B decays to radially excited D mesons in heavy quark effective theory

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Semileptonic transitions $\bar{B} \to D^{(n)}\ell\bar{\nu}$, where $D^{(n)}(n \neq 0)$ is a radially excited meson, have recently attracted much attention as a way to understand some puzzles between theory and data. In the heavy quark limit the inelastic Isgur-Wise function vanishes at zero recoil, $\xi^{(n)}(1) = 0(n \neq 0)$. We consider here the $1/m_Q$ corrections within heavy quark effective theory. We find simple formulas that involve the derivative of the inelastic IW function at $w = 1$, $\xi^{(n)'}(1)(n \neq 0)$. We propose a number of ways of isolating this important quantity in practice, with emphasis on lattice QCD. We formulate also a generalization to the inelastic case of Luke's theorem.

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This paper is devoted to the discussion of $1/m_Q$ corrections in heavy quark effective theory (HQET) for the heavy quark current transitions between the ground state and radially excited states, e.g. $\bar{B} \to D^{(n)}(n \neq 0)$.

These transitions have been the object of some attention in recent years, in the view of solving some puzzles of semileptonic B decays [1,2].

Rather detailed calculations of transition form factors to heavy meson radial excitations have been performed in Refs. [3,4] in the framework of a relativistic quark model. Within another quark model scheme, Ref. [5] is also very interesting in this respect.

The IW function $\xi^{(n)}(w)$ for the transitions $0 \to n$ satisfies the normalization condition

$$\xi^{(n)}(1) = \delta_{n,0}, \tag{1}$$

that expresses the normalization of the elastic IW function and the orthogonality between the initial and the final states for $n \neq 0$.

Our purpose here is to study in detail the subleading $1/m_Q$ corrections in HQET at zero recoil, following the paper by Falk and Neubert devoted to the elastic case [6].

Besides the leading inelastic Isgur-Wise function $\xi^{(n)}(w)(n > 0)$, we consider the first order $1/m_Q$ corrections, that are of two types, corresponding to corrections to the heavy quark current, and to corrections to the leading order HQET Lagrangian.

Consider first the leading and subleading corrections to the quark current

$$\bar{c}\Gamma b \to \bar{h}_c\Gamma h_b + \frac{1}{2m_c}\bar{h}_c\Gamma\gamma^\alpha(iD_\alpha h_b) + \frac{1}{2m_b}\overline{(iD_\alpha h_c)}\gamma^\alpha\Gamma h_b, \tag{2}$$

where Γ is a Dirac matrix. The matrix elements of the relevant operators read [6]:

$$\langle D^{(n)}(v')|\bar{h}_c\Gamma h_b|\bar{B}(v)\rangle = -\xi^{(n)}(w)Tr[\bar{\mathcal{D}}\Gamma\mathcal{B}], \tag{3}$$

$$\langle D^{(n)}(v')|\bar{h}_c\Gamma\gamma^\alpha(iD_\alpha h_b)|\bar{B}(v)\rangle = -\mathrm{Tr}[\xi_\alpha^{(b)(n)}(v,v')\bar{\mathcal{D}}\Gamma\gamma^\alpha\mathcal{B}], \tag{4}$$

$$\langle D^{(n)}(v')|\overline{(iD_\alpha h_c)}\gamma^\alpha\Gamma h_b|\bar{B}(v)\rangle = -\mathrm{Tr}[\bar{\xi}_\alpha^{(c)(n)}(v',v)\bar{\mathcal{D}}\gamma^\alpha\Gamma\mathcal{B}], \tag{5}$$

where $\bar{A} = \gamma^0 A^\dagger\gamma^0$, $\mathcal{B} = P_+(-\gamma_5)$, $\mathcal{D} = P'_+(-\gamma_5)$ are given in terms of the projectors $P_+ = \frac{1+\not{v}}{2}$, $P'_+ = \frac{1+\not{v}'}{2}$, where v and v' are the initial and final four-velocities, $D^{(n)}(v')$ denotes the pseudoscalar $D^{(n)}$ or vector $D^{(n)*}$ final states, and the functions $\xi_\alpha^{(Q)(n)}(v,v')$ $(Q = b, c)$ read :

$$\xi_\alpha^{(Q)(n)}(v,v') = \xi_+^{(Q)(n)}(w)(v+v')_\alpha + \xi_-^{(Q)(n)}(w)(v-v')_\alpha - \xi_3^{(Q)(n)}(w)\gamma_\alpha. \tag{6}$$

From the equations of motion and the matrix element of the divergence of the current one obtains

$$\xi_-^{(b)(n)}(w) = \frac{\bar{\Lambda}w - \bar{\Lambda}^{(n)}}{2(w-1)}\xi^{(n)}(w), \tag{7}$$

$$\xi_-^{(c)(n)}(w) = -\frac{\bar{\Lambda} - \bar{\Lambda}^{(n)}w}{2(w-1)}\xi^{(n)}(w). \tag{8}$$

Taking the limit $\bar{\Lambda}^{(n)} \to \bar{\Lambda}$ while keeping w fixed, one finds the results of the elastic case, valid for all w, namely $\xi_-^{(b)}(w) = \xi_-^{(c)}(w) = \frac{\bar{\Lambda}}{2}\xi(w)$, where we have omitted the superindex (0) corresponding to the ground state.

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In the inelastic case, taking the limit of relations (7), (8) for $w \to 1$, one obtains

$$\xi_-^{(b)(n)}(1) = -\xi_-^{(c)(n)}(1) = -\frac{\Delta E^{(n)}}{2}\xi^{(n)\prime}(1), \quad (n > 0). \quad (9)$$

There are two types of $1/m_Q$ Lagrangian perturbations, depending on the insertion of the Lagrangian in the initial b quark or in the final c quark. Both types of perturbations, $\langle D^{(n)}(v')|i\int dxT[J^{cb}(0), \mathcal{L}_v^{(b)}(x)]|B(v)\rangle$ and $\langle D^{(n)}(v')|i\int dxT[J^{cb}(0), \mathcal{L}_{v'}^{(c)}(x)]|B(v)\rangle$, even in the case of the b insertion, depend on the radial quantum number n because the final state is radially excited. The current $J^{cb}(0)$ denotes $J^{cb} = \bar{h}_{v'}^{(c)}\Gamma h_v^{(b)}$, where Γ is a Dirac matrix, and the Lagrangian is given by $\mathcal{L}_v^{(Q)}(x) = \frac{1}{2m_Q}[O_{kin,v}^{(Q)} + O_{mag,v}^{(Q)}]$, with $O_{kin,v}^{(Q)} = \bar{h}_v^{(Q)}(iD)^2 h_v^{(Q)}$, $O_{mag,v}^{(Q)} = \frac{g_s}{2}\bar{h}_v^{(Q)}\sigma_{\alpha\beta}G^{\alpha\beta}h_v^{(Q)}$.

Generalizing the notation of Ref. [6], considering now radial excitations in the final state $\bar{B} \to D^{(n)(*)}\ell\nu$ ($n = 0$ is the elastic case), the different form factors read, up to first order in $1/m_Q$:

$$h_+^{(n)} = \xi^{(n)} + \epsilon_c L_1^{(c)(n)} + \epsilon_b L_1^{(b)(n)}, \quad (10)$$

$$h_-^{(n)} = \epsilon_c L_4^{(c)(n)} - \epsilon_b L_4^{(b)(n)}, \quad (11)$$

$$h_V^{(n)} = \xi^{(n)} + \epsilon_c(L_2^{(c)(n)} - L_5^{(c)(n)}) + \epsilon_b(L_1^{(b)(n)} - L_4^{(b)(n)}), \quad (12)$$

$$h_{A_1}^{(n)} = \xi^{(n)} + \epsilon_c\left(L_2^{(c)(n)} - \frac{w-1}{w+1}L_5^{(c)(n)}\right) + \epsilon_b\left(L_1^{(b)(n)} - \frac{w-1}{w+1}L_4^{(b)(n)}\right), \quad (13)$$

$$h_{A_2}^{(n)} = \epsilon_c(L_3^{(c)(n)} + L_6^{(c)(n)}), \quad (14)$$

$$h_{A_3}^{(n)} = \xi^{(n)} + \epsilon_c(L_2^{(c)(n)} - L_3^{(c)(n)} - L_5^{(c)(n)} + L_6^{(c)(n)}) + \epsilon_b(L_1^{(b)(n)} - L_4^{(b)(n)}), \quad (15)$$

where $\xi^{(n)}(w)$ is the Isgur-Wise (IW) function for the transition between the ground state and the radially excited states $0 \to n$, $L_i^{(b)(n)}(w) (i = 1,4)$ are elastic subleading form factors at order $\epsilon_b = 1/2m_b$, and $L_i^{(c)(n)}(w) (i = 1, \ldots, 6)$ are subleading form factors at order $\epsilon_c = 1/2m_c$.

Extending the notation of [6], one finds for the different functions $L_i^{(Q)(n)}(w)$ $(Q = b, c)$ $(i = 1, 2, 3)$:

$$\begin{aligned} L_1^{(Q)(n)} &= A_1^{(Q)(n)} + 2(w-1)A_2^{(Q)(n)} + 3A_3^{(Q)(n)} \\ L_2^{(Q)(n)} &= A_1^{(Q)(n)} - A_3^{(Q)(n)} \\ L_3^{(Q)(n)} &= -2A_2^{(Q)(n)} \end{aligned} \quad (16)$$

and

$$\begin{aligned} L_4^{(b)(n)} &= -\frac{\bar{\Lambda}w - \bar{\Lambda}^{(n)}}{w-1}\xi^{(n)}(w) + 2\xi_3^{(b)(n)}, \\ L_5^{(b)(n)} &= -\frac{\bar{\Lambda}w - \bar{\Lambda}^{(n)}}{w-1}\xi^{(n)}(w), \\ L_6^{(b)(n)} &= -\frac{2}{w+1}\left(\frac{\bar{\Lambda}w - \bar{\Lambda}^{(n)}}{w-1}\xi^{(n)}(w) + \xi_3^{(b)(n)}\right), \end{aligned} \quad (17)$$

and similarly :

$$\begin{aligned} L_4^{(c)(n)} &= \frac{\bar{\Lambda} - \bar{\Lambda}^{(n)}w}{w-1}\xi^{(n)}(w) + 2\xi_3^{(c)(n)}, \\ L_5^{(c)(n)} &= \frac{\bar{\Lambda} - \bar{\Lambda}^{(n)}w}{w-1}\xi^{(n)}(w), \\ L_6^{(c)(n)} &= -\frac{2}{w+1}\left(-\frac{\bar{\Lambda} - \bar{\Lambda}^{(n)}w}{w-1}\xi^{(n)}(w) + \xi_3^{(c)(n)}\right). \end{aligned} \quad (18)$$

At order $1/m_Q$, in the transitions $0 \to n$ with $n \neq 0$, we have many more form factors than in the elastic case. However, some constraints still exist.

For the current type form factors, we must emphasize the relations for $L_5^{(b)(n)}(w)$ and $L_5^{(c)(n)}(w)$ that become, at zero recoil:

$$\begin{aligned} &L_5^{(b)(n)}(1) = -L_5^{(c)(n)}(1) = \Delta E^{(n)}\xi^{(n)\prime}(1) \\ &(\Delta E^{(n)} = \bar{\Lambda}^{(n)} - \bar{\Lambda}). \end{aligned} \quad (19)$$

and the constraints that follow from the Eqs. (17), (18):

$$\begin{aligned} L_4^{(b)(n)}(w) + (w+1)L_6^{(b)(n)}(w) &= 3L_5^{(b)(n)}(w), \\ L_4^{(c)(n)}(w) + (w+1)L_6^{(c)(n)}(w) &= 3L_5^{(c)(n)}(w). \end{aligned} \quad (20)$$

A remark is in order here. The inelastic relation (19) is not symmetric in the exchange $b \leftrightarrow c$ because the radial excitation occurs in the final charmed state. This gives an opposite sign between $L_5^{(b)(n)}(1)$ and $L_5^{(c)(n)}(1)$ for $n > 0$. This differs from the elastic case, for which one obtains $L_5^{(b)(0)}(w) = L_5^{(b)(0)}(w) = L_5(w)$ by taking the limit $\bar{\Lambda}^{(n)} \to \bar{\Lambda}^{(0)} = \bar{\Lambda}$ at w fixed.

For the form factors of the Lagrangian type we do not have Luke's theorem [7], but weaker generalizations of it. Consider the transition between charmed pseudoscalars $D \to D^{(n)}$ through the elastic current $\bar{c}\gamma^0 c$ at zero recoil $w = 1$. From (10) one has $h_+^{(n)}(1) = \xi^{(n)}(1) + \epsilon_c(L_1^{(c)(n)}(1) + L_1^{(b)(n)}(1))$.

The different superscripts (b) and (c) mean simply that the initial and final states are different, although the functions $L_1^{(c)(n)}, L_1^{(b)(n)}$ are independent of the heavy quark masses.

Since $\bar{c}\gamma^0 c$ is the c quark number operator and one has the normalization (1), relation (20) reads $\delta_{n,0} = \delta_{n,0} + \epsilon_c(L_1^{(c)(n)}(1) + L_1^{(b)(n)}(1))$, and it follows

$$L_1^{(c)(n)}(1) + L_1^{(b)(n)}(1) = 0. \tag{21}$$

Considering likewise the transition between charmed vector mesons $D^* \to D^{(n)*}$ through the elastic current $\bar{c}\gamma^0 c$ at zero recoil $w = 1$, one has $h_1^{(n)}(1) = \xi^{(n)}(1) + \epsilon_c(L_2^{(c)(n)}(1) + L_2^{(b)(n)}(1))$, which implies

$$L_2^{(c)(n)}(1) + L_2^{(b)(n)}(1) = 0. \tag{22}$$

Relations (21) and (22) are generalizations of Luke's theorem [7] to radially excited inelastic transitions, and for the elastic case $0 \to 0$ one has $L_i^{(c)(0)}(1) = L_i^{(b)(0)}(1) = L_i(1) (i = 1, 2)$, and therefore $L_1(1) = L_2(1) = 0$.

The IW function for the inelastic transition $\xi^{(n)}(w) (n \neq 0)$ vanishes at zero recoil, Eq. (1). Therefore, it would be very interesting to have information on the derivative at $w = 1$, at least for the first radial excitation. We have a number of suggestions for this purpose.

(i) Using the second relation (19) and (12), (13) one has

$$h_V^{(n)}(1) - h_{A_1}^{(n)}(1) = -\epsilon_c L_5^{(c)}(1) + O(\epsilon_b) = \epsilon_c \Delta E^{(n)} \xi^{(n)\prime}(1) + O(\epsilon_b). \tag{23}$$

Therefore, up to $1/m_b$ corrections, this difference of form factors is proportional to $\Delta E^{(n)} \xi^{(n)\prime}(1)$, and could give information on the derivative of the IW function at zero recoil.

Although difficult, one could in principle also envisage a lattice calculation of the difference of form factors at zero recoil (23) in the heavy quark limit for the B meson ($m_b \to \infty$), and thus isolate the desired quantity.

(ii) Using the same argument, one could as well consider the ratio of the form factors,

$$\frac{h_V^{(n)}(1)}{h_{A_1}^{(n)}(1)} = -\epsilon_c L_5^{(c)}(1) + O(\epsilon_b) + O(\epsilon_c^2) = \epsilon_c \Delta E^{(n)} \xi^{(n)\prime}(1) + O(\epsilon_b) + O(\epsilon_c^2), \tag{24}$$

that could also provide $\xi^{(n)\prime}(1)$ up to higher corrections. For example, varying the masses m_b, m_c on the lattice, one could isolate this derivative.

(iii) Another possibility would be to use the identities (3)–(5), the definition (6) and the results (7), (8). After some algebra, one finds for $n \geq 0$:

$$\langle D^{(n)}(v')|\bar{h}_c i\overleftarrow{D}_\alpha (v - v')^\alpha h_b|\bar{B}(v)\rangle = (w + 1)(\bar{\Lambda} - \bar{\Lambda}^{(n)} w)\xi^{(n)}(w), \tag{25}$$

$$\langle D^{(n)}(v')|\bar{h}_c i D_\alpha (v - v')^\alpha h_b|\bar{B}(v)\rangle = (w + 1)(\bar{\Lambda} w - \bar{\Lambda}^{(n)})\xi^{(n)}(w), \tag{26}$$

that gives for $n > 0$:

$$\lim_{w\to 1} \frac{\langle \overleftarrow{D}_\alpha(v')|\bar{h}_c i\overleftarrow{D}_\alpha (v - v')^\alpha h_b|\bar{B}(v)\rangle}{2(w - 1)} = -\Delta E^{(n)} \xi^{(n)\prime}(1), \tag{27}$$

$$\lim_{w\to 1} \frac{\langle D^{(n)}(v')|\bar{h}_c i D_\alpha (v - v')^\alpha h_b|\bar{B}(v)\rangle}{2(w - 1)} = -\Delta E^{(n)} \xi^{(n)\prime}(1). \tag{28}$$

Although difficult, a consideration of these matrix elements on the lattice could in principle isolate the interesting quantity $\Delta E^{(n)} \xi^{(n)\prime}(1)$.

(iv) There is also a straightforward systematic method, namely to consider on the lattice a form factor like $h_+^{(n)}(w)$ (10) and vary $1/m_c$, $1/m_b$ to have access to the inelastic form factor, following the general argument proposed in Ref. [8]. In this work, in the static limit for the b quark, $h_+^{(n)}(w)$ has been clearly isolated for the first radial excitation, providing some information on this form factor. Of course, to get the derivative $\xi^{(n)\prime}(1)$ one would need to consider several values of w near the zero recoil point $w = 1$.

On the other hand, the problems that one encounters for radial excitations within the Bakamjian-Thomas (BT) relativistic quark model have been exposed in [9]. We summarize here the results. We have checked that Eqs. (20) are satisfied at zero recoil in the BT model, as well as the generalizations of Luke's theorem (21), (22). However, the relation (19), that involves the difference $\bar{\Lambda}^{(n)} - \bar{\Lambda}$, is not satisfied in the model.

We have considered the $1/m_Q$ corrections of form factors for semileptonic transitions between B mesons and radially excited D mesons. In particular, we have found some relations that involve the derivative of the inelastic Isgur-Wise function at zero recoil $\xi^{(n)\prime}(1) (n > 0)$, and formulated a generalization of Luke's theorem in this inelastic case. Moreover, we have proposed some methods to isolate the derivative $\xi^{(n)\prime}(1) (n > 0)$, that could in principle be determined from physical form factors or lattice QCD.

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