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Rescaling of quantized skyrmions: From nucleon to heavy flavored baryons

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The role of rescaling (expansion or squeezing) of quantized Skyrmions is studied for the spectrum of baryons beginning with nucleon and $\Delta(1232)$ and with flavors strangeness, charm, or beauty. The expansion of Skyrmions due to the centrifugal forces has influence on the masses of baryons with u and d flavors (N and especially Δ). In the chirally symmetrical case [Adkins, Nappi, and Witten, Nucl. Phys. **B228**, 552 (1983)], numerical results are confirmed analytically. The rescaling of Skyrmions has some influence on the spectrum of strange baryons, is more important for the case of charm, and is crucial for baryons with a beauty quantum number, where strong squeezing takes place. Two competing tendencies are clearly observed: expansion of Skyrmions when the isospin (or spin) of the baryon increases and squeezing with the increasing mass of the flavor. For beauty the method seems to be not satisfactory, because the ratio $r_b = F_B/F_\pi$ necessary to obtain the observed masses of baryons is different for Λ_b and Σ_b and should be considerably greater than this ratio (theoretical), given in the literature. The approach itself for the case of beauty can be considered as some kind of a "toy model"; however, the necessity of rescaling of quantized Skyrmions is firmly established. Some pentaquark states with hidden strangeness, charm, or beauty are considered as well.

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I. INTRODUCTION

Studies of the baryon spectrum are one of the important directions of elementary particle physics, and the chiral soliton approach [1,2] provides an attractive possibility for this, after different kinds of quark models. The pioneer papers are well known [3,4], where the static properties of baryons [nucleon and $\Delta(1232)$] have been calculated within an accuracy of about 30% of experimental values.

Two parameters of the model—pion decay constant F_π and the Skyrme constant e—were defined in Refs. [3,4] by fitting the masses of the nucleon and $\Delta(1232)$, which allowed one to calculate other properties of these baryons. The classical mass of the soliton, Skyrmion with baryon (winding) number $B = 1$, was obtained by direct minimization of the static energy functional. The energy (mass) of the quantized states (baryons) is the sum of the classical mass and isospin-dependent quantum correction. The chiral fields configuration for each of the quantized states does not satisfy the Euler-Lagrange equation; therefore, the sum of the classical mass of the Skyrmion plus the isospin- (spin-) dependent quantum correction could be minimized further. This issue has not been discussed in Refs. [3,4], because the quantum correction turned out to be small enough for the nucleon—less than ∼10%—although greater and much more important for the $\Delta(1232)$, because it is responsible for the mass splitting between nucleon and Δ.

The $SU(3)$ extension of the model has been proposed somewhat later in Ref. [5], which allowed one to calculate the mass splittings between components of the $SU(3)$ multiplets of baryons. The $SU(3)$ violating mass term, added to the model Lagrangian, was considered as a small enough perturbation, similar to the quantum correction in the $SU(2)$ model [3], although in some cases this correction is even greater. Therefore, the problem that the energy of the quantized state is not the minimal energy appears here again, even in greater scale.

In present paper, we investigate this problem using the simple variant of the quantization scheme, proposed by Westerberg and Klebanov [6] and slightly modified in Refs. [7,8].

For the quantized states with u and d flavors [nucleon and $\Delta(1232)$], the expansion of the state due to the centrifugal forces takes place, which decreases considerably the mass splitting between N and Δ. This effect, established numerically for several variants of the model, is confirmed analytically in the chirally symmetrical case of Ref. [3] and leads to certain changes of the model parameters defined in Ref. [3] to fit the masses of the nucleon and $\Delta(1232)$.

We study numerically the influence of the change of the scale (dimension) of the whole Skyrmion on the energy of quantized states with strangeness (strange hyperons) and with heavy flavors, charm, or beauty. The squeezing of the

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Skyrmion, attracting heavy flavor, leads to a considerable decrease of the energy (mass) of the quantized state with a charm quantum number. This effect is striking for beauty, but not so important for strangeness, where it is compensated to a large extent by the tendency of expansion due to the contribution of isospin- and spin-dependent terms in the mass of the state.

Features of the chiral soliton approach are described briefly in the next section, where some static characteristics of the Skyrmion are presented. The quantization scheme is described in Sec. III, where the moments of inertia of the Skyrmion, their scaling properties, and flavor excitation energies are given as well. The spectrum of baryon states is presented in Sec. IV. The masses of some positive parity pentaquarks with hidden flavor (strangeness, or charm, or beauty) are estimated in Sec. V within the same approach. The final section contains some conclusions and discussion of prospects. An analytical treatment of rescaling quantized Skyrmions in the case of chiral symmetry is presented in the Appendix, where required changes of the model parameters, defined in Ref. [3], are estimated as well.

II. FEATURES OF THE CHIRAL SOLITON APPROACH; SOME STATIC PROPERTIES OF THE SKYRMION

The starting point of the chiral soliton approach (CSA), as well as of the chiral perturbation theory, is the effective chiral Lagrangian written in terms of the chiral fields incorporated into the unitary matrix $U \in SU(2)$ in the original variant of the model [1,2], $U = \cos f + i \sin f \vec{\tau}\vec{n}$, $n_z = \cos\alpha$, $n_x = \sin\alpha\cos\beta$, $n_y = \sin\alpha\sin\beta$, where functions f (the profile of the Skyrmion) and angular functions α and β in the general case are the functions of three coordinates x, y, and z. To get the states with flavor s, c, or b, we make an extension of the basic $U \in SU(2)$ to $U \in SU(3)$ with (u,d,s), (u,d,c), or (u,d,b) degrees of freedom.

It is convenient to write the Lagrangian density of the model in terms of the left (or right) chiral derivative

$$l_\mu = \partial_\mu U U^\dagger = -U\partial_\mu U^\dagger, \tag{1}$$

$$\mathcal{L} = -\frac{F_\pi^2}{16} l_\rho l_\rho + \frac{1}{e^2}[l_\rho l_\tau]^2 + \frac{F_\pi^2 m_\pi^2}{16}\mathrm{Tr}(U + U^\dagger - 2), \tag{2}$$

where F_π is the pion decay constant, its experimental value is now $F_\pi \simeq 185$ MeV [9], and e is the constant introduced by Skyrme [1]. It can be defined experimentally as well, but the allowed interval for this parameter is wide enough presently. Meson properties—mass, decay constants—are input of the model, and baryon properties are deduced from meson properties, according to Refs. [1–3]. This, the original variant of the model [1], where soliton stabilization takes place due to the fourth-order term in the Lagrangian density (2), is called now the $SK4$ variant.

Mass splittings within $SU(3)$ multiplets of baryons are due to the term in the Lagrangian [10] (see also [7]):

$$\mathcal{L}^{br} = \frac{F_\pi^2 \tilde{m}_D^2}{24}\mathrm{Tr}(1 - \sqrt{3}\lambda_8)(U + U^\dagger - 2) + \frac{F_D^2 - F_\pi^2}{48}\mathrm{Tr}(1 - \sqrt{3}\lambda_8)(U l_\mu l^\mu + l_\mu l^\mu U^\dagger), \tag{3}$$

where λ_8 is the $SU(3)$ Gell-Mann matrix and $\tilde{m}_D^2 = F_D^2 m_D^2/F_\pi^2 - m_\pi^2$ includes the $SU(3)$-symmetry violation in flavor decay constants, as well as in meson masses. m_D denotes the mass of the kaon, D meson, or B meson, for strangeness, charm, or beauty; similar holds for F_D.

The Wess-Zumino term in the action of the model is crucially important in the quantization procedure [6] (see below), but it cannot be presented as the integral of some Lagrangian density over space-time; it can be presented as the differential form [2] (see [7,8] as well).

In this section, we present some static properties of the Skyrmion which are necessary to perform the procedure of the $SU(3)$ quantization and to obtain the spectrum of states with definite quantum numbers. The quantity Γ, proportional to the sigma term,

$$\Gamma(\lambda) \simeq \lambda^3 \frac{F_\pi^2}{2}\int (1 - c_f) d^3 r, \tag{4}$$

plays an important role in any of known quantization model, in a rigid (or soft) rotator model, and in the bound state model, which simplified version we exploit here. The scaling properties of this quantity (i.e., the behavior under a change of the dimension of the soliton $r \to \lambda r$) are shown, which will be important in our consideration. Numerically, for the baryon number $B = 1$ configuration, $\Gamma(\lambda = 1) \sim 5\ \mathrm{GeV}^{-1}$. The moments of inertia of the Skyrmion, the isotopical $\Theta_I \sim (5\text{–}6)\ \mathrm{GeV}^{-1}$ and the flavor $\Theta_F \sim (2\text{–}3)\ \mathrm{GeV}^{-1}$, play an important role as well; see, e.g., [7,8], and references there. All moments of inertia $\Theta \sim N_c$, where N_c is the number of colors of underlying QCD [2]. Expressions for the moments of inertia will be given in the next section.

One of the main advantages of the CSA[1]consists in the possibility to consider baryonic states with different flavors—strange, charmed, or beautiful—and with different atomic (baryon) numbers from a unique point of view, using one and the same set of the model parameters. The properties of the system are evaluated as a function of external quantum numbers which characterize the system as a whole, whereas the hadronic content of the state plays a secondary role. This is in close correspondence with the standard experimental situation where, e.g., in the missing mass experiments the spectrum of states is measured at fixed external quantum numbers—strangeness or other flavor, isospin, etc. The so-called deeply bound antikaon-nuclei states have been

[1]It is the authors' opinion, probably not accepted widely.

TABLE I. Numerical values of the quantities used in present paper, which have definite scaling behavior. t_1, t_3, f_1, f_3, Γ, and $\tilde{\Gamma}$ are in GeV^{-1}, and m_1, m_2, and m_3 are in MeV. The first line corresponds to the original parametrization of Ref. [3], $F_\pi = 129$ MeV, $e = 5.45$[2] the second line corresponds to the parametrization of Ref. [4], $F_\pi = 108$ MeV, $e = 4.84$; and the third line corresponds to the parametrization of the Siegen University group with $F_\pi = 186$ MeV, $e = 4.12$.

	t_1	f_1	t_3	f_3	Γ	$\tilde{\Gamma}$	m_1	m_{-1}	m_3
ANW	2.039	⋯	3.078	⋯	⋯	⋯	432	432	0
AN	3.216	⋯	1.902	⋯	⋯	⋯	357	470	38
Siegen	3.180	0.83	2.370	1.20	4.80	15.6	759	897	46

considered from this point of view in Ref. [11] not in contradiction with data (this is probably one of most striking examples).

Remarkably, the moments of inertia of Skyrmions carry information about their interactions. Probably the first example of how it works are the moments of inertia of the toroidal $B = 2$ bi-Skyrmion [12]. The orbital moment of inertia Θ_J is greater than the isotopic moment of inertia Θ_I; as a result, the quantized state with the isospin $I = 0$ and spin $J = 1$ (analogue of the deuteron) has smaller energy than the state with $I = 1$, $J = 0$ (quasideuteron, or nucleon-nucleon scattering state), in qualitative agreement with the experimental observation that deuteron is bound stronger.

In the pioneer paper [3], the masses of the nucleon and Δ isobar have been fitted, and as a result the pion decay constant turned out to be considerably lower than the experimental value $F_\pi \simeq 185$ MeV. Later, another approach has been developed, in particular, by the Siegen University theory group (Holzwarth, Schwesinger, Walliser, and Weigel). The idea is that the value of the classical mass of the Skyrmion is controlled by poorly known loop corrections of the order of N_c^0, or so-called Casimir energy [13]. Therefore, it makes more sense to calculate the differences of masses of baryons with different quantum numbers, like the difference of masses of the nucleon and $\Delta(1232)$ isobar (as it was made first in Ref. [3]), nucleon and hyperons, i.e., mass splittings inside $SU(3)$ multiplets of baryons, calculated first in Ref. [5].

The classical mass of the Skyrmion is calculated usually with the pion mass term included in the Lagrangian and consists of three parts which scale differently:

$$M_{cl} = m_1\lambda + m_{-1}/\lambda + m_3\lambda^3 \tag{5}$$

with

$$m_1 = F_\pi^2 \frac{\pi}{2}\int \left(f'^2 + 2\frac{s_f^2}{r^2}\right) r^2 dr,$$

$$m_{-1} = \frac{2\pi}{e^2}\int \frac{s_f^2}{r^2}\left(2f'^2 + \frac{s_f^2}{r^2}\right) r^2 dr,$$

$$m_3 = \pi F_\pi^2 m_\pi^2 \int (1 - c_f) r^2 dr = \frac{m_\pi^2}{2}\Gamma, \tag{5a}$$

which satisfy the Derrick relation

$$m_1 + 3m_3 = m_{-1};$$

see Table I.

III. RIGID OSCILLATOR QUANTIZATION MODEL, MOMENTS OF INERTIA OF THE SKYRMION

We shall use the following mass formula for the quantized state derived in Ref. [6] for the quantization scheme used here:

$$M(B = 1, F, I, J) = M_{cl} + |F|\omega_F + \Delta E_{1/N_c}. \tag{6}$$

The flavor (antiflavor) excitation energies are

$$\omega_F = \frac{3}{8\Theta_F}(\mu_F - 1); \qquad \bar{\omega}_F = \frac{3}{8\Theta_F}(\mu_F + 1) \tag{7}$$

with

$$\mu_F = \left[1 + \frac{16[\bar{m}_D^2\Gamma + (F_D^2 - F_\pi^2)\tilde{\Gamma}]\Theta_F}{9}\right]^{1/2}, \tag{8}$$

$$\tilde{\Gamma} = \frac{1}{4}\int c_f[c_f(\vec{\partial} f)^2 + s_f^2(\vec{\partial} n_i)^2] d^3r; \tag{9}$$

see [7,8]. Evidently, $\tilde{\Gamma} \sim \lambda$ under the scaling procedure. The so-called flavored moment of inertia Θ_F and isospin (spin) inertia Θ_I are given below.

Different terms in (6) scale differently as the number of colors in this expression:

$$M_{cl} \sim N_c, \qquad \omega_F \sim N_c^0,$$

all moments of inertia $\Theta \sim N_c$.

The difference between flavor and antiflavor excitation energies appears just due to the Wess-Zumino-Witten term in the action [2] mentioned above. Previously, estimates of the flavor excitation energies were made mostly in the perturbation theory; i.e., the flavor excitation energy has been simply added to the Skyrmion energy. This is not justified, however, when the flavor excitation energy is large. Here we include this energy into a simplified minimization procedure which is made by means of the

[2] We used the values of the soliton mass and the moment of inertia given in Ref. [3] by formulas after Eq. (9).

change of the soliton dimension (rescaling of the soliton). This procedure takes into account the main degree of freedom of the $B = 1$ Skyrmion (hedgehog) and Skyrmions given by the rational map ansatz [14] which has been applied successfully to describe some properties of nuclei. As we show here, the rescaling leads to a considerable decrease of the energy of states, beginning with the $\Delta(1232)$. A similar (although not the same) modification of the quantized Skyrmion was made, in particular, by Kopeliovich, Schwesinger, and Stern [15] to improve the description of strange dibaryon configurations.

The hyperfine splitting correction to the energy of states, which is formally of the $1/N_c$ order in the number of colors, has been obtained previously in Ref. [6] and reproduced in Refs. [7,8]:

$$\Delta E_{1/N_c} = \frac{1}{2\Theta_I}[c_F I_r(I_r+1) + (1-c_F)I(I+1) + (\bar{c}_F - c_F)I_F(I_F+1)], \tag{10}$$

where I is the isospin of the state, I_F is the isospin carried by a flavored meson (K, D, or B meson, for unit flavor $I_F = 1/2$), and I_r can be interpreted as a "right" isospin or an isospin of the basic nonflavored configuration. The hyperfine splitting constants

$$c_F = 1 - \frac{\Theta_I(\mu_F - 1)}{2\Theta_F \mu_F}, \qquad \bar{c}_F = 1 - \frac{\Theta_I(\mu_F - 1)}{\Theta_F \mu_F^2}. \tag{11}$$

This correction is considered usually as a small one, but it should be included into the minimization procedure, especially when isospin I is large. Here we include this correction to the masses for all baryons.

At large enough m_D the expansion can be made:

$$\mu_F \simeq \frac{4\bar{m}_D(\Gamma\Theta_F)^{1/2}}{3} + \frac{3}{8\bar{m}_D\Gamma\Theta_F};$$

therefore,

$$\omega_F \simeq \frac{1}{2}\bar{m}_D\left(\frac{\Gamma}{\Theta_F}\right)^{1/2} - \frac{3}{8\Theta_F}. \tag{12}$$

Here we take the ratio of decay constants $F_K/F_\pi \simeq 1.1928$, $F_D/F_\pi \simeq 1.58$ according to the analysis performed by Rosner, Stone, and Van de Water in Ref. [9]. The value of $r_b = F_B/F_\pi$ given in Ref. [9], $r_b \simeq 1.45$, is a theoretical one, not confirmed experimentally. Our results presented in this paper suggest that the ratio F_B/F_π should be greater, between 2 and 2.6.

The flavored moment of inertia equals (we added the rescaling factor—some power of the parameter λ to make evident the behavior under the rescaling procedure $r \to r\lambda$)

$$\Theta_F = \lambda f_1 + \lambda^3 f_3^{(0)} \frac{F_D^2}{F_\pi^2} = \Theta_F^{(0)} + \lambda^3 f_3^{(0)}\left(\frac{F_D^2}{F_\pi^2} - 1\right) \tag{13}$$

with

$$f_1 = \frac{\pi}{2e^2}\int (1 - c_f)\left(f'^2 + 2\frac{s_f^2}{r^2}\right) r^2 dr;$$
$$f_3^{(0)} = \frac{\pi}{2}F_\pi^2 \int (1 - c_f) r^2 dr. \tag{14}$$

In the integrands, f denotes the profile function of the soliton (Skyrmion). Here we show explicitly the dependence of different parts of the inertia on the rescaling parameter λ. In Table I, we present numerical values for f_1, f_3, t_1, and t_3 and other quantities used to perform calculations of the masses of quantized states.

There is a simple connection between the total moment of inertia in the $SK4$ variant of the model, the Θ_F, and the sigma term:

$$\Theta_F^{\text{tot}} = \frac{F_D^2}{4F_\pi^2}\Gamma + \Theta_F = \frac{F_D^2}{F_\pi^2} f_3^{(0)} + f_1. \tag{15}$$

Similarly, the isotopic moment of inertia Θ_I within the rational map approximation can be written as

$$\Theta_I = \lambda t_1 + \lambda^3 t_3 \tag{16}$$

with

$$t_1 = \frac{4\pi}{3}\int \frac{2s_f^2}{e^2}\left(f'^2 + \frac{s_f^2}{r^2}\right) r^2 dr,$$
$$t_3 = \frac{2\pi}{3}F_\pi^2 \int s_f^2 r^2 dr. \tag{17}$$

The value of isotopic inertia for the $B = 1$ Skyrmion (which coincides with the value of usual spin inertia) Θ_I was defined in many papers, beginning with Refs. [3,4], but the distribution of Θ_I between t_1 and t_3 was not known. For this reason, we recalculated the static properties of the unit hedgehog Skyrmion. The results are shown in Table I. It turned out that the ratio t_3/t_1 is different for different parametrizations; the largest is for the chiral symmetric case of Ref. [3], as shown in Table I.

IV. RESCALING OF MASSES OF THE LOWEST BARYONS

For the $B = 1$ Skyrmion, the static configuration of lowest energy is of the so-called hedgehog type; i.e., it possesses generalized spherical symmetry, and its profile function f is spherically symmetrical. This type of configuration was proposed at first in Ref. [1] and used in Ref. [3]. We did not find, and to our knowledge, up to now there is no idea which kind of configuration could provide the energy of the $B = 1$ configuration lower than the hedgehog-type configuration.[3] The situation is quite different for Skyrmions with baryon numbers $B \geq 2$. In these cases, known Skyrmions of lowest energy have a form quite different from the hedgehog one (i.e., with generalized spherical symmetry). For $B = 2$ the Skyrmion of lowest energy has a toruslike form of the mass and baryon number distribution [12]. The form of many Skyrmions with $B \geq 2$ is well described within the rational map approximation, as established in Ref. [14].

For these reasons, we investigated first the influence of the change of the Skyrmion dimension without changes of the symmetry properties. The results turned out to be striking.

We are using the following expressions for the masses of baryons (for M_N and M_Δ they were given at first in Refs. [3,4], for hyperons in Ref. [6]):

$$M_N = M_{cl} + \frac{3}{8\Theta_I}; \qquad M_\Delta = M_{cl} + \frac{15}{8\Theta_I}; \tag{18}$$

$$M_\Lambda = M_{cl} + \frac{3}{8\Theta_I} + \omega_F - \frac{3(\mu_F - 1)}{8\mu_F^2 \Theta_F};$$
$$M_\Sigma = M_{cl} + \frac{3}{8\Theta_I} + \omega_F - \frac{3(\mu_F - 1)}{8\mu_F^2 \Theta_F} + \frac{\mu_F - 1}{2\mu_F \Theta_F}. \tag{19}$$

The flavor inertia $\Theta_F^{(0)}$ is the same for all three flavors [see Eq. (13)], but the quantity μ_F and the flavor excitation energy ω_F are different for different flavors.

It is not difficult algebraic work to define the parameters of the model F_π and e in the chiral symmetry limit of Ref. [3]. It has been obtained in Ref. [3] for the soliton mass $M_{cl} = (5M_N - M_\Delta)/4 = aF_\pi/e$, $a = 36.5$, and for the mass splitting between $\Delta(1232)$ and nucleon, $\Delta_M = M_\Delta - M_N = 3/(2\Theta_I)$, with the moment of inertia Θ_I (λ in notations of Refs. [3,4]), $\Theta_I = \lambda = 2\pi b/(3e^3 F_\pi)$, the constant $b = 50.9$ [3], as indicated after Eq. (9) of Ref. [3]. It follows then immediately that

$$F_\pi = \left[\frac{\pi b \Delta_M (5M_N - M_\Delta)^3}{144 a^3}\right]^{1/4},$$
$$e = \left[\frac{16\pi a b \Delta_M}{9(5M_N - M_\Delta)}\right]^{1/4}. \tag{20}$$

Recall that in Eq. (20) M_N and M_Δ should be taken from the experiment, and one obtains then $F_\pi \simeq 129$ MeV, $e \simeq 5.45$ [3]. After rescaling, there are no such simple relations, but F_π and e should be somewhat greater, $F_\pi \sim 140$ MeV; see the Appendix of the present paper.

[3]The question, are there some configurations for the $B = 1$ Skyrmion which have the static energy lower than the hedgehog one, has been raised by a referee of the present paper.

TABLE II. The values of $\lambda_{\min}$ at the minimal total energy (mass) of quantized baryon states. The decrease of masses δM due to the change of λ from 1 to $\lambda_{\min}$ is given in MeV. The differences of masses $M_B - M_N$, theoretical and experimental values, are presented as well (in MeV). The first two lines correspond to the parametrization in the chiral symmetry limit, considered at first in Ref. [3]. Lines 3 and 4 correspond to the parametrization of Ref. [4], the pion mass included into the Lagrangian. Other calculations were performed taking the Siegen parametrization, $F_\pi = 186$ MeV, $e = 4.12$ [10].

	$\lambda_{\min}$	δM	$M_B - M_N$	$(M_B - M_N)_{\exp}$
N_{ANW}	1.1534	11.5	…	…
$\Delta(1232)_{ANW}$	1.4956	155	150	293
N_{AN}	1.0978	6.0	…	…
$\Delta(1232)_{AN}$	1.3558	100	199	293
N	1.0568	3.5	…	…
$\Delta(1232)$	1.2312	66	208	293
Λ_s	0.8545	24	247	177
Λ_c	0.5594	129	1290	1347
$\Lambda_b(r_b = 1.45)$	0.2929	1991	3104	4680
$\Lambda_b(r_b = 2.60)$	0.2226	2784	4705	4680
Σ_s	1.0262	0.6	369	251
Σ_c	0.8983	9.1	1684	1515
$\Sigma_b(r_b = 1.45)$	0.2843	1242	4041	4874
$\Sigma_b(r = 1.95)$	0.2507	1438	4946	4874

Expressions for masses of Λ and Σ hyperons in Eq. (19) are natural generalizations of those for the nucleon and Δ, given in Refs. [3,4]. The description of the masses of strange hyperons Λ_s and Σ_s is not perfect in Table II, because the configuration mixing, i.e., the mixing between the states with the same isospin and strangeness but which belong to different $SU(3)$ multiplets, is not taken into account in our approach. Moreover, after rescaling these states cannot mix, because they have different properties (dimensions, in particular). Satisfactory agreement with data has been obtained in Ref. [10] just due to including such mixing into consideration. An improvement of the fit is certainly possible in our case as well, by changing the model parameters, first of all.

For the case of baryons with u and d flavors, the expansion of the configuration takes place due to centrifugal forces. Technically, it appears from the contribution of the spin- or isospin-dependent term in the energy of quantized state, which contains the isotopic (or spin) inertia Θ_I in the denominator. This moment of inertia consists of two terms proportional to the scale factor λ, or λ^3, and an increase of λ (expansion of the Skyrmion) leads to the decrease of energy.

As can be seen from Table II, in the case of $N - \Delta$ baryons, the largest effect due to a rescaling (expansion) of the Skyrmion takes place for the chirally symmetric variant of the model, considered in Ref. [3]: the nucleon mass decreases by 11.5 MeV, but the mass of $\Delta(1232)$ drops by 155 MeV, which makes the $\Delta - N$ mass difference

~150 MeV, considerably lower than experimental value 293 MeV. The effect of rescaling is not so striking for the model [4], where the physical pion mass is included into consideration. Even somewhat smaller is the effect of expansion for the Siegen model, which, therefore, seems to be more realistic.

The flavor excitation energies are proportional to the mass $\bar{m}_D$, which is large for charm or beauty [see Eq. (12)], and to $\sqrt{\Gamma} \sim \lambda^{3/2}$, and this explains why $\lambda_{\min}$ is so small for beauty (that means squeezing is strong).

As can be seen in Table II, masses of beautiful baryons Λ_b and Σ_b obtained with the value of $r_b = 1.45$ given in Ref. [9] are too low in comparison with the data. The values $r_b \simeq 2.6$ and $r_b \simeq 1.95$ are more preferable to get the masses of Λ_b and Σ_b near the experimental values. The considerable difference between these values is a clear indication that the approach used here is not satisfactory for the case of a beauty quantum number.

V. ESTIMATES OF THE MASSES OF PENTAQUARKS WITH HIDDEN FLAVOR

For the case of pentaquarks with hidden flavor, i.e., containing the pair of quark and antiquark or the pair of D and $\bar{D}$ (or K and $\bar{K}$, or B and $\bar{B}$) mesons, we take in Eq. (10) $I_r = I$ and $I_F = 0$ and come to the energy (mass) of the state

$$M_{P_F} = M_{cl} + \frac{3\mu_F}{4\Theta_F} + \frac{I(I+1)}{2\Theta_I}, \qquad (21)$$

which we minimize numerically. Some results are shown in Table III.

Tables II and III illustrate well two competing tendencies for quantized Skyrmion states: the squeezing with increasing flavor excitation energy and the expansion due to centrifugal forces which become stronger with increasing spin (isospin). For beauty, squeezing dominates in all cases considered here.

The states considered in Table III have positive parity, as a consequence of the quantization scheme used, and isospin which coincides with the right isospin and equals the spin of the state—because the quantized configuration of fields is of the hedgehog type. Pentaquarks with $I = J = 1/2$ could belong to the antidecuplet of the corresponding $SU(3)$ group, $(p, q) = (0, 3)$, those with $I = J = 3/2$ could belong to the $\{27\}$-plet, $(p, q) = (2, 2)$, and pentaquarks with $I = J = 5/2$ could belong to the $\{35\}$-plet with $(p, q) = (4, 1)$.

To obtain the masses of pentaquarks predicted by this simplified model, one should add the nucleon mass, 939 MeV, to the numbers of the fourth column of Table III; see the last column. The hidden strangeness pentaquark states, presented in Table III, have masses by few hundreds of MeV greater than such states discussed previously in connection with the low-lying positive strangeness pentaquark $\Theta^+(1540)$; see, e.g., the discussion in Ref. [16]. For example, $M[P_s(J = 1/2)] = 1892$ MeV. The hidden charm pentaquark state has a mass near to the mass of the state observed by the LHCb Collaboration, $M(P_c) \simeq 4450$ MeV [17]: $M[P_c(J = 3/2)] \simeq 4553$ MeV; see Table III.

TABLE III. The values of $\lambda_{\min}$ at the minimal total energy (mass) of some pentaquark states. The decrease of masses δM due to the change of λ from 1 to $\lambda_{\min}$ is given in MeV, similar to Table II. The differences of masses $M_B - M_N$, theoretical values only, are presented as well (in MeV). Calculations are performed taking parametrization $F_\pi = 186$ MeV, $e = 4.12$ [10].

$B(I = J)$	$\lambda_{\min}$	δM	$M_P - M_N$	M_P
$P_s(1/2)$	1.0600	5.2	953	1892
$P_s(3/2)$	1.2068	63	1166	2105
$P_s(5/2)$	1.3931	247	1422	2361
$P_c(1/2)$	0.7297	82	3261	4200
$P_c(3/2)$	0.9993	0.0	3614	4553
$P_c(5/2)$	1.2814	105	3959	4898
$P_b(1/2, r_b = 1.45)$	0.3263	2980	7505	8444
$P_b(1/2, r_b = 1.95)$	0.2843	3729	8969	9908
$P_b(3/2, r_b = 1.45)$	0.3812	2025	8730	9669
$P_b(3/2, r_b = 1.95)$	0.3324	2564	10403	11342
$P_b(5/2, r_b = 1.45)$	0.4973	885	10321	11260
$P_b(5/2, r_b = 1.95)$	0.4357	1139	12279	13218

VI. CONCLUSIONS AND PROSPECTS

We have demonstrated that a considerable decrease of the energy of quantized Skyrmion states (baryons) takes place due to a change of the Skyrmion dimension (rescaling). Even for baryons with (u, d) flavors, nucleon and $\Delta(1232)$ isobar, the expansion of the Skyrmion due to centrifugal force decreases the mass splitting between N and Δ considerably and destroys the fit of masses made in Refs. [3,4]. This fit could be recovered by some increase of the parameters of the model—towards a better agreement with the data. We proved this analytically for the chirally symmetrical case of Ref. [3] (see Appendix), and certain technical work is necessary to study if it is possible in other cases.

The change of the Skyrmion dimensions leads to a considerable lowering of the energy (mass) of the quantized states with quantum numbers charm or beauty. For strangeness, the effect takes place as well, but not so important—small enough in most of the cases. In our estimates, we used a simple and transparent variant of the quantization procedure, originally proposed in Ref. [6] and modified later in Ref. [8]. It is quite obvious that the effect is important in any variant of the quantization scheme, but we do not pretend here to provide the accurate description of the rescaling phenomenon; improvements and modifications of the approach could be necessary for this in many cases.

There are two competing tendencies, to decrease the dimension of the Skyrmion when the flavor excitation energy becomes large and to expand the Skyrmion due to centrifugal forces when the spin (or isospin) becomes greater. For a large enough spin (isospin) of the state, the expansion takes place due to centrifugal force—instead of squeezing. For strangeness, this effect dominates already for the Σ_s hyperon (see Table II) and takes place for all hidden strangeness pentaquarks and for a hidden charm pentaquark with $J = 5/2$ (Table III).

There is some discrepancy in the description of masses of beauty baryons Λ_b and Σ_b, which indicates that the method itself is limited in its applicability. To describe the mass of the Λ_b baryon the ratio $r_b = F_B/F_\pi$ should be about 2.6, but to obtain a satisfactory description of the mass of the Σ_b baryons there should be $r_b \sim 2.0$. The value of r_b given in the literature is $r_b \simeq 1.45$ [9], and this is a theoretical value, not confirmed experimentally. In spite of this big problem, the necessity to include the rescaling of the quantized Skyrmion seems to be without doubt.

The investigation of the role of rescaling is a viable problem for quantized multi-Skyrmions with baryon number $B = 2, 3, \ldots$, etc. The disappearance of some flavored bound states, which have been interpreted as possible hypernuclei, is expected.

Studies of the $SK6$ variant of the model, where Skyrmion stabilization takes place due to the sixth-order term (in chiral derivatives) in the Lagrangian density, proportional to the baryon number density squared (see, e.g., [18]), are of interest. We shall consider this variant of the model elsewhere.

Modifications and improvements of the approach, including its fine-tuning, seem to be of interest.

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APPENDIX: ANALYTICAL TREATMENT OF RESCALING; REQUIRED CHANGES OF THE MODEL PARAMETERS

It is possible and instructive to perform analytical treatment of the rescaling phenomenon, if the expected change of the Skyrmion dimension is not large. We shall write the rescaling factor as $\lambda = 1 + \delta'$, make an expansion in δ' assuming that it is small, and then find the value of δ' and a decrease of the mass (energy) of the quantized state.

In the case of the chiral symmetry [3] $m_3 = 0$, $m_1 = m_{-1} = M_{cl}/2$, and expansion of masses of quantized states in a small parameter δ' is

$$M_N = m_1 + m_{-1}(1+\delta'^2) + \frac{3}{8\Theta_I}\left[1 - \delta'\frac{t_1+3t_3}{\Theta_I} + \delta'^2\left(\frac{t_1+3t_3}{\Theta_I}\right)^2 - \delta'^2\frac{3t_3}{\Theta_I}\right], \tag{A1}$$

$\Theta_I = t_1 + t_3$. For $\Delta(1232)$ we have a similar relation:

$$M_\Delta = m_1 + m_{-1}(1+\delta'^2) + \frac{15}{8\Theta_I}\left[1 - \delta'\frac{t_1+3t_3}{\Theta_I} + \delta'^2\left(\frac{t_1+3t_3}{\Theta_I}\right)^2 - \delta'^2\frac{3t_3}{\Theta_I}\right]. \tag{A2}$$

Evidently, the contribution of the classical mass becomes greater after rescaling, because the classical mass is minimized by itself (the Derric condition is satisfied by this reason).

In both cases we should minimize the addition to the mass of the state which has the form

$$\delta M = -A\delta' + B\delta'^2. \tag{A3}$$

Evidently,

$$\delta'_{\min} = \frac{A}{2B}, \tag{A4}$$

and the decrease of the energy (mass) of the state due to rescaling is

$$\delta M = -\frac{A^2}{4B}. \tag{A5}$$

For a nucleon we have

$$A_N = \frac{3}{8\Theta_I^2}(t_1+3t_3),$$
$$B_N = m_{-1} + \frac{3}{\Theta_I^3}(t_1^2 + 3t_1t_3 + 6t_3^2), \tag{A6}$$

for the Δ isobar

$$A_\Delta = \frac{15}{8\Theta_I^2}(t_1+3t_3),$$
$$B_\Delta = m_{-1} + \frac{15}{\Theta_I^3}(t_1^2 + 3t_1t_3 + 6t_3^2). \tag{A7}$$

As a result, we obtain numerically

$$A_N \simeq 161.45\ \text{MeV}, \qquad B_N \simeq 655.44\ \text{MeV};$$
$$\delta'_N \simeq 0.123; \qquad \delta M_N \simeq -9.94\ \text{MeV};$$
$$A_\Delta \simeq 807.25\ \text{MeV}, \qquad B_\Delta \simeq 1549.2\ \text{MeV};$$
$$\delta'_\Delta \simeq 0.261; \qquad \delta M_\Delta \simeq -105.2\ \text{MeV}, \tag{A8}$$

in moderate agreement with the results of numerical minimization, presented in Table II.

The next question of interest to be addressed here is the change of the model parameters F_π and e which is necessary to compensate the decrease of masses of the nucleon and Delta due to rescaling. We will show for the case of the chiral symmetry considered at first in Ref. [3] that some increase of the parameter F_π is needed to compensate the decrease of the masses of N and Δ due to centrifugal forces. This consideration is valid if the changes of masses are small, $\delta M_N \ll M_N$, $\delta M_\Delta \ll M_\Delta$, and the changes of the model parameters are small as well, $\delta F_\pi \ll F_\pi$, $\delta e \ll e$.

Let us denote new model parameters as

$$F_\pi^r = F_\pi + \delta F; \qquad e^r = e + \delta e, \tag{A9}$$

and then the condition of compensation is

$$\begin{aligned} M_N &= a\frac{F_\pi^r}{e^r} + \frac{9}{16\pi b}(e^r)^3 F_\pi^r - \delta M_N; \\ M_\Delta &= a\frac{F_\pi^r}{e^r} + \frac{45}{16\pi b}(e^r)^3 F_\pi^r - \delta M_\Delta, \end{aligned} \tag{A10}$$

where δM_N and δM_Δ are given in (A8) (for convenience we operate here and below with absolute values of δM).

It is convenient to denote the changes of certain combinations of the model parameters in the following way:

$$\delta\left(\frac{F_\pi}{e}\right) = C, \qquad \delta(F_\pi e^3) = Q, \tag{A11}$$

which leads to the following equality:

$$\begin{aligned} \delta F_\pi &= \frac{1}{4e^3}(3Ce^4 + Q); \\ \delta e &= \frac{1}{4Fe^2}(-Ce^4 + Q). \end{aligned} \tag{A12}$$

Functions C and Q can be connected with decreases of the nucleon and Delta masses by the relations

$$aC + b_N Q = \delta M_N, \qquad aC + b_\Delta Q = \delta M_\Delta, \tag{A13}$$

with $b_N = 9/(16\pi b) \simeq 0.00352$, $b_\Delta = 5b_N \simeq 0.0176$, which leads to

$$C = \frac{b_\Delta \delta M_N - b_N \delta M_\Delta}{a(b_\Delta - b_N)}, \qquad Q = \frac{\delta M_\Delta - \delta M_N}{b_\Delta - b_N} \tag{A14}$$

and

$$\begin{aligned} \delta F_\pi = \frac{1}{4ae^3(b_\Delta - b_N)}[&a(\delta M_\Delta - \delta M_N) \\ &- 3e^4(b_N \delta M_\Delta - b_\Delta \delta M_N)]. \end{aligned} \tag{A15}$$

This gives numerically $\delta F_\pi \simeq 8.9$ MeV.

For the constant e, we have similarly

$$\begin{aligned} \delta e = \frac{1}{4aFe^2(b_\Delta - b_N)}[&a(\delta M_\Delta - \delta M_N) \\ &+ e^4(b_N \delta M_\Delta - b_\Delta \delta M_N)] \end{aligned} \tag{A16}$$

and numerically

$$\delta e \simeq 0.463.$$

After these corrections, we have new values of the model parameters:

$$F_\pi^r \simeq 137.9 \text{ MeV}, \qquad e^r \simeq 5.91, \tag{A17}$$

F_π being somewhat nearer to the experimental value $F_\pi^{\exp} \simeq 185$ MeV.

A similar analytical consideration of rescaling in other variants of the model, e.g., within the variant of Ref. [4], is possible but technically more complicated.

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