Relic gravitational waves from quintessential inflation

Safia Ahmad,[1,*] R. Myrzakulov,[2,†] and M. Sami[1,2,3,‡]

[1]*Centre for Theoretical Physics, Jamia Millia Islamia, New Delhi-110025, India*
[2]*Eurasian International Center for Theoretical Physics, Eurasian National University, Astana 010008, Kazakhstan*
[3]*Maulana Azad National Urdu University, Gachibowli, Hyderabad-500032, India*

(Received 17 May 2017; published 15 September 2017)

We study relic gravitational waves in the paradigm of quintessential inflation. In this framework, irrespective of the underlying model, inflation is followed by the kinetic regime. Thereafter, the field energy density remains subdominant before the onset of acceleration. We carry out model-independent analysis to obtain the temperature at the end of inflation and the estimate for the upper bound on the Hubble parameter to circumvent the problem due to relic gravitational waves. In this process, we use Planck 2015 data to constrain the inflationary phase. We demonstrate that the required temperature can be produced by the mechanism of instant preheating. The generic feature of the scenario includes the presence of the kinetic regime after inflation, which results in the blue spectrum of gravitational wave background at high frequencies. We discuss the prospects of detection of relic gravitational wave background in the advanced LIGO and LISA space-born gravitational wave missions. Finally, we consider a concrete model to realize the paradigm of quintessential inflation and show that inflationary as well as postinflationary evolution can be successfully described by the inflaton potential, $V(\phi) \propto \mathrm{Exp}(-\lambda\phi^n/M_{\mathrm{Pl}}^n)(n > 1)$, by suitably constraining the parameters of the model.

DOI: 10.1103/PhysRevD.96.063515

I. INTRODUCTION

Accelerated expansion has played an important role in the history of our Universe. It is a common belief that the Universe has gone through a phase of rapid expansion dubbed *inflation* [1–4] at early times and it started accelerating once again around the present epoch [5–10]. The standard model of the Universe, therefore, should be complemented by two phases of acceleration. Inflation is a beautiful paradigm that not only resolves the inconsistencies of the hot big bang model but also provides a mechanism of generation of primordial perturbations required for structure formation. Late-time cosmic acceleration is needed to address the age problem in the standard model. The phenomenon was detected in 1998 by supernovae observations [11,12] and was supported by other probes indirectly thereafter [13–15]. Similar confirmation for cosmological inflation is not available at present.

Cosmic acceleration is a generic phenomenon in our Universe that manifests at early and late times, keeping the thermal history intact. Often, these two phases are described independently. As for late-time acceleration, a variety of scalar field models has been investigated in the literature since 1998. At the background level, the cosmology community seems to converge on the cosmological constant as the underlying cause of said phenomenon. Even if acceleration is caused by a slowly rolling scalar field, the latter is not distinguished from the cosmological constant at the background level. As for perturbations, the study of large scale structure might reveal if there is something beyond ΛCDM ($\Lambda +$ cold dark matter). The current observational constraints related to inflation are tight such that many popular models are on the verge of being ruled out. We should emphasize that in a successful model inflation should be followed by an efficient reheating also.

It is interesting to ask whether a successful model of inflation can also describe late-time acceleration without interfering with the thermal history of Universe or if an inflaton can be dark energy. Unification of inflation with late-time acceleration or dark energy is termed *quintessential inflation* [16–33]. It is indeed challenging to describe inflation and late-time acceleration using a single scalar field. At the onset, it sounds possible if the field potential is shallow at early and late stages and steep for most of the history of the Universe. First, the model should comply with all the observational constraints related to inflation and should give rise to successful reheating, which is itself a difficult task. As we pointed out, the current observational constraints are quite tight, putting many known models in tension. Second, the conventional reheating mechanism [16,34–40] is not applicable to this class of models as the field potential is typically the runaway type. However, in this case, one can invoke an alternative mechanism dubbed instant preheating [41–44]. It is desirable that we have a scaling regime [45] after inflation allowing inflaton to go into hiding till late times, which is necessary to keep the

*safia@ctp-jamia.res.in
†rmyrzakulov@gmail.com
‡samijamia@gmail.com

thermal history intact. To what extent the field should be hidden is decided by the nucleosynthesis constraint. In this case, unlike the thawing picture, evolution, to a great extent, is independent of initial conditions.

The generic feature of quintessential inflation includes the presence of the kinetic regime [21,25,44] that follows inflation. The duration of the regime depends upon the temperature at the end of inflation. The energy density of relic gravitational waves [44,46–54] as compared to the field energy density enhances during the kinetic regime and might challenge the nucleosynthesis constraint at the commencement of the radiative regime. The distinguished signature of the unification, irrespective of the underlying model, includes the production of relic gravitational waves with the blue spectrum. Clearly, an alternative reheating mechanism is needed in this case to circumvent the problem.

Let us note that the generalized exponential potential $V(\phi) \propto \mathrm{Exp}(-\lambda\phi^n/M_{\rm Pl}^n), (n > 1)$ [55] can successfully realize the paradigm of quintessential inflation. This potential has a remarkable property: it is shallow around $\phi = 0$ and steep thereafter. Thanks to an additional parameter n compared to standard exponential, the model can comply with observational constraints on inflation. Second, the model shares the property of a simple steep exponential potential [20,21,28,56–58] at late times; namely, the approximate scaling regime is a late-time attractor in this case as $\Gamma = V''V/V'^2 \to 1$ for large values of the field.

Last but not least, we need to exit from the scaling regime around the present epoch. This can be achieved by invoking nonminimal coupling to massive neutrino matter (see Refs. [30,55,59–65] and references therein for details). The coupling appears at late stages as neutrinos turn nonrelativistic. As a result, the effective potential for the field acquires a minimum where the field can settle, causing an exit from the scaling regime to late-time cosmic acceleration.

In this paper, we first carry out a model-independent analysis to obtain the temperature at the end of inflation and the estimate on the upper bound of the Hubble parameter to circumvent the problem due to relic gravitational waves. We also investigate an alternative reheating mechanism suitable to the scenario under consideration. Finally, we discuss a model of quintessential inflation that can successfully realize the paradigm. Throughout this paper, $M_{\rm Pl}$ denotes the reduced Planck mass.

II. RELIC GRAVITATIONAL WAVE SPECTRUM

In this section, we shall study relic gravitational waves in the scenario of quintessential inflation. The evolution of tensor perturbations, h_{ij}, is given by the Klein-Gordon equation

$$\Box h_{ij} = 0 \to \ddot{\varphi}_k(\tau) + 2\frac{\dot{a}}{a}\varphi_k(\tau) + k^2\varphi_k(\tau) = 0;$$
$$h_{ij} \sim \varphi_k e^{ikx} e_{ij}, \tag{1}$$

where e_{ij} is the polarization tensor, τ ($d\tau = dt/a$) is the conformal time, and k is the comoving wave number. For simplicity, we shall consider the exponential inflation, namely, $a = \tau_0/\tau$ and $H_{\rm in} = -1/\tau_0$. The positive frequency solution of Eq. (1) in the adiabatic vacuum corresponds to the "in" state, $\varphi_{\rm in}^{(+)}(k,\tau)$,

$$\varphi_{\rm in}^{(+)}(k,\tau) = (\pi\tau_0/4)^{1/2}(\tau/\tau_0)^{3/2}H_{3/2}^{(2)}(k\tau), \tag{2}$$

where $H_{3/2}^{(2)}$ is the Hankel function of the second kind.

In a scenario of quintessential inflation, the Universe enters into the kinetic phase with a stiff equation of state parameter soon after the inflation ends [21,25]. This transition involves the nonadiabatic change of space-time geometry. Assuming for simplicity that the postinflationary dynamics is described by the power law expansion, i.e., $a = (t/t_0)^p \equiv (\tau/\tau_0)^{1/2-\mu}$ where $\mu \equiv (3(w-1)/2(3w+1))$ with w being the postinflationary equation of state parameter. Let us note that $\mu = 0$ in the kinetic regime ($w = 1$). As for the "out" state, it contains both positive and negative frequency solutions of (1),

$$\varphi_{\rm out} = \alpha\varphi_{\rm out}^{(+)} + \beta\varphi_{\rm out}^{(-)}, \tag{3}$$

where α and β designate the Bogoliubov coefficients [21] and

$$\varphi_{\rm out}^{(+,-)} = (\pi\tau_0/4)^{1/2}(\tau/\tau_0)^{\mu}H_{|\mu|}^{(2,1)}(k\tau). \tag{4}$$

We then estimate the energy density of relic gravitational waves [21,48],

$$\rho_g = \langle T_{00}\rangle = \frac{1}{\pi^2 a^2}\int dk k^3|\beta|^2. \tag{5}$$

During the kinetic regime, $|\beta_{\rm kin}|^2 \sim (k\tau_{\rm kin})^{-3}$, using then Eq. (5) and the fact that $H_{\rm in} = -1/\tau_0$, we obtain

$$\rho_g = \frac{32}{3\pi}h_{\rm GW}^2\rho_b\left(\frac{\tau}{\tau_{\rm kin}}\right), \tag{6}$$

where ρ_b denotes the background energy density and contains radiation and stiff scalar matter in the kinetic regime. We have hereby assumed that the generation of radiation takes place at the end of inflation thanks to some mechanism to be specified in the subsequent section.

Since, at the equality of radiation and scalar field energy densities ($\tau = \tau_{\rm rh}$),

$$\frac{\tau_{\rm rh}}{\tau_{\rm kin}} = \left(\frac{T_{\rm kin}}{T_{\rm rh}}\right)^2 \tag{7}$$

and $\rho_b = 2\rho_r$, we find using Eq. (6)

$$\left(\frac{\rho_g}{\rho_r}\right)_{\rm rh} = \frac{64}{3\pi} h_{\rm GW}^2 \left(\frac{T_{\rm end}}{T_{\rm rh}}\right)^2, \tag{8}$$

where $h_{\rm GW}$ is the dimensionless amplitude of the gravitational waves given by

$$h_{\rm GW}^2 = \frac{H_{\rm inf}^2}{8\pi M_{\rm Pl}^2} = \frac{V_{\rm inf}}{24\pi M_{\rm Pl}^4}, \tag{9}$$

where $V_{\rm inf}$ is the value of the inflationary potential at the time the cosmological scales exit the horizon and can be fixed by imposing the COBE normalization [66,67].

Equations (7) and (8) imply that a longer kinetic regime, i.e., large $\tau_{\rm rh}$, would correspond to smaller $T_{\rm rh}$ and hence the larger value of ρ_g/ρ_r at equality. As given in Ref. [21], it can be shown from (5) that for $w > 1/3$, $\rho_g \propto 1/a^4$ and for $w < 1/3$, $\rho_g \propto \rho_b$; during the radiation era also, ρ_g approximately tracks the background. For $w = 1/3$ (radiation), log factor appears.[1]

For simplicity,[2] we hereby assume that the kinetic regime ($\rho_\phi \propto 1/a^6$) immediately follows inflation; thus, we can use the approximation, $H_{\rm end} \simeq H_{\rm kin}$ and $T_{\rm end} \simeq T_{\rm kin}$. Since $T_{\rm rh} \sim T_{\rm end}(a_{\rm end}/a_{\rm rh})$, we find

$$\left(\frac{\rho_\phi}{\rho_r}\right)_{\rm end} = \left(\frac{T_{\rm end}}{T_{\rm rh}}\right)^2 \tag{10}$$

and the following important relation:

$$\left(\frac{\rho_\phi}{\rho_r}\right)_{\rm end} = \frac{3\pi}{64}\left(\frac{\rho_g}{\rho_r}\right)_{\rm rh} \frac{1}{h_{\rm GW}^2}. \tag{11}$$

From COBE normalization, we find that

$$V_{\rm inf}^{1/4} = 0.013 r^{1/4} M_{\rm Pl}, \tag{12}$$

where r is the scalar-to-tensor ratio. Combining this with Eq. (9) and inserting it into Eq. (11), we find

$$\left(\frac{\rho_\phi}{\rho_r}\right)_{\rm end} = \frac{9\pi^2}{8}\left(\frac{\rho_g}{\rho_r}\right)_{\rm rh} \frac{M_{\rm Pl}^4}{V_{\rm inf}} \lesssim \frac{3.88\times 10^6}{r}, \tag{13}$$

where we have used the constraint on the ratio, $(\rho_g/\rho_r)_{\rm rh} \lesssim 0.01$ as imposed by nucleosynthesis. We can compute the temperature at the end of inflation by using the fact that the cosmological scales, k, that cross the horizon during inflation and the scales that reenter it at a later time satisfy the relation $k = a_{\rm inf} H_{\rm inf} = a_0 H_0$, which implies that

$$\frac{k}{a_0 H_0} = \frac{a_{\rm inf}}{a_{\rm end}} \frac{a_{\rm end}}{a_0} \frac{H_{\rm inf}}{H_0} \Rightarrow e^{-\mathcal{N}} \frac{T_0}{T_{\rm end}} \frac{H_{\rm inf}}{H_0} = 1. \tag{14}$$

For $\mathcal{N} = 65$, we find that

$$T_{\rm end} = 1.99 \times 10^{15} r^{1/2}\ {\rm GeV}. \tag{15}$$

Now, the upper bound on the value of the Hubble parameter at the end of inflation, $H_{\rm end}$, can be obtained from Eq. (13), and using the fact that $\rho_{\rm r,end} = T_{\rm end}^4$,

$$H_{\rm end} \lesssim 1.85 \times 10^{15} r^{1/2}\ {\rm GeV}. \tag{16}$$

However, the bound (16) should be consistent with the estimate for $H_{\rm inf}$ obtained after using COBE normalization, namely, $H_{\rm inf} \simeq 2.37 \times 10^{14} r^{1/2}$ GeV. In general, the ratio $H_{\rm inf}/H_{\rm end}$ is more than a few, in the realistic scenarios with graceful exit. Clearly, $H_{\rm inf} = H_{\rm end}$ in case of de Sitter. Thus, we can take $H_{\rm end} \lesssim H_{\rm inf}$ for estimates, which is clearly a more conservative bound than (16). The bound on $H_{\rm end}$ in turn gives us the bound on $V_{\rm end}$, which is $V_{\rm end} \lesssim 1.15 \times 10^{-6} r M_{\rm Pl}^4$. Again, we know that $V_{\rm end}$ can at most take the value of $V_{\rm inf}$.

Since the potential in the model of quintessential inflation is generically the runaway type, the conventional reheating mechanism does not operate in this case. Gravitational particle production is a known possibility. After inflation, the geometry of space-time undergoes a nonadiabatic process, which gives rise to particle production. Let us note that gravitational particle production gives rise to the energy density [16,34]

$$\rho_{\rm r,end} \simeq 0.01 g_p H_{\rm end}^4, \tag{17}$$

where g_p is the number of different particles produced, which is typically ∼10–100, and since $H_{\rm end} \lesssim H_{\rm inf}$, we find that

$$\left(\frac{\rho_\phi}{\rho_r}\right)_{\rm end} \simeq \frac{\rho_{\phi,\rm end}}{0.01 g_p H_{\rm end}^4} \gtrsim \frac{2.1\times 10^{10} g_p^{-1}}{r}. \tag{18}$$

In the aforementioned model-independent discussion, we have used $H_{\rm end} \simeq H_{\rm inf}$. For an exact relation between these quantities, we need a concrete model. However, in models with $r \ll 1$ such as the Starobinsky inflation model [1], $H_{\rm inf}$ is close to $H_{\rm end}$. Thus, in general, the respective energy density produced due to gravitational particle production misses the nucleosynthesis constraint imposed by relic gravitational waves (13) by at least 2 orders of magnitude, which is related to the fact that the temperature of radiation due to gravitational particle production $T_{\rm end}^g$ is somewhat less than $T_{\rm end}$ (15), namely, $T_{\rm end}^g \simeq 2.63 \times 10^{14} r^{1/2}$ GeV. In the scenario under consideration, due to the steep postinflationary behavior of the potential, the field enters the kinetic regime soon after inflation ends and remains

[1]Precisely, $\rho_g \propto a^{-4}\log(\tau/\tau_0) \propto a^{-4}\log(a)$, but we have ignored the logarithmic factor as its contribution is negligible during evolution.

[2]It is supported by numerical simulation.

there for a long time in view of the aforementioned. Clearly, the problem of relic gravitational waves in quintessential inflation is associated with the longevity of the kinetic regime. During this time, the energy density in gravitational waves enhances compared to the field energy density challenging the nucleosynthesis constraint at the commencement of the radiative regime. We naturally require a more efficient process to circumvent the problem; one should then look for a suitable reheating mechanism.

We are now in a position to discuss the gravitational wave spectrum. The fractional spectral energy density parameter of the relic gravitational wave is defined as

$$\Omega_{\rm GW}(k) = \frac{\tilde{\rho}_{\rm g}(k)}{\rho_{\rm c}}, \tag{19}$$

where $\rho_{\rm c}$ denotes the critical energy density (see Ref. [21] for details) and $\tilde{\rho}_{\rm g}(k)$ is the spectral energy density

$$\tilde{\rho}_{\rm g}(k) \propto k^{1-2|\mu|}; \qquad \mu = \frac{3}{2}\left(\frac{w-1}{3w+1}\right). \tag{20}$$

From the above equation, we see that during the kinetic regime, i.e., for $w = 1$, $\rho_{\rm g}(k) \propto k$, giving rise to a blue spectrum in the wave spectrum. This is a generic feature of any model of quintessential inflation in which the scalar field enters into the kinetic regime after inflation.

We also have the relations

$$\Omega_{\rm GW}^{(\rm MD)} = \frac{3}{8\pi^3} h_{\rm GW}^2 \Omega_{\rm m0}\left(\frac{\lambda}{\lambda_{\rm h}}\right)^2, \qquad \lambda_{\rm MD} < \lambda \leq \lambda_{\rm h}, \tag{21}$$

$$\Omega_{\rm GW}^{(\rm RD)}(\lambda) = \frac{1}{6\pi} h_{\rm GW}^2 \Omega_{\rm r0}, \qquad \lambda_{\rm RD} < \lambda \leq \lambda_{\rm MD}, \tag{22}$$

$$\Omega_{\rm GW}^{(\rm kin)}(\lambda) = \Omega_{\rm GW}^{(\rm RD)}\left(\frac{\lambda_{\rm RD}}{\lambda}\right), \qquad \lambda_{\rm kin} < \lambda \leq \lambda_{\rm RD}, \tag{23}$$

with

$$\lambda_{\rm h} = 2cH_0^{-1}, \tag{24}$$

$$\lambda_{\rm MD} = \frac{2\pi}{3}\lambda_{\rm h}\left(\frac{\Omega_{\rm r0}}{\Omega_{\rm m0}}\right)^{1/2}, \tag{25}$$

$$\lambda_{\rm RD} = 4\lambda_{\rm h}\left(\frac{\Omega_{\rm m0}}{\Omega_{\rm r0}}\right)^{1/2}\frac{T_{\rm MD}}{T_{\rm end}}, \tag{26}$$

$$\lambda_{\rm kin} = cH_{\rm kin}^{-1}\left(\frac{T_{\rm end}}{T_0}\right)\left(\frac{H_{\rm kin}}{H_{\rm end}}\right)^{1/3} \simeq cH_{\rm end}^{-1}\left(\frac{T_{\rm end}}{T_0}\right), \tag{27}$$

where “MD,” “RD,” and “kin” designate matter, radiation, and kinetic energy dominated epochs; H_0, $\Omega_{\rm m0}$, and $\Omega_{\rm r0}$ are the present values of the Hubble parameter, dimensionless density parameters corresponding to matter energy density, and radiation energy density; and $T_{\rm end}$ and $H_{\rm end}$ denote the temperature and Hubble parameter at the end of inflation, respectively. We note that in Eq. (22) we again ignore the insignificant logarithmic correction.

III. INSTANT PARTICLE PRODUCTION AND PREHEATING

We hereby emphasize once again that the gravitational particle production is quite inefficient for circumventing the problem due to relic gravitational waves; one therefore should look for an alternative mechanism. One of the alternatives suitable to the class of models similar to the one we are going to discuss in the next section is based upon instant particle production dubbed instant preheating, which proceeds as follows. We assume that the scalar field, ϕ, interacts with some other scalar field, χ, which interacts with the Fermion field, ψ,

$$\mathcal{L}_{\rm int} = -\frac{1}{2}g^2\phi^2\chi^2 - h\bar{\psi}\psi\chi, \tag{28}$$

where g and h denote the coupling constants, assumed to be positive for convenience, and $g, h < 1$ for the perturbation treatment to be applicable. In this case, χ does not have a bare mass. However, the χ field has an effective mass that grows with ϕ as $m_\chi = g|\phi|$. The Lagrangian is specially designed to give rise to this feature. As inflation ends, the field ϕ enters the kinetic regime as the potential is very steep in the postinflationary regime. Consequently, the field ϕ rolls down its potential fast soon after inflation ends. Since the mass of χ depends upon ϕ, we can shift the field as $\phi \to \phi' = \phi - \phi_{\rm end}$ such that the effective mass of χ vanishes at the end of inflation. However, the Lagrangian (28) does not obey shift symmetry, and thus we need to assume an enhanced symmetry in the Lagrangian, which can be achieved by adding a suitable counterterm to (28). It is important to check how m_χ changes with time around the transition from inflation to the kinetic phase. The nonadiabatic change in m_χ is crucial for particle production. Indeed, the production of χ particles takes place when the adiabaticity condition is violated, i.e.,

$$|\dot{m}_\chi| \gtrsim m_\chi^2 \Rightarrow |\dot{\phi}| \gtrsim g\phi^2. \tag{29}$$

The above condition is satisfied if

$$|\phi| \lesssim |\phi_p| = \sqrt{\frac{\dot{\phi}_{\rm end}}{g}}. \tag{30}$$

The equation-of-state parameter for the inflaton field is given by

$$\omega_\phi = \frac{\dot{\phi}^2 - 2V}{\dot{\phi}^2 + 2V}. \tag{31}$$

Now, using the fact that inflation ends when ω_ϕ increases to $-1/3$, we find that

$$\dot{\phi}_{\rm end} = \sqrt{V_{\rm end}}. \tag{32}$$

Considering the fact that $\phi_p \lesssim M_{\rm Pl}$, Eq. (30) gives us the bound on the coupling g,

$$\frac{\dot{\phi}_{\rm end}}{g} \lesssim M_{\rm Pl}^2 \to g \gg \frac{1}{M_{\rm Pl}^2}\sqrt{V_{\rm end}}. \tag{33}$$

In addition, the production time of χ particles is given by

$$\delta t_p \sim \frac{|\phi|}{\dot{\phi}} = (g\dot{\phi}_{\rm end})^{-1/2}. \tag{34}$$

The uncertainty relation allows us to obtain the momentum, $k_p \simeq (\delta p)^{-1} \simeq \sqrt{g\dot{\phi}_{\rm end}}$. Following Refs. [39,42], the occupation number of χ is given by

$$n_k \sim \exp\left(-\frac{\pi k^2}{k_p^2}\right). \tag{35}$$

which allows us to estimate the number density of χ particles,

$$N_\chi = \frac{1}{(2\pi)^3}\int_0^\infty n_k d^3\vec{k} = \frac{(g\dot{\phi}_{\rm end})^{3/2}}{(2\pi)^3}, \tag{36}$$

and their total energy density

$$\rho_\chi = N_\chi m_\chi = \frac{g^2 V_{\rm end}}{8\pi^3}. \tag{37}$$

Assuming that at the end of inflation the produced energy is thermalized instantaneously, we obtain

$$\left(\frac{\rho_\phi}{\rho_r}\right)_{\rm end} \simeq \frac{12\pi^3}{g^2}. \tag{38}$$

The above equation combined with Eq. (13) gives the lower bound on the coupling g,

$$g \gtrsim 9.78 r^{1/2} \times 10^{-3}. \tag{39}$$

Now, since

$$\delta t_p H_{\rm end} \simeq \frac{1}{M_{\rm Pl}\sqrt{2g}}(V_{\rm end})^{1/4} < 9.29\times 10^{-2} \Rightarrow \delta t_p \ll H_{\rm end}^{-1}, \tag{40}$$

the expansion is negligible during the particle production. It should be noted that, since $\phi_p \lesssim 0.13 M_{\rm Pl}$, the production of particles commences immediately after the inflation ends.

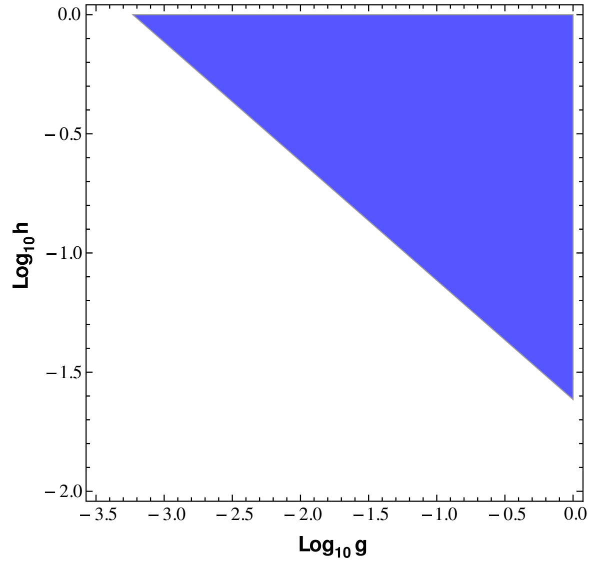

FIG. 1. Figure shows the parameter space of (g, h), while the shaded region gives the allowed values of the parameters for efficient preheating to occur. We have used $r = 10^{-3}$ to plot this figure.

In the postinflationary regime, the scalar field ϕ rolls down its potential faster, and hence the effective mass $g|\phi|$ of the scalar field χ grows, which facilitates the decay of χ particles into fermions. The decay rate is given by

$$\Gamma_{\bar{\psi}\psi} = \frac{h^2 m_\chi}{8\pi} = \frac{h^2}{8\pi} g|\phi|. \tag{41}$$

Around the end of inflation, $\rho_\phi \propto 1/a^2$ and diminishes more slowly than ρ_χ with the expansion of the Universe.[3] We now should arrange the decay rate of χ into matter fields such that the decay takes place before the backreaction of χ on ϕ evolution becomes important, which implies that

$$\Gamma_{\bar{\psi}\psi} \gg H_{\rm end} \Rightarrow h^2 \gtrsim 8\pi\frac{H_{\rm end}}{g|\phi|}. \tag{42}$$

For $\phi \lesssim M_{\rm Pl}$, the above condition gives us the lower bound on the coupling h, namely, $h \gtrsim 0.13 g^{-1/2} r^{1/4}$. In Fig. 1, we depict the allowed values of g and h, which shows that we have a wide parameter space for an efficient preheating to occur.

Figure 2 illustrates the spectral energy density ($\Omega_{\rm GW}$) of the relic gravitational wave background for the temperature $T_{\rm end} = 6.29 \times 10^{13}$ GeV, which corresponds to $g = 1.96 \times 10^{-3}$ (for $r \simeq 10^{-3}$) of the coupling, along with the

[3] Immediately after inflation ends ($\omega_\phi = -1/3$), $\rho_\phi \sim 1/a^2$; thereafter, it redshifts faster and enters the kinetic regime. Since the kinetic regime establishes quickly after inflation, the estimates change insignificantly if we identify the end of inflation with the commencement of the kinetic regime.

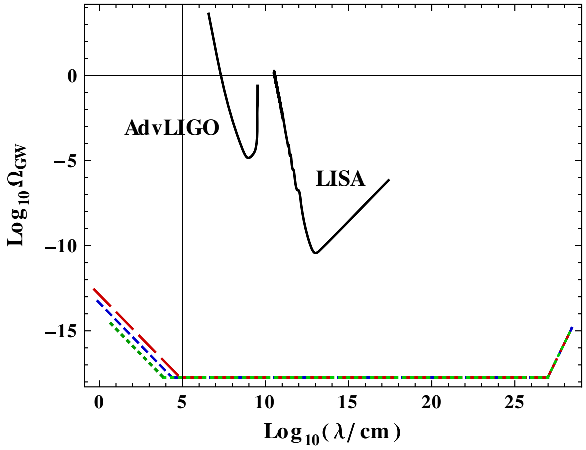

FIG. 2. Spectral energy density of the relic gravitational wave background for different temperatures have been plotted. The blue (dashed) line corresponds to $T_{\rm end} = 6.29 \times 10^{13}$ GeV obtained using COBE normalization, while the red (long dashed) and green (dotted) lines have been plotted using the model we have considered in Sec. IV for temperatures $T_{\rm end} = 2.09 \times 10^{13}$ and 2.09×10^{14} GeV, respectively. The numerical values of model parameters are taken to be $n = 6$, $r = 10^{-3}$ and $\lambda = 8.08 \times 10^{-6}$. Also, we have considered $\mathcal{N} = 65$ (behavior does not seem to change significantly for the variation of $\mathcal{N}$), where black solid curves represent the expected sensitivity curves of Advanced LIGO and LISA.

sensitivity curve of AdvLIGO [68,69] and LISA [70,71]. The figure shows that the spectrum is blue at high frequencies.

IV. SUCCESSFUL MODEL OF QUINTESSENTIAL INFLATION

We shall now check whether we can realize the aforementioned requirement in a particular model of quintessential inflation. To this effect, we shall consider a model based upon the following generalized exponential potential with two parameters n and λ,

$$V(\phi) = V_0 e^{-\lambda\phi^n/M_{\rm Pl}^n}, \qquad n > 1. \tag{43}$$

Using the expressions of slow-roll parameters,

$$\epsilon \equiv \frac{M_{\rm Pl}^2}{2}\left(\frac{V_\phi}{V}\right)^2, \qquad \eta \equiv M_{\rm Pl}^2 \frac{V_{\phi\phi}}{V}, \tag{44}$$

we have the standard expressions for the scalar spectral index (n_s) and tensor-to-scalar ratio (r) as

$$n_s - 1 = -6\epsilon + 2\eta, \qquad r = 16\epsilon. \tag{45}$$

In the flat Friedmann-Robertson-Walker (FRW) background, the slow-roll parameter, ϵ, for the generalized exponential potential (43) is given by

$$\epsilon = \frac{1}{2} n^2 \lambda^2 \left(\frac{\phi}{M_{\rm Pl}}\right)^{2n-2}, \tag{46}$$

which shows that slow roll takes place for $\phi/M_{\rm Pl} \ll 1$, whereas the potential is steep for large values of the field. Using the condition for the end of inflation, namely, $\epsilon|_{\phi=\phi_{\rm end}} = 1$, and the expression for the number of the e-foldings,

$$\mathcal{N} = \int_t^{t_{\rm end}} H dt' = -M_{\rm Pl}^{-2} \int_\phi^{\phi_{\rm end}} \frac{V(\phi')d\phi'}{dV(\phi')/d\phi'} \tag{47}$$

$$= \frac{1}{n\lambda(n-2)}\left[\left(\frac{\phi}{M_{\rm Pl}}\right)^{2-n} - \left(\frac{2}{n^2\lambda^2}\right)^{\frac{2-n}{2n-2}}\right], \tag{48}$$

we estimate the numerical value of ϕ at the commencement of inflation,

$$\frac{\phi}{M_{\rm Pl}} = \left[n(n-2)\lambda\mathcal{N} + \left(\frac{2}{n^2\lambda^2}\right)^{\frac{2-n}{2n-2}}\right]^{\frac{1}{2-n}}. \tag{49}$$

We can then eliminate ϕ from the slow-roll parameter, ϵ, in favor of model parameters n and λ and the number of e-folds by inserting the above expression for ϕ as

$$\epsilon = \frac{1}{2} n^2\lambda^2 \left[n\lambda(n-2)\mathcal{N} + \left(\frac{\sqrt{2}}{n\lambda}\right)^{\frac{2-n}{n-1}}\right]^{\frac{2(n-1)}{2-n}}. \tag{50}$$

We shall use the relation (50) in the subsequent discussion for the estimation of $V_{\rm end}$. Current observations, namely, the Planck 2015 results, impose constraints on the model parameters n and λ. Indeed, the predictions of the model are within the 2σ bound, provided that $n \gtrsim 5$ and $\lambda \lesssim 10^{-4}$; see Refs. [55,72].

The radiation energy produced due to the production of the particle at the end of inflation [16,34],

$$\rho_{\rm r,end} \simeq 0.01 g_p H_{\rm end}^4, \tag{51}$$

can now be calculated exactly. Choosing $r = 10^{-3}$, $\mathcal{N} = 65$, and $n = 6$ and making use of Eqs. (45) and (50), we find that $\lambda = 8.08 \times 10^{-6}$. In addition, using COBE normalization, we can estimate $V_0 = 3.4 \times 10^{-8} r M_{\rm Pl}^4$ and $V_{\rm end} = 5.39 \times 10^{-9} r M_{\rm Pl}^4$ so that

$$\left(\frac{\rho_\phi}{\rho_r}\right)_{\rm end} \simeq \frac{\rho_{\phi,\rm end}}{0.01 g_p H_{\rm end}^4} \simeq \frac{1.11 \times 10^{11} g_p^{-1}}{r}. \tag{52}$$

Clearly, the radiation energy density is very low in this case, only one part in 10^{12} (assuming $g_p \simeq 100$ and $r \simeq 10^{-3}$) as compared to the field energy density at the end of inflation, and violates the constraint on $(\rho_\phi/\rho_r)_{\rm end}$ put by nucleosynthesis (13). Also note that for this model the ratio $H_{\rm inf}/H_{\rm end}$, for $r \simeq 10^{-3}$, is found to be 1.87, which implies that $H_{\rm inf}$ is very close to $H_{\rm end}$ for $r \ll 1$ as we have mentioned in Sec. II. This ratio is larger for a larger value of r. For the model under consideration, one can invoke the instant preheating mechanism as explained in the previous section. In this case, the lower bound on the temperature at the end of inflation can be calculated using Eq. (11) together with the constraint on ratio, $(\rho_g/\rho_r)_{\rm rh} \lesssim 0.01$, due to nucleosynthesis as

$$\rho_{\rm r,end} \gtrsim \frac{9V_0^2}{M_{\rm Pl}^4}\exp\left[-\lambda\left(n\lambda(n-2)\mathcal{N} + \left(\frac{\sqrt{2}}{n\lambda}\right)^{\frac{2-n}{n-1}}\right)^{\frac{n}{2-n}} + \left(\frac{\sqrt{2}}{n\lambda}\right)^{\frac{n}{n-1}}\right]$$

$$\Rightarrow T_{\rm end} = \rho_{\rm r,end}^{1/4} \gtrsim 4.9 r^{1/2} \times 10^{14}\ \text{GeV}, \qquad (53)$$

which is consistent with the model-independent estimate obtained earlier. Now that we have a lower bound on $T_{\rm end}$, using Eq. (14), we can obtain a bound on the number of e-foldings in this model, namely, $\mathcal{N} \lesssim 66.4$. Note that the number of e-foldings depends on the temperature at the end of inflation and would be even more than this for the temperature of radiation due to gravitational particle production.

Also, the lower bound on the coupling constant h, for $n = 6$, is found to be $h \gtrsim 0.12 g^{-1/2} r^{1/4}$. Clearly, the model under consideration successfully implements the paradigm of quintessential inflation. Last but not least, we should emphasize that it is challenging to find a model that can implement all the requirements of the scenario listed in the introduction.

V. CONCLUSIONS

In the scenario of quintessential inflation, the field enters the steep region of the potential soon after inflation ends, such that $\rho_\phi \propto a^{-6}$ *à la* the kinetic regime. The duration of this phase, which generally follows inflation in the scenario of the quintessential inflation, irrespective of the field potential, depends upon the temperature at the end of inflation. A longer kinetic regime or the lower value of $T_{\rm end}$ implies the enhancement of the energy density in relic gravitational waves compared to the field energy density and might challenge the nucleosynthesis constraint at the commencement of the radiative regime. The temperature at the end of inflation can be estimated from $H_{\rm inf}$, the value of which is fixed using COBE normalization. The estimate on the temperature is found to be $T_{\rm end} \simeq 1.99 r^{1/2} \times 10^{15}$ GeV. We certainly need a reheating mechanism other than the standard one, which is not operative in the case of quintessential inflation. The gravitational particle production is a quantum mechanical process of particle creation from the vacuum. The leading contribution to the energy density of created particles in this process comes from the epoch when the transition from acceleration to deceleration takes place, namely, from the end of inflation. Using the estimate for $H_{\rm end} \simeq H_{\rm inf}$, we estimated the temperature of radiation created during this process. The radiation temperature so estimated turns out to be less than $T_{\rm end}$, challenging the nucleosynthesis constraint.

To set the goal, we implemented the instant preheating, which is based upon the assumption that ϕ interacts with an auxiliary scalar field χ with the coupling g, which then interacts with the matter field with the coupling strength h. We have found a wide range in the parameter space, (g, h), which allows us to obtain the desired temperature (see Fig. 1).

The generic feature of the paradigm of quintessential inflation includes the relic gravitational wave background with the blue spectrum produced during the transition from inflation to the kinetic regime. In Fig. 2, we depict the spectral energy density of relic gravitational waves for the temperature at the end of inflation, which we have estimated model independently, as well as for the temperatures we have calculated from the model we have considered. We have also plotted the proposed sensitivity curves for advanced LIGO and LISA. As seen in the figure, the blue spectrum appears at high frequencies, which clearly distinguishes the paradigm of quintessential inflation from conventional inflation.

We show that the aforementioned requirements of quintessential inflation, which are model independent, can be successfully met by a model based upon the inflaton potential, $V \propto \text{Exp}(-\lambda\phi^n/M_{\rm Pl}^n), (n > 1)$. This potential has an interesting property; namely, its slope goes as ϕ^{n-1}, giving rise to slow roll for small ϕ, whereas it exhibits steep behavior at late stages for large values of the field. As demonstrated in Refs. [55,72], the model under consideration leads to a viable postinflationary dynamics. In particular, the scaling solution is an attractor of the dynamics, namely, an approximate scaling solution, despite $n > 1$ in (43). In this case, the late-time exit from scaling regime to cosmic acceleration may be successfully realized by invoking nonminimal coupling of the field with massive neutrino matter [55,72]. The present scenario, therefore, is of great interest; it provides a successful unification of inflation and late-time cosmic acceleration.

ACKNOWLEDGMENTS

We thank Md. Wali Hossain, C. Q. Geng, C. C. Lee, and E. N. Saridakis for useful discussions. S. A. acknowledges DST, Government of India, for financial support through Inspire Fellowship (DST/INSPIRE FELLOWSHIP/2012/614). M. S. thanks the Eurasian International Center for Theoretical Physics, Eurasian National University, Astana, for hospitality where the work was initiated. M. S. also thanks Maulana Azad National Urdu University, Gachibowli, Hyderabad, where part of the work was completed.

[1] A. A. Starobinsky, Phys. Lett. **91B**, 99 (1980).
[2] A. A. Starobinsky, Phys. Lett. **117B**, 175 (1982).
[3] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[4] A. D. Linde, Phys. Lett. **129B**, 177 (1983).
[5] E. J. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006).
[6] V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D **15**, 2105 (2006).
[7] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).
[8] M. Sami, arXiv:0901.0756.
[9] M. Sami, Curr. Sci. **97**, 887 (2009).
[10] M. Sami and R. Myrzakulov, Int. J. Mod. Phys. D **25**, 1630031 (2016).
[11] A. G. Riess *et al.* (Supernova Search Team Collaboration), Astron. J. **116**, 1009 (1998).
[12] S. Perlmutter *et al.* (Supernova Cosmology Project Collaboration), Astrophys. J. **517**, 565 (1999).
[13] A. E. Lange *et al.* (Boomerang Collaboration), Phys. Rev. D **63**, 042001 (2001).
[14] C. L. Bennett *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **148**, 1 (2003).
[15] P. A. R. Ade *et al.* (BICEP2 and Planck Collaborations), Phys. Rev. Lett. **114**, 101301 (2015).
[16] B. Spokoiny, Phys. Lett. B **315**, 40 (1993).
[17] P. J. E. Peebles and A. Vilenkin, Phys. Rev. D **60**, 103506 (1999).
[18] M. Peloso and F. Rosati, J. High Energy Phys. 12 (1999) 026.
[19] K. Dimopoulos, Nucl. Phys. B, Proc. Suppl. **95**, 70 (2001).
[20] E. J. Copeland, A. R. Liddle, and J. E. Lidsey, Phys. Rev. D **64**, 023509 (2001).
[21] V. Sahni, M. Sami, and T. Souradeep, Phys. Rev. D **65**, 023518 (2001).
[22] A. S. Majumdar, Phys. Rev. D **64**, 083503 (2001).
[23] K. Dimopoulos and J. W. F. Valle, Astropart. Phys. **18**, 287 (2002).
[24] M. Giovannini, Phys. Rev. D **67**, 123512 (2003).
[25] M. Sami and V. Sahni, Phys. Rev. D **70**, 083513 (2004).
[26] M. Sami and N. Dadhich, Tomsk State Pedagogical University Bulletin **44N7**, 25 (2004).
[27] R. Rosenfeld and J. A. Frieman, J. Cosmol. Astropart. Phys. 09 (2005) 003.
[28] M. W. Hossain, R. Myrzakulov, M. Sami, and E. N. Saridakis, Phys. Rev. D **90**, 023512 (2014).
[29] M. W. Hossain, R. Myrzakulov, M. Sami, and E. N. Saridakis, Phys. Rev. D **89**, 123513 (2014).
[30] M. W. Hossain, R. Myrzakulov, M. Sami, and E. N. Saridakis, Int. J. Mod. Phys. D **24**, 1530014 (2015).
[31] K. Dimopoulos and C. Owen, J. Cosmol. Astropart. Phys. 06 (2017) 027.
[32] E. Guendelman, E. Nissimov, and S. Pacheva, Bulgarian Journal of Physics **44**, 15 (2017).
[33] J. Rubio and C. Wetterich, arXiv:1705.00552.
[34] L. H. Ford, Phys. Rev. D **35**, 2955 (1987).
[35] A. D. Dolgov and A. D. Linde, Phys. Lett. **116B**, 329 (1982).
[36] L. F. Abbott, E. Farhi, and M. B. Wise, Phys. Lett. **117B**, 29 (1982).
[37] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994).
[38] Y. Shtanov, J. H. Traschen, and R. H. Brandenberger, Phys. Rev. D **51**, 5438 (1995).
[39] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. D **56**, 3258 (1997).
[40] A. H. Campos, H. C. Reis, and R. Rosenfeld, Phys. Lett. B **575**, 151 (2003).
[41] G. N. Felder, L. Kofman, and A. D. Linde, Phys. Rev. D **59**, 123523 (1999).
[42] G. N. Felder, L. Kofman, and A. D. Linde, Phys. Rev. D **60**, 103505 (1999).
[43] A. H. Campos, J. M. F. Maia, and R. Rosenfeld, Phys. Rev. D **70**, 023003 (2004).
[44] H. Tashiro, T. Chiba, and M. Sasaki, Classical Quantum Gravity **21**, 1761 (2004).
[45] P. J. Steinhardt, L. M. Wang, and I. Zlatev, Phys. Rev. D **59**, 123504 (1999).
[46] L. P. Grishchuk, Zh. Eksp. Teor. Fiz. **67**, 825 (1974) [Sov. Phys. JETP **40**, 409 (1975)].
[47] A. A. Starobinsky, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 719 (1979) [JETP Lett. **30**, 682 (1979)].
[48] V. Sahni, Phys. Rev. D **42**, 453 (1990).
[49] T. Souradeep and V. Sahni, Mod. Phys. Lett. A **07**, 3541 (1992).
[50] M. Gasperini and M. Giovannini, Phys. Rev. D **47**, 1519 (1993).
[51] M. Gasperini and M. Giovannini, Phys. Lett. B **282**, 36 (1992).
[52] M. Giovannini, Phys. Rev. D **60**, 123511 (1999).
[53] M. Giovannini, Classical Quantum Gravity **16**, 2905 (1999).
[54] D. Langlois, R. Maartens, and D. Wands, Phys. Lett. B **489**, 259 (2000).
[55] C. Q. Geng, M. W. Hossain, R. Myrzakulov, M. Sami, and E. N. Saridakis, Phys. Rev. D **92**, 023522 (2015).
[56] F. Lucchin and S. Matarrese, Phys. Rev. D **32**, 1316 (1985).
[57] B. Ratra, Phys. Rev. D **45**, 1913 (1992).
[58] M. Sami, P. Chingangbam, and T. Qureshi, Phys. Rev. D **66**, 043530 (2002).
[59] C. Wetterich, Phys. Rev. D **89**, 024005 (2014).
[60] C. Wetterich, Phys. Dark Universe **2**, 184 (2013).
[61] C. Wetterich, Phys. Lett. B **726**, 15 (2013).
[62] C. Wetterich, Phys. Lett. B **736**, 506 (2014).
[63] C. Wetterich, Lect. Notes Phys. **892**, 57 (2015).
[64] L. Amendola, M. Baldi, and C. Wetterich, Phys. Rev. D **78**, 023015 (2008).
[65] C. Wetterich, Phys. Lett. B **655**, 201 (2007).
[66] E. F. Bunn and M. J. White, Astrophys. J. **480**, 6 (1997).
[67] E. F. Bunn, A. R. Liddle, and M. J. White, Phys. Rev. D **54**, R5917 (1996).
[68] LIGO Home Page, http://www.ligo.caltech.edu/.
[69] https://dcc.ligo.org/LIGO-T0900288/public.
[70] LISA Home Page, http://lisa.nasa.gov/.
[71] http://www.srl.caltech.edu/~shane/sensitivity/.
[72] C. Q. Geng, C. C. Lee, M. Sami, E. N. Saridakis, and A. A. Starobinsky, J. Cosmol. Astropart. Phys. 06 (2017) 011.