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Overcharging higher-dimensional black holes with point particles

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We investigate the possibility of overcharging spherically symmetric black holes in spacetime dimensions $D > 4$ by the capture of a charged particle. We generalize Wald's classic result that extremal black holes cannot be overcharged. For nearly extremal black holes, we also generalize Hubeny's scenario by showing that overcharging is possible in a small region of parameter space. We check how D affects the overcharging parameter space and find that this appears to shrink in the large-D limit, which suggests that overcharging becomes increasingly difficult in higher dimensions.

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I. INTRODUCTION

The cosmic censorship conjecture [1] is a long-standing open question in classical general relativity that continues to attract interest. Loosely speaking, the conjecture asserts that singularities arising from gravitational collapse are always hidden behind event horizons. It is useful in so much as it affords general relativity predictability as a classical theory. Its violation would compel input from a more complete and as-yet-undiscovered theory—quantum gravity—in order to make predictive statements on observables pertaining to events at or near singularities.

That it remains a challenging open conjecture is due in part to its lack of a rigorous formulation. In spite of this, or perhaps because of this, there has been no shortage of efforts to find counterexamples [2–21]. Nonetheless, the cosmic censorship conjecture is widely believed to be true, and the challenge to those who adhere to it is to explain why the counterexamples flounder upon closer scrutiny, and more importantly, to understand the mechanism that enforces cosmic censorship. Most of these counterexamples require some degree of fine-tuned initial data. An exception, however, can be found in higher dimensions. Higher-dimensional ($D > 4$) black hole analogs such as black strings, black rings, and p-branes have long been demonstrated to be unstable to small perturbations, and recent numerical evidence appears to indicate that they lead to the formation of naked singularities [22–24]. The end state of this instability is widely considered to be the only known generic violation of cosmic censorship so far [25].

A common approach to unveiling the singularities of black holes is by making them absorb point particles with certain properties. The seminal work by Wald [10] was the first to seriously explore this possibility. Black hole overcharging is the process by which a test particle of sufficient mass m, charge q, and energy E falls down a charged black hole of mass M and charge Q, overcomes the electrostatic repulsion between them, and is absorbed by the black hole, ultimately resulting in a spacetime that no longer corresponds to a black hole but a naked singularity. Wald demonstrated that when one tries to overcharge an extremal Kerr-Newman black hole in this way, however, the electrostatic repulsion prevents the particle from crossing the event horizon [10]. Many years later, Hubeny [15] revisited this scenario and discovered that if one starts with a nearly extremal Reissner-Nordström black hole instead of an extremal one, then overcharging is possible.

The same situation holds for spinning black holes. A black hole is overspun when it absorbs a test body with enough angular momentum such that the resulting metric after absorption is that of a naked singularity. Again, in Ref. [10], it was demonstrated that overspinning is impossible for an extremal Kerr-Newman black hole because the test body cannot overcome the spin-spin repulsion between the particle and the black hole. In the same spirit as Hubeny, Jacobson and Sotiriou [16] found that starting with near-extremality evades this restriction.

Overcharging and overspinning share the same fine-tuning flaw of other mechanisms for creating naked singularities; only for an infinitesimally narrow region of parameter space do they succeed. A more serious shortcoming though, already acknowledged in Refs. [15,16], is that both analyses rely on the test-particle approximation. It has been widely believed then that finite-charge and finite-mass corrections to the dynamics of the point particle drastically affect the outcomes of both scenarios [27]. These corrections are known as self-force effects. The self-force on a particle moving in a curved spacetime arises from the interaction between the particle and the fields it produces. In general, it cannot be computed straightforwardly [28–33], though the technology for such computations has progressed tremendously in recent years and remains an active area of research [34].

A more complete picture of the influence of the electromagnetic self-force on the Hubeny overcharging scenario was revealed by the work of Zimmerman *et al.* [20]. There it was shown that the self-force prevents the test

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particle from crossing the event horizon, becoming strongly repulsive as the charged particle gets close to the event horizon, and creating a turning point in the trajectory of the particle just as it is about to overcharge the black hole. Colleoni *et al.* reached similar conclusions in their careful study of the inclusion of gravitational self-force effects in the Kerr overspinning scenario [35]. This corroborates and completes the work by Barausse *et al.* that looked into the impact of dissipative self-force effects [36,37]. The emerging picture thus confirms the expectation that four-dimensional black holes are immune to overcharging or overspinning by point particles, and that indeed it is the self-force that acts as the cosmic censor in these scenarios.

In this short paper, we contribute to this developing narrative by extending the Hubeny overcharging scenario to higher-dimensional black holes. Indeed, the fact that generic violations of cosmic censorship occur in the non-linear evolution of higher-dimensional black objects suggests that higher dimensions might be a more fertile arena for seeking out violations. However, earlier work on extending the overspinning process to higher dimensions by Bouhmadi-Lopez *et al.* concluded that potentially destructive point particles with large angular momenta are not captured by the extremal Myers-Perry black holes. Higher-dimensional overspinning in the extremal case cannot succeed. To the best of our knowledge, the analogous overcharging scenario is yet to be extended to dimensions $D > 4$. (See, however, Ref. [9] for an overcharging by charged thin shells.) We find that, just like in four dimensions, overcharging an extremally charged black hole is impossible in the test-particle limit. We also show that Hubeny's conclusions in $D = 4$ extend to higher dimensions as well: there exist charged test particles that can overcharge a nearly extremal charged Schwarzschild-Tangherlini black hole.

Very recent work by Sorce and Wald [38] showed quite generally that overcharging and overspinning of nearly extremal black holes cannot occur, though it is not clear if this statement extends to all dimensions. For the extremal case (including higher dimensions), Natario *et al.* [39] showed that overcharging and overspinning fails for test fields satisfying the null energy condition at the horizon.

The rest of the paper proceeds as follows. To set the stage, we first briefly review the Hubeny overcharging scenario in a nearly extremal Reissner-Nordström black hole. We then look at generalizing this situation to a charged Schwarzschild-Tangherlini black hole. We work out the kinematics of charged particle infall in this background. From these we derive the conditions for overcharging these black holes—what we call generalized Hubeny inequalities—and show that these cannot be satisfied when the black hole is extremal, but can be satisfied for a small region of parameter space when the black hole is nearly extremal. These generalize the Wald and Hubeny results to $D > 4$. We also check how the overcharging parameter space depends on D. Finally, we summarize with a discussion of our results.

Throughout this paper our metric signature is mostly plus $(-,+,+,\cdots,+,+)$. For consistency with past work in $D = 4$, we adopt geometric units in which $G_D = c = 1$.

II. THE HUBENY SCENARIO

We first recall the Reissner-Nordström (RN) line element in the usual Schwarzschild coordinates, which describes the spacetime of a charged, asymptotically flat solution to the Einstein-Maxwell equations. The line element reads

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2, \tag{1}$$

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \tag{2}$$

and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric on the unit two-sphere. This solution represents a black hole of mass M and charge Q possessing an event horizon at $r = r_+ := M + \sqrt{M^2 - Q^2}$. The black hole supports an electromagnetic field and four-potential whose only nonzero components are $F_{tr} = -Q/r^2$ and $A_t = -Q/r$, respectively.

When $Q > M$, there is no event horizon, and the Reissner-Nordström solution bears its curvature singularity located at $r = 0$ to the outside universe. The case $Q = M$ represents an extremal RN black hole, and when $(Q - M)/M \ll 1$, we have a nearly extremal RN black hole. Hubeny parametrizes near-extremality by relating the mass and charge of the black hole as $Q = M - 2\epsilon^2$ and requiring that $0 < \epsilon \ll 1$. Extremal RN black holes are then those for which $\epsilon = 0$.

The Hubeny scenario consists of a test charge, with mass m and charge q, falling radially towards a nearly extremal Reissner-Nordström black hole. This radial infall proceeds according to the equation of motion

$$ma^\alpha = qF^\alpha{}_\beta u^\beta, \tag{3}$$

whereby the charged particle is met by the electrostatic repulsion coming from the black hole. For overcharging to occur, the charged particle must overcome this repulsion and accrete its energy E and charge q onto the black hole so that the latter's final mass and charge are $M + E$ and $Q + q$, respectively. When

$$Q + q > M + E, \tag{4}$$

the final state is said to be overcharged.

For the particle to cross the event horizon, its four-velocity, $u^\alpha = (\dot{t}, \dot{r}, 0, 0)$, must satisfy the conditions

(i) $\dot{r}^2 > 0 \ \forall \ r \geq r_+$, and
(ii) $\dot{t} > 0 \ \forall \ r > r_+$.

The overdot means differentiation with respect to the particle's proper time. Condition (i) simply means that no turning point exists in its trajectory before it enters the horizon. The particle must have sufficient energy E to overcome electrostatic repulsion. Condition (ii) ensures that the four-velocity is future-pointing at the horizon. Taking Eq. (4) and conditions (i) and (ii) together, one can derive the following inequalities:

$$q > \frac{r_+ - Q}{2}, \tag{5a}$$

$$\frac{qQ}{r_+} < E < q + Q - M, \tag{5b}$$

$$m < Q\sqrt{\frac{2MEq - Q(E^2 + q^2)}{Q(M^2 - Q^2)}}. \tag{5c}$$

These are the conditions for overcharging, and are constraints on the particle parameter space $\{m, E, q\}$. Particles with mass, energy, and charge $\{m, E, q\}$ within this domain are then said to produce an overcharged final state, i.e. a naked singularity, from a RN black hole of mass and charge $\{M, Q\}$.

In addition to these conditions, one must bear in mind that the test-particle approximation has to be valid, i.e., the test particle's stress-energy tensor must not significantly disturb the background spacetime. Thus, the particle parameters $\{m, E, q\}$ have to be much smaller than black hole parameters $\{M, Q\}$.

In the extremal limit, the inequalities (5a)–(5c) reduce to

$$q > 0, \tag{6a}$$

$$q < E < q, \tag{6b}$$

$$m < \infty, \tag{6c}$$

which admit no solution. We therefore recover Wald's result that extremal RN black holes cannot be overcharged.If the RN black hole is nearly extremal, so that $Q = M(1 - 2\epsilon^2)$, Hubeny showed how to obtain a solution to Eqs. (5a)–(5c). In particular, for any nonzero $\epsilon \ll 1$ and for $M = 1$, the choices

$$q = a\epsilon, \qquad a > 1, \tag{7a}$$

$$E = a\epsilon - 2b\epsilon^2, \qquad 1 < b < a, \tag{7b}$$

$$m = c\epsilon, \qquad c < \sqrt{a^2 - b^2} \tag{7c}$$

will satisfy the above inequalities [15,40].

III. OVERCHARGING IN D DIMENSIONS

In this section, we show that in D dimensions the Hubeny inequalities generalize to

$$q > r_+^{D-3}\left(\frac{M - \omega_D Q}{\omega_D r_+^{D-3} - Q/(D-3)}\right), \tag{8}$$

$$\frac{qQ}{(D-3)r_+^{D-3}} < E < \omega_D(Q + q) - M, \tag{9}$$

and,

$$m < \omega_D Q\sqrt{\frac{2MEq - Q(E^2 + \omega_D^2 q^2)}{Q(M^2 - \omega_D^2 Q^2)}}, \tag{10}$$

where

$$r_+^{D-3} = \frac{M}{(D-3)\omega_D^2}\left(1 + \sqrt{1 - \frac{\omega_D^2 Q^2}{M^2}}\right), \tag{11}$$

and ω_D is defined as

$$\omega_D = \sqrt{\frac{(D-2)}{(D-3)}\frac{\Omega_{(D-2)}}{8\pi}}. \tag{12}$$

Here $\Omega_{(D-2)}$ is the volume of the unit $(D-2)$-sphere defined as $\Omega_{(D-2)} = 2\pi^{(D-1)/2}/\Gamma((D-1)/2)$. It is easy to check that $\omega_D = 1$ in $D = 4$ and that these inequalities correctly reduce to the Hubeny inequalities in $D = 4$.

A. Einstein-Maxwell action

The Einstein-Maxwell equations in D dimensions arise from the action

$$S = S_{\rm g} + S_{\rm EM} + S_{\rm m} + S_{\rm int}, \tag{13}$$

where

$$S_{\rm g} = \frac{\kappa_0}{G_D}\int d^D x\sqrt{-g}R, \tag{14}$$

$$S_{\rm EM} = -\kappa_0\int d^D x\sqrt{-g}F^{\mu\nu}F_{\mu\nu}, \tag{15}$$

$$S_{\rm m} = -m\int_\gamma d\tau, \tag{16}$$

$$S_{\rm int} = \int d^D x\sqrt{-g}A_\mu j^\mu. \tag{17}$$

We choose the value of κ_0 to be

$$\kappa_0 \coloneqq \frac{1}{4\Omega_{(D-2)}}. \tag{18}$$

For a particle with charge q moving along a worldline γ, which is parametrized by $x^\mu = z^\mu(\lambda)$ for some arbitrary parameter λ,

$$j^\mu = q\int_\gamma d\lambda \frac{dz^\mu}{d\lambda}\frac{\delta^{(D)}(x^\mu - z^\mu(\lambda))}{\sqrt{-g}}. \tag{19}$$

This action yields the field equations

$$G_{\alpha\beta} = 8\pi G_D T_{\alpha\beta}, \tag{20}$$

$$F^{\alpha\beta}{}_{;\beta} = \Omega_{(D-2)} j^\alpha, \tag{21}$$

$$ma^\alpha = qF^\alpha{}_\beta u^\beta, \tag{22}$$

upon imposing stationarity of the action with respect to $g_{\alpha\beta}$, A_μ and $z^\mu(\lambda)$, respectively. The stress-energy tensor in our chosen units is given by

$$T_{\alpha\beta} = \frac{1}{\Omega_{(D-2)}}\left(F_{\alpha\mu}F^\mu_\beta - \frac{1}{4}g_{\alpha\beta}F^{\mu\nu}F_{\mu\nu}\right). \tag{23}$$

B. Charged Schwarzschild-Tangherlini black holes in D dimensions

The D-dimensional analogue to the Reissner-Nordström solution, or the charged Schwarzschild-Tangherlini solution, is again parametrized by a mass M and charge Q. Its line element is given by

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2_{D-2} \tag{24}$$

where

$$f(r) = 1 - \frac{\mu}{r^{D-3}} + \frac{\xi^2}{r^{2(D-3)}}, \tag{25}$$

and

$$\mu = \frac{16\pi M}{(D-2)\Omega_{(D-2)}}, \tag{26}$$

$$\xi = \left(\frac{8\pi}{\Omega_{(D-2)}(D-2)(D-3)}\right)^{1/2} Q, \tag{27}$$

with M and Q being the Arnowitt-Deser-Misner (ADM) mass and charge of the black hole. Finally,

$$d\Omega^2_{D-2} = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \cdots + \sin^2\theta_1 \cdots \sin^2\theta_{D-3} d\theta^2_{D-2}$$

is the line element of the unit $(D-2)$-sphere. When $D = 4$, this solution reduces to the expected Reissner-Nordström solution i.e. Eqs. (1)–(2). This solution supports an electromagnetic field and potential whose only nonzero components are $F_{tr} = -Q/r^{D-2}$ and $A_t = -Q/((D-3)r^{D-3})$.

The metric function f has an outer root

$$r_+^{D-3} = \frac{\mu}{2}\left(1 + \sqrt{1 - \frac{4\xi^2}{\mu^2}}\right), \tag{28}$$

which locates the event horizon. The location of the event horizon in terms of the black hole mass and charge is

$$r_+^{D-3} = \frac{8\pi M}{(D-2)\Omega_{(D-2)}}\left(1 + \sqrt{1 - \frac{(D-2)}{(D-3)}\frac{\Omega_{(D-2)}}{8\pi}\frac{Q^2}{M^2}}\right). \tag{29}$$

While taking note of the definition of ω_D as

$$\omega_D = \sqrt{\frac{(D-2)}{(D-3)}\frac{\Omega_{(D-2)}}{8\pi}}, \tag{30}$$

we can rewrite the location of the horizon as

$$r_+^{D-3} = \frac{M}{(D-3)\omega_D^2}\left(1 + \sqrt{1 - \frac{\omega_D^2 Q^2}{M^2}}\right). \tag{31}$$

An event horizon exists only when

$$M \geq \omega_D Q. \tag{32}$$

The extremal state occurs when the equality holds in relation (32). The overcharged state occurs when this is violated or when

$$Q > \omega_D^{-1} M. \tag{33}$$

C. Crossing conditions for charged particle infall

The setup for overcharging proceeds exactly as in Hubeny. We consider a particle of a certain mass m and charge q radially falling into the black hole with just the right parameters so that it crosses the horizon r_+ and produces a spacetime that violates Eq. (32). Our goal now is to generalize the Hubeny inequalities in Eqs. (5a)–(5c) to D spacetime dimensions.

The equation of motion for the charged particle remains $ma^\alpha = qF^\alpha{}_\beta u^\beta$. For a radial trajectory $z^\alpha = (T(\tau), R(\tau), 0, \ldots, 0)$, where (τ) is the proper time, the point charge has a velocity

$$u^\alpha = \frac{dz^\alpha}{d\tau} = (\dot{T}(\tau), \dot{R}(\tau), 0, \ldots, 0), \tag{34}$$

and momentum given by

$$p_\alpha = \left(-mf\dot{T} - \frac{qQ}{(D-3)r^{D-3}}, f^{-1}\dot{R}, 0, \dots, 0\right). \quad (35)$$

Associated with the time-like Killing vector of the spacetime, $\xi^\alpha_{(t)} = (1, 0, \dots, 0)$, is a constant of motion given by

$$E = -p_\alpha \xi^\alpha_{(t)} = mf\dot{T} + \frac{qQ}{(D-3)r^{D-3}}. \quad (36)$$

From this, we get

$$\dot{T} = \frac{1}{mf}\left(E - \frac{qQ}{(D-3)r^{D-3}}\right). \quad (37)$$

Moreover, from the normalization of the velocity, $u_\alpha u^\alpha = -1$, the equation of motion for $R(\tau)$ becomes

$$\dot{R}^2 = \frac{1}{m^2}\left(E - \frac{qQ}{(D-3)r^{D-3}}\right)^2 - f(r). \quad (38)$$

For the particle to cross the horizon, it is sufficient to require that $\dot{T} > 0$ for $r > r_+$ and $\dot{R}^2 > 0$ for all $r \geq r_+$. Evaluating Eq. (38) at $r = r_+$, we get

$$E > \frac{qQ}{(D-3)r_+^{D-3}}. \quad (39)$$

From Eq. (38), the condition $\dot{R}^2 > 0$ for all $r > r_+$ can be written as

$$m^2 < \frac{1}{f(r)}\left(E - \frac{qQ}{(D-3)r^{D-3}}\right)^2, \quad \forall\ r > r_+. \quad (40)$$

The minimum of the right-hand side occurs at

$$r_m^{D-3} = \frac{MQq - Q^2E}{(D-3)\omega_D^2 qQ - (D-3)ME}. \quad (41)$$

Substituting this back into Eq. (40), we get

$$m < \omega_D Q\sqrt{\frac{2MEq - Q(E^2 + \omega_D^2 q^2)}{Q(M^2 - \omega_D^2 Q^2)}}. \quad (42)$$

To summarize, the crossing conditions $\dot{T} > 0$ and $\dot{R}^2 > 0$ for all $r > r_+$ lead to the two inequalities

$$E > E_{\text{min}} \coloneqq \frac{qQ}{(D-3)r_+^{D-3}}, \quad (43a)$$

$$m < m_{\text{max}} \coloneqq \omega_D Q\sqrt{\frac{2MEq - Q(E^2 + \omega_D^2 q^2)}{Q(M^2 - \omega_D^2 Q^2)}}. \quad (43b)$$

Equation (43a) ensures that the right-hand side of Eq. (43b) is real-valued. Taken together, these inequalities guarantee that a charged particle characterized by $\{m, E, q\}$ will cross the event horizon. Now we seek to ascertain the form of the D-dimensional RN metric after it absorbs the charged particle.

D. Overcharging condition

Like all previous works [10,15,16,41], we assume that the particle energy E fully accretes to the ADM mass of the black hole. The ADM mass upon absorption of the particle then simply increases as $M \to M + E$ while the ADM charge increases as Q to $Q + q$. This assumption misses out on all radiative/self-force effects, which of course lie outside the test-particle approximation.

The line element of the spacetime after absorption of the charged particle becomes

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{D-2}^2, \quad (44a)$$

where

$$f(r) = 1 - \frac{16\pi}{(D-2)\Omega_{D-2}}\frac{M+E}{r^{D-3}} + \frac{8\pi}{\Omega_{(D-2)}(D-2)(D-3)}\frac{(Q+q)^2}{r^{2(D-3)}}. \quad (44b)$$

The location of the horizons for this new line element (44) is given by

$$r_\pm^{D-3} = \frac{M+E}{(D-3)\omega_D^2}\left(1 \pm \sqrt{1 - \frac{\omega_D^2(Q+q)^2}{(M+E)^2}}\right). \quad (45)$$

The line element describes the spacetime around a black hole when $r_\pm$ are real or

$$M + E \geq \omega_D(Q+q). \quad (46)$$

A naked singularity is described by the RN line element when

$$Q + q > \omega_D^{-1}(M+E). \quad (47)$$

This can be rewritten to give an upper bound for E

$$E < E_{\text{max}} \coloneqq \omega_D(Q+q) - M. \quad (48)$$

Together with Eq. (39), Eq. (48) can be written as

$$\frac{qQ}{(D-3)r_+^{D-3}} < E < \omega_D(Q+q) - M, \quad (49)$$

allowing us to derive a lower bound for the charge,

$$\frac{qQ}{(D-3)r_+^{D-3}} < \omega_D(Q+q) - M, \tag{50}$$

or

$$q > q_{\min} := r_+^{D-3}\left(\frac{M-\omega_D Q}{\omega_D r_+^{D-3} - Q/(D-3)}\right). \tag{51}$$

This completes the derivation of the generalized Hubeny inequalities.

E. Overcharging extremal black holes

We now use these inequalities to constrain the parameter space for a charged particle that is about to fall towards an extremal black hole. In the extremal limit, $M = \omega_D Q$, the horizon location becomes

$$r^{D-3} = r_+^{D-3} = \frac{M}{(D-3)\omega_D^2} = \frac{Q}{(D-3)\omega_D}, \tag{52}$$

and the system of inequalities in Eqs. (8), (9), and (10) reduces to

$$q > 0, \tag{53}$$

$$\omega_D q < E < \omega_D q, \tag{54}$$

$$m < \infty, \tag{55}$$

which does not have a solution. This confirms that Wald's result remains correct for higher-dimensional extremal black holes. This was already anticipated by the general arguments made in Ref. [39].

F. Overcharging in the nearly extremal case

Looking now at the near-extremal case, we note that near-extremality in D dimensions can be parametrized using an extremality parameter $0 < \epsilon \ll 1$ as

$$M \equiv 1, \qquad Q \equiv \omega_D^{-1} - 2\epsilon^2. \tag{56}$$

Similar to Hubeny [15], we can deal with the inequalities (8), (9), and (10), perturbatively in ϵ, and show that a solution to them (i.e. a choice of m, E, and q satisfying them) is always possible for any $0 < \epsilon \ll 1$.

The expression for the event horizon then reduces to

$$r_+^{D-3} = \frac{1 + 2\sqrt{\omega_D}\epsilon}{(D-3)\omega_D^2}, \tag{57}$$

while the lower bound for q becomes

$$q > \frac{\epsilon + 2\sqrt{\omega_D}\epsilon^2}{\sqrt{\omega_D} + \omega_D\epsilon}. \tag{58a}$$

Expanding in ϵ, this becomes

$$q > \omega_D^{-1/2}\epsilon + \epsilon^2 + \mathcal{O}(\epsilon^3), \tag{59}$$

which can be satisfied by the choice

$$q = A\epsilon, \qquad A > \omega_D^{-1/2}. \tag{60}$$

At $D = 4$, this reduces to

$$q = a\epsilon, \qquad a > 1, \tag{61}$$

the same solution to the Hubeny inequalities in the near-extremal case in the lowest order.

Inserting Eq. (60) into the energy constraints, Eqs. (39) and (48), we get

$$E < \omega_D(A\epsilon - 2\epsilon^2), \tag{62a}$$

$$E > \omega_D A(\epsilon - 2\epsilon^2\sqrt{\omega_D} + 2\omega_D\epsilon^3 + \mathcal{O}(\epsilon^4)), \tag{62b}$$

which can be satisfied with the choice

$$E = \omega_D(A\epsilon - 2B\epsilon^2), \qquad 1 < B < \sqrt{\omega_D}A. \tag{63}$$

This also reduces to the $D = 4$ case where

$$E = a\epsilon - 2b\epsilon^2, \qquad 1 < b < a. \tag{64}$$

Inserting Eqs. (56), (60), and (63) into Eq. (42), we get

$$m < \epsilon\sqrt{A^2\omega_D^2 - B^2\omega_D} - \epsilon^2\frac{AB\omega_D^{3/2}}{\sqrt{A^2\omega_D - B^2}} + \mathcal{O}(\epsilon^3). \tag{65}$$

This can be satisfied with the choice

$$m = C\epsilon, \qquad C < \sqrt{A^2\omega_D^2 - B^2\omega_D}. \tag{66}$$

Again, this reduces to Hubeny's mass constraint in $D = 4$,

$$m = c\epsilon, \qquad c < \sqrt{a^2 - b^2}. \tag{67}$$

This demonstrates that it is quite easy to find a solution to the generalized Hubeny inequalities for the case of a nearly extremal black hole for any dimension, D.

To summarize, the overcharging solution to nearly extremal black holes can be parametrized as $q = A\epsilon$, $E = \omega_D(A\epsilon - 2B\epsilon^2)$, $m = C\epsilon$, for

$$A > \omega_D^{-1/2}, \tag{68}$$

$$1 < B < \sqrt{\omega_D}A, \tag{69}$$

$$C < \sqrt{A^2\omega_D^2 - B^2\omega_D}. \tag{70}$$

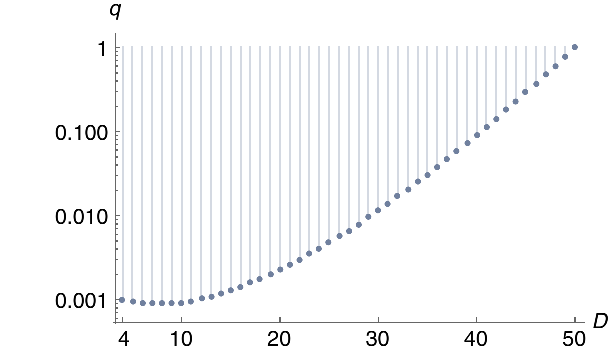

FIG. 1. Allowed parameter space in q for a nearly extremal $D \geq 4$ BH with $\epsilon = 0.001$. The dots represent the lower bound, $q_{\min}$, for different dimensions $D \geq 4$.

Note that we can reparametrize this region by letting $A = \alpha/\sqrt{\omega_D}, B = \beta, C = \gamma\sqrt{\omega_D}$. Then, the overcharging parameter region for $D > 4$ simplifies to

$$\alpha > 1, \tag{71}$$

$$1 < \beta < \alpha, \tag{72}$$

$$\gamma < \sqrt{\alpha^2 - \beta^2}, \tag{73}$$

which is in fact identical to the original overcharging parameter space identified by Hubeny in $D = 4$. This means that a solution to the inequalities exists in all dimensions $D \geq 4$.

Though a solution always exists for any D, it is of interest to look at how the overcharging parameter regions in $\{m, E, q\}$ are affected as $D \to \infty$. We can do this by looking at the generalized Hubeny inequalities directly.

The inequality in Eq. (8) provides a D-dependent lower bound on q. This lower bound is plotted in Fig. 1 for $\epsilon = 0.001$. We see from this that the smallest charge allowing for overcharging, $q_{\min}$, increases with D. Therefore, the required charge eventually becomes too large to satisfy the test-particle assumption, $\omega_D q \ll M \sim \omega_D Q$. (Note that $q_{\min}/Q \sim \omega_D^{-1/2}$ as $D \to \infty$.) In the energy sector, the inequality in Eq. (9) allows for energies within a range that also depends on D and q. In Fig. 2, this range is plotted as a function of D for various choices of q and the same ϵ as in Fig. 1. We see from this that the allowed energy range, $\Delta E = E_{\max} - E_{\min}$, generically shrinks to zero as $D \to \infty$. Finally, the upper bound on the mass m is plotted against D in Fig. 3 for various choices of q and E, which shows it going to zero in the same limit. We infer from these figures that, for any fixed ϵ, the available parameter space for overcharging rapidly shrinks as $D \to \infty$. Perhaps more importantly, in going to ever higher dimensions we eventually run afoul with the starting assumptions of a Hubeny-type analysis. In particular, the rapid growth of $q_{\min}$ as $D \to \infty$ throws into question the test-particle assumption at some sufficiently large D.

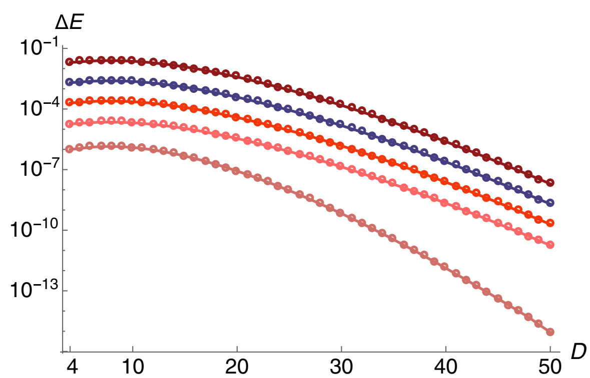

FIG. 2. Width of parameter space, $\Delta E = E_{\max} - E_{\min}$, for a nearly extremal BH with $\epsilon = 0.001$ and a specific value of q. From top to bottom: $q = 10000q_{\min}$, $1000q_{\min}$, $100q_{\min}$, $10q_{\min}$, and $q_{\min} + \epsilon/2$. As $D \to \infty$, ΔE decreases exponentially.

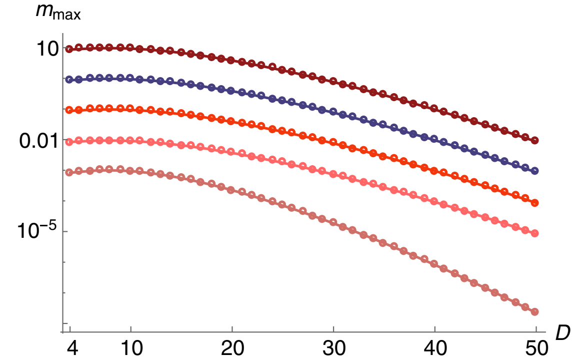

FIG. 3. $m_{\max}$ for a nearly extremal BH with $\epsilon = 0.001$ and different values of q and E. For the same choice of q in Fig. 2, we take E to be $E = (E_{\min} + E_{\max})/2$. From top to bottom: $q = 10000q_{\min}$, $1000q_{\min}$, $100q_{\min}$, $10q_{\min}$, and $q_{\min} + \epsilon/2$. As $D \to \infty$, $m_{\max}$ also decreases exponentially.

Interestingly, overcharging appears most favorable in $D = 8$. This intriguingly coincides with the dimension at which the volume of a two-sphere Ω_{D-2} is largest. We are unable to provide a simple physical explanation for this coincidence at this point.

IV. SUMMARY AND CONCLUSION

The goal in this work was to explore the possibility of overcharging of black holes by point particles in higher dimensions. To this end, we studied the radial infall of a charged particle in a charged, spherically symmetric, D-dimensional black hole. Like Hubeny, we reduced the conditions leading to cosmic censorship violation in this scenario into a set of generalized Hubeny inequalities. We further learned that these inequalities cannot be satisfied in extremal black holes, but that they can be satisfied when the black holes are nearly extremal. Thus, the results of Wald and Hubeny for $D = 4$ remain true in higher dimensions.

These results are not entirely surprising. The interaction between a charged black hole and an infalling test charge consists of a competition between their gravitational attraction and electromagnetic repulsion. Wald's classic no-go result in the extremal case ($Q = M$) can be taken to mean that these competing effects precisely cancel. On the other hand, starting with a black hole charge just shy of extremality [i.e. $Q = M(1 - 2\epsilon^2)$] weakens the electromagnetic repulsion sufficiently enough to allow some test charges to cross the horizon and overcharge the black hole. In higher dimensions, the strengths of both gravitational attraction and electromagnetic repulsion scale precisely in the same way as $\sim r^{-(D-2)}$. So the previous considerations concerning the balance between these two effects can be expected to remain true.

There may be little doubt that our conclusions are an artifact of the test-particle approximation, just as in $D = 4$. Destroying the event horizon is a violation not only of cosmic censorship but also the second and third laws of black hole mechanics, which ought to hold in higher dimensions [42]. Beyond these theoretical considerations, for any process that violates cosmic censorship, we are always left to ask about what possible mechanism might prevent the violation. For scenarios involving point particles, the natural choice of cosmic censor is the backreacting self-force, the calculation of which, in higher dimensions, is a subject still in its infancy, and is an active area of research [43–46]. Our work can be viewed as a strong invitation to pursue higher-dimensional self-force calculations.

There are indications that the self-force in higher dimensions gets divergently repulsive as the particle approaches the horizon [43], which would mean that none of the charged particles we identify as overcharging do, in fact, cross the horizon. However, the matter is far from settled [44]. It would be interesting to see how this story unfolds, as self-force calculations in higher dimensions mature to the state that has been reached in $D = 4$. We leave this problem for future work.

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