[1] Similar results have been obtained for supersymmetric theories in asymptotic AdS_3 spaces [16–18].

The enhancement of the symmetry algebra of flat three dimensional gravity has been extended to the $N = 1$ [19] and $N = 2$ [20] supersymmetric cases. In [21] all possible N—extended quantum Super BMS_3 algebras were found as a well-defined Inönü-Wigner contraction of the super-Virasoro algebras. The $N = 4$ and $N = 8$ algebras also possess nontrivial U(1) and non-Abelian internal R—symmetries. The scaling proposal for the R-charges was the main ingredient of this construction.

The purpose of the present paper is to find the $N = 4$ super BMS_3 algebra i.e. the algebra of three dimensional $N = 4$ flat supergravity theory at null infinity, by a direct asymptotic symmetry analysis *a' la* [19], i.e. by finding the appropriate boundary conditions to impose on the fields. This provides a check of the algebra found in [21] while also validating the proposed scaling of the R—charges. We leave the similar analysis for the $N = 8$ Super BMS_3 to a future project [22].

The $N = 4$ Super BMS_3 algebra obtained, which is the central result of this paper, is given in Eq. (3.26). The agreement with the result of [21], as will be clear from the detailed analysis done in later sections, works out it a nontrivial way. In fact, it was noticed long back in [23] that the presence of R-symmetry in the extended super-conformal algebra leads to nonlinearities in the asymptotic symmetry algebra. Those nonlinearities can be canceled only by appropriate Sugawara shift of the stress tensor. We will explicitly show how this issue arises when computing the asymptotic AdS algebras and its Inönü-Wigner contraction which gives the $N = 4$ Super BMS_3 algebra (3.26).

The paper is organized as follows: in the second section, we present the action for three dimensional $N = 4$ supergravity theory. In the next section, we derive the $N = 4$ super BMS_3 algebra by choosing the appropriate boundary conditions for the fields. In Sec. IV, we present the bounds on the energy of asymptotically flat solutions of the theory, imposed by supersymmetry. We also solve the asymptotic and global Killing spinor equations, and provide explicit solutions. Finally in the last section, we show how the asymptotic algebra is obtained by an appropriate flat limit of the asymptotic AdS_3 algebra. Our notations, conventions

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[1]A free field realization of this algebra was first obtained in [15].

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and some details of the computations are presented in the Appendices.

II. CONSTRUCTION OF THE ACTION

In three space-time dimensions, a gravity theory with (non)zero cosmological constant possesses a Chern-Simons formulation. For a three dimensional gauge field $\mathcal{A} = \mathcal{A}_\mu dx^\mu$, the Chern-Simons action is given by,

$$I[\mathcal{A}] = \frac{k}{4\pi}\int \left\langle \mathcal{A}, d\mathcal{A} + \frac{2}{3}\mathcal{A}^2 \right\rangle, \tag{2.1}$$

where $\langle , \rangle$ denotes the invariant bilinear form that one constructs from the symmetry algebra of the corresponding theory (see Appendix B for details on how to build this bilinear form).

As mentioned in the introduction, in this paper we want to construct the asymptotic symmetry algebra of three dimensional $N = 4$ flat supergravity theory. The bulk symmetry algebra for this theory consists of bosonic generators $\mathcal{J}_a, P_a, (a = 0, 1, 2), \mathcal{R}$, $\mathcal{S}$ and Majorana fermionic generators $\mathcal{Q}^{1\pm}_\alpha, \mathcal{Q}^{2\pm}_\alpha, (\alpha = \pm\frac{1}{2})$. The commutation relations are

$$\begin{aligned}
&[\mathcal{J}_a, \mathcal{J}_b] = \epsilon_{abc}\mathcal{J}^c, \qquad [\mathcal{J}_a, P_b] = \epsilon_{abc}P^c,\\
&[\mathcal{J}_a, \mathcal{Q}^{1,2\pm}_\alpha] = \frac{1}{2}(\Gamma_a)^\beta{}_\alpha \mathcal{Q}^{1,2\pm}_\beta,\\
&[\mathcal{R}, \mathcal{Q}^{1\pm}_\alpha] = \pm\frac{1}{2}\mathcal{Q}^{1\pm}_\alpha, \qquad [\mathcal{R}, \mathcal{Q}^{2\pm}_\alpha] = \mp\frac{1}{2}\mathcal{Q}^{2\pm}_\alpha,\\
&\{\mathcal{Q}^{1\pm}_\alpha, \mathcal{Q}^{1\mp}_\beta\} = -\frac{1}{2}(C\Gamma^a)_{\alpha\beta}P_a \mp \frac{1}{2}C_{\alpha\beta}\mathcal{S},\\
&\{\mathcal{Q}^{2\pm}_\alpha, \mathcal{Q}^{2\mp}_\beta\} = -\frac{1}{2}(C\Gamma^a)_{\alpha\beta}P_a \pm \frac{1}{2}C_{\alpha\beta}\mathcal{S}.
\end{aligned} \tag{2.2}$$

Here, $\mathcal{S}$ is a possible central extension of the super Poincaré algebra while $\mathcal{R}$ acts as a proper R-symmetry. One can construct the invariant nondegenerate bilinear form for this algebra (see Appendix B) whose nonzero elements are,[2]

$$\langle \mathcal{J}_a, P_b\rangle = \eta_{ab}, \quad \langle \mathcal{Q}^{1,2\pm}_a, \mathcal{Q}^{1,2\mp}_\beta\rangle = C_{\alpha\beta}, \quad \langle \mathcal{R}, \mathcal{S}\rangle = -1. \tag{2.3}$$

To write down the action for this supergravity theory, one expands the gauge field in terms of the basis generators as,

$$\mathcal{A} = e^a P_a + \omega^a \mathcal{J}_a + \sum_{\alpha=\pm}\psi^{1\alpha}_\pm \mathcal{Q}^{1\pm}_\alpha + \sum_{\alpha=\pm}\psi^{2\alpha}_\pm \mathcal{Q}^{2\pm}_\alpha + \upsilon\mathcal{R} + \sigma\mathcal{S}, \tag{2.4}$$

where, e^a is the vielbein field, ω^a is the corresponding dual spin connection, $\psi^{1\alpha}_\pm$, $\psi^{2\alpha}_\pm$ are Majorana gravitini and υ, σ are internal gauge fields. With this, we can readily write down the action for $N = 4$ asymptotically flat supergravity theory as,

$$S = \frac{k}{4\pi}\int 2e^a R_a - \sigma d\upsilon - \upsilon d\sigma + \sum_{a=\pm}\bar{\psi}^1_a D\psi^1_{-a} + \sum_{a=\pm}\bar{\psi}^2_a D\psi^2_{-a} \tag{2.5}$$

where

$$\begin{aligned}
D\psi^1_\pm &= d\psi^1_\pm + \frac{1}{2}\omega^a\Gamma_a\psi^1_\pm \pm \frac{1}{2}\upsilon\psi^1_\pm,\\
D\psi^2_\pm &= d\psi^2_\pm + \frac{1}{2}\omega^a\Gamma_a\psi^2_\pm \mp \frac{1}{2}\upsilon\psi^2_\pm,
\end{aligned} \tag{2.6}$$

$$R^a = d\omega^a + \frac{1}{2}\epsilon^a{}_{bc}\omega^b\omega^c. \tag{2.7}$$

The invariance of the action S under the supersymmetry (2.2) can be straightforwardly checked by using the transformations,

$$\delta\mathcal{A} = d\lambda^{\rm susy} + [\mathcal{A}, \lambda^{\rm susy}], \qquad \lambda^{\rm susy} = \theta^{1\alpha}_\pm\mathcal{Q}^{1\pm}_\alpha + \theta^{2\alpha}_\pm\mathcal{Q}^{2\pm}_\alpha.$$

which explicitly read:

$$\begin{aligned}
\delta e^a &= \frac{1}{2}(\bar{\theta}^1_+\Gamma^a\psi^1_- + \bar{\theta}^1_-\Gamma^a\psi^1_+ + \bar{\theta}^2_+\Gamma^a\psi^2_- + \bar{\theta}^2_-\Gamma^a\psi^2_+),\\
\delta\omega^a &= 0,\\
\delta\psi^{1\alpha}_\pm &= d\theta^{1\alpha}_\pm + \frac{1}{2}\omega^a\Gamma_a\theta^{1\alpha}_\pm \pm \frac{1}{2}\upsilon\theta^{1\alpha}_\pm = D\theta^{1\alpha}_\pm,\\
\delta\psi^{2\alpha}_\pm &= d\theta^{2\alpha}_\pm + \frac{1}{2}\omega^a\Gamma_a\theta^{2\alpha}_\pm \mp \frac{1}{2}\upsilon\theta^{2\alpha}_\pm = D\theta^{2\alpha}_\pm,\\
\delta\sigma &= \mp\frac{1}{2}(\bar{\psi}^1_\pm\theta^1_\mp - \bar{\psi}^2_\pm\theta^2_\mp), \qquad \delta\upsilon = 0.
\end{aligned}$$

The algebra of supersymmetry closes on-shell into a general coordinate transformation, a Lorentz transformation (with dualized parameter $\lambda^a = \epsilon^{abc}\Lambda_{bc}$) and a supersymmetry transformation with parameters $\varepsilon_\pm = -\xi^\nu\psi^1_{\nu\pm}$ and $\vartheta_\pm = -\xi^\nu\psi^2_{\nu\pm}$:

$$\begin{aligned}
&[\delta(\varepsilon^1_+, \varepsilon^1_-, \vartheta^1_+, \vartheta^1_-), \delta(\varepsilon^2_+, \varepsilon^2_-, \vartheta^2_+, \vartheta^2_-)]\\
&\quad = \delta_{\rm Lor}(\lambda^a = -\xi^\nu\omega_\nu{}^a) + \delta_{\rm susy}(\varepsilon_+, \varepsilon_-, \vartheta_+, \vartheta_-)\\
&\qquad + \delta_{\rm g.c.}\Big(\xi^\nu = -\frac{1}{2}(\bar{\varepsilon}^2_-\Gamma^\nu\varepsilon^1_+ + \bar{\varepsilon}^2_+\Gamma^\nu\varepsilon^1_- + \bar{\vartheta}^2_-\Gamma^\nu\vartheta^1_+\\
&\qquad + \bar{\vartheta}^2_+\Gamma^\nu\vartheta^1_-)\Big).
\end{aligned} \tag{2.8}$$

The dynamical equations are

[2]Our conventions are summarized in Appendix A.

$$T^a = -\frac{1}{2}(\bar{\psi}^1_+\Gamma^a\psi^1_- + \bar{\psi}^2_+\Gamma^a\psi^2_-) \quad D\psi^{1,2}_\pm = D\bar{\psi}^{1,2}_\pm = 0,$$
$$\mathrm{d}\upsilon = F_\upsilon = 0 \qquad 2F_\sigma + (\bar{\psi}^1_-\psi^1_+ - \bar{\psi}^2_+\psi^2_-) = 0,$$

with the torsion tensor $T^a = \mathrm{d}e^a + \varepsilon^a{}_{bc}\omega^b\omega^c$.

In this paper, we are interested in finding the asymptotic symmetry algebra for the above theory at null infinity. To do so, we change frame for the generators, as was done in [20]. The new generators $\{M_n, \mathcal{L}_n, q^{1,2\pm}_\alpha, \mathcal{R}, \mathcal{S}\}$ are related to the previous ones by the following relations[3]:

$$M_n = P_a U^a_n, \qquad \mathcal{L}_n = \mathcal{J}_a U^a_n,$$
$$q^{1\pm}_\alpha = \sqrt{2}\mathcal{Q}^{1\pm}_\alpha, \qquad q^{2\pm}_\alpha = \sqrt{2}\mathcal{Q}^{2\pm}_\alpha,$$

with $(\mathcal{R}, \mathcal{S})$ remaining unchanged. In terms of these generators, the super Poincaré algebra reads as:

$$[\mathcal{L}_n, \mathcal{L}_m] = (n-m)\mathcal{L}_{n+m}, \qquad [\mathcal{L}_n, M_m] = (n-m)M_{n+m},$$
$$[M_n, M_m] = 0, \qquad [\mathcal{L}_n, q^{1,2\pm}_\alpha] = \left(\frac{n}{2} - \alpha\right)q^{1,2\pm}_{n+\alpha},$$
$$[M_n, q^{1,2\pm}_\alpha] = 0, \qquad [\mathcal{S}, q^{1,2\pm}_\alpha] = 0,$$
$$[\mathcal{R}, q^{1\pm}_\alpha] = \pm\frac{1}{2}q^{1\pm}_\alpha, \qquad [\mathcal{R}, q^{2\pm}_\alpha] = \mp\frac{1}{2}q^{2\pm}_\alpha,$$
$$\{q^{1\pm}_\alpha, q^{1\mp}_\beta\} = M_{\alpha+\beta} \pm (\alpha-\beta)\mathcal{S},$$
$$\{q^{2\pm}_\alpha, q^{2\mp}_\beta\} = M_{\alpha+\beta} \mp (\alpha-\beta)\mathcal{S}. \tag{2.9}$$

In the next section, we find the right asymptotic gauge field to finally arrive at the asymptotic symmetry group for this three dimensional $N = 4$ flat super-gravity.

III. $N = 4$ BMS$_3$ ASYMPTOTIC ALGEBRA

The aim of this paper is to find the asymptotic symmetry algebra for a specific set of boundary conditions of the gauge field. The procedure is well defined and has been used in the literature, for both asymptotically flat and AdS theories. The boundary conditions need to (i) extend the ones of the purely gravitational sector so as to include the bosonic solutions of interest, mentioned in the previous section, and (ii) relaxed enough so as to enlarge the set of asymptotic symmetries from $N = 4$ super-Poincaré to its $N = 4$ super-BMS extension. Obviously, they also fix the form of the metric which is, in the usual BMS gauge with Eddington-Finkelstein coordinates (u, r, φ):

$$\mathrm{d}s^2 = \eta_{ab}e^a e^b = \mathcal{M}\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + \mathcal{N}\mathrm{d}u\mathrm{d}\varphi + r^2\mathrm{d}\varphi^2 \tag{3.1}$$

The gauge fields at the boundary is hence chosen in a radial gauge,

$$\mathcal{A} = b^{-1}(a + d)b, \qquad b = \exp\left(\frac{r}{2}M_{-1}\right) \tag{3.2}$$

where now $a(u, \varphi) = a_\varphi \mathrm{d}\phi + a_u \mathrm{d}u$ reads:

$$a_u = M_1 - \frac{1}{4}\mathcal{M}M_{-1} - \frac{i\rho}{2}\mathcal{S}$$
$$a_\varphi = \mathcal{L}_1 - \frac{1}{4}\mathcal{M}\mathcal{L}_{-1} - \frac{1}{4}\mathcal{N}M_{-1} - \frac{i\phi}{2}\mathcal{S} - \frac{i\rho}{2}\mathcal{R}$$
$$-\frac{1}{4}(\Psi^1_+ q^{1+}_- - \Psi^1_- q^{1-}_-) + \frac{1}{4}(\Psi^2_+ q^{2+}_- - \Psi^2_- q^{2-}_-). \tag{3.3}$$

The various charges appearing in the above expression asymptotically will only have u and φ dependence. The asymptotic symmetries correspond to the set of gauge transformations that preserve this behavior together with the dynamical equations:

$$\mathrm{d}a + \frac{1}{2}[a, a] = 0, \qquad \delta a = \mathrm{d}\Lambda + [a, \Lambda], \tag{3.4}$$

where, the parameter Λ is Lie-algebra valued and depends on various arbitrary functions of u and φ,

$$\Lambda = \Upsilon^n\mathcal{L}_n + \xi^n M_n + \zeta^{1\alpha}_+ q^{1+}_\alpha + \zeta^{1\alpha}_- q^{1-}_\alpha + \zeta^{2\alpha}_+ q^{2+}_\alpha + \zeta^{2\alpha}_- q^{2-}_\alpha + \lambda_\mathcal{R}\mathcal{R} + \lambda_\mathcal{S}\mathcal{S}. \tag{3.5}$$

From the equations of motion one gets the following differential identities:

$$\partial_\varphi\mathcal{M} = \partial_u\mathcal{N}, \qquad \partial_u\mathcal{M} = 0 \qquad \partial_u\rho = 0, \tag{3.6}$$

$$\partial_\varphi\rho = \partial_u\phi, \qquad \partial_u\Psi^1_\pm = 0, \qquad \partial_u\Psi^2_\pm = 0. \tag{3.7}$$

Similar identities exist also for the parameters,

$$\partial_u\xi^n = \partial_\varphi\Upsilon^n, \qquad \partial_u\Upsilon^n = 0, \qquad \partial_u\zeta^{1\alpha}_\pm = 0, \tag{3.8}$$

$$\partial_u\zeta^{2\alpha}_\pm = 0, \qquad \partial_u\lambda_\mathcal{S} = \partial_\varphi\lambda_\mathcal{R}, \qquad \partial_u\lambda_\mathcal{R} = 0. \tag{3.9}$$

Here, we see that fields and parameters are not independent of each other.

Next we start the analysis of the gauge variation condition, which constrain the parameters even further:

$$\xi^0 = -\partial_\varphi\xi^+ + r\Upsilon^+, \qquad \Upsilon^0 = -\partial_\varphi\Upsilon^+,$$
$$\Upsilon^- = \frac{1}{2}\partial^2_\varphi\Upsilon^+ - \frac{1}{4}\mathcal{M}\Upsilon^+ = 0 \tag{3.10}$$

[3] Our conventions are summarized in Appendix A and Appendix D.

$$\xi^{-} = \frac{1}{2}\partial_\varphi^2\xi^{+} - \frac{1}{4}(\mathcal{M}\xi^{+} + \mathcal{N}\Upsilon^{+}) - \frac{1}{8}(\Psi^1_+\zeta^{1+}_- - \Psi^1_-\zeta^{1+}_+) + \frac{1}{8}(\Psi^2_+\zeta^{2+}_- - \Psi^2_-\zeta^{2+}_+) \tag{3.11}$$

where we made multiple use of the above identities. The constraints on the fermionic parameters are

$$\zeta^{1-}_\pm = -\partial_\varphi\zeta^{1+}_\pm \mp \frac{1}{4}\Psi^1_\pm\Upsilon^{+} \pm \frac{i}{4}\rho\zeta^{1+}_\pm \qquad \zeta^{2-}_\pm = -\partial_\varphi\zeta^{2+}_\pm \pm \frac{1}{4}\Psi^2_\pm\Upsilon^{+} \mp \frac{i}{4}\rho\zeta^{2+}_\pm$$

From now on, then we will use $\zeta^{1,2+}_\pm = \zeta^{1,2}_\pm$. Finally we write down the variation of the fields. For bosonic fields, we get,

$$\begin{aligned}
\delta\mathcal{M} &= -2\partial_\varphi^3\Upsilon^{+} + 2\mathcal{M}\partial_\varphi\Upsilon^{+} + \partial_\varphi\mathcal{M}\Upsilon^{+},\\
\delta\mathcal{N} &= -2\partial_\varphi^3\xi^{+} + 2\mathcal{M}\partial_\varphi\xi^{+} + 2\mathcal{N}\partial_\varphi\Upsilon^{+} + \partial_\varphi\mathcal{M}\xi^{+}, + \partial_\varphi\mathcal{N}\Upsilon^{+}\\
&\quad + \frac{1}{2}\left(\partial_\varphi\Psi^1_+\zeta^1_- + 3\Psi^1_+\partial_\varphi\zeta^1_- - \partial_\varphi\Psi^1_-\zeta^1_+ - 3\Psi^1_-\partial_\varphi\zeta^1_+ + \frac{i}{2}\Psi^1_+\zeta^1_-\rho + \frac{i}{2}\Psi^1_-\zeta^1_+\rho\right)\\
&\quad - \frac{1}{2}\left(\partial_\varphi\Psi^2_+\zeta^2_- + 3\Psi^2_+\partial_\varphi\zeta^2_- - \partial_\varphi\Psi^2_-\zeta^2_+ - 3\Psi^2_-\partial_\varphi\zeta^2_+ - \frac{i}{2}\Psi^2_+\zeta^2_-\rho - \frac{i}{2}\Psi^2_-\zeta^2_+\rho\right),\\
\delta\phi &= 2i\partial_\varphi\lambda_{\mathcal{S}} - \frac{\mathrm{i}}{2}(\Psi^1_+\zeta^1_- + \Psi^1_-\zeta^1_+) - \frac{\mathrm{i}}{2}(\Psi^2_+\zeta^2_- + \Psi^2_-\zeta^2_+),\\
\delta\rho &= 2i\partial_\varphi\lambda_{\mathcal{R}}.
\end{aligned} \tag{3.12}$$

For the fermionic fields we get:

$$\begin{aligned}
\delta\Psi^1_\pm &= \pm 4\partial_\varphi^2\zeta^1_\pm + \left(\partial_\varphi\Psi^1_\pm\Upsilon^{+} + \frac{3}{2}\Psi^1_\pm\partial_\varphi\Upsilon^{+}\right) - \mathrm{i}(\partial_\varphi\rho\zeta^1_\pm + 2\rho\partial_\varphi\zeta^1_\pm) \mp \mathcal{M}\zeta^1_\pm \mp \frac{\mathrm{i}}{4}\Psi^1_\pm\rho\Upsilon^{+} \mp \frac{1}{2}\lambda_{\mathcal{R}}\Psi^1_\pm \mp \frac{1}{4}\rho^2\zeta^1_\pm,\\
\delta\Psi^2_\pm &= \mp 4\partial_\varphi^2\zeta^2_\pm + \left(\partial_\varphi\Psi^2_\pm\Upsilon^{+} + \frac{3}{2}\Psi^2_\pm\partial_\varphi\Upsilon^{+}\right) - \mathrm{i}(\partial_\varphi\rho\zeta^2_\pm + 2\rho\partial_\varphi\zeta^2_\pm) \pm \mathcal{M}\zeta^2_\pm \pm \frac{\mathrm{i}}{4}\Psi^2_\pm\rho\Upsilon^{+} \pm \frac{1}{2}\lambda_{\mathcal{R}}\Psi^2_\pm \pm \frac{1}{4}\rho^2\zeta^2_\pm.
\end{aligned} \tag{3.13}$$

The variation of the canonical generators that corresponds to the asymptotic symmetries of this theory spanned by fields can be obtained in the canonical approach. In the case of a Chern-Simons theory in three dimensions, they are given by,

$$\delta\mathcal{C} = -\frac{k}{2\pi}\int\langle\Lambda, \delta\mathcal{A}_\varphi\rangle d\varphi \tag{3.14}$$

This expression is linear in the fields variations and it reads explicitly

$$\begin{aligned}
\delta\mathcal{C} = -\frac{k}{4\pi}\int(&\Upsilon^{+}\delta\mathfrak{J} + T\delta\mathcal{M} + \delta\Psi^1_+\zeta^1_- - \delta\Psi^1_-\zeta^1_+\\
&- \delta\Psi^2_+\zeta^2_- + \delta\Psi^2_-\zeta^2_+ + i\lambda_{\mathcal{R}}\delta\phi + i\lambda_{\mathcal{S}}\delta\rho)\mathrm{d}\varphi,
\end{aligned} \tag{3.15}$$

where we have used the supertraces suitable for the current basis (derived from relations in Appendix B and Appendix D):

$$\langle\mathcal{L}_n, M_m\rangle = \gamma_{nm}, \quad \langle q^{1,2\pm}_\alpha, q^{1,2\mp}_\beta\rangle = 2C_{\alpha\beta}, \quad \langle\mathcal{R}, \mathcal{S}\rangle = -1, \tag{3.16}$$

and solved (3.6) and (3.8):

$$\mathcal{N} = \mathfrak{J}(\varphi) + u\partial_\varphi\mathcal{M}, \qquad \xi^{+} = T(\varphi) + u\partial_\varphi\Upsilon^{+}. \tag{3.17}$$

Under some mild regularity assumptions for the variations, we can readily read off the charge from the above variation formula as,

$$\begin{aligned}
\mathcal{C} &= -\frac{k}{4\pi}\int(\Upsilon^{+}\mathfrak{J} + T\mathcal{M} + \Psi^1_+\zeta^1_- - \Psi^1_-\zeta^1_+\\
&\qquad - \Psi^2_+\zeta^2_- + \Psi^2_-\zeta^2_+ + i\lambda_{\mathcal{R}}\phi + i\lambda_{\mathcal{S}}\rho)\mathrm{d}\varphi\\
&= -\frac{2}{k}\sum_n \Upsilon^{+}_{-n}\mathfrak{J}_n + T_{-n}\mathcal{M}_n + \Psi^{1+}_n\zeta^{1-}_{-n} - \Psi^{1-}_n\zeta^{1+}_{-n}\\
&\qquad - \Psi^{2+}_n\zeta^{2-}_{-n} + \Psi^2_n\zeta^{2+}_{-n} + \mathrm{i}\lambda^{\mathcal{R}}_{-n}\mathcal{R}_n + \mathrm{i}\lambda^{\mathcal{S}}_{-n}\mathcal{S}_n
\end{aligned} \tag{3.18}$$

where $\zeta^{1,2\pm}_n$ are now the modes of $\zeta^{1,2}_\pm$. We can derive the asymptotic symmetry algebra of this configuration using the asymptotic charge and its variation as given above. In particular, the Poisson brackets among various modes of the fields can be obtained using the formula

$$\{\mathcal{C}[\lambda_1], \mathcal{C}[\lambda_2]\}_{PB} = \delta_{\lambda_1}\mathcal{C}[\lambda_2]. \tag{3.19}$$

It reads:

$$
\begin{aligned}
\mathrm{i}\{\mathfrak{J}_n, \mathfrak{J}_m\} &= (n-m)\mathfrak{J}_{n+m}, \\
\mathrm{i}\{\mathfrak{J}_n, \mathcal{M}_m\} &= (n-m)\mathcal{M}_{n+m} + \frac{c_M}{12} n^3 \delta_{n+m,0} \\
\mathrm{i}\{\mathfrak{J}_n, \Psi_r^{1,2\pm}\} &= \left(\frac{n}{2} - r\right)\Psi_{r+n}^{1,2\pm} \mp \frac{1}{4}[\Psi^{1,2\pm}\mathcal{S}]_{n+r} \\
\mathrm{i}\{\mathcal{R}_n, \Psi_r^{1\pm}\} &= \pm\frac{1}{2}\Psi_{r+n}^{1\pm}, \\
\mathrm{i}\{\mathcal{R}_n, \Psi_r^{2\pm}\} &= \mp\frac{1}{2}\Psi_{r+n}^{2\pm}, \\
\mathrm{i}\{\mathcal{R}_n, \mathcal{S}_m\} &= \frac{c_M}{12} n \delta_{n+m,0} \\
\{\Psi_r^{1+}, \Psi_s^{1-}\} &= \mathcal{M}_{r+s} + (r-s)\mathcal{S}_{r+s} + \frac{1}{4}[\mathcal{S}\mathcal{S}]_{r+s} \\
&\quad + \frac{c_M}{6} r^2 \delta_{r+s,0} \\
\{\Psi_r^{2+}, \Psi_s^{2-}\} &= \mathcal{M}_{r+s} - (r-s)\mathcal{S}_{r+s} + \frac{1}{4}[\mathcal{S}\mathcal{S}]_{r+s} \\
&\quad + \frac{c_M}{6} r^2 \delta_{r+s,0},
\end{aligned}
\tag{3.20}
$$

where $c_M = 12k$ and the modes are defined as follows:

$$
\begin{aligned}
\mathfrak{J}_n &= \frac{k}{4\pi}\int d\varphi e^{in\varphi}\mathfrak{J}, \\
\mathcal{M}_n &= \frac{k}{4\pi}\int d\varphi e^{in\varphi}\mathcal{M}, \\
\mathcal{R}_n &= \frac{k}{4\pi}\int d\varphi e^{in\varphi}\phi, \\
\mathcal{S}_n &= \frac{k}{4\pi}\int d\varphi e^{in\varphi}\rho, \\
\Psi_r^{1,2\pm} &= \frac{k}{4\pi}\int d\varphi e^{ir\varphi}\Psi^{1,2\pm}, \\
[\Psi^{1,2\pm}\mathcal{S}]_r &= \frac{k}{4\pi}\int d\varphi e^{ir\varphi}\Psi^{1,2\pm}\rho, \\
[\mathcal{S}\mathcal{S}]_\alpha &= \frac{k}{4\pi}\int d\varphi e^{i\alpha\varphi}\rho\rho, \\
\delta_{n,0} &= \frac{1}{2\pi}\int d\varphi e^{in\varphi},
\end{aligned}
\tag{3.21}
$$

and similarly for the parameters. To get the Poisson brackets, we need the inverse relations among the fields and the modes as well. For example, for fields $\mathfrak{J}(\varphi)$ and $\mathcal{M}(\varphi)$, the inverse relations are given by:

$$
\mathfrak{J}(\varphi) = \frac{2}{k}\sum_n e^{-in\varphi}\mathfrak{J}_n, \quad \mathcal{M}(\varphi) = \frac{2}{k}\sum_n e^{-in\varphi}\mathcal{M}_n. \tag{3.22}
$$

Here we notice that the Poisson bracket $\{\mathfrak{J}_n, \psi_r^{1,2,\pm}\}$ contains a spurious term (the last term) while $i\{\mathfrak{J}_n, \mathcal{R}_m\}$ is zero. Also, the Poisson bracket $\{\Psi^{1,2+}, \Psi^{1,2-}\}$ contains a quadratic term in the $\mathcal{S}$ generator. Hence, at this stage, the algebra looks quite different from the one derived in [21]. The resolution of these differences resides on a simple argument: since we are dealing with a theory with one internal U(1) symmetry, the physical energy-momentum tensor should have a contribution from the corresponding U(1) current. Thus it is important to add a Sugawara-like term to $\mathfrak{J}_n$ as follows:

$$
\hat{\mathfrak{J}}_n = \mathfrak{J}_n + \frac{1}{2}(\mathcal{R}\mathcal{S})_n; \tag{3.23}
$$

With these shifts of the modes, some spurious terms get canceled or absorbed and the new Poisson brackets read:

$$
\begin{aligned}
i\{\hat{\mathfrak{J}}_n, \psi_r^{1,2\pm}\} &= \left(\frac{n}{2} - r\right)\psi_{r+n}^{1,2\pm}, \\
i\{\hat{\mathfrak{J}}_n, \mathcal{R}_m\} &= -m\mathcal{R}_{m+n}, \\
i\{\hat{\mathfrak{J}}_n, \mathcal{S}_m\} &= -m\mathcal{S}_{m+n}.
\end{aligned}
\tag{3.24}
$$

Finally, we also perform a shift on $\mathcal{M}_n$

$$
\hat{\mathcal{M}}_n = \mathcal{M}_n + \frac{1}{4}[\mathcal{S}\mathcal{S}]_n \tag{3.25}
$$

to absorb the quadratic term in $\mathcal{S}$ in the $\{\Psi, \Psi\}$ bracket. Also:

$$
i\{\hat{\mathcal{M}}_n, \mathcal{R}_m\} = -m\mathcal{S}_{n+m}.
$$

The underlying justification for these shifts will become clear in the last section. In the next subsection, instead we present the final result for the $N = 4$ super-BMS algebra which is in agreement with [21].

A. The BMS algebra

Here, we present the main result of this paper, the algebra of three dimensional $N = 4$ flat super-gravity at null infinity, namely the $N = 4$ super BMS_3 algebra. From the nonzero Poisson brackets, we can write down the final form of the algebra. The rule that we follow is

$$
i\{,\}_{PB} \to [,] \quad \text{and} \quad \{,\}_{PB} \to \{,\}.
$$

The algebra in terms of commutators and anti-commutators is given by:

$$
\begin{aligned}
&[\hat{\mathfrak{J}}_n, \hat{\mathfrak{J}}_m\} = (n-m)\hat{\mathfrak{J}}_{n+m} + \frac{c_J}{12} n^3 \delta_{n+m,0},\\
&[\mathcal{R}_n, \mathcal{R}_m] = \frac{c_J}{12} n \delta_{n+m,0}\\
&[\hat{\mathfrak{J}}_n, \hat{\mathcal{M}}_m\} = (n-m)\hat{\mathcal{M}}_{n+m} + \frac{c_M}{12} n^3 \delta_{n+m,0},\\
&[\mathcal{R}_n, \mathcal{S}_m] = \frac{c_M}{12} n \delta_{n+m,0}\\
&[\hat{\mathfrak{J}}_n, \mathcal{R}_m] = -m\mathcal{R}_{n+m}, [\hat{\mathfrak{J}}_n, \mathcal{S}_m] = -m\mathcal{S}_{n+m},\\
&[\hat{\mathcal{M}}_n, \mathcal{R}_m] = -m\mathcal{S}_{n+m}\\
&[\hat{\mathfrak{J}}_n, \Psi_r^{1,2\pm}\} = \left(\frac{n}{2} - r\right)\Psi_{r+n}^{1,2\pm},\\
&[\mathcal{R}_n, \Psi_r^{1\pm}] = \pm\frac{1}{2}\Psi_{r+n}^{1\pm}, [\mathcal{R}_n, \Psi_r^{2\pm}] = \mp\frac{1}{2}\Psi_{r+n}^{2\pm},\\
&\{\Psi_r^{1+}, \Psi_s^{1-}\} = \hat{\mathcal{M}}_{r+s} + (r-s)\mathcal{S}_{r+s} + \frac{c_M}{6} r^2 \delta_{r+s,0}\\
&\{\Psi_r^{2+}, \Psi_s^{2-}\} = \hat{\mathcal{M}}_{r+s} - (r-s)\mathcal{S}_{r+s} + \frac{c_M}{6} r^2 \delta_{r+s,0}.
\end{aligned}
\tag{3.26}
$$

Here, we have presented the most generic possible quantum extension of the algebra by allowing a possible central extension to the $[\hat{\mathfrak{J}}_n, \hat{\mathfrak{J}}_m]$ and $[\mathcal{R}_n, \mathcal{R}_m]$ commutator. We also notice that, after adding suitable shifts to $\mathfrak{J}_n$ and $\mathcal{M}_n$, we obtain the same algebra presented in [21].

IV. ENERGY BOUND AND KILLING SPINORS

In this section, we look for the energy bounds for three dimensional $N = 4$ asymptotically flat supergravity theories. We find the asymptotic symmetries that leave the asymptotic background unchanged. Finally, we find the global killing spinors for this system.

A. Supersymmetry energy bound

As it is well known, supersymmetry imposes constraints on the energy of supersymmetric states. One can find it from the super algebra. Specifically for our case, considering antiperiodic boundary conditions on the fermions,[4] we see that the global part of the algebra consists of $(\mathfrak{J}_m, \hat{\mathcal{M}}_m, \Psi_r^{1,2\pm}, \mathcal{R})$, where $m = -1, 0, 1$ and $r = \pm\frac{1}{2}$. For the quantum theory, following [26–28], we consider all possible positive-definite combinations of the supercharges $\Psi_{\pm 1/2}^{1,2\pm}$ and get:

$$
\hat{\mathcal{M}}_0 = \frac{1}{4}\sum_{\substack{i=1,2\\ \alpha=\pm 1/2}} \Psi_\alpha^{i+}\Psi_{-\alpha}^{i-} + \Psi_{-\alpha}^{i-}\Psi_\alpha^{i+} - \frac{k}{2} \geq -\frac{k}{2} = -\frac{1}{8G}. \tag{4.1}
$$

Here, it is important to note that, we have derived the above bound for $\hat{\mathcal{M}}_0$, the shifted charge. Unlike $\mathcal{M}_0$, the latter satisfies this nicer bound. This implies that, for extended supersymmetric cases, the right physical charge at null infinity corresponds to $\hat{\mathcal{M}}$ It is also very clear that for the Minkowski vacuum for which $\hat{\mathcal{M}}_0 = \mathcal{M}_0 = -\frac{1}{8G}$ as all the other fields, including the R- and S-symmetry gauge fields are vanishing, the bound is saturated. Hence, Minkowski space is certainly a ground state for this theory.

B. Asymptotic killing spinors

To study the asymptotic supersymmetries that preserve the asymptotically flat backgrounds, we impose that both the gravitinos and their generic variation be zero, at infinity. This is known as "asymptotic Killing spinor equation." One hence has to solve the simplified version of Eqs. (3.13), i.e.:

$$
\partial_\varphi^2 \zeta_\pm^i \mp \frac{\mathrm{i}}{2}\rho\partial_\varphi\zeta_\pm^i - \frac{1}{4}\left(\mathcal{M} + \frac{1}{4}\rho^2\right)\zeta^i = 0, \tag{4.2}
$$

where $i = 1, 2$ and we assumed $\partial_\varphi \rho = 0$ and $\mathcal{M}$ constant. The general solutions to the above equations read:

$$
\begin{aligned}
\zeta_+^i &= e^{-\mathrm{i}\frac{\rho}{4}\varphi}\left(c_1^i e^{\frac{\sqrt{\mathcal{M}}}{2}\varphi} + c_2^i e^{-\frac{\sqrt{\mathcal{M}}}{2}\varphi}\right)\\
\zeta_-^i &= e^{\mathrm{i}\frac{\rho}{4}\varphi}\left(d_1^i e^{\frac{\sqrt{\mathcal{M}}}{2}\varphi} + d_2^i e^{-\frac{\sqrt{\mathcal{M}}}{2}\varphi}\right)
\end{aligned}
\tag{4.3}
$$

for arbitrary $c_{1,2}^i$ and $d_{1,2}^i$ constant spinors. The solutions are well defined, given the periodicity of φ only when $\mathcal{M} = -n^2$ and $n > 0$, a strictly positive integer without loss of generality.

For $n = 1$, $\rho = 0$ we find the Killing spinors for the Minkowski vacuum, $\mathcal{M} = -1$. For $n > 1$, the energy bound is violated and we have angular defect solutions [29].

C. Global killing vectors

We end this section with the study of global killing spinors. These describe globally defined supersymmetry transformations that leave the pure bosonic solution in the asymptotic region invariant. Depending on the range of the mass parameter, the pure bosonic zero mode solutions include cosmological solutions [30,31], stationary conical defects solutions [29], the Minkowski spacetime and angular excess solutions of [32,33]. The global Killing spinor equations is given as,

$$
D\zeta_\pm^1 = \left(\mathrm{d} + \omega \pm \frac{1}{2}\upsilon\right)\zeta_\pm^1 = 0. \tag{4.4}
$$

From the gauge field (3.3), we obtain the values of the spin connection and the R-gauge field:

[4]We have not studied the Ramond boundary conditions for the fermions, more can be found in [19,24,25].

$$\omega = \frac{1}{2}\omega^n \tilde{\Gamma}_n = \Lambda^{-1}\mathrm{d}\Lambda, \qquad \Lambda = \exp\left(\frac{1}{2}\left(\tilde{\Gamma}_{+1} - \frac{\mathcal{M}}{4}\tilde{\Gamma}_{-1}\right)\varphi\right),$$

The general solution of this equation is obtained from the solution of the homogeneous equation ($\upsilon = 0$) that was already solved in [19], given as:

$$\zeta^1_{\mathrm{hom}} = \Lambda^{-1}\zeta^1_0 = \begin{pmatrix} \cosh\left(\frac{\sqrt{\mathcal{M}}}{2}\varphi\right) & -\frac{\sqrt{\mathcal{M}}}{2}\sinh\left(\frac{\sqrt{\mathcal{M}}}{2}\varphi\right) \\ -\frac{2}{\sqrt{\mathcal{M}}}\sinh\left(\frac{\sqrt{\mathcal{M}}}{2}\varphi\right) & \cosh\left(\frac{\sqrt{\mathcal{M}}}{2}\varphi\right) \end{pmatrix}\zeta^1_0 \tag{4.5}$$

with ζ^1_0 constant spinors and we have suppressed the indices $\pm$. indices $\pm$. The solution of the inhomogeneous equation with nonzero υ is of the form:

$$\zeta^{1,2}_{\pm\mathrm{gen}} = \Lambda^{-1}(\zeta^{1,2}_0 + \zeta^{1,2}_{\pm}(x)). \tag{4.6}$$

By explicitly plugging in the above (4.4) we get:

$$\mathrm{d}\zeta^{1,2}_{\pm}(x) = \pm\frac{\mathrm{i}}{2}\phi\mathrm{d}\varphi(\zeta^{1,2}_0 + \zeta^{1,2}_{\pm}(x)) \tag{4.7}$$

where we identified $\upsilon = -\mathrm{i}\frac{\phi}{2}$ from the form asymptotic gauge field. This differential equation is immediately solved by:

$$\partial_r\zeta^{1,2}_{\pm} = \partial_u\zeta^{1,2}_{\pm} = 0$$
$$\zeta^{1,2}_{\pm}(\varphi) = \tilde{\zeta}^1 e^{\pm\mathrm{i}\frac{1}{2}\phi\varphi} - \zeta^1_0 \tag{4.8}$$

with $\tilde{\zeta}^1$ constant spinor. Thus the final solution for the global killing spinors takes the form:

$$\zeta^1_{\pm\mathrm{gen}} = \Lambda^{-1}\tilde{\zeta}^1 e^{\mp\mathrm{i}\frac{1}{2}\phi\varphi}. \tag{4.9}$$

For the second Killing spinor, the equation has simply the signs of the R-symmetry gauge field flipped, which corresponds only to the sign of the exponential flipped. Like the asymptotic case, the Killing spinors are globally well-defined when $\mathcal{M} = -n^2$, with n being positive integer. A more detailed discussion can be found in [19,20,34].

V. SUPER BMS$_3$ AS A FLAT LIMIT OF ASYMPTOTICALLY SUPER-AdS$_3$ SUPERGRAVITY

It is well known that the flat asymptotic algebra can be obtained by taking an appropriate limit (or contraction) of two copies of the asymptotic AdS algebras. In [21], we adopted this limiting procedure to derive all possible supersymmetric extensions of the BMS$_3$ algebras by considering the limit of the mixed sectors of the superconformal algebra. When the R-symmetry is present, there exist two possible combinations for the $R-$ charge generators, the democratic and the despotic scalings. The former was excluded because the R-generators did not rotate the supercharges, so that left us with one well-defined combination of the R-symmetry generators of the two super-Virasoro sectors, which led to a $N = 4$ super-BMS$_3$ algebra. In this paper, we have re-derived this algebra as given in (3.26) by a direct analysis of the gauge field boundary conditions. The result is in complete agreement with the results of [21] after considering suitable shifts in two generators, as shown in the last section. The reason behind these shifts is discussed below.

As it turns out, the asymptotic AdS algebra considered in [21] initially contains nonlinear terms in R—charge generators. The asymptotic symmetry algebra of N—extended AdS Supergravity theories were first discussed in [23] and those are not the usual superconformal algebras. For completeness, we shall present again those results, including the Sugawara shifts of the Virasoro generators, by using the Chern-Simons formulation of AdS gravity.

A. Asymptotic symmetry algebra for (2,0) and (0,2) AdS supergravity

There are two inequivalent locally supersymmetric extensions of general relativity with negative cosmological constant in three spacetime dimensions containing an R-symmetry, known as the (2,0) and (0,2) theories. The bulk symmetry algebras for both the theories are presented in Appendix C. Here, we formulate them as a Chern-Simons theory with appropriate gauge group Osp$(2|2,\mathbb{R})$. The action is a functional of two independent connections A_+ and A_-:

$$I = I[A_+] + I[A_-], \tag{5.1}$$

where, $I[A]$ is defined earlier in (2.1). Here, we have defined $x^{\pm} = u/l \pm \varphi$, where, l is the identical AdS radius in both sectors. Hence, the (2,0) sector asymptotically only depends on x^+ and the (0,2) sector depends on x^-. The asymptotic behaviour of the gauge fields can be taken to be

$$A_+ = \left(L_1 + \frac{r}{l}L_0 + \frac{r^2}{4l^2}L_{-1} - \frac{1}{2}\mathfrak{L}_+ L_{-1} - \frac{1}{2}\psi_+\mathcal{Q}^+_- + \frac{1}{2}\psi_-\mathcal{Q}^-_- - i\phi^A_R R\right)\mathrm{d}x^+ + \frac{\mathrm{d}r}{2l}L_{-1}$$
$$\bar{A}_- = \left(\bar{L}_{-1} - \frac{r}{l}\bar{L}_0 + \frac{r^2}{4l^2}\bar{L}_1 - \frac{1}{2}\mathfrak{L}_-\bar{L}_1 - \frac{1}{2}\bar{\psi}_+\bar{\mathcal{Q}}^+_+ + \frac{1}{2}\bar{\psi}_-\bar{\mathcal{Q}}^-_+ - i\bar{\phi}^A_R\bar{R}\right)\mathrm{d}x^- + \frac{\mathrm{d}r}{2l}\bar{L}_1, \tag{5.2}$$

and again from the dynamical equations we get the trivial constraints:

$$\partial_- \mathfrak{L}_+ = \partial_- \psi_\pm = \partial_- \phi_R^A = 0,$$
$$\partial_+ \mathfrak{L}_- = \partial_+ \bar{\psi}_\pm = \partial_+ \bar{\phi}_R^A = 0. \tag{5.3}$$

The asymptotic symmetries for these systems are generated by the asymptotic gauge transformations $\delta A_\pm = \mathrm{d}\Lambda_\pm + [A_\pm, \Lambda_\pm]$ for both gauge fields, where the transformation parameters are given as,

$$\Lambda_+ = \chi^n L_n + \epsilon_+^\alpha \mathcal{Q}_\alpha^+ + \epsilon_-^\alpha \mathcal{Q}_\alpha^- + \lambda_R^A R$$
$$\Lambda_- = \bar{\chi}^n L_n + \bar{\epsilon}_+^\alpha \bar{\mathcal{Q}}_\alpha^+ + \bar{\epsilon}_-^\alpha \bar{\mathcal{Q}}_\alpha^- + \bar{\lambda}_R^A \bar{R}. \tag{5.4}$$

The variation at infinity constrains some parameters and also fixes the variation of various fields appearing in the asymptotic gauge fields. Below, we present the relations in the (2,0) sector, where fields and parameters are only a function of x_+:

$$\chi^0 = -\mathcal{Y}' + \frac{r}{l}\mathcal{Y}$$
$$\chi^- = \frac{1}{2}\mathcal{Y}'' - \frac{r}{2l}\mathcal{Y}' + \left(\frac{r^2}{4l^2} - \frac{1}{2}\mathfrak{L}_+\right)\mathcal{Y} - \frac{1}{4}(\psi_+\epsilon_- - \psi_-\epsilon_+)$$
$$\epsilon_+^- = -\epsilon_+' + \frac{r}{2l}\epsilon_+ - \frac{1}{2}\psi_+\mathcal{Y} + \frac{1}{2}\mathrm{i}\phi_R^A\epsilon_+$$
$$\epsilon_-^- = -\epsilon_-' + \frac{r}{2l}\epsilon_- + \frac{1}{2}\psi_-\mathcal{Y} - \frac{1}{2}\mathrm{i}\phi_R^A\epsilon_-$$

where we called $\chi^+ = \mathcal{Y}$, $\epsilon_+^+ = \epsilon_+$ and $\epsilon_-^+ = \epsilon_-$. The variations read:

$$\delta\mathfrak{L}_+ = -\mathcal{Y}''' + 2\mathfrak{L}_+\mathcal{Y}' + \mathfrak{L}_+'\mathcal{Y} + \frac{1}{2}(\psi_+'\epsilon_- + 3\psi_+\epsilon_-')$$
$$- \frac{1}{2}(\psi_-'\epsilon_+ + 3\psi_-\epsilon_+') + \frac{1}{2}\mathrm{i}(\psi_+\epsilon_-{}^A\phi_R + \psi_-\epsilon_+\phi_R^A)$$
$$\delta\psi_+ = 2\epsilon_+'' + \psi_+'\mathcal{Y} + \frac{3}{2}\psi_+\mathcal{Y}' - \mathrm{i}(\phi_R^{A\prime}\epsilon_+ + 2\phi_R^A\epsilon_+')$$
$$- \mathfrak{L}_+\epsilon_+ - \frac{1}{2}\mathrm{i}\psi_+\phi_R^A\mathcal{Y} - \frac{1}{2}\lambda_R^A\psi_+ - \frac{1}{2}\phi_R^A\phi_R^A\epsilon_+$$
$$\delta\psi_- = -2\epsilon_-'' + \psi_-'\mathcal{Y} + \frac{3}{2}\psi_-\mathcal{Y}' - \mathrm{i}(\phi_R^{A\prime}\epsilon_- + 2\phi_R^A\epsilon_-')$$
$$+ \mathfrak{L}_+\epsilon_- + \frac{1}{2}\mathrm{i}\psi_-\phi_R\mathcal{Y} + \frac{1}{2}\lambda_R^A\psi_- + \frac{1}{2}\phi_R^A\phi_R^A\epsilon_-$$
$$\delta\phi_R^A = \mathrm{i}\lambda_R^{A\prime} - \frac{1}{2}\mathrm{i}\psi_+\epsilon_- - \frac{1}{2}\mathrm{i}\psi_-\epsilon_+.$$

Now following the same procedure as before, the asymptotic symmetry algebra for (2,0) asymptotically AdS supergravity theory is straightforwardly found. The nontrivial supertrace elements are

$$\langle L_n, L_m\rangle = \frac{1}{2}\gamma_{nm}, \qquad \langle \mathcal{Q}_\alpha^\pm, \mathcal{Q}_\beta^\mp\rangle = C_{\alpha\beta}, \qquad \langle R, R\rangle = -\frac{1}{2}, \tag{5.5}$$

from which the generic charge reads

$$\mathcal{Q}[\mathcal{Y}, \epsilon_\mp, \lambda_R] = -\frac{k_l}{4\pi}\int \mathfrak{L}_+\mathcal{Y} + \psi_+\epsilon_- - \psi_-\epsilon_+ + \mathrm{i}\phi_R^A\lambda_R^A$$
$$= -\frac{2}{k_l}\sum_n \mathfrak{L}_n^+\mathcal{Y}_{-n} + \psi_n^+\epsilon_{-n}^- - \psi_n^-\epsilon_{-n}^+ + \mathrm{i}R_n\lambda_{-n}^A \tag{5.6}$$

with $\epsilon_n^\pm$ modes of $\epsilon_\pm$. The nontrivial Poisson brackets are given as:

$$\mathrm{i}\{\mathfrak{L}_n^+, \mathfrak{L}_m^+\}_{PB} = (n-m)\mathfrak{L}_{n+m}^+ + \frac{c}{12}n^3\delta_{n+m,0}$$
$$\mathrm{i}\{R_n, R_m\}_{PB} = \frac{k_l}{2}n\delta_{m+n,0} = \frac{c}{12}n\delta_{m+n,0}$$
$$\mathrm{i}\{\mathfrak{L}_n^+, \psi_\alpha^\pm\}_{PB} = \left(\frac{n}{2} - \alpha\right)\psi_{\alpha+n}^\pm \mp \frac{1}{2}[\Psi^\pm R]_{n+\alpha}$$
$$\mathrm{i}\{R_n, \psi_\alpha^\pm\}_{PB} = \pm\frac{1}{2}\psi_{\alpha+n}^\pm$$
$$\{\psi_\alpha^+, \psi_\beta^-\}_{PB} = \mathfrak{L}_{\alpha+\beta}^+ + (\alpha-\beta)R_{\alpha+\beta} + \frac{1}{2}[RR]_{\alpha+\beta} + \frac{c}{6}\alpha^2\delta_{\alpha+\beta}, \tag{5.7}$$

where the modes are defined as follows:

$$\mathfrak{L}_n^+ = \frac{k_l}{4\pi}\int d\varphi e^{in\varphi}\mathfrak{L}_+, \qquad R_n = \frac{k_l}{4\pi}\int d\varphi e^{in\varphi}\phi_R^A,$$
$$\psi_\alpha^\pm = \frac{k_l}{4\pi}\int d\varphi \psi^\pm e^{i\alpha\varphi}, \qquad [\psi^\pm R]_\alpha = \frac{k_l}{4\pi}\int d\varphi e^{i\alpha\varphi}\psi^\pm\phi_R^A,$$
$$[RR]_\alpha = \frac{k_l}{4\pi}\int d\varphi e^{i\alpha\varphi}\phi_R^A\phi_R^A. \tag{5.8}$$

Now, we need to redefine the generator $\mathfrak{L}_n$ by adding a term bilinear in the $\mathcal{R}$-current:

$$\mathfrak{L}_n \to \hat{\mathfrak{L}}_n = \mathfrak{L}_n + \frac{1}{2}(RR)_n. \tag{5.9}$$

This is a Sugawara shift on the Stress-tensor in presence of internal currents. The effect of this shift is shown below, where we write down the quantum (anti)commutator for the theory using the same convention as (III A):

$$[\hat{\mathfrak{L}}_n^+, \hat{\mathfrak{L}}_m^+] = (n-m)\hat{\mathfrak{L}}_{n+m}^+ + \frac{c}{12}n^3\delta_{n+m,0},$$
$$[R_n, \psi_\alpha^\pm] = \pm\frac{1}{2}\psi_{n+\alpha}^\pm$$
$$\{\psi_\alpha^+, \psi_\beta^-\} = \hat{\mathfrak{L}}_{\alpha+\beta}^+ + (\alpha-\beta)R_{\alpha+\beta} + \frac{c}{6}\alpha^2\delta_{\alpha+\beta,0}$$
$$[\hat{\mathfrak{L}}_n^+, R_m] = -mR_{n+m}, \qquad [\hat{\mathfrak{L}}_n^+, \psi_\alpha^\pm] = \left(\frac{n}{2} - \alpha\right)\psi_{n+\alpha}^\pm,$$
$$[R_n, R_m] = \frac{c}{12}n\delta_{n+m,0}. \tag{5.10}$$

One can carry on similar computation for the (0,2) sector and in this case, the constraints are

$$\bar{\chi}^0 = \bar{\mathcal{Y}}' - \frac{r}{l}\mathcal{Y}$$
$$\bar{\chi}^+ = \frac{1}{2}\bar{\mathcal{Y}}'' - \frac{r}{2l}\bar{\mathcal{Y}}' + \left(\frac{r^2}{4l^2} - \frac{1}{2}\mathfrak{L}_-\right)\bar{\mathcal{Y}} + \frac{1}{4}(\bar{\psi}_+\bar{\epsilon}_- - \bar{\psi}_-\bar{\epsilon}_+)$$
$$\bar{\epsilon}_+^{\pm} = \bar{\epsilon}'_+ - \frac{r}{2l}\bar{\epsilon}_+ - \frac{1}{2}\bar{\psi}_+\bar{\mathcal{Y}} - \frac{1}{2}\mathrm{i}\bar{\phi}_R^A\bar{\epsilon}_+$$
$$\bar{\epsilon}_-^{\pm} = \bar{\epsilon}'_- - \frac{r}{2l}\bar{\epsilon}_- + \frac{1}{2}\bar{\psi}_-\bar{\mathcal{Y}} + \frac{1}{2}\mathrm{i}\bar{\phi}_R^A\bar{\epsilon}_-$$

where we called $\bar{\chi}^- = \bar{\mathcal{Y}}$, $\bar{\epsilon}_i^- = \bar{\epsilon}_i$. The variations read:

$$\delta\mathfrak{L}_- = -\bar{\mathcal{Y}}''' + 2\mathfrak{L}_-\bar{\mathcal{Y}}' + \mathfrak{L}'_-\bar{\mathcal{Y}} - \frac{1}{2}(\bar{\psi}'_+\bar{\epsilon}_- + 3\bar{\psi}_+\bar{\epsilon}'_-)$$
$$+ \frac{1}{2}(\bar{\psi}'_-\bar{\epsilon}_+ + 3\bar{\psi}_-\bar{\epsilon}'_+) - \frac{1}{2}\mathrm{i}(\bar{\psi}_+\bar{\epsilon}_-\bar{\phi}_R^A + \bar{\psi}_-\bar{\epsilon}_+\bar{\phi}_R^A)$$
$$\delta\bar{\psi}_+ = -2\bar{\epsilon}''_+ + \bar{\psi}'_+\bar{\mathcal{Y}} + \frac{3}{2}\bar{\psi}_+\bar{\mathcal{Y}}' + \mathrm{i}(\bar{\phi}_R^{A\prime}\bar{\epsilon}_+ + 2\bar{\phi}_R^A\bar{\epsilon}'_+)$$
$$+ \mathfrak{L}_-\bar{\epsilon}_+ - \frac{1}{2}\mathrm{i}\bar{\psi}_+\bar{\phi}_R^A\bar{\mathcal{Y}} - \frac{1}{2}\bar{\lambda}_R^A\bar{\psi}_+ + \frac{1}{2}\bar{\phi}_R^A\bar{\phi}_R^A\bar{\epsilon}_+$$
$$\delta\bar{\psi}_- = 2\bar{\epsilon}''_- + \bar{\psi}'_-\bar{\mathcal{Y}} + \frac{3}{2}\bar{\psi}_-\bar{\mathcal{Y}}' + \mathrm{i}(\bar{\phi}_R^{A\prime}\bar{\epsilon}_- + 2\bar{\phi}_R^A\bar{\epsilon}'_-)$$
$$- \mathfrak{L}_-\bar{\epsilon}_- + \frac{1}{2}\mathrm{i}\bar{\psi}_-\bar{\phi}_R^A\bar{\mathcal{Y}} + \frac{1}{2}\bar{\lambda}_R^A\bar{\psi}_- - \frac{1}{2}\bar{\phi}_R^A\bar{\phi}_R^A\bar{\epsilon}_-$$
$$\delta\bar{\phi}_R^A = \mathrm{i}\bar{\lambda}_R^{A\prime} + \frac{1}{2}\mathrm{i}\bar{\psi}_-\bar{\epsilon}_- + \frac{1}{2}\mathrm{i}\bar{\psi}_-\bar{\epsilon}_+.$$

The nonzero supertraces elements are

$$\langle\bar{L}_n, \bar{L}_m\rangle = -\frac{1}{2}\gamma_{nm}, \qquad \langle\bar{\mathcal{Q}}_\alpha^{\pm}, \bar{\mathcal{Q}}_\beta^{\mp}\rangle = -C_{\alpha\beta},$$
$$\langle\bar{R}, \bar{R}\rangle = \frac{1}{2}, \tag{5.11}$$

and the charge of the barred sector reads

$$\bar{Q}[\bar{\mathcal{Y}}, \bar{\epsilon}_{\mp}, \bar{\lambda}_R] = -\frac{k_l}{4\pi}\int \mathfrak{L}_-\bar{\mathcal{Y}} - \bar{\psi}_+\bar{\epsilon}_- + \bar{\psi}_-\bar{\epsilon}_+ + \mathrm{i}\bar{\phi}_R^A\bar{\lambda}_R^A$$
$$= -\frac{2}{k_l}\sum_n \mathfrak{L}_n^-\bar{\mathcal{Y}}_{-n} - \bar{\psi}_n^+\bar{\epsilon}_{-n}^- + \bar{\psi}_n^-\bar{\epsilon}_{-n}^+ + \mathrm{i}\bar{R}_n\bar{\lambda}_{-n}^A \tag{5.12}$$

with $\bar{\epsilon}_n^{\pm}$ modes of $\bar{\epsilon}_{\pm}$. Finally, the asymptotic form of the Poisson brackets between various modes take identical form of the (2,0) sector as,

$$\mathrm{i}\{\mathfrak{L}_n^-, \mathfrak{L}_m^-\}_{PB} = (n-m)\mathfrak{L}_{n+m}^- + \frac{\bar{c}}{12}n^3\delta_{m+n,0}$$
$$\mathrm{i}\{\bar{R}_n, \bar{R}_m\}_{PB} = \frac{k_l}{2}n\delta_{m+n,0} = \frac{\bar{c}}{12}n\delta_{m+n,0}$$
$$\mathrm{i}\{\mathfrak{L}_n^-, \bar{\psi}_\alpha^{\pm}\}_{PB} = \left(\frac{n}{2} - \alpha\right)\bar{\psi}_{\alpha+n}^{\mp} \pm \frac{1}{2}[\bar{\Psi}^{\pm}\bar{R}]_{n+\alpha}$$
$$\mathrm{i}\{\bar{R}_n, \bar{\psi}_\alpha^{\pm}\}_{PB} = \pm\frac{1}{2}\bar{\psi}_{\alpha+n}^{\pm}$$
$$\{\bar{\psi}_\alpha^+, \bar{\psi}_\beta^-\}_{PB} = \mathfrak{L}_{\alpha+\beta}^- + (\alpha-\beta)\bar{R}_{\alpha+\beta} + \frac{1}{2}[\bar{R}\,\bar{R}]_{\alpha+\beta}$$
$$+ \frac{c}{6}\alpha^2\delta_{\alpha+\beta} \tag{5.13}$$

where the modes are defined as follows:

$$\mathfrak{L}_n^- = \frac{k_l}{4\pi}\int d\varphi e^{-in\varphi}\mathfrak{L}_-, \quad \bar{R}_n = \frac{k_l}{4\pi}\int d\varphi e^{-in\varphi}\bar{\phi}_R^A,$$
$$\bar{\psi}_\alpha^{\pm} = \frac{k_l}{4\pi}\int d\varphi\bar{\psi}^{\pm}e^{-i\alpha\varphi}, \quad [\bar{\psi}^{\pm}\bar{R}]_\alpha = \frac{k_l}{4\pi}\int d\varphi e^{-i\alpha\varphi}\bar{\psi}^{\pm}\bar{\phi}_R^A,$$
$$[\bar{R}\,\bar{R}]_\alpha = \frac{k_l}{4\pi}\int d\varphi e^{-i\alpha\varphi}\bar{\phi}_R^A\bar{\phi}_R^A. \tag{5.14}$$

Notice that the definition of Fourier transform in barred and unbarred sectors are different. This is ultimately due to the fact that the two sectors depend exclusively on x^- and x^+ respectively, so that one can expand all the arguments in power series of $1/l$, and the fields and charges in the barred sector will depend on $-\varphi$. Finally using the same convention for writing the suitable quantum commutators in the barred sector, the asymptotic symmetry algebras for the generators of the barred sector, i.e. of (0,2) three dimensional AdS theory takes exactly identical form as the one for (2,0) three dimensional AdS theories presented in (5.10). Here also, we required a Sugawara shift of the stress-tensor as

$$\mathfrak{L}_n^- \to \hat{\mathfrak{L}}_n^- = \mathfrak{L}_n^- + \frac{1}{2}(\bar{R}\,\bar{R})_n \tag{5.15}$$

to get the final form of the algebra which is identical to (5.10).

Note that we started with identical copies of bulk symmetry algebras for (2,0) and (0,2) sectors as given in Appendix C and, as a consequence, the asymptotic algebras of the modes of the conserved charges are also identical, differences in the sign of the Fourier modes notwithstanding. It was shown in [15] that by properly combining the two algebras, one immediately obtains the modes of [(3.21)] and the corresponding BMS algebra [(3.26)].

B. $N = 4$ super-BMS$_3$ from $N = (2,2)$ super-AdS$_3$

In this section, we shall explicitly show the relations between the generators, gauge fields components and gauge parameters of two copies of the super conformal algebras and the flat $N = 4$ BMS_3 algebra. As it is easy to understand, in fact, the latter quantities can be obtained from the linear combinations of the former ones. Before doing so, let us recall that a Inönü-Wigner contraction of two copies of Super-conformal algebra gives us the Super-Poincare algebra. The contraction is defined in the large AdS radius limit $l \to \infty$. The level of the corresponding Chern-Simons actions are related as $k_l = k \cdot l$. The generators of the flat algebra can be obtained from the AdS ones as,

$$\mathcal{L}_n = L_n - \bar{L}_{-n}, \qquad M_n = \frac{L_n + \bar{L}_{-n}}{l}, \qquad \mathcal{R} = R - \bar{R},$$
$$\mathcal{S} = \frac{R + \bar{R}}{l} \qquad q^{1\pm}_\alpha = \sqrt{\frac{2}{l}}\mathcal{Q}^{\pm}_\alpha, \qquad q^{2\pm}_\alpha = \sqrt{\frac{2}{l}}\bar{\mathcal{Q}}^{\pm}_{-\alpha}$$

It is easy to check that the asymptotic gauge field and the gauge transformation parameter of the flat theory is obtained from the AdS ones in the limit $l \to \infty$ as,

$$\mathcal{A} = A_+ + A_-, \qquad \Lambda = \Lambda_+ + \Lambda_-. \tag{5.16}$$

which can be immediately decomposed in the sum of the gauge variations of the two superconformal sectors, up the remembering that the unbarred and barred sectors are functions of x^+ and x^- coordinate respectively. We further need to use the following maps of various charges, whose algebra is indeed the asymptotic super-BMS_3 algebra we have derived before[5]:

$$\mathcal{M} = \mathfrak{L}_+ + \mathfrak{L}_-, \qquad \mathcal{N} = l(\mathfrak{L}_+ - \mathfrak{L}_-), \qquad \phi = l(\phi^A_R - \bar{\phi}^A_R),$$
$$\rho = \phi^A_R + \bar{\phi}^A_R, \qquad \psi^\alpha_\pm = \frac{1}{\sqrt{2l}}\Psi^{1\alpha}_\pm, \qquad \bar{\psi}^{-\alpha}_\pm = \frac{1}{\sqrt{2l}}\Psi^{2\alpha}_\pm, \tag{5.17}$$

for the parameters:

$$\epsilon^\alpha_\pm = \sqrt{\frac{2}{l}}\zeta^{1\alpha}_\pm, \qquad \bar{\epsilon}^{-\alpha}_\pm = \sqrt{\frac{2}{l}}\zeta^{2\alpha}_\pm, \qquad \Upsilon^n = \frac{\chi^n - \bar{\chi}^{-n}}{2},$$
$$\xi^n = l\frac{\chi^n + \bar{\chi}^{-n}}{2}, \qquad \lambda_\mathcal{R} = \frac{\lambda^A_R - \bar{\lambda}^A_R}{2}, \qquad \lambda_\mathcal{S} = l\frac{\lambda^A_R + \bar{\lambda}^A_R}{2}.$$

To obtain (3.6)–(3.7) from (5.3) one needs to make use of the change of variables identity:

$$\partial_\varphi = \partial_+ - \partial_-, \qquad \partial_u = \frac{\partial_+ + \partial_-}{l} \tag{5.18}$$

and similarly for the constraints on the parameters. Finally, as proposed in [21], the three dimensional $N = 4$ BMS algebra (3.26) can be obtained by two identical copies of asymptotic (2,0) and (0,2) AdS_3 algebras with the following identification for the charges and their Fourier modes

$$\mathfrak{J}_m = \lim_{\epsilon\to 0}(\mathfrak{L}^+_m - \mathfrak{L}^-_{-m}), \qquad \hat{\mathcal{M}}_m = \lim_{\epsilon\to 0}\epsilon(\mathfrak{L}^+_m + \mathfrak{L}^-_{-m}),$$
$$\Psi^{1,\pm}_r = \lim_{\epsilon\to 0}\sqrt{\epsilon}\,\psi^\pm_r, \qquad \Psi^{2,\pm}_r = \lim_{\epsilon\to 0}\sqrt{\epsilon}\,\bar{\psi}^\pm_{-r},$$
$$c_J = \lim_{\epsilon\to 0}(c - \bar{c}), \qquad c_M = \lim_{\epsilon\to 0}\epsilon(c + \bar{c}),$$
$$\mathcal{R}_m = \lim_{\epsilon\to 0}(R_m - \bar{R}_{-m}), \qquad \mathcal{S}_m = \lim_{\epsilon\to 0}\epsilon(R_m + \bar{R}_{-m}), \tag{5.19}$$

where $\epsilon = \frac{1}{l}$. The above identification follows directly from relation (5.17) and definitions of various modes as given in (3.21), (5.8), and (5.14).

Finally we end this section by justifying the Sugawara shifts on the two flat algebra generators $\mathfrak{J}$ and $\mathcal{M}$ to obtain the algebra (3.26). If we think of the BMS_3 algebra as a limit of two copies of AdS_3 algebras, then it is obvious to realize why both $\mathfrak{J}$ and $\mathcal{M}$ require a shift. Writing those shifts in terms of fields we have:

$$\hat{\mathfrak{L}}_+ = \mathfrak{L}_+ + \frac{1}{2}\phi^2_A, \qquad \hat{\mathfrak{L}}_- = \mathfrak{L}_- + \frac{1}{2}\bar{\phi}^2_A. \tag{5.20}$$

It is easy to check that, the BMS_3 charges $\mathfrak{J}$ and $\mathcal{M}$, which are combinations of the two above AdS_3 charges $\mathfrak{L}_\pm$ will pick up certain shifts. In particular the shift in $\mathcal{M}$ comes out as,

$$\begin{aligned}\mathcal{M} &= (\mathfrak{L}_+ + \mathfrak{L}_-)\\ &= (\hat{\mathfrak{L}}_+ + \hat{\mathfrak{L}}_-) - \frac{1}{2}(\phi^2_A + \bar{\phi}^2_A)\\ &= \hat{\mathcal{M}} - \frac{1}{4}(\rho^2 + (\phi/l)^2)\end{aligned}$$

where we used the definitions of the R- and S-symmetry gauge fields. Similarly, one obtains the shift for $\mathfrak{J}$ (more care needs to be exercised in that case,where it is crucial to expand the Virasoro R-symmetry fields in powers of $1/l$). In the limit $l \to \infty$, we finally get the shifts (in terms of the modes) as:

$$\hat{\mathfrak{J}}_n = \mathfrak{J}_n + \frac{1}{2}[\mathcal{R}\mathcal{S}]_n, \qquad \hat{\mathcal{M}}_n = \mathcal{M}_n + \frac{1}{4}[\mathcal{S}\mathcal{S}]_n. \tag{5.21}$$

These are indeed the correct Sugawara shifts for the BMS_3 generators that simplify the algebra notably. The most important simplification happens at the level of the anticommutator of the supercharges, as the nonlinear term $[\mathcal{S}\mathcal{S}]$ is immediately absorbed inside $\mathcal{M}$.

[5]Again, when writing down the BMS charges in terms of Virasoro modes, the presence of the Chern-Simons level is crucial to obtain the correct scaling.

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APPENDIX A: CONVENTIONS

In this paper we follow conventions similar to [20]. We will list them here to maintain the paper self-contained. The antisymmetric Levi-Civita symbol has component $\epsilon_{012} = +1$ and the tangent space metric is the 3D Minkowski metric

$$\eta_{ab} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{A1}$$

The Γ-matrices satisfying the three dimensional Clifford algebra $\{\Gamma_a, \Gamma_b\} = 2\eta_{ab}$ are

$$\Gamma_0 = i\sigma_2, \qquad \Gamma_1 = \sigma_1, \qquad \Gamma_2 = \sigma_3, \tag{A2}$$

with σ_i the Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A3}$$

Finally, the charge conjugation matrix $C = i\sigma_2$, or explicitly

$$C_{\alpha\beta} = \varepsilon_{\alpha\beta} = C^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{A4}$$

Throughout this paper the fermionic indices α, β run over $-, +$ (contrarily to [20] where they run over $+, -$). The supercharges are also taken to be Grassmann quantities, as are the fermion parameters and the gravitini. All spinors in this work are Majorana and the Majorana conjugate of a spinor ψ^α is $\bar{\psi}_\alpha = C_{\alpha\beta}\psi^\beta$. Our conventions imply that we can use the identities

$$\Gamma_a\Gamma_b = \epsilon_{abc}\Gamma^c + \eta_{ab}\mathbb{1}, \quad \Gamma^{a\alpha}{}_\beta\Gamma_a{}^\gamma{}_\delta = 2\delta^\alpha_\delta\delta^\gamma_\beta - \delta^\alpha_\beta\delta^\gamma_\delta, \tag{A5}$$

$$C^T = -C, \qquad C\Gamma_a = -(\Gamma_a)^T C. \tag{A6}$$

In verifying the closure of the supersymmetry algebra on the fields and the off-shell invariance of the action, the three dimensional Fierz relation is useful.

$$\zeta\bar{\eta} = -\frac{1}{2}\bar{\eta}\zeta\mathbb{1} - \frac{1}{2}(\bar{\eta}\Gamma^a\zeta)\Gamma_a, \tag{A7}$$

Other useful identities are

$$\bar{\psi}\Gamma_a\eta = \bar{\eta}\Gamma_a\psi \qquad \bar{\psi}\Gamma_a\epsilon = -\bar{\epsilon}\Gamma_a\psi$$

where ψ, η are Grassmannian one-forms, while ϵ is a Grassmann paramter. It is sometimes convenient to change basis of the tangent space to one more suited for the $isl(2)$ algebra in the bosonic sector of flat space supergravity. We do this by choosing a map to bring the generators of $SO(2,1)$ ($[J_a, J_b] = \epsilon_{abc}J^c$) to those of $SL(2,\mathbf{R})$ satisfying $[L_n, L_m] = (n-m)L_{n+m}$. This defines a matrix $U^a{}_n$ as a map from the tangent space metric η_{ab} with $a, b = \{0,1,2\}$ to the metric γ_{nm} defined in (A11) with $n, m = \{-1, 0, +1\}$, satisfying

$$L_n = J_a U^a{}_n. \tag{A8}$$

An explicit representation of $U^a{}_n$ that does the job is for instance

$$U^a{}_n = \begin{pmatrix} -1 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{A9}$$

In this basis the gamma matrices satisfy a Clifford algebra with

$$\{\tilde{\Gamma}_m, \tilde{\Gamma}_n\} = 2\gamma_{nm} \equiv 2\begin{pmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ -2 & 0 & 0 \end{pmatrix}$$

$$\text{with:} \quad n, m = -1, 0, +1. \tag{A10}$$

A real representation for the gamma matrices with n, m indices can be obtained by taking $\tilde{\Gamma}_n = U^a{}_n\Gamma_a$, or explicitly:

$$\tilde{\Gamma}_{-1} = -(\sigma_1 + i\sigma_2) = \begin{pmatrix} 0 & -2 \\ 0 & 0 \end{pmatrix}, \tag{A11}$$

$$\tilde{\Gamma}_0 = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A12}$$

$$\tilde{\Gamma}_{+1} = \sigma_1 - i\sigma_2 = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}. \tag{A13}$$

In addition to the Clifford algebra (A10), the gamma matrices now satisfy the commutation relations

$$[\tilde{\Gamma}_n, \tilde{\Gamma}_m] = 2(n-m)\tilde{\Gamma}_{n+m}, \tag{A14}$$

which is the $sl(2,\mathbb{R})$ algebra.

APPENDIX B: CONSTRUCTION OF THE SUPERTRACE ELEMENTS

In this appendix, we shall outline the procedure to obtain the supertrace elements for a given algebra. Below, we present the computation for (2,0) AdS algebra, that is presented in the last appendix. Super trace element is computed from nondegenerate bilinear form of a given algebra. For this, we construct a quadratic scalar combination of all the generators and impose that it commutes with all the generators, so that it is a Casimir operator. The construction of this quadratic scalar invariant is quite easy. Let us focus on the (2,0) algebra first, and find its nonzero supertrace elements. Now let us start with the most generic possible bilinear form W:

$$W = a\eta^{ab}J_aJ_b + bC^{\alpha\beta}Q^+_\alpha Q^-_\beta + \bar{b}C^{\alpha\beta}Q^-_\alpha Q^+_\beta + cC^{\alpha\beta}Q^+_\alpha Q^+_\beta + \bar{c}C^{\alpha\beta}Q^-_\alpha Q^-_\beta + dRR \tag{B1}$$

By demanding that W commutes with all the generators of the (2,0) super algebra, we can fix the factors $(a,b,\bar{b},c,\bar{c},d)$. In this process, we need to make sure that the final Casimir is nondegenerate. We will use the identities:

$$[AB,C]=A[B,C]+[A,C]B, \quad [AB,C]=A\{B,C\}-\{A,C\}B, \quad C_{\alpha\beta}C^{\beta\gamma}=\delta^\gamma_\alpha, C\Gamma=(C\Gamma)^T, \quad \Gamma C=(\Gamma C)^T. \tag{B2}$$

The parameters $(a,b,\bar{b},c,\bar{c},d)$ get fixed as,

$$a=b=\bar{b}=-d, \qquad c=\bar{c}=0. \tag{B3}$$

So overall, the invariant reads:

$$W = a(\eta^{ab}J_aJ_b + C^{\alpha\beta}Q^+_\alpha Q^-_\beta + C^{\alpha\beta}Q^-_\alpha Q^+_\beta - RR). \tag{B4}$$

From W we extract all the supertrace elements (see also book by Blagojevic M. "Gravitation and gauge symmetries," Appendix L), by taking the inverse of the matrices η^{ab}, $C^{\alpha\beta}$, and $\mathbb{I}$:

$$\langle J_a, J_b\rangle = \frac{1}{a}\eta_{ab}, \qquad \langle Q^+_\alpha, Q^-_\beta\rangle = \langle Q^-_\alpha, Q^+_\beta\rangle = \frac{1}{a}C_{\alpha\beta}, \qquad \langle R,R\rangle = -\frac{1}{a}. \tag{B5}$$

Similarly for the (0,2) sector, the supertrace element is given as,

$$\langle \bar{J}_a, \bar{J}_b\rangle = \frac{1}{\bar{a}}\eta_{ab}, \qquad \langle \bar{Q}^\pm_\alpha, \bar{Q}^\mp_\beta\rangle = \frac{1}{\bar{a}}C_{\alpha\beta}, \qquad \langle \bar{R},\bar{R}\rangle = -\frac{1}{\bar{a}} \tag{B6}$$

where we have kept the overall factors a, d in the supertrace of both the sectors, because they correspond to an overall normalization in the action. These constant factors get fixed to $a=-\bar{a}=-2$ for the bosonic action to contain the Einstein-Hilbert term. The super Poincaré generators $\mathcal{J}_a$, P_a, $\mathcal{Q}^{1,2,\pm}_r$, $\mathcal{R}$, and $\mathcal{S}$ are given in terms of the super conformal generators in Appendix D. Hence, we can find the supertrace elements for flat generators as linear combinations of the AdS_3 supertrace-elements (the factor of $1/l$ is absorbed in the Chern-Simons level of the action, hence neglected below):

$$\begin{aligned}
\langle \mathcal{J}_a, P_b\rangle &= \langle (J_a+\bar{J}_a),(J_a-\bar{J}_a)\rangle = \eta_{ab},\\
\langle \mathcal{Q}^{1\pm}_\alpha, \mathcal{Q}^{1\mp}_\beta\rangle &= (\sqrt{2})^2\langle Q^\pm_\alpha, Q^\mp_\beta\rangle = C_{\alpha\beta},\\
\langle \mathcal{Q}^{2\pm}_\alpha, \mathcal{Q}^{2\mp}_\beta\rangle &= (\sqrt{-2})^2\langle \bar{Q}^\pm_\alpha, \bar{Q}^\mp_\beta\rangle = C_{\alpha\beta},\\
\langle \mathcal{R},\mathcal{S}\rangle &= \langle (R-\bar{R}), \qquad (R+\bar{R})\rangle = -1.
\end{aligned} \tag{B7}$$

(see Appendix D for the change of basis of the generators). When dealing with the asymptotic algebra,the overall factor is necessary to obtain the correct normalization of the charges.

APPENDIX C: THE (0,2) AND (2,0) ADS SECTORS

Below we present the $N=(2,0)$ and (0,2) superconformal algebras, the global bulk algebras for the corresponding AdS supergravity theories.

$$\begin{aligned}
&[J_a,J_b]=\epsilon_{abc}J^c, \qquad [J_a,R]=0, \qquad [R,R]=0,\\
&[J_a,Q^\pm_\alpha]=\frac{1}{2}(\Gamma_a)^\beta{}_\alpha Q^\pm_\beta, \qquad [R,Q^\pm_\alpha]=\pm\frac{1}{2}Q^\pm_\alpha,\\
&\{Q^+_\alpha,Q^-_\beta\}=-\frac{1}{2}(C\Gamma^a)_{\alpha\beta}J_a-\frac{1}{2}C_{\alpha\beta}R, \qquad \{Q^\pm_\alpha,Q^\pm_\beta\}=0.
\end{aligned} \tag{C1}$$

$$\begin{aligned}
&[\bar{J}_a,\bar{J}_b]=\epsilon_{abc}\bar{J}^c, \qquad [\bar{J}_a,\bar{R}]=0, \qquad [\bar{R},\bar{R}]=0,\\
&[\bar{J}_a,\bar{Q}^\pm_\alpha]=\frac{1}{2}(\Gamma_a)^\beta{}_\alpha \bar{Q}^\pm_\beta, \qquad [\bar{R},\bar{Q}^\pm_\alpha]=\pm\frac{1}{2}\bar{Q}^\pm_\alpha,\\
&\{\bar{Q}^+_\alpha,\bar{Q}^-_\beta\}=-\frac{1}{2}(C\Gamma^a)_{\alpha\beta}\bar{J}_a-\frac{1}{2}C_{\alpha\beta}\bar{R}, \qquad \{\bar{Q}^\pm_\alpha,\bar{Q}^\pm_\beta\}=0.
\end{aligned} \tag{C2}$$

Here, a, $b=0$, 1, 2 and $\alpha,\beta=\pm\frac{1}{2}$. Our convention for $(\Gamma_a)^\beta{}_\alpha$ and $C_{\alpha\beta}$ are presented in the first appendix. With gauge fields

$$A = A^a J_a + \sum_{i=\pm} \psi_i^\beta Q_\beta^i + \phi_R R,$$
$$\bar{A} = \bar{A}^a \bar{J}_a + \sum_{i=\pm} \eta_i^\beta \bar{Q}_\beta^i + \bar{\phi}_R \bar{R} \tag{C3}$$

where $A^a = \omega + \frac{1}{l} e^a$ and $\bar{A}^a = \omega - \frac{1}{l} e^a$, one can build the supersymmetric action:

$$S = \frac{1}{16\pi G} \int \Big[2e_a R^a + \frac{2}{l^2} e + \frac{l}{2} \bar{\psi}_- D\psi_+ + \frac{l}{2} \bar{\psi}_+ D\psi_- - \frac{l}{2} \bar{\eta}_- D\eta_+ - \frac{l}{2} \bar{\eta}_+ D\eta_- + \frac{1}{4} (\bar{\psi}_+ e_a \Gamma^a \psi_- + \bar{\psi}_- e_a \Gamma^a \psi_+ - \bar{\eta}_+ e_a \Gamma^a \eta_- - \bar{\eta}_- e_a \Gamma^a \eta_+) - \frac{l}{2} (\phi_R \mathrm{d}\phi_R - \bar{\phi}_R \mathrm{d}\bar{\phi}_R) \Big] \tag{C4}$$

by using the supertrace elements (B6). The covariant derivatives read:

$$D\psi_+ = \mathrm{d}\psi_+ + \frac{1}{2} \phi_R \psi_+ + \frac{1}{2} \omega \Gamma \psi_+,$$
$$D\psi_- = \mathrm{d}\psi_- - \frac{1}{2} \phi_R \psi_- + \frac{1}{2} \omega \Gamma \psi_-,$$
$$D\eta_+ = \mathrm{d}\eta_+ + \frac{1}{2} \bar{\phi}_R \eta_+ + \frac{1}{2} \omega \Gamma \eta_+,$$
$$D\eta_- = \mathrm{d}\eta_- + \frac{1}{2} \bar{\phi}_R \eta_- + \frac{1}{2} \omega \Gamma \eta_-.$$

To obtain the flat action (2.5), we take the limit $l \to \infty$ combined with the following redefinitions for the fermions and R-symmetry generators:

$$\psi_\pm^\alpha \to \sqrt{\frac{2}{l}} \psi_\pm^{1\alpha}, \qquad \eta_\pm^\alpha \to \sqrt{-\frac{2}{l}} \psi_\pm^{2\alpha},$$
$$\phi_R \to \left(\frac{\sigma}{l} + \upsilon \right), \qquad \bar{\phi}_R \to \left(\frac{\sigma}{l} - \upsilon \right). \tag{C5}$$

APPENDIX D: SUM UP ALL THE NOTATIONS AND CHANGE OF BASIS

In this appendix, we sum up the notations for the two basis for AdS and flat algebra that we have used in our computation. Although required relations are mentioned in the main draft, here we sum them up in a compact form for future reference. First we write down the notations for various generators ,fields and gauge transformation parameters in all four cases:

AdS supergravity	generator	L_n	$\bar{L}_n$	$\mathcal{Q}_\alpha^\pm$	$\bar{\mathcal{Q}}_\alpha^\pm$	R	$\bar{R}$
	gauge fields	A^n	$\bar{A}^n$	$\psi_\pm^{A\alpha}$	$\bar{\psi}_\pm^{A\alpha}$	ϕ_R^A	$\bar{\phi}_R^A$
	parameters	χ^n	$\bar{\chi}^n$	$\epsilon_\pm^\alpha$	$\bar{\epsilon}_\pm^\alpha$	λ_R^A	$\bar{\lambda}_R^A$

AdS supergravity	generator	J_a	$\bar{J}_a$	$Q_\alpha^\pm$	$\bar{Q}_\alpha^\pm$	R	$\bar{R}$
	gauge fields	A^a	$\bar{A}^a$	$\psi_\pm^\alpha$	$\bar{\psi}_\pm^\alpha$	ϕ_R	$\bar{\phi}_R$
	parameters	...	...	$\varepsilon_\pm^\alpha$	$\vartheta_\pm^\alpha$	λ_R	$\bar{\lambda}_R$

Poincaré supergravity	generator	$\mathcal{J}_a$	P_a	$\mathcal{Q}_\alpha^{1\pm}$	$\mathcal{Q}_\alpha^{2\pm}$	$\mathcal{R}$	$\mathcal{S}$
	gauge fields	ω^a	e^a	$\psi_\pm^{1\alpha}$	$\psi_\pm^{2\alpha}$	$\tilde{\phi}$	$\tilde{\rho}$
	parameters	...	...	$\theta_\pm^{1\alpha}$	$\theta_\pm^{2\alpha}$	$\lambda_\mathcal{R}$	$\bar{\lambda}_\mathcal{S}$

Poincaré supergravity	generator	$\mathcal{L}_n$	M_n	$q_\alpha^{1\pm}$	$q_\alpha^{2\pm}$	$\mathcal{R}$	$\mathcal{S}$
	gauge fields	ω^n	e^n	$\Psi_\pm^{1\alpha}$	$\Psi_\pm^{2\alpha}$	ϕ	ρ
	parameters	Υ^n	ξ^n	$\zeta_\pm^{1\alpha}$	$\zeta_\pm^{2\alpha}$	$\lambda_\mathcal{R}$	$\bar{\lambda}_\mathcal{S}$

Next we present the relations between the generators, fields and parameters for the above cases:

Relation between the two AdS basis:

$$L_n = J_a U_n^a, \quad \bar{L}_n = \bar{J}_a U_n^a, \quad \mathcal{Q}_\alpha^\pm = \sqrt{2} Q_\alpha^\pm, \quad \bar{\mathcal{Q}}_\alpha^\pm = \sqrt{2} \bar{Q}_\alpha^\pm \tag{D1}$$

R and $\bar{R}$ remain unchanged.

Relation between the AdS and flat basis:

$$P_a = \frac{J_a - \bar{J}_a}{l}, \quad \mathcal{J}_a = J_a + \bar{J}_a, \quad \mathcal{Q}_\alpha^{1\pm} = \sqrt{\frac{2}{l}} Q_\alpha^\pm, \quad \mathcal{Q}_\alpha^{2\pm} = \sqrt{-\frac{2}{l}} \bar{Q}_\alpha^\pm, \quad \mathcal{R} = R - \bar{R}, \quad \mathcal{S} = \frac{R + \bar{R}}{l},$$
$$A^a = \omega^a + \frac{1}{l} e^a, \quad \bar{A}^a = \omega^a - \frac{1}{l} e^a \quad \psi_\pm^{1\alpha} = \sqrt{\frac{l}{2}} \psi_\pm^\alpha, \quad \psi_\pm^{2\alpha} = \sqrt{-\frac{l}{2}} \eta_\pm^\alpha, \quad \sigma = l \frac{\phi_R + \bar{\phi}_R}{2}, \quad \upsilon = \frac{\phi_R - \bar{\phi}_R}{2}$$
$$\theta_\pm^{1\alpha} = \sqrt{\frac{l}{2}} \epsilon_\pm^\alpha, \quad \theta_\pm^{2\alpha} = \sqrt{-\frac{l}{2}} \vartheta_\pm^\alpha, \quad \lambda_\mathcal{R} = \frac{\lambda_R - \bar{\lambda}_R}{2}, \quad \lambda_\mathcal{S} = l \frac{\lambda_R + \bar{\lambda}_R}{2} \tag{D2}$$

Relation between the two flat basis:

$$M_n = P_a U_n^a, \quad \mathcal{L}_n = \mathcal{J}_a U_n^a, \quad q_\alpha^{1\pm} = \sqrt{2} Q_\alpha^{1\pm}, \quad q_\alpha^{2\pm} = \sqrt{2} Q_{-\alpha}^{2\pm}, \tag{D3}$$

$\mathcal{R}$ and $\mathcal{S}$ remain unchanged.

Relation between flat and AdS basis:

$$M_n = \frac{L_n + \bar{L}_{-n}}{l}, \qquad \mathcal{L}_n = L_n - \bar{L}_{-n}, \qquad q_\alpha^{1\pm} = \sqrt{\frac{2}{l}}\mathcal{Q}_\alpha^{\pm}, \qquad q_\alpha^{2\pm} = \sqrt{\frac{2}{l}}\bar{\mathcal{Q}}_{-\alpha}^{\pm} \qquad \mathcal{R} = R - \bar{R}, \qquad \mathcal{S} = \frac{R + \bar{R}}{l},$$
$$\psi_\pm^{A\alpha} = \frac{1}{\sqrt{2l}}\Psi_\pm^{1,\alpha}, \qquad \bar{\psi}_\pm^{A-\alpha} = \frac{1}{\sqrt{2l}}\Psi_\pm^{\alpha}, \qquad \phi = l(\phi_R^A - \bar{\phi}_R^A), \qquad \rho = \phi_R^A + \bar{\phi}_R^A, \qquad \lambda_{\mathcal{R}} = \frac{\lambda_R^A - \bar{\lambda}_R^A}{2}, \qquad \lambda_{\mathcal{S}} = l\frac{\lambda_R^A + \bar{\lambda}_R^A}{2},$$
$$\Upsilon^n = \frac{\chi^n - \bar{\chi}^{-n}}{2}, \qquad \xi^n = l\frac{\chi^n + \bar{\chi}^{-n}}{2}, \qquad \epsilon_\pm^\alpha = \sqrt{\frac{2}{l}}\zeta_\pm^{1\alpha}, \qquad \bar{\epsilon}_\pm^{-\alpha} = \sqrt{\frac{2}{l}}\zeta_\pm^{2\alpha}. \tag{D4}$$

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