Phenomenology with F-theory $SU(5)$

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We explore the low-energy phenomenology of an F-theory-based $SU(5)$ model which, in addition to the known quarks and leptons, contains Standard Model (SM) singlets and vectorlike color triplets and $SU(2)$ doublets. Depending on their masses and couplings, some of these new particles may be observed at the LHC and future colliders. We discuss the restrictions by Cabibbo-Kobayashi-Maskawa matrix constraints on their mixing with the ordinary down quarks of the three chiral families. The model is consistent with gauge coupling unification at the usual supersymmetric GUT scale; dimension-five proton decay is adequately suppressed, while dimension-six decay mediated by the superheavy gauge bosons is enhanced by a factor of 5–7. The third generation charged fermion Yukawa couplings yield the corresponding low-energy masses in reasonable agreement with observations. The hierarchical nature of the masses of lighter generations is accounted for via nonrenormalizable interactions, with the perturbative vacuum expectation values (VEVs) of the SM singlet fields playing an essential role.

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I. INTRODUCTION

Models originating from string theory constructions often contain SM singlets and vectorlike fields which can mix with the light spectrum and are therefore natural candidates for predicting rare processes that might be discovered in future experiments at the LHC and elsewhere. F-theory models [1], in particular, have the necessary ingredients to describe in a simple and convincing manner a complete picture of such new phenomena. One of the most appealing grand unified theories incorporating these features in an F-theory context is $SU(5)$.[1] Indeed, on breaking F-$SU(5)$ to SM symmetry, one ends up with the Minimal Supersymmetric Standard Model (MSSM) spectrum augmented by scalar fields and vectorlike states, which are remnants of the underlying GUT representations. In this framework, it is possible to retain gauge coupling unification even in the presence of some additional fields, provided that these form complete multiplets of $SU(5)$. In view of the ongoing experimental searches and possible future signatures, in this work we reconsider some issues regarding the exotic part of these models.

We start with a brief review of the basic features of an $SU(5)$ model [12] derived in an F-theory framework and, in particular, in the context of the spectral cover. We derive an effective theory model by imposing a Z_2 monodromy and identify the complex surfaces where the chiral matter and Higgs can be accommodated in the quotient theory. We assume a hypercharge flux breaking of the $SU(5)$ symmetry down to the SM one, and proceed with a specific assignment of the MSSM representations on these matter curves and then work out the spectrum and the superpotential. After fixing the necessary free parameters [such as flux units and singlet vacuum expectation values (VEVs)], we proceed with the investigation of the exotic massless spectrum left over from higher dimensional fields. We then derive their superpotential couplings and analyze the implications for baryon number violating decays as well as other rare processes. We examine the possibility that these states remaining massless at low energies is consistent with gauge coupling unification, and we discuss the physics implications of the TeV scale exotic states.

II. F-$SU(5)$

We consider the elliptically fibered case where the highest smooth singularity in Kodaira's classification is associated with the exceptional group of E_8 [20,21]. We assume 7-branes wrapping an $SU(5)$ divisor and interpret this as the GUT symmetry of the effective model. Under these assumptions

$$E_8 \supset SU(5)_{GUT} \times SU(5)_{\perp}, \qquad (1)$$

where the first factor is interpreted as the well-known $SU(5)_{GUT}$ and the second factor is usually denoted as $SU(5)_{\perp}$.

The MSSM spectrum and possible exotic fields descend from the decomposition of the E_8 adjoint which, under the assumed breaking pattern (1), decomposes as follows:

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[1]For F-theory model building reviews and early references see [2–5]. For an incomplete list including more recent research papers see [6–37].

$$248 \to (24,1) + (1,24) + (10,5) + (\bar{5},10) + (5,\overline{10}) + (\overline{10},\bar{5}). \quad (2)$$

Thus, matter transforms in bifundamental representations, with the GUT 10-plets lying in the fundamental of $SU(5)_\perp$, and the $\bar{5}, 5$-plets lying in the antisymmetric representation of $SU(5)_\perp$.

We note in passing that the flipped $SU(5)$ can also be obtained within the same context, by splitting $SU(5))_\perp$ to $U(1)_X \times SU(4)_\perp$. In this case the hypercharge is defined by the formula $Y = \frac{1}{5}(x + \frac{1}{6}y)$, where x denotes the $U(1)_X$ charge and y that of the Abelian factor inside $SU(5)$. If we recall that the flipped model $SU(5) \times U(1)_X$ is embedded in $SO(10)$, the analogue of the decomposition of (2) can be expressed with respect to the breaking pattern

$$E_8 \supset SO(10) \times SU(4)_\perp \to SU(5) \times SU(4)_\perp \times U(1)_X$$

as follows:

$$\begin{aligned}248 &\to (45,1) + (16,4) + (\overline{16},\bar{4}) + (10,6) + (1,15)\\ &\to (24,1)_0 + (1,15)_0 + (1,1)_0 + (1,4)_{-5}\\ &\quad + (1,\bar{4})_5 + (10,4)_{-1} + (10,1)_4\\ &\quad + (10,\bar{4})_1 + (10,1)_{-4} + (\bar{5},4)_3 + (\bar{5},6)_{-2}\\ &\quad + (5,\bar{4})_{-3} + (5,6)_2. \end{aligned} \quad (3)$$

Returning to the $SU(5)$ model, we choose to work in the Higgs bundle picture (the spectral cover approach). In this context the properties of the GUT representations with respect to the spectral cover are described by a degree-five polynomial [6],

$$\mathcal{C}_5\colon\ \sum_{k=0}^{5} b_k s^{5-k} = 0, \quad (4)$$

where the b_k coefficients that carry the information of the internal geometry and their homologies are given by $[b_n] = \eta - nc_1$ (with $\eta = 6c_1 - t$), where $c_1 = c_1(S)$ is the first Chern class of the tangent bundle and $-t$ that of the normal to the surface S. The roots of the equation are identified as the weight vectors $t_{1,\ldots,5}$ satisfying the standard $SU(N)$ constraint ($N = 5$ in the present case),

$$\sum_{i=1}^{5} t_i = 0. \quad (5)$$

Under t_i the matter curves acquire specific topological and symmetry properties inherited by the fermion families and Higgs fields propagating there. We denote the matter curves accommodating the 10-plets, 5-plets of $SU(5)$, and singlets emerging from $SU(5)_\perp$ adjoint decomposition as $\Sigma_{10_{t_i}}, \Sigma_{5_{t_i+t_j}}, \Sigma_{1_{t_i-t_j}}$. Correspondingly, the possible representations residing on these matter curves are denoted by

$$\Sigma_{10_{t_i}}\colon 10_{t_i}, \overline{10}_{-t_i}, \quad \Sigma_{5_{t_i+t_j}}\colon \bar{5}_{t_i+t_j}, 5_{-t_i-t_j}, \quad \Sigma_{1_{t_i-t_j}}\colon 1_{t_i-t_j},$$

where, as far as 5-plets and singlets are concerned, we must have $t_i \neq t_j$.

Working in the framework of spectral cover, while assuming distinct roots t_i of (4), one may further consider the breaking $SU(5)_\perp \to U(1)^4_\perp$. Then, the invariant tree-level superpotential couplings are of the form

$$\begin{aligned}\mathcal{W} \supset\ & h_1 10_{t_i} 10_{t_j} 5_{-t_i-t_j} + h_2 10_{t_i} \bar{5}_{t_j+t_k} \bar{5}_{t_l+t_m}\\ &+ h_3 1_{t_i-t_j} 5_{-t_i-t_k} \bar{5}_{t_j+t_m} + h_4 1_{t_i-t_j} 1_{t_j-t_k} 1_{t_k-t_i},\end{aligned} \quad (6)$$

where $h_{1,2,3,4}$ represent the Yukawa strengths. In each of the above terms, the sum of the t_i "charges" should add up to zero. Hence, in the second term $t_i + t_j + t_k + t_l + t_m = 0$, which unambiguously implies that all indices in the term proportional to the Yukawa coupling h_2 should differ from each other (due to the fact that $t_1 + t_2 + t_3 + t_4 + t_5 = 0$).

Returning to the polynomial (4), although its coefficients b_n belong to a certain field (holomorphic functions), the roots t_i do not necessarily do so. Solutions, in general, imply branch cuts and, as a result, certain roots might be interrelated. The simplest case is if two of them are subject to a Z_2 monodromy, say,[2]

$$Z_2\colon\ t_1 = t_2. \quad (7)$$

From the point of view of the effective field theory model, the appearance of the monodromy is a welcome result since it implies rank-one mass matrices for the fermions. Indeed, under the Z_2 monodromy, the coupling

$$\mathcal{W} \supset 10_{t_1} 10_{t_2} 5_{-t_1-t_2} \xrightarrow{Z_2} 10_{t_1} 10_{t_1} 5_{-2t_1} \quad (8)$$

ensures a top-quark mass at tree level, while the remaining mass matrix entries are expected to be generated from nonrenormalizable terms. After this brief description of the basic features, we proceed in the next section with the analysis of the implications of the hypercharge flux on the symmetry breaking and the massless spectrum of $SU(5)$.

III. HYPERCHARGE FLUX BREAKING OF $SU(5)$

The Z_2 monodromy implies that the spectral cover polynomial factorizes as follows:

$$\begin{aligned}&b_0 s^5 + b_2 s^3 + b_3 s^2 + b_4 s + b_5\\ &\quad = (a_1 + a_2 s + a_3 s^2)(a_4 + a_5 s)(a_6 + a_7 s)(a_8 + a_9 s),\end{aligned} \quad (9)$$

where all a_i are assumed in the same field as b_n's. Thus, while the roots of the three monomials on the right-hand

[2]For various choices of monodromies, see [7,8,10,11].

TABLE I. Field content under $SU(5)$, their charges under the "perpendicular" $U(1)_{t_i}$'s, their homology class, and flux restrictions. For convenience, only the properties of 10,5 are shown. $\overline{10}, \bar{5}$ are characterized by $t_i \to -t_i$. Note that the fluxes satisfy $N = N_7 + N_8 + N_9$ and $\sum_i M_{10_i} + \sum_j M_{5_j} = 0$, while $\chi = \chi_7 + \chi_8 + \chi_9$.

Curve	Field	$U(1)_i$	Defining equation	Homology	$U(1)_Y$ flux	$U(1)$ flux
$\Sigma_{10^{(1)}}$:	10_3	t_1	a_1	$\eta - 2c_1 - \chi$	$-N = 0$	$M_{10_1} = 1$
$\Sigma_{10^{(2)}}$:	10_1	t_3	a_4	$-c_1 + \chi_7$	$N_7 = -1$	$M_{10_2} = 1$
$\Sigma_{10^{(3)}}$:	10_2	t_4	a_6	$-c_1 + \chi_8$	$N_8 = 1$	$M_{10_3} = 1$
$\Sigma_{10^{(4)}}$:	$10_2'$	t_5	a_8	$-c_1 + \chi_9$	$N_9 = 0$	$M_{10_4} = 0$
$\Sigma_{5^{(0)}}$:	5_{H_u}	$-2t_1$	$a_{578} + a_{479} + a_{569}$	$-c_1 + \chi$	$N = 0$	$M_{5_{H_u}} = 1$
$\Sigma_{5^{(1)}}$:	$\bar{5}_2$	$t_1 + t_3$	$a_1 - c(a_{478} + a_{469})$	$\eta - 2c_1 - \chi$	$-N = 0$	$M_{5_1} = -1$
$\Sigma_{5^{(2)}}$:	$\bar{5}_3$	$t_1 + t_4$	$a_1 - c(a_{568} + a_{469})$	$\eta - 2c_1 - \chi$	$-N = 0$	$M_{5_2} = -1$
$\Sigma_{5^{(3)}}$:	5_x	$-t_1 - t_5$	$a_1 - c(a_{568} + a_{478})$	$\eta - 2c_1 - \chi$	$-N = 0$	$M_{5_3} = n$
$\Sigma_{5^{(4)}}$:	$\bar{5}_1$	$t_3 + t_4$	$a_{56} + a_{47}$	$-c_1 + \chi - \chi_9$	$N - N_9 = 0$	$M_{5_4} = -1$
$\Sigma_{5^{(5)}}$:	$\bar{5}_{H_d}$	$t_3 + t_5$	$a_{58} + a_{49}$	$-c_1 + \chi - \chi_8$	$N - N_8 = -1$	$M_{5_{H_d}} = 0$
$\Sigma_{5^{(6)}}$:	$\bar{5}_{\bar{x}}$	$t_4 + t_5$	$a_{78} + a_{49}$	$-c_1 + \chi - \chi_7$	$N - N_7 = 1$	$-n - 1$
	θ_{12}	0	$\cdots$	$\cdots$	$\cdots$	M_{12}
$\Sigma_{5^{(6)}}$:	θ_{ij}	$t_i - t_j$	$\cdots$	$\cdots$	$\cdots$	M_{ij}
	θ_δ	0	$\cdots$	$\cdots$	$\cdots$	M_δ

side of (9) are rational functions in this field, it is assumed that the two roots of the binomial $(a_1 + a_2 s + a_3 s^2)$ cannot be written in terms of functions in the same field.

The $b_n(a_i)$ relations are easily extracted by identifying coefficients of the same powers in s and are of the form $b_n = \sum a_i a_j a_k a_l$, where the indices satisfy $i + j + k + l + n = 24$. Therefore, given the homologies $[b_n]$, the corresponding ones for the a_i coefficients satisfy $[a_i] + [a_j] + [a_k] + [a_l] = [b_n]$. Solving the resulting simple linear system of equations, it turns out that these can be determined in terms of the known classes c_1, $-t$, and three arbitrary ones (dubbed here $\chi_{6,7,8}$), which will be treated as free parameters [10]. Each matter curve is associated with a defining equation involving products of a_i's and, as such, it belongs to a specific homological class which subsequently is used to determine the flux restriction on it. If $\mathcal{F}_Y$ represents the hypercharge flux, we will require the vanishing of $\mathcal{F}_Y \cdot c_1 = \mathcal{F}_Y \cdot (-t) = 0$, so that all can be expressed in terms of three free (integer parameters) defined by the restrictions

$$N_7 = \mathcal{F}_Y \cdot \chi_7, \quad N_8 = \mathcal{F}_Y \cdot \chi_8, \quad N_9 = \mathcal{F}_Y \cdot \chi_9. \tag{10}$$

To construct a specific model, we start by assuming that a suitable $U(1)_X$ flux [where the Abelian factor $U(1)_X$ lies outside $SU(5)$ GUT] generates chirality for the 10 and $\bar{5}$ representations. Next, the hypercharge flux breaks $SU(5)$ down to the SM and, at the same time, it splits the $10, \overline{10}$'s and $5, \bar{5}$'s into different numbers of SM multiplets. If some integers M_{10}, M_5 are associated with the $U(1)_X$ flux, and some linear combination N_y of $N_{7,8,9}$ represents the corresponding hyperflux piercing a given matter curve, the 10-plets and 5-plets split according to

$$10_{t_i} = \begin{cases} \text{Representation} & \text{flux units} \\ n_{(3,2)_{1/6}} - n_{(\bar{3},2)_{-1/6}} & = M_{10} \\ n_{(\bar{3},1)_{-2/3}} - n_{(3,1)_{2/3}} & = M_{10} - N_y \\ n_{(1,1)_{+1}} - n_{(1,1)_{-1}} & = M_{10} + N_y \end{cases}, \tag{11}$$

$$5_{t_i} = \begin{cases} \text{Representation} & \text{flux units} \\ n_{(3,1)_{-1/3}} - n_{(\bar{3},1)_{+1/3}} & = M_5 \\ n_{(1,2)_{+1/2}} - n_{(1,2)_{-1/2}} & = M_5 + N_y. \end{cases} \tag{12}$$

As already discussed, depending on the restrictions of the flux on the matter curves Σ_j, there are certain conditions on the corresponding hypercharge flux, denoted as N_{y_j} (for the specific matter curve Σ_j). These are deduced from the topological properties of the coefficients a_i as well as the fluxes.

For a given choice of the flux parameters M_i, N_{y_j}, the most general spectrum and its properties under the assumption of a Z_2 monodromy are exhibited in Table I. The first column shows the available matter curves and the assumed chiral state propagating on it. The chirality is fixed by the specific choice of M_i, N_{y_j} flux coefficients shown in the last two columns of Table I. The second column shows the charge assignments, $\pm t_i$ for the 10-plets, and $\pm(t_i + t_j)$, $\pm(t_i - t_j)$ for the 5-plets and singlets respectively. For this particular arrangement, the structure of the fermion mass matrices exhibits a hierarchical form, consistent with the experimentally measured masses and mixings [12]. In the present work, we will explore other interesting phenomenological implications of this model. The defining equations are shown in the fourth column where, for brevity, the notation $a_{ijk\ldots} = a_i a_j a_k \ldots$ is used. The next column indicates the homologies, the sixth column their associated integers expressing the restrictions of flux on the corresponding matter curves, and the last column lists a choice of M_i values consistent with a chiral $SU(5)$ spectrum. Notice that the flux integers are subject to the restrictions [10] $N=N_7+N_8+N_9$ and $\sum_i M_{5_i}+\sum_j M_{10_j}=0$. In the minimal case $n=0$ and there are no extra $5+\bar{5}$ pairs. Furthermore, the multiplicities M_{ij}, M_δ of singlet fields are not determined in the context of the spectral cover and are left arbitrary.

IV. SPECTRUM OF THE EFFECTIVE LOW-ENERGY THEORY

A comprehensive classification of the resulting spectrum is shown in Table II where, in the first column, the $SU(5)$ properties are shown. The third column shows the accommodation of the SM representations with their corresponding t_i charges [associated with the perpendicular $U(1)$'s] given in column 2. Column 4 includes the exotics which, for the specific choice of parameters, involves the triplet pair $D+D^c$ and, in principle, n copies of $5+\bar{5}$ representations. In the minimal case we set $n=0$, but perturbativity allows values up to $n\leq 4$. In the modified version of the model we allow for $n\neq 0$ and explore the phenomenological implications. Note that restrictions on the number of vectorlike 5-plets arise when the model is embedded in an E_6 framework [26–28]. In the last column of the table, we have also introduced a Z_2 matter parity to the MSSM field as well as the singlets.

TABLE II. Field content under $SU(5)\times U(1)_{t_i}$. The third column shows the MSSM spectrum and the fourth column displays the predicted exotics. The R-parity assignments appear in the last column. We use assignments $10_{t_i}, 5_{-t_i-t_j}$ with $\overline{10}, \bar{5}$ characterized by opposite values, $t_i\to -t_i$ etc. The fluxes eliminated components of the $SU(5)$ multiplets, giving rise to incomplete representations. There are also n copies of $5+\bar{5}$ multiplets.

Irrep	$U(1)_i$	SM spectrum	Exotics	R-parity
10_1	t_3	Q_1, u_1^c, u_2^c	$\cdots$	$-$
10_2	t_4	Q_2, e_1^c, e_2^c	$\cdots$	$-$
10_3	t_1	Q_3, u_3^c, e_3^c	$\cdots$	$-$
$\bar{5}_1$	t_3+t_4	d_1^c, ℓ_1	$\cdots$	$-$
$\bar{5}_2$	t_1+t_3	d_2^c, ℓ_2	$\cdots$	$-$
$\bar{5}_3$	t_1+t_4	d_3^c, ℓ_3	$\cdots$	$-$
5_{H_u}	$-2t_1$	H_u	D	$+$
$\bar{5}_{H_d}$	t_3+t_5	H_d	$\cdots$	$+$
5_x	$-(t_1+t_5)$	$\cdots$	$(H_{u_i}, D_i)_{i=1,\ldots,n}$	$+$
$\bar{5}_{\bar{x}}$	t_4+t_5	$\cdots$	$D^c+(H_{d_i}, D_i^c)_{i=1,\ldots,n}$	$+$
$\theta_{12,21}$	0		S(singlet)	$-$
θ_{14}	t_1-t_4		$\langle\theta_{14}\rangle=V_1\equiv v_1 M_{\rm GUT}$	$+$
θ_{15}	t_1-t_5		$\langle\theta_{15}\rangle=V_2\equiv v_2 M_{\rm GUT}$	$+$
θ_{43}	t_4-t_3		$\langle\theta_{43}\rangle=V_3\equiv v_3 M_{\rm GUT}$	$+$
$\theta_{ij}^\perp$	t_i-t_j		$\langle\theta_{ij}^\perp\rangle=0$	$+$

Before proceeding with the main part of our paper we present a few remarks about R-parity in supersymmetric models. A discrete Z_2 R-parity is often invoked in four-dimensional supersymmetric $SU(5)$ models in order to eliminate rapid proton decay mediated by the supesrymmetric partners of the SM quarks and leptons. If left unbroken, this discrete symmetry also yields an attractive candidate for cold dark matter, namely, the lightest neutralino. It is perhaps worth noting that this Z_2 symmetry naturally appears if we employ an $SO(10)$ GUT which is broken down to $SU(3)_c\times U(1)_{em}$ by utilizing only tensor representations [38].

The question naturally arises: how do string-theory-based unified models avoid rapid proton decay? In the ten-dimensional $E_8\times E_8$ heterotic string framework [39], the compactification process utilizes Calabi-Yau manifolds which typically yields non-Abelian discrete symmetries that may contain the desired R-parity ([40] and references therein.)

In F-theory models discrete symmetries including R-parity may arise from a variety of sources. They can emerge from Higgsing $U(1)$ symmetries in F-theory compactifications, or from a nontrivial Mordell-Weil group associated with the rational sections of the elliptic fibration, first invoked in [29] and further discussed in several works, including [30–33]. More generally, Z_n symmetries are associated with Calabi-Yau manifolds whose geometries are associated with the Tate-Shafarevich group [34]. Finally, they may appear as geometric properties of the construction in the spectral cover picture [36]. Based on the existence of such possibilities, in the present model we implement the notion of R-parity assuming that it is associated with some symmetry of geometric origin.

A. Matter curves and fermion masses

Returning to the description of the emerging effective model, for further clarification we include a few more details. Initially, in the covering theory there are five matter curves,[3] but due to monodromy $Z_2: t_1=t_2$, two of them are identified and thus they are reduced to four. Similarly, the ten $\Sigma_{5_{t_i+t_j}}$ reduce to seven matter curves. Furthermore, there are 24 singlets from the decomposition of the adjoint of $SU(5)_\perp$ denoted with $\theta_{ij}, i,j=1,2\ldots,5$, and 20 of them live on matter curves defined by t_i-t_j, while four are "chargeless." However, because of the Z_2 monodromy among the various identifications, $\theta_{i1}\equiv\theta_{i2}$ and $\theta_{1j}\equiv\theta_{2j}$, the following two singlets,

$$\theta_{12}=\theta_{21}\to S, \tag{13}$$

[3]Recall from (2), $\Sigma_{10_i}, i=1,2,\ldots,5$ that the 10-plets transform in the fundamental and 5-plets in the antisymmetric representation of $SU(5)_\perp$.

are equivalent to one singlet S with zero charge. The remaining singlets with nonzero charges are

$$\theta_{13},\theta_{14},\theta_{15},\theta_{34},\theta_{35},\theta_{45}, \quad \text{and} \quad \theta_{31},\theta_{41},\theta_{51},\theta_{43},\theta_{53},\theta_{54}.$$

The following singlets acquire nonzero VEVs which help in realizing the desired fermion mass textures:

$$\begin{aligned} \langle\theta_{14}\rangle &\equiv V_1 \equiv v_1 M_{\mathrm{GUT}} \neq 0, \\ \langle\theta_{15}\rangle &\equiv V_2 \equiv v_2 M_{\mathrm{GUT}} \neq 0, \\ \langle\theta_{43}\rangle &\equiv V_3 \equiv v_3 M_{\mathrm{GUT}} \neq 0. \end{aligned} \tag{14}$$

All other singlets (designated with $\theta_{ij}^{\perp}$ in Table II) have zero VEVs. Using the SM Higgs and singlet VEVs given by (14), we obtain hierarchical quark and charged mass textures,

$$\begin{aligned} M_u &\propto \begin{pmatrix} v_1^2 v_3^2 & v_1^2 v_3 & v_1 v_3 \\ v_1^2 v_3 & v_1^2 & v_1 \\ v_1 v_3 & v_1 & 1 \end{pmatrix} \langle H_u\rangle, \\ M_{d,\ell} &= \begin{pmatrix} v_1^2 v_3^2 & v_1 v_3^2 & v_1 v_3 \\ v_1^2 v_3 & v_1 v_3 & v_1 \\ v_1 v_3 & v_3 & 1 \end{pmatrix} \langle H_d\rangle, \end{aligned} \tag{15}$$

where the Yukawa couplings are suppressed for simplicity.

1. Neutrino sector

The tiny masses accompanied by the relatively large mixings of the neutrinos, as indicated by various experiments, can find a plausible solution in the context of the seesaw mechanism and the existence of family symmetries. In the present F-$SU(5)$ GUT model, the SM singlet fields such as θ_{ij} form Yukawa terms invariant under the additional family symmetries described above and could be the natural candidates for the right-handed neutrinos. Furthermore, observing that the right-handed neutrino mass scale is of the order of the Kaluza-Klein scale in string compactifications, a minimal scenario would be to associate the right-handed neutrinos with the Kaluza-Klein (KK) modes [7] of these singlet fields, $\theta_{ij}^{KK} \to N_R$. An obstruction to this interpretation is that in the covering theory these singlets θ_{ij} transform in the complex representation, so that $\theta_{ij}^{KK} = N_R, \theta_{ji}^{KK} = N_R^c$ and the mass term becomes $M_{KK} N_R N_R^c$, but there are no corresponding Dirac mass terms for both N_R and N_R^c. However, in the quotient theory under the Z_2 monodromy $t_1 = t_2$, the KK modes $\theta_{12}^{KK} \equiv \theta_{21}^{KK}$ transform in the real representation, so that for any KK-level the corresponding modes $N_{R_k} = N_{R_k}^c \to \nu_k^c$ are identified and a seesaw mechanism is possible. Hence, the nonrenormalizable term $5_{-t_1-t_2}\bar{5}_{t_1+t_4}\theta_{14}\theta_{21}^{KK}$ under the Z_2 monodromy is identified with $5_{-2t_1}\bar{5}_{t_1+t_4}\theta_{14}\theta_{21}^{KK} \to 5_{h_u}\bar{5}_3\theta_{14}\nu^c$ and so on. Therefore, under the above assumptions, the KK modes corresponding to right-handed neutrinos couple to the following combination of the left-handed neutrino components:

$$5_{H_u}(\bar{5}_1\theta_{14}^2\theta_{43} + \bar{5}_2\theta_{14}\theta_{43} + \bar{5}_3\theta_{14}). \tag{16}$$

The interesting fact is that the right-handed neutrinos are associated with a specific class of wave functions [7] such that the emerging mass hierarchy is milder than that of the charged leptons and quarks. It is shown that the mass matrix obtained in this way [7] can accommodate the two large mixing angles observed in atmospheric and solar neutrino experiments.

B. Mass terms for the doublets and triplets

Returning to the content of Table II, we observe that there is still freedom to accommodate additional vectorlike 5-plets which respect all the required conditions. Hence, aiming to accommodate potential diphoton resonances and other possible experimental signatures of exotic matter beyond the MSSM spectrum, in the present construction we assume the existence of $5 + \bar{5}$ pairs and discuss possible implications of the exotic states. As already explained, the Z_2 monodromy allows a tree-level coupling for the top quark $10_3 10_3 5_{H_u}$. Furthermore, from the specific accommodation of the fermion generations listed in Table II, we observe that a tree-level coupling for the bottom quark is also available. A geometric perspective of the Yukawa couplings in the internal manifold is depicted in Fig. 1. All other mass entries are generated from nonrenormalizable terms [12].

Regarding the 5-plets accommodating the MSSM Higgs, we observe that the flux splits the doublet from the triplet in the Higgs sector. As a result, the MSSM μ term

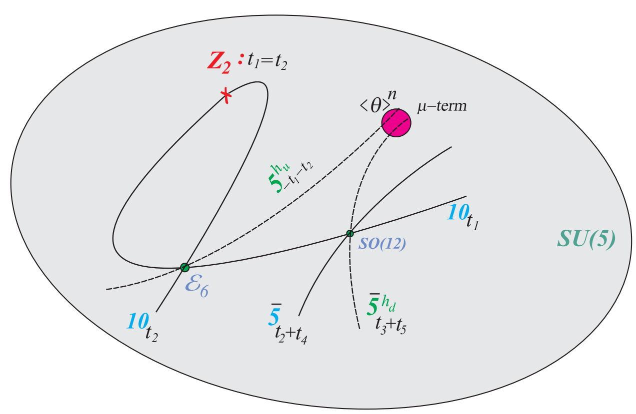

FIG. 1. The trilinear top and bottom Yukawa couplings at the triple intersections of the matter curves with symmetry enhancements $\mathcal{E}_6$ and $SO(12)$, respectively. Under a Z_2 monodromy we obtain identifications such as $10_{t_1} = 10_{t_2}$ so that a "diagonal" top Yukawa coupling can be realized. A μ-term emerges only from nonrenormalizable (suppressed) contributions. $\langle\theta\rangle^n$ stands for the ratio of singlet VEVs divided by the high (compactification) scale.

$$\frac{\theta_{14}\theta_{43}\theta_{15}}{M_{GUT}^2}\bar{5}_{t_3+t_5}5_{-2t_1} \to \frac{V_1V_2V_3}{M_{GUT}^2}H_uH_d \to \mu H_uH_d \qquad (17)$$

does not involve masses for the triplet fields. Fermion mass hierarchies require at least that the singlet VEV $V_1 = \langle\theta_{14}\rangle \gtrsim \mathcal{O}(10^{-1})M_{GUT}$, so that the MSSM μ parameter can be kept light for $v_2 \cdot v_3 \ll v_1$.

In the general case, we need to take into account the extra doublet pairs emerging from the 5-plets remaining in the zero-mode spectrum. As an illustrative example, we take only one additional vectorlike pair of 5-plets, that is $n = 1$. In this case the available couplings are

$$5_{H_u}\bar{5}_{H_d}\theta_{14}\theta_{43}\theta_{15}/M_{GUT}^2 + 5_{H_u}\bar{5}_{\bar{x}}\theta_{14}\theta_{15}/M_{GUT} + 5_x\bar{5}_{H_d}\theta_{14}\theta_{43}/M_{GUT} + 5_x\bar{5}_{\bar{x}}\theta_{14}.$$

The Higgs mass matrix in the basis $\mathcal{L}\supset(H_d,H_d')M_H\binom{H_u}{H_u'}$ is

$$M_H \propto V_1\begin{pmatrix} v_3v_2 & v_3 \\ v_2 & 1\end{pmatrix}, \qquad (18)$$

where the Yukawa couplings are suppressed to avoid clutter. This implies a light Higgs mass term $\mu \sim V_1v_2v_3$ and a heavy one $M_H \sim V_1$.

The triplet mass terms emerge from different couplings:

$$\theta_{14}\theta_{15}\bar{5}_{t_4+t_5}5_{-2t_1}/M_{GUT} + \epsilon\theta_{14}\bar{5}_{t_4+t_5}5_{-t_1-t_5} \to \theta_{14}\theta_{15}\bar{5}_{\bar{x}}5_{H_u}/M_{GUT} + \epsilon\theta_{14}\bar{5}_{\bar{x}}5_x. \qquad (19)$$

Hence, written in a matrix form

$$\mathcal{L}_D \supset (5_{H_u}, 5_x)M_D\begin{pmatrix}\bar{5}_{H_d}\\ \bar{5}_{\bar{x}}\end{pmatrix},$$

where the triplet mass matrix is $M_D = V_1\left(\begin{smallmatrix} v_2 & \epsilon \\ \epsilon' v_2 & 1\end{smallmatrix}\right)$, and the parameters $\epsilon \simeq \epsilon' \lesssim 1$ stand for corrections when more than one matter multiplet are on the same matter curve. The eigenmasses also depend on the singlet VEVs and will be discussed in conjunction with proton decay in the subsequent sections.

In addition to these superpotential couplings, the vector pairs $5 + \bar{5}$ generate superpotential terms with the matter fields

$$10_3\bar{5}_{\bar{x}}\bar{5}_2, \qquad 10_3\bar{5}_{\bar{x}}(\bar{5}_1\theta_{14} + \bar{5}_3\theta_{34}),$$
$$10_1\bar{5}_{\bar{x}}(\bar{5}_1\theta_{14}\theta_{43} + \bar{5}_2\theta_{43} + \bar{5}_3)\theta_{14},$$
$$10_2\bar{5}_{\bar{x}}(\bar{5}_1\theta_{14} + \bar{5}_2 + \bar{5}_3\theta_{34})\theta_{14}, \qquad (20)$$

where the nonrenormalizable terms are assumed to be scaled by appropriate powers of M_{GUT}. In the next sections we will explore possible phenomenological consequences of (20). However, we note that it is feasible to eliminate such couplings from the Lagrangian by introducing a different R-parity assignment for the color triplets.

It is worth mentioning some differences and similarities with models arising in the context of orbifolds and heterotic strings. In [41], for example, the hierarchy of fermion families results from the number of powers of Yukawa couplings allowed under certain selection rules and suitable assumptions on the VEVs, while here the textures of Yukawa matrices display a Froggatt-Nielsen form. On the other hand, there are similarities in obtaining the doublet-triplet splitting mass matrix of the Higgs sector.

V. GAUGE COUPLING UNIFICATION

The presence of additional vectorlike pairs of color triplets and Higgsinos with masses in the TeV range affect the renormalization group running of the gauge couplings and the fermion masses. The existence of complete $5 + \bar{5}$ $SU(5)$ multiplets at the TeV scale may enhance processes that could be observed in future searches, while they can be consistent with perturbative gauge coupling unification as long as their number is less than four. Threshold corrections from KK modes and fluxes play a significant role too [42–44]. Under certain circumstances [43], (for example, when the matter fields are localized on genus one surfaces) the KK threshold effects can be universal, resulting in a common shift of the gauge coupling constant at the GUT scale. This has been analyzed in some detail in Ref. [43] and will not be elaborated further. However, in F-theory constructions, there are additional corrections associated with nontrivial line bundles [45,46]. More precisely, assuming that the $SU(5)$ is generated by D7-branes wrapping a del Pezzo surface, the gauge flux quantization condition [47] implies that D7-branes are associated with a nontrivial line bundle $\mathcal{L}_a$. On the other hand, the breaking of $SU(5)$ occurs with a nontrivial hypercharge flux $\mathcal{L}_Y$ supported on the del Pezzo surface (but with a trivial restriction on the Calabi-Yau fourfold so that the associated gauge boson remains massless). The flux threshold corrections to the gauge couplings associated with these two line bundles can be computed by dimensionally reducing the Chern-Simons action. If we define

$$y = \frac{1}{2}\mathrm{Re}S\int c_1^2(\mathcal{L}_a), \qquad x = -\frac{1}{2}\mathrm{Re}S\int c_1^2(\mathcal{L}_Y), \qquad (21)$$

where $c_1(\mathcal{L})$ denotes the first Chern class of the corresponding line bundle and $S = e^{-\phi} + iC_0$ is the axion-dilaton field (and $g_{IIB} = e^{\phi}$), the flux corrections to the gauge couplings are expressed as follows:

$$\frac{1}{a_3(M_U)} = \frac{1}{a_U} - y \qquad (22)$$

$$\frac{1}{a_2(M_U)} = \frac{1}{a_U} - y + x \qquad (23)$$

$$\frac{1}{a_1(M_U)} = \frac{1}{a_U} - y + \frac{3}{5}x, \tag{24}$$

where a_U represents the unified gauge coupling. From (22)–(24) we observe that the corrections from the $\mathcal{L}_a$ line bundle are universal and therefore y can be absorbed in a redefinition of a_U. On the other hand, hypercharge flux thresholds expressed in terms of x are not universal and destroy the gauge coupling unification at the GUT scale M_U. Notice that in order to eliminate the exotic bulk states $(3,2)_5 + (\bar{5},2)_{-5}$ emerging from the decomposition of 24, we need to impose $\int c_1^2(\mathcal{L}_Y) = -2$, and therefore we find the simple form $x = e^{-\phi} = \frac{1}{g_{IIB}}$. The value of the gauge coupling splitting has important implications on the mass scale of the color triplets discussed in the previous section. In the following we will explore this relation within the matter and Higgs field context of the present model.

We assume that the color triplets $D + D^c \in 5_H + \bar{5}_H$ receive masses at a scale M_X, while the complete $5 + \bar{5}$ extra multiplets obtain masses at a few TeV. The renormalization group equations take the form

$$\frac{1}{a_i(M_U)} = \frac{1}{a_i(M_U)} + \frac{b_i^x}{2\pi}\log\frac{M_U}{M_X} + \frac{b_i}{2\pi}\log\frac{M_X}{\mu}\cdot \tag{25}$$

It can be readily checked that the GUT values of the gauge coupling satisfy

$$\frac{5}{3}\frac{1}{a_1(M_U)} = \frac{1}{a_2(M_U)} + \frac{2}{3}\frac{1}{a_2(M_U)}\cdot \tag{26}$$

Assuming n_D pairs of $(D + D^c)$ and n_V vectorlike 5-plets, the beta functions are

$$b_3^x = -3 + n_V + n_D, \qquad b_2^x = 1 + n_V,$$
$$b_1^x = \frac{33}{5} + \frac{2}{5}n_D + n_V \tag{27}$$

$$b_3 = -3 + n_V, \qquad b_2 = 1 + n_V, \qquad b_1 = \frac{33}{5} + n_V\cdot \tag{28}$$

Using (22), (23), and (26), we find

$$\log\frac{M_U}{M_X} = \frac{2\pi}{\beta_x}\frac{1}{\mathcal{A}} - \frac{\beta}{\beta_x}\log\frac{M_X}{\mu}, \tag{29}$$

where we introduced the definitions

$$\beta = \frac{5}{3}(b_1 - b_3) + (b_3 - b_2) \tag{30}$$

$$\beta_x = \frac{5}{3}(b_1^x - b_3^x) + (b_3^x - b_2^x) \tag{31}$$

$$\frac{1}{\mathcal{A}} = \frac{5}{3}\frac{1}{a_1} - \frac{1}{a_2} - \frac{2}{3}\frac{1}{a_3} = \frac{1 - 2\sin^2\theta_W}{a_e} - \frac{2}{3}\frac{1}{a_3}\cdot \tag{32}$$

Notice that for the particular spectrum, β_x, β are equal ($\beta_x = \beta = 12$) and independent of the number of multiplets n_D and n_V. Then, from (29) we find that the unification scale is

$$M_U = e^{\frac{2\pi}{12\mathcal{A}}}M_Z \approx 2.04\times 10^{16}\ \mathrm{GeV}, \tag{33}$$

i.e., independent of n_V, n_D and the intermediate scale M_X.

To unravel the relation between the scale M_X and the parameter x, we proceed as follows. First, we subtract (23) from (22):

$$\begin{aligned} x &= \frac{1}{a_2} - \frac{1}{a_3} + \frac{b_3^x - b_2^x}{2\pi}\frac{M_U}{M_X} + \frac{b_3 - b_2}{2\pi}\frac{M_X}{\mu} \\ &= \frac{1}{a_2} - \frac{1}{a_3} - \frac{4 - n_D}{2\pi}\frac{M_U}{M_X} - \frac{4}{2\pi}\frac{M_X}{\mu}\cdot \end{aligned} \tag{34}$$

Using (29) and the fact that in our model $n_D = 1$, we find

$$\log\frac{M_X}{\mu} = 2\pi\left(\frac{6\sin^2\theta_W - 1}{4a_e} - \frac{5}{6}\frac{1}{a_3} - x\right)\cdot \tag{35}$$

This determines the relation between the parameter $x = e^{-\phi}$ and the scale M_X, where the Higgs triplets become massive. We can use the expression for M_U to express the M_X scale as follows:

$$\log\frac{M_X}{M_U} = 2\pi\left(\frac{5\sin^2\theta_W - 1}{3a_e} - \frac{7}{9}\frac{1}{a_3} - x\right)\cdot \tag{36}$$

To determine the value of the GUT coupling a_U we use (22), (29), and (34) to find

$$\frac{1}{a_U} + x = \frac{1}{a_2} - \frac{b_2^x}{\beta_x}\frac{1}{\mathcal{A}} = \frac{1}{a_2} - \frac{1 + n_V}{12}\frac{1}{\mathcal{A}}\cdot \tag{37}$$

For the present application, we allow three pairs of 5-plets, $n_V = 3$, and we obtain the relation

$$\frac{1}{a_U} = \frac{5\sin^2\theta_W - 1}{3a_3} + \frac{2}{9}\frac{1}{a_3} - x\cdot \tag{38}$$

Substitution of (38) in (36) gives an elegant and very suggestive formula:

$$M_X = e^{2\pi\left(\frac{1}{a_U} - \frac{1}{a_3}\right)}M_U\cdot \tag{39}$$

We observe that in order to have $M_X \le M_U$, we always need $a_U \ge a_3 \approx \frac{1}{8.5}$. We depict the main results in the figures that follow. In Fig. 2 we show the variation of the color triplets' decoupling scale versus the range of

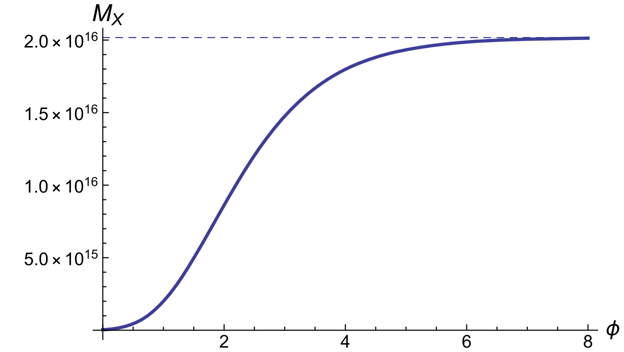

FIG. 2. Variation of the M_X scale with respect to the dilaton field. For the chosen range of $\phi \in (0, \infty)$, (strong g_{IIB} coupling regime), there is a lower bound $M_X \sim 10^{13}$ GeV.

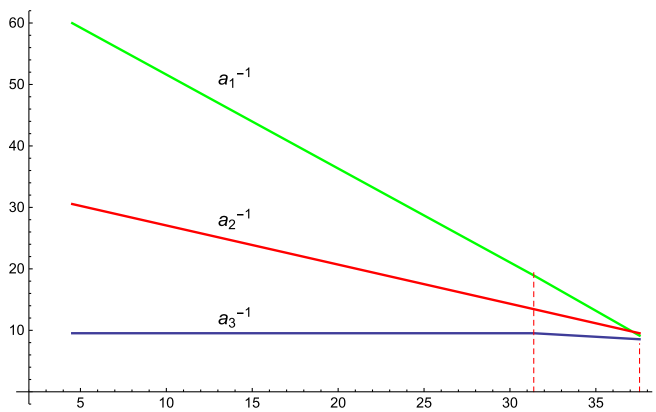

FIG. 3. Gauge coupling running in the presence of flux thresholds and the triplet's decoupling scale M_X.

values of the dilaton and, in Fig. 3, we plot the inverse SM gauge couplings, taking into account the thresholds of the color triplets.

A. Renormalization Group Equations for Yukawa couplings

These modifications in the gauge sector and, in particular, the large g_U value compared to that of the standard MSSM unification scenario ($g_U \sim 1/25$ in MSSM) are expected to have a significant impact on the evolution of the Yukawa couplings. On the other hand, in F-theory constructions the Yukawa coupling strengths at the unification scale are computed analytically and can be expressed in terms of the geometric properties of the internal six-dimensional compact space and the fluxes of the particular construction. For the sake of argument, we assume that all three $5 + \bar{5}$ surplus matter fields receive masses in the TeV range, with $\tan\beta$ values ~48–50 and $M_{GUT} \sim 2\times 10^{16}$ GeV. Then, according to [48], the top mass, in particular, is achieved for Yukawa coupling $h_t(M_{GUT}) \gtrsim 0.35$, which is significantly lower than the value ~0.6 obtained in the case of RG running with the beta functions for the MSSM spectrum.

Turning now to F-theory predictions, as we have seen, the Yukawa couplings are realized at the intersections of three matter curves. The properties of the corresponding matter fields in a given representation R are captured by the wave function Ψ_R whose profile is obtained by solving the equations of motion (EoM) [1]. It is found that the solution exhibits a Gaussian profile picked along the matter curve supporting the particular state, $\Psi_R \propto f(z_i)e^{M_{ij}z_i\bar{z}_j}$. Here $z_{1,2}$ are local complex coordinates, the "matrix" M_{ij} takes into account background fluxes, and $f(z_j)$ is a holomorphic function. The value of the Yukawa coupling results from integrating over the overlapping wave functions. Thus, for the up/down Yukawa couplings,

$$h_t \propto \int \Psi_{10}\Psi_{10}\Psi_{5_{H_u}} dz_1\wedge d\bar{z}_1\wedge dz_2\wedge d\bar{z}_2,$$
$$h_b \propto \int \Psi_{10}\Psi_{\bar{5}}\Psi_{\bar{5}_{H_d}} dz_1\wedge d\bar{z}_1\wedge dz_2\wedge d\bar{z}_2. \tag{40}$$

The top Yukawa coupling is realized at the intersection where the symmetry is enhanced to E_6, while the bottom and τ Yukawa couplings are associated with triple intersections of $SO(12)$ enhancements. We note in passing that the corresponding solution of the EoM providing the wave function for the up-type quark coupling is rather involved because of the monodromy and must be solved in a nontrivial background, where the notion of the T-brane is required [49]. Using appropriate background fluxes, we can break $\mathcal{E}_6$ to $SU(5)$, while the latter can break down to the SM gauge group with the hypercharge flux. To estimate the top Yukawa coupling, one has to perform the corresponding integration (40). Varying the various flux parameters involved in the corresponding wave functions, it is found that the top quark Yukawa takes values in the interval $h_t \sim [0.3–0.5]$, in agreement with previous computations [50,51], and hence the desired value $h_t \sim 0.35$ can be accommodated.[4]

In the present approach, the bottom and τ Yukawa couplings are formed at a different point of the compact space where the symmetry enhancement is $SO(12)$. Proceeding in analogy with the top Yukawa, one can adjust the flux breaking mechanism to achieve [50,51] the successive breaking to $SU(5)$ and $SU(3)\times SU(2)\times U(1)$. Further, for certain regions of the parameter space, one can obtain $h_{b,\tau}$ values in agreement with those predicted by the renormalization group evolution [52].

VI. DECAY OF VECTORLIKE TRIPLETS

While analyzing the spectrum in Sec. IV, we have seen that the existence of vectorlike triplets is a frequently

[4]Notice that the predicted values $g_U \sim 0.1$, $h_t \sim 0.3–0.5$ differ from those in the minimal MSSM unification scenario. In view of these modifications it would be interesting to reconsider the stability of the Higgs vacuum; such an analysis, however, goes beyond the objectives of the present work.

occurring phenomenon. They can be produced in pairs at LHC through their gauge couplings to gluons. However, such exotic particles are not yet observed and must decay through higher dimensional operators through mixing with the MSSM particles.

We start with the minimal model by setting $n = 0$, in which case the only states beyond the MSSM spectrum are D^c, D found in the $\bar{5}_{\bar{x}}$ and 5_{H_u}, respectively. We will consider the case of their mixing with the third family which enhances their decays, due to the large Yukawa coupling compared to the two lighter generations. The available Yukawa couplings which mix the down-type triplets are

$$\begin{aligned}\mathcal{W} \supset\ & \lambda 10_{t_1}\bar{5}_{t_1+t_4}\bar{5}_{t_3+t_5} + \lambda_1 10_{t_1}\bar{5}_{t_4+t_5}\bar{5}_{t_3+t_5}\theta_{15}/M_{GUT} \\ & + \lambda_2 \bar{5}_{t_4+t_5} 5_{-2t_1}\theta_{14}\theta_{15}/M_{GUT} \\ & \to \lambda 10_3\bar{5}_3\bar{5}_{H_d} + \lambda_1 10_3\bar{5}_{H_d}\bar{5}_{\bar{x}} v_2 + \lambda_2\bar{5}_{\bar{x}}5_{H_u} v_1 v_2, \end{aligned} \tag{41}$$

where the nonrenormalizable terms are scaled by the appropriate powers of the compactification scale or the GUT scale. These terms generate a mixing matrix of the third generation down quark and D^c, D, which can be cast in the form

$$\mathcal{L}_Y \supset (Q_3, D) M_D \begin{pmatrix} b^c \\ D^c \end{pmatrix}, \quad M_D \propto \begin{pmatrix} \frac{\lambda}{\sqrt{2}} v_d & \frac{\lambda_1}{\sqrt{2}} v_2 v_d \\ 0 & \lambda_2 v_1 v_2 \end{pmatrix},$$

where v_d stands for the down Higgs VEV scaled by the GUT scale. This nonsymmetric matrix M_D is diagonalized by utilizing the left and right unitary matrices

$$M_D^\delta = V_L^\dagger M_D V_R,$$

implying

$$M_D^{\delta\,2} = V_L^\dagger M_D M_D^\dagger V_L = V_R^\dagger M_D^\dagger M_D V_R,$$

where

$$M_D M_D^\dagger = \begin{pmatrix} \frac{1}{2}\lambda^2 v_d^2 + \frac{1}{2}\lambda_1^2 v_2^2 v_d^2 & \frac{\lambda_1\lambda_2}{\sqrt{2}} v_1 v_2^2 v_d \\ \frac{\lambda_1\lambda_2}{\sqrt{2}} v_1 v_2^2 v_d & \lambda_2^2 v_1^2 v_2^2 \end{pmatrix} \tag{42}$$

and

$$M_D^\dagger M_D = \begin{pmatrix} \frac{1}{2}\lambda^2 v_d^2 & \frac{1}{2}\lambda\lambda_1 v_2 v_d^2 \\ \frac{1}{2}\lambda\lambda_1 v_2 v_d^2 & \frac{1}{2}\lambda_1^2 v_2^2 v_d^2 + \lambda_2^2 v_1^2 v_2^2 \end{pmatrix}. \tag{43}$$

Following standard diagonalization procedures, in the limit $v_{1,2} \gg v_d$, we find that the left mixing angle is

$$\tan 2\theta_L = \frac{\sqrt{2}\lambda_1\lambda_2 v_1 v_2^2 v_d}{-\frac{1}{2}\lambda^2 v_d^2 - \frac{1}{2}\lambda_1^2 v_2^2 v_d^2 + \lambda_2^2 v_1^2 v_2^2} \approx \frac{\sqrt{2}\lambda_1 v_d}{\lambda_2 v_1}, \tag{44}$$

and for the right-handed mixing we obtain

$$\tan 2\theta_R = \frac{\lambda\lambda_1 v_2 v_d^2}{-\frac{1}{2}\lambda^2 v_d^2 + \frac{1}{2}\lambda_1^2 v_2^2 v_d^2 + \lambda_2^2 v_1^2 v_2^2} \approx \frac{\lambda\lambda_1 v_d^2}{\lambda_2^2 v_1^2 v_2}. \tag{45}$$

From these, we find

$$\tan(2\theta_R) \approx \frac{\lambda v_d}{\sqrt{2}\lambda_2 v_1 v_2}\tan(2\theta_L).$$

For the assumed hierarchy of VEVs we see that the left mixing prevails. The mixing is restricted by Cabibbo-Kobayashi-Maskawa matrix constraints and the contributions of the heavy triplets to the oblique parameters S, T, which have been measured with precision in LEP experiments (for detailed computations see [53]). A rough estimate would give the upper bounds $\tan 2\theta_L \sim 0.1, \tan 2\theta_R \sim 0.3$, which can be easily satisfied for the v_1, v_2 values used in this work.

A. Proton decay

In this model the dimension-five proton decay R-parity violating tree-level couplings of the form $10_f\bar{5}_f\bar{5}_f$ are absent due to the t_i charge assignments of matter fields. However, nonrenormalizable terms that could lead to suppressed baryon and lepton number violating processes may still appear. A class of these operators has the general structure

$$\lambda_{\rm eff} 10_i\bar{5}_{t_j+t_k}\bar{5}_{t_l+t_m},; \quad \lambda_{\rm eff} \sim \langle\theta_{pq}^n\rangle, \quad i, i, j, l, m \neq 5, \tag{46}$$

where θ_{pq}^n represents products of singlet fields required to cancel the nonvanishing combinations of $t_{i,j..}$ charges. Notice, however, that for the particular family assignment in this model none of $t_{i,j,k,l,m}$ in (46) is t_5 and therefore, to fulfill the condition $\sum_{k=1}^5 t_k = 0$ some singlet $\theta_{5s} \equiv 1_{t_5-t_s}$, with $s = 1, 2, 3, 4$, must always be involved.[5] In the present model no singlet of this kind acquires a nonzero VEV, namely $\langle\theta_{5s}\rangle \equiv 0$, and hence dimension-four operators are suppressed.

However, as already pointed out, additional Yukawa terms give rise to new tree-level graphs mediated by color triplets. Such graphs induce dimension-five operators of the form

[5] Notice however, that all possible higher order R-parity violating terms

$$\begin{aligned} & 10_1\bar{5}_1(\bar{5}_1\theta_{14}\theta_{53} + \bar{5}_2\theta_{53} + \bar{5}_3\theta_{54})\theta_{13} \\ & + 10_1\bar{5}_2(\bar{5}_2\theta_{43} + \bar{5}_3)\theta_{53} + 10_2\bar{5}_3\bar{5}_3\theta_{54} \end{aligned}$$

can be eliminated due to the R-parity assignment of the singlets θ_{ij} shown in Table II.

$\frac{1}{M_{\rm eff}}QQQ\ell$, $\frac{1}{M_{\rm eff}}u^c u^c d^c e^c$, where $M_{\rm eff}$ is an effective color triplet mass $M_{\rm eff} \geq M_{GUT} \sim 2.0 \times 10^{16}$ GeV [54–57]. Here, because of the missing triplet mechanism described in the previous section, the D, D^c triplets develop masses through mixing with other heavy triplets D_i, D_i' emerging from the decomposition of the additional $5 + \bar{5}$-pairs. Also, several couplings are realized as higher order nonrenormalizable terms so that, in practice, an effective triplet mass $M_{\rm eff}$ is involved which, with suitable conditions on the triplet mixing, could be of the order of the GUT scale. For the case of the Higgsino exchange diagram, for example, with a Higgsino mass identified with the supersymmetry breaking scale M_S, the proton lifetime is estimated to be [57]

$$\tau_p \approx 10^{35}(\sqrt{2}\sin 2\beta)^4 \left(\frac{0.1}{C_R}\right)^2 \left(\frac{M_S}{10^2\ \mathrm{TeV}}\right)^2 \left(\frac{M_{D_{\rm eff}}}{10^{16}\ \mathrm{GeV}}\right)^2, \tag{47}$$

where the coefficient $C_R \geq 0.1$, taking into account the renormalization group effects on the masses. From (47) we infer that with an effective triplet mass $\gtrsim M_{GUT}$ and a relatively high supersymmetry breaking scale, proton decay can be sufficiently suppressed in accordance with the Super-Kamiokande bound on the proton lifetime.

To estimate the effects of these operators in this model, we consider the triplet mass matrix derived in the previous section,

$$M_T = \begin{pmatrix} \lambda\theta_{14}\theta_{15} & \epsilon'\theta_{14}\theta_{15} \\ \epsilon\theta_{14} & \theta_{14} \end{pmatrix}\theta_{14} \to \begin{pmatrix} \lambda v_2 & \epsilon' v_2 \\ \epsilon & 1 \end{pmatrix}\langle\theta_{14}\rangle, \tag{48}$$

with $v_2 = \frac{\langle\theta_{15}\rangle}{M_{GUT}}$ and $v_1 = \frac{\langle\theta_{14}\rangle}{M_{GUT}}$ as defined in (14). As before, the left and right unitary matrices V_L, V_R, as well as the eigenmasses, are determined by $M_T^{\delta\,2} = V_L^\dagger M_T M_T^\dagger V_L = V_R^\dagger M_T^\dagger M_T V_R$, where, in general, $M_T M_T^\dagger$ and $M_T^\dagger M_T$ are Hermitian but, for simplicity, we will take to be symmetric, $M^2 \sim \left(\begin{smallmatrix} a & b \\ b & d \end{smallmatrix}\right)\langle\theta_{14}\rangle^2$, with real entries and triplet eigenmasses $M_{1,2}^2 = \frac{1}{2}(a + d \pm \sqrt{4b^2 + (a-d)^2})\langle\theta_{14}\rangle^2$.

In Fig. 4 a representative graph is shown mediated by the color triplets, leading to the dominant proton decay mode $p \to K^+\bar{\nu}$. The mass insertion (red bullet) in the graph is

$$\lambda\frac{\langle\theta_{14}\theta_{15}\rangle}{M_{GUT}} \equiv \langle\Phi\rangle.$$

After summing over the eigenstates, one finds that the effective mass involved is

$$\frac{1}{M_{\rm eff}^0} \propto \sum_j V_{1j}\frac{\langle\Phi\rangle}{M_j^2}V_{j2}^\dagger \to \left(\lambda\frac{v_2}{v_1}\frac{1}{M_{GUT}}\right)\frac{b}{ad - b^2}, \tag{49}$$

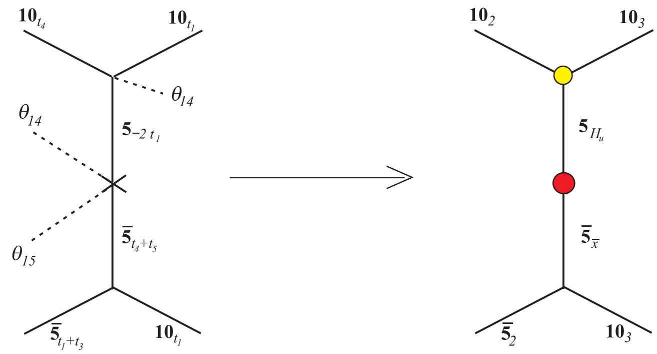

FIG. 4. Diagram leading to proton decay. Red and yellow circles represent the nonrenormalizable couplings discussed in the text.

while there is an additional suppression factor $v_1 = \langle\theta_{14}\rangle/M$ from the nonrenormalizable term (yellow bullet in the graph). Finally $\frac{1}{M_{\rm eff}} \sim \frac{v_1}{M_{\rm eff}^0}$.

For the V_L mixing, assuming reasonable values for the parameters $\epsilon, \epsilon' < 1$, while taking $v_1 \sim O(10^{-1})$ and $\lambda \sim 1$, we find

$$M_{\rm eff} \sim \frac{v_2}{v_1}M_{GUT}.$$

For the V_R case we find

$$M_{\rm eff} \sim \frac{M_{GUT}}{\epsilon}.$$

For a supersymmetry breaking scale M_S in the TeV region, we conclude that the lifetime of the proton is consistent with the experimental bounds for an effective mass $M_{\rm eff}$ a few times larger than M_{GUT} which can be satisfied for[6] $v_2 > v_1$ and $\epsilon < 1$.

There are implications for the μ term given by $\mu \sim v_1 v_2 v_3$ [see Eq. (17)]. Since v_1, v_2 cannot be small, in order to sufficiently suppress this term we must have $v_3 = \langle\theta_{43}\rangle/M_{GUT} \ll 1$. On the other hand, the smallness of v_3 suppresses also the mass scale of the lighter generations and might lead to inconsistences with the experimental values. We should recall, however, that there are significant contributions to the fermion masses from one-loop gluino exchange diagrams [58], implying masses of the order $m_{u/d} \propto \frac{a_3}{4\pi}A_q\frac{m_{\tilde{g}}m_{t/b}}{m_{\tilde{q}}^2}$ for the up/down quarks, where A_q, $m_{\tilde{g}}$, $m_{\tilde{q}}$ are, respectively, the trilinear parameter, the gaugino, and squark masses.

We have already stressed that the presence of additional vectorlike 5-plets in the model under consideration is

[6] Since the mass insertion $\langle\Phi\rangle 5_H \bar{5}_{\bar{x}} \propto v_2$ one would expect that for $v_2 \ll 1$ the contribution of the graph to proton decay would be small. However, the element b cancels the effect because it is also proportional to $b \propto v_2$.

compatible with a smaller value of the unified gauge coupling $g_U \sim 10^{-1}$ at the GUT scale. This has significant implications for the proton decay rate which occurs through the exchange of the gauge bosons (dimension-six operators). For the well-known case $p \to e^+\pi^0$, the lifetime is estimated to be [57,59,60]

$$\tau(p\to e^+\pi^0) \approx 8\times 10^{34}\ \text{years} \times \left(\frac{a_U^0}{a_U}\times\frac{2.5}{A_R}\times\frac{0.015\ \text{GeV}^3}{a_H}\right)^2 \times \left(\frac{M_V}{10^{16}\ \text{GeV}}\right)^4. \quad (50)$$

The various quantities in the above formula are as follows: $a_U^0 = \frac{1}{25}$ is assumed to be the value of the unified gauge coupling in the minimal $SU(5)$, while a_U stands for its value for the present model, which is taken to be $a_U \approx \frac{1}{8.5}$ (see Sec. V). The factor A_R takes into account the various renormalization effects, a_H is the hadronic matrix element, and M_V denotes the mass of the gauge boson mediating the process $p \to e^+\pi^0$. Comparing with the recent experimental limit [61] $\tau(p\to e^+\pi^0) \geq 1.6\times 10^{34}$ years, we find a lower bound on the mass of the gauge boson $M_V \geq 1.14\times 10^{16}$ GeV, which is exciting since it just below the GUT scale predicted in this model $M_U \approx 2.04\times 10^{16}$ GeV. Note, however, that by invoking discrete symmetries (such as the Z_{4R} of Refs. [62]), dimension-five operators contributing to proton decay are automatically removed.

B. Variation with new physics predictions accessible at LHC

In this section we consider the possibility of predicting new physics phenomena (such as diphoton events) from relatively light (~TeV) scalars and triplets. The model discussed so far cannot accommodate a process such as the diphoton event, since there is no direct coupling D^cDS with a light singlet S. Indeed, the only singlet coupled to D^c, D is θ_{14}, which acquires a large VEV and decouples. To circumvent this we briefly present a modification of the above model by assuming the following nonzero VEVs,

$$v_1 = \langle\theta_{13}\rangle, \qquad v_2 = \langle\theta_{34}\rangle, \qquad v_3 = \langle\theta_{43}\rangle, \quad (51)$$

and we maintain the same assignments for the fermion generations listed in Table II. The mass matrices for the up/down quarks and charged leptons are given by

$$m_u \sim \begin{pmatrix} v_1^2 & v_1^2 v_2 & v_1 \\ v_1^2 v_2 & v_1^2 v_2^2 & v_1 v_2 \\ v_1 & v_1 v_2 & 1 \end{pmatrix} h_t\langle H_u\rangle,$$

$$m_{d,\ell} \sim \begin{pmatrix} v_1^2 & v_1 v_3 & v_1 \\ v_1^2 v_2 & v_1 & v_1 v_2 \\ v_1 & v_3 & 1 \end{pmatrix} h_b\langle H_d\rangle, \quad (52)$$

where, as before, we have suppressed the Yukawa couplings expected to be of $\mathcal{O}(1)$. We observe that the matrices exhibit the expected hierarchical structure. Assuming a natural range of the VEVs and Yukawa couplings we estimate that the fermion mass patterns are consistent with the observed mass spectrum.

With this modification, the singlet θ_{14} is not required to acquire a large VEV and it can remain as a light singlet $\theta_{14} = S'$. Through its superpotential coupling

$$\theta_{14}\bar{5}_{\bar{x}}5_x \to S'(D''^c D + H'_u H'_d),$$

where D''^c stands for the linear combination $D''^c = \cos\phi D^c + \sin\phi D'^c$, S' could contribute to diphoton emission.

VII. SUMMARY

F-theory appears to be a natural and promising framework for constructing unified theories with predictive power. The $SU(5)$ GUT model in particular appears to be the most economic unified group containing all those necessary ingredients to accommodate vectorlike fermions that might show up in future experiments. Therefore, in light of possible new physics at the LHC experiments, in this paper, we reconsidered a class of F-theory $SU(5)$ models, aiming to concentrate on the specific predictions and low-energy implications.

In the F-theory framework, after the $SU(5)$ breaking down to the Standard Model gauge symmetry, we end up with the MSSM chiral mass spectrum, the Higgs doublet fields, and usually a number of vectorlike exotics, as well as neutral singlet fields. We point out that we dispense with the use of large Higgs representations for the $SU(5)$ symmetry breaking since the latter takes place by implementing the mechanism of the hypercharge flux. The corresponding $U(1)_Y$ gauge field remains massless by requiring the hypercharge flux to be globally trivial. As a result of these requirements, the spectrum of the effective theory and the additional Abelian symmetries accompanying the GUT group are subject to certain constraints. In addition to the $SU(5)$ GUT group, the model is subject to additional symmetry restrictions emanating from the perpendicular "spectral cover" $SU(5)_\perp$ group, which in the effective theory reduces down to Abelian factors according to the "breaking" chain

$$SU(5)_\perp \supset U(1)_\perp^4 \xrightarrow{Z_2} U(1)_\perp^3,$$

where Z_2 is the monodromy action, chosen for this particular class of models under discussion.[7] A suitable choice of fluxes along these additional Abelian factors is responsible

[7]For the $SU(5)_\perp$ spectral cover symmetry, the possible monodromies fall into a discrete subgroup of the Weyl group $W(SU(5)_\perp) \sim S_5$, with S_5 being the permutation symmetry of five objects.

for the chirality of the $SU(5)$ GUT representations and their propagation on the specific matter curves presented in this paper.

In practice, the effects of the remaining spectral cover symmetry in the low-energy effective theory are described by a few integers (associated with fluxes) and the charge roots t_i, $i = 1, 2, \ldots, 5$ of the spectral cover fifth-degree polynomial, where two of them, namely, $t_{1,2}$, are identified under the action of the monodromy Z_2: $t_1 \leftrightarrow t_2$ applied in this work.

The implementation of the hyperflux symmetry breaking mechanism has additional interesting effects. As is known, chiral matter and Higgs fields reside on the intersections [i.e., Riemann surfaces, dubbed here as matter curves and characterized by the remaining $U(1)$ factors through the charges t_i] of seven branes with those wrapping the $SU(5)$ singularity. In general the various intersections are characterized by distinct geometric properties and as a consequence flux restricts differently on each of them, while implying splittings of the $SU(5)$ representation content in certain cases. As a result, in the present model doublet Higgs fields are accommodated on matter curves which split the $SU(5)$ representations, realizing an effective doublet-triplet splitting mechanism in a natural manner. More precisely, this amounts to removing one triplet from the initial Higgs curve with the simultaneous appearance (excess) of another one on a different matter curve. This displacement, however, is enough to allow a light mass term for the Higgs doublets while heavy triplet-antitriplet mass terms originate from different terms, leading to suppression of baryon number violating processes. Chiral fermion generations are chosen to be accommodated on different matter curves, so that a Froggatt-Nielsen-type mechanism is implemented to generate the required hierarchy. Furthermore, certain Kaluza-Klein modes are associated with the right-handed neutrino fields, implementing the seesaw mechanism through appropriate mass terms with their left-handed counterparts.

The additional spectrum in the present model consists of neutral singlet fields as well as color triplets and Higgs-like doublets comprising complete $SU(5)$ vectorlike pairs in $5 + \bar{5}$ multiplets, characterized by nontrivial t_i charges. Some singlet fields are allowed to acquire VEVs at the TeV scale, inducing masses of the same order for the vectorlike exotics through the superpotential terms. Such light exotics contribute to the formation of resonances, producing an excess of diphoton events which could be discovered in future LHC experiments. A Renormalization Group Equation analysis shows that the resulting spectrum is consistent with gauge coupling unification and the predictions of the third family Yukawa couplings.

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[1] C. Beasley, J. J. Heckman, and C. Vafa, GUTs and exceptional branes in F-theory—I, J. High Energy Phys. 01 (2009) 058.
[2] T. Weigand, Lectures on F-theory compactifications and model building, Classical Quantum Gravity **27**, 214004 (2010).
[3] J. J. Heckman, Particle physics implications of F-theory, Annu. Rev. Nucl. Part. Sci. **60**, 237 (2010).
[4] G. K. Leontaris, Aspects of F-theory GUTs, Proc. Sci., CORFU 2011 (**2011**) 095 [arXiv:1203.6277].
[5] A. Maharana and E. Palti, Models of particle physics from type IIB string theory and F-theory: A review, Int. J. Mod. Phys. A **28**, 1330005 (2013).
[6] R. Donagi and M. Wijnholt, Higgs bundles and UV completion in F-theory, Commun. Math. Phys. **326**, 287 (2014).
[7] V. Bouchard, J. J. Heckman, J. Seo, and C. Vafa, F-theory and neutrinos: Kaluza-Klein dilution of flavor hierarchy, J. High Energy Phys. 01 (2010) 061.
[8] J. Marsano, N. Saulina, and S. Schafer-Nameki, Monodromies, fluxes, and compact three-generation F-theory GUTs, J. High Energy Phys. 08 (2009) 046.
[9] R. Blumenhagen, T. W. Grimm, B. Jurke, and T. Weigand, Global F-theory GUTs, Nucl. Phys. **B829**, 325 (2010).
[10] E. Dudas and E. Palti, On hypercharge flux and exotics in F-theory GUTs, J. High Energy Phys. 09 (2010) 013.
[11] I. Antoniadis and G. K. Leontaris, Building SO(10) models from F-theory, J. High Energy Phys. 08 (2012) 001.
[12] G. K. Leontaris and G. G. Ross, Yukawa couplings and fermion mass structure in F-theory GUTs, J. High Energy Phys. 02 (2011) 108.
[13] K. S. Choi and J. E. Kim, Supersymmetric three family chiral SU(6) grand unification model from F-theory, Phys. Rev. D **83**, 065016 (2011).
[14] J. Borchmann, C. Mayrhofer, E. Palti, and T. Weigand, Elliptic fibrations for $SU(5) \times U(1) \times U(1)$ F-theory vacua, Phys. Rev. D **88**, 046005 (2013).
[15] M. Cvetič, A. Grassi, D. Klevers, and H. Piragua, Chiral Four-Dimensional F-theory Compactifications With SU(5) and multiple U(1)-factors, J. High Energy Phys. 04 (2014) 010.
[16] J. Borchmann, C. Mayrhofer, E. Palti, and T. Weigand, SU(5) tops with multiple U(1)s in F-theory, Nucl. Phys. **B882**, 1 (2014).

[17] S. Krippendorf, D. K. M. Pena, P. K. Oehlmann, and F. Ruehle, Rational F-theory GUTs without exotics, J. High Energy Phys. 07 (2014) 013.
[18] M. Cvetič, R. Donagi, D. Klevers, H. Piragua, and M. Poretschkin, F-theory vacua with $\mathbb{Z}_3$ gauge symmetry, Nucl. Phys. **B898**, 736 (2015).
[19] L. Lin and T. Weigand, G 4-flux and standard model vacua in F-theory, Nucl. Phys. **B913**, 209 (2016).
[20] C. Vafa, Evidence for F theory, Nucl. Phys. **B469**, 403 (1996).
[21] D. R. Morrison and C. Vafa, Compactifications of F theory on Calabi-Yau threefolds. 1, Nucl. Phys. **B473**, 74 (1996).
[22] F. Baume, E. Palti, and S. Schwieger, On E_8 and F-theory GUTs, J. High Energy Phys. 06 (2015) 039.
[23] J. Jiang, T. Li, D. V. Nanopoulos, and D. Xie, $F - su(5)$, Phys. Lett. B **677**, 322 (2009).
[24] J. Jiang, T. Li, D. V. Nanopoulos, and D. Xie, Flipped $SU(5)xU(1)_X$ models from F-theory, Nucl. Phys. **B830**, 195 (2010).
[25] G. K. Leontaris and Q. Shafi, Diphoton resonance in F-theory inspired flipped SO(10), Eur. Phys. J. C **76**, 574 (2016).
[26] J. C. Callaghan, S. F. King, G. K. Leontaris, and G. G. Ross, Towards a realistic F-theory GUT, J. High Energy Phys. 04 (2012) 094.
[27] J. C. Callaghan and S. F. King, E_6 models from F-theory, J. High Energy Phys. 04 (2013) 034.
[28] J. C. Callaghan, S. F. King, and G. K. Leontaris, Gauge coupling unification in E_6 F-theory GUTs with matter and bulk exotics from flux breaking, J. High Energy Phys. 12 (2013) 037.
[29] V. Braun and D. R. Morrison, F-theory on genus-one fibrations, J. High Energy Phys. 08 (2014) 132.
[30] L. Lin, C. Mayrhofer, O. Till, and T. Weigand, Fluxes in F-theory compactifications on genus-one fibrations, J. High Energy Phys. 01 (2016) 098.
[31] L. B. Anderson, I. Garcia-Etxebarria, T. W. Grimm, and J. Keitel, Physics of F-theory compactifications without section, J. High Energy Phys. 12 (2014) 156.
[32] D. Klevers, D. K. M. Pena, P. K. Oehlmann, H. Piragua, and J. Reuter, F-theory on all toric hypersurface fibrations and its Higgs branches, J. High Energy Phys. 01 (2015) 142.
[33] M. Cvetic, R. Donagi, D. Klevers, H. Piragua, and M. Poretschkin, F-theory vacua with $\mathbb{Z}_3$ gauge symmetry, Nucl. Phys. **B898**, 736 (2015).
[34] M. Cvetic, A. Grassi, and M. Poretschkin, Discrete symmetries in heterotic/F-theory duality and mirror symmetry, J. High Energy Phys. 06 (2017) 156.
[35] Y. Kimura, Discrete gauge groups in F-theory models on genus-one fibered Calabi-Yau 4-folds without section, J. High Energy Phys. 04 (2017) 168.
[36] A. Karozas, S. F. King, G. K. Leontaris, and A. Meadowcroft, Discrete family symmetry from F-theory GUTs, J. High Energy Phys. 09 (2014) 107.
[37] M. C. Romão, A. Karozas, S. F. King, G. K. Leontaris, and A. K. Meadowcroft, MSSM from F-theory SU(5) with Klein monodromy, Phys. Rev. D **93**, 126007 (2016).
[38] T. W. B. Kibble, G. Lazarides, and Q. Shafi, Strings in SO (10), Phys. Lett. **113B**, 237 (1982).
[39] P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Vacuum configurations for superstrings, Nucl. Phys. **B258**, 46 (1985).
[40] G. Lazarides, P. K. Mohapatra, C. Panagiotakopoulos, and Q. Shafi, Phenomenology with a three generation superstring model, Nucl. Phys. **B323**, 614 (1989); N. Ganoulis, G. Lazarides, and Q. Shafi, Classification of three generation superstring models according to their discrete symmetries, Nucl. Phys. **B323**, 374 (1989).
[41] J. E. Kim, B. Kyae, and S. Nam, The anomalous U(1) global symmetry and flavors from an $\mathrm{SU}(5) \times \mathrm{SU}(5)'$ GUT in Z_{12-I} orbifold compactification, arXiv:1703.05345.
[42] R. Donagi and M. Wijnholt, Breaking GUT groups in F-theory, Adv. Theor. Math. Phys. **15**, 1523 (2011).
[43] G. K. Leontaris and N. D. Vlachos, On the GUT scale of F-theory SU(5), Phys. Lett. B **704**, 620 (2011).
[44] A. Hebecker and J. Unwin, Precision unification and proton decay in F-theory GUTs with high scale supersymmetry, J. High Energy Phys. 09 (2014) 125.
[45] R. Blumenhagen, Gauge Coupling Unification in F-Theory Grand Unified Theories, Phys. Rev. Lett. **102**, 071601 (2009).
[46] G. K. Leontaris and N. D. Tracas, Gauge coupling flux thresholds, exotic matter and the unification scale in F-SU(5) GUT, Eur. Phys. J. C **67**, 489 (2010).
[47] R. Blumenhagen, V. Braun, T. W. Grimm, and T. Weigand, GUTs in type IIB orientifold compactifications, Nucl. Phys. **B815**, 1 (2009).
[48] A. Hebbar, G. K. Leontaris, and Q. Shafi, Masses of Third Family Vector-like Quarks and Leptons in Yukawa-unified E_6, Phys. Rev. D **93**, 111701 (2016).
[49] S. Cecotti, C. Cordova, J. J. Heckman, and C. Vafa, T-branes and monodromy, J. High Energy Phys. 07 (2011) 030.
[50] A. Font, F. Marchesano, D. Regalado, and G. Zoccarato, Up-type quark masses in SU(5) F-theory models, J. High Energy Phys. 11 (2013) 125.
[51] A. Font, Yukawa couplings in string theory: The case for F-theory GUT's, J. Phys. Conf. Ser. **651**, 012009 (2015).
[52] M. C. Romão, A. Karozas, S. F. King, G. K. Leontaris, and A. K. Meadowcroft, R-parity violation in F-theory, J. High Energy Phys. 11 (2016) 081.
[53] J. A. Aguilar-Saavedra, R. Benbrik, S. Heinemeyer, and M. Perez-Victoria, Handbook of vectorlike quarks: Mixing and single production, Phys. Rev. D **88**, 094010 (2013).
[54] T. Goto and T. Nihei, Effect of RRRR dimension five operator on the proton decay in the minimal SU(5) SUGRA GUT model, Phys. Rev. D **59**, 115009 (1999).
[55] M. U. Rehman, Q. Shafi, and J. R. Wickman, Minimal Supersymmetric Hybrid Inflation, Flipped SU(5) and proton decay, Phys. Lett. B **688**, 75 (2010).
[56] R. Dermisek, A. Mafi, and S. Raby, SUSY GUTs under siege: Proton decay, Phys. Rev. D **63**, 035001 (2001).
[57] J. Hisano, D. Kobayashi, T. Kuwahara, and N. Nagata, Decoupling Can Revive Minimal Supersymmetric SU(5), J. High Energy Phys. 07 (2013) 038.
[58] T. Banks, Supersymmetry and the Quark Mass Matrix, Nucl. Phys. **B303**, 172 (1988).
[59] H. Murayama and A. Pierce, Not even decoupling can save minimal supersymmetric SU(5), Phys. Rev. D **65**, 055009 (2002).
[60] B. Dutta, Y. Gao, T. Ghosh, I. Gogoladze, T. Li, Q. Shafi, and J. W. Walker, Diphoton Excess in Consistent

Supersymmetric SU(5) models with vector-like particles, arXiv:1601.00866.

[61] M. Miura (Super-Kamiokande Collaboration), Search for proton decay via $p \to e^+\pi^0$ and $p \to \mu^+\pi^0$ in 0.31 megaton $\cdot$ years exposure of the Super-Kamiokande Water Cherenkov Detector, Phys. Rev. D **95**, 114505 (2017).

[62] H. M. Lee, S. Raby, M. Ratz, G. G. Ross, R. Schieren, K. Schmidt-Hoberg, and P. K. S. Vaudrevange, A unique $\mathbb{Z}_4^R$ symmetry for the MSSM, Phys. Lett. B **694**, 491 (2011); J. E. Kim, Abelian discrete symmetries Z_N and Z_{nR} from string orbifolds, Phys. Lett. B **726**, 450 (2013).