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Ultralight axion in supersymmetry and strings and cosmology at small scales

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Dynamical mechanisms to generate an ultralight axion of mass $\sim 10^{-21}$–10^{-22} eV in supergravity and strings are discussed. An ultralight particle of this mass provides a candidate for dark matter that may play a role for cosmology at scales of 10 kpc or less. An effective operator approach for the axion mass provides a general framework for models of ultralight axions, and in one case recovers the scale 10^{-21}–10^{-22} eV as the electroweak scale times the square of the hierarchy with an $O(1)$ Wilson coefficient. We discuss several classes of models realizing this framework where an ultralight axion of the necessary size can be generated. In one class of supersymmetric models an ultralight axion is generated by instanton-like effects. In the second class higher-dimensional operators involving couplings of Higgs, standard model singlets, and axion fields naturally lead to an ultralight axion. Further, for the class of models considered the hierarchy between the ultralight scale and the weak scale is maintained. We also discuss the generation of an ultralight scale within string-based models. In the single-modulus Kachru-Kallosh-Linde-Trivedi moduli stabilization scheme an ultralight axion would require an ultralow weak scale. However, within the large volume scenario, the desired hierarchy between the axion scale and the weak scale is achieved. A general analysis of couplings of Higgs fields to instantons within the string framework is discussed and it is shown that the condition necessary for achieving such couplings is the existence of vector-like zero modes of the instanton. Some of the phenomenological aspects of these models are also discussed.

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I. INTRODUCTION

Recently it has been proposed [1,2] that an ultralight boson candidate for dark matter [sometimes referred to as fuzzy dark matter (FDM)], with a mass of $\mathcal{O}(10^{-22})$ eV, can properly explain cosmology at scales of 10 kpc or less.[1] Such an ultralight particle was identified with an axion[2] with a decay constant in the range $10^{16} \leq F \leq 10^{18}$ GeV. It was shown that an axion of the size needed could be generated via instanton effects. See Refs. [10–50] for recent works related to ultralight axions.

We emphasize, as did the authors of Ref. [1], that this ultralight axion is not the QCD axion (for work related to the QCD axion see e.g. Ref. [51]). In the latter case the axion mass $m_a \simeq \Lambda_{\rm QCD}^2/F$ depends on one parameter, since $\Lambda_{\rm QCD} \simeq 200$ MeV is known. For relic QCD axions produced by misalignment, this sets an upper bound $F \lesssim 10^{12}$ GeV. The axion considered here is another axion, perhaps a string axion (see e.g. Ref. [52]), that is not necessarily related to gauge dynamics in any way. Instead, its effective Λ is set by nonperturbative effects, such as string instantons, and therefore $m_a \simeq \Lambda^2/F$ depends on two parameters. This allows for greater freedom in the axion mass and relic abundance, and such axions are ubiquitous in string theory [53,54].

In this work we discuss explicit models where an ultralight axion can arise. We will study the axion mass scale using effective operators and will account for the scale $O(10^{-22})$ eV in terms of the electroweak scale and the hierarchy. We will also exhibit the emergence of such a light particle both in supergravity effective field theory and then in the framework of a specific class of string-motivated models. The outline of the paper is as follows. In Sec. II we discuss the general issue of the mass scale of the axion using an effective operator approach. In Sec. III we discuss field-theoretic models based on supersymmetry (SUSY) and supergravity that lead to an ultralight axion. In Sec. IV we discuss the possibility of realizing the axion within more general string frameworks. Specifically we consider the large volume scenario (LVS) moduli stabilization scheme, and show that the desired hierarchy between the axion scale and the weak scale can be achieved. In Sec. V we discuss conditions within string theory that allow the possibility of coupling axions with higher-dimensional Higgs operators via D-brane instantons. The phenomenology of these models is discussed in Sec. VI and we conclude in Sec. VII.

II. THE MASS SCALE OF THE ULTRALIGHT AXION

An apparent conspiracy of scales exists [1] between the observed dark matter relic abundance, astrophysical

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[1]Alternative possibilities for cosmology at small scales include complex dynamics or baryonic physics. However, in this work we focus on the approach involving an ultralight boson.

[2]For early work on axions see Refs. [3–9].

observations, and common properties of axions in string theory. Specifically, if one considers an axion in string theory with a string scale decay constant $O(10^{16})$ GeV and demands that misalignment produces an axion relic abundance matching the observed dark matter relic abundance $\Omega h^2 = .12$, then the axion must be ultralight with a mass m_a of $O(10^{-22})$ eV. This is the relevant mass scale for accounting for a variety of astrophysical observations, as discussed in Ref. [1].

From an ultraviolet perspective, however, it is preferable to turn this logic around: if the mass scale $m_a \simeq 10^{-22}$ eV could be motivated by theoretical considerations, then a misalignment-produced axion, with a string scale decay constant, would give a derivation of the observed relic abundance. In Ref. [1] this was achieved by tuning an instanton action to obtain the mass, which is possible in string theory but depends critically on moduli stabilization. In this section we will instead study axion masses utilizing symmetry arguments and effective field theory, motivating $m_a \simeq 10^{-22}$ eV.

The effective operator V_a in the scalar potential that gives the axion its mass must respect all of the symmetries of the theory. In particular, the axion itself has a perturbative continuous shift symmetry that is expected to be (and typically is in concrete constructions) broken to a discrete shift symmetry by instantons. This consideration leads to $V_a \sim \cos(a/F)$ [55]. The coefficient of this periodic term must also respect all symmetries of the theory. Since any theory consistent with observations respects at least standard model gauge invariance, it is natural to decompose V_a as

$$V_a = \tilde{A}\mathcal{O}_H \mathcal{O}_V \cos(a/F), \tag{1}$$

where $\mathcal{O}_H$ is a hidden sector operator, and $\mathcal{O}_V$ is a visible sector operator that contains only standard model [or minimal supersymmetric standard model (MSSM)] fields or a standard model singlet s that couple to the Higgs. For this term to give the axion a mass, both coefficient operators must receive vacuum expectation values (VEVs), where one or both could be the identity operator. Defining $A \coloneqq \tilde{A}\langle\mathcal{O}_H\rangle$ and recognizing that if $\mathcal{O}_V$ obtains a VEV it can[3] only involve powers of s and $(h^\dagger h)$, we write[4]

$$V_a = A\frac{s^{2m}(h^\dagger h)^{2k}}{\Lambda^{4k+2m-4}}\cos(a/F). \tag{2}$$

We note that in supersymmetric formulations higher-dimensional operators with integer powers in the superpotential will naturally lead to even integer powered higher-dimensional operators. For this reason we take the powers of s and of $(h^\dagger h)$ to be $(2m, 2k)$ where (m, k) are integers or half-integers.

Equation (2) gives rise to an axion mass

$$m_a = A^{\frac{1}{2}}\langle h\rangle \left(\frac{\langle s\rangle}{\langle h\rangle}\right)^m \left(\frac{\langle h\rangle}{\Lambda}\right)^{n-1}\left(\frac{\Lambda}{F}\right), \tag{3}$$

where $n = 2k + m$ and where Λ is some ultraviolet cutoff. The precise axion mass depends on model-dependent details that determine the precise values of A, F, and Λ, but if A is not too small and F is near the high-scale cutoff, as motivated by string theory, and also $\langle s\rangle \sim \langle h\rangle \sim \Lambda_{\rm EW}$, we have the approximate mass equation

$$m_a \simeq \Lambda_{\rm EW}\left(\frac{\Lambda_{\rm EW}}{\Lambda}\right)^{n-1}. \tag{4}$$

For a high-scale cutoff $\Lambda \simeq 10^{18}$ GeV and $\Lambda_{\rm EW} \simeq 10^2$ GeV, this gives

$$\begin{aligned} m_a &\simeq 10^{27}\ \text{eV} \quad \text{for } n = 0,\\ m_a &\simeq 10^{11}\ \text{eV} \quad \text{for } n = 1,\\ m_a &\simeq 10^{-5}\ \text{eV} \quad \text{for } n = 2,\\ m_a &\simeq 10^{-21}\ \text{eV} \quad \text{for } n = 3, \end{aligned} \tag{5}$$

and we therefore have four different regimes for axion masses: high scale, electroweak scale, neutrino scale, and ultralight scale. Note that the mass scale relevant for ultralight axion dark matter has arisen out of known mass scales in nature. In Sec. III we will show that a potential of the form (2) arises naturally from a superpotential, in which case the appearance of the singlet is related to having integral powers of superfields.[5]

We wish to emphasize that Eq. (2) is an effective field theory proposal which is motivated by our desire to realize the result of Hui *et al.* on the ultralight axion from concrete models. This is what we set out to do in the rest of the paper. Thus concrete analyses of some of the possibilities discussed above for various values of A, F, and Λ will be presented in Sec. III, but we would like to make some brief comments here. One critical aspect of the Sec. VI analysis will address the fact that A in string theory is typically exponentially suppressed by the volume of an internal cycle in a Calabi-Yau manifold. From this perspective, the authors of Ref. [1] used the $n = 0$ case and fine-tuned this exponential to obtain the axion mass $O(10^{-22})$ eV. This requires a large internal cycle and depends on moduli stabilization. We are simply proposing that the same small scale can be obtained by trading instanton suppression for

[3] In the MSSM we could use $(h^\dagger_{u,d}h_{u,d})^k$ and similar conclusions would hold.

[4] In supergravity and strings with strong dynamics a fermion condensate of appropriate power could replace the $s^{2m}(h^\dagger h)^{2k}$ factor in Eq. (2).

[5] See Ref. [56] for a method of generating such operators from a discrete symmetry.

the electroweak hierarchy. In particular, we will see that reasonable values of A in string theory can be accommodated in this framework. In Sec. V we will discuss how operators of the schematic form (1) may arise from D-brane instanton corrections to the superpotential in which vector-like instanton zero modes play a crucial role.

III. THE AXION IN SUPERSYMMETRY AND SUPERGRAVITY MODELS

In this section we construct explicit supersymmetric models that generate an ultralight axion. The ultralight nature of the axion is due to a perturbatively exact shift symmetry which is broken by a small amount (relative to other scales in the model) by nonperturbative effects such as instantons. The construction of a superpotential at the perturbative level that respects invariance under a $U(1)$ shift symmetry $S \to e^{i\lambda} S$ for a field S can be achieved with extra matter charged under the standard model and $U(1)$, and in this case terms in the superpotential involving S and the extra matter can be written such that the superpotential is neutral under the shift symmetry [3,4]. Alternately one may make the MSSM fields charged under $U(1)$ and introduce terms in the superpotential involving S and the MSSM fields [8].

Here we take an alternative approach where we introduce two fields S_1 and S_2 which are $SU(3) \times SU(2)_L \times U(1)_Y$ singlets but are oppositely charged under the global $U(1)$ symmetry, i.e., under a global $U(1)$ transformation one has

$$S_1 \to e^{i\lambda} S_1, \qquad S_2 \to e^{-i\lambda} S_2, \tag{6}$$

so that $S_1 S_2$ is neutral under the $U(1)$. We consider a superpotential of the form

$$W_s = \mu_0 S_1 S_2 + \frac{\lambda_s}{2M} (S_1 S_2)^2. \tag{7}$$

The superfields S_i (i = 1, 2) have the expansion

$$S_i = \phi_i + \theta \xi_i + \theta\theta F_i, \tag{8}$$

where ϕ_i is a complex scalar containing the axion and the saxion, ξ_i is the axino and F_i is the auxiliary field. Here we write

$$\phi_i = (\rho_i^0 + \rho_i) e^{i a_i/\rho_i^0}, \qquad i = 1, 2, \tag{9}$$

where the ρ_i are expansions about the VEVs ρ_i^0. The higher-dimensional operator in Eq. (7) is needed to give a VEV to the scalar component of ϕ_i. The F-term equations of motion give the constraint[6]

$$\mu_0 + \left(\frac{\lambda_s}{M}\right)(\rho_1^0 \rho_2^0) = 0. \tag{10}$$

Further one finds $F \equiv \rho_1^0 = \rho_2^0$. Thus we may write ϕ_i in the form

$$\phi_i = (F + \rho_i) e^{i a_i/F}, \qquad i = 1, 2. \tag{11}$$

It is useful to define the combination of axion fields a_1 and a_2 so that

$$a_\pm = \frac{1}{\sqrt{2}} (a_1 \pm a_2). \tag{12}$$

Here one finds that Eq. (7) leads to the following potential for a_+:

$$V = 4F^2 \mu_0^2 \left[1 - \cos\left(\frac{\sqrt{2} a_+}{F}\right)\right]. \tag{13}$$

Equation (13) gives a_+ a mass $m_{a_+} = 2\sqrt{2}\mu_0$. One may also check that the saxion field ρ_+ defined so that $\rho_\pm = (\rho_1 \pm \rho_2)/\sqrt{2}$ and the axino fields ξ_+ where $\xi_+ = (\xi_1 \pm \xi_2)/\sqrt{2}$ also have exactly the same mass. Thus the superpotential in Eq. (7) gives rise to an entire massive chiral multiplet ρ_+, a_+, ξ_+, as required by supersymmetry. We also note that the axion a_- still possesses a continuous shift symmetry, and thus no potential is generated for a_- and so it remains massless. The same applies to ρ_- and ξ_-. Thus one combination of the original chiral fields becomes massive while the orthogonal combination remains massless. We now turn to the generation of a mass for a_-. To give a_- mass we need to include contributions in the superpotential which break the continuous shift symmetry. We will discuss two classes of models. For one class we will use an instanton-type contribution and for the other class we will use higher-dimensional operators, which couple the Higgs fields and standard model singlets to the axion fields, which breaks the continuous shift symmetry.

We begin by considering models of the first type. Here we take a superpotential of the form

$$W = W_s + W_n, \qquad W_n = A(e^{-\alpha S_1} + e^{-\alpha S_2}), \tag{14}$$

where W_s is as defined by Eq. (7) and W_n violates the shift symmetry. In this case the equations of motion give

$$\mu_0 F + \left(\frac{\lambda_s}{M}\right) F^3 - \alpha A e^{-\alpha F} = 0. \tag{15}$$

Retaining only the dependence on a_- the axion potential takes the form

[6]We will assume throughout this section that the axions are stabilized at zero, which we will find to be a consistent assumption.

$$V(a_-) = 2\alpha^2 A^2 e^{-2\alpha F} e^{-\alpha F \cos(a_-/\sqrt{2}F)} \times [1 - \cos(\alpha F \sin(a_-/\sqrt{2}F))]. \quad (16)$$

We note that the form of the axion potential is not of the standard form $\cos(ca)$. However, it reduces to it when we expand $\sin(a_-/\sqrt{2}F)$ about $a_- = 0$ and retain the first term in the expansion. Thus an expansion of the potential, and using the condition $\alpha F \gg 1$, is needed to simulate an instanton-like effect and leads to a mass term for a_- of the form

$$m_{a_-} \simeq \alpha^2 A e^{-\alpha F}. \quad (17)$$

Using numbers consistent with Ref. [1], i.e., $F = 10^{17}$ GeV, $\alpha^2 A = 10^{12}$ GeV, $\alpha F = 99$, one finds $m_{a_-} = 10^{-21}$ eV. A similar analysis holds for the saxion ρ_- and the axino ξ_- which develop a mass of similar size. We assume that μ_0 is on the order of the electroweak scale. Since $F = 10^{17}$ GeV, this requires λ_s to be $O(10^{-12})$.[7]

Next we discuss the case when the shift symmetry is broken by a higher-dimensional operator involving couplings to the Higgs, standard model singlets, and the axion fields. As an organizing principle we consider supersymmetric models with three sectors: visible, hidden and an overlap sector between the hidden and the visible sectors with interactions suppressed by the Planck mass[8] so that

$$W = W_{\rm vis} + W_{\rm hid} + W_{vh}, \quad (18)$$

where $W_{\rm vis}$ contains fields in the visible sector, $W_{\rm hid}$ contains fields in the hidden sector and $W_{\rm vih}$ contains the overlap. In this analysis we assume that $W_{\rm vis}$ contains the fields H_1, H_2, and S, where S is a standard model singlet like the one used in the next-to-MSSM and does not possess any shift symmetry and $W_{\rm hid}$ contains the axion fields S_1, S_2 discussed above. Here we take

$$W_{\rm vis} = \mu_s S^2 + \lambda_0 S H_1 H_2,$$
$$W_{\rm hid} = \mu_0 S_1 S_2 + \frac{\lambda_s}{2M}(S_1 S_2)^2,$$
$$W_{vh} = \frac{\lambda}{M} S_1 S_2 H_1 H_2 + \frac{c}{M^{n-2}}(S_1 + S_2) S^n. \quad (19)$$

We assume that the Higgs fields develop VEVs due to sources in the visible sector not considered here. The effects of W_{vh} on the VEVs of S, S_1, S_2 are small because of Planck mass suppression. Thus, to the lowest order, one can see that the minimization condition in the S sector gives $\langle S\rangle \sim \lambda_0 \langle v_1 v_2\rangle/\mu_s$. We assume μ_s is of order the electroweak scale, which implies $v_0 \equiv \langle S\rangle$ is of the same order.

Next we focus on the F-term equations in the S_1 and S_2 sectors. Here we find

$$\mu_0 \rho_1^0 + \frac{\lambda_s}{M}(\rho_1^0)^2 \rho_2^0 + \frac{\lambda}{M}\rho_1^0 v_1 v_2 + \frac{c}{M^{n-2}} v_0^n = 0,$$
$$\mu_0 \rho_2^0 + \frac{\lambda_s}{M}\rho_1^0 (\rho_2^0)^2 + \frac{\lambda}{M}\rho_2^0 v_1 v_2 + \frac{c}{M^{n-2}} v_0^n = 0. \quad (20)$$

From Eq. (20) we deduce $F = \rho_1^0 = \rho_2^0$, which results in the constraint

$$\mu_0^2 F + \frac{\lambda_s}{M} F^3 + \frac{\lambda}{M} F v_1 v_2 + \frac{c}{M^{n-2}} v_0^n = 0. \quad (21)$$

The axion potential results from the term $\sum_{i=1,2} |\partial W/\partial S_i|^2$. Retaining only the dependence on a_- we find

$$V(a_-) = 4c^2 \left(\frac{v_0^n}{M^{n-2}}\right)^2 \left(1 - \cos\left(\frac{a_-}{\sqrt{2}F}\right)\right). \quad (22)$$

Equation (22) leads to a mass for a_- of the form

$$m_{a_-} = \sqrt{2} c \frac{v_0^n}{F M^{n-2}} = \Lambda_{\rm EW}\left(\frac{\Lambda_{\rm EW}}{\Lambda}\right)^{n-1}, \quad (23)$$

where $v_0 \sim \Lambda_{\rm EW}$, and $\Lambda = (F M^{n-2})^{1/n-1}$.

We now show that the term $|\partial W/\partial S|^2$ does not contribute to the a_- mass. The S-dependent terms in the superpotential are given by

$$W(S) = \mu_s S^2 + \lambda_0 S H_1 H_2 + \frac{c}{M^{n-2}}(S_1 + S_2) S^n. \quad (24)$$

The F-term equation in this sector reads

$$2\mu_s S_0 + \lambda_0 v_1 v_2 + \frac{nc}{M^{n-2}}(\rho_1^0 + \rho_2^0) S_0^{n-1} = 0. \quad (25)$$

Using the result deduced above that $\rho_a^0 = F = \rho_2^0$, the axion potential from this sector is given by

$$V_S(a_1, a_2) = |2\mu_0 S_0 + \lambda_0 v_1 v_2 + \frac{nc}{M^{n-2}} F(e^{ia_1/F} + e^{ia_2/F}) S_0^{n-1}|^2. \quad (26)$$

Applying Eq. (25) to Eq. (26) we have

$$V_S(a_1, a_2) = \left|\frac{nc}{M^{n-2}} F(e^{ia_1/F} - 1 + e^{ia_2/F} - 1) S_0^{n-1}\right|^2. \quad (27)$$

[7]This choice of λ_s, though small, is protected from renormalization by supersymmetry. We note also that this size of λ_s can be generated in string perturbation theory; e.g. in type IIA disc instantons can generate suppressions of the form e^{-A}, where A is the disc area. This effect is distinct from the Euclidean D-brane instantons that we consider elsewhere.

[8]Supersymmetric models of this sort with three sectors have been considered in previous works; see, e.g., Ref. [57].

From the above we deduce that the a_--dependent part of the potential is

$$V_S(a_1, a_2) = \left|\frac{ncS_0^{n-1}}{M^{n-2}}F\right|^2 \Big[2\cos(\sqrt{2}a_-/F) - 8\cos(a_-/\sqrt{2}F)\Big], \tag{28}$$

which gives a vanishing mass for a_-. Therefore $|F_S|^2$ does not contribute to the mass of a_-. Finally we consider the potential for a_- generated by the terms $\sum_{i=1,2}|\frac{\partial W}{\partial H_i}|^2$. Here we find

$$V_S(a_-) = \sum_{i=1,2}\left|\lambda_0 SH_i + \frac{\lambda}{M}S_1S_2H_i\right|^2, \tag{29}$$

which gives a vanishing contribution to $V(a_-)$. Superpotentials of the type considered in W_{vh} in Eq. (19) can be generated in string models as discussed in Sec. V.

When supersymmetry is promoted to supergravity [58,59] and supersymmetry breaking is taken into account, one will generate soft terms and the potential will have the form

$$V = \sum_i\left|\frac{\partial W}{\partial \phi_i}\right|^2 + V_{\rm soft}, \tag{30}$$

where ϕ_i are all the fields that enter in the superpotential and $V_{\rm soft}$ are terms such as $m_0^2\sum_i\phi_i\phi^\dagger$ and trilinear terms. In this case one finds that the dominant term that contributes to the axion a_- mass is

$$m_{a_-}^2 = qh^2\left(\frac{h}{\Lambda}\right)^{n-1}, \tag{31}$$

where q is an $\mathcal{O}(1)$ number, and we assume $\mu_0 \sim s \sim h$. Taking $h \sim \Lambda_{\rm EW}$, we then have

$$m_{a_-} \simeq \Lambda_{\rm EW}\left(\frac{\Lambda_{\rm EW}}{\Lambda}\right)^{m-1}, \tag{32}$$

where $m = (n+1)/2$. Here $m = 3$ requires $n = 5$.

A. Models with higher-dimensional Higgs-axion couplings

Next we discuss the case when the shift symmetry is broken by a higher-dimensional operator involving couplings of the Higgs and S_i. Here we assume a superpotential of the form

$$W = \mu_0 S_1S_2 + \frac{\lambda_s}{2M}(S_1S_2)^2 + \frac{\lambda}{M}S_1S_2H_1H_2 + \frac{c}{M^{2k-2}}(S_1+S_2)(H_1H_2)^k. \tag{33}$$

Next using the superpotential of Eq. (33) and after spontaneous breaking which gives VEVs to S_i and also assuming that H_i develop VEVs, the axion a_- potential can be obtained as discussed in the previous analysis and one gets

$$V(a_-) = \left[\left(\frac{2}{M^{2k-2}}c(v_1v_2)^k\right)^2 + \left(\frac{cF}{M^{2k-2}}(v_1v_2)^{k-1}\right)^2 \times (v_1^2+v_2^2)\right]\left(1-\cos\left(\frac{a_-}{\sqrt{2}F}\right)\right). \tag{34}$$

For the case $k = 2$ the first term in the brackets on the right-hand side of Eq. (34) is small relative to the second which gives an axion mass

$$M_{a_-} = c(v_1^2+v_2^2)^{1/2}\left(\frac{M}{F}\right)\left(\frac{(v_1v_2)^{1/2}}{M}\right)^{2k-2}. \tag{35}$$

This is of the same form as Eq. (4) with $n = 2k-1$ and for $k = 2$ one has $n = 3$ which gives the ultralight axion. We note that after soft terms are taken into account we will have a result similar to Eq. (32).

As a final example we consider a model where the axion couples directly to the Higgs fields, via a nonperturbative term in the superpotential. We present this model because it is a very simple realization of the organizing principle of Sec. II involving higher-dimensional Higgs-axion couplings. In this example the axion a is the imaginary part of a complex modulus $T = \tau + ia$, whose potential is generated nonperturbatively. This class of models is ubiquitous in string theory, and we will explore the details of string embeddings in Secs. IV and V. We consider a superpotential of the form

$$W = W_0 + \mu H_1H_2 + \Lambda^{3-2n}(H_1H_2)^n e^{-T/F}, \tag{36}$$

where W_0 is a constant obtained from integrating out heavy fields. The axion appears in the potential only via the H_1 and H_2 F-terms, and a quick calculation shows the mass of a takes the form

$$m_a = 2\left(\frac{h}{\Lambda}\right)^n\sqrt{n\mu\frac{\Lambda^3}{F^2}e^{-\tau}}. \tag{37}$$

Taking $F \sim \Lambda$ to be a high scale and $h \sim \mu \sim \Lambda_{\rm EW}$, we have

$$m_a = 2\sqrt{n}\left(\frac{\Lambda_{\rm EW}}{\Lambda}\right)^n\sqrt{\Lambda_{\rm EW}\Lambda e^{-\tau}}. \tag{38}$$

Furthermore, if we take $\Lambda e^{-\tau} \sim \Lambda_{\rm EW}$, we find

$$m_a \simeq \Lambda_{\mathrm{EW}} \left(\frac{\Lambda_{\mathrm{EW}}}{\Lambda} \right)^n. \quad (39)$$

Here taking $n = 2$ provides the desired ultralight mass for the axion. In many string models [60–62] the μ term in the superpotential is generated nonperturbatively, so we find it plausible that additional nonperturbative effects could generate this coupling at the same scale. Alternatively, it may be possible for the instanton that generates the higher-order Higgs coupling to be in the same homology class as the instanton that generates the μ term; in this case the relationship $\Lambda e^{-\tau} \sim \Lambda_{\mathrm{EW}}$ is automatic. We leave the study of these important global issues to future work.

We note that *a priori* it may seem strange that there would be a relationship between the axion mass, the electroweak scale, and a high scale–such as the grand unified theory (GUT) scale or the Planck scale–since the electroweak scale and the Planck scale do not care much about the axion mass. However, such asymmetric relationships between very low-mass scales and high-mass scales are already known to us. In the standard model the electron mass $m_e \sim 0.5$ MeV arises from the electroweak scale $v_{\mathrm{EW}} \sim 250$ GeV via the relation $m_2 \sim \lambda_{\mathrm{yuk}} v_{\mathrm{EW}}$. Similarly in GUT theories the neutrino mass $m_\nu \sim v_{\mathrm{EW}}^2/M_{\mathrm{Pl}}$. Thus while the electroweak scale does not care much about the electron mass, the electron mass does care about the electroweak scale. Similarly the electroweak mass and the Planck scale do not care about the neutrino mass, but the neutrino mass is sensitive to both. The relationship of the axion mass to the electroweak scale and the high scale Λ of Eq. (39) is very similar to the case of the neutrino.

IV. AXIONS IN A SIMPLIFIED STRING MODEL

The authors of Ref. [1] suggested that the FDM model of dark matter could be embedded in a string compactification, and the necessary mass and axion decay constant are natural from a stringy point of view. To make a precise statement one should scan over an ensemble of vacua and use the distribution of axion masses and decay constants to estimate the frequency in which parameters consistent with FDM occur. Unfortunately, while it is well known how to calculate axion decay constants even when the number of moduli is large (cf. Ref. [63]), calculating the masses requires intimate knowledge of nonperturbative effects, which are currently only partially calculable. In addition, moduli stabilization with a large number of moduli is notoriously difficult.

It is therefore our goal to find a realistic simplified model to demonstrate that embedding FDM in string theory is consistent with moduli stabilization, and does not remove us from the regime of validity of the effective theory.

A typical four-dimensional (4D) effective supergravity (SUGRA) theory constructed from a string compactification has scalar fields known as moduli. These fields arise from reducing the metric and various p-form gauge fields along appropriate p-cycles in the internal space X. A virtually universal class of moduli are the Kähler moduli, whose vacuum expectation values parametrize complexified volumes of holomorphic cycles in X. We consider a compactification of IIB string theory on a Calabi-Yau orientifold X, which yields an effective $\mathcal{N} = 1$ SUGRA theory in 4D. Type IIB string theory has a four-form gauge field C_4 in ten dimensions, and dimensionally reducing C_4 along a holomorphic four-cycle (divisor) in X yields an axion in the 4D theory. This axion pairs with the volume modulus of the four-cycle in a complex scalar field, which is the lowest component of a chiral superfield. The Kähler moduli T^i are written as

$$T^i = \frac{1}{2} \int_{D^i} J \wedge J + i \int_{D^i} C_4 \equiv \tau^i + i\theta^i, \quad (40)$$

where D^i is the corresponding divisor with volume modulus τ^i and axion θ^i, and J is the Kähler form on X. The theory typically has other moduli besides Kähler moduli, including the complex structure moduli U and the holomorphic axio-dilation $S = e^{-\phi} + iC_0 \equiv S_1 + iS_2$.[9] The tree-level Kähler potential takes the form

$$K = -\log(S + \bar{S}) - 2\log(\mathcal{V}) + K_{cs}(U, \bar{U}). \quad (41)$$

The complex structure moduli and holomorphic axio-dilaton acquire masses via the tree-level flux superpotential [64]

$$W_{\mathrm{Tree}} = \int_X G_3 \wedge \Omega, \quad (42)$$

where G_3 is a particular flux on X, and Ω is the holomorphic (3, 0)-form. We will assume that S and U are stabilized at a high scale by W_{Tree}. The Kähler moduli, on the other hand, only appear in the superpotential nonperturbatively [65]. Including these nonperturbative effects, the superpotential then takes the form

$$W = W_0 + \sum_a A_a e^{-q_i^a T^i}, \quad (43)$$

where $W_0 = \langle W_{\mathrm{Tree}} \rangle$, and the matrix q_i^a is a matrix of rational numbers. The full $D = 4, \mathcal{N} = 1$ SUGRA potential takes the form [58,59]

$$V = e^{\kappa^2 K}(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3\kappa^2 |W|^2),$$
$$D_i W = W_{,i} + \kappa^2 K_{,i} W. \quad (44)$$

In the analysis below we set $\kappa = 1$.

[9] Here the variables S, S_1, and S_2 are not to be confused with the ones from Sec. III.

In the Kachru-Kallosh-Linde-Trivedi (KKLT) moduli stabilization scheme [66] the weak scale m_s is related to the hidden sector W_0 so that $m_s = e^{K/2}|W_0|$ in gravity-mediated breaking of supersymmetry (see Ref. [59] and the references therein). Therefore, in realizing an ultralight axion in KKLT one finds the string scale to be far below the electroweak scale. Thus, we see that single-modulus KKLT is incompatible with an ultralight axion.

In order to get around this we must modify the theory, by introducing more fields and/or by considering further corrections to the potential. A particularly simple way to introduce an additional scale is to consider the first nonvanishing α' correction to the Kähler potential. This correction was computed in Ref. [67], and the corrected Kähler potential takes the form[10]

$$K = -\log(S+\bar{S}) - 2\log(\mathcal{V}+\alpha) + K_{cs}(U,\bar{U}), \quad (45)$$

where $\alpha = \frac{1}{2}\xi S_1^{3/2}$, $\xi = \zeta(3)\chi/2(2\pi)^3$, and χ is the topological Euler characteristic of X. The LVS [51,68] is a multimodulus (≥ 2) stabilization scheme that uses the α' correction, along with a nonperturbative effect, to realize a hierarchy of scales.

Here we will consider the simplest case, where the number of Kähler moduli, which is counted by the Hodge number $h^{1,1}(X)$, equals two. It was shown in Ref. [69] that the volume of all $h^{1,1} = 2$ Calabi-Yau manifolds can be written in the *strong cheese* form, such that

$$\mathcal{V} = \eta(\tau_b^{3/2} - \tau_s^{3/2}). \quad (46)$$

Here τ_b is the big (or large) cycle, which controls the overall volume (size of the cheese), and τ_s is a small cycle (a hole in the cheese). The constant η is typically an $\mathcal{O}(1)$ number, which depends on the intersection numbers of X. We will take $\eta = 1/9\sqrt{2}$ for concreteness, as in the $\mathbb{P}^4_{1,1,1,6,9}$ Calabi-Yau hypersurface. Each of these volume moduli pairs with an axion, so we have two complex scalars $T_s = \tau_s + i\theta_s$ and $T_b = \tau_b + i\theta_b$.

In the LVS the overall volume is taken to be large, with τ_s left small, so that $V \sim \tau_b^{3/2}$, and

$$\frac{\tau_s}{\tau_b} \ll 1, \qquad \frac{\alpha}{\mathcal{V}} \ll 1. \quad (47)$$

In this regime the Kähler potential can be expanded as

$$K \approx -2\log(\mathcal{V}) - 2\frac{\alpha}{\mathcal{V}}. \quad (48)$$

In the standard LVS the cycle τ_b is taken to be large enough to effectively ignore any nonperturbative effects that depend on τ_b. The superpotential then takes the form

$$W = W_0 + A_s e^{-a_s T_s}. \quad (49)$$

The axion θ_b is massless in this approximation, as it does not appear in the potential. Of course, it is expected that a nonperturbative correction to the potential will generate a mass of θ_b. In an $\mathcal{N}=1$ SUGRA model the mass for θ_b can be generated by a correction to either the superpotential or the Kähler potential (or both). Let us first consider a correction to the superpotential, of the form

$$\Delta W = A e^{-a_b T_b}. \quad (50)$$

At large volume (large τ_s) this correction is negligible compared to the terms in Eq. (49), and will therefore not affect the stabilization of τ_b, τ_s, or θ_s. However, Eq. (50) provides the only term in W that explicitly depends on θ_b, and will therefore be the leading-order operator that generates a mass for θ_b, in the absence of additional corrections. However, this term will be quite suppressed, and so one must consider whether this correction truly is leading order. Holomorphy, along with the shift symmetry of the axion, constrain ΔW to take the form derived in Ref. [65]:

$$\Delta W = \sum_i A_i e^{-q^i_j T^j}, \quad (51)$$

where the q^i_j are rational numbers, and $-q^i_j \tau^j$ is a positive rational multiple of the volume of a divisor.

However, holomorphy does not constrain the Kähler potential, and the corrections can take a more general form. It is beyond the scope of this work to explicitly calculate any such corrections; instead, we believe the following assumptions are well motivated:

(1) ΔK is periodic in θ_b.
(2) ΔK is generated by instantons that are charged under C_4, namely, Euclidean D3- and anti-D3-branes.
(3) The nonperturbative correction preserves the logarithmic form of the Kähler potential.

If one assumes that the correction is generated by a Euclidean D3- or anti-D3-brane, wrapping a cycle γ, then we expect the correction to the Kähler potential to take the form

$$\Delta K = \frac{A}{\mathcal{V}} e^{-S} f(\theta_b), \quad (52)$$

where f is a periodic function of θ_b. Here S is the instanton action, which we expect to go roughly as the volume of the brane. In order to solve the equations of motion γ should be a *locally* volume-minimizing representative of its class $[\gamma]$, with volume vol(γ), and so $S \simeq \mathrm{vol}(\gamma)$. However, since this instanton is correcting the Kähler potential, and not the superpotential, γ does not need to have the minimal volume in the class $[\gamma]$, as it is not necessarily a holomorphic representative. Therefore, $\mathrm{vol}(\gamma) \geq \tau_\gamma$, where τ_γ is the

[10]In this note we work exclusively in the Einstein frame.

minimal volume of $[\gamma]$. Without an explicit calculation we see no reason to assume that the inequality $\mathrm{vol}(\gamma) \geq \tau_\gamma$ cannot be saturated by at least some corrections to the Kähler potential. If this is the case the correction in Eq. (52) could provide corrections to V of the same order as those in Eq. (50). We will assume this is not the case, but it is important to understand these corrections further in the future.

Under the assumption that the correction to W given in Eq. (50) provides the leading-order term for θ_b, the scalar potential[11] takes the form

$$V = \left(\frac{12\sqrt{2}|A_s|^2 a_s^2 \sqrt{\tau_s} e^{-2a_s\tau_s}}{\mathcal{V}S_1} + \frac{2|A_s W_0| a_s \tau_s e^{-a\tau_s}}{\mathcal{V}^2 S_1}\cos(a_s\theta_s) + \frac{2a_b\tau_b|A_b W_0|}{\mathcal{V}^2 S_1} e^{-a_b\tau_b}\cos(a_b\theta_b) + \xi\frac{3|W_0|^2\sqrt{S_1}}{8\mathcal{V}^3} + \frac{4a_b a_s \tau_b \tau_s |A_b \bar{A}_s|}{\mathcal{V}^2 S_1} e^{-a_s\tau_s - a_b\tau_b}\cos(a_b\theta_b - a_s\theta_s)\right). \tag{53}$$

This form is derived in Appendix A. The axions are stabilized at $\theta_b = \pi/a_b, \theta_s = \pi/a_s$. A nonsupersymmetric anti–de Sitter minimum of the potential is found approximately at $a\tau_s \sim \ln\mathcal{V}$. For a concrete example we consider the following parameters:

$$h^{1,1} = 2, \quad h^{2,1} = 171, \quad W_0 = 10^{-12}, \quad A_s = A_b = 1, \quad a_s = a_b = 2\pi/6, \quad S_1 = 10.71. \tag{54}$$

These numbers are well motivated in weakly coupled IIB string theory. Calabi-Yau manifolds with a hierarchy in $h^{1,1}$ and $h^{2,1}$ are quite common, and the dual Coxeter number 6 appearing in a_s and a_b corresponds to an SO(8) gauge group, which is consistent with our weak coupling assumption. Here we also have $S_1 = 1/g_s$, so in this example $g_s \approx 0.1$ is small. Inserting these parameters into Eq. (53) and minimizing the potential, we find the volume[12] is stabilized at $\mathcal{V} = 187$. The small cycle is stabilized at $\tau = 32.5$. One might be concerned that a volume of $\mathcal{O}(100)$ is too small for the $1/\mathcal{V}$ expansion of the Kähler potential to be valid, but in this example the correction is at the percent level, so we expect the approximation to be good.[13] Using the parameters in Eq. (54) we find a light axion mass of 3.9×10^{-22} eV. The mass of the other axion is approximately 26 TeV, and the masses of the saxions are 590 GeV and 280 TeV. The fermions masses are 13 and 26 TeV. Both axion decay constants are $\mathcal{O}(10^{16})$ GeV. Importantly, the gravitino mass, which is the order parameter for SUSY breaking, is not too large, at approximately 13 TeV. It would be difficult to argue for SUSY as a solution to the hierarchy problem if the gravitino mass was near the Planck scale.

While the potential in Eq. (53) is a toy model for a real string compactification, with all relevant corrections computed, our analysis demonstrates that a mass scale for the lightest axion of $\mathcal{O}(10^{-22})$ eV is arguably consistent with moduli stabilization and a realistic electroweak scale. Of course, further study of both nonperturbative and perturbative corrections to the Kähler potential, such as those in Ref. [70], and the superpotential is important in understanding how FDM could be embedded in string theory.

[11]We set $K_{cs} = 0$ for simplicity, and absorb any phase of W_0 into the axions.

[12]In this section we express all of our volumes in the appropriate units of α'.

[13]While the relative smallness of the perturbation to the Kähler potential is a necessary condition for the LVS approximation to be valid, it is not sufficient, due to the nontrivial Kähler geometry. We have checked that the higher-order terms are subleading.

V. ULTRALIGHT AXION COUPLINGS TO THE HIGGS IN STRING THEORY

In this section we discuss how operators of the form $(h^\dagger h)^n \cos(a/F)$ may arise in string theory, focusing on nonperturbative corrections to the superpotential [65]. Some of the concepts implicit in previous sections will be repeated here in order to present a more complete picture of instanton corrections to the superpotential in string theory.

We begin by reviewing well-known facts about instantons in string theory. Nonperturbative corrections to the superpotential may arise from gauge dynamics, Euclidean D-brane instantons, M2-brane instantons, or worldsheet instantons, depending on the situation. For example, in type IIB compactifications, in particular in KKLT and the LVS, Euclidean D3 (ED3) instantons may generate such corrections, and Euclidean D2 (ED2) instantons and M2-brane instantons provide similar corrections in type IIA and M-theory compactifications. The nonperturbative contribution to the superpotential from a single instanton is typically written in the schematic form

$$W_{np} = A(\phi)e^{-T}, \tag{55}$$

where T is a modulus appropriate to the compactification, e.g. a Kähler modulus in type IIB compactifications, where $\langle \mathrm{Re}(T)\rangle = \mathrm{vol}(D)$, where D is the internal cycle wrapped by the instanton, and the axion a is $\mathrm{Im}(T)$. $A(\phi)$ is an instanton prefactor that depends on other moduli. These couplings do not couple a to the Higgs, and therefore are not of the desired type.

More general classes of brane instantons exist [60–62] in which the instanton prefactor may also contain gauge-invariant combinations of chiral supermultiplets charged under gauge groups. Such corrections arise due to the presence of additional instanton zero modes when D intersects some other cycle D' wrapped by spacetime-filling branes that carry nontrivial gauge sectors. We write the general form of these corrections as

$$W_{np} = A(\phi)\mathcal{O}_H\mathcal{O}_V e^{-T}, \tag{56}$$

where the visible sector operator $\mathcal{O}_V$ contains only MSSM superfields, whereas $\mathcal{O}_H$ may have charged fields beyond the MSSM, which could live in a hidden sector separated from the visible sector in the extra dimensions. One important aspect of these instantons is that they may generate the leading coupling in $\mathcal{O}_H\mathcal{O}_V$, if $\mathcal{O}_H\mathcal{O}_V$ on its own is forbidden by an anomalous $U(1)$ symmetry. For example, in weakly coupled type II compactifications the top-quark Yukawa coupling 10105 of a Georgi-Glashow $SU(5)$ GUT is always forbidden in perturbation theory, as are the flavor-diagonal Majorana mass terms for right-handed neutrinos. Obtaining these superpotential couplings therefore *requires* nonperturbative effects, such as the ones described.

For concreteness, we will restrict our attention to ED3 instantons in type IIB compactifications, though similar statements regarding vector-like zero modes and higher-dimensional operators should hold in other contexts as well.

We would like to study situations under which an ultralight axion mass can arise from an effective operator of the schematic form (1), which itself arises from an instanton contribution to the superpotential. For this to happen, holomorphy and gauge invariance dictate that the nonperturbative superpotential contains a term[14]

$$W_{ax} = A\frac{(H_1H_2)^n}{M_s^{2n-3}}e^{-T}. \tag{57}$$

Whether or not such a term exists depends on the detailed structure of the instanton zero modes. These include ED3-ED3 zero modes, as well as ED3-D7 zero modes that arise from ED3 intersections with spacetime-filling D7-branes that give rise to the Higgs fields H_1 and H_2. Of particular importance are the fermionic zero modes, so-called λ modes, in the ED3-D7 sector.

For example, if the μ term H_1H_2 is forbidden by an anomalous $U(1)$ symmetry, a nonperturbative effective operator of the form

$$AM_sH_1H_2e^{-T} \tag{58}$$

may generate it nonperturbatively [60–62], where the effective μ parameter $\mu_{\text{eff}} = AM_s e^{-\langle \text{Re}(T)\rangle}$ may be at the electroweak scale depending on the expectation value of the stabilized field T. In this way, ED3 instantons give a solution to the μ problem. Generating such an operator that is forbidden in perturbation theory by an anomalous $U(1)$ symmetry requires a chiral excess of λ modes and an associated shift of T under the anomalous $U(1)$, so that the entire operator is gauge invariant. In such a case the axion in T becomes the longitudinal component of the massive Z' boson associated to the anomalous $U(1)$, which has a string scale mass via the Stückelberg mechanism. See Refs. [71,72] for systematic phenomenological studies in this context.

We now turn to issues related to the effective operators that are of interest for this paper. The central issue is to identify instanton corrections of the right form that ensure that the axion is not eaten in a Stückelberg mechanism, as then it would have a string scale mass. Therefore the operators $(H_1H_2)^n$ must not be forbidden by an anomalous $U(1)$, and correspondingly the instanton must have at most vector-like λ modes, i.e. the modes have index zero. Using the instanton calculus of Ref. [60], an instanton on a divisor D with Kähler modulus T and a single vector-like pair $\lambda\bar{\lambda}$ with an appropriate structure of ED3-ED3 zero modes generates an effective operator of the form

$$\int d^4xd^2\theta \int d\lambda d\bar{\lambda} AM_s^3 e^{-T+\lambda H_1H_2\bar{\lambda}/M_s^2+\cdots}$$
$$\supset \int d^4xd^2\theta AM_sH_1H_2e^{-T}, \tag{59}$$

which is precisely W_{ax} in the $n=1$ case. More generally, there may be n pairs of vector-like zero modes $\lambda_i\bar{\lambda}_i$, in which case there are more Grassmann integrals, and we have

$$\int d^4xd^2\theta \int d\lambda_1 d\bar{\lambda}_1 \ldots d\lambda_n d\bar{\lambda}_n AM_s^3 e^{-T+a_{ij}\lambda_i H_1H_2\bar{\lambda}_j/M_s^2+\cdots}$$
$$\supset \int d^4xd^2\theta det(a_{ij})A\frac{(H_1H_2)^n}{M_s^{2n-3}}e^{-T}, \tag{60}$$

which is precisely W_{ax}. Thus, we see that a superpotential operator W_{ax} of the desired form may be generated if there is an instanton with n pairs of vector-like zero modes $\lambda\bar{\lambda}$. The $n=2$ case is quite similar to the nonperturbative Weinberg operator LH_2LH_2 studied in Ref. [73], since L and H_1 have the same quantum numbers under the MSSM gauge group.

The appearance W_{ax}, then, depends crucially on the structure of vector-like instanton zero modes, and we would like to consider when such zero modes exist. In Appendix B we provide the details of a relevant string-motivated example to demonstrate the existence of such zero modes.

VI. PHENOMENOLOGY

As discussed in Ref. [1], the relic density of the ultralight axion arises from misalignment, where after inflation the axion begins to oscillate around its minimum. Initially the

[14]One could easily incorporate the field S, considered in Sec. II, in this effect, but we omit it here for simplicity of discussion.

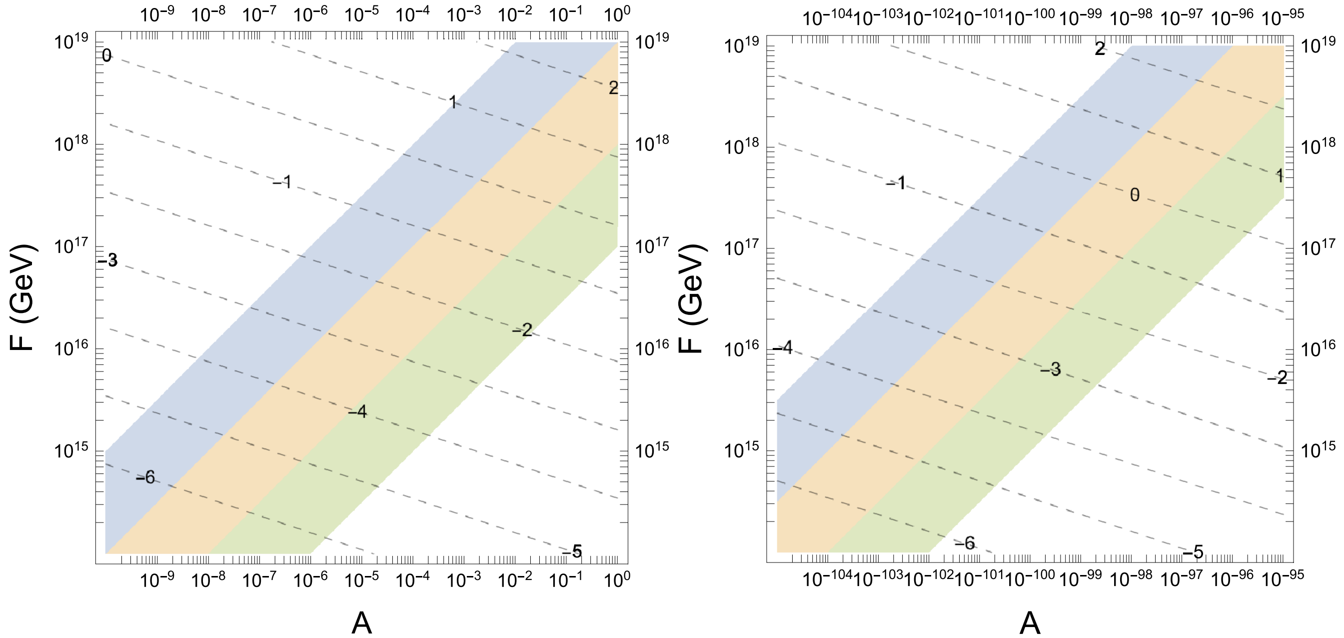

FIG. 1. Axion relic abundance and mass as a function of axion decay constant F and Wilson coefficient A. The dashed contours denote the axion relic abundance and are labeled by $\log_{10}(\Omega_{ax}h^2)$; the -1 contour is the observed relic abundance. The blue, orange, and green bands are mass regions 10^{-23} eV $\le m_a \le 10^{-22}$ eV, 10^{-22} eV $\le m_a \le 10^{-21}$ eV, and 10^{-21} eV $\le m_a \le 10^{-20}$ eV, respectively, so that the $m_a = 10^{-22}$ eV line is the boundary between the blue and orange bands. Left: The $n = 3$ case, which accommodates the relic abundance and mass by using the electroweak hierarchy. Right: The $n = 0$ case, which accommodates these solely with instanton suppression.

axion field is assumed to have a value close to the decay constant, which leads to a relic density

$$\Omega_a \sim 0.1\left(\frac{m_a}{10^{-22}\ \mathrm{eV}}\right)^{1/2}\left(\frac{F}{10^{17}\ \mathrm{GeV}}\right)^2, \tag{61}$$

consistent with WMAP [74] and Planck [75] if $m_a \simeq 10^{-22}$ eV and $F \simeq 10^{17}$ GeV. If $\langle s\rangle \simeq \langle h\rangle$ or $m = 0$ [see Eq. (3)], the effective operator (2) of Sec. II accounts for this mass scale in the $n = 3$ case with a Wilson coefficient $A = 1$ and an ultraviolet cutoff and axion decay constant of size $\Lambda = F = M_{pl}$, in which case

$$m_a \simeq \Lambda_{\mathrm{EW}}\left(\frac{\Lambda_{\mathrm{EW}}}{M_{pl}}\right)^2 \simeq 10^{-21}\ \mathrm{eV}. \tag{62}$$

From Eq. (61), we see that with this axion decay constant the relic abundance is oversaturated.

A sub-Planckian axion decay constant and suppressed coefficient A may give rise to the correct relic abundance and relevant axion mass, though, and this is well motivated by ultraviolet considerations. The analysis is simplified by the assumption that $\langle s\rangle \simeq \langle h\rangle$ or $m = 0$, in which case the axion mass in Eq. (3) becomes

$$m_a = A^{\frac{1}{2}}\langle h\rangle\left(\frac{\langle h\rangle}{\Lambda}\right)^{n-1}\left(\frac{\Lambda}{F}\right). \tag{63}$$

Given this simplifying assumption, in Fig. 1 we plot the axion mass and relic abundance as a function of F and A in the cases $n = 3$ and $n = 0$. In the $n = 0$ ($n = 3$) case the relic abundance $\Omega_a h^2 = \Omega_{\mathrm{obs}}h^2 = .12$ and axion mass $m_a = 10^{-22}$ eV arise for $F = 2\times 10^{17}$ GeV and $A \simeq 10^{-100}$ ($A \simeq 5\times 10^{-4}$). From the perspective of this effective operator, the authors of Ref. [1] studied the $n = 0$ case and used a large instanton suppression to account for the relic abundance and ultralight axion. We see that the $n = 3$ case may also do so by utilizing the electroweak hierarchy to account for the small mass scale, rather than a very large instanton suppression. From the figure we also see that smaller values of A and F are also permitted in the case that the ultralight axion is a subcomponent of the dark matter.

As discussed in Sec. III, Eq. (23) gives an axion mass of the desired size for the case $n = 3$ and from Eq. (61) we find that the same mass then gives the desired relic density. Thus as mentioned in Sec. III one may call this the $n = 3$ miracle. We discuss now the remaining fields arising from S, S_1, S_2 that appear in Sec. III. The field S has cubic interactions with the Higgs fields and assuming its mass to be larger than the Higgs it decays into MSSM fields and does not contribute to the relic density. We are then left with the fields a_+, ρ_+, ξ_+ and ρ_-, ξ_-. To discuss their disposition we need to look at their couplings to the Higgs given in

Eq. (19).[15] After S_1 and S_2 develop VEVs, the interaction $S_1S_2H_1H_2$ in Eq. (19) will generate an effective μH_1H_2 term where $\mu = (\lambda_0 S_0 + \frac{\lambda F^2}{M})$. For any reasonable phenomenology μ must be electroweak size. Noting the size of F as given in Sec. III we infer $\lambda \sim 10^{-12}$. While we have no fundamental explanation for the smallness of λ, we note that the desired size is technically natural.[16]

Every supersymmetric model of an ultralight axion will be accompanied by a scalar saxion ρ and fermion partner axino. In general, the saxion may give rise to cosmological problems if it dominates the energy density of the Universe through the time of big bang nucleosynthesis (BBN). However, many UV completions give rise to Planck-suppressed operators that lead to a saxion decay rate

$$\Gamma_\rho = \frac{c}{4\pi} \frac{m_\phi^3}{M_{pl}^2}. \tag{64}$$

As is well known, if $m_\phi \gtrsim 50$ TeV then the saxion decays prior to BBN. Throughout, we assume that UV completions of our models give rise to such operators and scalars of this mass, in order to avoid spoiling BBN. In addition, there are constraints on dark radiation production via modulus decay during reheating, cf. Refs. [76–78].

Let us discuss these ideas in the specific case of three of the models of Sec. III, which have heavy fields ρ_+, a_+, ξ_+. Here we assume that they, as well as ρ_- (which acquires a mass through soft breaking), have masses $\sim 10^5$ GeV. Such a mass assures their decay before BBN. For specificity let us discuss the ρ_+ decay. Here the relevant term arises from the couplings in Eq. (19) and is

$$W_3 = \frac{\sqrt{2}\lambda F}{M} \rho_+ H_1 H_2 + \cdots. \tag{65}$$

The interaction above allows for the decay $\rho_+ \to \tilde{H}_1 \tilde{H}_2$ with a lifetime consistent with the BBN constraints. The lifetimes of a_+ and the axino ξ_+ are similar. Thus the decays of fields ρ_+, a_+, ξ_+ are all consistent with the BBN constraints and do not play a role in any further discussion. To decay ρ_- we consider the coupling

$$L_{\rho F} = -\frac{1}{4} f_r(S_-) F^a_{\mu\nu} F^a_{\mu\nu}, \tag{66}$$

where $f_r(S_-)$ is the real part of the kinetic energy function in supergravity [58,59,79]. Using the interaction of Eq. (66) the decay width of ρ_- to gauge bosons is given by [79]

$$\Gamma(\rho_- \to gg) \simeq \frac{n_g d_f}{128\pi} \frac{m_{\rho_-}^3}{M_P^2}, \tag{67}$$

where $d_f \sim 1$; note that this effective operator has realized a decay rate of the form in Eq. (64). There is an identical contribution arising from the decay into gauginos. For $n_g = 4$ for the electroweak gauge bosons and for a ρ_- mass of 10^5 GeV one gets a decay lifetime consistent with BBN.

We assume that in the MSSM sector there exists a term which violates R-parity which makes the neutralino unstable. Thus the only remaining dark matter particles are the axion a_- and the axino ξ_-. There is no efficient production mechanism to generate the relic density for ξ_- comparable to the a_- and thus dark matter is dominated by the ultralight axion whose relic density is given by Eq. (61).

Tests of axionic dark matter have been discussed over the years (see, e.g., Refs. [53,80]). Tests specific to ultralight axions may come from more studies of cosmology at small scales [81]. It has also been suggested that future gravitational-wave detectors such as LISA, eLISA and DECIGO could be used for the detection of ultralight axionic dark matter [82–86].

VII. CONCLUSION

Recently it has been proposed that a boson with a de Broglie wavelength of 1 kpc may help resolve problems in cosmology at scales of the order of 10 kpc. A possible candidate is an ultralight axion with a mass in the range 10^{-21}–10^{-22} eV. In this work we discussed models within the framework of supersymmetry, supergravity and strings where an ultralight axion of the desired mass may arise. In Sec. II we presented an effective operator analysis of the axion mass, noting its possible dependence upon the expectation values of the Higgs field and singlets s that couple to the Higgs. We saw that for one effective operator the relevant axion mass arises as the electroweak scale times the square of the electroweak hierarchy; this arose in the case $n = 3$, where n is an integer parameter in the effective operator. In Sec. III we discussed two classes of supersymmetric models. In one class the shift symmetry is broken by instanton-type effects and the instanton action can be fine-tuned to generate the desired axion mass. In the second class of models it was shown that higher-dimensional operators constructed out of Higgs fields, standard model singlet fields and the axion fields which violate the shift symmetry naturally lead to an ultralight axion of size 10^{-21}–10^{-22} eV. Quite remarkably it was shown in Ref. [1] that such an ultralight axion leads to a relic density consistent with WMAP [74]. In the analysis given in Sec. III it was shown that for the case when the shift symmetry is broken by higher-dimensional operators involving Higgs fields, standard model singlet fields and the axion fields both the mass of the axion and correspondingly the relic density consistent with WMAP

[15] Another term bilinear in the MSSM fields which can be added to the superpotential is $\frac{\lambda'}{M} S_1 S_2 L H_2$. However, in the analysis here we focus on the term exhibited in Eq. (19).

[16] A λ this size may be generated by the same mechanism as discussed in footnote 5.

arise naturally for the case $n = 3$. The possibility of generating an ultralight axion within string-based models was also discussed.

In arguing for the consistency of such an embedding one must show that the model is consistent with moduli stabilization, and that the operators necessary to generate the mass scale are well motivated from a UV perspective, for reasonable choices of Wilson coefficients. As an ultralight axion is inconsistent with KKLT moduli stabilization, due to the ultralow electroweak scale that is required, we considered the large volume scenario moduli stabilization scheme. In the large volume scenario a hierarchy between the axion scale and the weak SUSY scale can be achieved. We next turned to demonstrating that the necessary effective operators could be generated in string theory with a reasonable degree of generality. To ensure that higher-dimensional Higgs-instanton operators which violate shift symmetry can be generated in string theory, the conditions necessary for the coupling of the instanton to Higgs fields were discussed. It was shown that the conditions require the existence of vector-like zero modes of the instanton. An illustrative example was given where such vector-like zero modes can arise. These considerations therefore demonstrate that an ultralight axion, of mass scale $\sim 10^{-21}$ eV, can plausibly be generated in string theory, in a regime of parameter space consistent with the validity of the effective field theory. Some phenomenological aspects of the models analyzed were discussed, including the dependence of the axion mass and relic abundance on the Wilson coefficient A and axion decay constant F, as well as cosmologically relevant decay channels.

The concrete models discussed here for the realization of ultralight dark matter may help in further investigations.

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APPENDIX A: DERIVATION OF THE LVS POTENTIAL

In this section we derive the form of the LVS potential, using the α'-corrected Kähler potential $K = -2\log(\mathcal{V} + c)$, where c is independent of the Kähler moduli. We first work with the classical Kähler potential $\tilde{K}$ given by $c = 0$, and then treat c as a perturbation, where $c/\mathcal{V} \ll 1$. The good Kähler coordinates on the moduli space of X are the complexified divisor volumes $T^i = \tau^i + i\theta^i$. However, the volume is most naturally expressed in terms of the dual coordinates t_i:

$$\mathcal{V} = \frac{1}{6}\kappa^{ijk} t_i t_j t_k. \tag{A1}$$

Here the κ^{ijk} are the divisor triple intersection numbers of X. The relationship between τ^i and the t_j is given by

$$\tau^i = \frac{\partial \mathcal{V}}{\partial t_i} = \frac{1}{2}\kappa^{ijk} t_j t_k. \tag{A2}$$

It is also useful to define the following symmetric matrix:

$$A^{ij} = \frac{\partial \tau^i}{\partial t_j} = \kappa^{ijk} t_{,}. \tag{A3}$$

We will denote the inverse of A^{ij} by A_{ij}, such that $A_{ik}A^{kj} = \delta^i_j$. We also note the useful identities

$$\begin{aligned} \tau^i t_i &= 3\mathcal{V}, \\ A^{ij} t_j &= 2\tau^i, \\ A_{ij}\tau^j &= \frac{1}{2} A_{ij} A^{jk} t_k = \frac{1}{2}\delta^k_i t_k = \frac{t_i}{2}. \end{aligned} \tag{A4}$$

The metric on Kähler moduli space is given by

$$\tilde{K}_{i\bar{j}} = \frac{\partial}{\partial T^i}\frac{\partial}{\partial \bar{T}^{\bar{j}}}\tilde{K}. \tag{A5}$$

However, since $\mathcal{V}$ only depends on the real parts of the T^i we can replace the holomorphic and antiholomorphic derivatives with real derivatives via

$$\frac{\partial}{\partial T^i} = \frac{1}{2}\left(\frac{\partial}{\partial \tau^i} + i\frac{\partial}{\partial \theta^i}\right) \to \frac{1}{2}\frac{\partial}{\partial \tau^i}, \tag{A6}$$

and similarly for the antiholomorphic derivatives. The Kähler connection is given by

$$\begin{aligned} \tilde{K}_i &= \frac{\partial}{\partial T^i}\tilde{K} = \frac{1}{2}\frac{\partial}{\partial \tau^i}(-2\log(\mathcal{V})) = -\frac{1}{\mathcal{V}}\frac{\partial \mathcal{V}}{\partial t_j}\frac{t_j}{\partial \tau^i} \\ &= -\frac{1}{\mathcal{V}}\tau^j A_{ij} = -\frac{t_i}{2\mathcal{V}}. \end{aligned} \tag{A7}$$

The metric takes the form

$$\tilde{K}_{i\bar{j}} = \frac{1}{4}\left(-\frac{A_{i\bar{j}}}{\mathcal{V}} + \frac{t_i t_{\bar{j}}}{2\mathcal{V}^2}\right), \tag{A8}$$

and the inverse metric is then

$$\tilde{K}^{i\bar{j}} = 4(-\mathcal{V}A^{i\bar{j}} + \tau^i \tau^{\bar{j}}). \tag{A9}$$

The $\mathcal{N} = 1$ SUGRA potential contains the contractions $\tilde{K}^{i\bar{j}}\tilde{K}_{\bar{j}}$. We have

$$\tilde{K}^{i\bar{j}}\tilde{K}_{,\bar{j}} = -4(-\mathcal{V}A^{i\bar{j}} + \tau^i\tau^{\bar{j}})\frac{t_{\bar{j}}}{2\mathcal{V}} = -\frac{2}{\mathcal{V}}(-2\tau^i\mathcal{V} + 3\tau^i\mathcal{V}) = -2\tau^i. \quad \text{(A10)}$$

Therefore

$$\tilde{K}^{i\bar{j}}\tilde{K}_i\tilde{K}_{,\bar{j}} = 2\tau^i\frac{t_i}{2\mathcal{V}} = 3. \quad \text{(A11)}$$

In the large-volume limit $\mathcal{V} \gg c$, we can write

$$K = -2\log(\mathcal{V} + c) \approx -2\log(\mathcal{V}) - 2\frac{c}{\mathcal{V}} \equiv \tilde{K} + \Delta K. \quad \text{(A12)}$$

Here ΔK can be treated as a perturbation to the classical Kähler potential $\tilde{K}$. The correction to the Kähler connection is then

$$\Delta K_i = \frac{1}{2}\frac{\partial}{\partial\tau^i}\Delta K = \frac{c}{\mathcal{V}^2}\frac{\partial\mathcal{V}}{\partial t_j}\frac{\partial t_j}{\partial\tau^i} = \frac{c}{\mathcal{V}^2}\tau^j A_{ji} = \frac{c}{2\mathcal{V}^2}t_i. \quad \text{(A13)}$$

The correction to the Kähler metric is then

$$\Delta K_{i\bar{j}} = \frac{1}{2}\frac{\partial}{\partial\tau^{\bar{j}}}\left(\frac{c}{2\mathcal{V}^2}t_i\right) = \frac{c}{4}\left(-\frac{2t_i}{\mathcal{V}^3}\frac{\partial\mathcal{V}}{\partial t_k}\frac{\partial t_k}{\partial\tau^{\bar{j}}} + \frac{1}{\mathcal{V}^2}A_{i\bar{j}}\right) = \frac{c}{4\mathcal{V}^2}\left(-\frac{t_i t_{\bar{j}}}{\mathcal{V}} + A_{i\bar{j}}\right). \quad \text{(A14)}$$

From this we can infer the correction to the inverse Kähler metric, via

$$K_{i\bar{j}}K^{\bar{j}k} = \delta_i^k = (\tilde{K}_{i\bar{j}} + \Delta K_{i\bar{j}})(\tilde{K}^{\bar{j}k} + \Delta K^{\bar{j}k}) \approx \delta_i^k + \Delta K_{i\bar{j}}\tilde{K}^{\bar{j}k} + \tilde{K}_{i\bar{j}}\Delta K^{\bar{j}k}, \quad \text{(A15)}$$

where in the last equality we have dropped terms of $\mathcal{O}(\Delta K_{i\bar{j}}^2)$. We then have

$$\Delta K^{i\bar{j}} = \tilde{K}^{i\bar{l}}\Delta K_{m\bar{l}}\tilde{K}^{m\bar{j}}. \quad \text{(A16)}$$

To evaluate this, we first calculate

$$\tilde{K}^{i\bar{l}}\Delta K_{m\bar{l}} = 4(-\mathcal{V}A^{i\bar{l}} + \tau^i\tau^{\bar{l}})\frac{c}{4\mathcal{V}^2}\left(-\frac{t_m t_{\bar{l}}}{\mathcal{V}} + A_{m\bar{l}}\right) = \frac{c}{\mathcal{V}^2}\left(2t_m\tau^i - \mathcal{V}\delta_m^i - 3t_m\tau^i + \frac{1}{2}t_m\tau^i\right) = -\frac{c}{\mathcal{V}^2}\left(\frac{1}{2}t_m\tau^i + \mathcal{V}\delta_m^i\right). \quad \text{(A17)}$$

We then have

$$\Delta K^{i\bar{j}} = \tilde{K}^{i\bar{l}}\Delta K_{m\bar{l}}\tilde{K}^{m\bar{j}} = -\frac{4c}{\mathcal{V}^2}\left(\frac{1}{2}t_m\tau^i + \mathcal{V}\delta_m^i\right)(-\mathcal{V}A^{m\bar{j}} + \tau^m\tau^{\bar{j}}) = -\frac{4c}{\mathcal{V}^2}\left(-\mathcal{V}\tau^i\tau^{\bar{j}} + \frac{3}{2}\mathcal{V}\tau^i\tau^{\bar{j}} - \mathcal{V}^2A^{i\bar{j}} + \mathcal{V}\tau^i\tau^{\bar{j}}\right) = -\frac{4c}{\mathcal{V}^2}\left(\frac{3}{2}\mathcal{V}\tau^i\tau^{\bar{j}} - \mathcal{V}^2A^{i\bar{j}}\right). \quad \text{(A18)}$$

We now calculate the $\mathcal{N} = 1$ SUGRA potential for our specific example. We will first use the tree-level Kähler potential, and then add the α' correction. We consider a superpotential of the form

$$W = W_0 + A_s e^{-a_s T_s} + A_b e^{-a_b T_b} \equiv W_0 + W_s + W_b. \quad \text{(A19)}$$

We then have $\partial_i W = -a_i A_i e^{a_i T_i} = -a_i W_i$, where there is no sum on $i \in \{T_b, T_s\}$. In addition, we will take W_0 to be much larger than the nonperturbative contributions to the superpotential, so that $K_i W \approx K_i W_0$. Using the no-scale structure $K^{i\bar{j}}K_i K_{\bar{j}} = 3$, we then have

$$e^{-K_{\text{total}}}V = K^{i\bar{j}}(\partial_i W\partial_{\bar{j}}\bar{W}) - (2\tau^i(\partial_i W)\bar{W}_0 + \text{c.c.}) \quad \text{(A20)}$$

where K_{total} is the Kähler potential for all the moduli. In the large-volume limit the relevant terms are

$$e^{-K_{\text{total}}}V \approx K^{s\bar{s}}a_s^2 W_s\bar{W}_s + K^{s\bar{b}}(a_s a_b W_s\bar{W}_b + \text{c.c.}) + (2a_s\tau^s(W_s)\bar{W}_0 + \text{c.c.}) + (2a_b\tau^b(W_b)\bar{W}_0 + \text{c.c.}). \quad \text{(A21)}$$

In the LVS we have $K^{s\bar{s}} \approx -4\mathcal{V}A^{s\bar{s}}$, and for our particular example $K^{s\bar{b}} = 4\tau^s\tau^{\bar{b}}$. Taking a volume of the form

$$\mathcal{V} = \frac{1}{9\sqrt{2}}(\tau_b^{3/2} - \tau_s^{3/2}), \quad \text{(A22)}$$

we have

$$\begin{pmatrix} A^{s\bar{s}} & A^{s\bar{b}} \\ A^{b\bar{s}} & A^{b\bar{b}} \end{pmatrix} = 6\sqrt{2}\begin{pmatrix} \sqrt{\tau_s} & 0 \\ 0 & -\sqrt{\tau_b} \end{pmatrix}.$$

Taking $K_{\rm cs} = 0$, we have

$$e^{K_{\rm total}} = \frac{1}{2\mathcal{V}^2 S1}, \tag{A23}$$

and so we can write the potential as

$$V = \Bigg(\frac{12\sqrt{2}|A_s|^2 a_s^2\sqrt{\tau_s}e^{-2a_s\tau_s}}{\mathcal{V}S_1} + \frac{2|A_s W_0|a_s\tau_s e^{-a\tau_s}}{\mathcal{V}^2 S_1}\cos(a_s\theta_s) + \frac{2a_b\tau_b|A_b W_0|}{\mathcal{V}^2 S_1}e^{-a_b\tau_b}\cos(a_b\theta_b) + \frac{4a_b a_s\tau_b\tau_s|A_b\bar{A}_s|}{\mathcal{V}^2 S_1}e^{-a_s\tau_s - a_b\tau_b}\cos(a_b\theta_b - a_s\theta_s)\Bigg), \tag{A24}$$

where we have absorbed any phase of W_0 into the axions. We now calculate the α' correction to the SUGRA potential, whose presence is crucial for the existence of a large-volume minimum. The term that is important in the LVS is given by the leading-order breaking of the no-scale structure, given schematically by $\Delta(K^{i\bar{j}}K_i K_{\bar{j}})|W_0|^2$. We have

$$\Delta(K^{i\bar{j}}K_i K_{\bar{j}}) = (\Delta K^{i\bar{j}})\tilde{K}_i\tilde{K}_{\bar{j}} + \tilde{K}^{i\bar{j}}(\Delta K_i)\tilde{K}_{\bar{j}} + \tilde{K}^{i\bar{j}}\tilde{K}_i(\Delta K_{\bar{j}}). \tag{A25}$$

We will calculate this term by term. First, we have

$$(\Delta K^{i\bar{j}})\tilde{K}_i\tilde{K}_{\bar{j}} = \frac{c}{\mathcal{V}^4}\left(\frac{3}{2}\mathcal{V}\tau^i\tau^{\bar{j}} - \mathcal{V}^2 A^{i\bar{j}}\right)t_i t_{\bar{j}} = \frac{c}{\mathcal{V}^4}\left(\frac{27}{2}\mathcal{V}^3 - 6\mathcal{V}^3\right) = \frac{15c}{2\mathcal{V}}. \tag{A26}$$

We also have

$$\tilde{K}^{i\bar{j}}(\Delta K_i)\tilde{K}_{\bar{j}} = -\frac{c}{\mathcal{V}^3}(-\mathcal{V}A^{i\bar{j}} + \tau^i\tau^{\bar{j}})t_i t_{\bar{j}} = -\frac{3c}{\mathcal{V}}. \tag{A27}$$

Putting it all together, and including the nontrivial factor of $e^{K_{\rm total}}$, we have

$$\Delta V = \frac{3c}{4\mathcal{V}^3 S_1}|W_0|^2. \tag{A28}$$

Plugging in $c = \frac{1}{2}\xi S_1^{3/2}$, we find

$$\Delta V = \xi\frac{3|W_0|^2\sqrt{S1}}{8\mathcal{V}^3}. \tag{A29}$$

The full potential then takes the form (53).

APPENDIX B: VECTOR-LIKE ZERO MODES FROM STRINGY INSTANTONS

In this section we present a string-motivated example to demonstrate the existence of the vector-like zero modes considered in Sec. V. Suppose that an ED3 and a D7-brane (or a stack of D7-branes) wrap divisors D and D' in a smooth Calabi-Yau threefold X that intersect along a curve $C \coloneqq D\cdot D'$. Both the instanton and the D7-brane may carry (1,1)-form worldvolume fluxes (or more generally holomorphic vector bundles), which may be written in terms of line bundles $\mathcal{L}_D$ and $\mathcal{L}_{D'}$ on D and D', respectively. Then the ED3-D7 instanton zero modes at the intersection are counted by the cohomology $h^i(C, K_C^{1/2}\otimes\mathcal{L})$, where $\mathcal{L} \coloneqq \mathcal{L}_D|_C \otimes \mathcal{L}_{D'}^{-1}|_C$.

As discussed, a necessary condition for obtaining couplings of the desired type is that there is no chiral excess of ED3-D7 zero modes on C, i.e.

$$\chi(C, K_C^{1/2}\otimes\mathcal{L}) = h^0(C, K_C^{1/2}\otimes\mathcal{L}) - h^1(C, K_C^{1/2}\otimes\mathcal{L}) = 0. \tag{B1}$$

Computing this index by applying the Hirzebruch-Riemann-Roch theorem, we have

$$\chi(C, K_C^{1/2}\otimes\mathcal{L}) = \int_C ch(K_C^{1/2}\otimes\mathcal{L})td(C) = \int_C (1 + c_1(K_C^{1/2}\otimes\mathcal{L}))(1 + c_1(C)/2) = \int_C c_1(\mathcal{L}), \tag{B2}$$

and we see that the index is zero when $c_1(\mathcal{L}) = 0$. By this we see that if $c_1(\mathcal{L}_D|_C) = c_1(\mathcal{L}_{D'}|_C)$ then $\chi(C, K_C^{1/2}\otimes\mathcal{L}) = 0$, i.e. we have at most vector-like instanton zero modes on C.

In such a case, determining whether there actually are vector-like instanton zero modes requires computing the cohomology, not just the index. This computation can be done by a variety of means, but as an existence proof we would like to present a simple example.

Consider the case where a divisor $D = \mathbb{P}^1\times\mathbb{P}^1$ is wrapped by an ED3 instanton that intersects a spacetime-filling D7-brane on another divisor D' at a degree (m, n) curve $C \subset D$, and there are no worldvolume fluxes, i.e. $\mathcal{L}_D = \mathcal{O}_D$ and $\mathcal{L}_{D'} = \mathcal{O}_{D'}$. The zero modes are counted by $h^i(C, K_C^{1/2})$, which has index zero, where

$K_C = (K_D + \mathcal{O}(C))|_C = \mathcal{O}(m-2, n-2)|_C$. Taking the square root, a Koszul sequence for $K_C^{1/2}$ is given by

$$0 \to \mathcal{O}_D\left(-\frac{m}{2}-1, -\frac{n}{2}-1\right) \to \mathcal{O}_D\left(\frac{m}{2}-1, \frac{n}{2}-1\right) \to K_C^{1/2} \to 0. \tag{B3}$$

By Serre duality, $h^i(D, \mathcal{O}_D(\frac{m}{2}-1, \frac{n}{2}-1)) = h^{2-i}(D, \mathcal{O}_D(-\frac{m}{2}-1, -\frac{n}{2}-1))$. Since a degree l line bundle on $\mathbb{P}^1$ has $l+1$ global sections, and therefore a degree $(k-1, l-1)$ line bundle on $\mathbb{P}^1 \times \mathbb{P}^1$ has kl global sections, $h^i(D, \mathcal{O}_D(\frac{m}{2}-1, \frac{n}{2}-1)) = (mn/4, 0, 0)$. Using the long exact sequence in cohomology associated to the Koszul sequence (B3), we obtain

$$h^i(C, K_C^{1/2}) = \left(\frac{mn}{4}, \frac{mn}{4}\right), \tag{B4}$$

which shows that there are vector-like instanton zero modes for general even m and n. For a more in-depth introduction to this type of computation, see e.g. Refs. [87,88].

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