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Bulk viscosity of strange quark matter in an enhanced perturbative QCD model

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The bulk viscosity is studied in a thermodynamically enhanced perturbative QCD (EPQ) model that includes the running of both the strong coupling and the strange quark mass. Similar to the results in other effective models, the bulk viscosity in the EPQ model is larger by 1–2 orders of magnitude compared with that in the MIT bag model. Comparison among the different models indicates that the similarity in orders is mainly due to the inclusion of interactions between quarks. When the temperature is relatively low and enters the equation of state of quark matter, the bulk viscosity can be enormously enlarged. For the analytical expression of bulk viscosity, a reasonable range of model parameters is found. Moreover, due to the enlarged bulk viscosity, the damping time of a strange star with about 2 times the solar mass can be as short as 10^{-3} second, showing that massive quark stars could reach stability quickly after some radial unstable activities.

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I. INTRODUCTION

It is generally believed that strange quark matter (SQM) composed of comparable numbers of up (u), down (d), and strange (s) quarks could be the true ground state of quantum chromodynamics (QCD) [1–6]. SQM may be produced in supernovae explosions [7], or be found in high-energy cosmic rays [8] and in the inner part of neutron stars. If it is absolutely stable at zero pressure and temperature, some or most of the known neutron stars could turn out to be the so-called strange stars; or if it is only metastable, then neutron stars could be hybrid stars with a strange-matter core covered by a nuclear-matter shell [9–12].

Because compact stars with masses between 1 and 2 times the solar mass are bound mainly by gravity rather than strong interactions, they have similar radii, and it is thus difficult to distinguish strange stars, hybrid stars, and neutron stars observationally [6,9,10,13]. It was thought that if a pion, kaon condensate, or quark matter is present in a neutron star, neutrino emission is faster than by the modified Urca process. However, in Ref. [14], it was demonstrated that the direct Urca process would lead to more rapid cooling than any other process.

One possible way to distinguish these three kinds of compact stars could be the studies of phenomena related to stellar rotations. It has been shown that the emission of gravitational radiation due to the r-mode instabilities in hot, young neutron stars severely limits the rotation period of these stars, but the r-mode does not play any role in young strange stars [15]. Another way is concerned with stellar vibration. The important issue for both ways is the time scale for the damping of vibrations, and of the gravitational radiation reaction instability. The time scale for the damping of these mechanisms is strongly influenced by the bulk viscosity of the component matter.

The bulk viscosity in quark matter, which stems from the strangeness-changing weak interaction or nonleptonic interaction $u + d \leftrightarrow s + u$, was investigated by Wang and Lu [16]. The results, using a method similar to previous papers [17,18], showed that the damping of vibrations in quark matter is more efficient than in π condensate. Sawyer found that the bulk viscosity arising from nonleptonic strangeness-changing quark-quark interactions is orders of magnitude larger than that for ordinary nuclear matter [19]. With a new rate expression for the reaction $u + d \leftrightarrow s + u$, Madsen calculated the bulk viscosity in the conventional bag model [20]. In recent decades, the bulk viscosity of SQM has been extensively investigated in many kinds of phenomenological models; e.g., the density-dependent quark mass model [21,22], the quasiparticle model [23,24], and so on [25–30].

Due to the asymptotic freedom of the strong interaction at very high densities, perturbative QCD models had been very useful in studying the properties of SQM and strange stars [2,31–34]. If the density is not that high, the perturbative method needs to be extended. However, naive extension to comparatively lower densities of the perturbation approach is questionable due to quark confinement. Recently, according to the fundamental requirement of

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thermodynamic consistency, we put forward an enhanced perturbative QCD model (EPQ) [35,36]. In the present paper, we derive the formula for bulk viscosity in the EPQ model and make comparisons among the different models. It is found that the bulk viscosities in the EPQ and quasiparticle models are similar in magnitude. The similarity is mainly due to the inclusion of the interactions between quarks. Moreover, on application of the obtained bulk viscosity, we calculate the damping time of massive quark stars.

The paper is organized as follows: In Sec. II, we first introduce the EPQ model and derive the expression of the bulk viscosity. In Sec. III, we give the numerical results for the bulk viscosity and explore the implication on the damping time of a massive strange quark star. Finally, a summary is given in Sec. IV.

II. FORMULA OF THE BULK VISCOSITY IN THE EPQ MODEL

A. Brief introduction to the EPQ model

In the EPQ model, the perturbative contributions to the thermodynamic potential density from u and d quarks at the first order of strong running coupling are, respectively,

$$\Omega_{\rm u} = -\frac{\mu_{\rm u}^4}{4\pi^2}(1-2\alpha), \qquad \Omega_{\rm d} = -\frac{\mu_{\rm d}^4}{4\pi^2}(1-2\alpha), \tag{1}$$

and that from the massive strange quarks is [32,37]

$$\begin{aligned}\Omega_{\rm s} = {}& \frac{-1}{4\pi^2}\left[\mu_{\rm s}\nu_{\rm s}\left(\mu_{\rm s}^2-\frac{5}{2}m_{\rm s}^2\right)+\frac{3}{2}m_{\rm s}^4 {\rm ach}\frac{\mu_{\rm s}}{m_{\rm s}}\right] \\ & + \frac{\alpha}{2\pi^2}\left[3\left(\mu_{\rm s}\nu_{\rm s}-m_{\rm s}^2{\rm ach}\frac{\mu_{\rm s}}{m_{\rm s}}\right)^2-2\nu_{\rm s}^4\right. \\ & \left. + m_{\rm s}^2\left(6\ln\frac{u}{m_{\rm s}}+4\right)\left(\mu_{\rm s}\nu_{\rm s}-m_{\rm s}^2{\rm ach}\frac{\mu_{\rm s}}{m_{\rm s}}\right)\right].\end{aligned} \tag{2}$$

Here $\mu_{\rm u}$, $\mu_{\rm d}$, and $\mu_{\rm s}$ are the chemical potentials of u, d, and s quarks, respectively; $\alpha \equiv \alpha_s/\pi = g^2/(4\pi^2)$ is the running coupling; and ach$x \equiv \ln(x+\sqrt{x^2-1})$ is the inverse hyperbolic cosine function. Because the mass of a u/d quark is much smaller than that of an s quark, we consider only the mass effect of strange quarks. For simplicity, we have used the notation $\nu_{\rm s} \equiv \sqrt{\mu_{\rm s}^2-m_{\rm s}^2}$, which can be regarded as the fermion momentum of s quarks. Because the electron does not participate in the strong interactions, its contribution to the thermodynamic potential density is simply

$$\Omega_{\rm e} = -\frac{\mu_{\rm e}^4}{12\pi^2}. \tag{3}$$

At one-loop level, the running coupling and running strange quark mass are, respectively, given by

$$\alpha(u) = \frac{1}{\beta_0 \ln(u^2/\Lambda^2)}, \qquad m_{\rm s} = \hat{m}_{\rm s}\alpha^{\gamma_0/\beta_0}, \tag{4}$$

where $\beta_0 = 11/4 - N_{\rm f}/6$, $\gamma_0 = 1$, and Λ and $\hat{m}_{\rm s}$ are the QCD scale parameters for the coupling and strange quark mass, respectively. In the following numerical calculations, we take $\Lambda = 146$ MeV and $\hat{m}_{\rm s} = 280$ MeV [36].

The number densities of u and d quarks and electrons are, respectively,

$$\rho_{\rm u} = \frac{\mu_{\rm u}^3}{\pi^2}(1-2\alpha), \qquad \rho_{\rm d} = \frac{\mu_{\rm d}^3}{\pi^2}(1-2\alpha), \qquad \rho_{\rm e} = \frac{\mu_{\rm e}^3}{3\pi^2}, \tag{5}$$

while that of s quarks is

$$\rho_{\rm s} = \frac{\nu_{\rm s}^3}{\pi^2} - \frac{2\alpha}{\pi^2}\nu_{\rm s}\left(\mu_{\rm s}\nu_{\rm s}+2m_{\rm s}^2-3m_{\rm s}^2\ln\frac{\mu_{\rm s}+\nu_{\rm s}}{u}\right). \tag{6}$$

In the EPQ model, an additional term Ω', determined by the thermodynamic consistency requirement, is included to consider the nonperturbative interactions among quarks. The details can be found in Ref. [36]. Another important thing in the EPQ model is the relation between the renormalization substraction point u and quark chemical potentials, which is usually given arbitrarily. However, in the EPQ model the expression for u is given as an implicit function of quark chemical potentials, i.e.

$$\frac{8\pi^2}{9}f_{1,0}(\mu_{\rm u},\mu_{\rm d},\mu_{\rm s},u) - \frac{N_{\rm f}}{C^4}u^4 = 0, \tag{7}$$

where $N_{\rm f} = 3$, C is an EPQ model parameter in the range $0 < 1/C < 1$ [36], and

$$\begin{aligned}f_{1,0} = {}& \frac{9}{8\pi^2}(\mu_{\rm u}^4+\mu_{\rm d}^4+\mu_{\rm s}^4)+\frac{75m_{\rm s}^4}{8\pi^2}{\rm ach}^2\left(\frac{\mu_{\rm s}}{m_{\rm s}}\right) \\ & + \frac{m_{\rm s}^2}{8\pi^2}\left[41\mu_{\rm s}^2-50m_{\rm s}^2+44\mu_{\rm s}\nu_{\rm s}\right. \\ & + 6\ln\frac{u}{m_{\rm s}}\left(17\mu_{\rm s}\nu_{\rm s}-25m_{\rm s}^2{\rm ach}\frac{\mu_{\rm s}}{m_{\rm s}}\right) \\ & \left. - 2{\rm ach}\frac{\mu_{\rm s}}{m_{\rm s}}(38m_{\rm s}^2+51\mu_{\rm s}\nu_{\rm s})\right].\end{aligned} \tag{8}$$

Now, taking into account the weak equilibrium

$$\mu_{\rm u}+\mu_{\rm e} = \mu_{\rm d} = \mu_{\rm s}, \tag{9}$$

the charge neutrality

$$\frac{2}{3}\rho_{\rm u} - \frac{1}{3}\rho_{\rm d} - \frac{1}{3}\rho_{\rm s} - \rho_{\rm e} = 0, \tag{10}$$

and the baryon-number conservation

$$n_{\mathrm{b}} = \frac{1}{3}(\rho_{\mathrm{u}} + \rho_{\mathrm{d}} + \rho_{\mathrm{s}}), \tag{11}$$

one can get the chemical potentials $\mu_i (i = \mathrm{u}, \mathrm{d}, \mathrm{s}, \text{and } \mathrm{e})$, and further obtain the energy density and pressure of SQM by

$$P = -\Omega, \tag{12}$$

$$E = \Omega + \sum_{i=\mathrm{u,d,s,e}} \mu_i n_i, \tag{13}$$

where the total thermodynamic potential density of the whole system is

$$\Omega = \sum_{i=\mathrm{u,d,s,e}} \Omega_i + \Omega'. \tag{14}$$

B. The bulk viscosity in the EPQ model

In this section, we deduce the expression of the bulk viscosity of strange quark matter in the EPQ model with the method used in mid-1980s [16,19] and later developed by many authors [20–24].

First, we assume the volume per unit mass of quark matter is v, which, due to vibration, changes periodically in time according to the following relation:

$$v(t) = v_0 + \Delta v \sin\left(\frac{2\pi t}{\tau}\right), \tag{15}$$

where v_0 is the equilibrium volume, Δv is the vibration amplitude, and τ is the oscillation period. Because of the smallness of the relative volume vibration amplitude, i.e. $\Delta v/v_0 \ll 1$, the pressure P can be expanded near the equilibrium pressure P_0, i.e.

$$P(t) = P_0 + \left(\frac{\partial P}{\partial v}\right)_0 \delta v + \left(\frac{\partial P}{\partial n_{\mathrm{d}}}\right)_0 \delta n_{\mathrm{d}} + \left(\frac{\partial P}{\partial n_{\mathrm{s}}}\right)_0 \delta n_{\mathrm{s}}. \tag{16}$$

Here only the leading terms are considered. The changes in particle number in the volume v due to the reaction $u + s \leftrightarrow u + d$ can be expressed as

$$\delta n_{\mathrm{d}} = -\delta n_{\mathrm{s}} = \int_0^t \frac{\mathrm{d}n_{\mathrm{d}}}{\mathrm{d}t}\mathrm{d}t. \tag{17}$$

If the reaction rate $\mathrm{d}n_{\mathrm{d}}/\mathrm{d}t$ is given, the mean dissipation rate of the vibration energy per unit mass can be obtained by

$$\left(\frac{\mathrm{d}w}{\mathrm{d}t}\right)_{\mathrm{av}} = -\frac{1}{\tau}\int_0^\tau P(t)\frac{\mathrm{d}v}{\mathrm{d}t}\mathrm{d}t. \tag{18}$$

As to the reaction rate $\mathrm{d}n_{\mathrm{d}}/\mathrm{d}t$, we still adopt the expression given in Ref. [20], viz.

$$\frac{\mathrm{d}n_{\mathrm{d}}}{\mathrm{d}t} \approx G_{\mathrm{C}}\mu_{\mathrm{d}}^5 \delta\mu(\delta\mu^2 + 4\pi^2 T^2)v_0, \tag{19}$$

where the constant quantity G_{C} can be connected to the weak-coupling constant G_{F} and the Cabibbo angle θ_{C} by

$$G_{\mathrm{C}} \equiv \frac{16}{5\pi^5} G_F^2 \sin^2\theta_C \cos^2\theta_C = 6.76 \times 10^{-26}\ \mathrm{MeV}^{-4}. \tag{20}$$

The chemical potential difference $\delta\mu \equiv \mu_{\mathrm{s}} - \mu_{\mathrm{d}}$ in Eq. (19) stems from the change of the Fermi surface of strange and down quarks. It can be derived with a process similar to that of obtaining the pressure in Eq. (16), giving

$$\delta\mu(t) = \left(\frac{\partial\delta\mu}{\partial v}\right)_0 \delta v + \left(\frac{\partial\delta\mu}{\partial n_{\mathrm{d}}}\right)_0 \delta n_{\mathrm{d}} + \left(\frac{\partial\delta\mu}{\partial n_{\mathrm{s}}}\right)_0 \delta n_{\mathrm{s}}, \tag{21}$$

where $\frac{\partial\delta\mu}{\partial x} = \frac{\partial\mu_{\mathrm{s}}}{\partial x} - \frac{\partial\mu_{\mathrm{d}}}{\partial x}$, $(x = v, n_{\mathrm{d}}, \text{ or } n_{\mathrm{s}})$. When calculating the energy dissipation rate in Eq. (18), the first two terms on the right-hand side of Eq. (16) can be ignored [16]. Then the third and fourth terms of $P(t)$ can be obtained from the standard thermodynamic relation, i.e.

$$\frac{\partial P}{\partial n_i} = -\frac{\partial\mu_i}{\partial v}\ (i = \mathrm{d}, \mathrm{s}). \tag{22}$$

According to the particle-number density in Eq. (5), the particle number per unit mass for d quarks is

$$n_{\mathrm{d}} = \frac{\mu_{\mathrm{d}}^3}{\pi^2}(1 - 2\alpha)v, \tag{23}$$

which immediately gives the differential form as

$$\mathrm{d}n_{\mathrm{d}} = \frac{\mu_{\mathrm{d}}^2 v}{\pi^2}\left[3(1-2\alpha) - 2\mu_{\mathrm{d}}\frac{\mathrm{d}\alpha}{\mathrm{d}u}\frac{\partial u}{\partial\mu_{\mathrm{d}}}\right]\mathrm{d}\mu_{\mathrm{d}} + \frac{\mu_{\mathrm{d}}^3}{\pi^2}(1-2\alpha)\mathrm{d}v, \tag{24}$$

where

$$\frac{\mathrm{d}\alpha}{\mathrm{d}u} = -\frac{2\beta_0\alpha^2}{u}, \qquad \frac{\partial u}{\partial\mu_{\mathrm{d}}} = \frac{\partial f_{1,0}/\partial\mu_{\mathrm{d}}}{\frac{9N_{\mathrm{f}}u^3}{2\pi^2C^4} - \frac{\partial f_{1,0}}{\partial u} - \frac{\partial f_{1,0}}{\partial m_{\mathrm{s}}}\frac{\mathrm{d}m_{\mathrm{s}}}{\mathrm{d}\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}u}}, \tag{25}$$

with

$$\frac{\mathrm{d}m_{\mathrm{s}}}{\mathrm{d}\alpha} = \frac{4\hat{m}_{\mathrm{s}}}{9}\alpha^{-5/9}. \tag{26}$$

The partial derivative of $f_{1,0}$ with respect to $\mu_{\rm d}$ in the second equation of Eq. (25) can also be calculated from Eq. (8) without much difficulty.

Letting $C_{\rm d} = 3(1-2\alpha) - 2\mu_{\rm d}\frac{{\rm d}\alpha}{{\rm d}u}\frac{\partial u}{\partial \mu_{\rm d}}$, one can obtain a more tightknit form of Eq. (24):

$$ {\rm d}n_{\rm d} = \frac{\mu_{\rm d}^2 C_{\rm d} v}{\pi^2}{\rm d}\mu_{\rm d} + \frac{\mu_{\rm d}^3}{\pi^2}(1-2\alpha){\rm d}v, \tag{27}$$

from which we immediately have

$$ \left(\frac{\partial \mu_{\rm d}}{\partial n_{\rm d}}\right)_0 = \frac{\pi^2}{\mu_{\rm d}^2 v_0 C_{\rm d}}, \qquad \left(\frac{\partial \mu_{\rm d}}{\partial v}\right)_0 = -\frac{\pi^2 \rho_{\rm d}}{\mu_{\rm d}^2 v_0 C_{\rm d}}. \tag{28}$$

Similarly, starting from the number of s quarks per unit mass

$$ n_{\rm s} = \left[\frac{\nu_{\rm s}^3}{\pi^2} - \frac{2\alpha}{\pi^2}\nu_{\rm s}\left(\mu_{\rm s}\nu_{\rm s} + 2m_{\rm s}^2 - 3m_{\rm s}^2 \ln\frac{\mu_{\rm s}+\nu_{\rm s}}{u}\right)\right]v, \tag{29}$$

one can get

$$ \left(\frac{\partial \mu_{\rm s}}{\partial n_{\rm s}}\right)_0 = \frac{\pi^2}{\mu_{\rm s}^2 v_0 C_{\rm s}}, \qquad \left(\frac{\partial \mu_{\rm s}}{\partial v}\right)_0 = -\frac{\pi^2 \rho_{\rm s}}{\mu_{\rm s}^2 v_0 C_{\rm s}}, \tag{30}$$

where $C_{\rm s}$ is defined to be

$$ C_{\rm s} \equiv \frac{1}{\mu_{\rm s}^2}\left[\frac{C_{\nu_{\rm s}}\mu_{\rm s}}{\nu_{\rm s}} + C_{\mu_{\rm s}} - \left(\frac{C_{\nu_{\rm s}} m_{\rm s}}{\nu_{\rm s}} + C_{m_{\rm s}}\right)\frac{{\rm d}m_{\rm s}}{{\rm d}\alpha}\frac{{\rm d}\alpha}{{\rm d}u}\frac{\partial u}{\partial \mu_{\rm s}} - C_\alpha \frac{{\rm d}\alpha}{{\rm d}u}\frac{\partial u}{\partial \mu_{\rm s}} - C_u \frac{\partial u}{\partial \mu_{\rm s}}\right], \tag{31}$$

with

$$\begin{aligned} C_\alpha &\equiv 2\mu_{\rm s}\nu_{\rm s}^2 + 4\nu_{\rm s}m_{\rm s}^2 - 6\nu_{\rm s}m_{\rm s}^2 \ln\frac{\mu_{\rm s}+\nu_{\rm s}}{u}, \\ C_{\nu_{\rm s}} &\equiv 3\nu_{\rm s}^2 - 4\alpha\mu_{\rm s}\nu_{\rm s} - 4\alpha m_{\rm s}^2 + \frac{6\alpha\nu_{\rm s}m_{\rm s}^2}{\mu_{\rm s}+\nu_{\rm s}} + 6\alpha m_{\rm s}^2 \ln\frac{\mu_{\rm s}+\nu_{\rm s}}{u}, \\ C_{m_{\rm s}} &\equiv 8\alpha\nu_{\rm s}m_{\rm s} - 12\alpha\nu_{\rm s}m_{\rm s}\ln\frac{\mu_{\rm s}+\nu_{\rm s}}{u}, \\ C_{\mu_{\rm s}} &\equiv \frac{6\alpha\nu_{\rm s}m_{\rm s}^2}{\mu_{\rm s}+\nu_{\rm s}} - 2\alpha\nu_{\rm s}^2, \quad \text{and} \quad C_u \equiv \frac{6\alpha\nu_{\rm s}m_{\rm s}^2}{u}. \end{aligned} \tag{32}$$

The partial derivatives in Eq. (31) are similar to Eqs. (25) and (26), only with the replacement of $\mu_{\rm d}$ by $\mu_{\rm s}$ in the second equation of Eq. (25).

According to Eqs. (16), (22), (28), and (30), one can get the pressure contributing to the dissipation energy $\delta P(t)$,

$$\begin{aligned} \delta P(t) &= \left(\frac{\partial P}{\partial n_{\rm d}}\right)_0 \delta n_{\rm d} + \left(\frac{\partial P}{\partial n_{\rm s}}\right)_0 \delta n_{\rm s} \\ &= \frac{\pi^2}{\mu_{\rm d}^2 v_0}\left(\frac{\rho_{\rm d}}{C_{\rm d}} - \frac{\rho_{\rm s}}{C_{\rm s}}\right)\int_0^t \frac{{\rm d}n_{\rm d}}{{\rm d}t}{\rm d}t, \end{aligned} \tag{33}$$

and the chemical difference $\delta\mu(t)$ in Eq. (21)

$$ \delta\mu(t) = \frac{\pi^2}{\mu_{\rm d}^2}\left(\frac{\rho_{\rm d}}{C_{\rm d}} - \frac{\rho_{\rm s}}{C_{\rm s}}\right)\frac{\Delta v}{v_0}\sin\left(\frac{2\pi t}{\tau}\right) - \frac{\pi^2}{\mu_{\rm d}^2 v_0} \times \left(\frac{1}{C_{\rm d}} + \frac{1}{C_{\rm s}}\right)\int_0^t \frac{{\rm d}n_{\rm d}}{{\rm d}t}{\rm d}t, \tag{34}$$

which is equivalent to

$$ \frac{{\rm d}\delta\mu(t)}{{\rm d}t} = \frac{\pi^2}{\mu_{\rm d}^2}\left(\frac{\rho_{\rm d}}{C_{\rm d}} - \frac{\rho_{\rm s}}{C_{\rm s}}\right)\frac{\Delta v}{v_0}\frac{2\pi}{\tau}\cos\left(\frac{2\pi t}{\tau}\right) - \frac{\pi^2}{\mu_{\rm d}^2 v_0}\left(\frac{1}{C_{\rm d}} + \frac{1}{C_{\rm s}}\right)\frac{{\rm d}n_{\rm d}}{{\rm d}t}. \tag{35}$$

Further, the mean dissipation rate of the vibration energy per unit mass reads

$$ \left(\frac{{\rm d}w}{{\rm d}t}\right)_{\rm av} = -\left(\frac{\Delta v}{v_0}\right)\left(\frac{2\pi}{\tau}\right)\frac{\pi^2}{\mu_{\rm d}^2\tau}\left(\frac{\rho_{\rm d}}{C_{\rm d}} - \frac{\rho_{\rm s}}{C_{\rm s}}\right)\int_0^\tau {\rm d}t \times \left(\int_0^t \frac{{\rm d}n_{\rm d}}{{\rm d}t}{\rm d}t\right)\cos\left(\frac{2\pi t}{\tau}\right). $$

The bulk viscosity can then be written as [16,20]

$$ \zeta \equiv 2\frac{({\rm d}w/{\rm d}t)_{\rm av}}{v_0}\left(\frac{v_0}{\Delta v}\right)^2\left(\frac{\tau}{2\pi}\right)^2. \tag{36}$$

Accordingly, we obtain the bulk viscosity in the EPQ model by

$$ \zeta = -2\left(\frac{v_0}{\Delta v}\right)\left(\frac{\tau}{2\pi}\right)\frac{\pi^2}{\mu_{\rm d}^2 v_0}\left(\frac{\rho_{\rm d}}{C_{\rm d}} - \frac{\rho_{\rm s}}{C_{\rm s}}\right)\frac{1}{\tau} \times \int_0^\tau {\rm d}t\left(\int_0^t \frac{{\rm d}n_{\rm d}}{{\rm d}t}{\rm d}t\right)\cos\left(\frac{2\pi t}{\tau}\right). \tag{37}$$

If the temperature is high enough, i.e. $2\pi T \gg \delta\mu$, the cubic term in Eq. (19) can be safely ignored. Therefore, the bulk viscosity can be solved analytically, giving

$$ \zeta_a = 2\pi^3 G_{\rm C}\mu_{\rm d}^3 T^2\left(\frac{v_0}{\Delta v}\right)\left(\frac{\rho_{\rm d}}{C_{\rm d}} - \frac{\rho_{\rm s}}{C_{\rm s}}\right)\frac{A\tau^3}{4\pi^2 + B^2\tau^2} \times \left[1 - \frac{2B\tau}{4\pi^2 + B^2\tau^2}(1 - e^{-B\tau})\right], \tag{38}$$

where we have used the symbol definitions

$$A \equiv \frac{\pi^2}{\mu_d^2}\left(\frac{\rho_d}{C_d}-\frac{\rho_s}{C_s}\right)\frac{\Delta v}{v_0}\frac{2\pi}{\tau} \tag{39}$$

and

$$B \equiv 4\pi^4 G_C \mu_d^3 T^2\left(\frac{1}{C_d}+\frac{1}{C_s}\right). \tag{40}$$

At low temperature, however, the term proportional to $\delta\mu^3$ becomes important and thus cannot be neglected. Generally in this case, to calculate the bulk viscosity, one has to simultaneously solve Eqs. (19), (35), and (37) numerically. The numerical results will be given in the following section.

III. NUMERICAL RESULTS AND DISCUSSIONS

A. Properties of bulk viscosity

In order to investigate the bulk viscosity of SQM in the EPQ model, we have to choose the values of the model parameter C, the relative volume vibration amplitude $\Delta v/v_0$, the oscillation period τ (in seconds), the density n_b (in fm^{-3}) or quark chemical potentials μ_q (in MeV), and the temperature T (in MeV). In Fig. 1, we show the bulk viscosity of SQM as a function of $\Delta v/v_0$ for the fixed $\tau = 10^{-3}$ s and $C = 1.25$ (solid curve), 2.0 (dashed curve), and 2.5 (dotted curve) at $T = 10^{-4}$ MeV and $\mu_d = \mu_s = 400$ MeV.

From Fig. 1, one can see that with the increasing model parameter C, the bulk viscosity slightly decreases at the same $\Delta v/v_0$, which implies that the uncertainty in the EPQ model parameter C does not influence the bullk viscosity significantly on one hand. On the other hand, due to large C corresponding to small strong coupling α, it can be well understood that the bulk viscosity of SQM decreases with increasing C. In addition, the bulk viscosity as a function of $\Delta v/v_0$ can be notably divided into three regimes. At small $\Delta v/v_0$, the bulk viscosity is almost constant. Then there is an increasing part where the bulk viscosity increases rapidly with increasing $\Delta v/v_0$. Finally, at high enough $\Delta v/v_0$, the bulk viscosity decreases with increasing $\Delta v/v_0$. It is obvious that the last decreasing part should not be taken seriously, because the expansion method should be valid at $\Delta v \ll v_0$.

At different baryon-number densities (or quark chemical potentials), the bulk viscosity is also different, as shown in Fig. 2. At relatively low temperatures, the difference of bulk viscosity at different densities is almost negligible, but at relatively high temperatures, the difference becomes large. Another important feature is that at relatively low $\Delta v/v_0$, as the temperature increases, the bulk viscosity first stays almost constant, then becomes large as well, but decreases when temperature gets too high. This situation can be more clearly illustrated from Fig. 3. Moreover, in the EPQ model, when temperature becomes high enough, the bulk viscosities are almost the same even if the relative

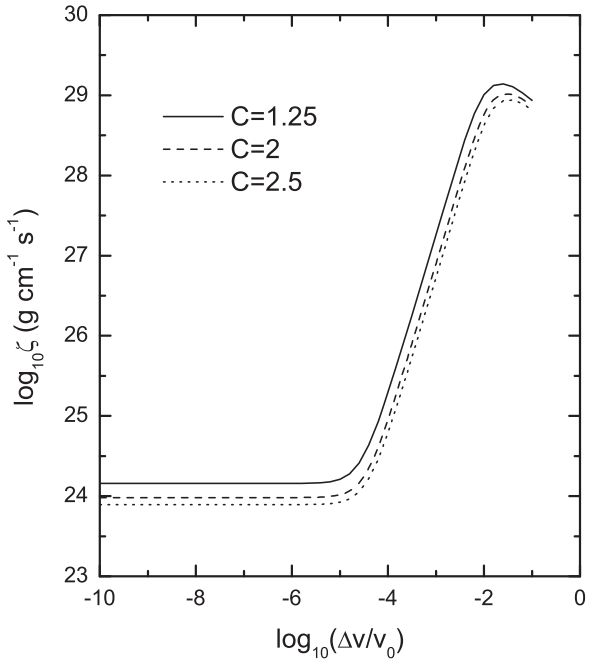

FIG. 1. Bulk viscosity as a function of the relative volume vibration amplitude $\Delta v/v_0$ at $C = 1.25$ (solid curve), 2.0 (dashed curve), and 2.5 (dotted curve) for $\mu_d = \mu_s = 400$ MeV. For all curves, the oscillation period is $\tau = 10^{-3}$ s, and the temperature is $T = 10^{-4}$ MeV.

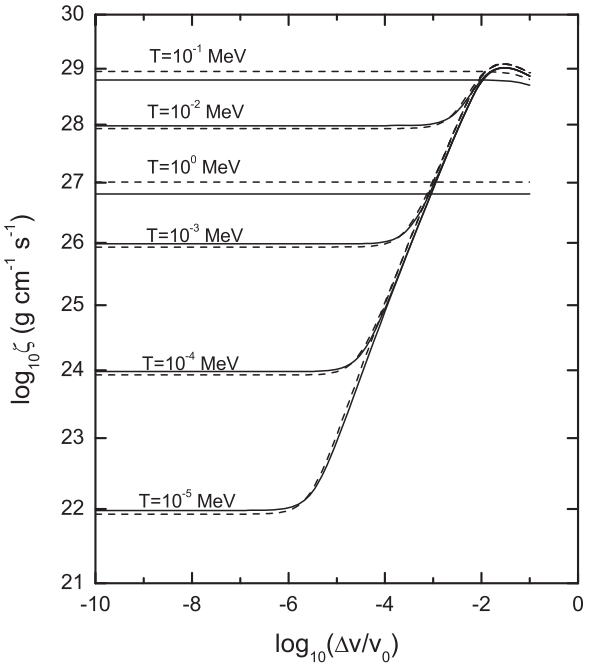

FIG. 2. Bulk viscosity as a function of the relative volume vibration amplitude for $\mu_d = \mu_s = 400$ MeV (solid line, baryon-number density $n_b = 0.568$ fm^{-3}) and $\mu_d = \mu_s = 362$ MeV (dashed line, baryon-number density $n_b = 0.400$ fm^{-3}) with the model parameter $C = 2$ and the vibration period $\tau = 10^{-3}$ s.

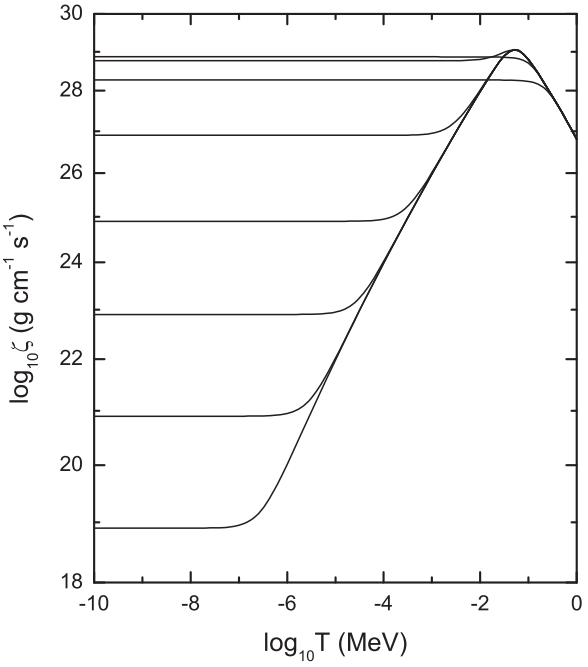

FIG. 3. Bulk viscosity as a function of temperature for $\mu_d = \mu_s = 400$ MeV with the fixed $C = 2$, $\tau = 10^{-3}$ s, and the respective $\Delta v/v_0$ values as 10^{-7}, 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{0}, 10^{-2}, and 10^{-1} for the curves from bottom to top.

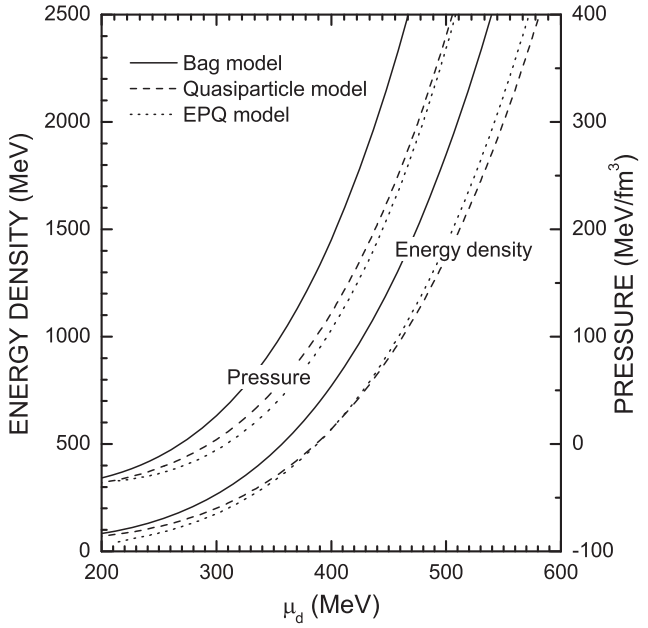

FIG. 4. Energy density and pressure as functions of chemical potential given in different models. Except for the bag model, the quasiparticle model used in Ref. [23] and the EPQ model employed in this paper all include the interactions between quarks in some way.

volume vibration amplitudes are very different. This seems different from the results given by Ref. [23]. The reason for this deviation stems from the fact that in Ref. [23] discrete values of the running coupling had been employed to calculate the temperature behavior of bulk viscosity, while in Fig. 3 all curves are plotted with the same running coupling.

Compared with the previous results in different models, the bulk viscosity in our model is analogous in the order of magnitude with that in those models including interactions (perturbative and/or confinement) between quarks, such as the quasiparticle model [22,23] and density-dependent quark mass model [24], which means that the bulk viscosity in the EPQ model is also much larger than that in the noninteracting quark-gas model, i.e. the conventional bag model [20].

Actually, in calculating the bulk viscosity, one usually has to solve the related equations at fixed chemical potentials. Therefore, to analyze the similarities shown in different effective models, we plot, in Fig. 4, the energy density on the left axis and pressure on the right axis as functions of the chemical potential μ_d. The quasiparticle model has various versions [38–41], and the one we adopt here for comparison purposes was suggested in Ref. [42] with thermodynamic improvement, and was applied to study the bulk viscosity in Ref. [23]. All the effective models in Fig. 4 have a bag constant B, and we take $B^{1/4} = 135$ MeV in numerical calculations. Additionally, in the bag model and quasiparticle model, the current mass of strange quarks is taken as $m_{s0} = 100$ MeV. The coupling constant in the quasiparticle model [24] is taken as $g = 3$. In the EPQ model, the additional model parameter is given as $C = 2$.

From Fig. 4, one can easily find that at the same chemical potential μ_d, the energy density and pressure in the quasiparticle model and EPQ model are very close. In the bag model, however, both the energy density and pressure deviate obviously from those in the quasiparticle model and EPQ model. As is well known, the effective mass in the quasiparticle model includes the medium effets, and in EPQ model, the running effect is taken into consideration. Moreover, in order to maintain thermodynamic consistency, an additional chemical-potential-dependent term is usually added to the thermodynamic potential density in both the quasiparticle model and the EPQ model. This extra term generally plays an important role at low densities, which, in some sense, reflects the confinement interactions between quarks. At relatively higher density, the perturbative interaction sets in. The quasiparticle and EPQ models contain the first-order perturbative interaction by considering the strong interaction coupling. The difference is that the EPQ model also includes the running of the strange quark mass. It is the interactions that lead to the similarities of the bulk viscosity shown in different effective models. These similarities are understandable with a view to the fact that, at the fixed chemical potential μ_d, the energy density and pressure in those models including the interactions

among quarks are much more similar than those in the bag model without interactions.

Naturally, the bulk viscosity depends on the temperature. There are two ways to achieve this temperature dependence. One way is by the reaction rate through Eq. (19), which has nothing to do with the strong-interaction models. The other way is crucially dependent on the EOSs of different models or on the derivatives in Eq. (22). Because the present EPQ model is limited at zero temperature, we tentatively investigate the temperature effect entering the EOS of quark matter on the bulk viscosity by plotting the bulk viscosity as a function of the temperature T in three different cases in Fig. 5. Case 1 (solid line) is the result of the currently employed EPQ model with the first-order interaction at zero temperature. Trying to investigate the effect of temperature entering the EOS on bulk viscosity, we first simplify the EPQ model by ignoring the interaction between quarks, i.e. setting $\alpha = 0$, and we refer to this case as case 2 (dashed line). Then, based on this case, the temperature effect is taken into consideration in the EOS of quark matter when the temperature is much smaller than the typical chemical potential, and we refer to this case as case 3 (the dotted line in Fig. 5). In this case, the contributions to the thermodynamic potential density from u, d, and s quarks are, respectively,

$$\Omega_{\rm u} = -\frac{1}{4\pi^2}[\mu_{\rm u}^4 + 2\pi^2 T^2 \mu_{\rm u}^2], \tag{41}$$

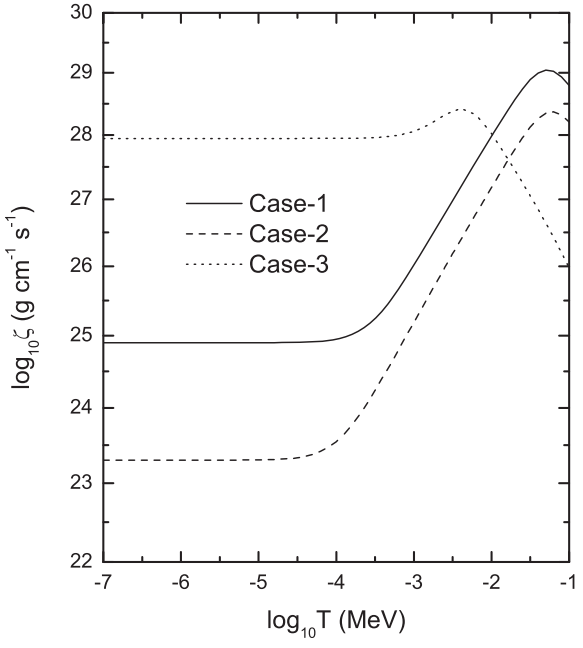

FIG. 5. Bulk viscosity as functions of temperature for three cases described in the text. In the numerical calculations, we use the chemical potential $\mu_{\rm d} = 400$ MeV, the oscillation time $\tau = 10^{-3}$ s, and the relative volume vibration amplitude $\Delta v/v_0 = 10^{-4}$.

$$\Omega_{\rm d} = -\frac{1}{4\pi^2}[\mu_{\rm d}^4 + 2\pi^2 T^2 \mu_{\rm d}^2], \tag{42}$$

$$\Omega_{\rm s} = -\frac{1}{4\pi^2}\left[\mu_{\rm s}\left(\mu_{\rm s}^2 - \frac{5}{2}m_{\rm s0}^2\right)\sqrt{\mu_{\rm s}^2 - m_{\rm s0}^2} + \frac{3}{2}m_{\rm s0}^4 \ln\frac{\mu_{\rm s} + \sqrt{\mu_{\rm s}^2 - m_{\rm s0}^2}}{m_{\rm s0}} + 2\pi^2 T^2 \mu_{\rm s}\sqrt{\mu_{\rm s}^2 - m_{\rm s0}^2}\right], \tag{43}$$

where $m_{\rm s0}$ is the current mass of strange quarks, rather than the running strange quark mass in the EPQ model. Equations (41)–(43) can be obtained from the model in Ref. [43] by ignoring terms containing the density- and temperature-dependent term $m_{\rm I}$ (the interaction part of the equivalent mass), then expanding to a logarithmic series, and finally taking the second order in temperature. They are consistent with the corresponding expressions via setting $\alpha_c = 0$ in Eq. (2) of Ref. [44]. Therefore, the case 3 model contains the temperature effect with no quark interactions involved. Repeating the procedure of calculating the bulk viscosity in Sec. II B, we can obtain the bulk viscosity in case 3 without much difficulty. The difference from case 2 to case 1 is the addition of quark interactions via running coupling and running strange quark mass. The bulk viscosity in case 1 is thus much larger than that in case 2. Comparing case 2 with case 3, we find that the temperature effect entering the EOS of quark matter can enormously enlarge the bulk viscosity. As just mentioned or directly seen from Eqs. (41)–(43), however, the case 3 model contains no interaction between quarks. If one would like to do full calculations with quark interaction included at finite temperature, the current EPQ model should be generalized self-consistently to finite temperature, which is beyond the scope of the current paper, but interestingly, a meaningful work in the future.

As mentioned once before, in order to obtain the bulk viscosity of SQM, one has to simultaneously solve Eqs. (19), (35), and (37) numerically, which is much more complicated than solving the analytical expression in Eq. (38) when the validity condition $2\pi T \gg \delta\mu$ is satisfied. So, in order to use Eq. (38) correctly, one has to find the reasonable range of $\delta\mu/2\pi T$. For this purpose, we define a new parameter Θ,

$$\Theta = \frac{\zeta - \zeta_a}{\zeta}, \tag{44}$$

where ζ_a is the bulk viscosity in Eq. (38). Obviously Θ reflects the accuracy of ζ_a compared with ζ.

In Fig. 6, we show Θ as a function of $\Delta v/v_0$ on the right axis by a dashed curve with different temperatures labeled on the curves. On the left axis, $|\delta\mu_{\rm max}|/2\pi T$ is also shown as a function of $\Delta v/v_0$ by solid lines with different temperatures labeled on the curves. Here $|\delta\mu_{\rm max}|$ represents

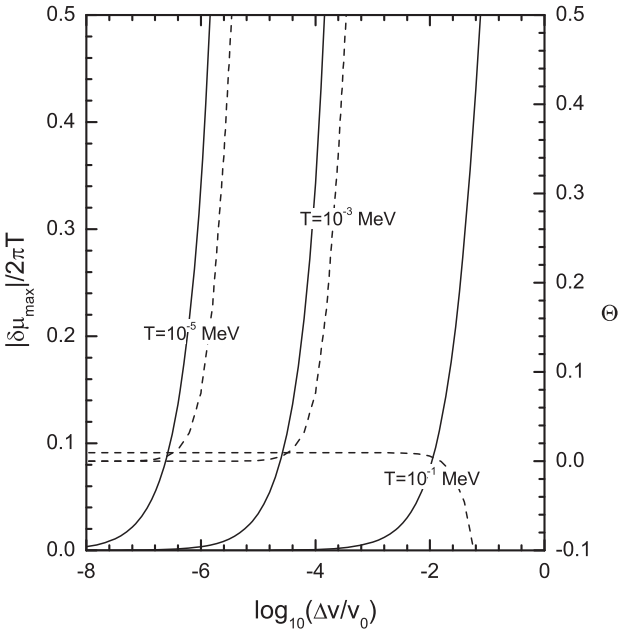

FIG. 6. Θ and $|\delta\mu_{\max}|/2\pi T$ as functions of $\Delta v/v_0$. The parameters used here are completely the same as those in Fig. 1 with $C = 2$.

the maxima of $\delta\mu$, from Eq. (35), which actually can be seen as a sinelike function of time t with a given $\Delta v/v_0$. From Fig. 6, one can readily find that with increasing $\Delta v/v_0$, Θ first stays almost zero, and after a critical $\Delta v/v_0$ increases (decreases) sharply, which means at low $\Delta v/v_0$, ζ_a is an excellent approximation of ζ, but it deviates from ζ quickly after the critical $\Delta v/v_0$. Meanwhile, in each case with different temperature, the critical $\Delta v/v_0$ corresponds to almost the same $|\delta\mu_{\max}|/2\pi T$, which is about 0.1. At the same time, the error caused by using the analytical expression instead of the integral expression is smaller than 2%, which is nearly negligible. So we conclude from this figure that the reasonable range of $|\delta\mu_{\max}|/2\pi T$ is

$$|\delta\mu_{\max}|/2\pi T \lesssim 0.1 \tag{45}$$

when temperature is lower than 10^{-1} MeV, and the corresponding reasonable maxima of $\Delta v/v_0$ can be roughly determined by

$$\log_{10}(\Delta v/v_0) \lesssim \log_{10}(T/\mathrm{MeV}) - 1, \tag{46}$$

which means that the higher the temperature is, the larger the applicable range of $\Delta v/v_0$. Besides that, the model parameter C and densities interested in astrophysics affect ζ slightly, and we also find that the variation of bulk viscosity keeps within 1 order of magnitude when τ changes from 10^{-4} s to 10^{-2} s. Therefore, this relation is not only valid in the EPQ model but is also correct in many other effective models [20–24]. In fact, because ζ_a is independent of $\Delta v/v_0$, the flat part of each curve in Figs. 1–3 actually represents ζ_a. And Eq. (46) can be easily verified by the values of $\Delta v/v_0$ and T of the inflection points, which are between the flat part and the increasing part in all curves. So, in practical concrete calculations, if both Eqs. (45) and (46) are satisfied, the analytical expression for bulk viscosity Eq. (38) can be safely used instead of solving the complicated integral expression Eq. (37).

B. Astrophysical application of bulk viscosity in the EPQ model

Recently, since two massive compact stars have been observed [45,46], a lot of works have been devoted to refining the EOSs of quark matter or hadronic matter to accommodate such massive compact stars [12,35,36,38,47–50]. In our previous work [36], we proposed an enhanced perturbative QCD model, and in this model we found that with proper model parameters, strange quark matter can be absolutely stable, and accordingly the maximum of strange quark stars can be as large as 2 $M_\odot$.

In the previous studies, a strange quark star with 1.4 $M_\odot$ has been used to study the damping time in different models, such as the density-dependent quark mass model [21,24] and the quasiparticle model [20]. All the results indicated that the damping time of a strange quark star is about 2–3 orders of magnitude lower, which implies that the quark star can reach stability earlier than in the MIT bag model. In what follows, we will roughly calculate the damping time for a massive strange quark star with about 2 $M_\odot$.

From Ref. [36], we know that in the EPQ model, when we choose the model parameters $C = 2$ and $B^{1/4} = 135$ MeV, we can obtain the massive strange star with 1.968 $M_\odot$ and 11.2 km in radius. Therefore, the average mass density ρ of the strange star can be roughly given by

$$\rho = \frac{M}{V} = \frac{3M}{4\pi R^3} \approx 6.652 \times 10^{14}\ \mathrm{g\,cm^{-3}}. \tag{47}$$

Here we have taken the mass of the Sun as $M_\odot = 1.9891 \times 10^{33}$ g.

The typical damping time for a strange star of constant density (an excellent approximation for a strange star, except very close to the gravitational instability limit) is [20]

$$\tau_D = 30^{-1}\rho R^2 \zeta^{-1}, \tag{48}$$

where ρ is the mass density and R is the radius of a quark star. On application of the obtained bulk viscosity in the preceding section, we can get the damping time of the massive quark star, and the results are shown in Fig. 7.

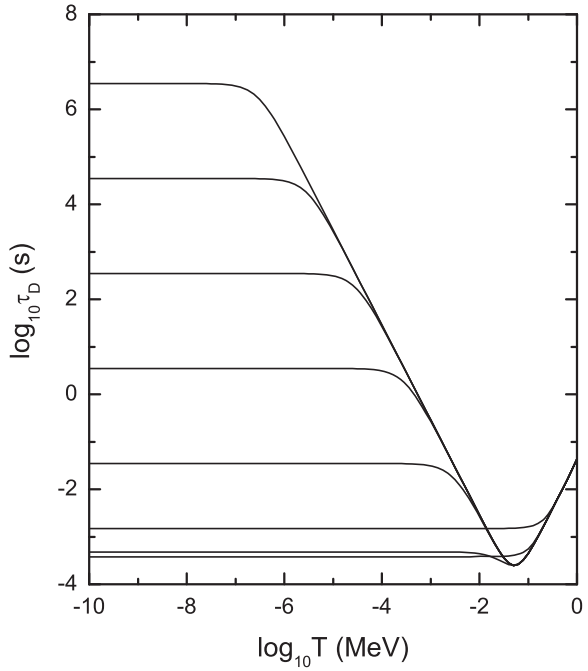

FIG. 7. Damping time for a star of 1.968 times the solar mass and with average density $\rho = 6.652 \times 10^{14}$ g cm^{-3}. Curves from bottom to top correspond to relative volume vibration amplitudes 10^{-1}, 10^{-2}, 1, 10^{-3}, etc., ending at 10^{-7}. The oscillation period τ is taken as 10^{-3} s.

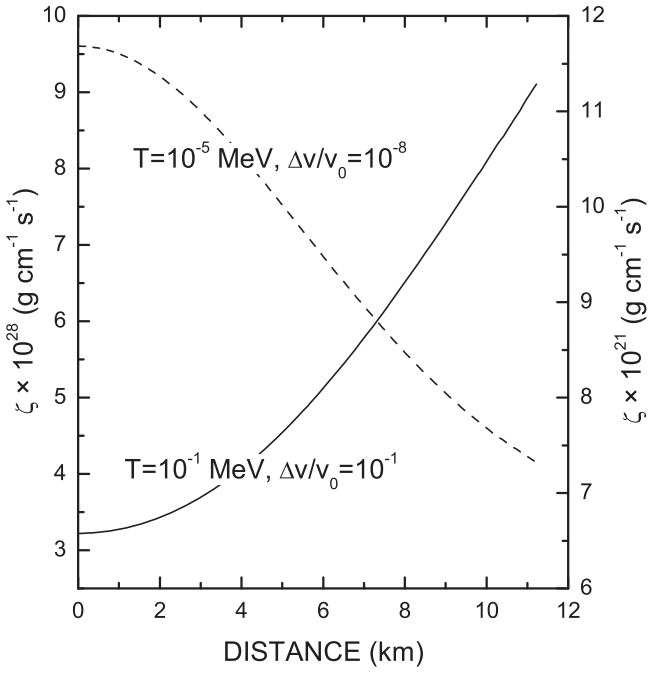

FIG. 8. The bulk viscosity as a function of the distance from the center of the quark star. The structure of the quark star is calculated with the model parameters $C = 2$ and $B_0^{1/4} = 135$ MeV. The dashed curve corresponds to the right axis, while the solid curve is on the left axis.

From Fig. 7, one can see that the lowest damping time can be shorter than 10^{-3} s, and if the vibration amplitude is around a unit, a quark star can reach stability in fractions of second after some radial unstable activities, such as the starquake. Even if the temperature of the star is very low and the vibration amplitude is tiny, the heat released by the viscous dissipation may enlarge the temperature and make the damping time go from the flat part to the decline part in Fig. 7, which means that after a relatively long time of weak oscillation, the star may reach stability in a short time.

It should be emphasized that the discussion here is based on rather crude estimates. The real situation could be much complicated. In Fig. 8, we give the bulk viscosity as a function of the distance from the center of the quark star. From this figure, it is easy to find that the bulk viscosity strongly depends on the temperature and $\Delta v/v_0$. When the temperature is high and $\Delta v/v_0$ is large, the bulk viscosity increases from the center to the surface of the star (solid line on the left axis). When both the temperature and $\Delta v/v_0$ are relatively small, the bulk viscosity decreases from the center to the surface of the star (dashed line on the right axis). Moreover, the order of magnitude of the bulk viscosity in these two cases varies widely. But for either case, the difference of the bulk viscosity between the center and the surface is small, within 1 order of magnitude. So the qualitative discussion about the damping time above is reliable.

IV. SUMMARY

We have calculated the bulk viscosity in the fully self-consistent EPQ model with running coupling and running strange quark mass. Similar to the results in the quasiparticle model and the density-dependent quark mass model, the bulk viscosity in the EPQ model is larger by 1–2 orders of magnitude than that in the conventional MIT bag model. The similarities in the magnitude of bulk viscosity originate from the fact that various models all include the interactions between quarks, which can lead to similar EOSs of quark matter. The temperature entering the EOS can, in principle, enlarge the bulk viscosity, especially when the temperature is relatively low. In order to avoid solving the complicated integral equations, we investigate the reasonable range of model parameters for valid use of the analytical expression for the bulk viscosity. On application of the obtained bulk viscosity in the EPQ model, we estimate the damping time of a massive quark star with mass about 2 $M_\odot$, and we find that the damping time can be as short as 10^{-3} s.

As we have already seen, the bulk viscosity can be strongly affected by temperature entering the EOS. Therefore, in order to have a fully consistent study on the bulk viscosity, the current EPQ model should be generalized to finite temperature. At the same time, compact stars usually have a strong magnetic field, which

should be considered as well [51]. Besides the equilibrium achieved by the nonleptonic process studied here, the leptonic processes $[u+e \leftrightarrow s(d)+\nu_e; s(d) \rightarrow u+e+\bar{\nu}_e]$ can also play an important role when the temperature approaches to 3 MeV [21]. Therefore, more detailed investigations on the bulk viscosity should be performed in the future.

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