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Supersymmetry of Bianchi attractors in gauged supergravity

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Bianchi attractors are near horizon geometries with homogeneous symmetries in spatial directions. We construct supersymmetric Bianchi attractors in $\mathcal{N} = 2, d = 4$, 5 gauged supergravity. In $d = 4$, we consider gauged supergravity coupled to vector and hypermultiplets. In $d = 5$, we consider gauged supergravity coupled to vector multiplets with a generic gauging of symmetries of the scalar manifold and the $U(1)_R$ gauging of the R-symmetry. Analyzing the gaugino conditions, we show that when the fermionic shifts do not vanish, there are no supersymmetric Bianchi attractors. This is analogous to the known condition that for maximally supersymmetric solutions, all of the fermionic shifts must vanish. When the central charge satisfies an extremization condition, some of the fermionic shifts vanish and supersymmetry requires that the symmetries of the scalar manifold are not gauged. This allows supersymmetric Bianchi attractors sourced by massless gauge fields and a cosmological constant. In five dimensions in the Bianchi I class, we show that the anisotropic $\mathrm{AdS}_3 \times \mathbb{R}^2$ solution is $1/2$ BPS (Bogomol'nyi-Prasad-Sommerfield). We also construct a new class of $1/2$ BPS Bianchi III geometries labeled by the central charge. When the central charge takes a special value, the Bianchi III geometry reduces to the known $\mathrm{AdS}_3 \times \mathbb{H}^2$ solution. For the Bianchi V and VII classes, the radial spinor breaks all of supersymmetry. We briefly discuss the conditions for possible massive supersymmetric Bianchi solutions by generalizing the matter content to include tensor, hypermultiplets, and a generic gauging on the R-symmetry.

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I. INTRODUCTION

In recent years, intensive research on extremal black holes in anti–de Sitter (AdS) space have unveiled relations between seemingly unrelated fields such as gravity and condensed matter systems. In AdS/CFT, extremal black holes provide the bulk gravitational description of zero temperature ground states in strongly coupled field theories [1]. At the quantum critical point, the field theory description is strongly coupled and exhibits phase transitions at zero temperature due to quantum fluctuations [2,3]. The presence of diverse phases in the field theory predict an equally large number of dual extremal geometries in the bulk. It is an interesting program to identify and classify various possible extremal geometries. Some of the earlier work in this direction have identified extremal geometries that exhibit Lifshitz and hyperscaling violations [4–9]. Of more recent research interest are extremal black branes dual to field theories with reduced symmetries [10–21]. Some of these examples are anisotropic and display interesting phenomena such as violation of the KSS bound [22] when the anisotropy becomes much larger than the temperature [23].

In five dimensions, homogeneous anisotropic extremal black brane geometries have been constructed in [13,14]. The metrics display manifest homogeneous symmetries in three spatial directions. It is well known that the Killing vectors that generate these symmetries form algebras that are isomorphic to real Lie algebras in dimension three. These real Lie algebras have been well studied and are well-known through the Bianchi classification [24,25]. The five dimensional geometries that display manifest homogeneous symmetries in three spatial directions are referred to as the "Bianchi attractors." These near horizon geometries are exact solutions to Einstein-Maxwell theories with massive/massless gauge fields and a cosmological constant.[1]

Black holes in $\mathcal{N} = 2$ supergravity exhibit a phenomenon known as the attractor mechanism [27–30]. In a black hole background, moduli fields flow to fixed point values at the horizon irrespective of their asymptotic values at spatial infinity. The fixed point values are determined entirely in terms of the charges carried by the black hole. As a result, the Bekenstein-Hawking entropy of the black hole is determined in terms of its charges. Although initial studies have

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[1]The terminology attractor is used because the horizon geometries solve the field equation exactly. Interpolating numerical solutions have been constructed in [26], justifying the terminology. However, analytic solutions are much harder to find.

focused on supersymmetric black holes, it has been realized that the attractor mechanism is a consequence of extremality [31]. Subsequently the attractor mechanism is generalized to nonsupersymmetric extremal black holes [32,33].

In recent years, an enormous effort has gone into generalizing the attractor mechanism to gauged supergravities [34–41]. Significant progress has been made especially for dyonic AdS_4 black holes [42]. Large N index computations in the dual twisted mass deformed ABJM theory find perfect matching of the microstate counting with the Bekenstein-Hawking entropy of the black hole [43,44]. It is interesting to ask if the attractor mechanism generalizes to black brane geometries in AdS. In this light, the first step is to embed these geometries in supergravity in order to study their properties such as supersymmetry and stability.

Some steps in this direction have been taken [35,45] and explicit examples of Bianchi attractors in $\mathcal{N} = 2$ gauged supergravity are constructed. However, it turns out that the geometries are nonsupersymmetric and are unstable under linearized fluctuations unless certain conditions are satisfied [46,47]. The conditions are such that there must exist a critical point of the effective potential, and the Hessian of the effective potential evaluated at the solution must have positive eigenvalues. For nonsupersymmetric extremal black hole solutions, the above two conditions are sufficient to guarantee a stable Bianchi attractor in gauged supergravity. However supersymmetric solutions always satisfy these conditions and guarantee stability.

In this work, we look for supersymmetric Bianchi attractor geometries in $\mathcal{N} = 2$ gauged supergravity. As a warmup, we study $d = 4$ gauged supergravity coupled to vector and hyper multiplets with a generic gauging of the symmetries of the hyper Kähler manifold. In four dimensions the homogeneous symmetries are along the two spatial directions, and the corresponding Lie algebras are of two types, namely Bianchi I and Bianchi II. Bianchi I geometries, such as AdS_5 [41] and $z = 2$ Lifshitz solution [48,49], are well-known solutions in this theory. In the Bianchi I case, we construct a $\frac{1}{4}$ BPS $AdS_2 \times \mathbb{R}^2$ geometry sourced by timelike gauge fields. In the Bianchi II case, a $\frac{1}{8}$ BPS $AdS_2 \times \mathbb{H}^2$ solution sourced by magnetic fields has been found recently in [50]. We construct a $AdS_2 \times \mathbb{H}^2$ solution sourced by timelike gauge fields and find that the radial spinor breaks all of the supersymmetry. The Bianchi I and Bianchi II classes we studied in four dimensions correspond to the symmetries of $\mathbb{R}^2$ and $\mathbb{H}^2$. These are the only possible Bianchi classes of metric that one can construct in $3 + 1$ dimensions with homogeneous symmetries in two spatial directions. Of course, there exist more general manifolds like T^2 [34,51], however they do not belong to the Bianchi class, and we do not consider them in our analysis.

In $d = 5$, there exist a richer class of Bianchi attractor geometries. We consider the $\mathcal{N} = 2$ gauged supergravity coupled to vector multiplets with a generic gauging of both symmetries of the very special manifold and the $U(1)_R$ subgroup of the $SU(2)_R$ symmetry group. From the gaugino conditions, we find that there are no supersymmetric Bianchi attractors when the fermionic shifts in the supersymmetry variations are nonvanishing.[2] This is in the same spirit as the general analysis for maximally supersymmetric solutions [41,52]. This result holds for a generic gauging of the scalar manifold, is dependent on the choice of the gauge field configuration that sources the solution, and is independent of the functional form of the Killing spinor. The basic argument is that the constant part of the Killing spinor should be a simultaneous eigenspinor of commuting matrices that can appear in the gaugino conditions. We find that for the known gauge field configurations that generate Bianchi type solutions, this does not happen in general. Independently, we have checked that a radial Killing spinor breaks supersymmetry.

When the central charge Z of the solution satisfies an extremization condition, some of the fermionic shifts in the gaugino variations vanish. This is a reasonable condition to impose for any plausible geometry that can be an attractor solution. Given this condition, supersymmetry invariance then requires that the effective mass term vanish at the attractor point.[3] This condition allows Bianchi attractor solutions sourced by massless gauge fields since at the attractor point, the "effective mass terms" in gauged supergravity are proportional to g^2. There are no further conditions from the gaugino variations and hence the supersymmetry of the solutions are entirely determined by the Killing spinor equation that follows from the gravitino variation. It is crucial to observe that the Killing spinor equation depends only on the gauge coupling constant of the R-symmetry gauging, hence it follows that the Killing spinor integrability conditions (see Eq. 31 of [35]) do not depend on the gauging of the scalar manifold.

We construct Bianchi solutions sourced by massless gauge fields and a cosmological constant in the Bianchi I, Bianchi III, Bianchi V, and Bianchi VII classes. In the Bianchi I case, we find the anisotropic $AdS_3 \times \mathbb{R}^2$ geometry recently studied in [23,53] to be $1/2$ BPS. We also construct a supersymmetric class of $1/2$ BPS Bianchi III geometries labeled by the central charge Z. When the central charge of the solution takes special values, the geometry reduces to the known $AdS_3 \times \mathbb{H}^2$ [54]. The Killing spinors in both of these cases come in pairs where one spinor is purely radial and the other spinor depends on both radial and transverse coordinates other than $\mathbb{R}^2/\mathbb{H}^2$ directions. Moreover, the constant part of the spinors are eigenspinors of the radial Dirac matrix in all of the above

[2]In gauged supergravity literature, the supersymmetry variations in the gaugino and hyperino that are proportional to the gauge coupling constant are referred to as fermionic shifts.

[3]One way to possibly avoid this is to consider tensor multiplets. We comment on this in Sec. IV C.

cases. For the Bianchi V and Bianchi VII classes, we find that the radial spinor breaks all supersymmetry. These are the main new results of this paper.

Finally, the presence of hyper and tensor multiplets can allow for some massive Bianchi attractor solutions in some special cases. In particular, our results from the gaugino and Killing spinor conditions for the nonsupersymmetric cases continues to hold even after including hypermultiplets and $SU(2)_R$ gauging as the Killing spinor equation is not affected seriously by this addition. However, addition of tensor multiplets will affect the analysis and depend crucially on the tensor field configuration in addition to new gaugino and hyperino conditions. We comment on the possibilities in Sec. IV C, leaving a detailed analysis for future work.

The paper is organized as follows. In Sec. II, we briefly describe homogeneous symmetries and motivate Bianchi attractors. Following this, we present the analysis for the $d = 4$ Bianchi attractors in $\mathcal{N} = 2$ gauged supergravity in Sec. III. We move on to the five-dimensional case in Sec. IV. In subsection Sec. IV A we present our main argument for the absence of massive supersymmetric Bianchi attractors in gauged supergravity with $U(1)_R$ gauging and gauging of the symmetries of the very special manifold. Subsequently we analyze the Killing spinor equations for the massless cases in Sec. IV B. In Sec. IV C, we comment on the possible generalizations and necessary conditions when hyper and tensor multiplets are included with generic gauging. We present our conclusions and summarize in Sec. V. In Appendix Sec. A, we provide useful supplementary material on spinors in $d = 4$, 5 and summarize our conventions.

II. HOMOGENEOUS SYMMETRIES AND BIANCHI ATTRACTORS

In this section, we describe the homogeneous symmetries in two and three dimensions classified by the Bianchi classification of Lie algebras. Towards the end we describe the "Bianchi attractors." These are proposed near horizon geometries of extremal black branes with homogeneous symmetries in the spatial directions [13]. Consider a manifold M endowed with a metric $g_{\mu\nu}$ that is invariant under a given set of isometries. The Killing vectors X_i that generate the isometries close to form an algebra

$$[X_i, X_j] = C_{ij}{}^k X_k, \tag{1}$$

where C^k_{ij} are structure constants and they obey the usual Jacobi identity. The symmetry group of the manifold is isomorphic to an abstract Lie group G, whose Lie algebra is generated by the algebra of Killing vectors.

A homogeneous manifold has identical metric properties at all points in space. Any two points on a homogeneous space are connected by a symmetry transformation. The symmetry group of a homogeneous space of dimension d is isomorphic to the group corresponding to d dimensional real Lie algebra [24,25]. On the other hand, given the real Lie algebra in a dimension d, it is possible to write the corresponding metric with manifest homogeneous symmetries as follows. First, one finds a basis of invariant vectors e_i that commute with the Killing vectors X_i

$$[X_i, e_i] = 0, \tag{2}$$

then the metric with homogeneous symmetries can be expressed in terms of one forms ω^i dual to the invariant vectors e_i as

$$ds^2 = g_{ij}\omega^i \otimes \omega^j, \tag{3}$$

where g_{ij} are constants. The invariant one forms satisfy the relation

$$d\omega^k = \frac{1}{2} C_{ij}{}^k \omega^i \wedge \omega^j, \tag{4}$$

where $C_{ij}{}^k$ are the same structure constants that appear in the algebra of the Killing vectors. The real Lie algebras of dimension three fall into nine classes and are given by the well known Bianchi classification. The structure constants and invariant one forms are listed in detail in [25] (or see Appendix A of [13]).

In this work, we investigate the supersymmetry conditions on various Bianchi attractor geometries described in [13]. The geometries have the general structure

$$ds^2 = -g_{tt}(r)dt^2 + g_{ij}(r)d\omega^i \wedge d\omega^j + dr^2, \tag{5}$$

where $i = 1, 2$ in case of four dimensions and the ω^i are invariant one forms corresponding to the homogeneous symmetries of two dimensional real Lie algebras described above. In five dimensions $i = 1, 2, 3$, and the corresponding ω^i are invariant one forms given by the usual Bianchi classification. The functions $g_{tt}(r)$ and $g_{ij}(r)$ have a general form $e^{\beta r}$, where β are positive exponents. These metrics can be constructed as solutions to Einstein-Maxwell theories with massive/massless gauge fields and a cosmological constant. As long as the matter stress-tensor preserves the symmetries of the metric, explicit solutions can be constructed for a wide range of parameters of the theory of interest.

III. BIANCHI ATTRACTORS IN $\mathcal{N} = 2$, $d = 4$ GAUGED SUPERGRAVITY

In this section, we describe $\mathcal{N} = 2, d = 4$ gauged supergravity with n_V vector and n_H hyper multiplets. We use the notations and conventions of [48,55], the relevant conventions are summarized in Appendix A 1. The gravity multiplet consists of a metric $g_{\mu\nu}$, a graviphoton A^0_μ and an

SU(2) doublet of gravitinos $(\psi_\mu^A, \psi_{\mu A})$ of opposite chirality, where $A = 1, 2$ is an SU(2) index. The vector multiplet consists of a complex scalar z^i, a vector A_μ^i, where $i = 1, 2\ldots n_V$ and an SU(2) doublet of gauginos $(\lambda^{iA}, \lambda_A^{\bar{i}})$ with opposite chirality. The hyper multiplets contain scalars q^X, where $X = 1, \ldots 4n_H$ and two hyperinos $(\zeta_\alpha, \zeta^\alpha)$, $(\alpha = 1\ldots 2n_H)$ of opposite chirality. The bosonic part of the Lagrangian of the $\mathcal{N} = 2$ theory takes the form

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}R + g_{i\bar{j}}D^\mu z^i D_\mu \bar{z}^{\bar{j}} + g_{XY}D_\mu q^X D^\mu q^Y \\ &+ i(\bar{N}_{\Lambda\Sigma}\mathcal{F}_{\mu\nu}^{-\Lambda}\mathcal{F}^{-\Sigma\mu\nu} - N_{\Lambda\Sigma}\mathcal{F}_{\mu\nu}^{+\Lambda}\mathcal{F}^{+\Sigma\mu\nu}) \\ &+ \mathcal{V}(z, \bar{z}, q), \end{aligned} \tag{6}$$

where $N_{\Lambda\Sigma}$ are the period matrices.[4] The self/antiself dual field strengths are defined as

$$\mathcal{F}_{\mu\nu}^{\pm\Lambda} = \frac{1}{2}\left(F_{\mu\nu}^\Lambda \pm \frac{i}{2}\epsilon_{\mu\nu\rho\sigma}F^{\Lambda\rho\sigma}\right), \tag{7}$$

where the usual field strength is defined as $F_{\mu\nu}^\Lambda = \frac{1}{2}(\partial_\mu A_\nu^\Lambda - \partial_\nu A_\mu^\Lambda)$. The gauge covariant derivatives are defined as

$$\begin{aligned} D_\mu z^i &= \partial_\mu z^i + K_\Lambda^i A_\mu^\Lambda \\ D_\mu q^X &= \partial_\mu q^X + A_\mu^\Lambda K_\Lambda^X(q), \end{aligned} \tag{8}$$

where K_Λ^i are Killing vectors that gauge the symmetries of the Kähler manifold, and K_Λ^X are Killing vectors that gauge the symmetries of the Quaternionic Kähler manifold. Gauging introduces a potential that is given by

$$\begin{aligned}\mathcal{V}(z, \bar{z}, q) = &((g_{i\bar{j}}K_\Lambda^i K_\Sigma^{\bar{j}} + 4g_{XY}K_\Lambda^X K_\Sigma^Y)\bar{L}^\Lambda L^\Sigma \\ &+ (g^{i\bar{j}}f_i^\Lambda f_{\bar{j}}^\Sigma - 3\bar{L}^\Lambda L^\Sigma)P_\Lambda^x P_\Sigma^x), \end{aligned} \tag{9}$$

where $f_i^\Lambda = (\partial_i + \frac{1}{2}\partial_i\mathcal{K})L^\Lambda$. Here $\mathcal{K}$ is the Kähler potential. The triplet P_Λ^x, $x = 1, 2, 3$ are real Killing prepotentials on the quaternionic Kähler manifold. The supersymmetry transformations of the fermionic fields are given by

$$\begin{aligned} \delta\psi_{\mu A} &= D_\mu\epsilon_A + iS_{AB}\gamma_\mu\epsilon^B + 2i(\mathrm{Im}N)_{\Lambda\Sigma}L^\Sigma\mathcal{F}_{\mu\nu}^{-\Lambda}\gamma^\nu\epsilon_{AB}\epsilon^B \\ \delta\lambda^{iA} &= iD_\mu z^i\gamma^\mu\epsilon^A - g^{i\bar{j}}\bar{f}_{\bar{j}}^\Sigma(\mathrm{Im}N)_{\Lambda\Sigma}\mathcal{F}_{\mu\nu}^{-\Lambda}\gamma^{\mu\nu}\epsilon^{AB}\epsilon_B + W^{iAB}\epsilon_B \\ \delta\zeta_\alpha &= i\mathcal{U}_X^{B\beta}D_\mu q^X\gamma^\mu\epsilon^A\epsilon_{AB}\epsilon_{\alpha\beta} + N_\alpha^A\epsilon_A, \end{aligned} \tag{10}$$

where

$$\begin{aligned} S_{AB} &= \frac{i}{2}(\sigma^r)_A{}^C\epsilon_{BC}P_\Lambda^r L^\Lambda \\ W^{iAB} &= \epsilon^{AB}k_\Lambda^i\bar{L}^\Lambda + i(\sigma_r)_C^B\epsilon^{CA}P_\Lambda^r g^{i\bar{j}}f_{\bar{j}}^\Lambda \\ N_\alpha^A &= 2\mathcal{U}_{\alpha X}^A K_\Lambda^X\bar{L}^\Lambda. \end{aligned} \tag{11}$$

In the above, $\mathcal{U}_{\alpha X}^A$ are vielbeins on the quaternionic manifold. The covariant derivative on the spinor ϵ_A is defined as

$$D_\mu\epsilon_A = \nabla_\mu\epsilon_A + \frac{i}{2}(\sigma^r)_A^B A_\mu^\Lambda P_\Lambda^r\epsilon_B, \tag{12}$$

where ∇_μ is the covariant derivative defined with respect to the usual spin connection. For the rest of the discussion, we assume a generic gauging of the symmetries of hypermultiplet manifold.

At the attractor point the scalars are independent of spacetime coordinates,

$$z^i = \text{const}, \qquad q^X = \text{const}. \tag{13}$$

The supersymmetry variations (10) at the attractor point then reduce to

$$\begin{aligned} \delta\psi_{\mu A} &= D_\mu\epsilon_A + iS_{AB}\gamma_\mu\epsilon^B + 2i(\mathrm{Im}N)_{\Lambda\Sigma}L^\Sigma\mathcal{F}_{\mu\nu}^{-\Lambda}\gamma^\nu\epsilon_{AB}\epsilon^B \\ \delta\lambda^{iA} &= -g^{i\bar{j}}\bar{f}_{\bar{j}}^\Sigma(\mathrm{Im}N)_{\Lambda\Sigma}\mathcal{F}_{\mu\nu}^{-\Lambda}\gamma^{\mu\nu}\epsilon^{AB}\epsilon_B + W^{iAB}\epsilon_B \\ \delta\zeta_\alpha &= i\mathcal{U}_X^{B\beta}K_\Lambda^X A_\mu^\Lambda\gamma^\mu\epsilon^A\epsilon_{AB}\epsilon_{\alpha\beta} + N_\alpha^A\epsilon_A. \end{aligned} \tag{14}$$

Setting the gravitino variations to zero, we get the Killing spinor equation

$$\begin{aligned} &\partial_\mu\epsilon_A + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\epsilon_A + \frac{i}{2}(\sigma_x)_A^B P_\Lambda^x A_\mu^\Lambda\epsilon_B + iS_{AB}\gamma_\mu\epsilon^B \\ &\quad + 2i(\mathrm{Im}N)_{\Lambda\Sigma}L^\Sigma\mathcal{F}_{\mu\nu}^{-\Lambda}\gamma^\nu\epsilon_{AB}\epsilon^B = 0. \end{aligned} \tag{15}$$

In the rest of the section, we evaluate the Killing spinor equation (15), the gaugino and hyperino equations on the background of Bianchi geometries and derive the conditions for supersymmetry.

A. Bianchi I

Metrics with Bianchi I symmetry in the spatial directions have been studied in the gauged supergravity literature, the simplest of them being the supersymmetric AdS_4 solution [41]. A supersymmetric Lifshitz solution with exponent $z = 2$ has also been constructed earlier in gauged supergravity by [48,49]. In this section, following the analysis of [48], we present the supersymmetry conditions for a simple Bianchi I type—$AdS_2 \times \mathbb{R}^2$ solution.[5]

[4]These are functions of z^i and can be expressed in terms of the sections $M_\Lambda = N_{\Lambda\Sigma}L^\Sigma$.

[5]Magnetic $AdS_2 \times \mathbb{R}^2$ solutions and their stability have been well explored in the literature (see for instance [56–58]).

The $AdS_2 \times \mathbb{R}^2$ metric has the form

$$ds^2 = \frac{R_0^2}{\sigma^2}(dt^2 - d\sigma^2) - R_0^2(dy^2 + d\rho^2). \quad (16)$$

The Killing vectors along the spatial directions $X_1 = \partial_y, X_2 = \partial_\rho$ generate the Bianchi I algebra

$$[X_1, X_2] = 0. \quad (17)$$

It is easy to construct this metric as a solution to the equations of motion that follow from the gauged supergravity action (6). It is supported by an electrically-charged gauge field whose ansatz we choose to be

$$A^\Lambda = \frac{E^\Lambda}{\sigma} dt. \quad (18)$$

It is straightforward to check that this configuration solves the equations of motion. The Killing spinor equations (15) evaluated in the above background are

$$\frac{\gamma^0 \sigma}{R_0}\partial_t \epsilon_A - \frac{\gamma^1}{2R_0}\epsilon_A + \frac{iG_A^B\gamma^0}{2R_0}\epsilon_B + iS_{AB}\epsilon^B + \frac{iN}{2R_0^2}\gamma^{01}\epsilon_{AB}\epsilon^B = 0 \quad (19)$$

$$\frac{\gamma^1 \sigma}{R_0}\partial_\sigma \epsilon_A + iS_{AB}\epsilon^B + \frac{iN}{2R_0^2}\gamma^{01}\epsilon_{AB}\epsilon^B = 0 \quad (20)$$

$$\frac{\gamma^2}{R_0}\partial_y \epsilon_A + iS_{AB}\epsilon^B - \frac{N}{2R_0^2}\gamma^{23}\epsilon_{AB}\epsilon^B = 0 \quad (21)$$

$$\frac{\gamma^3}{R_0}\partial_\rho \epsilon_A + iS_{AB}\epsilon^B - \frac{N}{2R_0^2}\gamma^{23}\epsilon_{AB}\epsilon^B = 0, \quad (22)$$

where we have defined

$$N = (\mathrm{Im}N_{\Lambda\Sigma})L^\Sigma E^\Lambda, \qquad G_A^B = (\sigma_x)_A^B P_\Lambda^x E^\Lambda \quad (23)$$

for brevity. We choose the following radial ansatz for the Killing spinor

$$\epsilon_A = f(\sigma)\chi_A, \quad (24)$$

where χ_A is a constant spinor. The difference of (20) and (19) leads to

$$\frac{\gamma^1 \sigma}{R_0}\partial_\sigma \epsilon_A + \frac{\gamma^1}{2R_0}\epsilon_A - \frac{iG_A^B\gamma^0}{2R_0}\epsilon_B = 0. \quad (25)$$

The above equation has a simple solution

$$f(\sigma) = \frac{1}{\sqrt{\sigma}}, \quad (26)$$

provided we impose the condition

$$E^\Lambda P_\Lambda^x = 0. \quad (27)$$

We note that this same condition has enabled a supersymmetric Lifshitz solution in 4d $\mathcal{N} = 2$ gauged supergravity [48]. Thus, the Killing spinor equations reduce to the algebraic conditions

$$-\frac{\gamma^1}{2R_0}\chi_A + iS_{AB}\chi^B + \frac{iN}{2R_0^2}\gamma^{01}\epsilon_{AB}\chi^B = 0 \quad (28)$$

$$iS_{AB}\chi^B - \frac{iN}{2R_0^2}\gamma^{01}\epsilon_{AB}\chi^B = 0, \quad (29)$$

where we have substituted $\gamma^{23} = -i\gamma^{01}\gamma_5$ and used $\gamma_5\epsilon^A = -\epsilon^A$. It is straightforward to recast the above equations into the projection conditions

$$\chi_A = \frac{2iN}{R}\epsilon_{AB}\gamma^0\chi^B \quad (30)$$

$$\chi_A = -4iRS_{AB}\gamma^1\chi^B. \quad (31)$$

These projection conditions are very similar to the conditions obtained for the 4d Lifshitz case by [48] (cf. Eqs. 67–68). Squaring the first projection condition (30) we get

$$|N| = \frac{R_0}{2}. \quad (32)$$

Mutual consistency of the two projectors leads to the equation

$$\chi_A = 4R_0 S_{AB}\gamma^{10}\epsilon^{BC}\chi_C, \quad (33)$$

whose self consistency gives the condition

$$\sum_{x=1}^{3}(P_\Lambda^x L^\Lambda)^2 = -\frac{1}{4R_0^2}. \quad (34)$$

Note that the triplet of Killing prepotentials P_Λ^x are real functions on the quaternionic manifold. However, the symplectic sections L^Λ are complex functions in general. For simplicity, we can choose the Killing prepotential to lie along the $x = 3$ direction.[6] Thus, the final projection

[6]Note that this sets

$$P_\Lambda^3 L^\Lambda = \frac{i}{2R_0}. \quad (35)$$

It is easy to check that this choice is consistent with the projection condition (31). Substituting the above in (31) we get

$$\chi_A = i(\sigma_3)_A{}^D\epsilon_{BD}\gamma^1\chi^B, \quad (36)$$

that is self consistent.

conditions that follow from the gravitino Killing spinor equations are

$$\chi_A = i\epsilon_{AB}\gamma^0\chi^B$$
$$\chi_A = (\sigma_3)_A{}^C\gamma^{10}\chi_C. \quad (37)$$

These are mutually self-consistent projection conditions, and together they preserve $\frac{1}{4}$ of the supersymmetry. We now proceed to analyze the gaugino and hyperino conditions in (14).

Setting the hyperino variation (14) to zero, we get the algebraic condition

$$i\mathcal{U}_X^{A\beta}K_\Lambda^X\frac{E^\Lambda}{R_0}\gamma^0\epsilon^B\epsilon_{BA}\epsilon_{\alpha\beta} + 2\mathcal{U}_{\alpha X}^A K_\Lambda^X\bar{L}^\Lambda\epsilon_A = 0. \quad (38)$$

We can use the $\frac{1}{4}$ BPS projectors (37) to simplify the above expression to get

$$\mathcal{U}_{X\alpha}^A K_\Lambda^X\left(\frac{E^\Lambda}{R_0} + 2\bar{L}^\Lambda\right)\chi_A = 0. \quad (39)$$

An obvious way to solve the condition is to set $E^\Lambda = -2\bar{L}^\Lambda R_0$. In fact, this leads to the correct equation of motion [second of (B7)]. However, this leads to an inconsistency with the known identity (see Eq. 4.38 of [55]) $\mathrm{Im}N_{\Lambda\Sigma}L^\Lambda\bar{L}^\Sigma = -\frac{1}{2}$ that is true for any $\mathcal{N}=2$ supergravity. Note that this was also observed earlier in [48] for the 4d Lifshitz solution. However, we can solve the hyperino conditions by choosing the Killing vectors to be degenerate on the quaternionic manifold. In other words,

$$K_\Lambda^X\left(\frac{E^\Lambda}{R_0} + 2\bar{L}^\Lambda\right) = 0. \quad (40)$$

The gaugino conditions in (14) upon using the $\frac{1}{4}$ BPS projections have the very simple form

$$g^{i\bar{j}}\bar{f}_{\bar{j}}^\Sigma\left((-\mathrm{Im}N)_{\Lambda\Sigma}\frac{E^\Lambda}{R_0^2} + iP_\Sigma^3\right) = 0. \quad (41)$$

This concludes the set of conditions that follow from supersymmetry requirements. To summarize, the final set of conditions for a $\frac{1}{4}$ BPS $\mathrm{AdS}_2\times\mathbb{R}^2$ solution are

$$E^\Lambda P_\Lambda^3 = 0, \qquad \mathrm{Im}N_{\Lambda\Sigma}L^\Lambda E^\Sigma = \frac{R_0}{2},$$
$$P_\Lambda^3 L^\Lambda = \frac{i}{2R_0}, \qquad K_\Lambda^X\left(\frac{E^\Lambda}{R_0} + 2\bar{L}^\Lambda\right) = 0,$$
$$g^{i\bar{j}}\bar{f}_{\bar{j}}^\Sigma\left(-\mathrm{Im}N_{\Lambda\Sigma}\frac{E^\Lambda}{R_0^2} + iP_\Sigma^3\right) = 0. \quad (42)$$

In addition, one has to impose the gauge field equations of motion (B4).

B. Bianchi II

In this section, we discuss the supersymmetry conditions for a Bianchi II ($\mathrm{AdS}_2\times E\mathrm{AdS}_2$) solution of the form

$$ds^2 = \frac{R_0^2}{\sigma^2}(dt^2 - d\sigma^2) - \frac{R_0^2}{\rho^2}(dy^2 + d\rho^2). \quad (43)$$

As discussed in Sec. II, the symmetries along the spatial directions correspond to that of $E\mathrm{AdS}_2$. Like the previous solution, the $\mathrm{AdS}_2\times E\mathrm{AdS}_2$ solution can also be constructed using a timelike gauge field (18) as a source, since it preserves the Bianchi II symmetry along the (y,ρ) directions. However, the electric solution is nonsupersymmetric unlike the magnetic case.[7] We present the details of the computation in Appendix Sec. B 2. We now move on to the five-dimensional case, where there is a wider variety of solutions with Bianchi symmetries in the spatial directions.

IV. BIANCHI ATTRACTORS IN $\mathcal{N}=2$, $d=5$ GAUGED SUPERGRAVITY

We begin with a brief introduction to $\mathcal{N}=2, d=5$ gauged supergravity coupled to n_V vector multiplets with a generic gauging of the very special manifold $\mathcal{S}$ and the $U(1)_R$ subgroup of the $\mathrm{SU}(2)_R$ symmetry group [52].[8] The bosonic part of the Lagrangian reads as

$$\hat{e}^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{4}a_{IJ}F_{\mu\nu}^I F^{J\mu\nu} - \frac{1}{2}g_{xy}(\phi)D_\mu\phi^x D^\mu\phi^y$$
$$+\frac{\hat{e}^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\tau}F_{\mu\nu}^I F_{\rho\sigma}^J A_\tau^K - \mathcal{V}(\phi), \quad (44)$$

where $\hat{e} = \sqrt{-\det g_{\mu\nu}}$. Here, ϕ^x are n_V+1 scalars in the vector multiplet. The metric g_{xy} is defined on the very special manifold that exists in the 5d theory. The gauge covariant derivatives are defined by

$$D_\mu\phi^x = \partial_\mu\phi^x + gA_\mu^I K_I^x(\phi)$$
$$D_\mu\psi_{\nu i} = \nabla_\mu\psi_{\nu i} + g_R A_\mu^I P_{Ii}^j\psi_{\nu j}, \quad (45)$$

where A_μ^I are the vectors in the vector multiplet and g is the gauge coupling constant. The covariant derivative ∇ is

[7] See [50] for magnetic black hole solutions interpolating between $\mathrm{AdS}_2\times\mathbb{H}^2$ and hyperscale violating solutions at infinity.

[8] Please note that in all of five dimensions, we use the mostly plus metric signature.

defined with the usual spin connection. For the purposes of this paper, we restrict ourselves to the case where the gauge group is Abelian. In the fermionic covariant derivatives, g_R is the $U(1)_R$ gauge coupling constant. The quaternionic prepotential for the $U(1)_R$ gauging is a singlet

$$P_{Iij} = -V_I \delta_{ij}, \tag{46}$$

where V_I are the Fayet-Illioupoulos parameters. Note that in the above expression, δ_{ij} does not play the role of ϵ_{ij} as a raising or lowering operator. The potential in the (44) is defined in terms of the quaternionic prepotentials

$$\mathcal{V}(\phi) = -g_R^2(2P_{ij}P^{ij} - P^a_{ij}P^{aij}), \tag{47}$$

with the definitions

$$P_{ij} = h^I P_{Iij}, \qquad P^a_{ij} = h^{aI} P_{Iij}, \qquad h^{aI} = f^a_x h^{xI}, \tag{48}$$

where f^a_x are vielbeins on the very special manifold $\mathcal{S}$. Note that the potential is unaffected by the gauging of $\mathcal{S}$. The addition of hypermultiplets and tensor multiplets will change the shape of the potential, however, to get the AdS vacuum, it is sufficient to gauge the $U(1)_R$ symmetry.

The bosonic sector of the supersymmetry transformations are

$$\delta_\epsilon \psi_{\mu i} = D_\mu \epsilon_i + \frac{i}{4\sqrt{6}} h_I F^{\nu\rho I}(\gamma_{\mu\nu\rho} - 4 g_{\mu\nu}\gamma_\rho)\epsilon_i + \frac{i}{\sqrt{6}} g_R \gamma_\mu \epsilon^j P_{ij}$$
$$\delta_\epsilon \lambda^a_i = -\frac{i}{2} f^a_x D_\mu \phi^x \gamma^\mu \epsilon_i + \frac{1}{4} h^a_I F^I_{\mu\nu}\gamma^{\mu\nu}\epsilon_i + g_R \epsilon^j P^a_{ij}. \tag{49}$$

The λ^a_i ($i = 1,\ 2$ and $a = 1, \ldots n_V$) are gauginos in the vector multiplets and ϵ_i is a symplectic majorana spinor. The covariant derivative is defined as

$$D_\mu \epsilon_i \equiv \partial_\mu \epsilon_i + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\epsilon_i + g_R A^I_\mu P_{Iij}\epsilon^j. \tag{50}$$

See Appendix Sec. A 2 for our notations and conventions of 5d gamma matrices.

We are interested in Bianchi type near horizon solutions to (49) that satisfy attractor conditions. It is well known that at the attractor point, the moduli are constants independent of spacetime coordinates

$$\phi^x = \text{const.} \tag{51}$$

The field equations that follow from (44) are given in [35]. The supersymmetry transformations at the attractor point take the form

$$\delta_\epsilon \psi_{\mu i} = D_\mu \epsilon_i + \frac{i}{4\sqrt{6}} h_I F^{\nu\rho I}(\gamma_{\mu\nu\rho} - 4 g_{\mu\nu}\gamma_\rho)\epsilon_i + \frac{i}{\sqrt{6}} g_R \gamma_\mu \epsilon^j P_{ij}$$
$$\delta_\epsilon \lambda^a_i = -\frac{i}{2} g f^a_x A^I_\mu K^x_I \gamma^\mu \epsilon_i + \frac{1}{4} h^a_I F^I_{\mu\nu}\gamma^{\mu\nu}\epsilon_i + g_R \epsilon^j P^a_{ij}. \tag{52}$$

In the following sections, we evaluate the spinor conditions on the Bianchi attractor backgrounds. As discussed in Sec. II, the Bianchi type metrics have the generic form[9]

$$ds^2 = \eta_{ab} e^a e^b = L^2(-e^{2\beta_t r} dt^2 + \eta_{ij}(r)\omega^i \otimes \omega^j + dr^2), \tag{53}$$

where e^a, $a = 0, \ldots 4$, are one forms, and L is a positive constant that measures the size of the spacetime. The ω^i, $i = 1, \ldots 3$ are one forms manifestly invariant under the homogeneous symmetries described by the Bianchi classification.

A. The gaugino conditions

In this section, we solve the gaugino conditions

$$\delta_\epsilon \lambda^a_i = -\frac{i}{2} g f^a_x A^I_\mu K^x_I \gamma^\mu \epsilon_i + \frac{1}{4} h^a_I F^I_{\mu\nu}\gamma^{\mu\nu}\epsilon_i - g_R \epsilon^j h^a_I V^I \delta_{ij} = 0, \tag{54}$$

where we have substituted (46) and (48). In gauged supergravity literature, the terms in the supersymmetry variations that are proportional to the gauge coupling constants are referred to as fermionic shifts. For maximal supersymmetry, all of the fermionic shifts in the gaugino conditions must vanish [35,52]. From the integrability conditions of Eq. 31 of [35], it follows that the only maximally supersymmetric Bianchi type solution is AdS_5. Our first result will be to argue that the above result is also true for solutions with matter, in this case the Bianchi type geometries. Then we will require some of the fermionic shifts to vanish and explore conditions for supersymmetric solutions.

First, we focus on the cases when none of the fermionic shifts vanish. Preserving some amount of supersymmetry from the gaugino and hyperino conditions requires that the algebraic conditions on the constant part of the spinor ζ_i be not too restrictive. In other words, the matrices that project out the various components of ζ_i must commute with one another. The projection conditions that can appear on the spinor in the equations (54) are entirely dependent on the gauge field configurations. Typically, the Bianchi type solutions are sourced by either timelike or spacelike massive gauge fields and a cosmological constant [13,14,35]. (In particular, see Appendix B of [13] for various choices of gauge field configurations that solve the

[9]In this coordinate system, the boundary of the Poincaré AdS metric lies at $r \to \infty$.

equations of motion.) At the attractor point, the scalars are constant and effective mass terms for the gauge fields

$$g^2 K_{IJ}(\phi) A^{I\mu} A^J_\mu \tag{55}$$

appear due to the presence of the gauge covariant derivatives in the supergravity action (44). Here, K_{IJ} is the Killing norm defined as $g_{xy} K^x_I K^y_J$. The mass terms are proportional to the norm of the Killing vectors and to the square of the gauge coupling constant. We analyze two possible cases separately below.

1. Nonvanishing fermionic shifts

To begin with, we keep our analysis very generic with respect to the gauging of the scalar manifold (model independent) but specific only to the field content that generates the solution. By this, we mean that there are no specific conditions that the Killing vectors on $\mathcal{S}$ are required to satisfy. We first consider the case where the gauge fields have only the time component turned on. The Bianchi metrics that have been constructed so far [13,14,35] are sourced by timelike or spacelike gauge fields. Timelike gauge fields are of the form

$$\begin{aligned} A &= A(r) dt \\ dA &= \partial_r A(r) dr \wedge dt. \end{aligned} \tag{56}$$

In order to solve the gaugino conditions (54), it is necessary to impose projection conditions on the constant part of the spinor ϵ_i. From the timelike gauge field configuration, it is clear that the following conditions have to be imposed in (54)[10]

$$\begin{aligned} \gamma_0 \epsilon_i &= \pm i \epsilon_i \\ \gamma_{04} \epsilon_i &= \pm \epsilon_i. \end{aligned} \tag{57}$$

The first projector appears in the $A^\mu \gamma_\mu$ terms, while the second appears in the $F^{\mu\nu}\gamma_{\mu\nu}$ terms. While each of the projectors is well defined, it is clear that the two conditions are mutually incompatible since the projections (57) are mutually orthogonal. Thus, when the fermionic shifts do not vanish, all solutions sourced by timelike gauge fields break supersymmetry. Thus, with a timelike gauge field, under gauging it is not possible to obtain supersymmetry preserving projection conditions. Note that this is completely independent of the functional dependence of the Killing spinor.

Let us now consider the case with gauge fields having spacelike components turned on. (For examples, see [13,14,35])

$$\begin{aligned} A &= A(x,r)\omega^i \\ dA &= \partial_r A(x,r) dr \wedge \omega^i + \partial_{x_j} A(x,r) dx^j \wedge \omega^i \\ &\quad + \frac{1}{2} A(x,r) C_{jk}{}^i \omega^j \wedge \omega^k, \end{aligned} \tag{58}$$

where $x = x^i, i = 1, 2, 3$ are the directions that have homogeneous symmetries. In this case, it is easy to see from (54) that the projections that can appear are

$$\begin{aligned} \gamma_i \epsilon_i &= \pm \epsilon_i \\ \gamma_{i4} \epsilon_i &= \pm i \epsilon_i \\ \gamma_{ij} \epsilon_i &= \pm i \epsilon_i. \end{aligned} \tag{59}$$

In any given configuration for the space like gauge field, the first projector always appears. Depending on the precise functional dependence, the second/third or both second and third projectors can appear. In any case, we see that the first projector in (59) is mutually orthogonal to both the second and third. Thus, even with a spacelike gauge field, under generic conditions it is not possible to obtain supersymmetry preserving conditions. Note that this too is completely independent of the functional dependence of the Killing spinor. Thus, when the fermionic shifts do not vanish, all massive Bianchi attractors are nonsupersymmetric in gauged supergravity with generic gauging of the scalar manifold.

For all of the solutions in this class, we have studied the Killing spinor equations independently and find that the radial spinor breaks supersymmetry. The solutions constructed in [35] are all of this type and are all nonsupersymmetric.

2. Vanishing fermionic shifts

The other possibilities to solve (54) are situations where some of the fermionic shifts vanish in special cases. From the studies of the attractor mechanism for black holes in $d = 5$ ungauged supergravity, it is known that attractor solutions solve the gaugino conditions [54,59] with the extremization of central charge

$$\partial_x(Z) = \partial_x(h^I Q_I) = 0, \qquad h^I V_I = 1. \tag{60}$$

Imposing the attractor conditions on (54),[11] we find that the gaugino conditions reduce to

$$\delta_\epsilon \lambda^a_i = -\frac{i}{2} g f^a_x A^I_\mu K^x_I \gamma^\mu \epsilon_i = 0. \tag{61}$$

Note that the square of this fermionic shift term is proportional to

[10]The spacetime coordinates are $x^\mu = (t, x^1, x^2, x^3, r)$, while the corresponding tangent space indices run over $a = 0, \ldots 4$.

[11]The FI parameters V_I are arbitrary and can be scaled to satisfy this condition.

$$g^2 g_{xy} A^I_\mu A^{\mu J} K^x_I K^y_J, \tag{62}$$

the mass term discussed in the introduction of this section. Thus, for preserving supersymmetry, we have to set the effective mass term to zero. This can be achieved in two ways.

(a) The trivial choice is $g = 0$ or no gauging of the scalar manifold $\mathcal{S}$.

(b) The other more nontrivial possibility is to find a Killing direction in $\mathcal{S}$ that satisfies $K^I_x Q_I = 0$ at the attractor point.

Note that for the class of models discussed in [60,61], studied earlier explicit solutions were constructed and analyzed in [35,46], and it can be checked that the condition $K^I_x Q_I = 0$ is not satisfied. However, note that using this condition would kill the effective mass terms in the field equations of motion (see Eq. 18, Eq. 22 of [35]), which is problematic and would only lead to massless solutions.

We pause here to briefly summarize the conclusions of this section. Analyzing the gaugino conditions, we have the results that in $\mathcal{N} = 2$ gauged supergravity with a generic gauging of the symmetries of scalar manifold and a $U(1)$ gauging of the $\mathrm{SU}(2)_R$ symmetry,

(a) There are no massive Bianchi attractor solutions that preserve any amount of supersymmetry for a generic gauging when the fermionic shifts do not vanish.

(b) When the extremization condition is met $\partial_x(Z) = 0$, supersymmetry allows only massless Bianchi solutions.[12]

(c) For massless Bianchi solutions, the gaugino conditions are completely solved by the attractor conditions (60), and there are no additional projection conditions. The amount of supersymmetry preserved is completely determined by the Killing spinor equations.

These solutions can be easily constructed in Einstein-Maxwell theory with a cosmological constant. Actually, all of them can be also constructed easily, for instance in the $U(1)_R$ gauged supergravity model studied in [47].

The last and final possibility for this section corresponds to vacuum solutions in the absence of matter. In this case, the gaugino conditions are trivial. The supersymmetry conditions are completely determined by the Killing spinor equation that follows from the gravitino variation. The solution space includes the well-known AdS_5 solution [41,52,60], Bianchi III $\mathrm{AdS}_3 \times \mathbb{H}^2$, and Bianchi V $\mathrm{AdS}_2 \times \mathbb{H}^3$ solutions, sourced only by a cosmological constant. The results of this section can get modified by addition of tensor and hyper multiplets. We comment on this briefly in Sec. IV C.

B. The gravitino conditions: Killing spinor equation

In this section, we analyze the gravitino Killing spinor equation for the Bianchi solutions sourced by massless gauge fields. We describe new supersymmetric Bianchi I and Bianchi III solutions in detail. We also find new nonsupersymmetric Bianchi V and Bianchi VII solutions, these are summarized in Appendix Sec. D.

For the $U(1)_R$ gauged supergravity (46), the Killing spinor equation we need to solve is of the form

$$D_\mu \epsilon_i + \frac{i}{4\sqrt{6}} h_I F^{\nu\rho I}(\gamma_{\mu\nu\rho} - 4 g_{\mu\nu}\gamma_\rho)\epsilon_i + \frac{i}{\sqrt{6}} g_R \gamma_\mu \epsilon_i{}^k \epsilon_k = 0, \tag{63}$$

where

$$D_\mu \epsilon_i \equiv \partial_\mu \epsilon_i + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\epsilon_i + g_R A^I_\mu V_I \epsilon_i{}^k \epsilon_k, \tag{64}$$

where we have used the attractor conditions (60). Note that we have used the notation $\epsilon^{kj}\delta_{ij} = \epsilon_i{}^k$, where $\epsilon_i{}^k$ is numerically same as $-\epsilon_{ik}$. It follows that $\epsilon_i{}^k \epsilon_i{}^l = -\delta_i{}^l$. We need to remember that δ_{ij} is just one component of the general triplet in P_{Iij}, and hence, one cannot use δ_{ij} or $\epsilon_i{}^k$ to raise or lower the R symmetry index [52,61].[13] In the following, we solve the Killing spinor equations for various Bianchi type geometries.

1. Supersymmetric Bianchi I: Anisotropic $\mathrm{AdS}_3 \times \mathbb{R}^2$

The anisotropic $\mathrm{AdS}_3 \times \mathbb{R}^2$ solution can be easily constructed with magnetic fields and a cosmological constant.[14] The metric has the simple form

$$e^0 = e^r dt, \qquad e^1 = e^r \omega^1, \qquad e^2 = \frac{|B|}{2}\omega^2,$$
$$e^3 = \frac{|B|}{2}\omega^3, \qquad e^4 = dr. \tag{65}$$

The magnetic fluxes in the x^2, x^3 directions generate anisotropy but preserve the rotational symmetries of $\mathbb{R}^2$. The solution (65) has been of considerable interest in computations of shear viscosities in anisotropic phases [23,53]. The invariant one forms

$$\omega^i = dx^i, \qquad i = 1, 2, 3, \tag{66}$$

and all commute with one another and satisfy $d\omega^i = 0$ of the Bianchi I algebra. In (65) $|B| = B^I B_I$ is the strength of the magnetic field. We choose our gauge field ansatz such that

[12] This possibility appears to be relaxed when tensor multiplets are included, we comment on this briefly in Sec. IV C

[13] We thank Antoine Van Proeyen for useful communication regarding this issue.

[14] See for example the isotropic solution in the $U(1)^3$ truncation of type II supergravity on S^5 by [57]. For general geometries of the type $\mathrm{AdS}_3 \times \Sigma_g$ in STU model of supergravity and their dual field theory interpretation see [62–65].

$$F^I_{x^2x^3} = B^I. \tag{67}$$

The Killing spinor equations in the background are of the form

$$\gamma_0 e^{-r}\partial_t\epsilon_i - \frac{1}{2}\gamma_4\epsilon_i - \frac{i}{\sqrt{6}}\left(\frac{Z}{2}\gamma_{23}\epsilon_i + g_R\epsilon_i{}^k\epsilon_k\right) = 0 \tag{68}$$

$$\gamma_1 e^{-r}\partial_{x^1}\epsilon_i + \frac{1}{2}\gamma_4\epsilon_i + \frac{i}{\sqrt{6}}\left(\frac{Z}{2}\gamma_{23}\epsilon_i + g_R\epsilon_i{}^k\epsilon_k\right) = 0 \tag{69}$$

$$\gamma_2\partial_{x^2}\epsilon_i + \frac{i}{\sqrt{6}}\frac{|B|}{2}(-Z\gamma_{23}\epsilon_i + g_R\epsilon_i{}^k\epsilon_k) = 0 \tag{70}$$

$$\gamma_3\partial_{x^3}\epsilon_i + \frac{i}{\sqrt{6}}\frac{|B|}{2}(-Z\gamma_{23}\epsilon_i + g_R\epsilon_i{}^k\epsilon_k) = 0 \tag{71}$$

$$\gamma_4\partial_r\epsilon_i + \frac{i}{\sqrt{6}}\left(\frac{Z}{2}\gamma_{23}\epsilon_i + g_R\epsilon_i{}^k\epsilon_k\right) = 0, \tag{72}$$

where $Z = h_I B^I$ is the central charge. In the above, we have chosen the following condition

$$B^I V_I = 0. \tag{73}$$

This condition is the five-dimensional analogue of (27).

It is easy to obtain the following differential equations from the above set

$$\begin{aligned}\gamma_0\partial_t\epsilon_i + \gamma_1\partial_{x^1}\epsilon_i &= 0\\ \gamma_2\partial_{x^2}\epsilon_i - \gamma_3\partial_{x^3}\epsilon_i &= 0\\ \gamma_4\partial_r\epsilon_i + \gamma_0 e^{-r}\partial_t\epsilon_i - \frac{1}{2}\gamma_4\epsilon_i &= 0\\ \gamma_4\partial_r\epsilon_i - \gamma_1 e^{-r}\partial_{x^1}\epsilon_i - \frac{1}{2}\gamma_4\epsilon_i &= 0.\end{aligned} \tag{74}$$

Notice the similarity of the above equations to the ones we have obtained in AdS_5 case (C4), except the second equation that suggests that the x^2, x^3 directions can scale differently as compared to the x^1 direction. There are two independent solutions to the above equations

$$\epsilon_i = e^{\frac{r}{2}}\zeta_i^+, \qquad \gamma_4\zeta_i^+ = \zeta_i^+ \tag{75}$$

$$\epsilon_i = (e^{-\frac{r}{2}} + e^{\frac{r}{2}}(t\gamma_0 + x^1\gamma_1 + \alpha(x^2\gamma_2 + x^3\gamma_3)))\zeta_i^-,$$
$$\gamma_4\zeta_i^- = -\zeta_i^-, \tag{76}$$

where α is a real parameter. The projection, due to the radial Dirac matrix, has the same effect as in the AdS case, namely the projector preserves one half of the supersymmetry in each of $\zeta^\pm$. Substituting the solution (76) in the x^2, x^3 equations (70)–(71) we find that $\alpha = 0$. Thus, the Killing spinor (76) is independent of the $\mathbb{R}^2$ directions.

The remaining equations give rise to the conditions

$$\frac{1}{2}\zeta_i^\pm + \frac{i}{\sqrt{6}}\left(\frac{Z}{2}\gamma_{23}\zeta_i^\pm + g_R\epsilon_i{}^k\zeta_k^\pm\right) = 0 \tag{77}$$

$$-Z\gamma_{23}\zeta_i^\pm + g_R\epsilon_i{}^k\zeta_k^\pm = 0. \tag{78}$$

It is easy to see that the above two equations give rise to the conditions

$$\begin{aligned}\gamma_{23}\zeta_i^\pm &= \epsilon_i{}^k\zeta_k^\pm\\ |Z| = |g_R| &= \frac{\sqrt{6}}{3}.\end{aligned} \tag{79}$$

The projection above breaks half of the remaining supersymmetries in each of $\zeta_\pm$. As a result, each of $\zeta_\pm$ generate $\frac{1}{4}$ of the supersymmetry. Thus the solution (65) is a $\frac{1}{2}$ BPS solution.

2. *Supersymmetric Bianchi III and* $AdS_3 \times \mathbb{H}^2$

In this section we construct a superymmetric Bianchi III type solution sourced by a massless gauge field

$$\begin{aligned}&e^0 = Le^{\beta r}dt, \qquad e^1 = L\omega^1, \qquad e^2 = Le^{\beta r}\omega^2,\\ &e^3 = L\omega^3, \qquad e^4 = Ldr,\end{aligned} \tag{80}$$

where the invariant one forms are

$$\omega^1 = e^{-x^1}dx^2, \qquad \omega^2 = dx^3, \qquad \omega^3 = dx^1. \tag{81}$$

The spatial part of the metric has the symmetries of $\mathbb{H}^2 \times \mathbb{R}$. The symmetry algebra, due to these Killing vectors, form the Bianchi III algebra in the Bianchi classification in three dimensions. The subalgebra generated by the Killing vectors of $\mathbb{H}^2$ generate the Bianchi II algebra in two dimensions.

We choose the gauge field to have components along the ω^1 direction

$$A^I = B^I e^1. \tag{82}$$

The Killing spinor equations evaluated in the background are $(Z = h_I B^I)$

$$e^{-\beta r}\gamma_0\partial_t\epsilon_i - \frac{\beta}{2}\gamma_4\epsilon_i - \frac{i}{\sqrt{6}}\left(\frac{1}{2}Z\gamma_{13}\epsilon_i + Lg_R\epsilon_i{}^k\epsilon_k\right) = 0 \tag{83}$$

$$\begin{aligned}&e^{x^1}\gamma_1\partial_{x^2}\epsilon_i - \frac{\gamma_3}{2}\epsilon_i + Lg_R B^I V_I\gamma_1\epsilon_i{}^k\epsilon_k\\ &\quad + \frac{i}{\sqrt{6}}(-Z\gamma_{13}\epsilon_i + Lg_R\epsilon_i{}^k\epsilon_k) = 0\end{aligned} \tag{84}$$

$$e^{-\beta r}\gamma_2\partial_{x^3}\epsilon_i+\frac{\beta}{2}\gamma_4\epsilon_i+\frac{i}{\sqrt{6}}\left(\frac{1}{2}Z\gamma_{13}\epsilon_i+Lg_R\epsilon_i{}^k\epsilon_k\right)=0 \tag{85}$$

$$\gamma_3\partial_{x^1}\epsilon_i+\frac{i}{\sqrt{6}}(-Z\gamma_{13}\epsilon_i+Lg_R\epsilon_i{}^k\epsilon_k)=0 \tag{86}$$

$$\gamma_4\partial_r\epsilon_i+\frac{i}{\sqrt{6}}\left(\frac{1}{2}Z\gamma_{13}\epsilon_i+Lg_R\epsilon_i{}^k\epsilon_k\right)=0. \tag{87}$$

As before we can obtain the following equations from above

$$\begin{aligned}\gamma_0\partial_t\epsilon_i+\gamma_2\partial_{x^3}\epsilon_i&=0\\ e^{-\beta r}\gamma_0\partial_t\epsilon_i-\frac{\beta\gamma_4}{2}\epsilon_i+\gamma_4\partial_r\epsilon_i&=0\\ e^{-\beta r}\gamma_2\partial_{x^3}\epsilon_i+\frac{\beta\gamma_4}{2}\epsilon_i-\gamma_4\partial_r\epsilon_i&=0\\ e^{x^1}\gamma_{13}\partial_{x^2}\epsilon_i+\partial_{x^1}\epsilon_i+\frac{\epsilon_i}{2}+Lg_RB^IV_I\gamma_{13}\epsilon_i{}^k\epsilon_k&=0.\end{aligned} \tag{88}$$

The AdS_3 part of the Killing spinor will preserve some supersymmetry provided we assume that the Killing spinor does not depend on the $\mathbb{H}^2$ part. We get the following conditions from the above set of equations

$$\epsilon_i=e^{\frac{\beta r}{2}}\zeta_i^+,\qquad \gamma_4\zeta_i^+=\zeta_i^+ \tag{89}$$

$$\epsilon_i=(e^{-\frac{\beta r}{2}}+e^{\frac{\beta r}{2}}(t\gamma_0+x^3\gamma_2))\zeta_i^-,\qquad \gamma_4\zeta_i^-=-\zeta_i^- \tag{90}$$

$$\gamma_{13}\zeta_i^\pm=\epsilon_i{}^k\zeta_k^\pm,\qquad 4L^2g_R^2(B_IV^I)^2=1. \tag{91}$$

As discussed in the previous sections, the two projectors above combine to break half of the total supersymmetries of the solution. Substituting the above relations in the Killing spinor equation we find

$$\frac{\beta}{2}\zeta_i^\pm+\frac{i}{\sqrt{6}}\left(\left(\frac{Z}{2}+Lg_R\right)\epsilon_i{}^k\zeta_k^\pm\right)=0 \tag{92}$$

$$(-Z+Lg_R)\epsilon_i{}^k\zeta_k^\pm=0. \tag{93}$$

Consistency of the above equations yields the conditions

$$Lg_R=Z,\qquad \beta=\sqrt{\frac{3}{2}}Z,\qquad 4L^2g_R^2(B_IV^I)^2=1. \tag{94}$$

Thus, we have a one parameter family of $\frac{1}{2}$ BPS Bianchi III solutions labeled by the central charge Z.

When the central charge takes the value (79) (the one corresponding to the $AdS_3\times\mathbb{R}^2$ solution), it follows from (94) that

$$L=1,\qquad \beta=1. \tag{95}$$

This is the $AdS_3\times\mathbb{H}^2$ solution constructed in [54].[15]

We have repeated the above analysis for electric Bianchi V, vacuum Bianchi V, and magnetic Bianchi VII cases. However, in these cases the radial spinor breaks all supersymmetry. The details are summarized in Appendix Sec. D. This concludes our analysis of supersymmetric solutions in gauged supergravity with $U(1)_R$ gauging. In the following section, we explore possible generalizations to find new supersymmetric solutions.

C. Including hyper and tensor multiplets

In this section, we briefly comment about the possibilities of new supersymmetric solutions due to addition of tensor or hypermultiplets. We will provide formal arguments as explicit solutions such as the ones constructed in [35] have not been explored yet in specific models with tensor/hypermultiplets. The addition of tensor/hyper multiplets modifies the supersymmetry transformations (49). Let us first consider the gravitino equation [52]

$$\begin{aligned}\delta_\epsilon\psi_{\mu i}&=D_\mu\epsilon_i\\ &\quad+\frac{i}{4\sqrt{6}}h_{\tilde M}\mathcal{H}^{\tilde M\nu\rho}(\gamma_{\mu\nu\rho}-4g_{\mu\nu}\gamma_\rho)\epsilon_i+\frac{i}{\sqrt{6}}g_R\gamma_\mu\epsilon^jP_{ij}\end{aligned} \tag{96}$$

where $\mathcal{H}^{\tilde M}_{\mu\nu}=\{F^I_{\mu\nu},B^J_{\mu\nu}\}$, $I=0\ldots n_V$ and $J=1,\ldots,n_T$, $B^J_{\mu\nu}$ is an antisymmetric tensor that belongs to the tensor multiplet. The scalars $h_{\tilde M}=\{h^I,h^J\}$ are similarly functions of scalars from the vector and tensor multiplets respectively. The addition of hypermultiplets allows more general R symmetry gauging of the full $SU(2)_R$ symmetry group,

$$P_{ij}(q)=h^IP_{Iij}(q)=h^IP_I^r(q)(\sigma_r)_{ij}, \tag{97}$$

where the potentials are now SU(2) valued functions of the hyperscalars in the hypermultiplet.

First, let us consider the case of hypermultiplets turned on, but no tensor multiplets. In this case, the only difference is that the quaternionic prepotential is a SU(2) triplet function of the hyperscalars instead of a singlet for the $U(1)_R$ case. Hence for $\mathcal{N}=2$ gauged supergravity with $SU(2)_R$ gauging, including vector and hypermultiplets, and a generic gauging of the symmetries of the very special manifold and the quaternionic Kähler manifold, the Killing spinor results that pertain to nonsupersymmetric solutions

[15] It is also possible to construct the vacuum $AdS_3\times\mathbb{H}^2$ solution. However, this breaks all supersymmetry, unlike the charged solution. (see Appendix Sec. D 1)

in Sec. IV B continue to hold.[16] Of course, this does not affect the gaugino conditions, but in addition, there are new conditions from hyperino equations. We will discuss them shortly.

With tensor multiplets turned on in addition there are more possibilities. If the tensor fields are oriented carefully, there are possibilities of subtle cancellations that can potentially lead to interesting new solutions with supersymmetry preserving projection conditions. However, in the models that have been studied before in [35], we have not found any such possibility. Nevertheless, this requires an independent analysis, and it is helpful to obtain some conditions from gaugino and hyperino conditions first to aid in this direction.

The addition of tensor multiplets also changes the analysis of the gaugino conditions in an interesting way. The gaugino equations acquire an additional term due to tensor multiplets [52]

$$\delta_\epsilon \lambda_i^{\tilde{a}} = -\frac{i}{2} g A_\mu^I f_{\tilde{x}}^{\tilde{a}} K_I^{\tilde{x}} \gamma^\mu \epsilon_i + \frac{1}{4} h_{\tilde{M}}^{\tilde{a}} \mathcal{H}_{\mu\nu}^{\tilde{M}} \gamma^{\mu\nu} \epsilon_i - g_R \epsilon^j P_{ij}^{\tilde{a}} + g\frac{\sqrt{6}}{4} h^I K_I^{\tilde{x}} f_{\tilde{x}}^{\tilde{a}} \epsilon_i = 0, \tag{98}$$

where $\tilde{x} = 0,\ n_V + n_T$ labels the moduli $\phi^{\tilde{x}}$ in the vector and tensor multiplets. The vielbeins $f_{\tilde{x}}^{\tilde{a}}$ live on the tangent space corresponding to the very special manifold $\mathcal{S}$. If we continue to impose a straightforward generalization of the attractor conditions (60)[17]

$$\partial_{\tilde{x}}((Q_I h^I(\phi^*) + B_J h^J(\phi^*)) = 0, \qquad h^I(\phi^*) P_I^{\tilde{x}}(q^*) = 1 \tag{99}$$

the gaugino equations reduce to

$$\delta_\epsilon \lambda_i^{\tilde{a}} = g f_{\tilde{x}}^{\tilde{a}} K_I^{\tilde{x}} \left(-i A_\mu^I \gamma^\mu + \frac{\sqrt{6}}{2} h^I \right) \epsilon_i = 0. \tag{100}$$

It can be solved for an electric solution by imposing the conditions

$$\gamma^0 \epsilon_i = \pm i \epsilon_i, \qquad g f_{\tilde{x}}^{\tilde{a}}(\phi^*) K_I^{\tilde{x}}(\phi^*)(\pm Q^I + 2\sqrt{6} h^I(\phi^*)) = 0, \tag{101}$$

or by imposing the conditions

$$Q^I K_I^{\tilde{x}}(\phi^*) = 0, \qquad h^I(\phi^*) K_I^{\tilde{x}}(\phi^*) = 0 \tag{102}$$

for either of electric or magnetic solutions. In addition, one also has the hyperino conditions [52] at the attractor point

$$\delta\zeta^A = g f_{Xi}^A K_I^X \left(-i A_\mu^I \gamma^\mu + \frac{\sqrt{6}}{2} h^I \right) \epsilon_i = 0. \tag{103}$$

In the above, K_I^X are similarly Killing vectors on the quaternionic manifold $\mathcal{Q}$, f_{XI}^A are vielbeins on $\mathcal{Q}$, and g is the gauge coupling constant for the gauging of the symmetries on $\mathcal{Q}$. Note that (103) is structurally similar to the gaugino condition (100) after imposing attractor like conditions (99). Thus, for electric solutions, we can impose

$$\gamma^0 \epsilon_i = \pm i \epsilon_i, \\ g f_{Xi}^A(q^*) K_I^X(q^*)(\pm Q^I + 2\sqrt{6} h^I(\phi^*)) = 0, \tag{104}$$

or by imposing the conditions

$$Q^I K_I^X(q^*) = 0, \qquad h^I(\phi^*) K_I^X(q^*) = 0 \tag{105}$$

for either of electric or magnetic solutions. It is interesting to note that the conditions in (102) and (105) namely,

$$h^I(\phi^*) K_I^{\tilde{x}}(\phi^*) = 0, \qquad h^I(\phi^*) K_I^X(q^*) = 0, \tag{106}$$

appear in flow equations that preserve supersymmetry in AdS (see Eq. 2.60 of [41]). So it seems reasonable to impose the above conditions to find Bianchi attractor solutions that potentially flow to an asymptotic AdS geometry. However the conditions

$$Q^I K_I^X(q^*) = 0, \qquad Q^I K_I^{\tilde{x}}(\phi^*) = 0 \tag{107}$$

are problematic, as they kill the effective mass terms in the field equations [35] and would still lead to massless solutions. Thus, one possibility to find more interesting massive Bianchi solutions in the $\mathcal{N} = 2$ theory with vector, tensor, and hypermultiplets with generic gauging, is to consider solutions sourced by timelike gauge fields. Then, the gaugino and hyperino equations are satisfied by the attractor condition (99), the projections (101), and (104). However, solving the Killing spinor equation would require great care in choosing the tensor field configuration, as we would require a projection condition on the spinor that would commute with that of (101) and (104). We have not found any such solution in the models considered earlier in [35,46]. Perhaps instead of trying to find explicit solutions and then verifying supersymmetry, it may be useful to carefully analyze the Killing spinor integrability conditions together with the flow conditions (106) to determine the possible supersymmetric Bianchi attractor solutions in this theory. We hope to report this in a future work.

[16] For the supersymmetric solutions Eq. (65) and Sec. IV B 2 addition of tensor and hypermultiplets imposes additional new relations from the hyperscalar equations and the tensor field equations of motion. Moreover, the parameter space is also enhanced, so one can possibly find new such solutions. It will be interesting to see if the solutions in Eq. (65) and Sec. IV B 2 continue to remain supersymmetric in suitable models.

[17] Here ϕ^* and q^* are constant attractor values of the moduli and B^I are the tensor charges.

V. SUMMARY

In this paper, we analyzed the supersymmetry of Bianchi attractors in $\mathcal{N}=2$ $d=4$, 5 gauged supergravity. In $d=4$, we studied the supersymmetry of Bianchi I and II attractors sourced by electric fields. In the Bianchi I case, we studied an $\mathrm{AdS}_2\times\mathbb{R}^2$ metric sourced by a timelike gauge field. We analyzed the gaugino and Killing spinor equations and found that the radial spinor and its projection condition preserved $1/4$ of the supersymmetry. In the Bianchi II case, we constructed an electric $\mathrm{AdS}_2\times\mathbb{H}^2$ solution and found that the radial spinor breaks all supersymmetry.[18] The main lesson we learned from this exercise is that the radial spinor plays an important role in preserving supersymmetry. These results are special cases of the more general analysis of [34,51].

In $d=5$ $\mathcal{N}=2$ gauged supergravity, we considered the theory with a generic gauging of symmetries of the scalar manifold and a $U(1)_R$ gauging of the R-symmetry. The Bianchi attractor geometries that can be constructed are sourced by massive or massless gauge fields. For a generic gauging of the scalar manifold and R-symmetry, when the fermionic shifts in the gaugino and hyperino conditions do not vanish, the projection conditions that need to be imposed on the Killing spinor depend entirely on the gauge field/field strength configuration. We showed that, for the known field configurations that source the Bianchi type geometries, there are no supersymmetric projections possible. Independently, we showed that the radial spinor breaks supersymmetry for all metrics of this class. Thus, for a generic gauging of the scalar manifold and when the fermionic shifts do not vanish, there are no supersymmetric Bianchi attractors. This result for Bianchi type geometries is similar to the result for maximally supersymmetric solutions [41,52].

When the central charge of the theory satisfies an extremization condition at the attractor point[19]

$$\partial_i Z=0, \qquad (108)$$

some of the fermionic shifts vanish. Supersymmetry invariance of the resultant equations allowed only massless solutions. This prompts the search for Bianchi type metrics sourced by massless gauge fields and cosmological constant. We constructed new Bianchi I, Bianchi III, Bianchi V, and Bianchi VII classes of solutions sourced by massless gauge fields and a cosmological constant. Since the gaugino conditions are completely solved in these cases, the supersymmetry preserved by the geometries are determined by the Killing spinor equation. In the Bianchi I class, we constructed an anisotropic $1/2$ BPS $\mathrm{AdS}_3\times\mathbb{R}^2$ solution where the anisotropy is generated by a magnetic field. The supersymmetry is entirely due to the AdS_3 part and the Killing spinor does not depend on the $\mathbb{R}^2$ directions. We also constructed a one parameter family of $1/2$ BPS Bianchi III geometries, labeled by the central charge. When the central charge of the Bianchi III geometry takes the same value as that of $\mathrm{AdS}_3\times\mathbb{R}^2$, the solution reduces to the known $1/2$ BPS $\mathrm{AdS}_3\times\mathbb{H}^2$ solution [54]. For the Bianchi V and Bianchi VII classes the radial spinor breaks all supersymmetry, and hence, these are nonsupersymmetric geometries. However, the parameters that characterize these solutions can be chosen in accordance with the stability criterion discussed in [47].

In Sec. IV C we explored the possible conditions to find more interesting Bianchi attractor geometries with massive gauge fields. Solutions with timelike gauge fields and suitable tensor field configurations may give rise to supersymmetry preserving projection conditions in the Killing spinor equation. However, this has not worked so far in the models considered in [35,46]. We hope to explore this more further in future works.

Having constructed some of the simplest supersymmetric Bianchi attractors, it is interesting to find such solutions in theories with more supersymmetry. It will also be interesting to uplift these solutions to higher dimensional supergravity. The Killing spinor equations suggest that, in most cases, if the geometry has an AdS_n part that factorizes, the corresponding Killing spinor is sufficient to preserve the supersymmetry of the whole solution. Having an AdS part may enable the construction of more general Bianchi attractor geometries. Finally, it will be most interesting to construct analytic solutions that interpolate to AdS. A related issue is the embeddability of the Bianchi algebra in the Poincaré or the conformal algebra. The Bianchi I and Bianchi VII algebras are subalgebras of the Poincaré algebra. The other Bianchi algebras have scaling type generators and may presumably be obtained from a truncation of the conformal algebra.

In this work, we studied Bianchi attractors in $d=4$, 5. Earlier works have constructed Bianchi attractors as generalized attractors in gauged supergravity [35,45,46]. In the studies of black holes in ungauged supergravity, there have been studies on the 4d/5d correspondence where the relation between the potential and critical points in $d=4$ and $d=5$ have been elucidated [67]. Similar studies have been performed for gauged supergravity relating black strings in $d=5$ and $\mathrm{AdS}_2\times S^2$ in $d=4$ [68]. It would be interesting to explore the relation between generalized attractor potentials in $d=4$ and $d=5$ and their critical points.

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[18]The magnetic $\mathrm{AdS}_2\times\mathbb{H}^2$ is known to be $\frac{1}{8}$ BPS [50,66].

[19]In the study of the attractor mechanism in $d=5$ ungauged supergravity, it is well known that central charge satisfies an extremization condition at the attractor points [59].

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APPENDIX A: CONVENTIONS

1. Gamma matrices and Spinors in four dimensions

The Clifford algebra in four spacetime dimensions is

$$\{\gamma_a, \gamma_b\} = 2\eta_{ab} \tag{A1}$$

with the metric convention $\eta_{ab} = \{+,-,-,-\}$.[20] The Dirac matrices in four dimensions can be chosen to be

$$\begin{aligned} \gamma^0 &= I_2 \otimes \sigma_1 \\ \gamma^1 &= i\sigma_1 \otimes \sigma_2 \\ \gamma^2 &= i\sigma_2 \otimes \sigma_2 \\ \gamma^3 &= i\sigma_3 \otimes \sigma_2, \end{aligned} \tag{A2}$$

where σ_i, $i = 1, 2, 3$ are the usual Pauli matrices, and I_2 is the two dimensional unit matrix. We define the chirality matrix to be $\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$ and the charge conjugation matrix $C = i\gamma^2\gamma^0 = \gamma^1\gamma^3$. The charge conjugation matrix C has the property $C^t = -C = C^{-1}$.

In four dimensions, we can impose the Weyl condition on a four component spinor such that

$$\begin{aligned} \gamma_5\epsilon_A &= \epsilon_A \\ \gamma_5\epsilon^A &= -\epsilon^A, \end{aligned} \tag{A3}$$

where the conjugate spinor is defined as

$$\epsilon^A = (\epsilon_A)^c = \gamma_0 C^{-1}(\epsilon_A)^* = -\gamma_0 C(\epsilon_A)^*. \tag{A4}$$

[20]We follow the conventions of [48] for $\mathcal{N} = 2, d = 4$ gauged supergravity.

We use the following decomposition of the spinors in some sections. Using the fact that $[\gamma_5, C] = 0$, we can decompose the spinor into simultaneous eigenstates of C and γ_5 as follows

$$\epsilon_A = \begin{pmatrix} 0 \\ C_A^+|+\rangle \end{pmatrix} + \begin{pmatrix} 0 \\ C_A^-|-\rangle \end{pmatrix}, \tag{A5}$$

where C_A^+ and C_A^- are complex coefficients. The two component states $|+\rangle, |-\rangle$

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}, \qquad |-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} \tag{A6}$$

are eigenstates of σ matrices

$$\sigma^1|\pm\rangle = \pm i|\mp\rangle, \qquad \sigma^2|\pm\rangle = \pm|\pm\rangle, \qquad \sigma^3|\pm\rangle = |\mp\rangle. \tag{A7}$$

2. Gamma matrices and spinors in five dimensions

In this section, we summarize our notations and conventions for spinors in five dimensions. We mostly follow our conventions of [52]. The Clifford algebra in five spacetime dimensions is

$$\{\gamma_a, \gamma_b\} = 2\eta_{ab} \tag{A8}$$

where the metric signature that is mostly plus. The Dirac matrices in five dimensions are

$$\begin{aligned} \gamma^0 &= -i\sigma_2 \otimes \sigma_3 \\ \gamma^1 &= -\sigma_1 \otimes \sigma_3 \\ \gamma^2 &= I_2 \otimes \sigma_1 \\ \gamma^3 &= I_2 \otimes \sigma_2 \\ \gamma^4 &= -i\gamma^0\gamma^1\gamma^2\gamma^3 = \sigma_3 \otimes \sigma_3 \end{aligned} \tag{A9}$$

where σ_i, $i = 1, 2, 3$ are the usual Pauli matrices and I_2 is the two dimensional unit matrix. The charge conjugation matrix C has the property $C^t = -C = C^{-1}$ and,

$$C\gamma^a C^{-1} = (\gamma^a)^t \tag{A10}$$

where $C = B\gamma^0$, with $B = \gamma^3$ such that $B^*B = -1$. The spinors in the theory carry an SU(2) index which is raised and lowered using ϵ_{ij}

$$X^j = \epsilon^{ji}X_i, \qquad X_j = X^i\epsilon_{ij,} \tag{A11}$$

with $\epsilon_{12} = \epsilon^{12} = 1$.

Spinors in $d = 5$ satisfy a symplectic majorana condition. To apply this condition, one needs $B^*B = -1$, even number of Dirac spinors ψ_i, $i = 1, \ldots, 2n$ and an antisymmetric real matrix Ω_{ij} with $\Omega^2 = -1_{2n}$. The symplectic majorana condition on a generic spinor reads as

$$\psi_i^* = \Omega_{ij}B\psi_j \tag{A12}$$

or equivalently [52] as

$$\bar{\psi}^i \equiv (\psi_i^*)^t \gamma^0 = (\psi^i)^t C. \tag{A13}$$

For $\mathcal{N} = 2$ supersymmetry $i = 1, 2$, and using $\Omega_{ij} = \epsilon_{ij}$ (A12) reads as

$$\psi_1^* = \gamma^3 \psi_2. \tag{A14}$$

Note that this condition does not reduce the degrees of freedom as compared to a single unconstrained Dirac spinor. This is because one needs at least a pair of Dirac spinors to apply the symplectic majorana condition (A12). However, it does make the R-symmetry manifest.

Antisymmetrization of indices in the Dirac matrices is done with the following convention

$$\gamma_{a_1 a_2 \ldots a_n} = \gamma_{[a_1 a_2 \ldots a_n]} = \frac{1}{n!} \sum_{\sigma \in P_n} \mathrm{Sign}(\sigma) \gamma_{a_{\sigma(1)}} \gamma_{a_{\sigma(2)}} \cdots \gamma_{a_{\sigma(n)}}. \tag{A15}$$

In $d = 5$ only I, γ_a, γ_{ab} form an independent set, other matrices are related by the general identity for $d = 2k + 3$

$$\gamma^{\mu_1 \mu_2 \ldots \mu_s} = \frac{-i^{-k+s(s-1)}}{(d-s)!} \epsilon^{\mu_1 \mu_2 \ldots \mu_s} \gamma_{\mu_{s+1} \ldots \mu_d}. \tag{A16}$$

We also list some useful identities involving various Dirac matrices [69],

$$\begin{aligned} [\gamma_a, \gamma_b] &= 2\gamma_{ab} \\ [\gamma_h, \gamma_{abc}] &= 2\gamma_{habc} \\ [\gamma_{abc}, \gamma_{egh}] &= \eta_{ef} \eta_{gp} \eta_{hk} (2\gamma_{abc}{}^{fpk} - 36\delta_{[ab}{}^{[fp} \gamma_{c]}{}^{k]}). \end{aligned} \tag{A17}$$

APPENDIX B: BIANCHI SOLUTIONS IN 4D GAUGED SUPERGRAVITY

In this section, we list the field equations of the Bianchi I ($\mathrm{AdS}_2 \times \mathbb{R}_2$) solution in $\mathcal{N} = 2, d = 4$ gauged supergravity. We are interested in an attractor type solution where the scalars (z, q) are constants independent of spacetime coordinates and only the hypermultiplets are charged under Abelian gauging. The field equations can be derived from an effective Lagrangian [48]

$$\begin{aligned} \mathcal{L}_{\mathrm{eff}} = &-\frac{1}{2} R + \mathrm{Im} N_{\Lambda\Sigma} F^{\Lambda}_{\mu\nu} F^{\Sigma\mu\nu} - \mathcal{V}(z, \bar{z}, q) \\ &+ g_{XY} K^X_\Lambda K^Y_\Sigma A^\Lambda_\mu A^{\mu\Sigma}. \end{aligned} \tag{B1}$$

1. Bianchi I: $\mathrm{AdS}_2 \times \mathbb{R}_2$

We write the $\mathrm{AdS}_2 \times \mathbb{R}_2$ in a convenient coordinate system as

$$ds^2 = \frac{R_0^2}{\sigma^2}(dt^2 - d\sigma^2) - R_0^2 (dy^2 + d\rho^2). \tag{B2}$$

This metric can be easily supported by an electric gauge field, we choose our gauge field ansatz to be

$$A^\Lambda = \frac{E^\Lambda}{\sigma} dt. \tag{B3}$$

The gauge field equations are

$$g_{XY} K^X_\Lambda K^Y_\Sigma E^\Lambda = 0. \tag{B4}$$

There are $n_v + 1$ equations for the $n_v + 1$ variables E_Λ. At the attractor point, the scalars are constants, as a result all the spacetime derivatives drop and the scalar field equations reduce to the extremization of an effective potential (attractor potential)

$$\begin{aligned} &\frac{\partial}{\partial q^X} \mathcal{V}_{\mathrm{eff}} = 0, \qquad \frac{\partial}{\partial z^i} \mathcal{V}_{\mathrm{eff}} = 0 \\ &\mathcal{V}_{\mathrm{eff}} = \mathcal{V}(z, \bar{z}, q) - g_{XY} K^X_\Lambda K^Y_\Sigma \frac{E^\Lambda E^\Sigma}{R_0^2} + \mathrm{Im} N_{\Lambda\Sigma} \frac{E^\Lambda E^\Sigma}{2R_0^4}. \end{aligned} \tag{B5}$$

There are n_V scalar equations for z^i and $4n_H$ hyperscalar equations for q^X. The Einstein equations are

$$\begin{aligned} 0 &= R_0^2 \mathcal{V}_{\mathrm{eff}} + 2g_{XY} K^X_\Lambda K^Y_\Sigma E^\Lambda E^\Sigma - \mathrm{Im} N_{\Lambda\Sigma} \frac{E^\Lambda E^\Sigma}{R_0^2} \\ 0 &= -R_0^2 \mathcal{V}_{\mathrm{eff}} + \mathrm{Im} N_{\Lambda\Sigma} \frac{E^\Lambda E^\Sigma}{R_0^2} \\ -\frac{1}{R_0^2} &= \mathcal{V}_{\mathrm{eff}}, \end{aligned} \tag{B6}$$

where $\mathcal{V}_{\mathrm{eff}}$ is defined in (B5). The above equations can be recast as the following conditions

$$\begin{aligned} \mathcal{V}(z, \bar{z}, q) &= -\frac{1}{2R_0^2}, \\ \frac{\mathrm{Im} N_{\Lambda\Sigma} E^\Lambda E^\Sigma}{R_0^2} &= -1, \\ g_{XY} K^X_\Lambda K^Y_\Sigma E^\Lambda E^\Sigma &= 0, \end{aligned} \tag{B7}$$

to be satisfied for a given specific model.

2. Nonsupersymmetric electric Bianchi II

In this section, we discuss the supersymmetry conditions for a Bianchi II ($\mathrm{AdS}_2 \times E\mathrm{AdS}_2$) solution of the form

$$ds^2 = \frac{R_0^2}{\sigma^2}(dt^2 - d\sigma^2) - \frac{R_0^2}{\rho^2}(dy^2 + d\rho^2). \tag{B8}$$

As discussed in Sec. II, the symmetries along the spatial directions correspond to that of $E\mathrm{AdS}_2$. Like the previous solution, the $\mathrm{AdS}_2 \times E\mathrm{AdS}_2$ solution can also be constructed using a timelike gauge field (18) as source since it preserves the Bianchi II symmetry along the (y, ρ) directions. The Killing spinor equations on this background are

$$\frac{\gamma^0\sigma}{R_0}\partial_t\epsilon_A-\frac{\gamma^1}{2R_0}\epsilon_A+\frac{iG_A^B\gamma^0}{2R_0}\epsilon_B+iS_{AB}\epsilon^B+\frac{iN}{2R_0^2}\gamma^{01}\epsilon_{AB}\epsilon^B=0,$$
$$\frac{\gamma^1\sigma}{R_0}\partial_\sigma\epsilon_A+iS_{AB}\epsilon^B+\frac{iN}{2R_0^2}\gamma^{01}\epsilon_{AB}\epsilon^B=0,$$
$$\frac{\gamma^2\rho}{R_0}\partial_y\epsilon_A-\frac{\gamma^3}{2R_0}\epsilon_A+iS_{AB}\epsilon^B-\frac{N}{2R_0^2}\gamma^{23}\epsilon_{AB}\epsilon^B=0,$$
$$\frac{\gamma^3\rho}{R_0}\partial_\rho\epsilon_A+iS_{AB}\epsilon^B-\frac{N}{2R_0^2}\gamma^{23}\epsilon_{AB}\epsilon^B=0, \tag{B9}$$

where we have defined the quantities N and G_A^B in (23).

Taking the difference of the first and second equations of (B9), and similarly the difference of the third and fourth equations in (B9), we get the pair of differential equations

$$\frac{\sigma}{R_0}(\gamma^0\partial_t-\gamma^1\partial_\sigma)\epsilon_A-\frac{\gamma^1}{2R_0}\epsilon_A+\frac{iG_A^B\gamma^0}{2R_0}\epsilon_B=0,$$
$$\frac{\rho}{R_0}(\gamma^2\partial_y-\gamma^3\partial_\rho)\epsilon_A-\frac{\gamma^3}{2R_0}\epsilon_A=0. \tag{B10}$$

Since the $\mathrm{AdS}_2\times E\mathrm{AdS}_2$ metric factorizes into a product form, with two radii ρ and σ, we choose a Killing spinor ansatz of the form

$$\epsilon_A=\frac{1}{\rho^m\sigma^n}\chi_A, \tag{B11}$$

where χ_A is a constant spinor, while m, n take real values. This form of the ansatz is also consistent with the Bianchi II symmetry of the metric. Substituting the above in (B10) we get the conditions

$$(2n-1)\gamma^1\epsilon_A+iG_A^B\gamma^0\epsilon_B=0,$$
$$(2m-1)\gamma^3\epsilon_A=0, \tag{B12}$$

that can be solved by

$$m=n=\frac{1}{2},\qquad E^\Lambda P_\Lambda^x=0. \tag{B13}$$

With the ansatz (B11) and the condition (B13), the remaining Killing spinor equations give the conditions

$$(\gamma^1+\gamma^3)\epsilon_A=4iR_0S_{AB}\epsilon^B$$
$$(\gamma^1-\gamma^3)\epsilon_A=\frac{2iN\gamma^{01}}{R_0}\epsilon_{AB}\epsilon^B. \tag{B14}$$

Unlike the $\mathrm{AdS}_2\times\mathbb{R}^2$ case, these conditions are not as simple to work with. However, we can simplify them by multiplying from the left by γ^1 and writing in terms of the charge conjugate matrix $C=\gamma^1\gamma^3$ as

$$(-1+C)\chi_A=4iR_0S_{AB}\gamma^{01}C(\chi_B)^*$$
$$(-C+1)\chi_A=\frac{2iN}{R_0}\epsilon_{AB}(\chi_B)^*, \tag{B15}$$

where we have used $\chi^B=-\gamma_0C(\chi_B)^*$. We now show that the above condition breaks all of supersymmetry. Since $[\gamma_5,C]=0$ (see Sec. A 1), it is convenient to use a decomposition of the spinor χ_A in a basis of simultaneous eigenstates of γ_5 and C as follows

$$\chi_A=\begin{pmatrix}0\\ C_A^+|+\rangle\end{pmatrix}+\begin{pmatrix}0\\ C_A^-|-\rangle\end{pmatrix}, \tag{B16}$$

where C_A^+ and C_A^- are complex coefficients[21] and $|\pm\rangle$ are eigenstates of the Pauli matrices. Substituting in the second equation in (B15), we obtain

$$(1-i)C_A^+|+\rangle+(1+i)C_A^-|-\rangle$$
$$=\frac{2iN}{R_0}\epsilon_{AB}((C_B^+)^*|-\rangle+(C_B^-)^*|+\rangle). \tag{B17}$$

Linear independence of the states $|+\rangle$ and $|-\rangle$ gives rise to the constraints

$$(1-i)C_A^+=\frac{2iN}{R_0}\epsilon_{AB}(C_B^-)^*$$
$$(1+i)C_A^-=\frac{2iN}{R_0}\epsilon_{AB}(C_B^+)^*. \tag{B18}$$

It is straightforward to see that both of these constraints cannot be satisfied simultaneously as their mutual consistency leads to

$$C_A^+\left(1+\frac{2i|N|^2}{R_0^2}\right)=0. \tag{B19}$$

Since $|N|^2$ is real, it follows that the only possible solution is that all the $C_A^\pm$ vanish and hence the metric (B8) breaks all the supersymmetry.

APPENDIX C: BIANCHI I SUPERSYMMETRIC AdS_5 IN GAUGED SUPERGRAVITY

As a warm up, let us begin our analysis with the simplest known AdS_5 metric written in terms of the one forms

$$e^0=Le^rdt,\qquad e^1=Le^r\omega^1,$$
$$e^2=Le^r\omega^2,\qquad e^3=Le^r\omega^3,\qquad e^4=Ldr, \tag{C1}$$

where L is the AdS scale. The invariant one forms

$$\omega^i=dx^i,\qquad i=1,2,3, \tag{C2}$$

[21]Since ($A=1$, 2) there are eight independent constants in ϵ_A as it should be for a $\mathcal{N}=2$ spinor in four dimensions.

and all commute with one another and satisfy $d\omega^i = 0$, a characteristic of the Bianchi I algebra. Since we are discussing the $U(1)_R$ case, the gaugino conditions are trivial.[22]

The Killing spinor equation (63) in the background (C1) reads as,

$$\begin{aligned} e^{-r}\gamma_0\partial_t\epsilon_i - \frac{1}{2}\gamma_4\epsilon_i - \frac{i}{\sqrt{6}}Lg_R\epsilon_i{}^k\epsilon_k &= 0, \\ e^{-r}\gamma_1\partial_{x^1}\epsilon_i + \frac{1}{2}\gamma_4\epsilon_i + \frac{i}{\sqrt{6}}Lg_R\epsilon_i{}^k\epsilon_k &= 0, \\ e^{-r}\gamma_2\partial_{x^2}\epsilon_i + \frac{1}{2}\gamma_4\epsilon_i + \frac{i}{\sqrt{6}}Lg_R\epsilon_i{}^k\epsilon_k &= 0, \\ e^{-r}\gamma_3\partial_{x^3}\epsilon_i + \frac{1}{2}\gamma_4\epsilon_i + \frac{i}{\sqrt{6}}Lg_R\epsilon_i{}^k\epsilon_k &= 0, \\ \gamma_4\partial_r\epsilon_i + \frac{i}{\sqrt{6}}Lg_R\epsilon_i{}^k\epsilon_k &= 0. \end{aligned} \tag{C3}$$

The following equations can be obtained after some algebraic manipulations

$$\begin{aligned} \gamma_0\partial_t\epsilon_i + \gamma_a\partial_{x^a}\epsilon_i &= 0, \\ \gamma_a\partial_{x^a}\epsilon_i - \gamma_b\partial_{x^b}\epsilon_i &= 0, \\ \gamma_4\partial_r\epsilon_i + e^{-r}\gamma_0\partial_t\epsilon_i - \frac{1}{2}\gamma_4\epsilon_i &= 0, \\ \gamma_4\partial_r\epsilon_i - e^{-r}\gamma_a\partial_{x^a}\epsilon_i - \frac{1}{2}\gamma_4\epsilon_i &= 0, \end{aligned} \tag{C4}$$

where $a = 1, 2, 3$. There are two independent solutions to the above equations

$$\epsilon_i = e^{\frac{r}{2}}\zeta_i^+, \qquad \gamma_4\zeta_i^+ = \zeta_i^+ \tag{C5}$$

$$\epsilon_i = (e^{-\frac{r}{2}} + e^{\frac{r}{2}}(x^m\gamma_m))\zeta_i^-, \qquad \gamma_4\zeta_i^- = -\zeta_i^-. \tag{C6}$$

Each of the spinors (C5) and (C6) preserves $\frac{1}{2}$ the supersymmetry and the full solution enjoys a $\mathcal{N} = 2$ supersymmetry. Substituting the above in (C3) we get the consistency condition

$$\zeta_i^\pm = \mp\frac{2i}{\sqrt{6}}Lg_R\epsilon_i{}^k\zeta_k^\pm. \tag{C7}$$

It follows that (note that $\epsilon_i{}^k\epsilon_k^l = -\delta_i^l$)

$$\left(1 - \frac{2}{3}L^2g_R^2\right)\zeta_i^\pm = 0. \tag{C8}$$

This of course is the equation of motion for AdS_5 metric, thus we see that supersymmetry conditions automatically guarantee the equation of motion.

[22]For more general AdS critical points see [41,60].

APPENDIX D: NONSUPERSYMMETRIC SOLUTIONS IN 5D GAUGED SUPERGRAVITY

1. Bianchi III—vacuum $AdS_3 \times \mathbb{H}^2$

In this section, we present the killing spinor equations for the vacuum $AdS_3 \times \mathbb{H}^2$ solution. This however breaks supersymmetry explicitly, unlike the charged case. The simplified equations (88) are

$$\begin{aligned} \gamma_0\partial_t\epsilon_i + \gamma_2\partial_{x^3}\epsilon_i &= 0, \\ e^{-r}\gamma_0\partial_t\epsilon_i - \frac{\gamma_4}{2}\epsilon_i + \gamma_4\partial_r\epsilon_i &= 0, \\ e^{-r}\gamma_2\partial_{x^3}\epsilon_i + \frac{\gamma_4}{2}\epsilon_i - \gamma_4\partial_r\epsilon_i &= 0, \\ e^{x^1}\gamma_{13}\partial_{x^2}\epsilon_i + \frac{\epsilon_i}{2} + \partial_{x^1}\epsilon_i &= 0. \end{aligned} \tag{D1}$$

Any solution necessarily depends on the $\mathbb{H}^2$ coordinates and breaks supersymmetry. The AdS_3 part of the equations [first three of (D1)] are solved by the usual

$$\begin{aligned} &\epsilon_i = e^{\frac{r}{2}}\zeta_i^+, \qquad \gamma_4\zeta_i^+ = \zeta_i^+ \\ &\epsilon_i = (e^{\frac{r}{2}}(\gamma_0 t + \gamma_2 x^3) + e^{-\frac{r}{2}})\zeta_i^-, \qquad \gamma_4\zeta_i^- = -\zeta_i^-, \end{aligned} \tag{D2}$$

whereas the $\mathbb{H}^2$ part of the equations [the last equation in (D1)] are solved by

$$\begin{aligned} &\epsilon_i = e^{-\frac{x^1}{2}}\zeta_i^-, \qquad \gamma_3\zeta_i^- = -\zeta_i^- \\ &\epsilon_i = (e^{-\frac{x^1}{2}}\gamma_1 x^2 + e^{\frac{x^1}{2}})\zeta_i^+, \qquad \gamma_3\zeta_i^+ = \zeta_i^+. \end{aligned} \tag{D3}$$

We see that $\zeta^\pm$ are required to be simultaneous eigenspinors of both γ_3 and γ_4 in order to solve the full set of equations (D1).[23] However, that is impossible since the matrices anticommute. Thus, the product space in the vacuum case breaks all supersymmetry. In the charged case, we are able to avoid the spinor being an eigenspinor of γ_3 due to the condition (91). This is consistent with the conclusion from the integrability condition eq. 31 of [35] that AdS_5 is the unique maximally supersymmetric vacuum solution in the theory.

2. Bianchi V

The Bianchi V solution constructed in [13] is of the form

$$\begin{aligned} &e^0 = Le^{\beta_t r}dt, \qquad e^1 = L\omega^1, \qquad e^2 = L\omega^2, \\ &e^3 = L\omega^3, \qquad e^4 = Ldr, \end{aligned} \tag{D4}$$

where the invariant one forms are given by

$$\omega^1 = e^{-x^1}dx^2, \qquad \omega^2 = e^{-x^1}dx^3, \qquad \omega^3 = dx^1. \tag{D5}$$

The Bianchi V geometry in this case has the form of $AdS_2 \times \mathbb{H}^3$. The metric is sourced by a massless timelike gauge field

[23]The general solution is a combination of (D2) and (D3).

$$A^I = E^I e^0. \quad \text{(D6)}$$

The Killing spinor equations in this background take the form ($Z = E^I h_I$)

$$\begin{aligned}
&e^{-\beta_t r}\gamma_0\partial_t\epsilon_i - \frac{\beta_t}{2}\gamma_4\epsilon_i + g_R L E^I V_I \gamma_0 \epsilon_i{}^k\epsilon_k \\
&\quad + \frac{i}{\sqrt{6}}(\beta_t Z\gamma_{04}\epsilon_i - L g_R \epsilon_i{}^k\epsilon_k) = 0, \\
&e^{x^1}\gamma_1\partial_{x^2}\epsilon_i - \frac{1}{2}\gamma_3\epsilon_i + \frac{i}{\sqrt{6}}\left(\frac{\beta_t}{2}Z\gamma_{04}\epsilon_i + L g_R\epsilon_i{}^k\epsilon_k\right) = 0, \\
&e^{x^1}\gamma_2\partial_{x^3}\epsilon_i - \frac{1}{2}\gamma_3\epsilon_i + \frac{i}{\sqrt{6}}\left(\frac{\beta_t}{2}Z\gamma_{04}\epsilon_i + L g_R\epsilon_i{}^k\epsilon_k\right) = 0, \\
&\gamma_3\partial_{x^1}\epsilon_i + \frac{i}{\sqrt{6}}\left(\frac{\beta_t}{2}Z\gamma_{04}\epsilon_i + L g_R\epsilon_i{}^k\epsilon_k\right) = 0, \\
&\gamma_4\partial_r\epsilon_i - \frac{i}{\sqrt{6}}(\beta_t Z\gamma_{04}\epsilon_i - L g_R\epsilon_i{}^k\epsilon_k) = 0.
\end{aligned} \quad \text{(D7)}$$

We can write down the following differential equations after some algebraic manipulations

$$\begin{aligned}
&e^{-\beta_t r}\gamma_0\partial_t\epsilon_i + \gamma_4\partial_r\epsilon_i - \frac{\beta_t}{2}\gamma_4\epsilon_i + g_R L E^I V_I\gamma_0\epsilon_i{}^k\epsilon_k = 0, \\
&\gamma_1\partial_{x^2}\epsilon_i - \gamma_2\partial_{x^3}\epsilon_i = 0, \\
&e^{x^1}\gamma_{13}\partial_{x^2}\epsilon_i + \partial_{x^1}\epsilon_i + \frac{\epsilon_i}{2} = 0, \\
&e^{x^1}\gamma_{23}\partial_{x^3}\epsilon_i + \partial_{x^1}\epsilon_i + \frac{\epsilon_i}{2} = 0.
\end{aligned} \quad \text{(D8)}$$

Following the arguments given in the previous section, we can solve the AdS_2 part of the equations [first in (D8)] by

$$\begin{aligned}
&\epsilon_i = e^{\frac{\beta_t r}{2}}\zeta_i^+, \qquad \gamma_4\zeta_i^+ = \zeta_i^+ \\
&\epsilon_i = (e^{\frac{\beta_t r}{2}}\gamma_0 t + e^{-\frac{\beta_t r}{2}})\zeta_i^-, \qquad \gamma_4\zeta_i^- = -\zeta_i^-,
\end{aligned} \quad \text{(D9)}$$

provided we set $E^I V_I = 0$. If $E^I V_I \neq 0$ in this case, even the radial spinor breaks all supersymmetry. Similarly the $\mathbb{H}^3$ part of the equations [last three of (D8)] can be solved by

$$\begin{aligned}
&\epsilon_i = e^{-\frac{x^1}{2}}\zeta_i^-, \quad \gamma_3\zeta_i^- = -\zeta_i^- \\
&\epsilon_i = (e^{-\frac{x^1}{2}}(\gamma_1 x^2 + \gamma_2 x^3) + e^{\frac{x^1}{2}})\zeta_i^+, \quad \gamma_3\zeta_i^+ = \zeta_i^+.
\end{aligned} \quad \text{(D10)}$$

Once again, we see that the $\zeta_\pm$ are required to be simultaneous eigenspinors of γ_3 and γ_4, that is impossible since the matrices do not commute.[24] Thus, the solution breaks all supersymmetry. The same arguments apply for the vacuum Bianchi V $AdS_2 \times \mathbb{H}^3$ solution.

[24] In this case too, the general solution of (D8) is a combination of (D9) and (D10).

3. Bianchi VII

The Bianchi VII metric is expressed in terms of the following one forms

$$\begin{aligned}
&e^0 = L e^{\beta_t r}dt, \qquad e^1 = L dx^1, \\
&e^2 = L e^{\beta r}(\cos(x^1)dx^2 + \sin(x^1)dx^3), \\
&e^3 = L\lambda e^{\beta r}(-\sin(x^1)dx^2 + \cos(x^1)dx^3), \\
&e^4 = L dr,
\end{aligned} \quad \text{(D11)}$$

where λ is a squashing parameter. The gauge field ansatz is of the form

$$A^I = B^I e^2, \quad \text{(D12)}$$

where B^I are constants. The Killing spinor equations in the above background take the form ($Z = h_I B^I$)

$$\begin{aligned}
&e^{-\beta_t r}\gamma_0\partial_t\epsilon_i - \frac{\beta_t}{2}\gamma_4\epsilon_i \\
&\quad + \frac{i}{\sqrt{6}}\left(\frac{Z}{2}\left(\beta\gamma_{24} - \frac{\gamma_{13}}{\lambda}\right)\epsilon_i - L g_R\epsilon_i{}^k\epsilon_k\right) = 0, \\
&\gamma_1\partial_{x^1}\epsilon_i - \frac{(1+\lambda^2)}{4\lambda}\gamma_{123}\epsilon_i \\
&\quad - \frac{i}{\sqrt{6}}\left(\frac{Z}{2}\left(\beta\gamma_{24} + \frac{2\gamma_{13}}{\lambda}\right)\epsilon_i - L g_R\epsilon_i{}^k\epsilon_k\right) = 0, \\
&e^{-\beta r}\gamma_2(\cos x^1\partial_{x^2} + \sin x^1\partial_{x^3})\epsilon_i \\
&\quad + \frac{(1-\lambda^2)}{4\lambda}\gamma_{123}\epsilon_i + \frac{\beta}{2}\gamma_4\epsilon_i + L g_R B^I V_I\gamma_2\epsilon_i{}^k\epsilon_k \\
&\quad + \frac{i}{\sqrt{6}}\left(\frac{Z}{2}\left(\frac{\gamma_{13}}{\lambda} + 2\beta\gamma_{24}\right)\epsilon_i + L g_R\epsilon_i{}^k\epsilon_k\right) = 0, \\
&e^{-\beta r}\gamma_3(-\sin x^1\partial_{x^2} + \cos x^1\partial_{x^3})\epsilon_i - \frac{(1-\lambda^2)}{4\lambda}\gamma_{123}\epsilon_i + \frac{\beta}{2}\gamma_4\epsilon_i \\
&\quad + \frac{i}{\sqrt{6}}\left(-\frac{Z}{2}\left(\beta\gamma_{24} + \frac{2\gamma_{13}}{\lambda}\right)\epsilon_i + L g_R\epsilon_i{}^k\epsilon_k\right) = 0, \\
&\gamma_4\partial_r\epsilon_i + \frac{i}{\sqrt{6}}\left(\frac{Z}{2}\left(\frac{\gamma_{13}}{\lambda} + 2\beta\gamma_{24}\right)\epsilon_i + L g_R\epsilon_i{}^k\epsilon_k\right) = 0.
\end{aligned} \quad \text{(D13)}$$

After a few algebraic steps, we get the following differential equations

$$\begin{aligned}
&e^{-\beta r}\gamma_3(-\sin x^1\partial_{x^2} + \cos x^1\partial_{x^3})\epsilon_i - \gamma_1\partial_{x^1}\epsilon_i \\
&\quad + \frac{\lambda}{2}\gamma_{123}\epsilon_i + \frac{\beta}{2}\gamma_4\epsilon_i = 0, \\
&e^{-\beta r}\gamma_2(\cos x^1\partial_{x^2} + \sin x^1\partial_{x^3})\epsilon_i \\
&\quad + \frac{(1-\lambda^2)}{4\lambda}\gamma_{123}\epsilon_i - \gamma_4\partial_r\epsilon_i + \frac{\beta}{2}\gamma_4\epsilon_i \\
&\quad + L g_R B^I V_I\gamma_2\epsilon_i{}^k\epsilon_k = 0,
\end{aligned} \quad \text{(D14)}$$

that are solved by the radial spinor

$$B^I V_I = 0, \qquad \gamma_{1234}\epsilon_i = \epsilon_i, \qquad \beta = \lambda$$
$$\epsilon_i = e^{\frac{(3\beta^2-1)}{4\beta}r}\zeta_i. \tag{D15}$$

Substituting (D15) back into (D13) we obtain

$$-\frac{\beta_t}{2}\gamma_4\epsilon_i + \frac{i}{\sqrt{6}}\left(\frac{Z}{2}\frac{(\beta^2-1)}{\beta}\gamma_{24}\epsilon_i - Lg_R\epsilon_i{}^k\epsilon_k\right) = 0,$$
$$-\left(\frac{1+\beta^2}{4\beta}\right)\gamma_4\epsilon_i + \frac{i}{\sqrt{6}}\left(\frac{Z}{2}\frac{(\beta^2+2)}{\beta}\gamma_{24}\epsilon_i - Lg_R\epsilon_i{}^k\epsilon_k\right) = 0,$$
$$\frac{3\beta^2-1}{4\beta}\gamma_4\epsilon_i + \frac{i}{\sqrt{6}}\left(\frac{Z}{2}\frac{(2\beta^2+1)}{\beta}\gamma_{24}\epsilon_i + Lg_R\epsilon_i{}^k\epsilon_k\right) = 0. \tag{D16}$$

The equations (D16) lead to the projections

$$\gamma_4\zeta_i = -i\epsilon_i{}^k\zeta_k, \qquad (\beta_t + 2\beta)^2 = 6L^2 g_R^2$$
$$\gamma_2\zeta_i = -i\zeta_i, \qquad \left(\frac{\beta^2-1}{\beta^2+1}\right)^2 = \frac{3Z^2}{2},$$
$$\beta_t = \frac{\beta^4 + 4\beta^2 - 1}{2\beta(1+\beta^2)}. \tag{D17}$$

It is clear that the additional projection condition due to γ_2 breaks all of the supersymmetry. Thus, the Bianchi VII solution (D11) is nonsupersymmetric. However, the Bianchi VII algebra is a subalgebra of the Poincaré algebra, and hence, it is also part of the super Poincaré algebra. It is possible that there are more general solutions in this class that may be supersymmetric.

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